MINI-PROJECT 2
Mass Transfer for Kidney Dialysis

BMEG 315 - Transport Phenomena in Biological Systems

Spring 2022

Due Date: Wednesday April 27, 2022 by 11:59 PM

Group Members:	Melina McCabe Tatum Peyerl Mauricio Ramirez Andrew Seman Raafay Uqaily

Describe in detail how each group member contributed to this project:

Part 1:
Mauricio Ramirez - Equations
Tatum Peyerl - Code

Part 2:
Andrew Seman - Equations
Melina McCabe - Code

Part 3:
Raafay Uqaily - Discussion

Objectives:

The objective of this project is to delve deeper into mass transport relevant to biological processes. Specifically, you will be investigating different kidney hemodialysis device designs and compare their effectiveness at removing waste products from blood.

Overall Instructions:

Maximum points will be awarded for those answers that demonstrate a clear understanding of the underlying concepts. Some sections may require math, drawings, images, code, plots, etc. For answers that require you to search the literature*, you must reference your sources within your answer and include them in a bibliography at the end of the project using MLA format.
*(*Wikipedia does not count as a source*)

Grading Rubric:

Part 1	12 pts
Part 2	12 pts
Part 3	12 pts
References/Bibliography	3 pts
Formatting and Legibility	3 pts
Individual Contribution to Project	8 pts
Total	50 pts

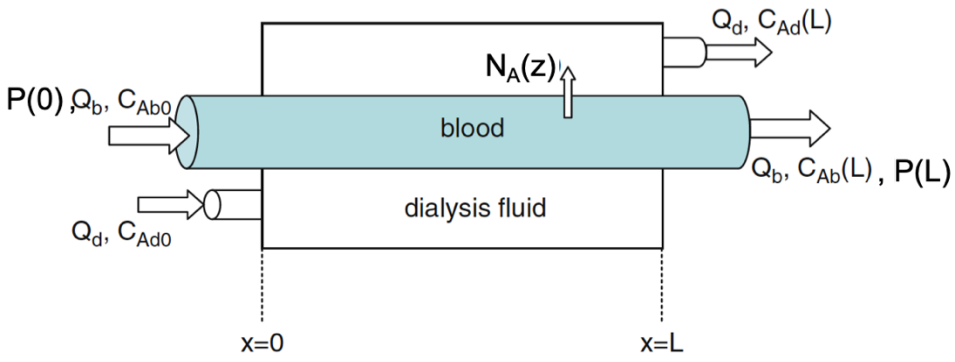
Individual contribution to the project will be assessed using a combination of contributions as detailed on the first page and feedback from group members assessed using a separate anonymous survey.

Introduction: What is a kidney dialysis device?

A kidney dialysis device (often called a hemodialyzer) replaces the function of the kidneys to filter out waste products and extra fluid from the blood. The typical flow rates of blood through the device can range between 150 and 500 mL/min, with the dialysis fluid usually flowing 1.5-2 times quicker. Most devices aim to reduce waste product concentration in the blood by 60% or more upon a single pass through the device.

Part 1: Dialyzer with co-current flow

Your goal is to design a hollow-fiber blood dialysis device (e.g., hemodialyzer) to remove waste products from the bloodstream in patients with poor kidney function. We will model this as shown below. The waste product "A" is initially at a high concentration in the blood but absent in the dialysis fluid. We will assume steady state, ignore radial variations in the concentration of "A" and only describe how "A" varies as a function along the length of the device.



The membrane we will use has a permeability coefficient for waste product "A" of 5.4 x 10^{-2} cm/min and you can assume that $\Phi = 1$. If the length of the fiber is 20 cm and using reasonable values of flow rates and fiber radii as found in Part 1:

1. Select an appropriate fiber radius, dialysis fluid flow rate, and blood flow rate so that at least 40% of the waste product "A" is removed from the blood in a single pass through the hemodialyzer.

