∈-NFA to NFA conversion

- Process of ∈-NFA to NFA conversion
- Practice problems
  1. Remove epsilon(∈) from the following NFA
  2. Convert the following NFA with ε to NFA without ε.
  3. Convert epsilon-NFA to NFA
  4. Consider the following epsilon NFA to NFA

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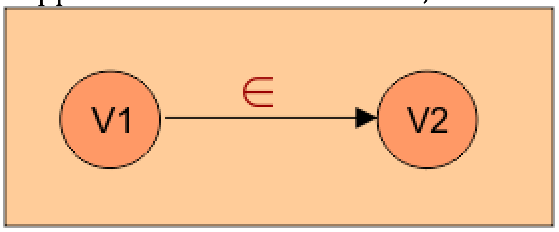
❑ ∈-NFA to NFA conversion

Elimination of Epsilon (ε) From NFA
If the epsilon exists between any two states in automata, then the removal of epsilon through some rules is called elimination of epsilon move from NFA.
Suppose there are two vertices, V1 and V2; the epsilon between them is below.



<u>Steps For Elimination Of Epsilon From NFA</u>

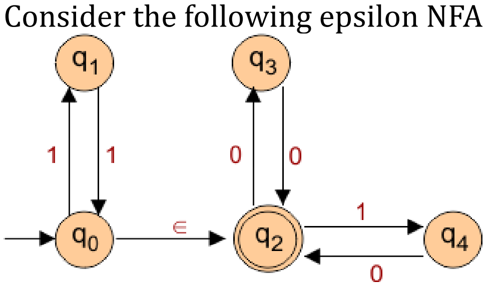
- Step 01: Find all edges starting from V2
- Step 02: Remove the epsilon. All Finding edges from V2 are Duplicated to V1 without changing edge labels.
- Step 03: if V1 is initial state, make V2 initial
- Step 04: if V2 is final state, make V1 final
- Step 05: Remove all dead states

Note: if more than one epsilon exists in epsilon-NFA, then first remove the epsilon that is far away from the initial state. After the Removal of one epsilon, remove the others till the removal of the epsilon is.

<u>Example of Elimination Of Epsilon From NFA</u>

<u>Example 01:</u>

Consider the following epsilon NFA



<u>Solution</u>

In the above epsilon NFA, suppose q0 is a V1 and q2 is V2 because epsilon exists between these two states. Now apply the steps of epsilon removal.

<u>Step 01</u>

- The first input, “0”, starts from V2 (q2) and goes to q3.
- The second input, “1”, starts from V2 (q2) and goes to q4

<u>Step 02</u>

Finding outgoing edges from V2 are “0” and “1”.

- First, Input “0” is going to q3 from V2 (q2). So, the first duplicate will go from V1 (q0) to q3.
- Second Input “1” is going to q4 from V2 (q2). So, the second duplicate will go from V1 (q0) to q4.

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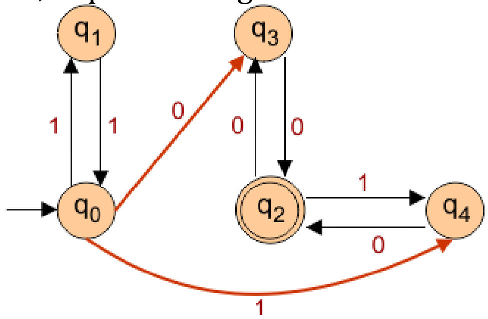
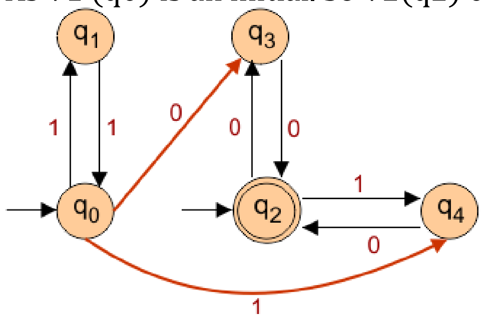
So, duplicated edges with their labels are given below



