Introduction to CFG (Context Free Grammar)

Introduction to CFG

A context Free Grammar (CFG) is a 4-tuple such that-

G = (V, T, P, S)

where-

V = Finite non-empty set of variables / non-terminal symbols

T = Finite set of terminal symbols

P = Finite non-empty set of production rules

S = Start symbol

Example-01: Find out the characteristics of CFG from the following expression: -

Expression → Expression + Term | Term Term → Term * Factor | Factor Factor → (Expression) | id

Variables V = {Expression, Term, Factor}

Terminals T = {+, *, (,), id}

Production rules P = {

Expression → Expression + Term | Term

Term → Term * Factor | Factor

Factor → (Expression) | id

}

Start Symbol S = {Expression}

Example-02: Write down the formal definition of CFG for the following expression: -

S → aSb / ∈

Variables V = {S}

Terminals T = {a, b, ∈}

Production rules P = S → aSb / ∈

Start Symbol S = {S}

Example-03: Write down the formal definition of CFG for the following expression: -

S → aAb / ∈ A → aAb / ∈

Consider a grammar G = (V, T, P, S) where-

Variables V = {S}

Terminals T = { a , b }

Production rules P = { S → aSbS , S → bSaS , S → ∈ }

Start Symbol S = { S }

Introduction to CFG (Context Free Grammar)

Question – 1:

a. Construct a CFG where L = {a^n | n >= 0}

Solution:

Regular Language, RL = {∈, a, aa, aaa, aaaaa,,,,,,,,,,,,,,,}

Regular Expression, RE = a*

CFG: S → ∈ | aS

b. Construct a CFG where L = {a^n | n >= 1}

Solution:

Regular Language, RL = {a, aa, aaa, aaaaa,,,,,,,,,,,,,,,}

Regular Expression, RE = a^+

CFG: S → a | aS

c. Construct a CFG where L = {set of all string over a, b}

Solution:

Regular Language, RL = {∈, a,b,bb,bbbb,aba,bbba,babab, aa, aaa, aaaaa,,,,,,,,,,,,,,,}

Regular Expression, RE = (a+b)*

CFG: S → ∈ | aS | bS

d. Construct a CFG where L = {set of all string length at least 2}

Solution:

Regular Language, RL = {aa,ab,ba,bb, aba, baa, aaa,bbbb, aaaaa,,,,,,,,,,,,,,,}

Regular Expression, RE = (a+b).(a+b).(a+b)*

CFG:

S → AAB

A→a|b

B→∈|aB|bB

e. Construct a CFG where L = {set of all string length at most 2}

Solution:

Regular Language, RL = {∈,b,a, aa,ab,ba,bb}

Regular Expression, RE = (a+b+∈) + (a+b+∈)

CFG:

S → AA

A →a|b|∈

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Question – 2:

a. Construct a CFG where L = {set of all string at least three 0's}

Solution:

Regular Language, RL = {000,10100,01001,00000101,,,,,,,,,,,,,,,}

Regular Expression, RE = (0+1)*. 0 .(0+1)* .0 .(0+1)*. 0. (0+1)*

CFG:

S → A0A0A0A

A → ∈ | 0A | 1A

b. Construct a CFG where L = start with 'a' and end with 'b'

Solution:

Regular Language, RL = {ab,abb,aab,abbbbb,abababab,,,,,,,,,,,}

Regular Expression, RE = a(a+b)*b

CFG:

S → aAb

A → ∈ | aA | bA

c. Construct a CFG where L = start and ends with same symbol

Solution:

Regular Language, RL = {a,aa, b,bb,aba,bab,abbbbbba,bbbbbbb,aaaaaa,,,,,,,,,,}

Regular Expression, RE = a(a+b)*a + b(a+b)*b

CFG:

S → aAa | bAb

A → ∈ | aA | bA

d. Construct a CFG where L = start and ends with different symbol

Solution:

Regular Language, RL = {ab,ba,aabbab,babbbba,abababab,,,,,,,,,,,,,,,,,}

Regular Expression, RE = a(a+b)*b + b(a+b)*a

CFG:

S → aAb | bAa

A → ∈ | aA | bA

e. Construct a CFG where L = even length of string

Solution:

Regular Language, RL = {aa,aaba,bb,bbba,bbbaaa,abababab,,,,,,,,,,,,}

Regular Expression, RE = ((a+b)(a+b))*

CFG:

S → BS|∈

B → AA

A → a|b

Introduction to CFG (Context Free Grammar)

Question – 3:

a. Construct a CFG where L = {$a^n b^{2n}$| n >= 1}

Solution:

Regular Language, RL = {abb, aabbbb, aaabbbbbb............}

CFG:

S → abb | aSbb

b. Construct a CFG where L = {$a^n b^m c^n$| n,m >= 1}

Solution:

Regular Language, RL = {abc,abbc,aabcc,aabbcc............}

CFG:

S → aSc | aAc

A→bA|∈

c. Construct a CFG where L = {$a^n b^m c^m d^n$| n,m >= 1}

Solution:

Regular Language, RL = {abcd,abbccd,aabcdd,aabbccdd ....}

CFG:

S → aSd | aAd

A→ bAc|bc

d. Construct a CFG where L = {$a^n b^n c^m d^m$| n,m >= 1}

Solution:

Regular Language, RL = {abcd,abccdd,aabbcd,aabbccdd ....}

CFG:

S → AB

A →aAb|ab

B →cBd|cd

e. Construct a CFG where L = {$a^{m+n} b^m c^n$| n,m >= 1}

Solution:

Regular Language, RL = {aabc,aaabcc,aaabbc ....}

CFG:

S → AB

A→ aAb|ab

B→cBd|cd