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THE VERSATILITY AND ABSOLUTE EFFICIENCY OF DETECTING MIRROR SYMMETRY IN RANDOM DOT DISPLAYS

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Abstract—The detection of mirror symmetry has been investigated by measuring discriminability (d') between two populations of dot displays that contain mirror pairs and random dots in different proportions. The difficulty of the task was varied by changing the proportions of paired dots in the two populations, and also by changing the accuracy of positioning the paired dots. Symmetry can be detected in brief exposures, monocularly or binocularly, when the axis is not vertical, and when the axis is not central in the visual field. The mechanism is therefore versatile. Its efficiency can be measured when the pairing is imperfect, for unpaired dots then fall where a pair is expected, thus causing the spurious appearance of a pair. When the pairing has an accuracy of about $\pm 6'$ vertically and horizontally an ideal mechanism would achieve d' values about double those attained by subjects; they are thus using 25% of the statistical information available, which is a high figure considering the versatility and complexity of the mechanisms required.

INTRODUCTION

The importance of symmetry as an organizing factor in the perception of a visual scene was well appreciated by the Gestalt school (see Köhler, 1929; Koffka, 1935). What intrigued them was the global nature of the forces by which symmetry exerts its influence, for the fact that the contour on one side of a figure is symmetric with the contour on the other side certainly influences how it is seen, yet this requires interactions across the whole extent of the figure. The means by which such global effects are achieved is a problem of special interest when one tries to account for them in terms of single units (Barlow, 1972), for the pattern-selective properties of neurons in sensory systems typically extend only over small receptive areas. The object of the investigations reported in this paper was to explore what human subjects can and cannot achieve in the recognition and discrimination of symmetry, in the hope that this would define more precisely the tasks which neural symmetry-detecting mechanisms must achieve.

The illustrations in Fig. 1 are a sample from the range of symmetrical objects that may be encountered, and they also show some of the different types of symmetry that exist and are steadily detected. Bilateral mirror symmetry, or reflection in a line, is the most widely recognized form, but Figs 1b–d illustrate examples of translation, and rotation that also generate strong impressions. Furthermore these forms can be combined, and the fascinating popular exposition by Weyl (1952) not only outlines the mathematical theory of symmetry groups but also illustrates their role in visual art.

In the examples shown in Fig. 1 the symmetry is such an integral part of the object depicted that it is not easy to separate the abstract property "symmetry" from the concrete object. This separation becomes easier when symmetry is added to a pattern which by itself has no meaning. Figure 2A shows an array of randomly positioned dots. In 2B each dot is replicated at a position symmetrical about the vertical midline of the array. In 2C each dot has been replicated at a position diagonally displaced by 1/20 of the width of the array, and in 2D each dot is replicated at a distance 1/20 of the array's width from the parent dot in a direction 45° clockwise from the line joining the dot to the centre of the array. Similar patterns have been generated by Glass (1969), Glass and Perez (1973) and Glass and Switkes (1976); as they have shown, the result of the different duplication operations is immediately apparent, and one form of such pattern has recently been investigated by Stevens (1978). The original array, Fig. 2A is shared by all four figures, but this common parentage is completely overwhelmed perceptually by the different symmetry of each display; indeed it takes careful inspection to confirm that 2A is present in 2B, 2C and 2D.

In the experiments to be described here arrays of random dots were used as the vehicle for symmetry not only because they themselves lack structure and thus allow the structure introduced by symmetry to be more evident, but also for another reason. The elements of these dot arrays are all suprathreshold and in most cases easily resolved from their neighbours. It is therefore reasonably certain that perception of the properties of the displays is not limited by image quality or failure of the retina to signal the display centrally. One can thus hope to investigate the abilities and limitations of central mechanisms, and the global or holistic nature of symmetry makes it a particularly interesting aspect of these central mechanisms.