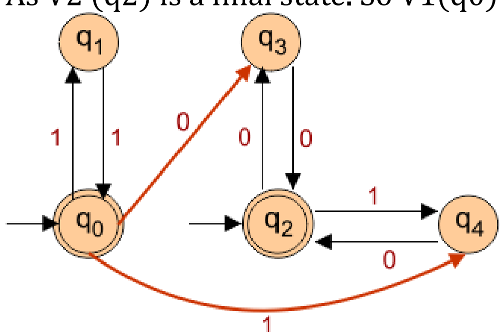
<u>Step 03</u>

As V1 (q0) is an initial. So V2(q2) changes to initial. As given below



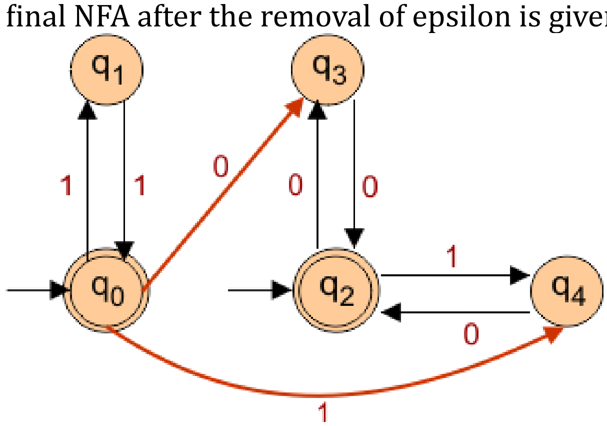
<u>Step 04</u>

As V2 (q2) is a final state. So V1(q0) changes to the final state. As given below



<u>Step 05</u>

An unreachable state is called a dead state. In the above NFA, there is no dead state. So, the final NFA after the removal of epsilon is given below.



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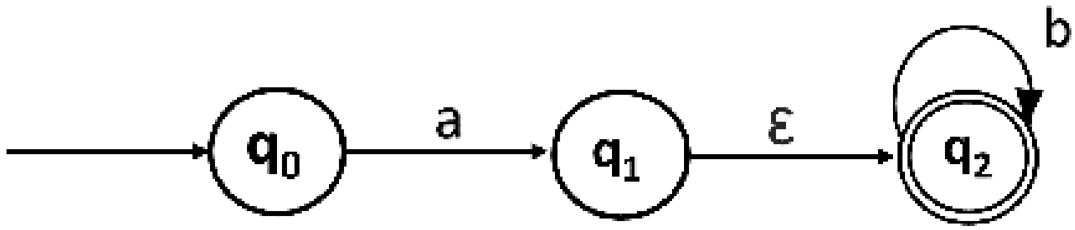
Example -2:

Convert the following NFA with ε to NFA without ε.



Solutions: We will first obtain ε-closures of q0, q1 and q2 as follows:

1. ε-closure(q0) = {q0}
2. ε-closure(q1) = {q1, q2}
3. ε-closure(q2) = {q2}

Now the δ' transition on each input symbol is obtained as:

δ'(q0, a) = ε-closure(δ(δ^(q0, ε),a))
= ε-closure(δ(ε-closure(q0),a))
= ε-closure(δ(q0, a))
= ε-closure(q1)
= {q1, q2}

δ'(q0, b) = ε-closure(δ(δ^(q0, ε),b))
= ε-closure(δ(ε-closure(q0),b))
= ε-closure(δ(q0, b))
= Φ

Now the δ' transition on q1 is obtained as:

δ'(q1, a) = ε-closure(δ(δ^(q1, ε),a))
= ε-closure(δ(ε-closure(q1),a))
= ε-closure(δ(q1, q2), a)
= ε-closure(δ(q1, a) ∪ δ(q2, a))
= ε-closure(Φ ∪ Φ)
= Φ

δ'(q1, b) = ε-closure(δ(δ^(q1, ε),b))
= ε-closure(δ(ε-closure(q1),b))
= ε-closure(δ(q1, q2), b)
= ε-closure(δ(q1, b) ∪ δ(q2, b))
= ε-closure(Φ ∪ q2)
= {q2}

The δ' transition on q2 is obtained as:

