Advanced Data Structures and Algorithms

Comprehensive Assignment Solutions

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1 Complexity Classes

Question 1. Define the class Co-NP. Explain the type of problems that belong to this complexity class.

Detailed Solution:

Definition

Co-NP is a complexity class containing decision problems for which the "No" instances can be efficiently verified. Formally, a language L is in Co-NP if its complement, $\bar{L}$ (the set of strings not in L), is in the class **NP**.

In simpler terms:

- **NP Problem:** "If the answer is YES, there exists a short proof (certificate) that I can check in polynomial time."
- **Co-NP Problem:** "If the answer is NO, there exists a short proof (counter-example) that I can check in polynomial time."

Visual Relationship of Complexity Classes

The relationship between P, NP, and Co-NP is often visualized using a Venn diagram. P is contained in the intersection of NP and Co-NP.

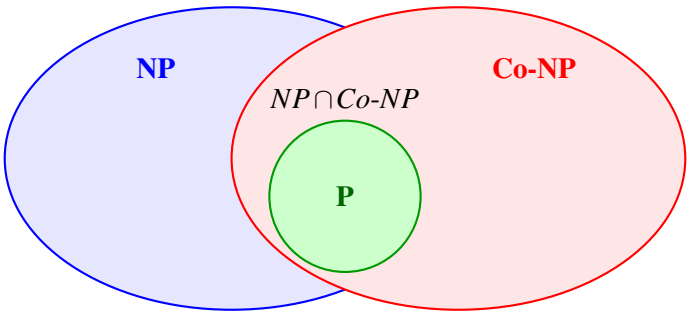

Diagram assumes $P \neq NP$ and $NP \neq Co\text{-}NP$

Type of Problems

Co-NP problems typically involve properties that must hold for **all** possible structures or assignments (Universality).

1. **TAUTOLOGY (Logic):**
   - *Problem:* Given a boolean formula ϕ, is it true for **every** possible assignment of truth values to variables?
   - *Why Co-NP:* If the answer is NO, there exists at least one assignment where ϕ is False. This single assignment serves as a succinct "counter-example" (certificate) that can be verified easily.
2. **UNSAT (Logic):**
   - *Problem:* Given a boolean formula ϕ, is it impossible to satisfy? (i.e., Is it false for all assignments?)
   - *Why Co-NP:* This is the direct complement of SAT. If the answer is NO (meaning it IS satisfiable), we can present a satisfying assignment as proof.
3. **GRAPH NON-ISOMORPHISM:**
   - *Problem:* Given two graphs G_1 and G_2, are they **not** isomorphic?
   - *Why Co-NP:* The complement problem is Graph Isomorphism (in NP). If the answer to Non-Isomorphism is NO (meaning they ARE isomorphic), the certificate is simply the mapping (bijection) between the vertices.
4. **NON-HAMILTONIAN GRAPH:**
   - *Problem:* Given a graph G, is it true that there is **no** Hamiltonian cycle?
   - *Why Co-NP:* If the answer is NO (meaning there IS a cycle), the certificate is simply the sequence of vertices forming the cycle.

Conclusion

The class Co-NP complements NP. While NP characterizes problems where "Yes" answers are easy to verify (existence proofs), Co-NP characterizes problems where "No" answers are easy to verify (universal truths). Most computer scientists believe $NP \neq Co\text{-}NP$, meaning that just because we can verify a solution quickly doesn't mean we can verify the *absence* of a solution quickly.
