Conversion from NFA with ε to DFA

- Process of Conversion from NFA with ε to DFA

- Practice problems

1. Convert the NFA with ε into its equivalent DFA.
2. Convert the given ε-NFA into its equivalent DFA.
3. Prove that there exists a DFA for every ε – NFA.
4. Convert the following NFA with ε to equivalent DFA.
5. Convert the given ε-NFA into its equivalent DFA
6. Convert the given ε-NFA into its equivalent DFA

Conversion from NFA with ε to DFA

❑ Conversion from NFA with ε to DFA

Non-deterministic finite automata(NFA) is a finite automata where for some cases when a specific input is given to the current state, the machine goes to multiple states or more than 1 states. It can contain ε move.

It can be represented as M = { Q, ∑, δ, q0, F}.

Where
Q: finite set of states
∑: finite set of the input symbol
q0: initial state
F: **final** state
δ: Transition function

NFA with ∈ move: If any FA contains ε transaction or move, the finite automata is called NFA with ∈ move.

ε-closure: ε-closure for a given state A means a set of states which can be reached from the state A with only ε(null) move including the state A itself.

Steps for converting NFA with ε to DFA:

Step 1: We will take the ε-closure for the starting state of NFA as a starting state of DFA.

Step 2: Find the states for each input symbol that can be traversed from the present. That means the union of transition value and their closures for each state of NFA present in the current state of DFA.

Step 3: If we found a new state, take it as current state and repeat step 2.

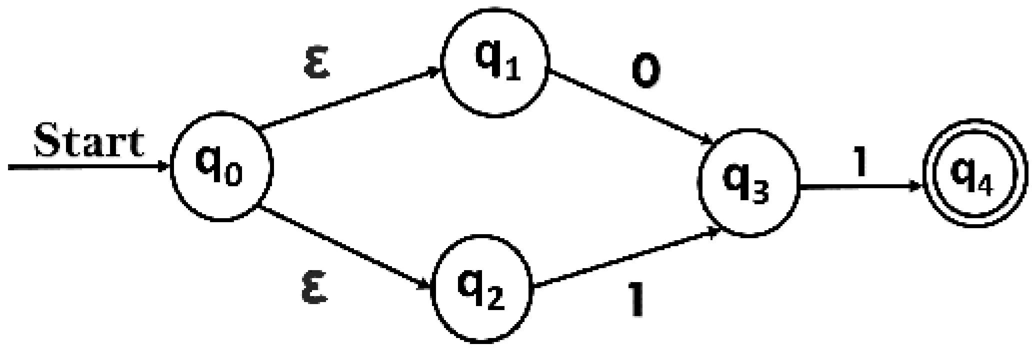
Step 4: Repeat Step 2 and Step 3 until there is no new state present in the transition table of DFA.

Step 5: Mark the states of DFA as a final state which contains the final state of NFA.

Conversion from NFA with ε to DFA

Example 1:

Convert the NFA with ε into its equivalent DFA.



Solution:

Let us obtain ε-closure of each state.

ε-closure {q0} = {q0, q1, q2}

ε-closure {q1} = {q1}

ε-closure {q2} = {q2}

ε-closure {q3} = {q3}

ε-closure {q4} = {q4}

Now, let ε-closure {q0} = {q0, q1, q2} be state A.

Hence

δ'(A, 0) = ε-closure {δ((q0, q1, q2), 0) }

= ε-closure {δ(q0, 0) ∪ δ(q1, 0) ∪ δ(q2, 0) }

= ε-closure {q3}

= {q3} **call it as state B**.

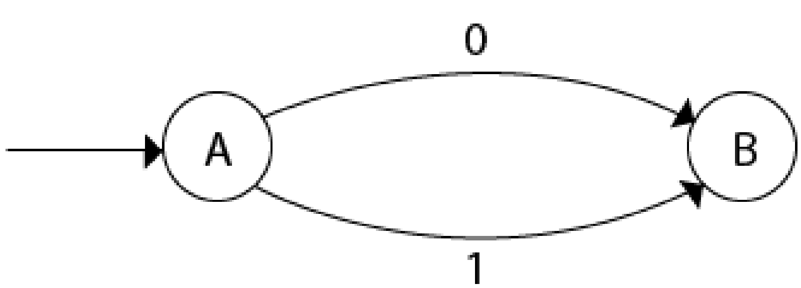
δ'(A, 1) = ε-closure {δ((q0, q1, q2), 1) }

= ε-closure {δ((q0, 1) ∪ δ(q1, 1) ∪ δ(q2, 1) }

= ε-closure {q3}

= {q3} = B.