$$N_{A,wall}(x) = P\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

$$Q_b = v_{x,b}A \rightarrow v_{x,b} = \frac{Q_b}{\pi R_{wall}^2}$$

$$Q_d = v_{x,d}A \rightarrow v_{x,d} = \frac{Q_b}{\pi R_{tissue}^2}$$

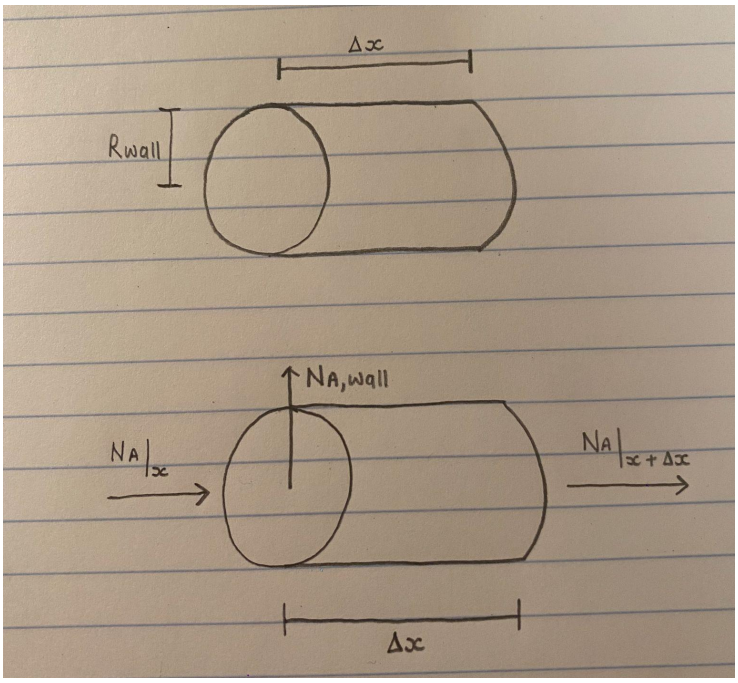
Boundary Conditions:

$$C_{Ab}(x = 0) = C_{Ab0}$$

$$C_{Ad}(x = 0) = 0$$

Blood:

Shell Diagram:



Shell-Balance Method:

$$\left(N_{Ab}|_x - N_{Ab}|_{x+\Delta x}\right)\pi R_b^2 - N_{A,wall}(x)\left(2\pi R_{wall}\Delta x\right) = 0$$

[Divide by volume = $\pi R_b^2 \Delta x$]

$$\frac{\left(N_{Ab}|_x - N_{Ab}|_{x+\Delta x}\right)}{\Delta x} - \frac{N_{A,wall}(x)}{R_{wall}} = 0$$

[Take limit as $\Delta x \to 0$]

$$0 = -\frac{dN_{Ab}}{dx} - \frac{2N_{A,wall}}{R}$$

$N_{Ab} = convection + diffusion$ (where diffusion is ignored)

$$N_{Ab} = C_A v_{x,b}$$

$$N_{Ab} = C_A \frac{Q_b}{\pi R_{wall}^2}$$

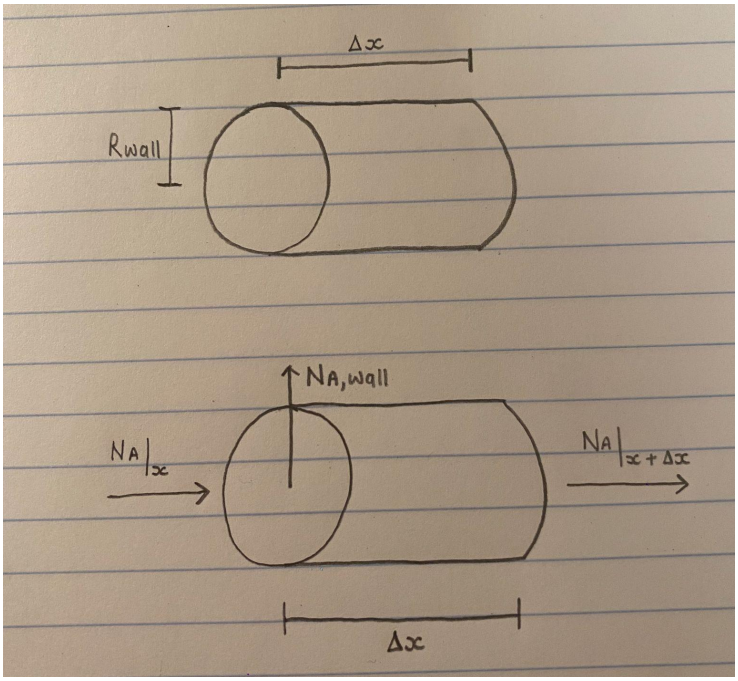
$$0 = -\frac{d}{dx}\left(C_{Ab}\frac{Q_b}{\pi R_{wall}^{\ 2}}\right) - \frac{2}{R}P\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

$$\frac{dC_{Ab}}{dx} + \frac{2\pi R_{wall}}{Q_b}P\left(C_{Ab}(x) - \phi C_{Ad}(x)\right) = 0$$

$$\frac{dC_{Ab}}{dx} = -\frac{2\pi PR_{wall}}{Q_b}\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

