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On the Flow Capacity of Automated Highways*

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Virtually all proposed systems for highway automation have at least one mode in common—steady-state car following. The nature of this mode is extremely important, as it can determine the upper limit of flow capacity of an automated highway. This limit is explored for a linear headway controller, and a fundamental relation between the effective vehicle response time and the permissible traffic stream density is obtained. The required intervehicular spacing with a linear headway controller is shown to be proportional to the effective vehicle time constant for small-signal inputs; thus, one can achieve small spacings and high flow rates by reducing this parameter to 1 sec or less. However, the vehicle is then highly responsive to small changes in lead-vehicle speed—possibly resulting in both passenger discomfort and poor fuel economy. These shortcomings can be avoided by using a linear velocity controller for automatic car following.

An examination of traffic conditions today—congested roadways, a large number of accidents and fatalities, and extremely powerful automobiles—indicates the need for improvement in our highway system. Unfortunately, these conditions will be much worse in the next decade, for it is predicted that the total number of vehicles registered in 1980 will be 62 percent greater than in 1960, and 75 percent more vehicle-miles will be traveled.[1] If one could look further ahead to the turn of the century, he would see vast

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sprawling supercities, with populations characterized by adequate incomes, longer life spans, and increased amounts of leisure time. One predictable result is greatly increased travel. The resulting traffic situation will be chaotic, unless some radical changes are instituted beforehand.

One promising approach for a partial solution to some of these problems is highway automation. This approach has been examined by a number of investigators, for in addition to the retention of the individual transportation unit, it appears that considerable improvements in highway capacity and safety can be achieved, as well as a considerable reduction in driver effort.

There are an extremely large number of possible systems for achieving such automation, and great care must be exercised so that an optimum or near-optimum one is chosen. However, there are certain functions that must be performed by virtually all such systems, and these can be profitably studied without regard to the over-all system configuration. One of these—automatic steady-state car following—is examined here in detail, and certain fundamental limitations on system performance are obtained.

CAR-FOLLOWING SYSTEM REQUIREMENTS

THE FOLLOWING performance restrictions must be imposed on the steady-state car following mode of an automatic vehicle control system:

1. The average separation between adjacent vehicles must not be excessive (the smaller the spacing at a fixed velocity, the greater the capacity of the system);

2. A controlled vehicle must be stable relative to the position of the nearest lead car;

3. Disturbances propagated to the rear must be attenuated or at least limited in amplitude (this is termed asymptotic stability);

4. The control system must not be required to respond so as to exceed the response capabilities of the vehicle; and

5. To ensure the comfort of the occupants, the acceleration and deceleration must normally not exceed 0.1 g.

CAR-FOLLOWING CONTROL SYSTEM THEORY

CONSIDER A line of identical automatically controlled vehicles that are following one another in a stable manner. It is assumed that the vehicles are initially in a steady-state situation with each vehicle separated from its two closest neighbors by a constant headway H_0, and all vehicles speeds are equal to V_s. Subsequently, disturbances are introduced into the traffic stream resulting in small deviations from the steady-state conditions. For simplicity, only the first two vehicles in the line will be considered; however, the results obtained also apply to the ith and $(i+1)$st vehicles.

The small-signal car-following model under investigation is one in which the acceleration (pv_2) of a controlled vehicle is made equal to a linear combination of relative velocity (v) and headway error. Thus,

$$pv_2 = k_1 v + k_2(h - k_3 v_1 - k_4 v_2), \quad (p \equiv d/dt) \quad (1)$$

where v_1 is the incremental speed change of the lead car, h is the corresponding incremental change in headway between the lead and controlled vehicles, and k_1, k_2, k_3, and k_4 are positive constants. Generally, either k_3 or k_4 is zero. Note that the controlled vehicle only responds to changes in state between itself and the nearest lead vehicle. Car-following models, which were obtained under full-scale conditions with an experimental automatically controlled vehicle, are closely approximated by equation (1).[2–4] These were obtained at average speeds of 40, 50, and 60 mph with speed variations of ± 4 mph about those values; thus, this equation is valid at least under such conditions.