The partial DFA will be



Conversion from NFA with ε to DFA

Now,

δ'(B, 0) = ε-closure {δ(q3, 0) }
= ϕ

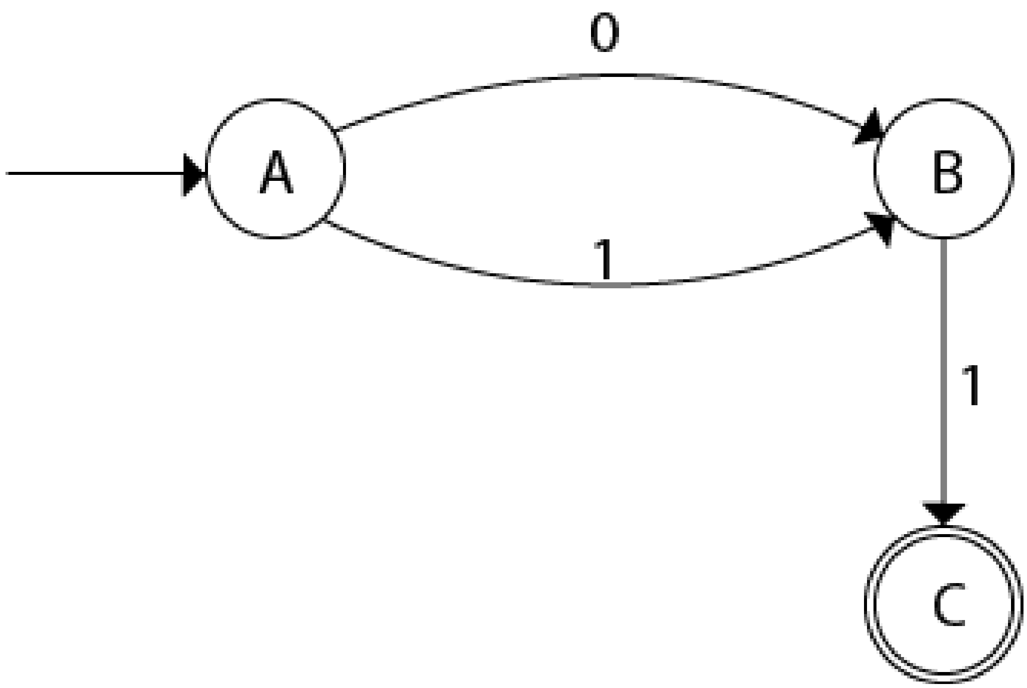
δ'(B, 1) = ε-closure {δ(q3, 1) }
= ε-closure {q4}
= {q4} **i.e. state C**

For state C:

δ'(C, 0) = ε-closure {δ(q4, 0) }
= ϕ

δ'(C, 1) = ε-closure {δ(q4, 1) }
= ϕ

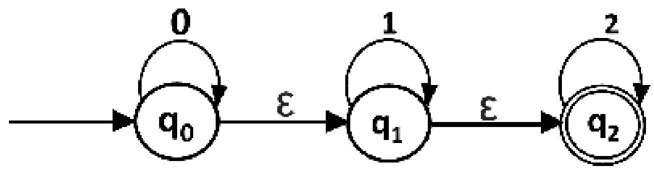
The DFA will be,



Conversion from NFA with ε to DFA

Example 2:

Convert the given NFA into its equivalent DFA.



Solution: Let us obtain the ε-closure of each state.

ε-closure(q0) = {q0, q1, q2}
ε-closure(q1) = {q1, q2}
ε-closure(q2) = {q2}

Now we will obtain δ' transition. Let ε-closure(q0) = {q0, q1, q2} call it as state A.

δ'(A, 0) = ε-closure{δ((q0, q1, q2), 0)}
= ε-closure{δ(q0, 0) ∪ δ(q1, 0) ∪ δ(q2, 0)}
= ε-closure{q0}
= {q0, q1, q2}

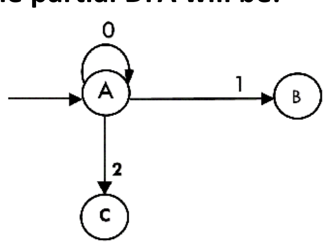
δ'(A, 1) = ε-closure{δ((q0, q1, q2), 1)}
= ε-closure{δ(q0, 1) ∪ δ(q1, 1) ∪ δ(q2, 1)}
= ε-closure{q1}
= {q1, q2} **call it as state B**

δ'(A, 2) = ε-closure{δ((q0, q1, q2), 2)}
= ε-closure{δ(q0, 2) ∪ δ(q1, 2) ∪ δ(q2, 2)}
= ε-closure{q2}
= {q2} **call it state C**

Thus we have obtained

δ'(A, 0) = A
δ'(A, 1) = B
δ'(A, 2) = C

The partial DFA will be:



Now we will find the transitions on states B and C for each input.