Dialysis Fluid:

Shell Diagram:



Shell-Balance Method:

$$\left(N_{Ad}|_x - N_{Ad}|_{x+\Delta x}\right)\pi R_b^2 + N_{A,wall}(x)\left(2\pi R_b \Delta x\right) = 0$$

[Divide by volume = $\pi R_b^2 \Delta x$]

$$\frac{\left(N_{Ad}|_x - N_{Ad}|_{x+\Delta x}\right)}{\Delta x} + \frac{2N_{A,wall}(x)}{R_{wall}} = 0$$

[Take limit as $\Delta x \to 0$]

$$0 = -\frac{dN_{Ad}}{dx} + \frac{2N_{A,wall}}{R}$$

$N_{Ad} = convection + diffusion$ (where diffusion is ignored)

$$N_{Ad} = C_A v_{x,b}$$

$$N_{Ad} = C_A \frac{Q_b}{\pi R_{wall}}$$

$$\frac{d}{dx}\left(C_{Ad}\frac{Q_d}{\pi R_{wall}^{\ 2}}\right) - \frac{2}{R}P\left(C_{Ab}(x) - \phi C_{Ad}(x)\right) = 0$$

$$\frac{dC_{Ad}}{dx} - \frac{2\pi R_{wall}}{Q_d}P\left(C_{Ab}(x) - \phi C_{Ad}(x)\right) = 0$$

$$\frac{dC_{Ad}}{dx} = \frac{2\pi P R_{wall}}{Q_d}\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

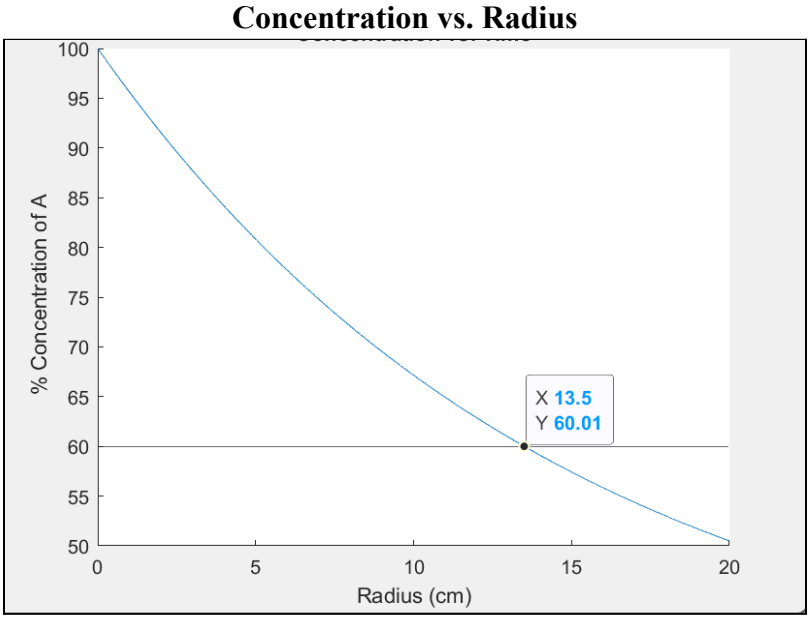
The two differential equations are shown below:

$$\frac{dC_{Ab}}{dx} = -\frac{2\pi P R_{wall}}{Q_b}\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

$$\frac{dC_{Ad}}{dx} = \frac{2\pi P R_{wall}}{Q_d}\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

These equations were solved using dsolve in MATLAB and then plotted for a length of 0cm to 20cm. To achieve this, a blood flow rate of 150 ml/min was used, and the dialysis fluid flow rate was decided to be double this, or 300 mL/min. A lower flow rate for blood and a higher flow rate for the dialysis fluid proved to be the best options for achieving the removal of 40% of the waste product with the lowest radius. These values were experimented with and results showed that higher blood flow rates and lower dialysis fluid flow rates increased the length of the radius needed to remove the waste product. Therefore the lowest value of 150 mL/min and highest value of 300 mL/min were chosen for the flow rate of blood and dialysis fluid respectively. At a fiber length of 20cm, the concentration of A can be seen to be around 50%.

