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Car-following: a historical review

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Abstract

In recent years, the topic of car-following has become of increased importance in traffic engineering and safety research. Models of this phenomenon, which describe the interaction between (typically) adjacent vehicles in the same lane, now form the cornerstone for many important areas of research including (a) simulation modelling, where the car-following model (amongst others) controls the motion of the vehicles in the network, and (b) the functional definition of advanced vehicle control and safety systems (AVCSS), which are being introduced as a driver safety aid in an effort to mimic driver behaviour but remove human error. Despite the importance of this area however, no overview of the models availability and validity exists. It is the intent of this paper therefore to briefly assess the range of options available in the choice of car-following model, and assess just how far work has proceeded in our understanding of what, at times, would appear to be a simple process. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Car-following; Microscopic simulation modelling; Calibration; Time-series

Contents

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1. Introduction

Car-following models, which describe the processes by which drivers follow each other in the traffic stream have been studied for almost half a century (Pipes, 1953). However, the many relationships available are usually crude and often not rigorously understood or proven. Car-following itself, forms one of the main processes in all microscopic simulation models as well as in modern traffic flow theory, which attempts to understand the interplay between phenomena at the individual driver level and global behaviour on a more macroscopic scale (e.g., Krauss, 1997). In recent years, the importance of such models has increased further, with 'normative' behavioural models forming the basis of the functional definitions of advanced vehicle control and safety systems (AVCSS). Other systems, such as autonomous cruise control (ACC), seek to replicate human driving behaviour through partial control of the accelerator, while removing potential hazards that may occur through driver misperception and reaction time. (Establishment of an understanding of normative driver behaviour was recently ranked as the second most important area for development out of 40 problem statements, by an expert human factors-AVCSS panel (ITS America, 1997).)

It is clear then that a detailed understanding of this key process is now becoming increasingly important as opportunities for using new techniques and technologies become available. This paper therefore seeks to provide a systematic re-examination of these models, their calibration to time-series data, and their evaluation.

2. Car-following models

2.1. Gazis–Herman–Rothery (GHR) model

The GHR model is perhaps the most well-known model and dates from the late fifties and early sixties. Its formulation is

$$a_n(t) = cv_n^m(t)\frac{\Delta v(t-T)}{\Delta x^l(t-T)}, \tag{1}$$

where a_n is the acceleration of vehicle n implemented at time t by a driver and is proportional to, v the speed of the nth vehicle, Δx and Δv, the relative spacing and speeds, respectively between the nth and $n-1$ vehicle (the vehicle immediately in front), assessed at an earlier time $t-T$, where T is the driver reaction time, and m, l and c are the constants to be determined.

The first prototype car-following model that would eventually lead to this formula was put forward in the late 50s by Chandler, Herman and Montroll (1958) at the General Motors research labs in Detroit (at the same time as worked by Kometani & Sasaki, 1958, in Japan). This was based on an intuitive hypothesis that a driver's acceleration was proportional to Δv, or deviation from a set following distance, $k - \Delta x$, which could itself be speed dependent. Initial calibration of this model used wire-linked vehicles to examine the responses of 8 test subjects to a 'realistic' speed profile of a lead vehicle (which varied from 10 to 80 mph), over 30 min on a test track. The analysis of the resulting data, assuming the presence of terms linear in both Δv and Δx led to two conclusions. Firstly that Δx contributed little to the following relationship (with a high certainty, $r^2 > 0.8$) and hence could be rejected (producing a sub case of the GHR model with $l = m = 0$), and secondly, that the scaling constant showed a high variation between subjects (0.17–0.74 s) as did T (1.0–2.2 s).

Rapid development to this model subsequently followed, with Herman, Montroll, Potts and Rothery (1959), suggesting that the difference of the r^2 value from unity for the Δv term may have been due to spontaneous fluctuations in the drivers acceleration which were impossible to avoid. They argued that the differences were representative of the maximum amount of control a driver was able to retain over the accelerator pedal, i.e., it would not be possible to apply exactly the correct pressure to produce the desired acceleration. Test track experiments using four subjects at a 'constant speed' showed that this fluctuation was of the order of $\pm 0.01g$ (with a small standard deviation of 2%), and was found to be speed independent. (Further details of this investigation may be found in Montroll (1959)).