Hence

δ'(B, 0) = ε-closure{δ((q1, q2), 0)}
= ε-closure{δ(q1, 0) ∪ δ(q2, 0)}
= ε-closure{ϕ}
= ϕ

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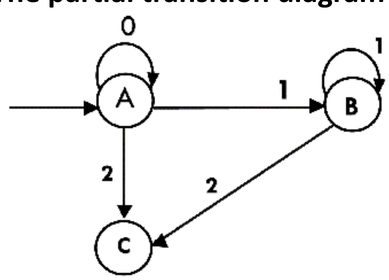
δ'(B, 1) = ε-closure{δ((q1, q2), 1)}
= ε-closure{δ(q1, 1) ∪ δ(q2, 1)}
= ε-closure{q1}
= {q1, q2} **i.e. state B itself**

δ'(B, 2) = ε-closure{δ((q1, q2), 2)}
= ε-closure{δ(q1, 2) ∪ δ(q2, 2)}
= ε-closure{q2}
= {q2} **i.e. state C itself**

Thus we have obtained

δ'(B, 0) = ϕ
δ'(B, 1) = B
δ'(B, 2) = C

The partial transition diagram will be



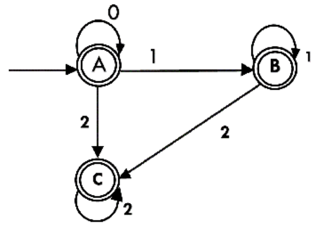
Now we will obtain transitions for C:

δ'(C, 0) = ε-closure{δ(q2, 0)}
= ε-closure{ϕ}
= ϕ

δ'(C, 1) = ε-closure{δ(q2, 1)}
= ε-closure{ϕ}
= ϕ

δ'(C, 2) = ε-closure{δ(q2, 2)}
= {q2}

Hence the DFA is



As A = {q0, q1, q2} in which final state q2 lies hence A is final state. B = {q1, q2} in which the state q2 lies hence B is also final state. C = {q2}, the state q2 lies hence C is also a final state.

Conversion from NFA with ε to DFA

Example 3: **Prove that there exists a DFA for every ε - NFA.**
Solution:

Step 1: Consider $M = (Q, \Sigma, \delta, q_0, F)$ is a NFA with ε. We have to convert this NFA with ε to equivalent DFA denoted by
$M_D = (Q_D, \Sigma, \delta_D, q_0, F_D)$
Then obtain,
ε - closure $(q_0) = \{P_1, P_2, P_3 ... P_n\}$ then $[P_1, P_2, P_3, ... P_n]$ becomes a start state of DFA.
Now $[P_1, P_2, P_3 ... P_n] \in Q_D$

Step 2: We will obtain δ transitions on $[P_1, P_2, P_3, ... P_n]$ for each input.
$\delta_D ([P_1, P_2, P_n], a) = \varepsilon$ - closure $(\delta (P_1, a) \cup \delta (P_2, a) \cup ... \delta (P_n, a))$
$= \cup^n_{i=1}$ ε - closure d (P_i, a)
where a is input $\in \Sigma$.

Step 3: The states obtained $[P_1, P_2, P_3, ... P_n] \in Q_D$. The states containing final state P_i is a final state in DFA.

Example 4: Convert the following NFA with ε to equivalent DFA.

