Journal of Financial Economics 110 (2013) 164–184

Contents lists available at ScienceDirect

Journal of Financial Economics

journal homepage: www.elsevier.com/locate/jfec

Investment shocks and the commodity basis spread ☆

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ARTICLE INFO

Article history:
Received 16 January 2012
Received in revised form
14 February 2013
Accepted 11 March 2013
Available online 3 May 2013

JEL classifications:
E23
E44
G12
G13

Keywords:
Commodity futures
Investment-based asset pricing
Investment shocks
Basis spread

ABSTRACT

I identify a "slope" factor in the cross section of commodity futures returns: high-basis commodity futures have higher loadings on this factor than low-basis commodity futures. Combined with a level factor (an index of commodity futures), this slope factor explains most of the average excess returns of commodity futures portfolios sorted by basis. More importantly, I find that this factor is significantly correlated with investment shocks, which represent the technological progress in producing new capital. I investigate a competitive dynamic equilibrium model of commodity production to endogenize this correlation. The model reproduces the cross-sectional futures returns and many asset pricing tests.

1. Introduction

This paper provides an empirical analysis of the cross-sectional commodity futures returns and then proposes a theoretical basis for the underlying macroeconomic risks that justify these cross-sectional returns. Recent studies have shown that commodity futures returns are predictable, even at a monthly horizon. Futures contracts written on commodities with a high basis (that is, those commodities with a "high" ratio of spot price to futures price) tend to have higher expected returns than futures contracts written on commodities with a low basis.[1] Erb and Harvey (2006) and Gorton, Hayashi, and Rouwenhorst (GHR, 2013) observe this result in the cross section. Specifically, GHR form two commodity futures portfolios sorted by the commodity's basis and find a 10% annual return spread between high- and low-basis portfolios. I refer to this result as the "basis spread" throughout this paper. This basis spread is very similar to the return spread between high and low forward rate currencies in the foreign exchange market (Lustig and Verdelhan, 2007), in the sense that a high forward rate is analogous to a high basis. Given the large economic magnitude of the basis spread, understanding its source is thus an important question.

To characterize the properties of commodity futures returns in the data, I split the commodity futures into

☆ For their many helpful suggestions, I am grateful to the members of my thesis committee: Frederico Belo, John Boyd, Bob Goldstein (chair), Jeremy Graveline, and Erzo G.J. Luttmer. I am especially grateful to an anonymous referee for constructive comments, which have significantly improved all aspects of this article. I have also benefited from comments by Philip Bond, Pierre Collin-Dufresne, Nicolae Garleanu, Joao Gomes, Jun Li, Xiaoji Lin, Dmitry Livdan, Lars Lochstoer (WFA discussant), Thomas Mertens (FMA discussant), Nikolai Roussanov, Jacob Sagi, Motohiro Yogo, and Jianfeng Yu. I thank the seminar participants at several universities, the WFA 2011 Meeting, and the 2010 FMA Doctoral Students Consortium. I also thank SAC Capital for the PhD Candidate Award for Outstanding Research in WFA 2011 and Joan Gieseke for excellent editorial assistance. All errors are my own.

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[1] See, for example, Fama and French (1987).

0304-405X/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.jfineco.2013.04.012

seven portfolios sorted by their basis. These portfolios are rebalanced at a monthly frequency. Consistent with GHR's results, I find that long positions in futures contracts of high-basis commodities offer approximately 10% annual excess returns relative to low-basis commodities. Furthermore, I find that the average portfolio return is increasing in basis. This is important evidence supporting that basis characterizes the risk of commodity futures.

I find a strong factor structure in commodity returns. In particular, two return factors can capture approximately 75% of the total variance of the returns of these portfolios. The first factor is a commodity market factor, which is defined as equally weighted commodity futures excess returns across commodities and maturities. I test a commodity Capital Asset Pricing Model (CAPM) using the portfolios as test assets. Test results show that the portfolios' risk exposure to this commodity market factor cannot explain the basis spread.[2] In particular, all of the portfolios have basically the same loadings on the market factor and thus cannot explain why excess returns vary dramatically with respect to basis.

The second factor is a "slope" factor, which is defined as the return spread between high- and low-basis portfolios.[3] The time-series asset pricing test results show that the commodity market and slope factors do a fairly good job of jointly explaining the average returns of the commodity futures portfolios. In particular, the loadings on the slope factor are monotonically increasing in basis, which suggests that the slope factor is a potential risk factor for commodity futures.

Furthermore, I identify a macroeconomic risk factor, namely, investment shocks, that is negatively correlated with this slope factor. Investment shocks represent the aggregate technological progress in producing new capital. Because investment shocks are associated with a negative price of risk (Papanikolaou, 2011), this novel finding suggests a potential risk-based explanation of the positive basis spread.

To rationalize these empirical findings, I propose an investment-based asset pricing model along the lines of Cochrane (1991) and Zhang (2005). In particular, I propose a model of many commodities which extends Kogan, Livdan, and Yaron (KLY, 2009) to reproduce the basis spread, the negative correlation between investment shocks and the slope factor, and several asset pricing test results. In the model, competitive firms produce commodities in the face of an exogenous demand curve. These producers optimize their investment in physical capital to maximize firm value.

The model rationalizes the basis spread via two key cross-sectional relationships. First, the model predicts that commodities subject to high demand induce producers to invest more heavily. Such producers are more sensitive to investment shocks compared to producers of commodities subject to low demand. In the cross section, commodity producers with high investment today have more new capital to be installed. The amount of capital to be installed faces more uncertainty due to investment shocks. Because the capital of producers determines the future supply and hence the prices of commodities in the future, the futures prices of high investment commodities are more sensitive to investment shocks than low investment ones.

This sensitivity, which determines the risk premium of futures contracts, can be quantified as futures betas to shocks. In particular, the sign of futures beta to investment shocks is negative in general. This is because a positive investment shock raises the efficiency of investment; it increases the future supply given the level of investment and hence depreciates commodity prices.

Overall, the futures prices of commodities with high demand are associated with high investment producers and thus load more negatively on investment shocks. With the negative risk price of investment shocks, this explains why high demand commodities are associated with high futures returns.

Second, the model implies that high demand commodities are associated with high investment producers and have high-basis futures curves. This prediction follows from KLY: theoretically, a T-month futures price can be decomposed into two parts: the expected spot price ($\mathrm{E}[P_T]$) and a risk premium ($-\mathrm{Cov}[M_T, P_T]/\mathrm{E}[M_T]$)

$$F_{0,T} = \frac{\mathrm{E}[M_T P_T]}{\mathrm{E}[M_T]} = \mathrm{E}[P_T] + \frac{\mathrm{Cov}[M_T, P_T]}{\mathrm{E}[M_T]}, \tag{1}$$

where M_T is the stochastic discount factor (SDF).[4] From Eq. (1), it is easy to see that the high basis or equivalently, a low futures price, can arise because of either the low expected spot price ($\mathrm{E}[P_T]$) or the more negative covariance ($\mathrm{Cov}[M_T, P_T]$) between the SDF and the spot price given the positive expected futures excess returns most of the time. In KLY, a high investment rate predicts high future supply and hence a low expected spot price ($\mathrm{E}[P_T]$). My model inherits this channel. In addition, as discussed in the above paragraph, my model also relates the commodity futures of high investment producers with a higher risk premium and hence a more negative covariance term ($\mathrm{Cov}[M_T, P_T]$). To summarize, my model relates the commodities of high investment producers with high basis through a low expected spot price and a high futures premium in the cross section of commodities.

Through these two key relationships, my model explains the positive basis spread and the negative correlation between the basis spread and investment shocks, as I find in the data. Quantitatively, the simulated basis spread is very close to the historical average (10%). The simulated data also replicate the negative correlation between the basis spread and investment shocks, as well as the results from several asset pricing tests.

The model prediction that high investment producers offer high futures returns does not contradict the fact that a high investment rate predicts low stock returns. Kogan and Papanikolaou (forthcoming-b) investigate the stock

[2] Hong and Yogo (2012) document (and I confirm here) substantial comovement in the futures returns across commodities and also provide evidence of an "aggregate commodity futures market premium."

[3] Lustig, Roussanov, and Verdelhan (2011) identify a similar "slope" factor to explain the forward rate puzzle in the foreign exchange market.

[4] The standard pricing equation of a futures contract is the risk-neutral expected spot price ($F_{0,T} = \mathrm{E}^Q[P_T]$). Throughout this paper, I price futures contracts under the real probability measure with SDF.

price implications of investment shocks and find that investment shocks can explain the low stock returns associated with high investment firms. A positive investment shock improves the efficiency of investment and hence increases stock prices and the future supply of commodities, thus deflating commodity prices. In the cross section, high investment stocks load more positively on investment shocks and hence are associated with lower returns. In contrast, commodities with high investment producers are more negatively exposed to investment shocks and thus are associated with high futures returns. The model can reconcile its implications with these empirical facts.

For parsimony, there is no inventory in my model. But adding many competitive inventory holders with a nonnegative inventory constraint as in Routledge, Seppi, and Spatt (RSS, 2000) does not change the main prediction of the model. The major part of the basis spread comes from the high average excess return (about 10% annually) of the high-basis portfolio. The futures curves of the commodities in this portfolio are strongly downward sloping. Their producers invest heavily, and hence, their expected spot prices are low relative to the current spot prices. With storage costs, inventory holders are reluctant to store any commodities in this portfolio and the inventories of these commodities are effectively "stocked out."[5] Therefore, their futures prices are only driven by the production and consumption of these commodities.

The rest of the paper proceeds as follows. Section 2 discusses the extant literature and clearly lays out the contributions of this paper with respect to that literature. Section 3 discusses the data and reports the empirical analysis and results. In Section 4, I propose an investment-based model for pricing futures contracts of many commodities. Section 5 calibrates the model. Section 6 analyzes the model solution. Section 7 reports the simulation results and comparisons. Section 8 concludes.

2. Related literature

This paper builds on a growing literature on commodity futures and investment-based asset pricing. To my knowledge, this is the first paper to explain cross-sectional futures returns without hedging pressure, and the first to rationalize cross-sectional futures returns with an investment-based model.

Classical theories explaining the predictability of commodity futures returns date back to the hedging pressure hypothesis of Keynes (1923). This hypothesis argues that speculators who take long positions in futures demand a positive risk premium from producers, who short the futures to lock in their future profits. This effect pushes down futures prices and thus raises basis. Hirshleifer (1988, 1990) solves an equilibrium model with this hedging pressure story by assuming that speculators face fixed setup costs, and producers are not able to issue equity on their future cash flows. These market imperfections lead to a risk premium for bearing idiosyncratic production risk.

The empirical evidence of the hedging pressure theory is mixed. Bessembinder (1992) finds evidence to support the theory in Hirshleifer (1988). De Roon, Nijman, and Veld (2000) identify the futures premium from cross-market hedging pressure. In contrast, GHR reject the hedging pressure hypothesis in the cross section of commodities. Hong and Yogo (2012) also observe that the predictability of futures returns from hedging pressure, which is measured as the net short positions taken by commercials, is insignificant in aggregate time series.

Other recent hedging pressure models include Acharya, Lochstoer, and Ramadorai (in press) and Etula (2010). Acharya et al. (in press) propose a limits-to-arbitrage production-based model with inventory to explain the predictability of oil futures returns from producers' default risk. Etula (2010) relates the commodity futures premiums to the risk-bearing capacity of brokers and dealers with another limits-to-arbitrage model.

In contrast to these papers, my model is free of hedging pressure. Instead, my model features investment shocks, which are important macroeconomic risks and well documented in macroeconomic literature (e.g., Greenwood, Hercowitz, and Krusell, 1997, 2000; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010). The stock return implications of investment shocks have been investigated by Papanikolaou (2011) and Kogan and Papanikolaou (2013a, 2013b). This paper extends the literature to study the commodity futures returns implications of investment shocks.

Structural models of commodities consist of two major categories: inventory- and production-based models. Examples of inventory models include Deaton and Laroque (1992, 1996) and RSS. The key assumption of these models is nonnegative constraints on aggregate inventory. Production-based models of commodities are widely developed as well. These models feature risk-neutral agents in order to explain empirical facts other than futures returns. Examples include Litzenberger and Rabinowitz (1995), who explain the frequent backwardation in oil futures; Carlson, Khokher, and Titman (2007), who investigate futures price dynamics with an exhaustible constraint and capital adjustment costs of producers; and KLY. Casassus, Collin-Dufresne, and Routledge (2009) construct a general equilibrium model to explain the time-varying risk premium in oil futures.