δ'(q2, a) = ε-closure(δ(δ^(q2, ε),a))
= ε-closure(δ(ε-closure(q2),a))
= ε-closure(δ(q2, a))
= ε-closure(Φ)
= Φ

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$\delta'(q2, b) = \varepsilon\text{-closure}(\delta(\delta\hat{\ }(q2, \varepsilon), b))$
$\quad = \varepsilon\text{-closure}(\delta(\varepsilon\text{-closure}(q2), b))$
$\quad = \varepsilon\text{-closure}(\delta(q2, b))$
$\quad = \varepsilon\text{-closure}(q2)$
$\quad = \{q2\}$

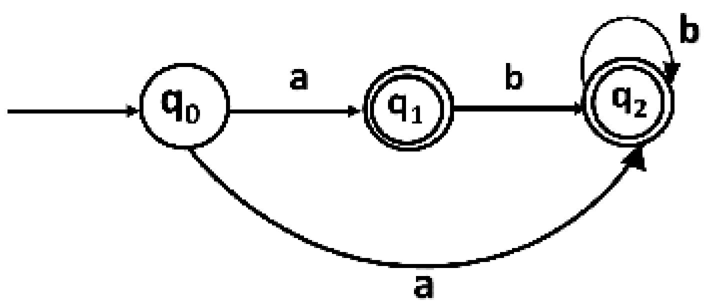
Now we will summarize all the computed δ' transitions:

$\delta'(q0, a) = \{q0, q1\}$
$\delta'(q0, b) = \Phi$
$\delta'(q1, a) = \Phi$
$\delta'(q1, b) = \{q2\}$
$\delta'(q2, a) = \Phi$
$\delta'(q2, b) = \{q2\}$

The transition table can be:

States	**a**	**b**
→q0	{q1, q2}	Φ
*q1	Φ	{q2}

State q1 and q2 become the final state as ε-closure of q1 and q2 contain the final state q2. The NFA can be shown by the following transition diagram:



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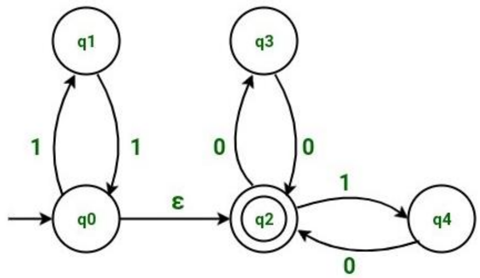
Example 3:

Convert epsilon-NFA to NFA. Consider the example having states q0, q1, q2, q3, and q4.



In the above example, we have 5 states named as q0, q1, q2, q3 and q4. Initially, we have q0 as start state and q2 as final state. We have q1, q3 and q4 as intermediate states.

Transition table for the above NFA is:

States/Input	Input 0	Input 1	Input epsilon
q0	–	q1	q2
q1	–	q0	–
q2	q3	q4	–
q3	q2	–	–
q4	q2	–	–

According to the transition table above,

- state q0 on getting input 1 goes to state q1.
- State q0 on getting input as a null move *(i.e. an epsilon move)* goes to state q2.
- State q1 on getting input 1 goes to state q0.
- Similarly, state q2 on getting input 0 goes to state q3, state q2 on getting input 1 goes to state q4.
- Similarly, state q3 on getting input 0 goes to state q2.
- Similarly, state q4 on getting input 0 goes to state q2.

We can see that we have an epsilon move from state q0 to state q2, which is to be removed. To remove epsilon move from state q0 to state q1, we will follow the steps mentioned below.

Step-1: Considering the epsilon move from state q0 to state q2. Consider the state q0 as vertex v1 and state q2 as vertex v2.