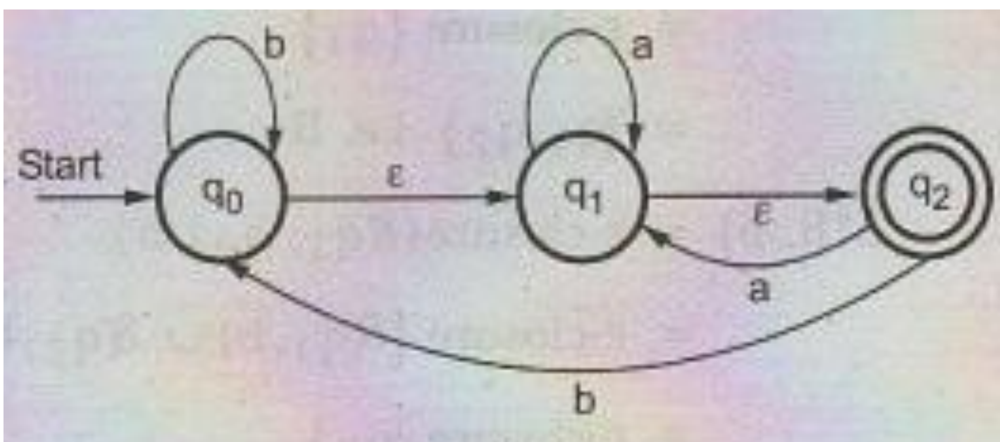


Fig. 1.11.1

Solution:
Step 1: To convert this NFA we will first find ε-closures.
ε - closures $\{q_0\} = \{q_0, q_1, q_2\}$
ε - closures $\{q_1\} = \{q_1, q_2\}$
ε - closures $\{q_2\} = \{q_2\}$

Conversion from NFA with ε to DFA

Step 2: Let us start from ε - closure of start state

ε - closure $\{q_0\} = \{q_0, q_1, q_2\}$ we will call this state as A.

Now let us find transitions on A with every input symbol.

δ (A, a) = ε-closure (δ (A, a))

= ε - closure (δ (q_0, q_1, q_2), a)

= ε - closure $\{(q_0, a) \cup \delta (q_1, a) \cup \delta (q_2, a)\}$

= ε - closure $\{q_1\}$

= $\{q_1, q_2)$. Let us call it as state B.

δ (A, b) = ε-closure (δ (A, b))

= ε - closure (δ (q_0, q_1, q_2), b)

= ε - closure $\{\delta (q_0, b) \cup \delta (q_1, b) \cup \delta (q_2, b)\}$

= ε - closure $\{q_0\}$

= $\{q_0, q_1, q_2\}$ i.e. A.

Hence we can state that,

δ' (A, a) = B

δ` (A, b) = A

Step 3: Now let us find transitions for state B = $\{q_1, q_2\}$

δ' (B, a) = ε - closure (δ (q_1, q_2), a)

= ε - closure $\{q_1\}$

= $\{q_1, q_2\}$ i.e. B

δ` (B, b) = ε - closure (δ ($q_1, q_2)$, b)

= ε - closure $\{\delta (q_1, b) \cup \delta (q_2, b)\}$

= ε - closure $\{q_0\}$

= (q_0, q_1, q_2) i.e. A.

Step 4: Hence the generated DFA is

	a	b
→ A	B	A
B	B	A

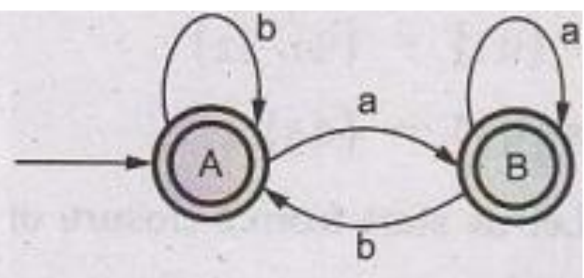


Fig. 1.11.2

Conversion from NFA with ε to DFA

Example 5: Convert the given ε-NFA into its equivalent DFA –

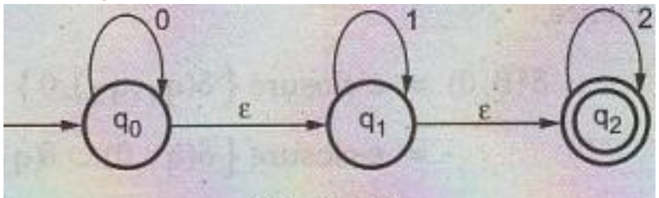


Fig. 1.11.3

Solution: Let us obtain ε - closure of each state.

ε - closure (q_0) = $\{q_0, q_1, q_2\}$

ε - closure (q_1)= $\{q_1, q_2\}$

ε - closure (q_2) = $\{q_2\}$

Now we will obtain d' transition. Let e-closure (q_0) = $\{q_0, q_1, q_2\}$ call it as state A.