We chose to start by investigating mirror sym-

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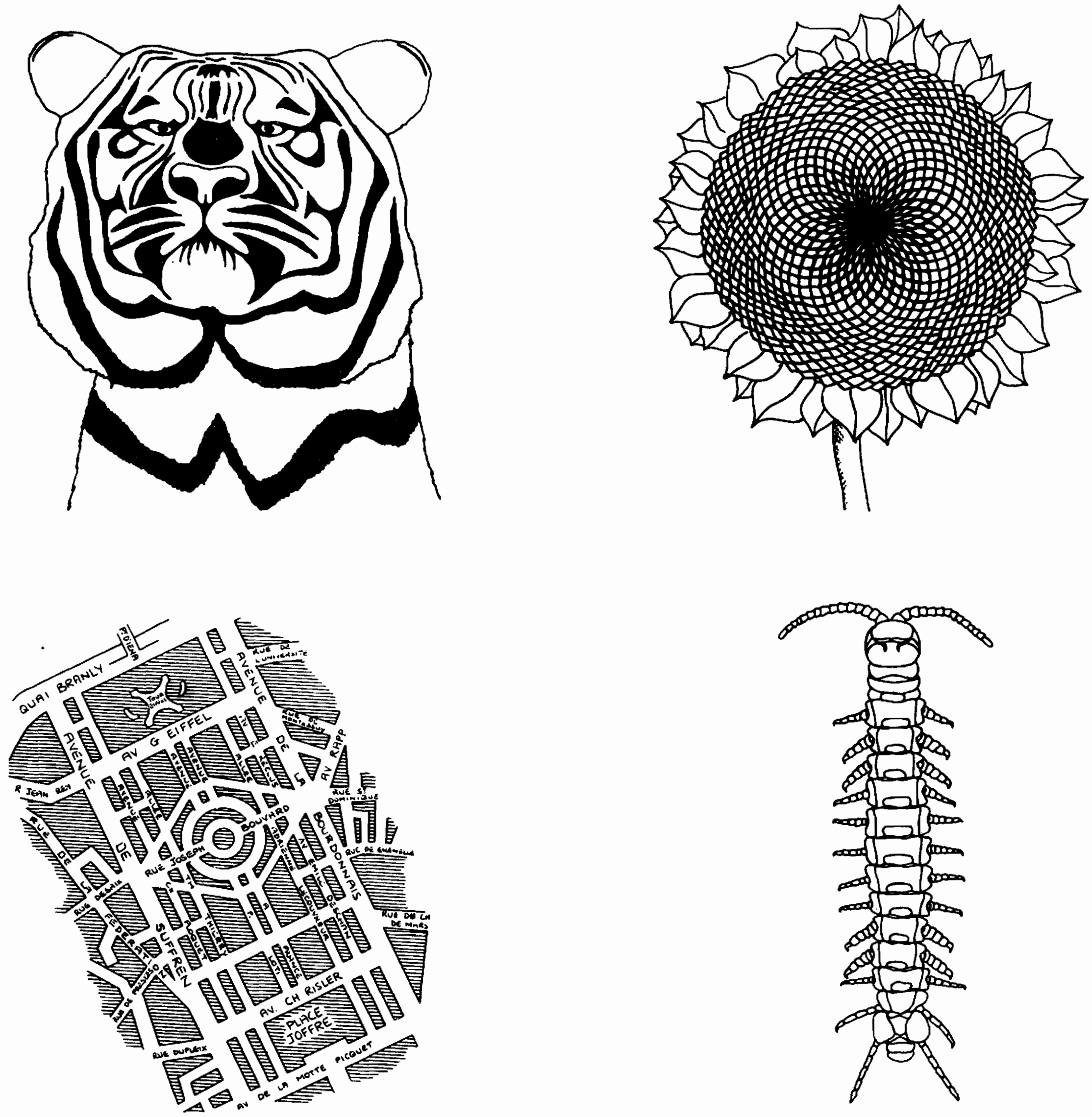


Fig. 1. Examples of symmetry.

metry. We thought at first that this might be immediately detectable only in an all-or-nothing fashion, when symmetry was complete and perfect, or that it might only be detectable with vertical orientation of the axis of symmetry, or only when this axis was central in the visual field. None of these assumptions proved to be correct, and the symmetry-detecting mechanism is thus rather a versatile one. Next we investigated the accuracy with which a point must be mirrored to contribute to the impression of symmetry. These measurements allow one to estimate how often the accidental occurrence of near-symmetrical dots contributes to errors in symmetry detection, and from this one can calculate the statistical efficiency of the symmetry detecting mechanism. Our general conclusion is that the mechanism is not only versatile, but also efficient, taking into account the complexity of the task and the statistical efficiency of perception of much simpler properties.

METHODS

Patterns of dots were generated by computer and visually displayed on D.E.C. GT44 oscilloscope display. For the current work only a few hundred dots were needed in each display, and the system could generate each picture comfortably in the 5 sec or so that elapsed between the response to one and the appearance of the next. Usually the subject sat 150 cm from the screen, at which distance 122 units of the 10 bit D/A's subtended 1°. The GT44 screen has good resolution and contrast but long persistence. The results of experiments in which the mirror axis was laterally displaced were confirmed using an oscilloscope with short persistence in place of the GT44 in order to avoid the possibility of moving the eyes to centralise the pattern.

The subjects were the authors, who both have good resolution at the viewing distance employed when using their normal corrections. The results shown in this paper are those achieved after considerable experience of each task. Some improvement with learning undoubtedly occurs, but it is not a very prominent effect and we have not done

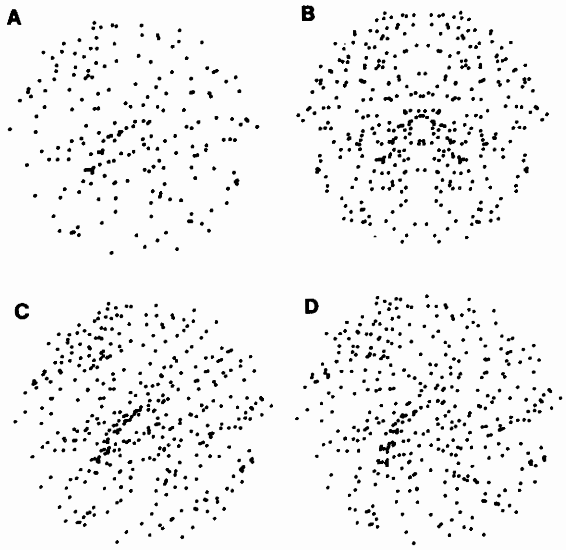


Fig. 2. A—200 dots are placed at random positions in a circular area. B—Each dot is repeated at its mirror position about the vertical midline. C—Each dot is repeated at a distance 1/20 of the diameter in an oblique direction. D—Each dot is repeated at 1/20 the diameter in a direction 45 anticlockwise from the line joining the dot to the centre. The impressions created by the pairing operations are immediately evident, but it takes detailed inspection to confirm that the dot pattern of A is contained in the other figures.

the special experiments that would be required to investigate its time course and extent. The order of individual trials was of course arranged to prevent learning or fatigue interfering with effects of other variables.