A study was previously made of certain variants of this equation by HERMAN, ET AL[5] who also considered the effects of system lag time. Here, on the basis of real-world dynamic models of this system,[2,3] it is assumed that the lag is negligible; i.e., the application of a corrective force will occur within a negligibly short time after a velocity difference and/or headway deviation occurs.

It should be noted that equation (1) defines an automatic longitudinal control system in one mode of operation, and the control law would be different for other modes such as overtaking and emergency braking.[6]

The over-all system function corresponding to equation (1) is

$$\frac{V_2(s)}{V_1(s)} = \frac{[(k_1 - k_2 k_3)/k_2]s + 1}{(s^2/k_2) + [(k_1 + k_2 k_4)/k_2]s + 1}, \qquad (s = \sigma + j\omega) \quad (2)$$

where

$$V_2(s) = \text{Laplace transform of } v_2(t),$$
$$V_1(s) = \text{Laplace transform of } v_1(t).$$

It is convenient to put this equation into standard form:

$$\frac{V_2(s)}{V_1(s)} = \frac{(1/\beta\omega_n)s + 1}{(s^2/\omega_n^2) + (2\zeta/\omega_n)s + 1}, \quad (3)$$

where ω_n is the undamped natural frequency of the system, ζ is the damping ratio, and β is a positive constant. The relations among these parameters and the k_i's are obtained by equating the corresponding coefficients in equations (2) and (3), and are as follows:

$$\frac{k_1 - k_2 k_3}{k_2} = \frac{1}{\beta\omega_n}, \quad (4)$$

$$k_2 = \omega_n^2 \tag{5}$$

and

$$\frac{k_1 + k_2k_4}{k_2} = \frac{2\zeta}{\omega_n}. \tag{6}$$

The performance of the system is determined by the triad (β, ζ, ω_n), and these quantities are constrained by the system requirements.

(a) Vehicle Separation Constraint

Consider the steady-state separation between adjacent vehicles. The relation between a steady-state change in headway (h_0) and the system parameters is obtained from equation (1) by setting $p = 0$ and recognizing that $v = 0$ in steady state. Further, the corresponding change in both lead car and following car speed is v_s. Thus, one obtains

$$h_0 = (k_3 + k_4)v_s,$$

or

$$h_0 = kv_s,$$

where

$$k = k_3 + k_4. \tag{7}$$

The new steady-state headway (H_f) is

$$\begin{aligned} H_f &= H_0 + h_0 \\ &= H_0 + kv_s. \end{aligned}$$

In order to avoid small headways for decreases in V_s $(v_s < 0)$, one must have

$$H_0 \geqq kV_s,$$

or

$$H_f \geqq k(V_s + v_s). \tag{8}$$

Thus the specifying of k is equivalent to specifying the maximum permitted flow per lane. If the equality is satisfied, then k is the time headway. Note that if no average changes or large variations in stream speed are permitted, it may not be necessary to satisfy equation (8).

It is convenient to express k in terms of (ζ, β, ω_n), which is easily done using equations (4)–(7),

$$k = \left(2\zeta - \frac{1}{\beta}\right)\frac{1}{\omega_n}. \tag{9}$$

(b) Stability Constraints

If this system is to be locally stable, it is only necessary that the coefficients of the denominator polynomial of equation (2) be positive. A necessary condition for asymptotic stability is[6]

$$\left|\frac{V_2(s)}{V_1(s)}\right|_{s=j\omega} \leqq 1 \quad \text{for all } \omega, \tag{10}$$

and this condition is satisfied provided

$$\sqrt{\zeta^2 - \left(\frac{1}{2\beta}\right)^2} \geqq 0.707 \tag{11}$$

as demonstrated in Appendix A. This relation is shown in Fig. 1, with the region above the curve corresponding to those doublets of (ζ, β) for which the system is asymptotically stable.