This paper extends KLY to price cross-sectional commodity futures returns. KLY find a U-shaped volatility in oil futures and propose an investment-based model featuring investment irreversibility and capacity constraints to explain this fact. I mainly add investment shocks to their model. Rather than focusing on volatility, this paper focuses on the average futures returns across commodities. Other differences in our models include separation of systematic and idiosyncratic demand shocks, and the removal of investment capacity constraints.

Reduced-form models are also widely estimated in the commodity futures literature. Different from structural models, spot price dynamics and convenience yield are exogenous in these models. See, for example, Schwartz (1997), Schwartz and Smith (2000), Casassus and Collin-Dufresne (2005), and Trolle and Schwartz (2009). This

[5] "Stocked out" is defined as zero inventory. The nonnegative inventory constraint binds in this case.

paper is also related to many other recent empirical papers in commodity futures, such as Gorton and Rouwenhorst (2006), Asness, Moskowitz, and Pedersen (2013), and Tang and Xiong (2012).

This paper extends the large growing literature on investment-based models to cross-sectional returns of commodity futures. Examples of investment-based models of stock and bond returns are Cochrane (1991), Gomes, Kogan, and Zhang (2003), Carlson, Fisher, and Giammarino (2004), Zhang (2005), Lettau and Wachter (2007), Liu, Whited, and Zhang (2009), Livdan, Sapriza, and Zhang (2009), Gomes, Kogan, and Yogo (2009), Bazdresch, Belo, and Lin (2009), Kuehn and Schmid (2011), Lin (2012), and Li (2013), among others.

3. Empirical analysis

In this section, I sort commodities by basis and split them into seven portfolios. The high-basis portfolio offers a return that is about 10% higher than that of the low-basis portfolio per annum. I refer to this 10% return spread as the basis spread. Then I analyze the systematic risk of these portfolios. The sample includes monthly close quotes of futures of maturities up to 12 months for 31 commodities from January 1970 to December 2008.

3.1. Data and variable definitions

The commodity futures price data are from the Commodity Research Bureau. The database includes a large panel of historical daily close quotes on futures contracts traded across many commodity markets worldwide.

The sample starts in January 1970 and ends in December 2008.[6] It includes monthly close quotes of futures contracts of maturities up to 12 months for a cross section of many commodities. The cross-sectional sample size ranges from 18 commodities in 1970 to 31 commodities in 2008 from four sectors: agriculture, energy, livestock, and metals. Futures contracts with maturities longer than 12 months are not liquidly traded and hence are excluded from the sample.

I define the futures excess return as the fully collateralized return of longing a futures contract. That is, at the time of signing a futures contract, the buyer has to deposit a cash amount that equals the present value of the futures price to eliminate counterparty risk. This cash amount, which is used as collateral, earns the risk-free rate. For commodity i, I denote the futures price at time t with maturity T as $F_{i,t,T}$. Following GHR, the futures excess return of longing this contract for one period is defined as

$$R^e_{i,t+1,T} = \frac{F_{i,t+1,T}}{F_{i,t,T}} - 1. \tag{2}$$

I exclude the return from holding a one-month futures contract to maturity in the sample because these returns are not purely financial and the underlying commodity must be delivered in order to earn the return.

Theoretically, the basis of a commodity, which measures the slope of the futures curve, is defined as the difference between its contemporaneous spot price and futures price with some maturity. Empirically, because spot and futures contracts are traded on separated markets and the nearest futures price is very close to the spot price due to the no-arbitrage condition, we usually use the nearest futures price to substitute for the spot price to compute the basis. Similar to GHR, I define the monthly basis as the log difference between the one-month ($T_1 = 1$) futures price and the 12-month ($T_2 = 12$) futures price divided by the difference in maturity as

$$B_{i,t} = \frac{\log(F_{i,t,T_1}) - \log(F_{i,t,T_2})}{T_2 - T_1}. \tag{3}$$

If the one-month futures price is not available, I use the price of the futures contract with the nearest maturity. The same applies to 12-month futures. The basis reported in the rest of this paper is annualized.

Table 1 reports the summary statistics of every individual commodity by sector in the sample. A commodity is in backwardation if its futures curve is downward sloping (the basis is positive). Otherwise, it is in contango. Crude oil futures become available in 1983. As noted by Litzenberger and Rabinowitz (1995), its futures curve is in backwardation about 70% of the time. The behavior of gold and silver futures is much closer to that of futures on financial assets rather than commodities. Their basis is close to a negative risk-free rate, and their futures curves have little probability of being in backwardation. Other commodities are associated with a heterogeneous average basis and frequency of backwardation.

3.2. Portfolios sorted by basis

I sort commodity futures contracts into seven portfolios by basis with a monthly rebalance frequency. When constructing these portfolios, I first sort available commodities by basis at the end of month t and then split them into portfolios by quantiles. At the end of the following month $t+1$, the futures contracts of these commodities are one month closer to their maturities. I compute the excess returns of these contracts from t to the end of month $t+1$, and aggregate these excess returns within each portfolio using equal weights to construct the portfolio excess return for each month. I repeat this strategy monthly.

Table 2 reports the key statistics of futures excess return and basis of these seven portfolios in percentages. Consistent with GHR's finding, the futures of high-basis commodities offer about 10% annual excess returns relative to low-basis commodities and are statistically significant with a Newey-West corrected t-stat of 2.15. Most of this return spread is due to the high excess return of the high-basis portfolio, since the low-basis portfolio has close to zero excess return in the historical average. This fact suggests a highly profitable trading strategy of longing the high-basis portfolio, whose annual Sharpe ratio is 51.35%.

[6] The database starts from 1959. In the early period, few commodities are traded. To ensure that there are at least three commodities within a portfolio to diversify idiosyncratic risk, I compose my sample starting from 1970.

Table 1
Summary statistics of commodity futures for every individual commodity in the sample.

The sample includes monthly close quotes of futures of maturities up to 12 months of 31 commodities from January 1970 to December 2008. N is the number of monthly observations available for a commodity. The basis column reports the historical average basis of a commodity. The "freq. of bw." column reports the frequency of a commodity futures curve that is in backwardation. A commodity is defined as being in backwardation if its basis is positive. Columns $E(R^e)$ and $\sigma(R^e)$ report the annualized historical average and standard deviation of futures excess returns of individual commodities with many maturities.

Sector	Commodity	Symbol	N	Basis	Freq. of bw.	$E[R^e]$	$\sigma[R^e]$	Sharpe ratio
Agriculture	Barley	WA	235	−3.66	27.66	−0.24	19.62	−1.21
	Butter	02	141	−3.68	33.33	3.66	27.22	13.46
	Canola	WC	377	−2.98	33.16	−0.18	19.82	−0.89
	Cocoa	CC	452	−2.61	25.22	4.52	30.32	14.90
	Coffee	KC	420	−2.57	36.90	6.00	36.52	16.44
	Corn	C-	468	−6.03	23.08	−0.01	23.35	−0.04
	Cotton	CT	452	−1.75	36.50	3.60	22.96	15.69
	Lumber	LB	468	−5.63	33.55	−1.13	22.80	−4.98
	Oats	O-	468	−5.65	31.20	0.44	28.90	1.53
	Orange juice	JO	448	−3.08	36.61	2.32	29.56	7.86
	Rough rice	RR	265	−7.56	26.04	−1.50	25.01	−6.01
	Soybean meal	SM	468	0.20	44.87	7.80	28.63	27.25
	Soybeans	S-	468	−0.58	37.18	5.99	26.25	22.81
	Wheat	W-	468	−2.88	38.68	2.79	23.76	11.72
Energy	Crude oil	CL	295	4.25	66.78	10.56	27.87	37.89
	Gasoline	RB	275	8.09	70.91	12.82	30.18	42.47
	Heating oil	HO	345	1.49	55.65	9.50	28.65	33.15
	Natural gas	NG	216	−3.63	43.06	8.66	34.63	25.00
	Propane	PN	247	5.53	55.47	14.28	34.18	41.77
	Unleaded gas	HU	250	8.62	71.20	16.02	29.24	54.78
Livestock	Broilers	BR	19	4.58	52.63	1.49	7.28	20.53
	Feeder cattle	FC	443	0.35	53.27	4.43	14.28	31.01
	Lean hogs	LH	468	2.66	59.40	7.98	22.34	35.70
	Live cattle	LC	468	0.46	50.64	4.55	14.92	30.46
Metals	Aluminum	AL	215	1.06	35.35	5.46	19.11	28.56
	Coal	QL	85	−1.55	34.12	6.20	30.02	20.65
	Copper	HG	412	0.52	41.75	4.62	25.50	18.12
	Gold	GC	400	−6.24	0.00	0.43	19.88	2.18
	Palladium	PA	362	−2.16	30.66	10.21	35.19	29.01
	Platinum	PL	410	−3.21	23.66	3.69	27.81	13.27
	Silver	SI	419	−6.51	1.19	0.44	32.09	1.37

Table 2
Key moments of the seven commodity portfolios sorted by basis.

The upper panel reports the moments of portfolio excess returns relative to the risk-free rate. The lower panel reports the historical averages and standard deviations of portfolio-level basis. Portfolios are rebalanced in the last trading day of every month. Futures excess return is defined as the fully collateralized return of longing a futures contract. Portfolio excess return is computed as the equally weighted futures excess return across all commodities within a portfolio. Basis measures the slope of a futures curve. It is defined as the annualized difference between log futures prices with the shortest maturity and the longest maturity available. Portfolio-level basis is computed as the equally weighted mean across all commodities within a portfolio. The t-stats report the statistical significance of the portfolio excess returns. They are adjusted using a Newey-West correction.

	Low	2	3	4	5	6	High	HML
				Futures excess return				
Mean	1.83	2.98	2.28	5.06	6.08	8.92	11.25	9.42
Std	20.67	18.99	17.02	19.01	17.23	18.20	21.90	27.12
Sharpe ratio	8.84	15.69	13.41	26.63	35.28	49.01	51.35	34.74
t-Stat	0.58	0.95	0.76	1.52	2.00	2.89	2.94	2.15
				Basis				
Mean	−18.59	−10.79	−6.57	−3.03	1.26	7.50	19.38	
Std	21.68	13.86	13.95	15.83	18.61	24.59	37.67	

More importantly, the portfolio excess return is increasing in basis. This suggests basis as a strong predictor of futures returns in the cross section of commodities. In addition, the transition probability matrix of the seven portfolios sorted by basis can be found in Online Appendix Table I. The frequent transition of commodities across portfolios indicates that the average return pattern is not because of some particular commodities but rather a characteristic of commodities. This characteristic is the basis.

3.3. Time-series tests of factor models

I introduce a commodity market factor Mkt_C and a high-minus-low HML_C factor to explain the excess returns for these portfolios. Multifactor models have been used to explain cross-sectional stock returns (Fama and French, 1992), bond returns (Fama and French, 1993), and forward returns in the foreign exchange market (Lustig, Roussanov, and Verdelhan, 2011).

First, I test a commodity CAPM and find that the commodity market factor alone fails to explain these portfolio returns. The market factor (Mkt_C) is defined as the equally weighted futures excess returns averaging across all commodities and maturities in the sample. Using the portfolios reported in Table 2 as test assets, I test this commodity CAPM with time-series regressions as

$$R^e_{j,t} = \alpha_j + \beta_{j,Mkt_C} Mkt_{C,t} + \epsilon_{j,t}, \tag{4}$$

where $j=1$–7 and "HML." The HML portfolio represents the strategy of longing the high-basis ($j=7$) portfolio and shorting the low-basis ($j=1$) portfolio. The t-stat of the abnormal return (α_j) tests the hypothesis that the model can explain the excess return of portfolio j.

The tests reject the commodity CAPM. The results of the time-series regression tests of the commodity CAPM are reported in Panel A of Table 3. The abnormal return (α_j) of the HML portfolio is significantly different from zero. In particular, the commodity market factor can explain only about 2% of the return of the HML portfolio, given that the average return is 9.42% and the abnormal return (Alpha) is 7.73%.