The subject's task was to view a pattern for a brief period, and then to signal from which of two parent populations it had been drawn. The two parent populations differed in the amount of bilateral symmetry they possessed; for instance one might contain 100 randomly positioned dots, while the other contained 60 randomly positioned dots and 40 that were in pairs about the midline. The subject knew the nature of the difference, and could call up examples from each population at will before the test period started. During the test period 100 samples were presented, the probability that it was from either population being 0.5 for each trial. After a correct decision nothing happened until the next picture appeared, but after a wrong decision a "beep" occurred; thus the subject was constantly reminded of the characteristics of the two populations.

The results of such trials were printed out as numbers and percentages correct for each category, and d' for distinguishing them was determined from Elliot's tables or the equivalent calculation, together with a calculated estimate of its sampling error.

The duration of exposure varied. In some trials it was 100 msec, when the probability of moving to a new fixation position during the exposure is small, but the standard GT44 display has such a long persistence that a second fixation on the after-glow might have been possible. Some tests have been duplicated, with similar results, on a display oscilloscope with short persistence, and we also tried to minimize its importance by having the screen illuminated so that the afterglow was less evident. In most cases the exposure was 500 msec, which seemed more relaxing for the subject since careful prefixation was not necessary. Subjectively the detection of symmetry appeared to be immediate, and changes of eye position did not seem necessary.

More details of particular experiments are given in the results section.

RESULTS

Sensitivity and versatility of symmetry detection

The first experiment was intended to test the sensitivity of the symmetry detecting mechanism by finding out how well one's capacity to detect it resisted dilution of the paired dots by randomly placed unpaired ones. Figure 3 shows a series each containing 100 dots. The top left has all the dots in pairs and is labelled 1.0. The top right labelled 0.7 has 70 in 35 symmetric pairs together with 30 randomly positioned dots, the lower left has 40 in pairs and 60 random, and the lower right 10 in pairs and 90 random. The impression of symmetry is strong in the top two, weak or absent in the lower ones. In order to obtain quantitative measures of this impression of symmetry we measured the discriminability of populations that differed only in the proportion P of dots that were in pairs, the proportion $(1 - P)$ being randomly placed. Some of these results are shown in Fig. 4. In this experiment the position of the pattern in the visual field was controlled by having the subject fixate a mark which was extinguished shortly before the target appeared. Exposure duration was 100 msec. The sequence of trials was arranged to minimize disturbing effects of learning or fatigue.

The points marked as crosses show the values of d' obtained when discriminating completely random $(P = 0)$ patterns from ones with the proportion symmetric shown on the abscissa; one can discriminate with $d' = 1$ when only 30 or 40% of the dots are in pairs. Now one can imagine a symmetry-detecting mechanism that would be sensitive enough to detect this degree of symmetry, but would be unable to signal varying degrees of symmetry above this level because it gave ungraded, all-or-nothing, responses. We therefore did the series of tests whose results are shown as the slanting dashed lines rising from the abscissa. In these experiments the subject had to discriminate populations differing by 0.3 in their proportions of symmetric dots, and the abscissa values at the ends of the lines show the lower and the upper proportions. The vertical rise of each line shows the discriminability measured as d'. It is clear that degree of symmetry can be used to discriminate over the whole range, being somewhat better for proportions approaching 1.

The two sets of results so far described are compatible. From the dashed lines one can obtain the average increment of d' for each 0.1 step, and by cumulating these values one obtains the continuous line. The crosses, which represent the discriminability of patterns with proportion X (plotted as abscissa) symmetric from all random $(P = 0)$, lie reasonably close to this line. In addition the discriminabilities of totally symmetric populations $(P = 1)$ from populations with X (abscissa) symmetric were measured. These are plotted (●) downwards (right ordinate) from the highest discriminability achieved, and also lie reasonably close to the line. We do not understand why the line is curved upwards, but it could result from the pos-

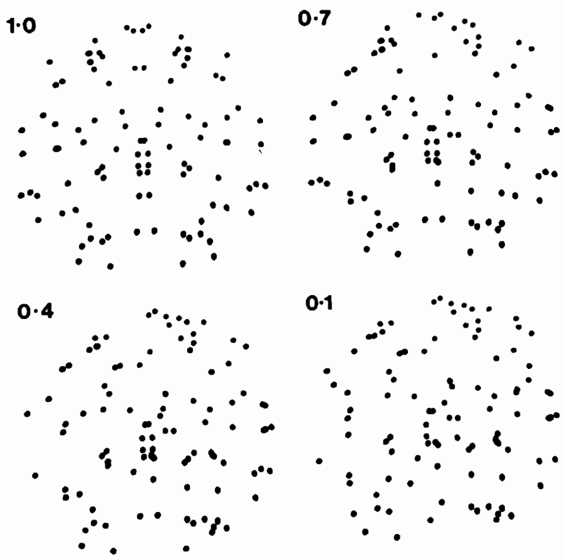


Fig. 3. Degradation of symmetry by substitution of randomly placed dots. Each pattern contains 100 dots with the proportion shown at top left belonging to pairs, the remainder being placed at random anywhere within the circular area. Symmetry is quite easy to detect when 60% of dots are random.

ition and orientation of the axis of symmetry being uncertain for low P, but well-defined for high P.