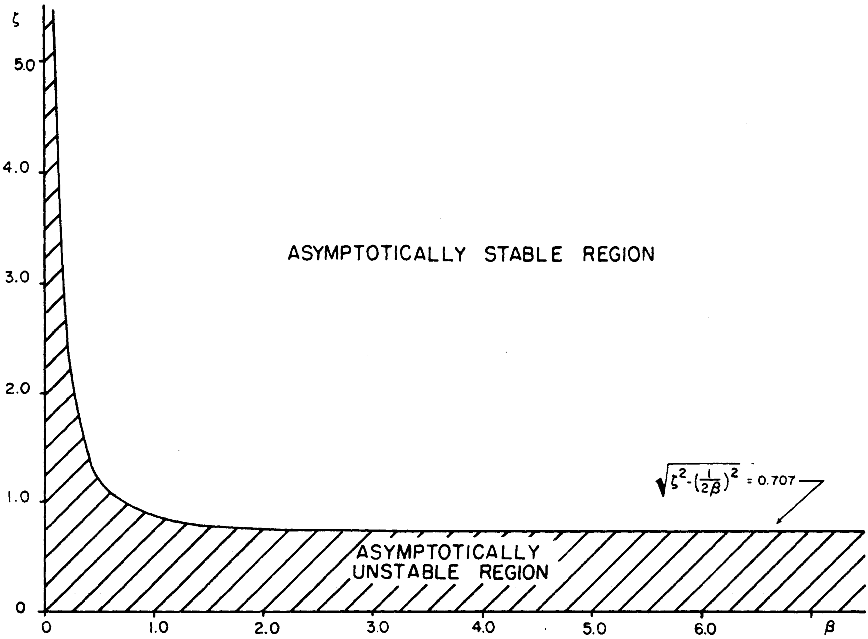

Fig. 1. Stability regions in ζ—β plane.

A controlled vehicle's response to small changes in lead-car speed can be described in terms of a small-signal vehicle time constant τ. This parameter, which can easily be obtained from the system function [equation (2)], is a measure of how fast the controlled vehicle can respond to a small-signal input. In classical control practice, such a quantity is usually obtained from an examination of the system-function response to a step input, and this approach is used here.

If a vehicle, initially traveling at a constant speed of V_{si}, were subjected to a command step change in speed $(V_{sf} - V_{si})$, its speed-time history would be as shown in either Figs. 2a or 2b (the form of the response observed in a given situation would, of course, depend on the coefficients of the system differential equation). The small-signal time constant is defined as the time required for the vehicle to reach 0.632 of the final incremental velocity value, and is easily obtained from a velocity-time trace as shown in Figs. 2a and 2b.

