Mathematics of Machine Learning - Questions and Solutions

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1.1 Prove that

$$\boldsymbol{p(x, y|z) = p(x|z)p(y|x, z)}$$

and also that

$$\boldsymbol{p(x|y, z) = \frac{p(y|x, z)p(x|z)}{p(y|z)}}$$

Let's start with the first equality On the right-hand side we have a product of two separate terms. By applying Bayes' rule once to each term we get

$$p(x|z)p(y|x, z) = p(x, z)p(z) * \frac{p((y|x)|z)}{p(z)} = p(x, z)p((y|x)|z)$$

Now if we apply Bayes' rule to the left-hand side we get

$$p(x, y|z) = p(x|(y|z))p(y|z) =$$

and another application gives us

$$p(x|(y|z))p(y|z)p$$