...y. The polar and
...ber represent the same

...left part of the equation as

...n θ

...l and ~~imag~~ imaginary
...s the following system of

...lculate θ, we can
... the 2nd equation by the
...to get.

...$\tan\left(\frac{b}{a}\right)$

Complex Polar → Complex Rectangular

~~$z = r e^{i\theta}$~~ $\quad z = r \cdot e^{i\theta} \qquad re^{i\theta} = r \cdot \cos\theta + i\sin\theta \cdot r = z$

$$z = \underbrace{r\cos\theta}_{\text{Real}} + \underbrace{i r \sin\theta}_{\text{imag}}$$

Multiplying Polar complex numbers.

<<Check computer notes, too lazy>>

Linear Algebra

$$\begin{bmatrix} a_{0,0} & a_{0,1} & \dots & a_{0,m-1} \\ a_{1,0} & \ddots & & \\ a_{2,0} & & & \\ \vdots & & & \\ a_{n-1,0} & & & a_{n-1,m-1} \end{bmatrix}$$ Syntax of a Matrix

~~Scalar~~ Scalar $= 1 \times 1$ matrix ex $[3] = 3$

Quantum computing uses complex-valued matrices: the elements of a matrix can be complex numbers. This

ex:

$$\begin{bmatrix} 1 & i \\ -2i & 3+4i \end{bmatrix}$$

Vectors

A vector is a $n \times 1$ matrix ex $V = \begin{bmatrix} 1 \\ 2i \\ 3+4i \end{bmatrix}$

Adding matrixes.

both matrixes need to be the same size

ex:

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ 10 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 12 & 3 \end{bmatrix}$$

note: same applies for Substraction

Also, $A + B = B + A$

$$(A+B) + C = A + (B+C)$$

Scalar Multiplication

rules:

$$x \cdot (yA) = (x \cdot y) \cdot A$$

$$x(A+B) = xA + xB$$

$$(x+y)A = xA + yA$$

ex $x = 10$ $A = \begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$

$$x \cdot A = \begin{bmatrix} 10 & 40 \\ -10 & 0 \end{bmatrix}$$

Matrix Multiplication

$$A_{n \cdot m} \cdot B_{m \cdot k} = C_{n \cdot k}$$

ex

$$\underset{2 \cdot 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}} \cdot \underset{3 \cdot 1}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} = \underset{2 \cdot 1}{\begin{bmatrix} 14 \\ 32 \end{bmatrix}}$$

Identity Matrix

ex

$$\underset{n \cdot n}{\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 1 \end{bmatrix}}$$

if $A_{n \cdot m}$

$$AI_m = I_n A = A$$

14
18
32

Inverse Matrix

If a matrix times A ...
equals In, then th...

$$A \cdot A^{-1} = I_n = \dots$$

A^{-1} acts as the multi...

Matrix Determinant

A square Matrix has a ...
with the determinant of ...
is invertible if and only ...

for a 2×2 m...

$$|A| = A_{0,0} \cdot A_{1,1} - A_{0,1} \cdot \dots$$

ex:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Inversion of matrix

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{|A|} \begin{bmatrix} d & \dots \\ -c & \dots \end{bmatrix}$$

2×2

...ame size

applies for Substitution

10 $A = \begin{bmatrix} 7 & 4 \\ -1 & 0 \end{bmatrix}$

$A = \begin{bmatrix} 10 & 40 \\ -10 & 0 \end{bmatrix}$

14
18
32

$I_n A = A$

Inverse Matrix

If a matrix A times another matrix (both of size $n \times n$) equals I_n, then this second Matrix $= A^{-1}$.

$$A \cdot A^{-1} = I_n = A^{-1} \cdot A$$

A^{-1} acts as the multiplicative inverse of A.