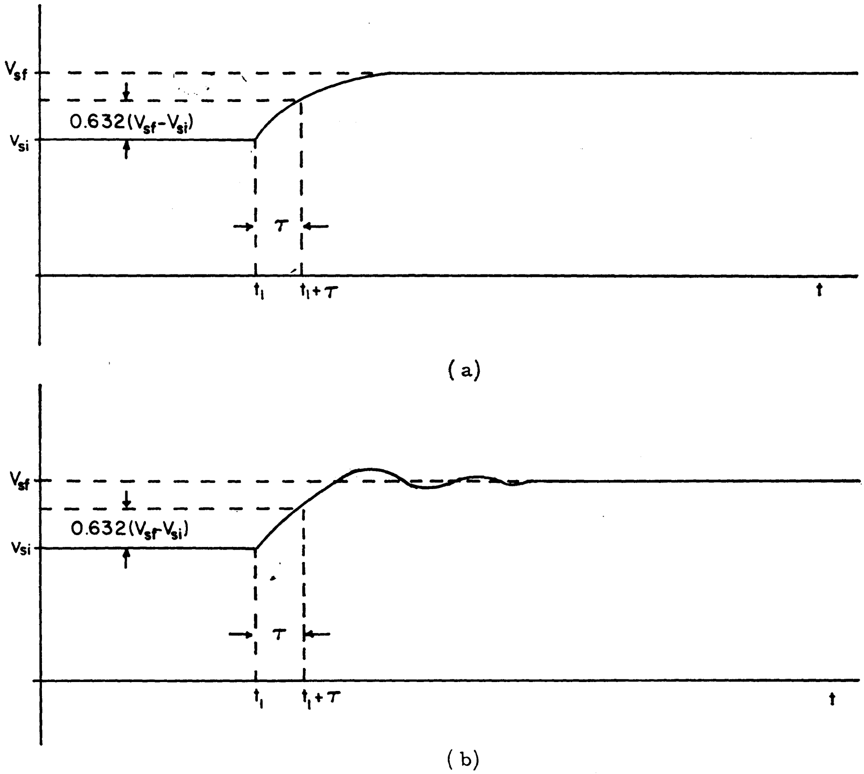

Fig. 2. Vehicle speed-time histories for a step command speed change.

This definition is equivalent to that ordinarily used for overdamped systems; however, it differs from that normally used for underdamped ones. This difference is due to the need to define a single parameter giving an accurate measure of the system rise time for all values of damping.

The values of τ for all cases of interest were obtained from digital-computer solutions of equation (3). Here, the system response to a command step change of velocity was examined for those doublets of (ζ, β) that lie in the stable region of Fig. 1, and τ was obtained directly from this response. The results are shown in Fig. 3 where the normalized time constant $\omega_n\tau$ is plotted versus β with ζ as a parameter.

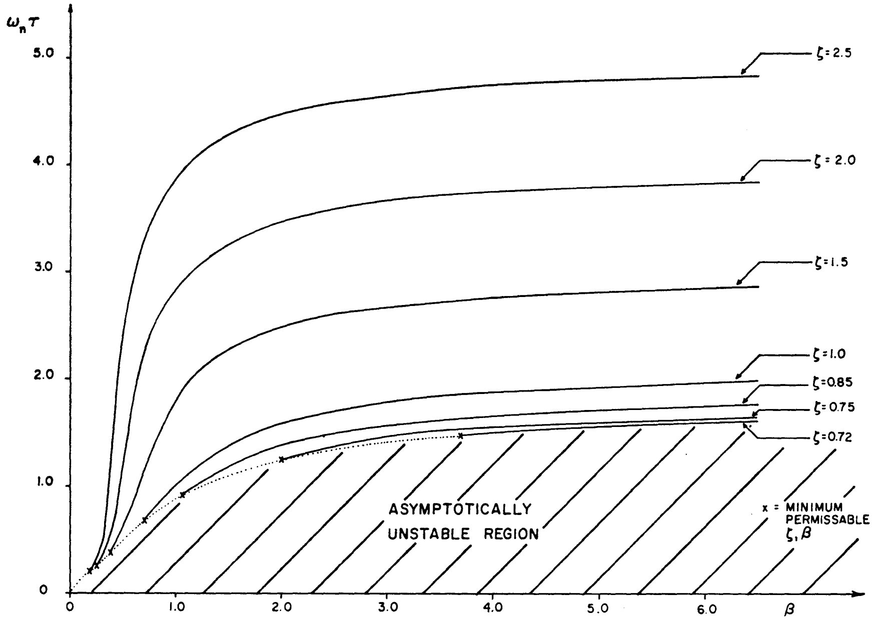

Fig. 3. System response characteristics.