We concluded from this experiment, together with partial replications of it, that symmetry is represented in the brain as a graded rather than a discrete or all-or-nothing property. Arbitrarily taking a just noticeable difference as a difference discriminable

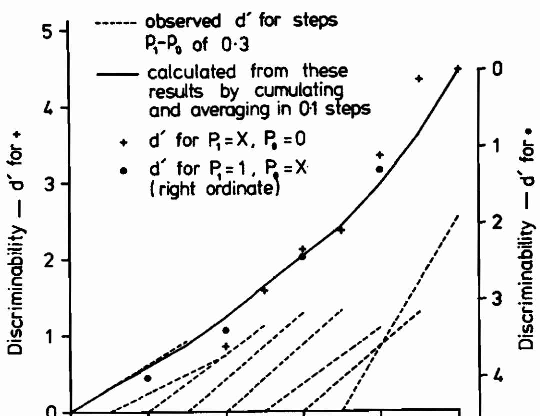


Fig. 4. Discriminability measured as d' (ordinate) between populations containing the proportions of symmetric dots given on the abscissa. The ends of the dotted lines show d' for proportions differing by 0.3, e.g. 0 and 0.3, 0.1 and 0.4. The continuous line is calculated from these as described in the text. +, Shows d' values for discriminating proportion on abscissa from proportion zero (i.e. all random). ●, Show d' values for discriminating between all paired and proportion on abscissa. These values are plotted downwards from the highest point, using the right-hand ordinate. The results are consistent with each other, and suggest that degree of symmetry is represented as a graded quantity, not in an all-or-none manner.

with $d' = 1$, there are 4 or 5 j.n.d.'s of symmetry in our conditions.

It was of some interest to test whether symmetry detection is greatly reduced when the patterns are viewed monocularly rather than binocularly. The experiment shown in Fig. 5 shows that there is no large effect.

In the experiments reported so far the axis of symmetry has been vertical and the pattern centrally fixated. The following experiments show that the mechanism is capable of detecting symmetry under a wider range of conditions. Figure 6 shows two experiments in which an array of random dots confined to a circular area was displayed for 100 msec just after extinguishing a central fixation mark. The axis of symmetry could be rotated to any angle but was constant throughout each run of 100 trials. The results for values from 0° (Horizontal) through 90° (Vertical) to 150° are shown. The task was to discriminate a population with 80 dots in pairs and 20 random ($P = 0.8$) from one in which all were random ($P = 0$). For both subjects the vertical axis (90°) is most easily discriminable, horizontal (0°, 180°) next, and oblique orientations are lower. Nonetheless symmetry can be detected at oblique axes, even with brief presentations.

The next experiment was designed to test whether bilateral symmetry could be detected when the axis was displaced to the right or left of the midline. Figure 7 shows an experiment on two subjects whose tasks were, as usual, to discriminate between patterns drawn from populations differing in their proportions (P_1 and P_0) of symmetric dots. This experiment was done first with $P_1 = 0.8$ and was repeated with $P_1 = 0.5$, P_0 being zero in both cases. For half the trials the pattern was not positioned with its vertical axis of symmetry in the midline, where the fixation mark had just been, but was displaced laterally by a predetermined distance. To avert anticipatory eye movements the direction of the displacement was also randomized, so that on any given trial the subject did not know if the pattern would appear centrally, or displaced to the right, or displaced to the left, though he did know the amount of displacement, and that half the trials would be central, half displaced. Each pattern contained 100 dots in a 2.1° square field.

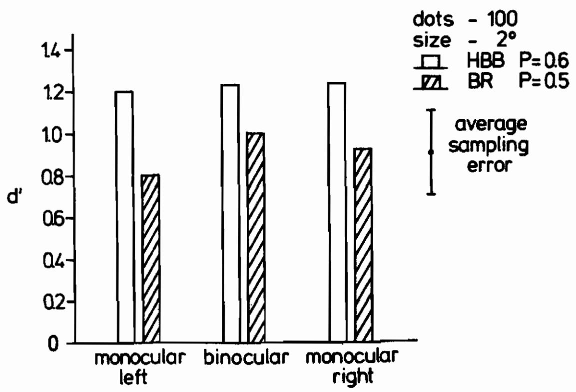


Fig. 5. Monocular and binocular performance in detecting symmetry for two subjects. There is no significant improvement with binocular vision.

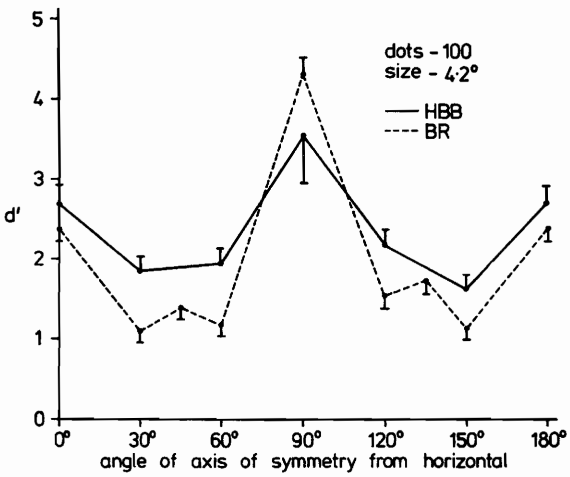


Fig. 6. Effect of the orientation of the axis of symmetry (measured in degrees clockwise from horizontal). 100 dots in a circular field with proportion 0.8 symmetric were to be discriminated from 100 randomly placed dots. Exposure time was 100 msec, and the axis was held constant for each run of 100 unknowns, and for an optional number of familiarizing trials. Discriminability is best for vertical axis and worst for obliques. 0° and 180° points are the same.

and a total of 200 trials were done in each run, 100 central and 100 displaced.