Gazis, Herman and Potts (1959) subsequently attempted to derive a macroscopic relationship describing speed and flow using the microscopic equation as a starting point. The mis-match between the macroscopic relationship they obtained from the microscopic equation, and other macroscopic relationships in use at the time, led to the hypothesis that the algorithm should be amended by the introduction of a $1/\Delta x$ term into the sensitivity constant ($c \to c/\Delta x$), in order to minimise the discrepancy between the two approaches. This now gave a model with $m = 0$ and $l = 1$. Herman and Potts (1959) performed a new series of wire linked vehicle experiments in order to calibrate the new formulation of the model, this time conducting tests on real roads in 3 of the main New York tunnels, using 11 subjects, over 4–16 runs, of on average, 4 min each. Δx was varied from ~ 15–50 m, and the data obtained produced a good fit to the new $m = 0$, $l = 1$ model, with r^2 values ranging from 0.8 to 0.98, with an average reaction time of $T = 1.2$ s and the new $c = 19.8$ (ft/s) (compared with a recalculated scalar from the first experiment of 27.4).

Subsequently, Edie (1960) attempted to match the $m = 0$, $l = 1$ model to new macroscopic data in a similar manner to Gazis, Herman and Potts, finding that another amendment should be made to the sensitivity constant, namely, the introduction of the velocity dependant term. This produced a new model with $m = 1$ and $l = 1$. This approach was used by Gazis, Herman and Rothery (1961) to investigate the sensitivity of their macroscopic relationships, to variations in the magnitude of the v and Δx terms, by introducing the general scaling constants m and l, respectively. Analysis based on 18 data sets found that all the combinations of m and l tested produced very similar r^2 values, with the most favourable combination falling between $m = 0$–2, and $l = 1$–2. (Edie's formulation was shown to be better at low flow due to its ability to predict a finite speed as density approaches zero.) This investigation was the first to propose that two separate relationships could be used in the description of traffic flow, one for non-congested, and one for congested traffic.

Several similar investigations occurred during the following 15 years, in the attempt to define the 'best' combination of m and l. Among these the most notable are the following:

(a) May and Keller (1967), using new data sets, found optimal integer solutions of $m = 1$, $l = 3$, (or assuming non-integer values, $m = 0.8$ and $l = 2.8$ with a scaling constant of approx 1.33×10^{-4}).
(b) Heyes and Ashworth (1972), in an attempt to relate the generalised GHR model to the perceptual models being initially investigated at the time by Michaels (detailed later), produced a relationship, where the stimulus is taken as $\Delta v/\Delta x^2$ and the sensitivity constant as the time headway Δt^P. This constant was evaluated using data from the Mersey tunnel in the UK as 0.8, essentially corresponding to an $m = -0.8$, $l = 1.2$ model.
(c) Ceder and May (1976), using a far larger number of data sets than ever before, found an optimum of $m = 0.6$ and $l = 2.4$. However, their main advance was in acknowledging the 'two regime' approach, that fitted the observed data better than using a single relationship. These relationships described behaviour in the uncongested regime by the use of $m = 0$ and $l = 3$, and in congested conditions by $m = 0$ and $l = 0$–1.

The next advance in microscopic calibration was made by Treiterer and Myers (1974), who used airborne film footage of a flow breakdown to monitor the paths of a large number of vehicles, from which they extracted measurements of v and Δx. Again assuming that behaviour may in some way be different according to what the driver is required to do, they split their analysis to separately consider the acceleration and deceleration phases of car-following, determining that two differing relationships could exist, one (acceleration) with $m = 0.2$, $l = 1.6$, and the other (deceleration) with $m = 0.7$ and $l = 2.5$. It is interesting to compare this with the finding of Hoefs (1972) who found $m = 1.5$, $l = 0.9$ for accelerating vehicles, $m = 0.2$ and $l = 0.9$ for those decelerating without braking, and $m = 0.6$, $l = 3.2$ for those decelerating using brakes, with a sensitivity constant that increases as one progresses throughout these types.

Ceder (1976, 1978) proposed yet another modification to the GHR model (initially in order to attain a better macroscopic fit), in which the traditional sensitivity term of $v^m/\Delta x^1$ was replaced by $A^{-S/\Delta x}/\Delta x^2$, where S is the jam spacing and A ~ 0 in free flow and between 1–10 in congested conditions. In a later publication Ceder (1979) attempts to microscopically justify his assumptions, based on the belief that the GHR equation is incorrect because it could not reproduce the 'spiral trajectories' observed by Gordon (1971) and Hoefs (1972). (This is not actually the case and spiral trajectories may be produced by the GHR model under suitable conditions.)

