Applications to Traffic Breakdown on Highways

Jevgenijs Kaupužs[1], Hans Weber[2], Julia Tolmacheva[3], Reinhard Mahnke[4]

[1] Institute of Mathematics and Computer Science, University of Latvia, 29 Rainja Boulevard, LV-1459 Riga, Latvia
kaupuzs@latnet.lv
[2] Department of Physics, Luleå University of Technology, S-97187 Luleå, Sweden
weber@ml.luth.se
[3] Theoretical Physics Department, Institute for Single Crystals, National Academy of Sciences Ukraine, 61001 Kharkov, Ukraine
julia@isc.kharkov.com
[4] Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany
reinhard.mahnke@physik.uni-rostock.de

1 Introduction to Physics of Traffic

During the last years researches into properties of vehicle ensembles on highways form a new branch of physics, called physics of traffic flow. On macroscopic scales the vehicle ensembles exhibit a wide class of phenomena like phase separation and phase transformations widely met in physical systems. Due to the steadily increasing traffic volume in cities and on highways, the mathematical modelling of these phenomena has attracted a great interest. Particularly, the topic of car–following has become of increased importance in traffic engineering and safety research.

2 Optimal Velocity Model (OVM) of Traffic Flow

We consider a traffic flow car–following model where N point–like cars are moving along a circular road of length L. The coordinate $x_i(t)$ and velocity $v_i(t)$ of each (i-th) car depending on time t are given by the Newton's equations of motion

$$\frac{dv_i}{dt} = \frac{1}{\tau}\left(v_{opt}(\Delta x_i) - v_i\right) \tag{1}$$

$$\frac{dx_i}{dt} = v_i \tag{2}$$

where the right hand side of (1) represents the relaxation force, given by the relaxation time τ and the headway distance to the next car $\Delta x_i = x_{i+1} - x_i$, as consistent with the optimal velocity model (OVM) proposed first by Bando

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et al. [BHN95]. According to (1), each car tends to adjust its velocity to the optimal one $v_{opt}(\Delta x)$ which is a monotonously increasing function of the headway distance Δx. The optimal velocity function can be evaluated from empirical traffic flow data [BHN95] . Several analytical approximations are valid which fulfil the conditions $v_{opt}(0) = 0$, which means that cars cannot move if they are close to each other (bumper–to–bumper), and $v_{opt}(\Delta x) \to v_{max}$ at $\Delta x \to \infty$, where v_{max} is the maximal velocity of driving. We shall use the optimal velocity function

$$v_{opt}(\Delta x) = v_{max} \, \frac{(\Delta x)^2}{D^2 + (\Delta x)^2} \tag{3}$$

proposed in [MK99], where D is the interaction distance at which the optimal velocity is half of the maximal one.

It is convenient to rewrite the equations of motion (1) and (2) in the dimensionless form

$$\frac{du_i}{dT} = u_{opt}(\Delta y_i) - u_i \tag{4}$$

$$\frac{dy_i}{dT} = \frac{1}{b} u_i \; , \tag{5}$$

where $T = t/\tau$ is the dimensionless time, $y_i = x_i/D$ is the dimensionless coordinate, $u_i = v_i/v_{max}$ is the dimensionless velocity, $b = D/(v_{max}\tau)$ is a dimensionless control parameter, and

$$u_{opt}(\Delta y) = \frac{(\Delta y)^2}{1 + (\Delta y)^2} \tag{6}$$

is the dimensionless optimal velocity function.

The system of differential equations (4) and (5) has been solved numerically by the fourth–order Runge–Kutta method, e. g. for a finite system of $N = 60$ cars. Periodic boundary conditions have been used corresponding to a rotary without exits which is a circular road of dimensionless lengths $\mathcal{L} = L/D$. First we have studied the long–time solution starting with an almost homogeneous distribution of staying cars ($u_i = 0$) at the initial time moment $T = 0$. Depending on the value of parameter b and the dimensionless density of cars $c = N/\mathcal{L}$, the phase trajectories in the headway–velocity space go either to the stable fixed point $\Delta y_i = \Delta y_{hom} = \mathcal{L}/N$, $u_i = u_{opt}(\Delta y_{hom})$ where all cars move with equal velocities and have identical headway distances, or to a limit cycle characterised by certain minimal velocity u_{min} and maximal velocity u_{max} and corresponding minimal and maximal headway distances. The fixed point corresponds to a homogeneous traffic, whereas the limit cycle – to a stop–and–go or congested traffic where a cluster (jam) of slowly moving cars coexists with a free–flow phase. The solid curve in Fig. 1 separates the regions in the b–c plane where the homogeneous traffic flow on an infinitely long road ($\mathcal{L} \to \infty$) is stable (above