From Fig. 7 it can be seen at once that quite high values of d' are obtained when the axis of symmetry is displaced to the left or right of the fixation position by up to 3°, though the performance on the displaced patterns is worse than on the centrally located patterns run at the same time. The variation in performance on these patterns with central axis of symmetry is disappointingly large, and reasons for this were apparent when performing the tests. It is certainly more difficult to assess symmetry when the position of its vertical axis is unknown, and this was the case even when the displacement was small, for under these conditions the displacement of the whole target was not very noticeable, and one was sometimes taken aback to realize that a pattern was symmetric, but with the axis in an unexpected position. Because of this complication we do not want to draw any more precise conclusion that that symmetry can be detected when the axis is displaced, but with reduced sensitivity compared with a central axis.

When one is trying to detect symmetry in arrays of random dots such as those shown in Fig. 3 one is subjectively aware that paired dots near the axis create a strong and vivid impression, whereas dots further away do not. The four dots about $\frac{1}{3}$ from the top in Fig. 3 illustrate this. Subjectively one would also judge that the "outline", if one can call it such, is important, for this can create the impression of a vase, or a butterfly, or some other symmetrical object. In order to find which dots contributed most to the discrimination of symmetry we divided the square frame within which dots appeared into six vertical slices, as shown in Fig. 8, and arranged for 16 or 17 dots to appear in each slice in all trials. For the "symmetric" trials, one of the 3 pairs of slices contained 34 dots in 17 paired positions, whilst the other four slices contained 16 or 17 randomly positioned dots. By measuring the discriminability of displays in which the symmetry, when present, was confined to one pair of slices we could find how the impression of symmetry was influenced by the position of a dot in the overall pattern. The results in Fig. 8 show that symmetry is best detected when next to the axis, worst when in the middle of each half figure, and higher again when the symmetrical dots lie near the edge of each half figure.

Accuracy and absolute efficiency of detecting smeared symmetry

The results described so far show that the mechanism for detecting bilateral symmetry works for a variety of orientations (Fig. 5) and positions (Fig. 7) of the axis. Also it gives a graded message about the degree of symmetry (Fig. 4) and is sensitive in so far as it is able to detect symmetry in spite of a mask of unpaired random dots (Fig. 3). But to specify its sensitivity in any real sense one needs to know what is the natural limit to the ability to detect symmetry, and how close the human perceptual system comes to this limit.

In general it is necessary to know as much as possible about an object that is to be detected before one can specify the ideal method of detecting it, and the property of symmetry is no exception. There would be great difficulties in specifying such a method for detecting symmetry among the natural objects that

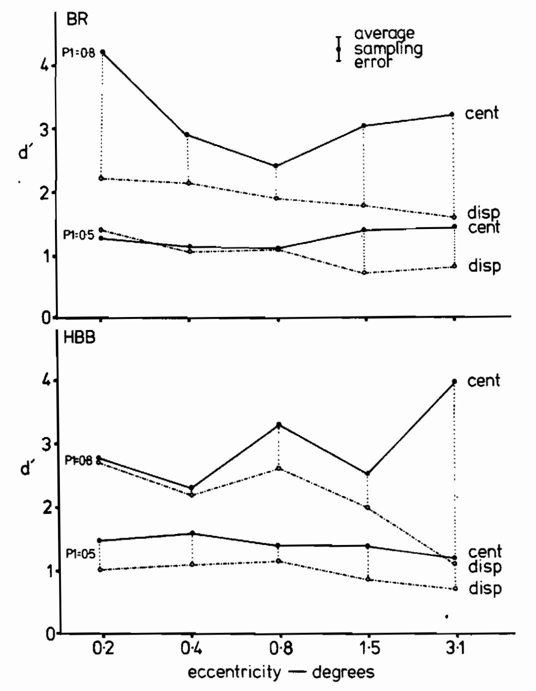
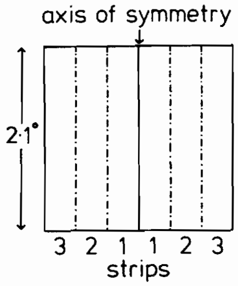


Fig. 7. Effect of displacement of the axis of symmetry. The two subjects fixed a mark; shortly after its disappearance the figure appeared for 100 msec either centrally (marked *cent*) or displaced either to left or right (marked *disp*) by the amount indicated on abscissa. In all except two cases performance was worse on the displaced pattern, by an amount indicated by the lengths of the dotted lines. The experiment was done with both 0.8 and 0.5 paired dots discriminated from all random.



	d'	
strips	BR	HB
1	4·6	3·6
2	0·97	0·76
3	1·8	2·7

Fig. 8. Symmetrical pairs of dots were placed in strips (1), (2) or (3) with random dots in the remainder. The resulting patterns were descriminated from a population in which the dots were random in all three strips. The results at right show that the pairs in the strips adjacent to the axis of symmetry are most easily discriminated, dots in the outer strips (3), which often delineate the outer border of the whole pattern, are next most effective, while those in the intermediate strips (2) are least effective, but nonetheless produce discrimination at d' of nearly 1. The average sampling error (calculated) for these estimates of d', based on 200 trials each, is ± 0.26.

are encountered, because we do not know what their properties are nor how they are distributed, but for the patterns produced by the computer we do have the necessary knowledge and the ideal method can be specified. The limiting factor in detecting the symmetry of these computer generated patterns will be brought out by first discussing briefly a non-ideal method which actually turns out to perform worse than human perception.