Matrix Determinant

A square Matrix has a property called the determinant, with the determinant of a matrix A being written as $|A|$. A matrix is invertible if and only if, determinant $\neq 0$

For a 2×2 matrix A, the determinant is defined as

$$|A| = A_{0,0} \cdot A_{1,1} - A_{0,1} \cdot A_{1,0}$$

ex:

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad |A| = 1 \cdot 4 + 2 \cdot 3 = 10$

Inversion of matrix

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2×2

Transpose

A^T, is essentially a reflection of the matrix across the diagonal:

$$A^T_{i,j} = A_{j,i}$$

Given a matrix $n \times m$, A, its transpose is the $m \times n$ matrix A^T

$$A = \begin{bmatrix} x_{0,0} & x_{0,1} & \cdots & x_{0,m-1} \\ x_{1,0} & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \\ x_{n-1,0} & x_{n-1,1} & \cdots & x_{n-1,m-1} \end{bmatrix}$$

then

$$A^T = \begin{bmatrix} x_{0,0} & x_{1,0} & \cdots & x_{n-1,0} \\ x_{0,1} & x_{1,1} & & x_{n-1,1} \\ \vdots & & & \\ x_{0,m-1} & x_{1,m-1} & & x_{n-1,m-1} \end{bmatrix}$$

ex

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

Symmetric Matrix

$A = A^T$ ex $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 4 & 3 \\ 5 & 3 & 6 \end{bmatrix}$

The transpose of a matrix product is equal to the product of transposed matrices, taken in reverse order

$$(AB)^T = B^T \cdot A^T$$

Matrix conjugates

denoted as $\overline{A}$, this operation makes sense of only for complex-valued matrices: it involves taking the complex conjugate of every element of the matrix.

$$A = \begin{bmatrix} z_{0,0} & z_{0,1} & \cdots & z_{0,m-1} \\ z_{1,0} & z_{1,1} & \cdots & z_{1,m-1} \\ \vdots & \vdots & \ddots & \vdots \\ z_{m-1,0} & z_{m-1,1} & \cdots & z_{n,m} \end{bmatrix}$$

same thing but $\begin{bmatrix} \overline{z} & \overline{z} & \overline{z} \\ \overline{z} & \overline{z} & \overline{z} \\ \overline{z} & \overline{z} & \overline{z} \end{bmatrix}$

reminder:

conjugate of a complex:

$z = a + bi =$

$\overline{z} = a - bi$

$$\overline{AB} = (\overline{A}) \cdot (\overline{B})$$

ex

return conjugate of A:

$$A = \begin{bmatrix} 1+5i & 2 \\ 3-6i & 4i \end{bmatrix} \quad \begin{bmatrix} 1-5i & 2-0i \\ 3-(-6i) & 0-4i \end{bmatrix}$$

Adjoint Matrix operation

$$A^{\dagger} = \overline{(A^T)} = \overline{(A)}^T$$

A Matrix known as a hermitian or self-adjoint if it equals its own adjoint for example, the following matrix is Hermitian

$$\begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix}$$

The adjoint matrix product can be calculated as

$$(AB)^{\dagger} = B^{\dagger} \cdot A^{\dagger}$$

ex:

$$A = \begin{bmatrix} 1 & i \\ -i & 2 \end{bmatrix} \qquad \bar{A} = \begin{bmatrix} 1-0i & 0-i \\ 0-(-i) & 2-0i \end{bmatrix} = \begin{bmatrix} 1 & -i \\ i & 2 \end{bmatrix}$$

$$(\bar{A})^T = \begin{bmatrix} 1 & i \\ i & 2 \end{bmatrix} \longrightarrow (\bar{A})^T = A = A^\dagger$$

Unitary Matrices

Unitary Matrices are very important for quantum computing.

A matrix is unitary when it is ~~invertible~~ invertible, and it's inverse is equal to it's adjoint: $U^{-1} = U^\dagger$.

That is, an $n \times n$ square matrix U is unitary if and only if $UU^\dagger = U^\dagger U = I_n$

ex is this matrix unitary?;

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \qquad \cancel{\frac{1}{\sqrt{2}} \cdot A^\dagger \cdot A = \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}} = \begin{bmatrix} 0 & \cdot \\ & \end{bmatrix}$$

$$A^\dagger = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} \qquad \cancel{A^{-1}?} \qquad A \cdot A^\dagger = \begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix} \begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$$

The online example is wrong.