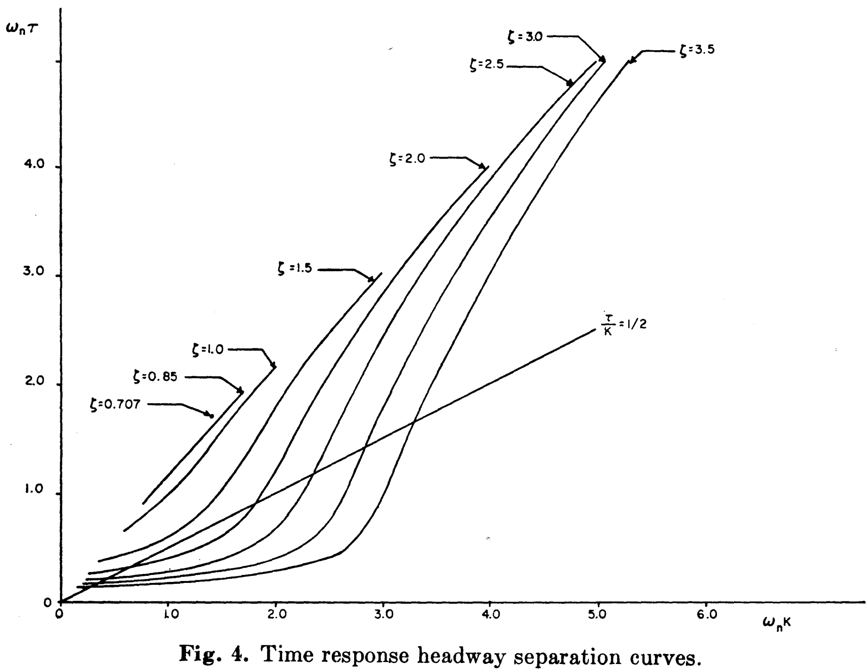

Fig. 4. Time response headway separation curves.

Note from the figure or equation (3) that three quantities—β, ζ, and ω_n—can be specified independently. Alternatively, one can specify

$$k = f(\beta, \zeta, \omega_n),$$

$$\tau = g(\beta, \zeta, \omega_n),$$

and one of the first triad; however, this choice is subject to the asymptotic stability constraint posed on ζ and β per equation (11).

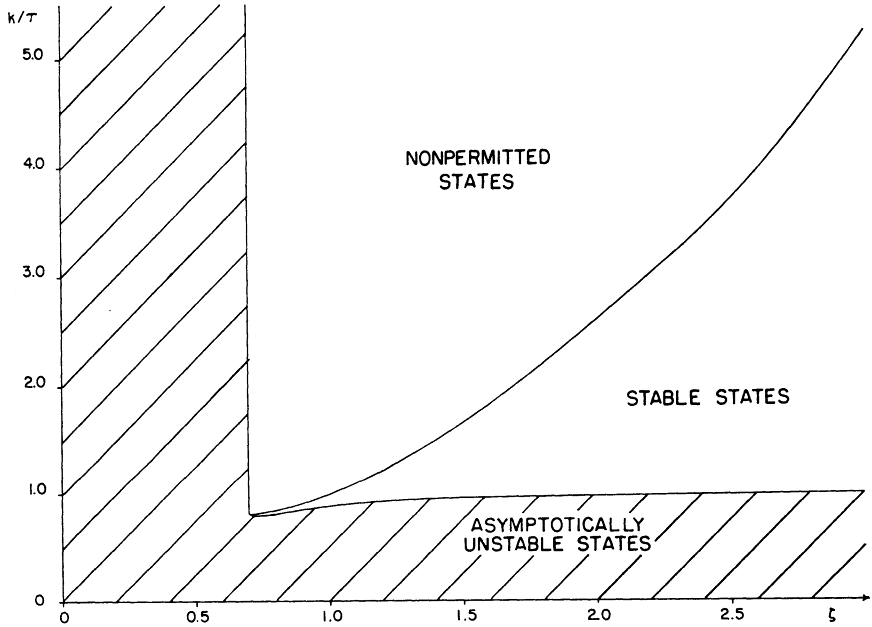

Fig. 5. Regions of vehicle system operation.