Given the failure of the commodity CAPM, I add a second factor (HML_C), which is defined as the return of the HML portfolio. I also use time-series regressions to test this two-factor model as

$$R^e_{j,t} = \alpha_j + \beta_{j,Mkt_C} Mkt_{C,t} + \beta_{j,HML_C} HML_{C,t} + \epsilon_{j,t}, \tag{5}$$

where $j=1$–7.

The time-series test results are reported at the lower part of Panel A of Table 3. All the abnormal returns (α_j) of these seven portfolios are statistically insignificant. This two-factor model passes these time-series regression tests. More interestingly, the portfolio loadings on the market factor (β_{j,Mkt_C}) are fairly flat across portfolios, whereas the portfolio loadings on the HML_C factor (β_{j,HML_C}) are monotonically increasing in basis. These findings show that the HML_C factor accounts for the most important part of the comovement among commodity futures returns along this dimension.

3.4. Cross-sectional tests of factor models

In addition to time-series tests, I also apply the two-stage regression method (Cochrane, 2005) and estimate the factor risk premiums. Panel B of Table 3 compares the estimation results of the commodity CAPM with the results of the two-factor model.

Intuitively, the two-stage regression method tests the hypothesis that cross-sectional differences in asset returns are due to cross-sectional differences in asset risk exposure. The two-stage regressions consist of two steps. First, we regress the time series of the excess returns of portfolio j on factors to estimate the vector of risk exposure (β_j) as

$$R^e_{j,t} = a_j + \beta'_j f_t + \epsilon_{j,t}. \tag{6}$$

Second, we can regress the portfolio average excess returns ($E[R^e_{j,t}]$) on its risk exposure (β_j) in the cross section of portfolios as

$$E[R^e_{j,t}] = \lambda' \beta_j + \alpha_j. \tag{7}$$

The slope of the cross-sectional regression is the vector of the factor risk premiums (λ). If the abnormal returns (α_j) significantly deviate from zero, the model fails to explain the returns of portfolios. The χ^2-test is used to test whether the average abnormal returns are statistically significant. I apply the Shanken correction (Shanken, 1992) to compute the t-stats and the χ^2-test in the second stage to adjust for the fact that β_j's are estimated in the first-stage time-series regression.

Estimation results of the second-stage regressions are reported in Panel B of Table 3. For the commodity CAPM, the R^2 of the cross-sectional regression is about 21.15%, and the root mean square error (RMSE), which approximates the average alpha of these portfolios, is 2.91% per annum. The Shanken-corrected p-values of the χ^2-tests are about 26.97%. The χ^2-test does not reject the commodity CAPM. However, the small R^2 indicates that the commodity market factor can only explain a small part of the futures returns in the cross section. Meanwhile, the pricing error (RMSE) is also large for this single-factor model.

The two-factor model (Commodity Mkt+HML) passes the cross-sectional tests and significantly improves the commodity CAPM in explaining these portfolio returns. The last row reports the average excess returns of the commodity market factor (6% per annum) and the HML_C factor (9.42% per annum). The estimated factor risk premiums (λ_{Mkt} and λ_{HML_C}) using all methods are very close to the average excess returns of the two factors. Theoretically, when factors are excess returns, factor risk premiums should equal their time-series averages. This no-arbitrage condition holds for the two factors among these portfolios. Statistically, factor risk premiums (λ) are significantly different from zero since the t-stat of the factor risk premium of the market factor is 2.6 and the t-stat of the factor risk premium of the HML_C factor is 2.33. The significance of λ indicates that both the market factor and the HML_C factor are priced by the investors. Meanwhile, the R^2 of the cross-sectional regressions increases to 66.73% from 21.15% for the commodity CAPM; and the pricing error (RMSE) reduces to 1.89% per annum. The Shanken-corrected p-values of the χ^2-tests are about 49.61%. The two-factor model passes the cross-sectional test. And all these cross-sectional statistics are improved significantly in comparison with the commodity CAPM.

Table 3
Time-series and cross-sectional tests of the commodity CAPM and a two-factor model.

The commodity CAPM uses the commodity market factor (Mkt_C). The two-factor model adds a high-minus-low factor. The commodity market factor is constructed as the equally weighted commodity futures across commodities and maturities in the sample. The high-minus-low factor is defined as the excess return of the high-basis relative to the low-basis portfolio (HML_C). Panel A reports the time-series regression results for seven basis-sorted portfolios. The t-stats are adjusted using a Newey-West correction. Panel B reports the cross-sectional regression results. The factor risk premiums (λ_{Mkt_C} and λ_{HML_C}) are estimated with two-stage regressions (Cochrane, 2005). I do not include a constant in the cross-sectional regression. The p-values are for the χ^2-test in the cross section. The p-values and t-stats in the cross-sectional tests are adjusted using a Shanken correction (Shanken, 1992). The alphas are annualized and reported in percentages.

Panel A: Factor betas

	Low	2	3	4	5	6	High	HML
				Commodity CAPM				
Alpha	−3.14	−2.97	−3.09	−1.69	0.45	2.75	4.59	7.73
(t-stat)	(−1.23)	(−1.42)	(−1.55)	(−0.95)	(0.23)	(1.46)	(1.86)	(1.98)
Mkt_C	0.79	0.95	0.86	1.08	0.90	0.99	1.06	0.27
(t-stat)	(7.43)	(15.59)	(15.00)	(13.69)	(14.09)	(13.98)	(9.92)	(1.45)
R^2 (%)	27.29	46.41	47.16	59.67	50.40	54.31	43.61	1.83
				Commodity Mkt and HML				
Alpha	0.94	−2.34	−2.92	−1.55	0.41	2.70	0.94	
(t-stat)	(0.57)	(−1.10)	(−1.43)	(−0.85)	(0.21)	(1.43)	(0.57)	
Mkt_C	0.94	0.97	0.87	1.08	0.90	0.98	0.94	
(t-stat)	(17.80)	(16.89)	(13.84)	(13.33)	(13.72)	(14.66)	(17.80)	
HML_C	−0.53	−0.08	−0.02	−0.02	0.00	0.01	0.47	
(t-stat)	(−15.64)	(−3.04)	(−0.77)	(−0.53)	(0.18)	(0.19)	(13.99)	
R^2 (%)	74.37	47.75	47.28	59.74	50.41	54.32	77.17	

Panel B: Factor risk premium

	λ_{Mkt_C}	λ_{HML_C}	R^2	RMSE	p-Value (χ^2 test)
			Commodity CAPM		
Est.	5.97		21.15	2.91	26.97
(t-stat)	(2.59)				
E[R^e]	6.25				
			Commodity Mkt and HML		
Est.	6.00	9.84	66.73	1.89	49.61
(t-stat)	(2.60)	(2.33)			
E[R^e]	6.25	9.42			

3.5. Investment shocks as a risk factor

Previous sections have empirically identified HML_C as a potential candidate for a risk factor in capturing the return comovement of these portfolios. Furthermore, I find empirical evidence that it is related to a well-identified macroeconomic risk called the investment shock.

Investment shocks represent the technological progress in producing new capital. They differ from productivity shocks, which impact the production of all goods including both capital goods and consumption goods. Investment shocks are well investigated in macroeconomics, as in Greenwood, Hercowitz, and Krusell (1997, 2000), Fisher (2006), and Justinano, Primiceri, and Tambalotti (2010). The stock pricing implications are studied by Papanikolaou (2011) and Kogan and Papanikolaou (2013a, 2013b), among others.

I use the relative price of investment goods and the *IMC* factor as two empirical proxies of investment shocks.[7] The price of investment goods relative to consumption goods is a direct macro measure for investment shocks, which were constructed by Gorton (1990) and extended by Israelsen (2010). A positive investment shock represents the technological progress in producing new capital (more supply

[7] I thank Dimitris Papanikolaou for sharing the data for investment shocks.

of investment goods) and hence a reduction in the relative price of investment goods. Empirically, this measure (z_t^{ξ}) is constructed as the negative innovation of the relative price of investment goods (ξ_t)

$$\log\ \xi_t = a_0 t + a_t t\mathbf{1}_{\{t>1982\}} + \rho \log\ \xi_{t-1} - z_t^{\xi}. \tag{8}$$

In the above equation, the growth rate of the investment goods price ($a_0 + a_t\mathbf{1}_{\{t>1982\}}$) is allowed to differ before and after 1982 because it experiences a sudden increase in its average rate of decline in 1982, as noted in Fisher (2006) and Justiniano et al. (2010). The second measure, the *IMC* factor, is defined as the excess returns of longing the stocks of investment good producers and shorting the stocks of consumption good producers following Papanikolaou (2011).

To relate the portfolio returns with investment shocks, I apply the two-stage regressions to a new two-factor model. The first factor is the commodity market factor. The second factor is the investment shock. Table 4 reports the two-stage regression results using both the relative price of investment goods and the IMC factor as proxies for investment shocks. The excess returns of commodity futures portfolios are annualized first before regressions in order to smooth the seasonality of commodity prices.

Panel A of Table 4 reports the time-series test results. The upper part of this panel reports the time-series

Table 4
Time-series and cross-sectional tests of two-factor models using the commodity market factor as the first factor and investment shocks as the second factor.

"Commodity Mkt and I-shock" uses the relative price of investment goods (*I-shock*) to measure investment shocks. "Commodity Mkt and IMC" uses the excess stock return of investment good producers relative to consumption good producers (*IMC*) to measure investment shocks. Panel A reports the time-series regression results for seven basis-sorted portfolios. The *t*-stats are adjusted using a Newey-West correction. Panel B reports the cross-sectional regression results. The factor risk premiums (λ_{Mkt_C}, λ_I, and λ_{IMC}) are estimated with two-stage regressions (Cochrane, 2005). I do not include a constant in the cross-sectional regression. The *p*-values are for the χ^2−test in the cross section. The *p*-values and *t*-stats in the cross-sectional tests are adjusted using a Shanken correction (Shanken, 1992). The alphas are annualized and reported in percentages.

Panel A: Factor betas

	Low	2	3	4	5	6	High	HML
				Commodity Mkt and I-shock				
Alpha	−1.83	−1.89	−3.36	−2.52	0.65	3.36	4.43	6.26
(*t*-stat)	(−0.68)	(−0.87)	(−1.15)	(−1.69)	(0.42)	(5.02)	(2.15)	(2.19)
Mkt_C	0.60	0.78	0.91	1.22	0.86	0.87	1.09	0.49
(*t*-stat)	(3.06)	(6.12)	(8.95)	(11.46)	(8.04)	(9.53)	(7.11)	(1.76)
I-shock	1.12	0.46	0.37	0.64	−0.57	−1.46	−0.07	−1.19
(*t*-stat)	(2.11)	(0.73)	(0.46)	(1.39)	(−1.06)	(−2.03)	(−0.11)	(−1.29)
R^2 (%)	29.45	42.96	47.08	72.46	57.34	56.92	47.87	8.21
				Commodity Mkt and IMC				
Alpha	−0.75	−1.89	−2.66	−3.13	0.87	2.45	4.34	5.09
(*t*-stat)	(−0.29)	(−0.79)	(−1.00)	(−1.78)	(0.44)	(1.69)	(1.72)	(1.43)
Mkt_C	0.55	0.79	0.87	1.26	0.84	0.91	1.09	0.55
(*t*-stat)	(3.01)	(5.57)	(6.56)	(9.67)	(7.87)	(8.23)	(4.84)	(1.54)
IMC	0.42	0.03	0.26	−0.16	0.04	−0.39	−0.03	−0.46
(*t*-stat)	(2.75)	(0.21)	(1.62)	(−1.96)	(0.28)	(−2.71)	(−0.17)	(−1.89)
R^2 (%)	36.85	42.73	49.51	72.96	56.94	61.79	47.90	12.21

Panel B: Factor risk premium

	λ_{Mkt_C}	λ_I	λ_{IMC}	R^2	RMSE	*p*-Value (χ^2 test)
			Commodity Mkt and I-shock			
Est.	6.33	−2.61		64.16	1.97	85.47
(*t*-stat)	(2.33)	(−2.07)				
			Commodity Mkt and IMC			
Est.	6.13		−5.80	46.50	2.40	46.31
(*t*-stat)	(2.20)		(−1.33)			

regression of the model with the relative price of investment goods (*I-shock*) as the second factor. The portfolio betas to the relative price of investment goods is decreasing in basis. The lower part of the panel reports the factor betas of the model with the IMC as the second factor. The portfolio betas to IMC is also decreasing in basis. Since we have found that the basis positively predicts futures returns in the cross section, the portfolio betas to investment shocks is negatively related to the returns of basis-sorted portfolios. The cross-sectional difference in futures returns can be attributed to the heterogeneity in the commodity risk exposure to investment shocks.