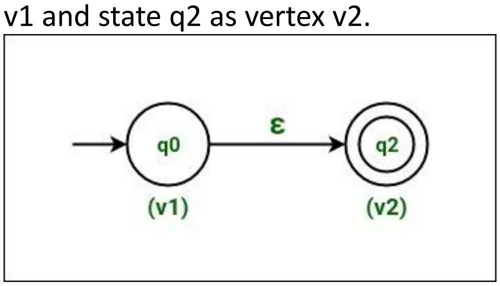


Figure – State q0 as vertex v1 and state q2 as vertex v2

Step-2: Now find all the moves that starts from vertex v2 *(i.e. state q2)*. After finding the moves, duplicate all the moves that start from vertex v2 *(i.e state q2)* with the same input to start from vertex v1 *(i.e. state q0)* and remove the epsilon move from vertex v1 *(i.e. state q0)* to vertex

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v2 *(i.e. state q2)*. Since state q2 on getting input 0 goes to state q3. Hence on duplicating the move, we will have state q0 on getting input 0 also to go to state q3.

Similarly state q2 on getting input 1 goes to state q4. Hence on duplicating the move, we will have state q0 on getting input 1 also to go to state q4.

So, NFA after duplicating the moves is:

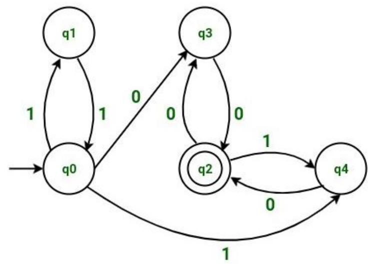

Figure – NFA on duplicating moves

Step-3: Since vertex v1 *(i.e. state q0)* is a start state. Hence we will also make vertex v2 *(i.e. state q2)* as a start state. Note that state q2 will also remain as a final state as we had initially. NFA after making state q2 also as a start state is:

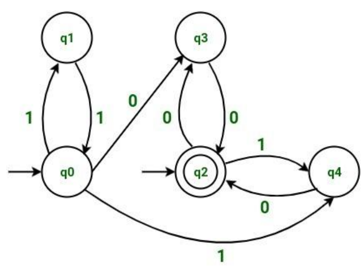

Figure – NFA after making state q2 as a start state

Step-4: Since vertex v2 *(i.e. state q2)* is a final state. Hence we will also make vertex v1 *(i.e. state q0)* as a final state. Note that state q0 will also remain as a start state as we had initially. After making state q0 also as a final state, the resulting NFA is:

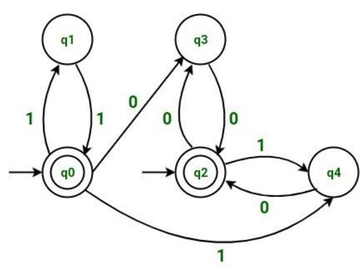

Figure – Resulting NFA (state q0 as a final state)The transition table for the above resulting NFA is:

States/Input	**Input 0**	**Input 1**
q0	q3	q1,q4
q1	–	q0
q2	q3	q4
q3	q2	–
q4	q2	–

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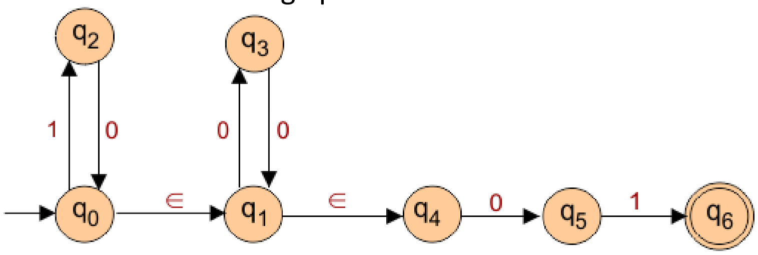
Example 4:

Consider the following epsilon NFA to NFA



Solution

In the above epsilon NFA, there are two epsilon exist. First, we remove the epsilon, away from the initial state. So, first, we remove the epsilon which exists in q1 and q4.

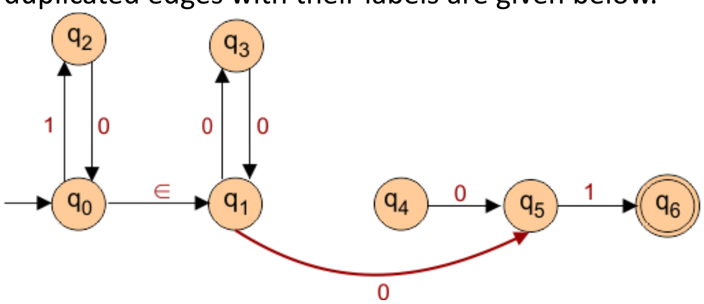
Suppose q1 is V1 and q4 is V2 because epsilon exists between these two states. Now apply the steps of epsilon removal.