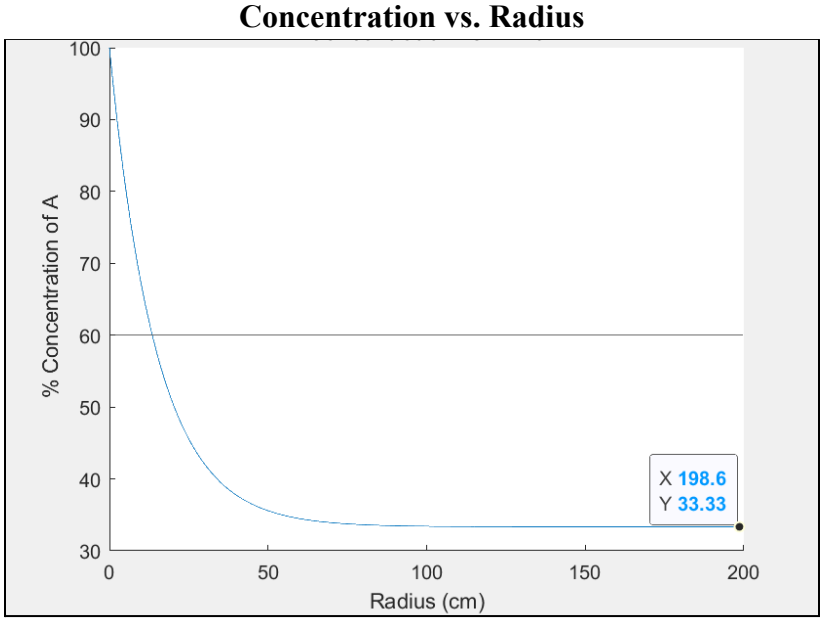
The radius that was found to be effective at removing at least 40% of the waste product was 13.5 cm. This can be shown in the plot below:



2. How low of concentration in the outgoing blood can you achieve using this setup?

For this problem, the radius of the blood vessel was changed to 200 cm in order to observe what would happen to the concentration of the blood as the radius approaches infinity. The flow rates for blood and the dialysis fluid were maintained the same as in the previous question at 150 mL/min and 300 mL/min respectively.

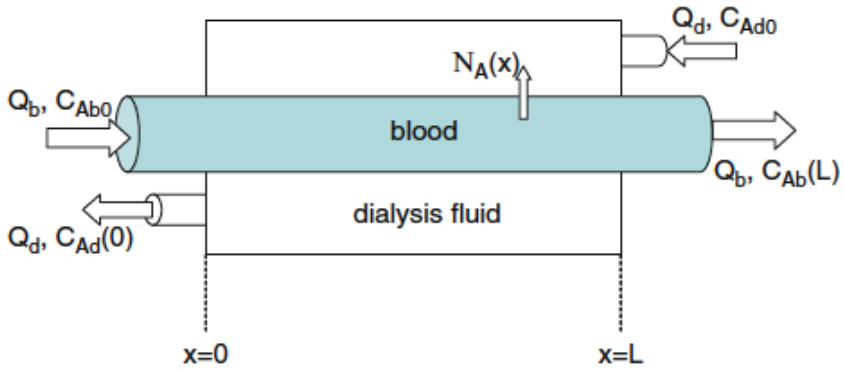
As the radius of the fiber increases, the outgoing blood approaches a waste concentration of 33.33%, meaning 66.67% of the waste concentration has been removed from the blood. This can be seen in the plot below:



Therefore, the lowest concentration of the waste product A in the outgoing blood that can be achieved using this setup is 33.33% of the original concentration. The plot represents the removal of the waste product in a co-current flow. This shows that with co-current flow, where both the dialysis fluid and blood flow in the same direction, it is difficult for the dialyzer to remove a lot of the waste product.

Part 2: Dialyzer with counter-current flow

You may have noticed that it's hard to get the dialyzer to remove a lot of the waste product in the configuration from Part 1 in which both the dialysis fluid and blood flow in the same direction (i.e., co-current flow). To overcome this design flaw, many devices have the fluids flow in opposite directions (i.e., counter-current) as shown below.