Suppose the task is to find whether any of the dots in the lower right pattern of Fig. 3 are symmetrically placed about the vertical midline of the figure. One would take each dot in turn on (say) the left half of the figure, calculate the co-ordinates of the mirror position, and check whether there is a dot there. These displays have 10 bit accuracy, so there are 2^{19} possible positions in each half field. Having specified a mirror position, there is clearly only a very small chance that it will be occupied by one of 50 randomly placed dots in that half field, and thus one should make very few errors in detecting even one pair of dots masked by 98 randomly placed ones in displays such as those of Fig. 3. However the key question is the accuracy with which the dots are placed. With 10 bit accuracy, 100% of paired presentations would be detected with 0.5% false positives. If the dots were positioned with 8 bit accuracy, the false positive rate would climb from 0.45 to 7.3%, and for 7 bit accuracy to 26%. The display normally subtends 2.1° at the subject's eye, so 7 bit accuracy corresponds to 1′, which is as high as the human visual system is likely to achieve.

Our experimental approach to this problem has been to produce patterns with smeared symmetry, made by reducing the accuracy with which pairs are placed, and then determine how this degradation influences the discrimination of symmetry. Figure 9 shows the principle on which "smeared" or imperfectly symmetrical figures are produced. The pair to a dot on the left is not placed at the exactly symmetric position on the right, but at a randomly chosen position within a box centred on that position.

Figures 10 and 11 show the results of two repetitions of an experiment in which the vertical and horizontal tolerance ranges of the paired dots were varied together. The subject's task was the usual one of judging whether a sample display came from the paired ($P_1 = 1$) or unpaired ($P_0 = 0$) populations, the paired ones now being imperfectly paired to a varying extent. There was a minor difference between the programs for the two experiments. For Fig. 10 the inaccurate pairing was done exactly as shown in Fig. 9; by error, it was always the dot on the right that was randomly perturbed, hence dots near the edges of the right half were sometimes displaced outside the area normally covered by the right half of the pattern. The resulting unintended asymmetry was never actually noticed by the subjects, but because the experiment is important it was repeated after correcting the program, and these results (which do not differ significantly) are shown in Fig. 11.

In Fig. 10, the percent correct is plotted to facilitate comparison with the probability calculations of the preceding and following discussion. As might be expected, the percent correct decreases as the accuracy of the pairing gets worse, and ultimately approaches 50%, the chance level. Both subjects get 75% correct when the tolerance area (Fig. 9) subtends about 0.4°, which is $\frac{1}{5}$ the total width of the patterns displayed. This shows that the mechanism is not highly accurate but counts dots as contributing towards the impression of symmetry provided they fall within the range of 12′ or so around the exactly symmetric position.

With a tolerance range of 24′ × 24′ there is a high probability a dot will occur by chance in the pair area for a dot, even with completely random patterns, so the method outlined above would perform very badly. The dotted line in Fig. 10 shows the expected percent correct if the occurrence of 1 dot in the tolerance range was sufficient to cause a pattern to be classified as symmetric. The example shows the importance of accidentally occurring spurious pairs in causing false positive responses and thereby limiting the discrimination of symmetry. This is the source of the "noise".

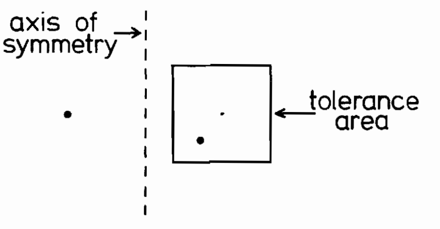


Fig. 9. Smeared mirror symmetry. Instead of pairing the left-hand dot at its exact mirror position (small dot) it is placed at a randomly selected position in a tolerance area. Unless the tolerance area is very small it is likely to contain other dots which were not placed as pairs, and random fluctuations in the number of these false pairs will set a limit to the reliable discrimination of paired and unpaired populations.

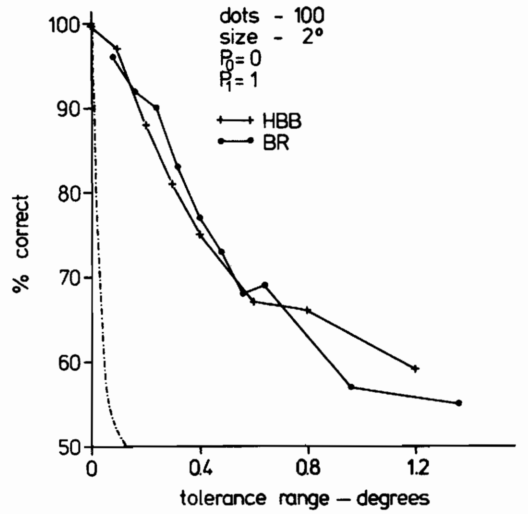


Fig. 10. The effect of inaccurate pairing on the discrimination of symmetrical from random patterns of 100 dots. Percent correct classification is plotted against the tolerance range (see Fig. 9) of the inaccurate pairing. 75% are still correctly classified when the tolerance range is 0.4°, or ±12′ both vertically and horizontally. The dotted line shows the calculated percent correct if a pattern was classified as "paired" whenever one or more dots fell in the tolerance range of another. Clearly the eye does better than this strategy.

The optimal procedure for discriminating the symmetric from the random patterns produced by the computer can now be specified. The tolerance area corresponding to each dot must be searched to find the total number of pairs that would qualify as deliberately generated pairs under the existing tolerance conditions. This number includes all those that were actually generated as pairs, plus a number of spurious pairs that result from a dot happening to fall within the tolerance range. The number of these spurious pairs will vary by chance from trial to trial, and will of course be larger when a larger tolerance area is being used; thus a random pattern may by chance contain more qualifying pairs than one which had been made as paired, and errors of discrimination are bound to occur.