Since the late 70s' the GHR model has seen less and less frequent investigation and use, with only two investigations being of note:
(a) Aron (1988) used an instrumented vehicle to collect data on car-following in a range of conditions in Paris. The data totals about 60 min, collected at an average speed of only 7 m/s and spacing of 14 m. In analysis, he splits the responses into three phases, finding for deceleration, $c = 2.45$, $m = 0.655$ and $l = 0.676$, while for 'steady state driving', $c = 2.67$, $m = 0.26$ and $l = 0.5$, and for acceleration $c = 2.46$, $m = 0.14$, and $l = 0.18$.
(b) Lastly, Ozaki (1993) used 90 min of data extracted from video film taken of a motorway from the 32nd floor of a city office building. This gave a 160-m field of view, and data were obtained on the passage of a total of 2000 vehicles. He concluded that the optimum parameter combinations are $c = 1.1$, $m = 0.9$ and $l = 1$ for deceleration and $c = 1.1$, $m = -0.2$ and $l = 0.2$ for acceleration. It should be noted that with such a small field of view it would only have been possible to extract a time-series for each vehicle of <10 s.

A summary of the varying parameter combinations to emerge from research on the 'GHR' equation is given in Table 1.

As we have seen, a great deal of work has been performed on the calibration and validation of the GHR model. However, it is now being used less frequently, significantly because of the large number of contradictory findings as to the correct values of m and l. This may be for two reasons. Firstly, following behaviour is likely to vary with traffic and flow conditions, and microscopic analysis at least confirms this in part, e.g., Rockwell and Treiterer (1966). Secondly, many of the empirical investigations have taken place at low speeds or in extreme stop start conditions, which may not reflect more general car-following behaviour. Removing the results of these experiments from Table 1 we produce in Table 2, the remaining experiments that may be considered to contribute.

Even within this reduced group there is a significant spread of values, and the fact that analysis within the first two experiments did not consider their data in differing phases would mean that the values obtained are in some sense averages. Next one should consider the experimental error likely to be present in the latter two experiments, where data was collected using either aerial

Table 1
Summary of optimal parameter combinations for the 'GHR' equation[a]

Source	m	l	Approach
Chandler et al. (1958)	0	0	Micro
Gazis, Herman and Potts (1959)	0	1	Macro
Herman and Potts (1959)	0	1	Micro
Helly (1959)	1	1	Macro
Gazis et al. (1961)	0–2	1–2	Macro
May and Keller (1967)	0.8	2.8	Macro
Heyes and Ashworth (1972)	−0.8	1.2	Macro
Hoefs (1972) (dcn no brk/dcn brk/acn)	1.5/0.2/0.6	0.9/0.9/3.2	Micro
Treiterer and Myers (1974) (dcn/acn)	0.7/0.2	2.5/1.6	Micro
Ceder and May (1976) (Single regime)	0.6	2.4	Macro
Ceder and May (1976) (uncgd/cgd)	0/0	3/0–1	Macro
Aron (1988) (dcn/ss/acn)	2.5/2.7/2.5	0.7/0.3/0.1	Micro
Ozaki (1993) (dcn/acn)	0.9/−0.2	1/0.2	Micro

[a] *Key*: dcn/acn: deceleration/acceleration; brk/no brk: deceleration with and without the use of brakes; uncgd/cgd: uncongested/congested; ss: steady state.

Table 2
Most reliable estimates of parameters within the GHR model[a]

Source	m	l	Approach
Chandler et al. (1958)	0	0	Micro
Herman and Potts (1959)	0	1	Micro
Hoefs (1972) (dcn no brk/dcn brk/acn)	1.5/0.2/0.6	0.9/0.9/3.2	Micro
Treiterer and Myers (1974) (dcn/acn)	0.7/0.2	2.5/1.6	Micro
Ozaki (1993) (dcn/acn)	0.9/−0.2	1/0.2	Micro

[a] *Key*: dcn/acn: deceleration/acceleration; brk/no brk: deceleration with and without the use of brakes.

techniques or data collected from a roof top. As the vehicle to be measured is such a long way from the point of recording, its size within each frame of film will be small, producing a large inaccuracy in the measured values of distance and separation, which must give rise to a steadily rising inaccuracy, that will be largest in the calculation of the resultant acceleration.

Thus, the only data which may be reliable may be that of Hoefs, which indicates that the sensitivity of ones deceleration to one's own speed increases as the 'importance' of the situation to one's safety increases, and that the sensitivity to $1/\Delta x$ increases as does one's concern with making progress (acceleration). These findings regarding speed dependence are generally in agreement with those of Treiterer and Myers, and Ozaki. However the findings on the sensitivity to $1/\Delta x$ differ considerably. The lack of conclusive evidence as to the behaviour of this equation has lead to its general demise, although the model has been 'resurrected' by Low and Addison (1995) who have started to experiment with a GHR model ($c = 0.3,\ m = 0,\ l = 1$) with an additional term, cubic in the distance between actual and desired separation ($c_2 = 30$). No calibration has been performed on this model to date.