Using similar design variables/constants as in Part 1, compare the effectiveness of the counter-current setup to the co-current setup in removing waste product "A" from the blood.

$$N_{A,wall}(x) = P\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

$$Q_b = v_{x,b}A \rightarrow v_{x,b} = \frac{Q_b}{\pi R_{wall}^2}$$

$$Q_d = v_{x,d}A \rightarrow v_{x,d} = \frac{Q_b}{\pi R_{tissuel}^2}$$

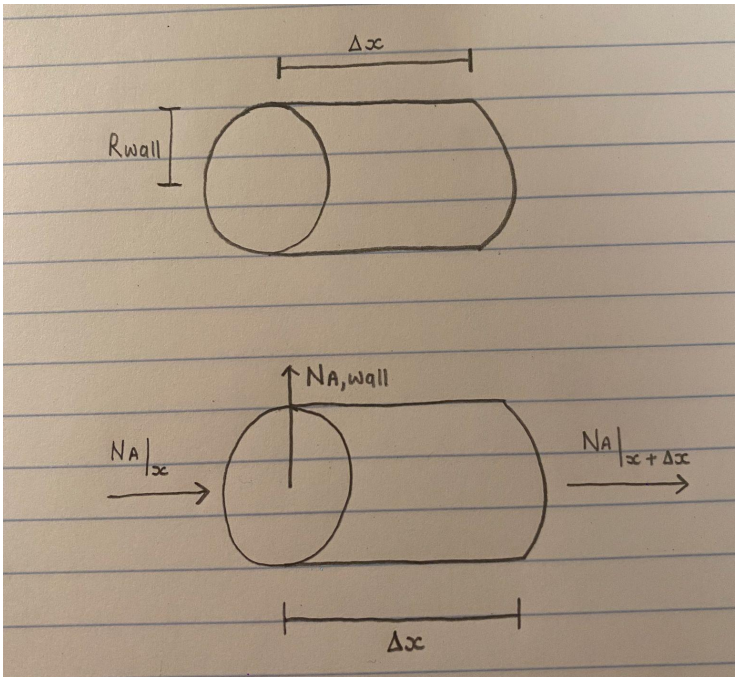
Boundary Conditions:

$$C_{Ab}(x = 0) = C_{Ab0}$$

$$C_{Ad}(x = L) = 0$$

Blood:

Shell Diagram:



Shell-Balance Method:

$$\left(-N_{Ab}|_{x+\Delta x}+N_{Ab}|_{x}\right)\pi R_b^2+N_{A,wall}(x)\left(2\pi R_{wall}\Delta x\right)=0$$

[Divide by volume = $\pi R_b^2 \Delta x$]

$$\frac{\left(N_{Ab}|_{x}+-|_{x+\Delta x}\right)}{\Delta x}-\frac{2N_{A,wall}(x)}{R_{wall}}=0$$

[Take limit as $\Delta x \to 0$]

$$0=-\frac{dN_{Ab}}{dx}-\frac{2N_{A,wall}}{R}$$

$N_{Ab}=convection+diffusion$, (where diffusion is ignored)

$$N_{Ab}=C_A v_{x,b}$$

$$N_{Ab}=C_A\frac{Q_b}{\pi R_{wall}^{\ 2}}$$

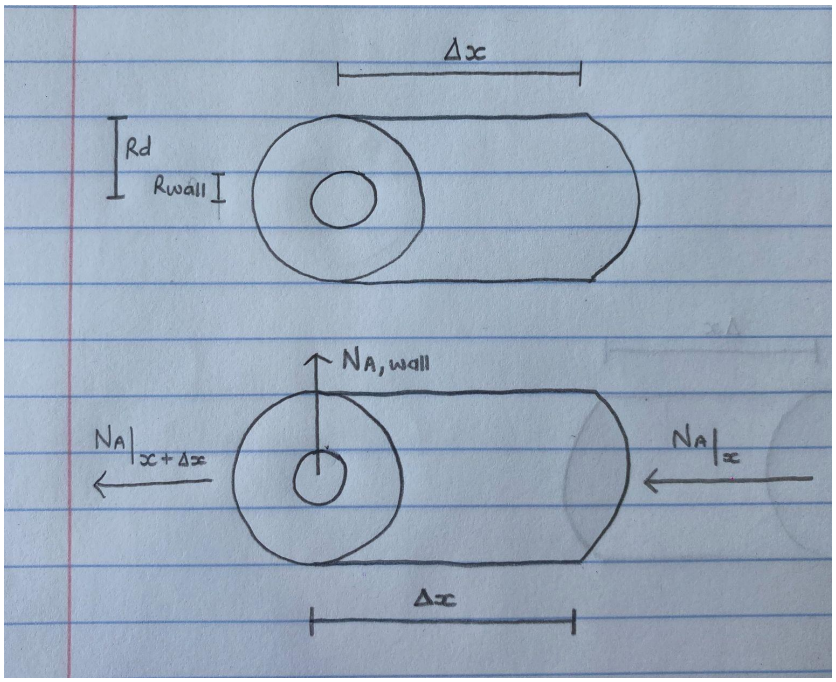
$$0=-\frac{d}{dx}\left(C_{Ab}\frac{Q_b}{\pi R_{wall}^{\ 2}}\right)-\frac{2}{R}P\left(C_{Ab}(x)-\phi C_{Ad}(x)\right)$$

