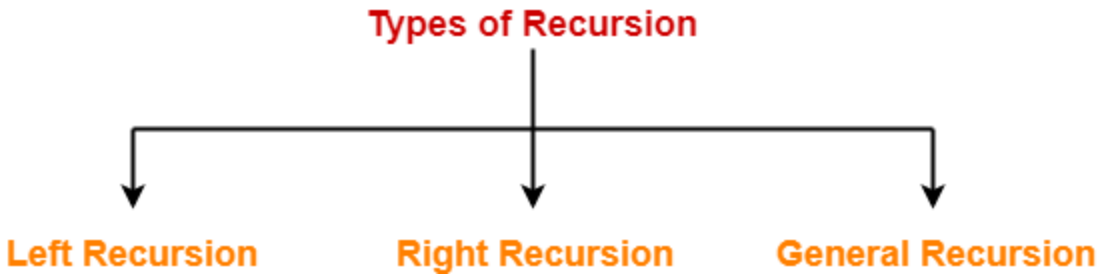
Left Recursion & Left Factoring

Recursion: Recursion can be classified into following three types-



1. Left Recursion: A production of grammar is said to have left recursion if the leftmost variable of its RHS is same as variable of its LHS. A grammar containing a production having left recursion is called as Left Recursive Grammar.
Example-
$S \rightarrow Sa / \in$
(Left Recursive Grammar)
Left recursion is considered to be a problematic situation for Top-down parsers. Therefore, left recursion has to be eliminated from the grammar.

Elimination of Left Recursion

Left recursion is eliminated by converting the grammar into a right recursive grammar.
If we have the left-recursive pair of productions-
$A \rightarrow A\alpha / \beta$
(Left Recursive Grammar)
where β does not begin with an A.

Then, we can eliminate left recursion by replacing the pair of productions with-
$A \rightarrow \beta A'$
$A' \rightarrow \alpha A' / \in$
(Right Recursive Grammar)

This right recursive grammar functions same as left recursive grammar.

2. Right Recursion: A production of grammar is said to have right recursion if the rightmost variable of its RHS is same as variable of its LHS. A grammar containing a production having right recursion is called as Right Recursive Grammar.
Example-
$S \rightarrow aS / \in$
(Right Recursive Grammar)
Right recursion does not create any problem for the Top-down parsers.
Therefore, there is no need of eliminating right recursion from the grammar.

Left Recursion & Left Factoring

PRACTICE PROBLEMS BASED ON LEFT RECURSION ELIMINATION-

Problem-01: Consider the following grammar and eliminate left recursion-

A → ABd / Aa / a
B → Be / b

Solution- The grammar after eliminating left recursion is-

A → aA'
A' → BdA' / aA' / ∈
B → bB'
B' → eB' / ∈

Problem-02: Consider the following grammar and eliminate left recursion-

E → E + E / E x E / a

Solution- The grammar after eliminating left recursion is-

E → aA
A → +EA / xEA / ∈

Problem-03: Consider the following grammar and eliminate left recursion-

E → E + T / T
T → T x F / F
F → id

Solution- The grammar after eliminating left recursion is-

E → TE'
E' → +TE' / ∈
T → FT'
T' → xFT' / ∈
F → id

Problem-04: Consider the following grammar and eliminate left recursion-

A → AAα / β

Solution- The grammar after eliminating left recursion is-

A → βA'
A' → AαA' / ∈

Problem-05: Consider the following grammar and eliminate left recursion-

A → Ba / Aa / c
B → Bb / Ab / d

Solution- This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from A → Ba / Aa / c
Eliminating left recursion from here, we get-
A → BaA' / cA'
A' → aA' / ∈
Now, given grammar becomes-
A → BaA' / cA'
A' → aA' / ∈
B → Bb / Ab / d

Step-02:

Substituting the productions of A in B → Ab, we get the following grammar-
A → BaA' / cA'
A' → aA' / ∈
B → Bb / BaA'b / cA'b / d

Step-03:

Now, eliminating left recursion from the productions of B, we get the following grammar-
A → BaA' / cA'
A' → aA' / ∈
B → cA'bB' / dB'
B' → bB' / aA'bB' / ∈
This is the final grammar after eliminating left recursion.

Problem-06: Consider the following grammar and eliminate left recursion-

S → S0S1S / 01

Solution- The grammar after eliminating left recursion is-

S → 01A
A → 0S1SA / ∈

Left Recursion & Left Factoring

Problem-07: Consider the following grammar and eliminate left recursion-

S → (L) / a
L → L , S / S

Solution- The grammar after eliminating left recursion is-

S → (L) / a
L → SL'
L' → ,SL' / ∈

Problem-08: Consider the following grammar and eliminate left recursion-

S → A
A → Ad / Ae / aB / ac
B → bBc / f

Solution- The grammar after eliminating left recursion is-

S → A
A → aBA' / acA'
A' → dA' / eA' / ∈
B → bBc / f

Problem-09: Consider the following grammar and eliminate left recursion-

X → XSb / Sa / b
S → Sb / Xa / a

Solution- This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from X → XSb / Sa / b

Eliminating left recursion from here, we get-

X → SaX' / bX'
X' → SbX' / ∈

Now, given grammar becomes-

X → SaX' / bX'
X' → SbX' / ∈
S → Sb / Xa / a

Step-02:

Substituting the productions of X in S → Xa, we get the following grammar-

X → SaX' / bX'
X' → SbX' / ∈
S → Sb / SaX'a / bX'a / a

Step-03:

Now, eliminating left recursion from the productions of S, we get the following grammar-

X → SaX' / bX'
X' → SbX' / ∈
S → bX'aS' / aS'
S' → bS' / aX'aS' / ∈

This is the final grammar after eliminating left recursion.