2.2. Safety distance or collision avoidance models (CA)

The original formulation of this approach dates to Kometani and Sasaki (1959). The base relationship does not describe a stimulus-response type function as proposed by the GHR model, but seeks to specify a safe following distance (through the manipulation of the basic Newtonian equations of motion), within which a collision would be unavoidable, if the driver of the vehicle in front were to act 'unpredictably'. The full original formulation is as follows:

$$\Delta x(t-T) = \alpha v_{n-1}^2(t-T) + \beta_l v_n^2(t) + \beta v_n(t) + b_0. \tag{2}$$

Data for calibration were generated by a pair of test vehicles driving on a city street, and collected using a cine film camera at the top of a roadside building. The observed road section covered, ~ 200 m and with an average speeds of <45 kph and a total of 22 test runs, it can be deduced that about 310 s of data was available for analysis, with a resolution of 1/8 s. The best fit to the above relationship, which was quite sharply peaked at an r^2 of 0.75, occurs for the following parameter set:

$$\Delta x(t-0.5) = 0.00028(-v_{n-1}^2(t-0.5) + v_n^2(t)) + 0.585 v_n(t) + 4.1. \tag{3}$$

A second experiment used a faster test track and speeds varied between 40 and 60 kph using 2 subjects, yielding best fit parameters of $T = 0.75$, $\beta = 0.78$ and $\beta_1 = -0.0084$ (40 times as large as before), although it should be noted that in this case the best r^2 for one of the subjects was 0.25 and 0.95 for the other.

The next major development of this model was made by Gipps (1981), in which he considered several mitigating factors that the earlier formulation neglected. These were that drivers will allow an additional 'safety' reaction time equal to $T/2$, (it can be shown that this condition is sufficient to avoid a collision under all circumstances), and that the kinetic terms in the above formula are related to braking rates of $-1/2b_n$, b_n the maximum braking rate that the driver of the nth vehicle wishes to use, and $-1/2b^*$, b^* the maximum braking rate of the $(n-1)$th vehicle that the nth driver believes is likely to be used. (Kometani's kinetic coefficients would describe a situation in which either driver is expected to brake at over 1700 m/s^2, although this may be because the experiment was conducted at such a low speed, and that subjects may have been given instructions which biased their driving behaviour.) Gipps offers no calibration of his parameters, but instead performed simulations using 'realistic' values ($b_n \sim b^* \sim -3$ m/s^2), finding that his model produced realistic behaviour on the propagation of disturbances, both for a vehicle pair and for a platoon of vehicles.

Since the developments by Gipps, the CA model continues to see widespread use in simulation models. These include, the UK DoTs SISTM model (McDonald, Brackstone & Jeffery, 1994), the SPEACS model, used in Italy and France as part of the PROMETHEUS programme (e.g., Broqua, Lerner, Mauro & Morello, 1991), INTRAS and CARSIM in the USA (e.g., Benekohal & Treiterer, 1989), and has recently seen use by Kumamoto, Nishi, Tenmoku and Shimoura (1995) in Japan. Part of the attractiveness of this model is that it may be calibrated using common sense assumptions about driver behaviour, needing (in the most part) only the maximal braking rates that a driver will wish to use, and predicts other drivers will use, to allow it to fully function. Although it produces acceptable results on, for example, comparing the simulated propagation of disturbances against empirical data, there are a number of problems which cannot easily be solved. For example, if one examines the 'safe headway' concept, we see that this is not a totally valid starting point, as in practice a driver may consider conditions several cars down stream, basing his assumption of how hard the vehicle in front will decelerate on the 'preview information' obtained.