$$-\frac{dC_{Ab}}{dx}-\frac{2\pi R_{wall}}{Q_b}P\left(C_{Ab}(x)-\phi C_{Ad}(x)\right)=0$$

$$\frac{dC_{Ab}}{dz} = -\frac{2\pi PR_{wall}}{Q_b}\left(C_{Ab}(x) - \phi C_{Ad}(x)\right)$$

Dialysis fluid:

Shell Diagram:



Shell-Balance Method:

$$\left(N_{Ad}|_{x+\Delta x} - N_{Ad}|_x\right)\pi(R_d^2 - R_b^2) + N_{A,wall}(x)\left(2\pi R_b \Delta x\right) = 0$$

[Divide by volume = $\pi(R_d^2 - R_b^2)\Delta x$]

$$\frac{\left(N_{Ad}|_{x+\Delta x} - N_{Ad}|_x\right)}{\Delta z} + \frac{N_{A,wall}(x)}{R_{wall}} = 0$$

[Take limit as $\Delta x \to 0$]

$$0 = \frac{dN_{Ad}}{dx} + \frac{2N_{A,wall}}{R}$$

$N_{Ad} = convection + diffusion$, (where diffusion is ignored)

$$N_{Ad} = C_A v_{x,b}$$

$$N_{Ad} = C_A \frac{Q_b}{\pi R_{wall}^2}$$

$$\frac{d}{dx}\left(C_{Ad}\frac{Q_d}{\pi R_{wall}^{\ 2}}\right)+\frac{2}{R}P\left(C_{Ab}(x)-\ \phi C_{Ad}(x)\right)=0$$

$$\frac{dC_{Ad}}{dx}+\frac{2\pi R_{wall}}{Q_d}P\left(C_{Ab}(x)-\ \phi C_{Ad}(x)\right)=0$$

$$\frac{dC_{Ad}}{dx}=-\frac{2\pi PR_{wall}}{Q_d}\left(C_{Ab}(x)-\phi C_{Ad}(x)\right)$$

The two differential equations are shown below:

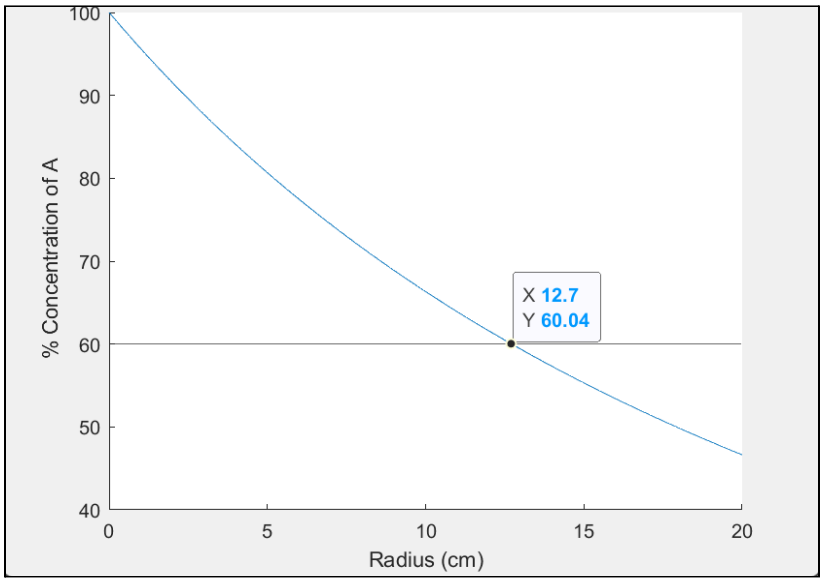
$$\frac{dC_{Ab}}{dx}=-\frac{2\pi PR_{wall}}{Q_b}\left(C_{Ab}(x)-\phi C_{Ad}(x)\right)$$

