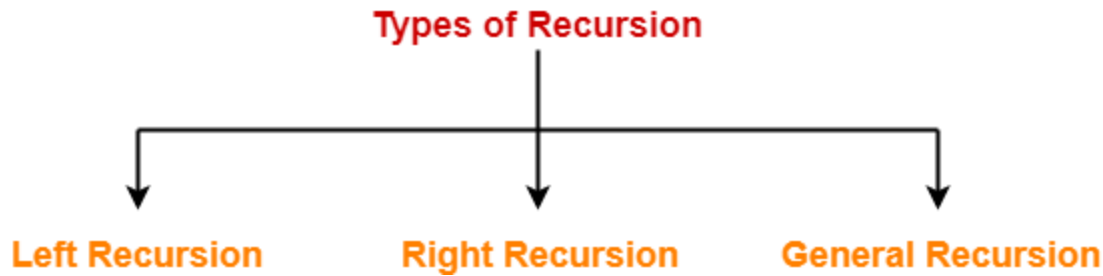


Removal of Left Recursion

Recursion: Recursion can be classified into following three types-



1. Left Recursion: A production of grammar is said to have left recursion if the leftmost variable of its RHS is same as variable of its LHS. A grammar containing a production having left recursion is called as Left Recursive Grammar.

Example-

$S \rightarrow Sa / \epsilon$

(Left Recursive Grammar)

Left recursion is considered to be a problematic situation for Top-down parsers. Therefore, left recursion has to be eliminated from the grammar.

Elimination of Left Recursion

Left recursion is eliminated by converting the grammar into a right recursive grammar.

If we have the left-recursive pair of productions-

$A \rightarrow A\alpha / \beta$

(Left Recursive Grammar)

where β does not begin with an A.

Then, we can eliminate left recursion by replacing the pair of productions with-

$A \rightarrow \beta A'$

$A' \rightarrow \alpha A' / \epsilon$

(Right Recursive Grammar)

This right recursive grammar functions same as left recursive grammar.

2. Right Recursion: A production of grammar is said to have right recursion if the rightmost variable of its RHS is same as variable of its LHS. A grammar containing a production having right recursion is called as Right Recursive Grammar.

Example-

$S \rightarrow aS / \epsilon$

(Right Recursive Grammar)

Right recursion does not create any problem for the Top-down parsers.

Therefore, there is no need of eliminating right recursion from the grammar.

Removal of Left Recursion

PRACTICE PROBLEMS BASED ON LEFT RECURSION ELIMINATION-

Problem-01: Consider the following grammar and eliminate left recursion-

$A \rightarrow ABd / Aa / a$

$B \rightarrow Be / b$

Solution- The grammar after eliminating left recursion is-

$A \rightarrow aA'$

$A' \rightarrow BdA' / aA' / \epsilon$

$B \rightarrow bB'$

$B' \rightarrow eB' / \epsilon$

Problem-02: Consider the following grammar and eliminate left recursion-

$E \rightarrow E + E / E \times E / a$

Solution- The grammar after eliminating left recursion is-

$E \rightarrow aA$

$A \rightarrow +EA / \times EA / \epsilon$

Problem-03: Consider the following grammar and eliminate left recursion-

$E \rightarrow E + T / T$

$T \rightarrow T \times F / F$

$F \rightarrow id$

Solution- The grammar after eliminating left recursion is-

$E \rightarrow TE'$

$E' \rightarrow +TE' / \epsilon$

$T \rightarrow FT'$

$T' \rightarrow \times FT' / \epsilon$

$F \rightarrow id$

Problem-04: Consider the following grammar and eliminate left recursion-

$A \rightarrow AA\alpha / \beta$

Solution- The grammar after eliminating left recursion is-

$A \rightarrow \beta A'$

$A' \rightarrow A\alpha A' / \epsilon$

Problem-05: Consider the following grammar and eliminate left recursion-

$A \rightarrow Ba / Aa / c$

$B \rightarrow Bb / Ab / d$

Solution- This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $A \rightarrow Ba / Aa / c$