2.3. Linear (Helly) models

Although the first model suggested by Chandler, Herman and Montroll as the first stage in the development of the GHR equation was linear, this class of models is generally attributed to Helly (1959). He proposed a model that included additional terms for the adaptation of the acceleration according to whether the vehicle in front (and the vehicle two in front) was braking. We shall return to these terms later, but the simplified model is as follows:

$$a_n(t) = C_1 \Delta v(t-T) + C_2(\Delta x(t-T) - D_n(t)), \quad D_n(t) = \alpha + \beta v(t-T) + \gamma a_n(t-T), \tag{4}$$

where $D(t)$ is a desired following distance. Helly borrowed directly from earlier work in his determination of C_1, produced by data taken from 14 drivers, with the best fit parameters almost all being produced at $r^2 > 0.8$, ranging from $T = 0.5$–2.2 and $C_1 = 0.17$–1.3, with averages of 0.75 and 0.5, respectively. Next C_2 was estimated by setting the Δv and Δx terms to be equal and opposite (producing no acceleration, i.e., free driving) when a vehicle detects a stationary object. This produced a final equation of

$$a = 0.5\Delta v(t-0.5) + 0.125(\Delta x(t-0.5) - D_n(t)), \quad D_n(t) = 20 + v(t-0.5) \tag{5}$$

(It is noteworthy that Helly commented that he believed that C_1 in future investigations should be made spacing dependant ($\sim 1/\Delta x$), and T speed dependant, essentially producing the $m = 0$, $l = 1$ GHR model).

The next major calibration of this model was performed by Hanken and Rockwell (1967) and Rockwell, Ernst and Hanken (1968), who conducted experiments on both 'free roads' (where each (of three) subjects experienced the same lead vehicle acceleration pattern), and on a congested urban freeway, where the subject was exposed to real traffic variations. Each of the three test runs per subject were conducted over a 10-min period using a wire linked vehicle, similar to those used by the earlier General Motors team. Each variable was sampled at 0.5 s intervals and experiments were conducted over a speed range of 20–60 mph with headway varying from 40 to 250 ft. Maximum r^2 was found for a time delay of 2 s, and the resultant following equation found, by partitioning the data, to be highly linear in nature.

$$\begin{aligned} a(t+T) &= 0.016 + 0.058(\Delta x - 134) + 0.498(v_{n-1} - 31.6) - 0.546(v_n - 31.6) \\ &= 0.058\Delta x + 0.498\Delta v - 0.048 v_{n-1} - 6.24. \end{aligned} \tag{6}$$

Some noteworthy results are that the time delay in the relationship with the speed of the front vehicle seems to decrease each time a run is made, giving some credence to the belief that some form of anticipation occurs. Later simulations show that although the model describes low acceleration patterns quite well, it gives significant errors when the magnitudes of the fluctuations are increased, producing noticeably higher headways than those observed.

The Helly model was used again, in the mid seventies when Bekey, Burnham and Seo (1977) attempted to derive a car-following model using traditional methods from the design of optimal control systems. The calibration performed is based on tracking 125 vehicles over 4 min (500 min total data) using the Ohio state aerial data (Treiterer & Myers, 1974). The model replicated trajectories quite well, but was a little too smooth in the transition region between acceleration and deceleration:

$$a(t+0.1) = 1.64(x - 1.14v) + 0.5v.$$

It was also noted that the exceptionally short response times were most probably due to 'look ahead' activity. This suggested that the driver assesses the behaviour of any two out of the three vehicles ahead, as a basis for following decisions.

Aron (1988) investigated the model further (along with the GHR), and split his data into phases of acceleration and deceleration. The dependence of the response to Δx was not found to change much between the phases and was found to be almost constant at 0.03, while that for Δv ranged from 0.36 for deceleration, through 1.1 in the steady state, to 0.29 for acceleration. More recently Xing (1995) has proposed a complex new model, partially related to both the Helly and GHR models, that contains four main terms, the first for 'standard' driving, the second for acceleration from a standing queue, the third for the effect of a gradient (which can be ignored for our purposes), and the fourth for use in a free flow regime where parameters for the first term are reduced accordingly:

$$\begin{aligned} a &= \alpha \frac{\Delta v(t-T)}{\Delta x(t-T_l)^l} + \beta \frac{\Delta x(t-T_2) - D_n(v(t-T_2))}{\Delta x(t-T_2)^m} - \gamma \sin\theta + \lambda(v_{\mathrm{Des}} - v_n) \\ D_n(v) &= a_0 + a_1 v + a_2 v^2 + a_3 v^3. \end{aligned} \tag{7}$$

The model was calibrated on aerial data taken on two separate days and observations made of vehicles moving over a over 500-m section of road, however no mention is made of the total number of vehicle minutes used in analysis. Earlier studies revealed that $l \sim m \sim 0$, reducing the equation to

$$a = 0.5\Delta v(t - 0.83) + 0.05(\Delta x(t - 3.43) - D_n(t - 0.83)), \quad D_n(t) = 7 + 0.5v(t) \tag{8}$$

It should be noted that the distribution of the significant parameters remaining have very high standard deviations. This model gives an extremely good fit to the observed trajectories, though care should be taken when comparing this model with other Helly-like models, because of the addition of the gradient term in the calibration. A summary of the varying parameter combinations to emerge from research on the 'Helly' model is given in Table 3.