Step 01:

- The only input "0" begins from V2 (q4) and goes to q5.

Step 02:

As the Input "0" is going to q5 from V2 (q2). So, a duplicate will go from V1 (q1) to q5. So, duplicated edges with their labels are given below.



Step 03:

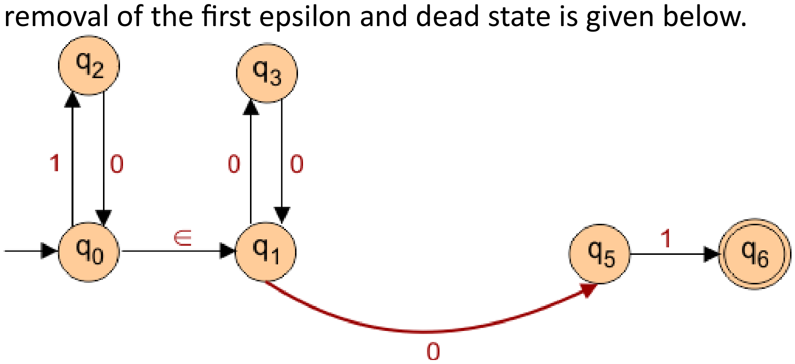
As V1 (q1) is not an initial state. So V2(q2) will be unchanged, and the NFA of Step 2 remain the Same.

Step 04:

As V2 (q4) is not a final state. So, V1(q1) will be unchanged, and the NFA of Step 2 remain the Same.

Step 05:

An unreachable state is called a dead state. In the above NFA, q4 is the dead state. So, NFA after removal of the first epsilon and dead state is given below.



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Now Repeat the Above 5 steps again for elimination of 2nd epsilon.

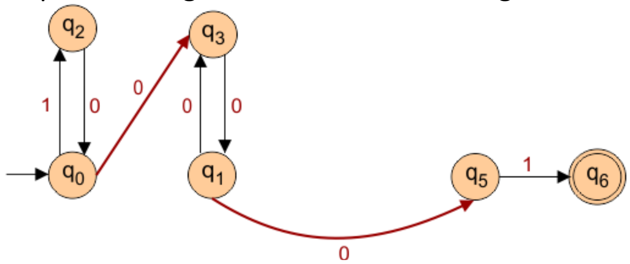
After the first 5 steps, epsilon- NFA, epsilon exists between q0 and q1. Again, suppose q0 is V1 and q2 is V2.

Again Step 01: Find all edges starting from V2

- The only input “0” begins from V2 (q2) and goes to q3.

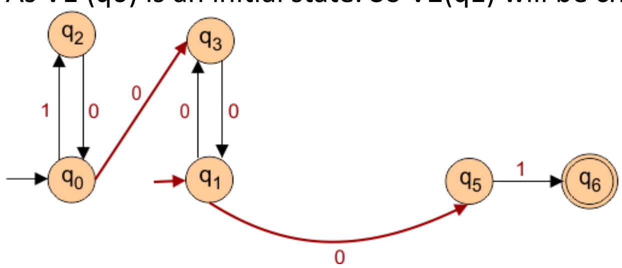
Step 02: Remove the epsilon. All Finding edges from V2 are Duplicated to V1 without changing edge labels.

As the Input “0” is going to q3 from V2 (q1). So, a duplicate will go from V1 (q0) to q3. So, duplicated edges with their labels are given below.



Step 03:

As V1 (q0) is an initial state. So V2(q1) will be changed to final. As given below



Step 04:

As V2 (q1) is not a final state. So, V1(q1) will remain unchanged, and the NFA of the above step3 will remain the same.

Step 05: Remove all dead states

No dead state was found. The above NFA of step 3 will remain the same. So, the final NFA after eliminating both epsilons and dead states is given below.