An especially convenient choice involves the specification of k, τ, and ζ. A normalized plot of $\omega_n\tau$ versus $\omega_n k$ with ζ as a parameter is given in Fig. 4. If one specifies k and τ, the ratio k/τ is fixed and is represented by a straight line in this figure. The intersection of this line and the curve corresponding to the chosen value of ζ gives one the appropriate values of $\omega_n k$ and $\omega_n\tau$. Then one has sufficient information to obtain β from equation (9), and the system is defined. For example, if $k = 4$ and $\tau = 2$, then $k/\tau = 2$ as shown in Fig. 4, and for $\zeta = 2$, note that $\omega_n k = 1.8$. Thus $\omega_n = 0.45$, and $\beta = 0.454$ per equation (9). Note that a second value is obtained for this choice of k, τ, and ζ, which can be handled in the same way. It should be observed from the figure that for a given choice of k and τ, one must choose ζ to be greater than a certain minimum value.

TRAFFIC-LANE FLOW CAPACITY

ONE CAN easily combine the system response characteristics shown in Fig. 3 with equation (9) so as to obtain a representation for the relation between k and τ. One such representation is shown in Fig. 5, which is divided into three regions—one corresponding to the unstable region shown in Fig. 1, one to the stable region, and the third to nonpermitted values of the triad k, τ, and ζ. The minimum permitted ratios of k/τ are defined by the bound-

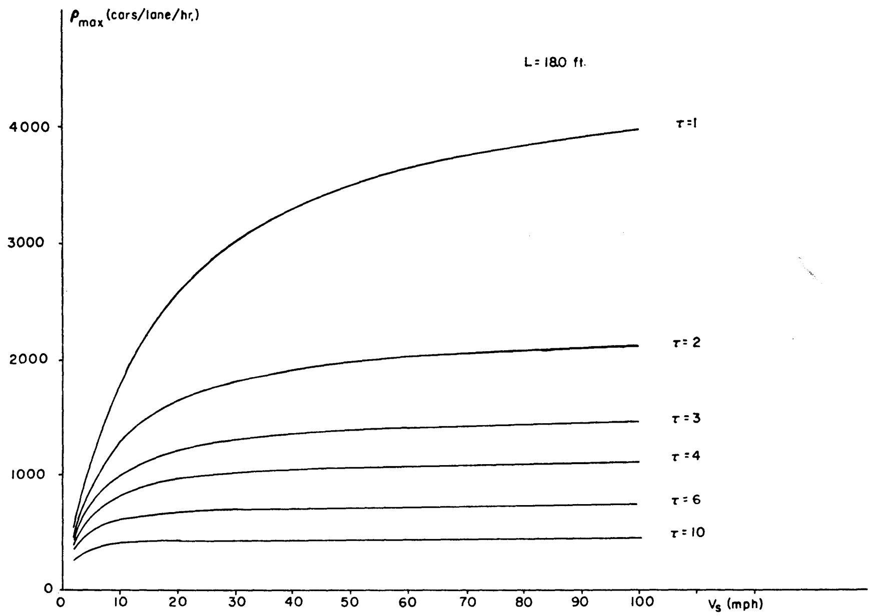

Fig. 6. Maximum flow-velocity curves.

ing curve between the stable and unstable regions. Note that the minimum allowed value of k/τ is 0.787; thus, if one chooses $\tau = 4.0$ sec, then $k =$ 3.148 sec. The resulting minimum required headway for $V_s = 102.5$ fps would be 323 ft. The curve definining the bound between the nonpermitted and stable regions may possibly be somewhat higher than shown. This uncertainty arises because of the quantized nature of the computer data search; however, it is of no practical importance.

With the aid of Fig. 5, one can relate the vehicle flow rate with τ. The basic flow equation is

$$\rho = \lambda V_s, \tag{12}$$

where

$$\rho = \text{flow (cars/lane/time)},$$

V_s = stream speed in mph,
λ = concentration (cars/lane/distance).

If the maximum allowed number of vehicles are on the highway

$$\lambda = \frac{5280}{L + k(1.465)V_s} \text{ (cars/lane/mile)}, \tag{13}$$

where L is vehicle length. Thus,

$$\rho = \frac{5280}{L/V_s + 1.465k} \text{ (cars/lane/hr)}. \tag{14}$$

It is apparent that k must be small if one is to achieve high flow rates; in particular, ρ is maximized when k has its minimum value. Note from Fig. 5 that this value is $k = 0.787\ \tau$; hence,

$$\rho_{max} = \frac{5280}{L/V_s + (1.465)(0.787)\tau}. \tag{15}$$

One can express this maximum flow relation as a family of curves as shown in Fig. 6. Here, ρ_{max} was chosen as the dependent variable, V_s as the independent variable, τ as a parameter, and L as 18 ft. Note from either the figure or equation (15) that a small τ is required for large flow rates—i.e., one must have a very responsive vehicle.