The criticisms applied to the GHR model can also be applied to the linear model, although there are two differences to note. Firstly there is a surprising degree of agreement between the various values found for the magnitude of response to Δv, and in all cases the magnitude of response to Δx is some 4–10 times less than the GHR model. As Δx is explicitly present as a term separate from v or Δv, we are also able to derive a desired distance relationship which is proportional to the closing speed divided by the speed, and which is consistent between researchers. Helly, gives a desired distance of $1 + (20/v) \sim 1.2$ s at typical speeds of 100 kph, while Hanken and Rockwell give $0.8 + (103/v) \sim 1.8$ s, Bekey, Burnham and Seo, 1.14 s and Xing, $0.5 + (7/v) \sim 0.7$ s. Despite this advantage over the GHR model, the linear model has little more general validity, either in form or in the degree of calibration obtained, with one of its few uses at present being within the SITRA-B model which concentrates on low speed traffic in urban networks (Aron, 1988).

A major strength of the Helly model, however, is the specific incorporation of 'error', an element of the original formulation that is often overlooked. Here, the model may be implemented such that once a specific acceleration/deceleration for a situation has been determined, the driver will not reassess the required acceleration until Δx (or Δv) disagrees substantially with its expected value, assuming a constant v_{n-1}. The estimated values are given as the sum of the actual value and the results of multiplying 'A' (an observation accuracy), by 'R' (a random number between -1 and 1), and mod(Δx, (v) as appropriate). 'Substantial' is defined as the condition where the magnitude of the discrepancy between estimated and actual spacing exceeds 'K' times the Desired spacing, with K and S related to acceleration noise, typically of the order of 0.25 and 0.125, respectively. The algebra for this process is lengthy but reduces to

$$|RA|\Delta x(t_D)| + (t - t_D)R'A'|\Delta v(t_D)|| > KD \Rightarrow t > \frac{KD/AR - \Delta x}{|\Delta v|} - \frac{\theta}{\partial\theta/\partial t} \tag{9}$$

Table 3
Summary of optimal parameter combinations for the 'Helly' equation[a]

Source	$C_1(\Delta v)$	C_2 (Δx)
Helly (1959)	0.5	0.125
Hanken and Rockwell (1967)	0.5	0.06
Bekey, Burnham and Seo (1977)	0.5	1.64
Aron (1988) (dcn/ss/acn)	0.36/1.1/0.29	0.03/0.03/0/03
Xing (1995)	0.5	0.05

[a] *Key*: dcn/acn: deceleration/acceleration; ss: steady state.

with, θ, the apparent angle of the vehicle in front, a variable whose importance will become clear in the next section.

2.4. Psychophysical or action point models (AP)

The first discussion of the underlying factors that would eventually lead to the construction of these models was given by Michaels (1963), who raised the concept that drivers would initially be able to tell they were approaching a vehicle in-front, primarily due to changes in the apparent size of the vehicle, by perceiving relative velocity through changes on the visual angle subtended by the vehicle ahead θ. The threshold for this perception is well-known in perception literature and given as, $\mathrm{d}/\mathrm{d}t(\sim \Delta v/\Delta x^2) \sim 6 \times 10^{-4}$. Once this threshold is exceeded, drivers will chose to decelerate until they can no longer perceive any relative velocity, and provided the threshold is not then re-exceeded, will base all their actions on whether they can then perceive any changes in spacing.

This second, spacing-based threshold (generically termed an ‘action point’) is particularly relevant at close headways where speed differences are always likely to be below threshold. Thus, for any changes to be noticeable, Δx must change by a ‘just noticeable distance’ (JND), related to Webers Law, i.e., that the visual angle must change by a set percentage, typically 10%. It is also noted that this threshold is $\sim$ 12% for the opening situation, which as a driver continually approaches and moves back from the vehicle in front, will lead to a gradual drifting apart. It is important to state that in crossing this last threshold, the driver will set a determined acceleration/deceleration and stay with it until they break another threshold, as the driver perceives no change in conditions, or at very least, no change in the rate of change. (A detailed formulation may be found in Lee & Jones, 1967). It is also likely that in this close-following area the driver is not fully able to control the acceleration/deceleration of his vehicle due to the very fine adjustments required. Motion is therefore governed by the use of a minimum value, theoretically the same as Montroll’s acceleration noise concept (Montroll, 1959).