Panel B of Table 4 reports the cross-sectional tests results. The estimated factor risk premiums of investment shocks are negative, which is consistent with Papanikolaou (2011). More specifically, the factor risk premium of the relative price of investment goods $\lambda_I = -2.61\%$ with t-stat of –2.07, which is statistically significant; the factor risk premium of the IMC factor $\lambda_{IMC} = -5.8\%$, which is also negative. In comparing with the cross-sectional test results of the commodity CAPM as reported in Panel B of Table 3, we can find that the two-factor model using the relative price of investment goods as the second factor captures the cross-sectional returns the best among the four linear factor models. The R^2 of the cross-sectional regression is 64.16%, the RMSE is 1.97%, and the Shanken-corrected p-value of the χ^2-test is 85.47%. In addition, the two-factor model using the IMC factor as the second factor can also explain the portfolio returns reasonably well.

Fig. 1 compares the performance of these four linear asset pricing models in matching the average portfolio returns. From these figures, it is easy to observe that adding the return of the high-minus-low portfolio or the investment shock as the second factor improves the model-predicted returns in matching the data. In these four figures, x-axis represents the historical average excess

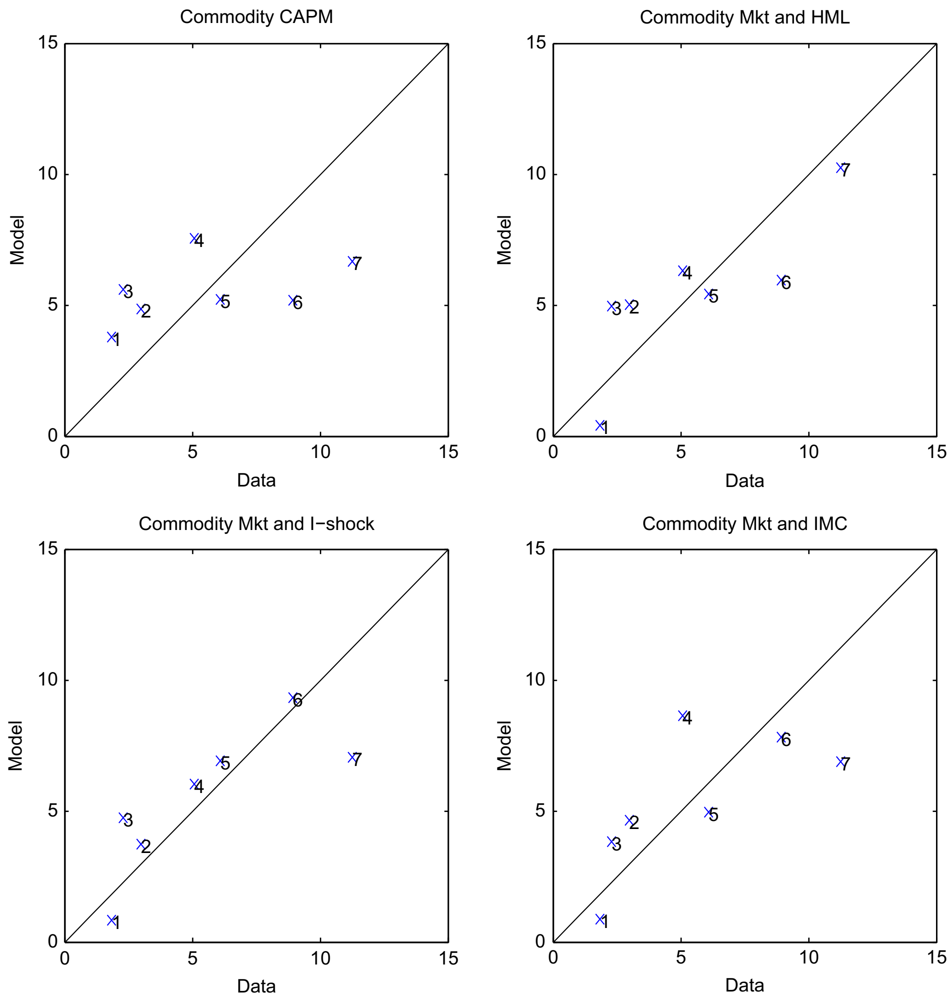

Fig. 1. Model-predicted and historical average excess returns of seven basis-sorted portfolios. These figures compare the performance of four linear models in explaining the historical returns of seven basis-sorted portfolios. The commodity CAPM uses the commodity market as a single factor. "Commodity Mkt and HML" uses the commodity market as the first factor and the excess return of the high-basis relative to the low-basis portfolio (HML_C) as the second factor. "Commodity Mkt and I-shock" uses the commodity market as the first factor and the relative price of investment goods (*I-shock*) as the second factor. "Commodity Mkt and IMC" uses the commodity market as the first factor and the excess stock return of investment good producers relative to consumption good producers (*IMC*) as the second factor.

Table 5
Historical correlation matrix among HML_C, *I-shock*, *IMC*, *TFP*, and Mkt_C factors.

HML_C is defined as the return spread of the two futures portfolios sorted by basis. *I-shock* is the relative price of investment goods. *IMC* is the excess stock return of investment good producers relative to consumption good producers. Mkt_C is the commodity market factor. The *t*-stats are adjusted using a Newey-West correction.

	HML_C	*I-shock*	*IMC*	*TFP*
I-shock	−18.34			
(*t*-stat)	(−1.84)			
IMC	−22.48	2.58		
(*t*-stat)	(−2.48)	(0.31)		
TFP	0.46	−7.11	14.92	
(*t*-stat)	(0.04)	(−0.43)	(3.42)	
Mkt_C	12.56	13.75	20.03	29.89
(*t*-stat)	(1.20)	(0.91)	(3.86)	(3.81)

returns of the seven portfolios sorted by basis and *y*-axis represents the expected portfolio excess returns predicted by models computed using Eq. (7). Theoretically, a 'perfect' model should predict that the seven data points lie exactly on the diagonal. In reality, some of the data points may deviate from the diagonal indicating the pricing errors. By comparing these four figures, we can see that the points in two-factor models are closer to the diagonal than the commodity CAPM. In particular, the points of the model using the relative price of investment shocks are closest to the diagonal among these four linear models.

In summary, investment shocks pass all these formal asset pricing tests as a fundamental risk for the basis spread.

In addition, I also investigate the relationship between the basis spread and investment shocks in time-series. Following GHR, I form two portfolios sorted by basis to further reduce the idiosyncratic risk. I redefine the HML factor as the return of longing the high-basis portfolio and shorting the low-basis portfolio. The historical correlation matrix among factors is reported in Table 5. In particular, the correlation between HML and the relative price of investment goods is −18.34% (*t*-stat = −1.84) and the correlation between HML and *IMC* is −22.48% (*t*-stat = −2.48). As a comparison, the correlation between HML and total-factor productivity (TFP) is only 0.46% (*t*-stat = 0.04) and the correlation between HML and the commodity market (Mkt_C) is 12.56% (*t*-stat = 1.20). The basis spread correlates with investment shocks more than TFP and the commodity market factor.

4. Model

In this section, I construct an investment-based dynamic equilibrium model extending KLY to endogenize the fact that the HML_C factor is negatively related to investment shocks. Through this channel, the model can replicate the basis spread, the failure of CAPM, and the two-stage regression tests of two-factor models in the previous empirical section.

4.1. Setup

In a competitive commodity production economy, there are *N* different commodities. I assume that commodities are nonstorable and mutually nonsubstitutable.[8] Each commodity has a large number of identical producers such that we can reduce their complex behavior into a single representative producer problem. The production technology follows a Cobb-Douglas production function as

$$Q^S_{j,t} = AK^{\alpha}_{j,t}. \tag{9}$$

Without loss of generality, I assume that production is deterministic $A = 1$ and that constant returns to scale $\alpha = 1$. Since this paper focuses on commodity price implications, we can combine the productivity and demand shocks of a commodity into a single demand shock, whereas capital share (α) and the price elasticity of demand can be combined into a single parameter in the model.

Following Fisher (2006), the capital of this producer accumulates as

$$K_{j,t+1} = (1-\delta)K_{j,t} + Y_t I_{j,t}. \tag{10}$$

Y_t represents the level of investment-specific technology. It is an aggregate shock that impacts the real investment of all commodity producers ($j = 1, 2, \ldots, N$). A positive shock raises the efficiency of investment (more future capital ($K_{j,t+1}$) given the same amount of investment ($I_{j,t}$)) and hence induces more investment.[9]

Firms own capital and produce commodities using the above technology. A producer's profit is generated from the revenue of selling its output commodity ($P_{j,t}Q^S_{j,t}$) deducted by the costs of investment

$$\pi_{j,t} = P_{j,t}Q^S_{j,t} - I_{j,t}, \tag{11}$$

where investment is assumed to be irreversible as

$$I_{j,t} \geq 0. \tag{12}$$

In the sensitivity analysis, I discuss how this assumption impacts the model-predicted basis spread.

Producers take commodity prices as given and make investment decisions to maximize firm value. The firm value of a producer of commodity *j* is defined as the present value of all future profits as

$$V_{j,0} = \max_{I_{j,t}} \quad E_0\left[\sum_{t=0}^{\infty}\left(\prod_{u=0}^{t} M_u\right)\pi_{j,t}\right] \tag{13}$$

$$\text{s.t.} \quad I_{j,t} \geq 0, \tag{14}$$

where M_{t+1} denotes the SDF from time t to $t+1$ and $M_0 = 1$.

The demand for a commodity is exogenous following KLY. Any commodity *j* faces two types of demand shocks: an aggregate demand shock (X_t) and an idiosyncratic demand shock ($Z_{j,t}$). The inverse demand function of commodity *j* is

[8] Including competitive inventory holders as in RSS does not change the model's prediction on the basis spread, as I argue in the introduction.

[9] Higher efficiency of investment is equivalent to lower costs of investment or lower price of investment goods, which is the empirical measure of investment shocks that I use in the previous section.

defined as

$$P_{j,t} = P(X_t, Z_{j,t}, Q^D_{j,t}) = \left(\frac{X_t Z_{j,t}}{Q^D_{j,t}} \right)^{\eta}, \tag{15}$$

where η measures the price elasticity of demand. For each commodity and each period, the market-clearing condition is that the supply of this commodity equals the demand of it as

$$Q^S_{j,t} = Q^D_{j,t}. \tag{16}$$

This economy has three types of exogenous shocks: aggregate demand shocks (X_t), idiosyncratic demand shocks ($Z_{j,t}$), and investment shocks (Y_t). Aggregate demand shocks drive the common movement of demand across commodities. Idiosyncratic demand shocks introduce heterogeneity across commodities in order to generate cross-sectional futures returns. Investment shocks are aggregate shocks, which are crucial in replicating the basis spread and many other asset pricing tests. I assume that log aggregate demand shocks ($\log X_t$) follow a random walk process. Log idiosyncratic demand shocks ($\log Z_{j,t}$) are assumed to follow an AR(1) process in order to generate a stationary distribution of firms in the cross section. Log investment shocks follow an AR(1) process as in Justiniano et al. (2010)

$$\log X_{t+1} = \log X_t + g_x + \sigma_x e_{t+1}, \tag{17}$$

$$\log Z_{j,t+1} = \rho_z \log Z_{j,t} + (1-\rho_z)\overline{z} + \sigma_z \epsilon_{j,t+1}, \tag{18}$$

$$\log Y_{t+1} = \rho_y \log Y_t + \sigma_y u_{t+1}. \tag{19}$$

Following Zhang (2005), the SDF is exogenous. This is a reduced-form way to model the consumer's problem and is common in the studies of cross-sectional stock returns. Deviating from the conditional CAPM framework [as in Zhang, 2005], my model is a two-factor model with constant prices of risk more similar to Eisfeldt and Papanikolaou (in press) and Kogan and Papanikolaou (2013a, 2013b). Both aggregate demand shocks and investment shocks are priced in this economy. I parameterize the SDF as

$$M_{t+1} = \frac{1}{\phi_t} \exp[-r_f - \gamma_x(\log X_{t+1} - \log X_t) - \gamma_y(\log Y_{t+1} - \log Y_t)], \tag{20}$$

where

$$\phi_t = \mathrm{E}_t[\exp[-\gamma_x(\log X_{t+1} - \log X_t) - \gamma_y(\log Y_{t+1} - \log Y_t)]] \tag{21}$$

is a compensator so that the SDF satisfies the no-arbitrage condition $\mathrm{E}_t[M_{t+1}] = e^{-r_f}$ over all states of the economy.[10] The risk-free rate (r_f) is a constant.

