Mathematics for Machine Learning - Chapter 3 Solutions

Edwin Fennell

Note - I'm not going to write out the questions here since they are very, very inefficiently posed and no way am I going to TeX all of that.

3.1 We can more straightforwardly represent $\langle x, y \rangle$ as

$$x^T \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} y$$

- Bilinearity - we note that tensor multiplication commutes with scalar multiplication and distributes over tensor addition. Therefore our function is linear over both x and y.
- Symmetry - the output of our function $\langle ., . \rangle$ is a scalar and thus is equal to its own transpose, so $\forall x, y$ we have

$$\langle x, y \rangle = x^T \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} y = \left(x^T \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} y \right)^T$$

$$= y^T \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}^T x = y^T \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} x = \langle y, x \rangle$$

- Positive definite - for some reason this chapter doesn't cover diagonalisation so I guess we'll do this manually.
  Consider an arbitrary $v = (v_1, v_2)^T \in \mathbb{R}$. Then

$$\langle v, v \rangle = v_1^2 - 2v_1v_2 + 2v_2^2 = (v_1 - v_2)^2 + v_2^2 \geq 0$$

  with equality iff $v_1 = v_2 = 0$.