The next point in the development of these models came through a series of perception-based experiments conducted in the early seventies, by researchers such as Evans and Rothery (1973), aimed at quantifying the thresholds that Michaels had suggested. These experiments required passengers in test vehicles to judge whether the gaps between themselves and the vehicle being followed were opening or closing, allowing only a set time to observe the target and come to a decision. In all, 1923 data points were collected for a response time of 1 s, and a further 247 for a 2 s exposure time, with analysis indicating that the chance of a correct judgement was likely to be a function of $v/\Delta x$ and the observation time. It was also noticed that the thresholds are subject to a negative response bias which increases with Δx, hence leading people to believe they are gaining on a vehicle when this is not actually the case. A review of the many investigations conducted in these areas at that time can be found in Evans and Rothery (1977), where it is shown that the wide body of research conducted on this topic during the seventies are all consistent from a statistical point of view.

The individual properties of these thresholds were first combined into a fully working simulation model by staff at IfV Karlsruhe in Germany and has been in progress continually since (for a review see Leutzbach & Wiedemann, 1986). (Other similar models have also been produced by

Burnham & Bekey, 1976; Lee, 1976, and more recently by Kumamoto et. al., 1995). The integrated model consists of the main thresholds as discussed above with the following:

(i) a relative speed threshold for the perception of 'closing', $\sim -3.1 \times 10^{-4}\Delta x$,

(ii) thresholds for the perception of opening (OPDV) and closing (CLDV) for small relative speeds (using Wiedemann's terminology (Wiedemann & Reiter, 1992)) of OPDV $= -5.2\times 10^{-4}$ and CLDV $= 6.9 \times 10^{-4}\Delta x$ respectively,

(iii) thresholds for perceiving increases and decreases in distance, $2.5 + 2.5v^{1/2}$ and $2.5 + 3.8v^{1/2}$, respectively.

Recent work by Reiter (1994) using an instrumented vehicle to measure the action points has resulted in the amendment of some of these parameters, finding that CLDV and OPDV have a totally different functional form, namely: CLDV $= -0.15 + 8.5 \times 10^{-4}\Delta x$ and $0.05 + 41.5\times 10^{-4}\Delta x$.

It is difficult to come to a firm conclusion as to the validity of these models, as although the entire system would seem to simulate behaviour acceptably, calibration of the individual elements and thresholds has been less successful. For example, since the sixties (with the exception of Reiter and macroscopic validation of model output by Fellendorf & Hoyer, 1997) little research work has been undertaken on the concepts involved in these models with the express intention of contributing to the compilation of a coherent model of driving behaviour. It is difficult therefore to either prove or disprove the validity of this model, although the basis upon which it is built is undoubtedly the most coherent, and best able to describe most of the features that we see in everyday driving behaviour. This approach, and derivatives of it, are now in use in the MISSION model (used in the CECs DRIVE1 programme, Wiedemann & Reiter, 1992), the related AS model used in the PROMETHEUS and fourth framework programmes (Benz, 1994) in Germany, along with recent advances being made by Fritzsche (1994) at Daimler Benz, and incorporation into PARAMICS-CM model in the UK, Cameron (1995).

2.5. Fuzzy logic-based models

The use of fuzzy logic within car-following models is worthy of mention as the latest distinct 'stage' in their development, as it represents the next logical step in attempting to accurately describe driver behaviour. Such models typically divide their inputs into a number of overlapping 'fuzzy sets' each one describing how adequately a variable fits the description of a 'term'. For example, a set may be used to describe and quantify what is meant by the term 'too close', where for example a separation of less than 0.5 s is definitely 'too close' and thus has a degree of truth or 'membership' of 1, while, a separation of 2 s is not close and is given a membership of 0, and intermediate values are said to exhibit 'degrees' of truth and have differing (fractional) degrees of membership. Once defined, it is possible to relate these sets via logical operators to equivalent fuzzy output sets (e.g., IF 'close' AND 'closing' THEN 'brake'), with the actual course of action being assessed from the modal value of the output set, calculated as the sum of all the potential outcomes.