[10] Even though ϕ_t has a closed-form solution, it has to be computed numerically to precisely compensate for the SDF, since we apply the SDF numerically when solving for the optimal investment rate, spot, and futures prices.

4.2. Futures price

The futures price is defined as the risk-neutral expected spot price. Buying a futures contract with maturity T written on commodity j costs zero at time t. The payoff at maturity is $P_{j,T} - F_{j,t,T}$. Therefore

$$0 = \mathrm{E}_t \left[\left(\prod_{u=t+1}^{T} M_u \right) (P_{j,T} - F_{j,t,T}) \right]. \tag{22}$$

To compute the futures prices recursively, we can rewrite the above equation as

$$F_{j,t,T} = \mathrm{E}^Q_t[F_{j,t+1,T}] = e^{r_f} \mathrm{E}_t[M_{t+1} F_{j,t+1,T}]. \tag{23}$$

The boundary condition is

$$F_{j,T,T} = P_{j,T}, \tag{24}$$

because the futures price converges to its spot price as the contract approaches maturity. This is the no-arbitrage condition for futures contracts. In computing the policy functions of the futures prices across maturities, I solve for market-clearing spot price dynamics first. Then, I compute the above expectation recursively to solve for futures prices with many maturities over all economic states.

4.3. Basis, futures returns, and risk exposures

The basis of a futures curve and futures returns follow the same definitions as in the empirical section. Three types of risk drive the futures returns in the model. Among them, aggregate demand shocks and investment shocks are systematic. Therefore, the risk exposure to these two types of risks determines the risk premiums of the futures contracts. We can compute futures risk exposure to aggregate demand shocks as

$$\beta^x_{j,t+1,T} = \frac{\mathrm{Cov}_t[R^e_{j,t+1,T}, \log X_{t+1} - \log X_t]}{\mathrm{Var}_t[\log X_{t+1} - \log X_t]} \tag{25}$$

and futures risk exposure to investment shocks as

$$\beta^y_{j,t+1,T} = \frac{\mathrm{Cov}_t[R^e_{j,t+1,T}, \log Y_{t+1} - \log Y_t]}{\mathrm{Var}_t[\log Y_{t+1} - \log Y_t]}. \tag{26}$$

4.4. Spot price dynamics and risk exposure

Because futures contracts are contingent claims written on spot prices (see Eq. (22)), spot price dynamics determine the risk exposure of futures contracts. From the model, we can derive the spot price dynamics as

$$\log P_{j,t+1} = \log P_{j,t} + g^P_{j,t} + \eta \sigma_x e_{t+1} + \eta \sigma_z \epsilon_{j,t+1}, \tag{27}$$

where $g^P_{j,t} = -\eta \log(1-\delta + Y_t i^*_{j,t}) + \eta g_x + \eta(1-\rho_z)(\overline{z} - \log Z_{j,t})$ is the predictable part of the changes in log spot prices and $i^*_{j,t}$ is the optimal investment rate. See Section A.1 of the Appendix for the derivation details. From the above equation, we can observe that the risk loading on investment shocks is approximately $-\eta i^*_{j,t}$, which is negative and dependent on the optimal investment $i^*_{j,t}$.[11]

[11] Because δ and $Y_t i^*_{j,t}$ are small, the first term in $g^P_{j,t}$ is approximately $\eta\delta - \eta Y_t i^*_{j,t}$.

Spot price dynamics are useful in interpreting how high basis predicts high futures returns through the investment channel. When the producer of commodity j invests heavily, $i^*_{j,t}$ is high. The predictable change of log spot prices ($g^P_{j,t}$) is low because it is a decreasing function of the investment rate. Hence, the expected spot price in the future is low relative to the current spot price. The basis, which is defined as the difference between the short- and long-term futures prices, is high. Meanwhile, under high investment ($i^*_{j,t}$), the predictable changes in log spot prices ($g^P_{j,t}$) face more negative exposure ($-\eta i^*_{j,t}$) to investment shocks (Y_t). Therefore, the expected futures return is high with the assumption that investment shocks are associated with a negative price of risk. The idiosyncratic demand shock ($z_{j,t+1}$) helps generate the cross section. This explains why high basis is associated with high futures returns in the cross section of commodities.

These spot price dynamics are very close to those in KLY. Deviating from theirs, I mainly introduce investment shocks (Y_t) into the dynamics. Secondly, I add AR(1) idiosyncratic demand shocks to generate a stationary cross-sectional distribution of firms and to price cross-sectional futures returns.

4.5. *The first-order conditions and a social planner's problem*

I solve the competitive equilibrium allocation by solving an equivalent social planner's problem following Lucas and Prescott (1971). I take this approach because the social planner's problem shares the same first-order conditions as the competitive equilibrium allocation.

In the competitive equilibrium allocation, producers take commodity prices as given. The first-order condition with respect to $K_{j,t+1}$ is

$$1-\mu_{j,t} = \mathrm{E}_t[M_{t+1}(P_{j,t+1} + (1-\delta)(1-\mu_{j,t+1}))], \tag{28}$$

where $\mu_{j,t}$ denotes the Lagrange multiplier of the investment irreversibility constraint of commodity j at period t. The detailed derivation is provided in the Online Appendix.

In the social planner's problem, the social planner chooses investment to maximize the present value of the social surplus net of investment costs. The social surplus is defined as

$$SS_{j,t} = \int_0^{Q^S_{j,t}} P(X_{j,t}, Z_{j,t}, q)\, dq = \frac{1}{1-\eta} X^\eta_{j,t} Z^\eta_{j,t} (Q^S_{j,t})^{1-\eta}. \tag{29}$$

The Bellman equation for the social planner is

$$SV_{j,t} = \max_{I_{j,t}} \quad \{SS_{j,t} - I_{j,t} + \mathrm{E}_t[M_{t,t+1} SV_{j,t+1}]\} \tag{30}$$

$$\text{s.t.} \quad I_{j,t} \geq 0. \tag{31}$$

Then, the first-order condition with respect to marginal changes in investment at time t is

$$1-\mu_{j,t} = \mathrm{E}_t\left[M_{t+1}\left(\frac{\partial SS_{j,t+1}}{\partial K_{j,t+1}} + (1-\delta)(1-\mu_{j,t+1})\right)\right]. \tag{32}$$

The two first-order conditions with respect to $K_{j,t+1}$ are identical because

$$\frac{\partial SS_{j,t+1}}{\partial K_{j,t+1}} = P_{j,t+1} \tag{33}$$

by design. Therefore, we can solve for the optimal investment by solving the equivalent social planner's problem.

I detrend the Bellman equation in Section A.2 and use value function iteration to solve for the optimal investment and spot price dynamics. The details of the numerical procedure are discussed in Section A.3.

5. Calibration

I calibrate my model at a monthly frequency with parameters from either existing literature or by matching the key moments of commodity futures.

The choices of parameters are reported in Table 6. The monthly capital depreciation rate is set as $\delta = 0.01$ from the empirical estimation in Cooper and Haltiwanger (2006). The literature offers a wide choice of price elasticity of demand for many types of goods. Caballero and Pindyck (1996) and Zhang (2005) choose $\eta = 0.5$ for consumption goods. Carlson, Khokher, and Titman (2007) set $\eta = 1$ for crude oil. KLY calibrate oil futures to get $\eta = 3.15$ using the simulated method of moments. I pick $\eta = 0.5$ to capture the aggregate price elasticity of commodity demand. Justiniano et al. (2010) estimate the monthly persistence of investment shocks $\rho_y = 0.9$ with a Bayesian approach. Papanikolaou (2011) assumes that investment shocks follow a random walk process. Hence, he implicitly sets $\rho_y = 1$. I set $\rho_y = 0.96$, which is in between the above two values, in the benchmark case. I follow the estimates from Justiniano et al. (2010) to set the monthly volatility of investment shocks to $\sigma_y = 0.04$. The volatility of the aggregate demand shock $\sigma_x = 0.08$ is chosen to match the commodity futures market volatility. ρ_z is set as 0.99 to capture the high persistency of idiosyncratic commodity demand. The volatility of the idiosyncratic demand shock $\sigma_z = 0.23$ is chosen to match portfolio volatility. I set the

Table 6
Parameter values of the benchmark model.
This table reports the calibrated parameter values of the benchmark model.

Parameter	Value	Description
Technology		
δ	0.01	Capital depreciation rate
η	0.5	Aggregate price elasticity of commodity demand
Exogenous shock		
g_x	0.083	Growth rate of aggregate demand (%)
σ_x	0.08	Volatility of aggregate demand shocks
ρ_y	0.96	Persistence of investment shocks
σ_y	0.04	Conditional volatility of investment shocks
ρ_z	0.99	Persistence of idiosyncratic demand shocks
$\overline{z}$	−7.5	Long-run mean of idiosyncratic demand shocks
σ_z	0.23	Conditional volatility of idiosyncratic demand shocks
Stochastic discount factor		
r_f	0.208	Real risk-free rate (%)
γ_x	0.6	Risk price of aggregate demand shocks
γ_y	−7	Risk price of investment shocks

monthly aggregate demand growth rate to be the same as the growth rate of aggregate productivity $g_x = 1\%/12 = 0.083\%$ and the monthly risk-free rate to be $r_f = 2.5\%/12$ from Kogan and Papanikolaou (forthcoming-b).[12] I set $\bar{z} = -7.5$ so that the long-term average capital stock is about 1.

Finally, the risk price of investment shocks (γ_y) is a key parameter in generating the basis spread. The choice of γ_y in the benchmark model is close to the value suggested by Kogan, Papanikolaou, and Stoffman (2012). I set the prices of risk as $\gamma_x = 0.6$ and $\gamma_y = -7$ to jointly match the commodity futures market premium and the average return of the HML portfolio. Kogan, Papanikolaou, and Stoffman (2012) solve a general equilibrium featuring heterogeneous agents and find that the risk price of investment shocks implied by the model is about $\mathrm{E}[\gamma_\xi(\omega)]/\sigma_\xi = -0.8/0.125 = -6.4$ where $\sigma_\xi = 0.125$ represents the annual volatility of investment shocks.[13]

6. Model solution

6.1. Policy functions

I solve the model using a value function iteration. The major policy functions are reported in Fig. 2. After detrending, the state variables of the model reduce to investment shocks (Y_t), idiosyncratic demand shocks ($Z_{j,t}$), and the detrended capital ($\hat{K}_t$). The policies such as basis, investment rate, futures expected return, and futures betas are functions of the model parameters and these three state variables. In these figures, I set Y_t to be its long-run mean. Two series of policies are plotted as functions of the detrended capital. One is associated with high demand commodities, whereas the other is associated with low demand commodities.

The first key relationship that we can observe from the two upper panels is that basis is increasing in investment. The upper left panel reports basis. It is decreasing in capital and idiosyncratic demand shocks. The upper right panel reports the optimal investment rate. Inherited from the standard investment-based model, the investment rate is decreasing in capital. High idiosyncratic demand shocks induce more investment from producers because of their persistency. By comparing the two upper panels in Fig. 2, we can see that high basis characterizes the commodities with high idiosyncratic demand and low producers'capital. And these commodities are exactly the output of producers with heavy investment.