The discriminability, using this optimum method, can be specified as d'_I, and this is simply $\Delta N/\sigma_N$, where ΔN is the average number of qualifying pairs in the "paired" patterns less the average number in "unpaired", and σ_N is the standard deviation of the number of qualifying pairs in the unpaired patterns. The calculation of d'_I is quite straightforward when the tolerance range is small, but the simple calculation based on the ratio of the tolerance area to the total area containing dots becomes very inaccurate when the tolerance is large, because the total area is then enlarged and non-uniformly covered with dots. It is quite simple, however, to estimate ΔN and σ_N from computer simulations of the ideal procedure, and this we have done.

If one knows the d' attainable ideally, and can also estimate the value attainable by human subjects, then one can specify the absolute efficiency of the performance achieved by the subjects. It is given by

$$F = (d'_E/d'_I)^2. \tag{1}$$

This is an estimate of the proportion of the available statistical information that is used by the subject. For a fuller discussion of the concept of efficiency see Rose (1948), Tanner and Birdsall (1958) or Barlow (1978) and the Discussion. One can regard the efficiency F as a measure of the extent to which signal/noise ratios present in the target are preserved in the sensory representation which the subject uses when making his decisions; the "signal" here is of course the symmetry of the patterns, and the "noise" is what obscures symmetry, in this case the occurrence of spurious symmetric pairs.

Figure 11 shows the repetition of Fig. 10 after correcting the trivial programming error mentioned above. The results were not substantially different, and are shown in Fig. 11 plotted as d'_E values for different tolerance ranges, which are here plotted on a logarithmic scale. In addition computer simulations were performed to determine d'_I, and the continuous line through the results of these trials is also shown; the right ordinate of Fig. 11 applies to this line.[2]

The ordinates for the ideal curve on the right edge of Fig. 11 have been chosen so that the experimental points lie close to the theoretical curve for tolerances greater than about 12′. This scale turns out to be exactly half that used for the experimental points, that is $d'_E = d'_I/2$ where the points fit the line. From expression (1) it follows that the efficiency F is 25% when the tolerance range is greater than about 12′ (±6′). We are uncertain about the efficiency at high tolerances partly because the low experimental d' values have large margins of error, and partly because of the theoretical inadequacy mentioned above.

[2] The basis of this "ideal" discrimination of symmetry is the total number of dots that fall within the tolerance range of another dot, the tolerance range being defined as in Fig. 9. It must be admitted that this is not quite the optimum measure for symmetry, for the following reason. When the tolerance range is large, the dots are not uniformly distributed over the whole square, either in the "symmetrical", or in the "random" populations, because of the decrease of dot density at the edge of the picture. Strictly speaking, therefore, the significance of a dot occurring within a tolerance range is also non-uniform, and ideally this should be taken account of. We have not done this and our estimated efficiencies are therefore slightly too high. However this cannot be important except for the largest tolerance ranges, and we have therefore ignored it.

DISCUSSION

Versatility of symmetry detection

Symmetry is a property of visual images that is apprehended immediately, without searching eye movements or introspective analysis. The results reported here show that the neural mechanism for achieving this is more versatile and efficient than we initially supposed. It can detect symmetry about non-vertical and non-horizontal axes, and when it is displaced from the midline, though with reduced sensitivity compared with detection when the axis is midline and vertical. Corballis and Roldan (1975)

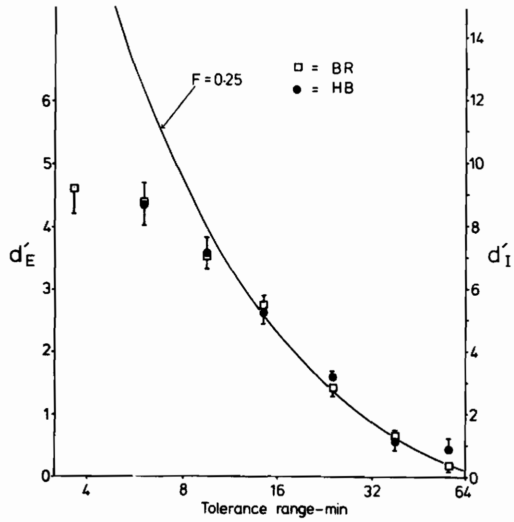


Fig. 11. A repetition of Fig. 10 with the results plotted as d'_E and the abscissa scale logarithmic. The continuous line shows the d'_I values (right-hand ordinate) obtained by Monte Carlo simulation of the process of searching all dot pairs exhaustively and counting those that qualify as symmetric under the tolerance range being used. If this process is performed with 25% efficiency, d' values are halved, and the left-hand scale is appropriate. For tolerances greater than about 12′ (±6′) the subject's performance matches that of the 25% efficient model.

reported that subjects could detect symmetry of six-dot patterns about horizontal and oblique axes, but with a slight prolongation of reaction time. They interpreted this prolongation as the time for "mental rotation", which is a difficult concept to attempt to understand in physiological terms. However the main result in that paper was to show that, with the head tilted to 45°, the position of minimum reaction time was also tilted, thus casting doubts on earlier suggestions that symmetry detection was best around the phenomenological vertical rather than the anatomically defined vertical (Goldmeier, 1937; Rock and Leaman, 1963).