Inner Products

Given 2 vectors V and W of the same size, their inner product $\langle V, W \rangle$ is defined as a product of matrices $V^\dagger$ and W

$$\langle V, W \rangle = V^\dagger \cdot W$$

A $1 \times n$ matrix (the adjoint of a $n \cdot 1$ matrix vector) multiplied by an $n \times 1$ vector results in a 1×1 scalar.

An immediate application ~~of~~ of the inner product in computing the vector norm.

The norm of vector V is defined as $\|V\| = \sqrt{\langle V, V \rangle}$

A vector is called normalized if its norm == 1

ex

$$\langle V, W \rangle = ? = V^\dagger \cdot W$$

$$V = \begin{bmatrix} -6 \\ 9i \end{bmatrix} \quad W = \begin{bmatrix} 3 \\ -8 \end{bmatrix}$$

~~$V^\dagger \cdot W = [$~~

$$V^T = \cancel{} = [-6 \;\; 9i] \quad \overline{V}^T = [-6 \;\; -9i] = V^\dagger$$

$$V^\dagger \cdot W = [-6 \;\; -9i] \cdot \begin{bmatrix} 3 \\ -8 \end{bmatrix} = \overset{1 \times 1}{[-18 + 72i]}$$

---

ex2: Return the normalized vector V, defined as $\frac{V}{\|V\|}$ where

$$V = \begin{bmatrix} -6 \\ 8i \end{bmatrix} \quad \|V\| = ?$$

$$\|V\| = \sqrt{\langle V, V \rangle} = \sqrt{V^\dagger \cdot V} = \sqrt{[-6 \;\; -8i] \cdot \begin{bmatrix} -6 \\ 8i \end{bmatrix}} = \sqrt{[36 + (-64i)]} = \sqrt{36 + 64} = 10$$

$$\frac{V}{\|V\|} = \begin{bmatrix} -6/10 \\ 8/10\, i \end{bmatrix}$$

Outer product

The outer product ~~to~~ of 2 vectors V and W are defined as $V \cdot W^t$. that is, the outer product of an $n \times 1$ vector and an $m \times 1$ vector is a $n \times m$ matrix. if we denote the outer product of V and W as X, $X_{i,j} = V_i \cdot \overline{W_j}$.

ex Find outer product of VW^t

$$V = \begin{bmatrix} -3i \\ 9 \end{bmatrix} \quad W = \begin{bmatrix} 9i \\ 2 \end{bmatrix}$$

$$VW^t = \underset{2 \times 1}{\begin{bmatrix} -3i \\ 9 \end{bmatrix}} \cdot \underset{1 \times 2}{\begin{bmatrix} -9i & 2 \end{bmatrix}} = \begin{bmatrix} -27 & -6 \\ -81 & 18 \end{bmatrix}$$

Tensor Product $\otimes$

ex return the tensor product of these 2 matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} \quad A \otimes B = \begin{bmatrix} 1 \cdot B & 2 \cdot B \\ 3 \cdot B & 4 \cdot B \end{bmatrix}$$

$$\begin{bmatrix} 1 \cdot 5 & 1 \cdot 6 & 2 \cdot 5 & 2 \cdot 6 \\ 1 \cdot 7 & 1 \cdot 8 & 2 \cdot 7 & 2 \cdot 8 \\ 3 \cdot 5 & 3 \cdot 6 & 4 \cdot 5 & 4 \cdot 6 \\ 3 \cdot 7 & 3 \cdot 8 & 4 \cdot 7 & 4 \cdot 8 \end{bmatrix} = \begin{bmatrix} 5 & 6 & 10 & 12 \\ 7 & 8 & 14 & 16 \\ 15 & 18 & 20 & 24 \\ 21 & 24 & 28 & 32 \end{bmatrix}$$