Eliminating left recursion from here, we get-

$A \rightarrow BaA' / cA'$

$A' \rightarrow aA' / \epsilon$

Now, given grammar becomes-

$A \rightarrow BaA' / cA'$

$A' \rightarrow aA' / \epsilon$

$B \rightarrow Bb / Ab / d$

Step-02:

Substituting the productions of A in $B \rightarrow Ab$, we get the following grammar-

$A \rightarrow BaA' / cA'$

$A' \rightarrow aA' / \epsilon$

$B \rightarrow Bb / BaA'b / cA'b / d$

Step-03:

Now, eliminating left recursion from the productions of B, we get the following grammar-

$A \rightarrow BaA' / cA'$

$A' \rightarrow aA' / \epsilon$

$B \rightarrow cA'bB' / dB'$

$B' \rightarrow bB' / aA'bB' / \epsilon$

This is the final grammar after eliminating left recursion.

Problem-06: Consider the following grammar and eliminate left recursion-

$S \rightarrow S0S1S / 01$

Solution- The grammar after eliminating left recursion is-

$S \rightarrow 01A$

$A \rightarrow 0S1SA / \epsilon$

Removal of Left Recursion

Problem-07: Consider the following grammar and eliminate left recursion-

$S \rightarrow (L) / a$
 $L \rightarrow L, S / S$

Solution- The grammar after eliminating left recursion is-

$S \rightarrow (L) / a$
 $L \rightarrow SL'$
 $L' \rightarrow ,SL' / \epsilon$

Problem-08: Consider the following grammar and eliminate left recursion-

$S \rightarrow A$
 $A \rightarrow Ad / Ae / aB / ac$
 $B \rightarrow bBc / f$

Solution- The grammar after eliminating left recursion is-

$S \rightarrow A$
 $A \rightarrow aBA' / acA'$
 $A' \rightarrow dA' / eA' / \epsilon$
 $B \rightarrow bBc / f$

Problem-09: Consider the following grammar and eliminate left recursion-

$X \rightarrow XSb / Sa / b$
 $S \rightarrow Sb / Xa / a$

Solution- This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $X \rightarrow XSb / Sa / b$

Eliminating left recursion from here, we get-

$X \rightarrow SaX' / bX'$
 $X' \rightarrow SbX' / \epsilon$

Now, given grammar becomes-

$X \rightarrow SaX' / bX'$
 $X' \rightarrow SbX' / \epsilon$
 $S \rightarrow Sb / Xa / a$

Step-02:

Substituting the productions of X in $S \rightarrow Xa$, we get the following grammar-

$X \rightarrow SaX' / bX'$
 $X' \rightarrow SbX' / \epsilon$
 $S \rightarrow Sb / SaX'a / bX'a / a$

Step-03:

Now, eliminating left recursion from the productions of S , we get the following grammar-

$X \rightarrow SaX' / bX'$
 $X' \rightarrow SbX' / \epsilon$
 $S \rightarrow bX'aS' / aS'$
 $S' \rightarrow bS' / aX'aS' / \epsilon$

This is the final grammar after eliminating left recursion.

Problem-10: Consider the following grammar and eliminate left recursion-

$S \rightarrow Aa / b$
 $A \rightarrow Ac / Sd / \epsilon$

Solution- This is a case of indirect left recursion.

Step-01:

First let us eliminate left recursion from $S \rightarrow Aa / b$

This is already free from left recursion.

Step-02:

Substituting the productions of S in $A \rightarrow Sd$, we get the following grammar-

$S \rightarrow Aa / b$
 $A \rightarrow Ac / Aad / bd / \epsilon$

Step-03:

Now, eliminating left recursion from the productions of A , we get the following grammar-

$S \rightarrow Aa / b$
 $A \rightarrow bdA' / A'$
 $A' \rightarrow cA' / adA' / \epsilon$

This is the final grammar after eliminating left recursion.