The initial use of this method (Kikuchi & Chakroborty, 1992) attempted to 'fuzzify' the traditional GHR model using Δx, Δv and a_{n-1}, as inputs, grouping these into 6, 6 and 12 natural language based sets. respectively. Each of these sets were taken to be triangular, and the Δx set was scaled according to v_{n-1} in order to incorporate a measure of time headway. The consequence of their rule base is that vehicle 'n' will accelerate at the same rate as $n-1$, plus a small term to account for Δv and Δx. Each term from the fuzzy inference is of the form:

$$\text{IF}\Delta x = \text{'ADEQUATE' THEN } a_{n,i} = (\Delta v_i + a_{n-1,i}\ x\ T)/\gamma,$$

with $T\,(=1)$ the reaction time and $\gamma(=2.5)$ the time in which the driver wishes to 'catch up' with vehicle $n-1$. If $\Delta x \neq$ ADEQUATE then the response is altered by sliding the membership function (making it larger or smaller) according to the degree of deviation from ADEQUATE. For each deviation to a shorter distance, a_i was reduced by -0.3 m/s^2 and, for each deviation to a longer distance, it was increased by a similar amount. Thus

$$a_{n,i} = (\Delta v_i + a_{n-1,i} x T)/\gamma + 0.3_{\Delta x}.$$

The model was then used to illustrate how the fuzzy logic system can be used to describe car-following, with, most importantly, local stability examined for a configuration of $\Delta x = 40$ m, and $a_{n-1} = -2.4$ m/s^2 from a speed of 13.3 m/s for 2 s in one case, and of -3.6 m/s^2 from 15.8 m/s in the second, and compared with traditional GHR results. It was demonstrated that the GHR model would produce differing headways according to the rate of deceleration and hence final speed. This is clearly in contradiction to what would be expected in practice. Additionally, the final following distance was shown to be dependant only on final speed, regardless of the original following distance or original speed. Although the model generally reflects the changes expected, its formulation is unrealistic for two reasons. The acceleration of a vehicle can be detected (it is highly debatable whether this is possible), and it has been found from the Helly model that any linear dependence on Δx is exceedingly small.

More recent work in this area includes that by Rekersbrink (1995), in fuzzifying the MISSION model, Yikai, Satoh, Itakura, Honda and Satoh (1993), formulating the MIcroscopic model for analysing TRAffic jaM (MITRAM) model, and work by Henn (1995). However, none of these approaches have attempted to calibrate the most important part of the model itself, the membership sets, which have only recently been investigated using on road subjectivity tests by Brackstone, McDonald and Wu (1997).

3. Concluding remarks

In the preceding section we have seen that the study of car-following models has been extensive, with conceptual bases supported by empirical data, but generally limited by the lack of time-series following behaviour. In many cases work has also been accomplished in investigation of model stability and the implications of each of the relationships to macroscopic flow characteristics. It is highly tempting to attempt to increase the realism of a chosen model by attempting to incorporate 'motivational' or attitudinal factors that may be able to explain the differences between drivers. There is certainly a body of evidence to support this suggestion (e.g., Howell, 1971; Gulian, Matthews, Glendon, Davies & Debney, 1989). However, there is little evidence to relate such features to observable 'dynamic' behaviour.

This pursuit of such model amendments and expansion does however, tend to distract from potentially the most important consensus that it is possible to draw from the above sections – namely, that although meaningful 'one off' experiments have been performed to calibrate models or features of models, little concerted work has been performed since the early sixties on the establishment of a 'complete' (basic) driver model.

With the increased interest in in-vehicle driving aids however, several vehicle manufacturers are now investing heavily in this area (e.g., Allen, Magdeleno, Serafin, Eckert & Sieja, 1997), but although undoubted progress is now being made, full availability of findings to the scientific community as a whole is not likely through commercial reasons. With the establishment of this area as a potential priority for further funding by US DoT research however (ITS America, 1997), and the establishment of a specific TRB task force regarding data needs (under Group A3A11) it is hoped that this area will soon receive long overdue attention. Systematic testing procedures and evaluation programmes have also generally been overlooked. Certainly a number of review activities have been undertaken in recent years regarding simulation model capability both in the US (Skabardonis, 1998) and in the EU (Brackstone & McDonald, 1991; SMARTEST, 1997), but with only a few exceptions (e.g., Benekohal, 1991; Bleile, 1997; McDonald, Brackstone & Sultan, 1998) little has been accomplished in establishing a concrete set of metrics which may be used to judge the performance of a model.

The evolution of car-following models therefore has clearly been slow, and although many would argue that they are sufficiently valid for the purposes for which we require them, there is a growing belief that this is not the case. Certainly there are potential pitfalls awaiting the unwary in the use of microscopic models (for an examination see, Brackstone & McDonald, 1996), and it is hoped that this article has in part expanded the understanding of the scientific community of the variety, and limitations of the tools available to them, and that this may increase care and overall scientific validity with which this area is approached in future.

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