Advanced Data Structures and Algorithms

Comprehensive Assignment Solutions

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1 Complexity Classes

Question 4. Is the 2-SAT problem NP-Hard? Can it be solved in polynomial time? Explain your reasoning.

Detailed Solution:

Answer

No, 2-SAT is **not NP-Hard** (assuming $P \neq NP$). It is in the complexity class **P** and can be solved in linear time.

Polynomial Time Solution Strategy

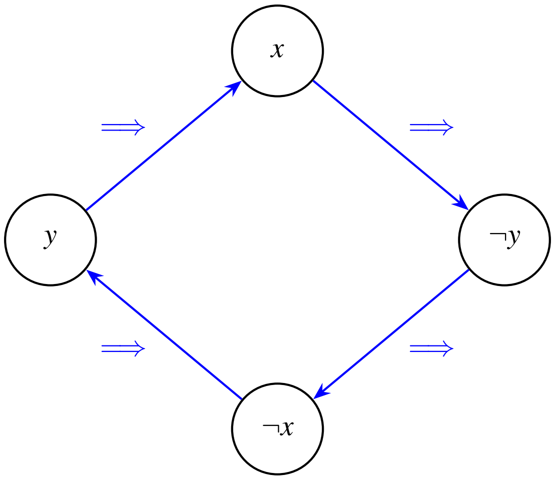
The 2-SAT problem (where every clause has exactly 2 literals) can be modeled using a directed graph called the **Implication Graph**.

Construction: For every clause $(A \vee B)$, we add two directed edges representing the logical implications:

1. If A is False, B must be True: $\neg A \implies B$.
2. If B is False, A must be True: $\neg B \implies A$.

Visualizing the Implication Graph

The graph below illustrates a scenario where a variable implies its own negation and vice versa via a cycle (Strongly Connected Component). This condition proves unsatisfiability.



Unsatisfiable Condition:
Path exists $x \rightsquigarrow \neg x$ AND $\neg x \rightsquigarrow x$.
They are in the same SCC.

Satisfiability Condition: The formula is unsatisfiable if and only if there exists a variable x such that there is a path from x to $\neg x$ **AND** a path from $\neg x$ to x. In graph theory terms, this means x and $\neg x$ belong to the same **Strongly Connected Component (SCC)**.

Algorithm

1. Build the implication graph. Nodes are literals $(x_i, \neg x_i)$.
2. Run an SCC finding algorithm (like **Kosaraju's** or **Tarjan's Algorithm**).
3. Check if any variable x_i and its negation $\neg x_i$ are in the same SCC.
4. If yes, return "Unsatisfiable". If no, return "Satisfiable".

Complexity

Building the graph takes linear time in the number of clauses. Finding SCCs takes $O(V+E)$ time. Thus, 2-SAT is solvable in $O(N+M)$ time, which is polynomial. This sharp contrast with 3-SAT (which is NP-Hard) is a famous example of the "boundary" of tractability.

Conclusion

The ability to map 2-SAT to a graph reachability problem allows us to bypass exponential search. Since graph connectivity (specifically SCCs) can be solved in linear time, 2-SAT resides firmly in P, unlike its general counterpart SAT or 3-SAT.