Problem-10: Consider the following grammar and eliminate left recursion-

S → Aa / b
A → Ac / Sd / ∈

Solution-This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from S → Aa / b

This is already free from left recursion.

Step-02:

Substituting the productions of S in A → Sd, we get the following grammar-

S → Aa / b
A → Ac / Aad / bd / ∈

Step-03:

Now, eliminating left recursion from the productions of A, we get the following grammar-

S → Aa / b
A → bdA' / A'
A' → cA' / adA' / ∈

This is the final grammar after eliminating left recursion.

Left Recursion & Left Factoring

❑ **Grammar With Common Prefixes-**

If RHS of more than one production starts with the same symbol, then such a grammar is called as **Grammar With Common Prefixes**.

Example-
$A \rightarrow \alpha\beta_1 / \alpha\beta_2 / \alpha\beta_3$
(Grammar with common prefixes)

- This kind of grammar creates a problematic situation for Top down parsers.
- Top down parsers can not decide which production must be chosen to parse the string in hand.

To remove this confusion, we use left factoring.

Left Factoring-

Left factoring is a process by which the grammar with common prefixes is transformed to make it useful for Top down parsers.

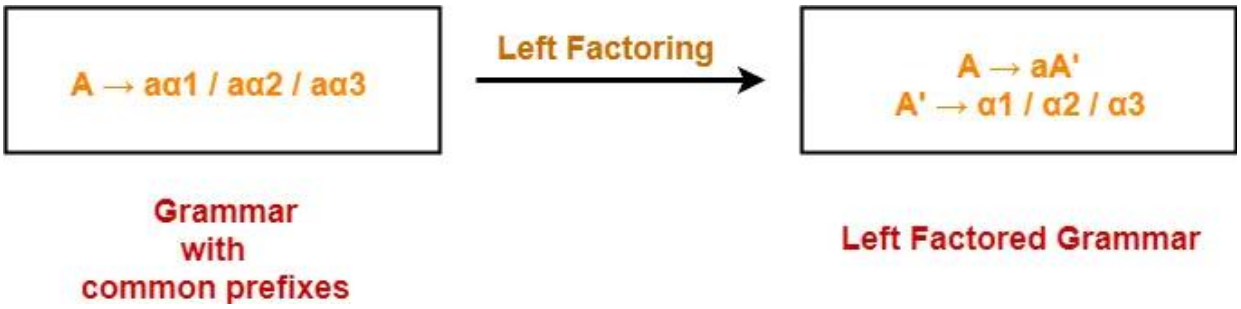
How?

In left factoring,

- We make one production for each common prefixes.
- The common prefix may be a terminal or a non-terminal or a combination of both.
- Rest of the derivation is added by new productions.

The grammar obtained after the process of left factoring is called as **Left Factored Grammar**.

Example-



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PRACTICE PROBLEMS BASED ON LEFT FACTORING-

Problem-01: Do left factoring in the following grammar-

S → iEtS / iEtSeS / a
E → b

Solution- The left factored grammar is-

S → iEtSS' / a
S' → eS / ∈
E → b

Problem-02: Do left factoring in the following grammar-

S → a / ab / abc / abcd

Solution- **Step-01:**

S → aS'
S' → b / bc / bcd / ∈

Again, this is a grammar with common prefixes.

Step-02:

S → aS'
S' → bA / ∈
A → c / cd / ∈

Again, this is a grammar with common prefixes.

Step-03:

S → aS'
S' → bA / ∈
A → cB / ∈
B → d / ∈

This is a left factored grammar.

Problem-03: Do left factoring in the following grammar-

S → bSSaaS / bSSaSb / bSb / a

Solution- **Step-01:**

S → bSS' / a
S' → SaaS / SaSb / b

Again, this is a grammar with common prefixes.

Step-02:

S → bSS' / a
S' → SaA / b
A → aS / Sb

This is a left factored grammar.

Problem-04: Do left factoring in the following grammar-

S → aSSbS / aSaSb / abb / b

Solution-

Step-01:

S → aS' / b
S' → SSbS / SaSb / bb

Again, this is a grammar with common prefixes.

Step-02:

S → aS' / b
S' → SA / bb
A → SbS / aSb

This is a left factored grammar.

Problem-05: Do left factoring in the following grammar-

S → aAd / aB
A → a / ab
B → ccd / ddc

Solution- The left factored grammar is-

S → aS'
S' → Ad / B
A → aA'
A' → b / ∈
B → ccd / ddc

To gain better understanding about Left Factoring,

Problem-06: Do left factoring in the following grammar-

A → aAB / aBc / aAc

Solution-

Step-01:

A → aA'
A' → AB / Bc / Ac

Again, this is a grammar with common prefixes.

Step-02:

A → aA'
A' → AD / Bc
D → B / c

This is a left factored grammar.