δ' (A, 0) = ε - closure { δ ((q_0, q_1, q_2), 0)}

= ε - closure {δ (q_0, 0) U δ (q_1, 0) U δ (q_2, 0)}

= ε - closure {q_0}

= {q_0, q_1, q_2} i.e. state A

δ' (A, 1) = ε - closure { δ ((q_0, q_1, q_2), 1)}

= ε - closure {(q_0,1) U δ (q_1,1) U δ (q_2,1)}

= ε - closure {q_1, q_2}

= {q_1, q_2} Call it as state B

δ' (A, 2) = ε - closure {δ ((q_0, q_1, q_2), 2)}

= ε - closure {δ (q_0, 2) U δ (q_1, 2) U δ (q_2, 2)}

= ε - closure {q_2}

= {q_2} Call it as state C.

Thus we have obtained,

δ` (A, 0) = A

δ` (A, 1) = B

δ` (A,2) = C

i.e.

Now we will find transitions on states B and C for each input.

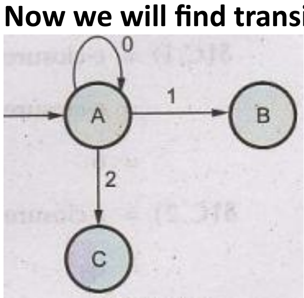


Fig. 1.11.4

Hence,

δ' (B, 0) = ε - closure {δ (q_1, q_2), 0}

= ε - closure {δ (q_1, 0) U δ (q_2, 0)}

= ε - closure {ϕ}

= ϕ

Conversion from NFA with ε to DFA

δ' (B, 1) = ε - closure {δ (q_1, q_2), 1}
= ε - closure {δ (q_1, 1) U δ (q_2, 1)}
= ε - closure {q_1}
= {q_1, q_2) i.e state B itself.
δ` (B, 2) = ε - closure {δ (q_1, q_2), 2}
= ε - closure {δ (q_1, 2) U δ (q_2, 2)}
= ε - closure {q_2}
= {q_2} i.e state C.
Hence,
δ` (B, 0) = ϕ
δ` (B, 1) = B
δ` (B, 2) = C

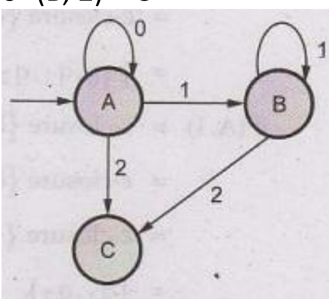


Fig. 1.11.5

The partial transition diagram will be,
Now we will obtain transitions for C:
δ` (C, 0) = ε - closure {δ (q_2, 0)}
= ε - closure {ϕ}
= ϕ
δ` (C, 1) = ε - closure {δ (q_2, 1)}
= ε - closure {ϕ}
= ϕ
δ` (C,2) = ε - closure {δ (q_2, 2)}
= q_2

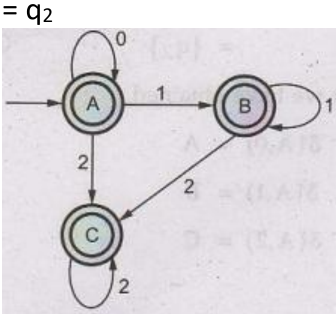
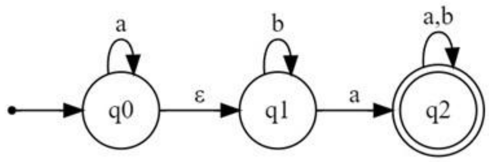


Fig. 1.11.6

Hence the DFA is,
As A = {q_0, q_1, q_2} in which final state q_2 lies hence A is final state in B = {q_1, q_2} the state q_2 lies hence B is also final state in C = {q_2}, the state q_2 lies hence C is also a final state.

Conversion from NFA with ε to DFA

Example 6: Convert the given ε-NFA into its equivalent DFA



Step 1: Finding the Initial State of the DFA

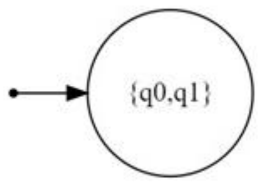
Identify the initial state of the epsilon NFA − In our example, the initial state is q0.

Calculate the epsilon closure of the initial state − The epsilon closure of a state includes all states reachable from that state using only epsilon moves.

- From q0, we can reach q1 with null (epsilon) move
- Therefore, the epsilon closure of q0 is {q0, q1}.