$$\frac{dC_{Ad}}{dx}=-\frac{2\pi PR_{wall}}{Q_d}\left(C_{Ab}(x)-\ \phi C_{Ad}(x)\right)$$

The two differential equations were solved using dsolve in MATLAB and then plotted for a length of 0cm to 20cm. The same values as above were used – 150mL/min for Qb and 300 mL/min for Qd. At a radius of 20cm, the concentration of the waste product "A" was 46% which is less than the 50% remaining from the setup in part 1.

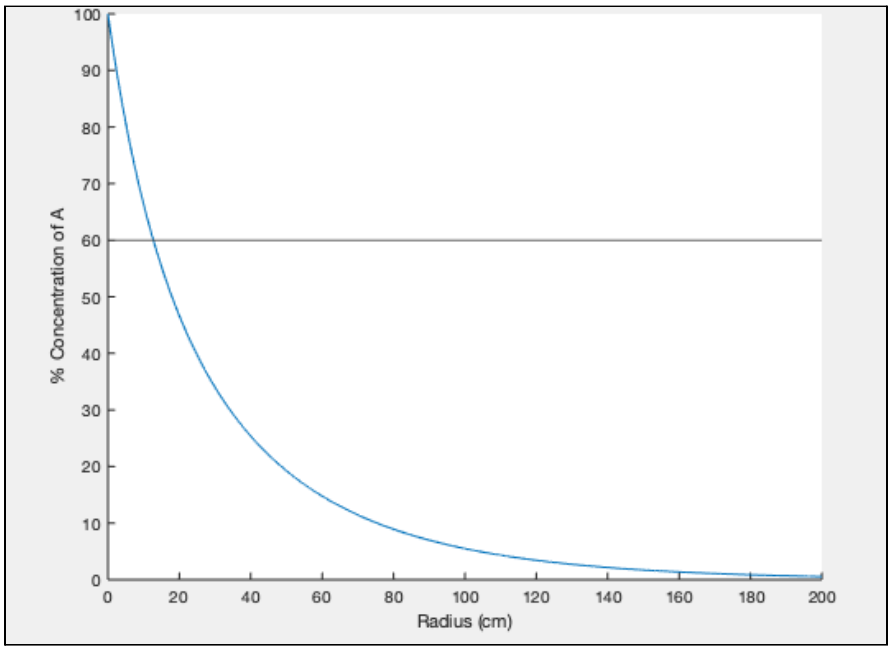
The radius that was found to be effective at removing at least 40% of the waste product (at which 60% concentration of waste product A was left) was 12.7cm, which was smaller than the result obtained from part 1, in which the radius of the fiber was 13.5 cm.This can be shown in the plot below:

Concentration vs. Radius



The maximum amount of waste product A that could be removed with a concurrent flow was then determined. At a radius of 200 cm, almost all of waste product A was removed from the blood as its concentration approached 0. This can be seen in the plot below:

Concentration vs. Radius



This is in contrast to the co-current design where still 33% of the drug remained at a very large radius of 200 cm. This proved that the counter-current setup was more effective than the co-current setup in removing waste product "A" from the blood.

Part 3: Parallel fiber design

In Parts 1 and 2, we analyzed the removal of waste product "A" from blood using a hollow-fiber design consisting of only one single fiber. In reality, most hollow-fiber kidney dialysis devices consist of many fibers in parallel. Perform a literature search to learn how these devices work and then discuss how they overcome limitations of the single-fiber design. In your answer, relate how the design parameters of such a device affect waste removal from the blood by discussing its effects on the transport equations from Part 1 and/or Part 2.

Patients with end stage renal disease typically undergo hemodialysis therapy to remove toxins from their blood through a dialyzer, which acts like an artificial kidney[1]. A hollow fiber dialyzer contains about 7,000 to 17,000 semipermeable hollow fibers that allow for the solute and fluid to transfer between the blood and dialysate. These fibers have a diameter of around 180 to 200 microns and have undulations that evenly distribute flow throughout the fiber bundle[2].