In order to ensure system compatibility with existing traffic, τ should be chosen such that it may be realized by the majority of vehicles in use on the highways. In practice, one encounters vehicle time constants ranging from 8 to 40 sec; however, if control compensation is used, the time constant can be reduced to 4 sec or less.[3,4]

CONCLUSIONS

It is clear that high flow rates cannot be obtained unless one has a very responsive vehicle ($\tau \leqq 1$). Further, the results presented in the last section were for the minimum allowed value of k/τ, which lies on the borderline of asymptotic instability. In practice, one would choose a larger value of k/τ and thus obtain lesser flow rates. Note from Fig. 6 that one must choose $\tau < 2$ to achieve a rate greater than 2000 cars/lane/hr. Results of current studies have indicated that the minimum achievable value of τ is approximately 1, and thus at a stream speed of 70 mph, one could have a maximum flow rate of 3750 vehicles/lane/hr.[3,4] However, one then would have a very responsive vehicle that would probably result in passenger discomfort, poor fuel economy, and a need for frequent tune ups. Thus, the effective vehicle time constant poses a definite upper limit on the capacity of an automated highway.

One approach to surmounting this limitation is via a velocity controller. This is obtained by setting $k_2 = 0$ in equation (1), and thus no headway information is used and the acceleration of the controlled vehicle is dependent only on relative velocity. The steady-state headway is arbitrary and can be specified by factors other than those discussed here. Another promising approach involves the use of a nonlinear controller.

This discussion has been for one particular control law; however, the same techniques are applicable to others. These include car-following systems utilizing information from both the nearest and next nearest forward neighbors, the nearest forward and trailing ones, and the two nearest forward and the nearest trailing car. Some of these alternate schemes together with the effects of system lags are currently being studied.

APPENDIX A

A Modified Criterion for Asymptotic Stability

A necessary condition for asymptotic stability is

$$\left|\frac{V_2(s)}{V_1(s)}\right|_{s=j\omega} \leqq 1 \quad \text{for all } \omega.$$

Thus, if one considers the system function

$$\frac{V_2(s)}{V_1(s)} = \frac{(k_1 - k_2k_3)[s + k_2/(k_1 - k_2k_3)]}{s^2 + (k_1 + k_2k_4)s + k_2}, \tag{A-1}$$

and converts it into standard form

$$\frac{V_2(s)}{V_1(s)} = \frac{[(1/\beta\omega_n)s + 1]}{(s^2/\omega_n{}^2) + (2\zeta/\omega_n)s + 1}, \tag{A-2}$$

it is required that

$$\left|\frac{V_2(s)}{V_1(s)}\right|_{s=j\omega} = \left\{\frac{(\omega/\beta\omega_n)^2 + 1}{[1 - (\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2}\right\}^{1/2} \leqq 1 \quad \text{for all } \omega.$$

This equation is easily rearranged into the following form

$$\left[\frac{1}{[1 - (\omega/\omega_n)^2]^2 + [(\zeta)^2 - (1/2\beta)^2][2\omega/\omega_n]^2}\right]^{1/2} \leqq 1.$$

The left-hand side of this equation represents a simple unity-feedback, second-order system with an effective damping ratio of

$$\sqrt{\zeta^2 - (1/2\beta)^2}.$$

If one examines a Bode plot of this side—which is included in virtually every standard text on classical servomechanism theory[7]—it is seen that the inequality is satisfied provided

$$\sqrt{\zeta^2 - (1/2\beta)^2} \geqq 0.707. \tag{A-3}$$

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