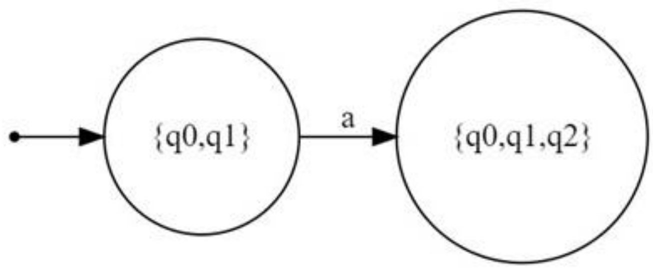
The epsilon closure of the initial state becomes the initial state of the DFA.

DFA so far will look like this −



Step 2: Determining Transitions for Each Input Symbol

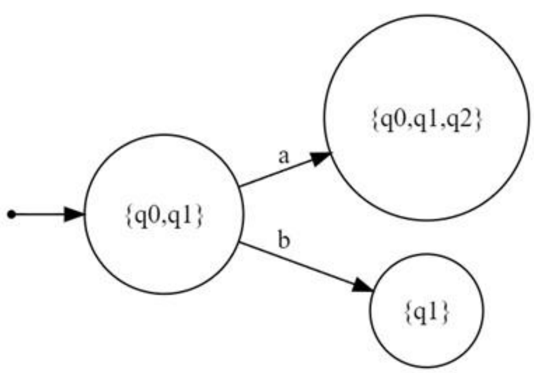
After processing 'a' from the initial state, the DFA will look like this −



Repeat the process for the input symbol 'b' from the initial state {Q0, Q1}

- From q0, using 'b', we can reach: q1 (using epsilon then 'b').
- From q1, using 'b', we can reach: q1.
- The union is {q1}.

After processing 'b' from the initial state, the DFA will look like this −



Conversion from NFA with ε to DFA

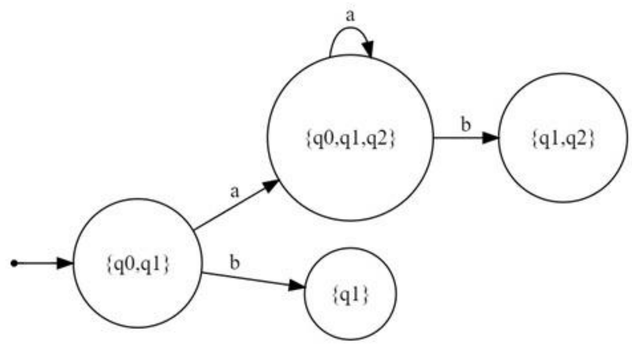
Step 3: Iterating for New States

We now have a new state in our DFA: {Q0, Q1, Q2}.

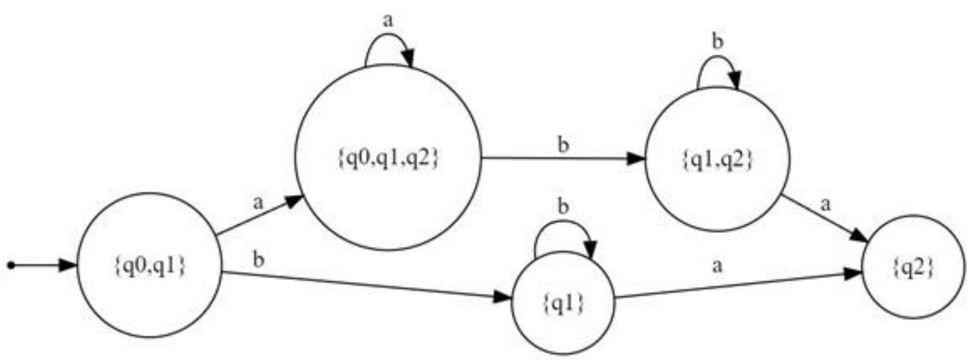
Repeat Step 2 for this new state and any subsequent new states generated, considering both input symbols 'a' and 'b'.

- From {Q0, Q1, Q2}, using 'a', we will reach the same set of states {Q0, Q1, Q2}.
- From {Q0, Q1, Q2}, using 'b', we can reach {Q1, Q2}.

After processing the new state, the DFA will look like this −



Continue this process until no new states are generated in the DFA.



Step 4: Identifying Final States in the DFA

Examine the final states of the epsilon NFA − In our example, the final state is q2.

In the DFA, any state that contains a final state from the epsilon NFA becomes a final state. In our DFA, the final states are: {q0, q1, q2} and {q1, q2}.

The conversion from epsilon NFA to DFA is now complete!