These fiber bundles are typically held together by the polyurethane potting material and are enclosed in the dialysate compartment. The header, which forms a barrier between the blood and the dialysate compartments, channels blood from the dialyzer inlet into the membrane fibers and back to the dialyzer outlet. Additionally, as shown in the figure below, both blood and dialysate typically flow in the opposite (counter-current) directions to maximize solute transfer through diffusion[2]. Benefits for the hollow fiber dialyzer include low resistance to blood flow and controlled ultrafiltration[3].

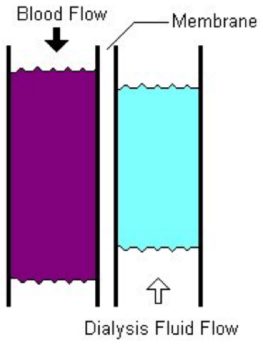


Figure 1. *Countercurrent Fluid Flow.*

Since the walls of the hollow fibers function as the dialysis membrane[4], multiple fibers both make the design compact and increase the surface area for diffusion[5]. A greater surface area allows for a faster removal of waste products. Additionally, another limitation of a single-fiber design that is overcome by a multiple-fiber design is the flow rate. With a single fiber, the radius needs to be very large to allow for significant removal of waste products, which drastically reduces the flow rate. This reduced flow rate can have negative consequences such as the formation of blood clots[6]. A multiple-fiber design, on the other hand, allows for faster flow rates as a result of the smaller fiber radii, resulting in a faster removal of waste products.

With regards to the design parameters of the device, as calculated above, a radius of 12.7cm (part 2) was more effective in removing 40% of the waste product in countercurrent flow as compared to a radius of 13.5cm (part 1) for a device removing the same % of waste product with co-current flow. Additionally, the countercurrent fluid flow device was able to remove almost 100% of the waste product from the blood as the fiber radius got infinitely larger as

compared to the co-current fluid flow device, which obtained a maximum waste product removal of 66.67%. The counter-current setup was therefore more effective in removing waste product "A" from the blood than the co-current setup, validating its use in traditional devices.

When comparing transport equations for both parts 1 and 2, the differential equation for blood was the same in both cases. The equation for dialysate was also similar with the only difference being a sign change in part 2 due to the opposite direction of fluid flow. Furthermore, when comparing the effect of radii on the differential equation for the rate of change of dialysate, even just 7000 fibers with a radius of 180 microns each had a rate of change 10 times greater than that of a single fiber with a radius of 12.7cm (as calculated in part 2).

Ultimately, this discussion highlighted the importance of a smaller radius, faster flow rate, higher surface area, and opposite fluid flow direction in the effective removal of waste products from blood when using a dialyzer.

References

[1]Beek, O.E.M. ter, et al. "Hollow Fiber Membranes for Long-Term Hemodialysis Based on Polyethersulfone-Slipskin™ Polymer Blends." *Journal of Membrane Science*, Elsevier, 15 Mar. 2020, https://www.sciencedirect.com/science/article/pii/S0376738820306475.

[2]America, Fresenius Medical Care North. "Understanding Dialyzer Types." *Fresenius Medical Care*, Fresenius Medical Care, North America, 13 Aug. 2021, https://fmcna.com/insights/articles/Understanding-Dialyzer-Types/.

[3]Themes, UFO. "Dialyzers, Dialysate, and Delivery Systems." *Nurse Key*, 24 July 2016, https://nursekey.com/dialyzers-dialysate-and-delivery-systems/.

[4]Waelchi, Kitty. "How Are Dialyzers Made?" *How Are Dialyzers Made?*, https://moviecultists.com/how-are-dialyzers-made.

[5]Yamashita, Akihiro C., and Kenji Sakurai. "Chapter: Dialysis Membranes - Physicochemical Structures and Features." *IntechOpen*, IntechOpen, 9 Sept. 2015, https://www.intechopen.com/chapters/48020.

[6]Fogelson, Aaron L, and Keith B Neeves. "Fluid Mechanics of Blood Clot Formation." *Annual Review of Fluid Mechanics*, U.S. National Library of Medicine, 1 Jan. 2015, https://www.ncbi.nlm.nih.gov/pmc/articles/PMC4519838/.