The second crucial relationship is that the futures expected return is positively related to investment. The lower left panel reports the expected excess return of longing 12-month futures for one month. Longing futures with other maturities for one month has an expected excess return of the same shape. The lower right panel reports the futures risk exposure beta to investment shocks (see Eq. (26) for definition). Futures β_y are mostly negative, which are consistent with the futures risk exposure to investment shocks implied by the spot price dynamics (see Eq. (27)). Because of the negative risk price of investment shocks, expected futures excess returns are mostly positive. Furthermore, the shapes of the expected futures return and futures β_y are very similar. Hence, the futures expected returns are mainly driven by β_y. In the cross section of commodities, high idiosyncratic demand and low capital commodities are more negatively correlated with investment shocks and hence offer higher futures returns. By comparing the lower panels with the upper panels, we can see that these commodities are associated with high investment in producers' physical capital and high basis. These cross-sectional relationships can be clearly observed from these policy functions.

6.2. Impulse response functions

Fig. 3 reports a cross-sectional comparison of the major impulse response functions of a low demand commodity to a negative investment shock with the functions of a high demand commodity to the same investment shock.

The two upper panels report the impulse response of the basis and the producers' investment rate to shocks. These functions show that a positive idiosyncratic demand shock raises the basis of the commodity and induces producers to invest heavily. In the meantime, those commodities with low idiosyncratic demand shocks have futures curves with a low basis, and their producers invest less.

The middle and lower four panels report the impulse response functions of short-term futures expected returns (two-month $\mathrm{E}[R^e]$), long-term futures expected returns (12-month $\mathrm{E}[R^e]$), short-term futures risk exposure to investment shocks (two-month β_y), and long-term futures risk exposure to investment shocks (12-month β_y). These panels show that high demand commodities are more negatively correlated with investment shocks (low β_y) and hence offer higher futures expected returns compared with low demand commodities. Along the maturity dimension, short-term futures are less negatively correlated with investment shocks (less negative β_y) and hence associated with less futures expected returns.

In summary, these figures clearly show the investment channel through which the model can explain the basis spread.

7. Simulation results

I use a Monte Carlo simulation to solve for the important moments predicted by the model. To neutralize the effect of initial conditions, I first simulate 35 commodities for 1,000 years. Then I take the end values of the cross

[12] The choice of μ_x does not affect the futures risk premium because it does not impact the correlation between futures returns and shocks.

[13] There is some difference in notation between Kogan, Papanikolaou, and Stoffman (2012) and this paper. In their paper, the SDF is written as $\frac{d\pi_t}{\pi_t} = \cdots - \gamma_\xi(\omega_t)\, dB_t^\xi$ where dB_t^ξ denotes a standard Brownian motion (volatility per unit of time$=1$) for investment shocks. Their model generates an average risk price of investment shocks over the stationary distribution of the state variable $\mathrm{E}[\gamma_\xi(\omega_t)] = -0.8$. In my paper, I specify the stochastic discount factor as $\log M_{t+1} = \cdots - \gamma_y(\log Y_{t+1} - \log Y_t)$ where the standard deviation of $(\log Y_{t+1} - \log Y_t)$ is the volatility of investment shocks. Therefore, γ_y in my paper is equivalent to $\mathrm{E}[\gamma_\xi(\omega)]/\sigma_\xi = -0.8/0.125 = -6.4$ in Kogan et al. (2012).

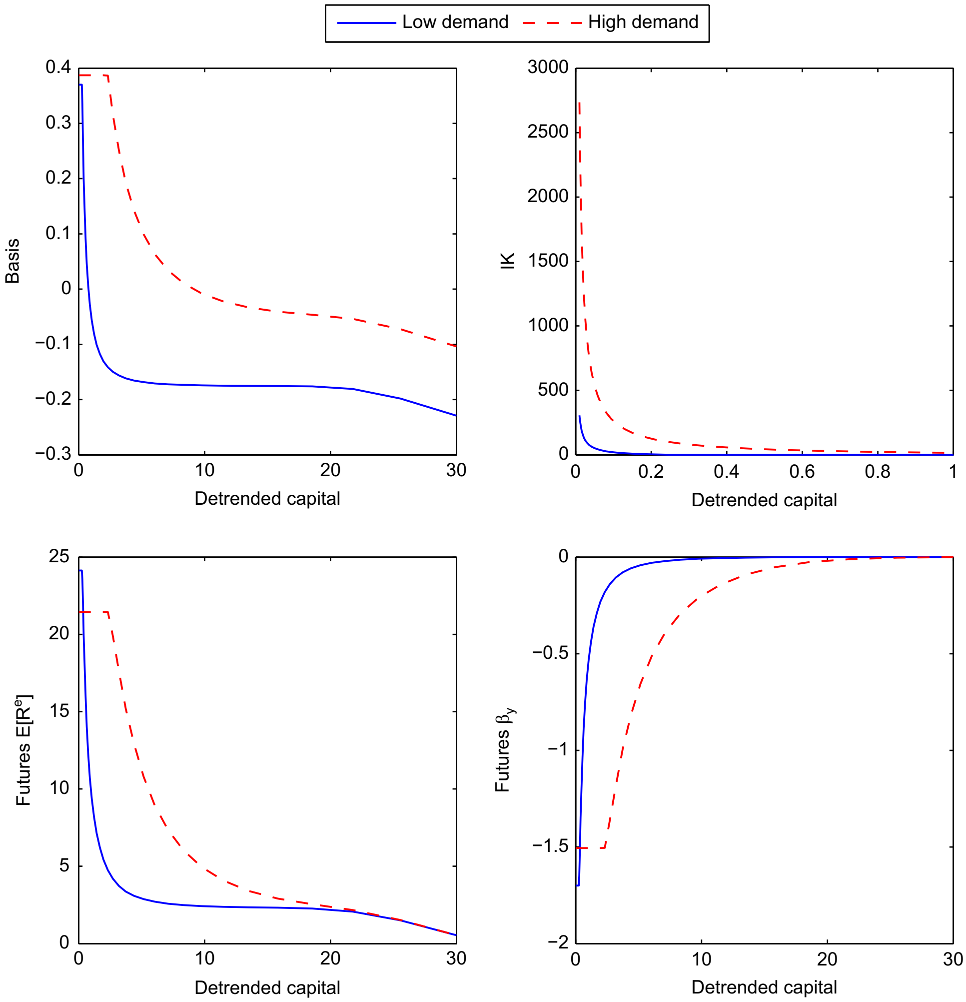

Fig. 2. Representative policy functions of basis, investment rate, expected futures excess return, and risk exposures (β_y) of the benchmark model. The basis is computed the same way as in the data. Expected futures excess return and risk exposure are for longing 12-month futures for one month. The solid curves represent the functions for low-demand commodities. The dashed curves represent the functions for high-demand commodities. Policy functions are annualized.

section of state variables from this simulation as the initial states of the economy. Using these initial states, I create 1,000 artificial panels, each of which includes monthly observations of 39 years of futures prices with maturities up to 12 months of 35 commodities. The same empirical analysis is applied to these simulated panel data to directly compare them with the results from the real ones. Key statistics (e.g., estimates and *t*-stats) are averages of the corresponding ones across these panels.

Panel A in Table 7 reports several important aggregate moments. The benchmark model replicates the commodity market average excess returns ($E[R^e] = 6.25\%$) and volatility ($\sigma[R^e] = 13.60\%$) very well. The model can also generate reasonably well time-series of aggregate basis which is defined as the equally weighted average of basis across all commodities in the sample. The model predicts average aggregate basis of −0.44% with volatility of 21.56%. These moments are very close to data.

7.1. Model-simulated basis-sorted portfolios

In this section, I investigate the model implications of the basis spread. Based on the 1,000 panels of simulated data, I sort the 35 commodities of each panel into seven portfolios by basis following the same procedure as in the empirical section. Then, I aggregate the simulated futures returns across commodities and maturities within each portfolio by taking equally weighted averages. The key moments of these portfolio returns are summarized in Panel B of Table 7.

The benchmark model reproduces the basis spread (10%) quantitatively. Panel B in Table 7 compares the average excess returns ($E[R^e]$), volatilities ($\sigma[R^e]$), and basis of these portfolios formed with both historical and simulated data. The portfolio average excess return is monotonically increasing in basis, and longing the high-basis portfolio earns a return that is about 10% higher than the low-basis

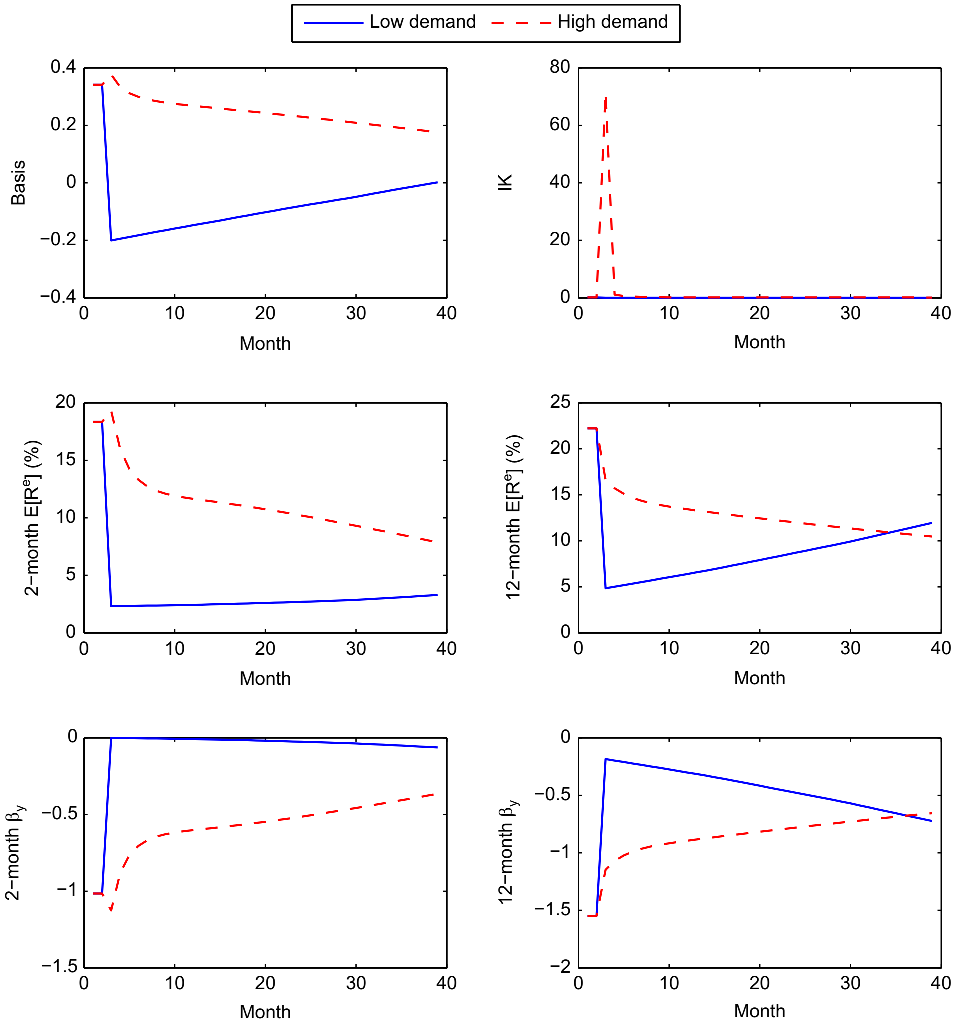

Fig. 3. Impulse response functions to idiosyncratic demand and investment shocks of the benchmark model. The solid curves represent the functions for low-demand commodities. The dashed curves represent the functions for high-demand commodities. Impulse response functions are annualized.

portfolio. The benchmark model reproduces these key patterns and matches the average and volatility of portfolio returns and portfolio-level basis quite well.

7.2. Model-simulated asset pricing tests

The model also replicates the failure of the commodity CAPM and the success of two-factor models in explaining the portfolios sorted in basis.

Table 8 compares the time-series test results of three factor models using the benchmark model simulated data with the test results of historical data. The left panel reports the estimates and statistics from historical data and the right panel reports the ones from simulated data. First, the benchmark model replicates the failure of the commodity CAPM. The commodity market factor is denoted by Mkt_C. The abnormal return (Alpha) of the HML portfolio (about 10.97%) in simulation is larger than 7.73% in historical data. Statistically, the abnormal return in the simulated data is also significant with a Newey-West corrected *t*-stat equal to 2.64. The simulated data reject the commodity CAPM as well. Second, the benchmark model reproduces the success of the two-factor model using the HML_C factor as a second factor. Alphas of all seven portfolios are close to zero and statistically insignificant. More importantly, in simulated data, the portfolio loading on the HML_C factor is also monotonically increasing in basis and the loadings on the market factor are quite flat across portfolios. These patterns are important empirical evidence that the two factors can capture the major comovements of these portfolio returns. And the benchmark model can replicate these patterns fairly well. Third, the benchmark model regenerates major test results of the second two-factor model using the commodity market factor and investment shocks. The simulated portfolio beta to investment shocks is decreasing in basis, the same as in real data. In particular, the simulated HML portfolio significantly negatively loads on investment shocks ($\beta_I = -0.73$ with t-stat$=-2.84$) whereas $\beta_I = -1.19$ in real data.

Table 9 compares the cross-sectional regression results of these three factor models. The left panel reports the

Table 7
Model-simulated moments of commodity futures market and basis-sorted portfolios.
Panel A compares the commodity market expected excess return (E[R^e]), volatility ($\sigma[R^e]$), and mean and volatility of aggregate basis in both historical and simulated data. Panel B compares the average excess returns (E[R^e]), volatilities ($\sigma[R^e]$), and basis of the seven portfolios sorted by basis and the HML portfolio from historical and model-simulated data.

Panel A: Commodity market

	Data	Benchmark
E[R^e]	6.25	6.43
$\sigma[R^e]$	13.60	13.66
Agg. basis	−0.74	−0.44
Vol. of agg. basis	19.07	21.56

Panel B: Basis-sorted portfolios

	Data			Benchmark		
Portfolio	E[R^e]	$\sigma[R^e]$	Basis	E[R^e]	$\sigma[R^e]$	Basis
Low	1.83	20.67	−18.59	2.76	23.50	−21.77
2	2.98	18.99	−10.79	3.14	21.81	−13.20
3	2.28	17.02	−6.57	4.74	20.53	−6.75
4	5.06	19.01	−3.03	5.74	19.39	−0.43
5	6.08	17.23	1.26	6.84	18.44	5.90
6	8.92	18.20	7.50	9.13	18.11	12.12
High	11.25	21.90	19.38	12.70	18.47	21.06
HML	9.42	27.12		9.94	26.42	

estimates and statistics from historical data and the right panel reports the ones from simulated data. In historical data, the estimated factor risk premium for the commodity market factor (λ_{Mkt_C}) is about 6% per annum and the factor risk premium for HML_C (λ_{HML_C}) is about 9.84% per annum. In the simulated data, the estimates of factor risk premium of the commodity market factor are 6.23% for the commodity CAPM. In the simulated data, the estimated factor risk premiums of the market factor and HML_C are 6.38% and 9.4%, which are also close to the average excess returns of these factors as reported in the right panel (E[R^e] = 6.43% for the market factor and E[R^e] = 9.94% for HML_C). The no-arbitrage conditions also hold among these portfolios in simulated data, the same as that found in historical data. The estimated factor risk premium of investment shocks in simulated data is negative, which shares the same sign as in the real data.

In addition, same as in historical data, the cross-sectional statistics such as R^2, pricing errors (RMSE), and Shanken-corrected p-value of χ^2-tests favor two-factor models over the commodity CAPM in simulated data.

7.3. Sensitivity of the model-implied basis spread

I study the two major ingredients which drive the basis spread in the benchmark model. The basis spread can be decomposed into the price of risk and the amount of risk. The price of risk is mainly controlled by the risk price of investment shocks (γ_y). The amount of risk is determined by the friction of producers' investment in physical capital.

I calibrate several important alternative specifications of the model to inspect the underlying economic mechanism of the model in generating the basis spread and the amount of risk. The model-implied amount of risk is measured by the portfolio betas to investment shocks estimated using time-series regressions while controlling the commodity market factor. In each specification, I change one parameter of the benchmark model while maintaining the others in order to study the changes in the excess returns of the simulated portfolios sorted by basis. Table 10 compares the average excess returns, volatilities, and the betas to investment shocks of seven portfolios sorted by basis across these calibrations.

First, I investigate how γ_y impacts the basis spread. Panel A of Table 10 reports two alternative specifications of the risk price of investment shocks (γ_y). In the benchmark model, the risk price of investment shocks is assumed to be negative, following Papanikolaou (2011). As a comparison, I solve the model with two other specifications of γ_y.

- *Specification* 1: I set $\gamma_y = 0$ so that investment shocks are not priced in this case. The model-implied amount of risk measured as the beta of the HML portfolio to investment shocks is −0.71 which is very close to the benchmark case ($\beta_I = -0.73$). However, the model-predicted basis spread (average excess return of the HML portfolio) is −0.77% which is close to zero.
- *Specification* 2: I set $\gamma_y = 7$ so that investment shocks are positively priced. The model predicts a negative basis spread (about −5.48% per annum) in contrast to the positive 9.42% basis spread in the data. The model-implied spread in beta to investment shocks is −0.47, which is relatively close to the benchmark case.

In summary, the risk price of investment shocks (γ_y) drives the basis spread but not the amount of risk (β_I) too much. In particular, a negative price of risk implies a positive basis spread. The assumption of a negative risk price of investment shocks is consistent with Papanikolaou

Table 8
Model-simulated time-series regression tests.

This table compares the time-series tests of three factor models in both historical and model-simulated data. These three factor models include the commodity CAPM, the two-factor model with the excess return of the high-basis relative to the low-basis portfolio (HML_C) as the second factor, and the two-factor model with the relative price of investment goods (*I*-shock) as the second factor. The *t*-stats are adjusted using a Newey-West correction. The alphas are annualized and reported in percentages.

	Data								Benchmark							
	Low	2	3	4	5	6	High	HML	Low	2	3	4	5	6	High	HML
Commodity CAPM																
Alpha	−3.14	−2.97	−3.09	−1.69	0.45	2.75	4.59	7.73	−4.16	−3.53	−1.78	−0.71	0.48	2.89	6.81	10.97
(*t*-stat)	(−1.23)	(−1.42)	(−1.55)	(−0.95)	(0.23)	(1.46)	(1.86)	(1.98)	(−1.51)	(−1.42)	(−0.77)	(−0.34)	(0.24)	(1.45)	(3.00)	(2.64)
Mkt_C	0.79	0.95	0.86	1.08	0.90	0.99	1.06	0.27	1.08	1.05	1.03	1.01	0.98	0.96	0.89	−0.19
(*t*-stat)	(7.43)	(15.59)	(15.00)	(13.69)	(14.09)	(13.98)	(9.92)	(1.45)	(14.92)	(16.72)	(18.56)	(20.27)	(21.54)	(20.31)	(16.25)	(−1.81)
R^2 (%)	27.29	46.41	47.16	59.67	50.40	54.31	43.61	1.83	39.71	43.25	46.64	50.57	53.09	52.05	43.79	1.43
Commodity Mkt and HML																
Alpha	0.94	−2.34	−2.92	−1.55	0.41	2.70	0.94		2.44	−3.08	−1.69	−1.04	−0.39	1.32	2.44	
(*t*-stat)	(0.57)	(−1.10)	(−1.43)	(−0.85)	(0.21)	(1.43)	(0.57)		(1.53)	(−1.22)	(−0.72)	(−0.49)	(−0.19)	(0.69)	(1.53)	
Mkt_C	0.94	0.97	0.87	1.08	0.90	0.98	0.94		0.97	1.04	1.03	1.01	1.00	0.98	0.97	
(*t*-stat)	(17.80)	(16.89)	(13.84)	(13.33)	(13.72)	(14.66)	(17.80)		(28.53)	(16.69)	(18.49)	(20.39)	(22.37)	(22.72)	(28.53)	
HML_C	−0.53	−0.08	−0.02	−0.02	0.00	0.01	0.47		−0.60	−0.04	−0.01	0.03	0.08	0.14	0.40	
(*t*-stat)	(−15.64)	(−3.04)	(−0.77)	(−0.53)	(0.18)	(0.19)	(13.99)		(−24.20)	(−1.28)	(−0.21)	(1.27)	(3.56)	(6.37)	(15.95)	
R^2 (%)	74.37	47.75	47.28	59.74	50.41	54.32	77.17		85.08	43.75	46.80	50.84	54.53	56.51	75.68	
Commodity Mkt and I-shock																
Alpha	−1.83	−1.89	−3.36	−2.52	0.65	3.36	4.43	6.26	−4.24	−3.92	−2.00	−0.71	0.37	3.16	7.33	11.57
(*t*-stat)	(−0.68)	(−0.87)	(−1.15)	(−1.69)	(0.42)	(5.02)	(2.15)	(2.19)	(−1.75)	(−1.68)	(−0.91)	(−0.40)	(0.26)	(1.79)	(3.34)	(3.00)
Mkt_C	0.60	0.78	0.91	1.22	0.86	0.87	1.09	0.49	1.10	1.12	1.08	1.01	0.99	0.92	0.78	−0.32
(*t*-stat)	(3.06)	(6.12)	(8.95)	(11.46)	(8.04)	(9.53)	(7.11)	(1.76)	(6.62)	(7.55)	(8.17)	(8.23)	(8.90)	(8.20)	(6.55)	(−1.38)
I−shock	1.12	0.46	0.37	0.64	−0.57	−1.46	−0.07	−1.19	0.26	0.23	0.16	0.06	−0.07	−0.17	−0.48	−0.73
(*t*-stat)	(2.11)	(0.73)	(0.46)	(1.39)	(−1.06)	(−2.03)	(−0.11)	(−1.29)	(1.42)	(1.49)	(1.15)	(0.58)	(−0.57)	(−1.30)	(−3.69)	(−2.84)
R^2 (%)	29.45	42.96	47.08	72.46	57.34	56.92	47.87	8.21	42.80	47.87	50.17	52.57	56.77	57.50	54.68	16.34

Table 9
Model-simulated cross-sectional regression tests.

This table compares the time-series tests of three factor models in both historical and model-simulated data. These three factor models include the commodity CAPM, the two-factor model with the excess return of the high-basis relative to the low-basis portfolio (HML_C) as the second factor, and the two-factor model with the relative price of investment goods (I-shock) as the second factor. The factor risk premiums are estimated with two-stage regressions (Cochrane, 2005). I do not include a constant in the cross-sectional regression. The p-values are for the χ^2−test in the cross section. The p-values and t-stats in the cross-sectional tests are adjusted using a Shanken (1992) correction.

	Data						Benchmark					
	λ_{Mkt_C}	λ_{HML}	λ_I	R^2	RMSE	p-Value (χ^2 test)	λ_{Mkt_C}	λ_{HML}	λ_I	R^2	RMSE	p-Value (χ^2 test)
Commodity CAPM												
Est.	5.97			21.15	2.91	26.97	6.23			−12.29	4.15	11.80
(t-stat)	(2.59)						(2.87)					
E[R^e]	6.25						6.43					
Commodity Mkt and HML												
Est.	6.00	9.84		66.73	1.89	49.61	6.38	9.40		47.75	2.76	27.53
(t-stat)	(2.60)	(2.33)					(2.94)	(2.20)				
E[R^e]	6.25	9.42					6.43	9.94				
Commodity Mkt and I-shock												
Est.	6.33		−2.61	64.16	1.97	85.47	6.24		−11.93	41.40	2.81	40.49
(t-stat)	(2.33)		(−2.07)				(3.10)		(−1.69)			

(2011) and Kogan, Papanikolaou, and Stoffman (2012), and the estimates from historical commodity futures data in the previous section. The risk price of investment shocks in the benchmark model ($\gamma_y = -7$) is close to the average price of risk of investment shocks ($\gamma_y = -6.4$) implied by the general equilibrium model with reasonable risk aversion in Kogan, Papanikolaou, and Stoffman (2012).

Second, the amount of risk for the basis spread is mainly driven by the investment friction. In order to investigate this effect quantitatively, I substitute the investment irreversibility constraint in the benchmark model with quadratic capital adjustment costs. The function of capital adjustment costs is specified as

$$S(I_{j,t}, K_{j,t}) = \frac{\theta(I_{j,t})}{2} \left(\frac{I_{j,t}}{K_{j,t}}\right)^2 K_{j,t}, \tag{34}$$

where

$$\theta(I_{j,t}) = \begin{cases} \theta_1, & I_{j,t} \geq 0 \\ \theta_2, & I_{j,t} < 0. \end{cases} \tag{35}$$

Panel B of Table 10 reports two alterative specifications of capital adjustment costs. These include the following:

- *Specification* 3: Following Zhang (2005), I specify asymmetric capital adjustment costs with ($\theta_1/\theta_2 = 15/150$). The model can only generate a very small basis spread 1.17% even with $\gamma_y = -7$. This is because the model can only generate $\beta_I = -0.17$ for the HML portfolio. This β_I is much smaller than the benchmark case.
- *Specification* 4: I impose symmetric adjustment costs by setting $\theta_1 = \theta_2 = 15$. In this case, the quadratic capital adjustment costs are symmetric around zero investment. β_I of the HML portfolio reduces further to −0.12. Hence, the basis spread decreases to 0.45% with $\gamma_y = -7$.

From these alternative specifications of the model, we learn that the specification of capital adjustment costs affects the quantity of risk for the basis spread. More asymmetric adjustment costs predict a larger β_I for the HML portfolio and hence a larger basis spread. This is because the shape of capital adjustment costs affects the cross-sectional distribution of futures betas to investment shocks.

8. Conclusion

This paper provides an empirical analysis of the cross-sectional commodity futures returns and then proposes a theoretical basis for the underlying macroeconomic risks that justify these cross-sectional returns. Empirically, I sort commodities into portfolios by basis and introduce a two-factor model to summarize the cross-sectional average returns of these portfolios. The two-factor model passes many empirical tests. Furthermore, I find that the cross-sectional return difference (the basis spread) is negatively correlated with investment shocks. With the negative risk price of investment shocks documented in Papanikolaou (2011), this finding suggests that the basis spread reflects a risk compensation to exposure to investment shocks.

I construct a dynamic equilibrium model extending KLY to replicate this negative correlation as well as the positive basis spread under the assumption of the negative risk price of investment shocks. The model also replicates the average returns of the seven portfolios sorted by basis and several asset pricing test results. Thus, this paper provides a potential macroeconomic risk-based explanation of the basis spread in contrast to the classic hedging pressure theory of the commodity futures premium (Keynes, 1923; and others).

Table 10
Sensitivity analysis: data- and model-simulated basis spreads across alternative calibrations.

This table compares the first two moments of the model-simulated futures excess returns and betas to investment shocks of commodity portfolios sorted by basis under alternative specifications. β_I denotes the portfolio return beta to investment shocks. Panel A reports the simulated results with alternative risk price of investment shocks (γ_y). Panel B reports the simulated results with alternative friction in producers' real investment.

Portfolio	Low	2	3	4	5	6	High	HML
Data								
$E[R^e]$	1.83	2.98	2.28	5.06	6.08	8.92	11.25	9.42
$\sigma[R^e]$	20.67	18.99	17.02	19.01	17.23	18.20	21.90	27.12
β_I	1.12	0.46	0.37	0.64	−0.57	−1.46	−0.07	−1.19
Benchmark								
$E[R^e]$	2.76	3.14	4.74	5.74	6.84	9.13	12.70	9.94
$\sigma[R^e]$	23.50	21.81	20.53	19.39	18.44	18.11	18.47	26.42
β_I	0.26	0.23	0.16	0.06	−0.07	−0.17	−0.48	−0.73
Panel A: Alternative risk price of investment shocks								
1. $\gamma_y=0$								
$E[R^e]$	1.99	1.84	2.17	1.61	1.34	0.79	1.21	−0.77
$\sigma[R^e]$	23.44	21.81	20.29	19.43	18.42	17.95	17.33	25.53
β_I	0.33	0.24	0.09	0.02	−0.08	−0.21	−0.38	−0.71
2. $\gamma_y=7$								
$E[R^e]$	2.02	0.72	−0.27	−1.80	−3.14	−2.94	−3.46	−5.48
$\sigma[R^e]$	23.58	21.71	20.60	19.51	18.76	18.03	16.67	25.31
β_I	0.28	0.19	0.10	−0.04	−0.15	−0.19	−0.18	−0.47
Panel B: Alternative specifications with capital adj. costs								
3. Asymmetric quadratic costs ($\theta_1/\theta_2=15/150$)								
$E[R^e]$	3.80	3.76	3.98	4.07	4.12	4.55	4.97	1.17
$\sigma[R^e]$	23.55	22.09	21.47	20.68	20.11	19.44	18.57	24.43
β_I	0.09	0.07	0.07	−0.05	−0.06	−0.03	−0.09	−0.17
4. Symmetric quadratic costs ($\theta_1/\theta_2=15/15$)								
$E[R^e]$	4.59	4.21	4.59	4.80	4.63	5.10	5.04	0.45
$\sigma[R^e]$	22.48	21.38	20.85	20.43	19.92	19.28	18.47	23.58
β_I	0.06	0.07	0.04	−0.02	−0.05	−0.01	−0.07	−0.12

The rationality of other cross-sectional return spreads in commodity futures such as the momentum effect (Erb and Harvey, 2006; GHR, 2013) and the "value premium" (Asness, Moskowitz, and Pedersen, 2013) would also be interesting topics to investigate in the future.

Appendix A

A.1. Derivation of the spot price dynamics

I derive the spot price dynamics (Eq. (27)) in this section. First, we can take the log of the inverse demand function (Eq. (15)) as

$$\log P_{j,t}=\eta\log X_t+\eta\log Z_{j,t}-\eta\log Q^D_{j,t}. \tag{36}$$

From the market-clearing condition ($Q^D_{j,t}=Q^S_{j,t}$) and the production technology ($Q^S_{j,t}=K_{j,t}$), we observe that the quantity of commodity j is determined by its producers' capital as

$$Q^D_{j,t}=Q^S_{j,t}=K_{j,t}. \tag{37}$$

We can take the difference of the log inverse demand function (Eq. (36)) over time as

$$\log P_{j,t+1}=\log P_{j,t}+\eta(\log X_{t+1}-\log X_t)$$
$$+\eta(\log Z_{j,t+1}-\log Z_{j,t})-\eta\log\left(\frac{K_{j,t+1}}{K_{j,t}}\right). \tag{38}$$

Furthermore, the dynamics of exogenous shocks (Eqs. (17) and (18)) and the dynamics of capital given the optimal investment rate ($i^*_{j,t}=I^*_{j,t}/K_{j,t}$) can be rewritten as

$$\log X_{t+1}-\log X_t=g_x+\sigma_x e_{t+1} \tag{39}$$

$$\log Z_{j,t+1}-\log Z_{j,t}=(1-\rho_z)(\overline{z}-\log Z_{j,t})+\sigma_z\epsilon_{j,t+1} \tag{40}$$

$$\frac{K_{j,t+1}}{K_{j,t}}=1-\delta+Y_t i^*_{j,t}. \tag{41}$$

Plugging these equations into Eq. (38), we can derive the log spot price dynamics

$$\log P_{j,t+1}=\log P_{j,t}-\eta\log(1-\delta+Y_t i^*_{j,t})+\eta g_x$$
$$+\eta(1-\rho_z)(\overline{z}-\log Z_{j,t})$$
$$+\eta\sigma_x e_{t+1}+\eta\sigma_z\epsilon_{j,t+1}, \tag{42}$$

as in Eq. (27).

A.2. The detrended model

I solve the social planner's problem using a value function iteration. This numerical method only applies to stationary models. The Bellman equation of the social planner is

$$SV_{j,t}=\max_{I_{j,t}}\left\{\frac{1}{1-\eta}X_t^\eta Z_{j,t}^\eta K_{j,t}^{1-\eta}-I_{j,t}+E_t[M_{t,t+1}V_{j,t+1}]\right\} \tag{43}$$

$$\text{s.t.}K_{j,t+1}=(1-\delta)K_{j,t}+Y_tI_{j,t} \tag{44}$$

$$I_{j,t}\geq 0. \tag{45}$$

Because the aggregate demand shocks follow a random walk process, I detrend all variables by letting $\hat{V}_{j,t}=V_{j,t}/X_t$, $\hat{K}_{j,t}=K_{j,t}/X_t$, and $\hat{I}_{j,t}=I_{j,t}/X_t$. The detrended value function of the producer reduces to a function of three state variables, investment shocks (Y_t), idiosyncratic shocks ($Z_{j,t}$), and the detrended capital ($\hat{K}_{j,t}$), as

$$\hat{V}(Y_t,Z_{j,t},\hat{K}_{j,t})=\max_{\hat{I}_{j,t}}\left\{\frac{1}{1-\eta}Z^\eta_{j,t}\hat{K}^{1-\eta}_{j,t}-\hat{I}_{j,t}\right.$$
$$\left.+E_t\left[M_{t+1}\frac{X_{t+1}}{X_t}\hat{V}(Y_{t+1},Z_{j,t},\hat{K}_{j,t+1})\right]\right\}. \tag{46}$$

The law of motion for the detrended capital is

$$\hat{K}_{j,t+1}=\frac{X_t}{X_{t+1}}[(1-\delta)\hat{K}_{j,t}+Y_t\hat{I}_{j,t}]. \tag{47}$$

From the inverse demand function, we can derive the spot price dynamics as

$$P_{j,t} = \left(\frac{Z_{j,t}}{\hat{K}_{j,t}}\right)^{\eta}, \tag{48}$$

which is mean reverting given that the detrended capital is stationary. This is consistent with the empirical fact that commodity spot prices are mean reverting. The state variables of the detrended model are $(Y_t, Z_{j,t}, \hat{K}_{j,t})$.

A.3. Numerical procedure details

The numerical solution of my model involves two steps: a value function iteration and a Monte Carlo simulation. The value function iteration solves for the optimal investment policy of producers $(\hat{I}(Y_t, Z_{j,t}, \hat{K}_{j,t}))$. Because this paper is focused on product price implications, in contrast to papers on stock prices, the value function itself is less important. Given the investment policy function, we can solve for the spot price using the detrended inverse demand function (Eq. (48)). Then, we can compute the futures prices over maturities by taking the expectation in Eq. (23) recursively starting from the boundary condition in Eq. (24). By simulating the futures price, capital, and exogenous shocks jointly, I create 1,000 panels of 35 commodities with futures prices with maturities up to 12 months. The model implications are performed on these panels of simulated data.

The first step of the value function iteration is to discretize the state space of exogenous shocks. The model has three shocks. The log investment shock (log Y_t) and the log idiosyncratic demand shock (log $Z_{j,t}$) are assumed to follow AR(1) processes. I follow previous literature (e.g., Zhang, 2005) in using the Rouwenhorst (1995) method to discretize this shock. Compared to Tauchen (1986), this method does a better job in discretizing highly persistent (ρ close to 1) shocks. The independent and identically distributed (i.i.d.) innovations e_t in the aggregate demand shock are discretized using the same procedure by setting $\rho = 0$. I use 15 grid points for log Y_t, seven grid points for e_t, and nine grid points for log $Z_{j,t}$.

Detrended capital $\hat{K}$ is discretized to be evenly spaced in its log space as $\log \hat{K}_j = \log \hat{K}_{j-1} + (\log \hat{K}_{\max} - \log \hat{K}_{\min})/(N-1)$. The upper bound $\log \hat{K}_{\max}$ and the lower bound $\log \hat{K}_{\min}$ of detrended capital are chosen so that simulated detrended capitals are always between the boundaries. I choose $N_K = 50$ for the coarse capital grid. For every value function iteration, a finer grid of capital is used to search for more precise optimal capital next period $(\hat{K}_{t+1})$ after the search on the coarse grid. I choose $N_K = 3{,}000$ for the finer grid near the optimal next-period capital $(\hat{K}_{t+1})$ on the coarse grid.

Monte Carlo simulations are performed on the grids of exogenous shocks $(e_t, \log Y_t, \log Z_{j,t})$ but off-grid for detrended capital $(\hat{K}_t)$. Interpolation is widely applied in both the value function iteration and the Monte Carlo simulation.

Appendix B. Supplementary material

Supplementary data associated with this article can be found in the online version at http://dx.doi.org.10.1016/j.jfineco.2013.04.012.

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