The literature is not clear on the ability to detect symmetry about axes displaced from the midline, for Julesz (1972) suggests it is not possible for complex random dot patterns, but is possible for simple figures. Our results show clearly that it is possible with random dot figures containing 100 dots, though it is not done as well as when the axis is midline. Julesz' statement that symmetry among the dots of random dot patterns is most evident for those that lie close to the axis of symmetry is confirmed in the six-slice experiment of Fig. 8. Bruce and Morgan (1975) found that violations of symmetry were more readily detected when they lay close to the axis of symmetry, and they emphasize that the proximity of points that have to be compared is an important factor in determining how easily the comparison is made, and may be responsible for the greater difficulty of detecting violations of repetition, rather than mirror symmetry. The results of Fig. 8 do show, however that the outer slices, containing the dots that form the outer contour of the figure, are important in generating the perception of symmetry.

Efficiency of symmetry detection

With regard to efficiency, we do not think there are any previous experiments which show how well, in absolute terms, a task as complex and abstract as the detection of symmetry is performed. It is worthwhile explaining the meaning of efficiency in rather more detail because it is a measure yielding much stronger and more immediate implications than others commonly used in psychophysics and psychology.

The concept of signal-to-noise ratio (S/N) is nowadays quite familiar: the detection of a specific message or "signal" is rendered difficult by unwanted disturbances, or "noise". When the signal and noise are measured in appropriate units their ratio determines the liability to err, that is the frequency of failures to detect the message, and of false alarms when the message is claimed to be present but is not. For a simple task, such as the detection of a dim light added to a constant background, one can calculate the S/N ratio for the light entering a subject's pupil. Now the signal-detection-theorist's measure of discriminability, d', can be regarded as an estimate of the S/N ratio of the sensory messages that the subject uses to make his decisions when performing the discrimination. Thus we can follow the fate of the physical message after it enters the eye: clearly in a highly efficient system the S/N ratio would be preserved, and we would expect the d' values achieved in a psychophysical experiment to approach the value of the S/N ratio in the messages entering the eye. In fact, in the case of detecting an increment of light on a steady background, it would be much lower because the eye fails to make use of all the quanta that enter the pupil and because there are other sources of inefficiency (Barlow, 1977). The important point is that there is a hard physical limit to performance, and one can determine how closely human vision approaches it.

In the above example the "noise" arose from fluctuations in the numbers of quanta entering the eye from the background light on which the added light, or "signal", was superimposed. It is probably unusual for perception to be limited by a physical source of noise that can be readily recognized, as in that case. Rather, perception fails through confusion of the specific signal with other messages that are not random, but are unwanted and unpredictable disturbances in the sense that they interfere with the detection of the sought-for signal. The value of the S/N ratio can be calculated in such cases if one can measure the extent to which the "noise" is liable to be mistaken for the signal, but this may be a very difficult problem. What is the signal that enables one to recognize a familiar face in a crowd, and what is the noise that interferes? Whatever the answers, the message "That is the face of my friend" comes through loud and clear, and it would certainly be interesting to know how the S/N ratio for this particular message was preserved.

The experiments reported here were undertaken as a first step in this direction. In considering the figure of 25% efficiency achieved in detecting symmetry in random dot patterns one must remember first that we hope to have avoided the losses from failure to absorb quanta. Each dot in our patterns has a very high S/N ratio, easily high enough to be reliably detected, so the noise arises, not from quantal fluctuations, but from deliberate randomization while forming the patterns. Nevertheless, 25% seems extraordinarily high in view of two facts. First, we have only optimized for the magnitude of tolerance range. It is almost certain that the tolerance for counting a dot as symmetric is much less when the dots are close to the midline axis of symmetry than when they are remote from it, and ideally one should have a tolerance graded with distance from the axis. Furthermore we have confined ourselves to uniform distributions and have not optimized for the shape of the tolerance area. Optimization along these lines might increase efficiency considerably by better matching the properties of the patterns produced by the computer to those of the human symmetry-detector.

The second reason that 25% seems high is that figures about 50% have not been obtained even on much simpler tasks (Barlow, 1978; Van Meeteren and Barlow, in preparation) involving the detection of changes in the average density of dots in random dot patterns. The detection of symmetry is certainly a more complex task than detecting changes of dot density; the latter can be done efficiently by just counting the numbers of dots in a comparatively small number of areas over the pattern, but to detect symmetry one needs to deal with the properties of pairs; since there are $n(n-1)/2$ different pairs among n objects it can be seen at once that the task is much more complex. It should be noted that, since the information about symmetry lies in the pairwise structure, the figure of 25% efficiency can be regarded as showing that $\frac{1}{4}$ of the *pairs* are used in the discrimination. This would imply that $\sqrt{1/4}$, or 50% of the dots are used, because the number of pairs is almost proportional to n^2, but the agreement of this figure with the highest figure obtained for detecting changes in dot density may be coincidental.

Mechanisms

The fact that surprisingly large inaccuracy in the placing of pairs can be tolerated suggests a model that is more plausible as the basis of the neurophysiological mechanism than the fully efficient process. This fully efficient process searches exhaustively through all pairs, counting those that qualify as symmetric. Instead, suppose the area is divided into subregions, and the numbers of dots in each subregion are counted. If the pattern is symmetric, the numbers in symmetrical placed subregions will be equal; if there is no symmetry, the difference between symmetric regions will vary as much as the difference between any other two regions, that is the variance of the difference will be twice the variance of the number in a single square.

The dotted curve in Fig. 12 shows the performance of a model in which the 2.1° square was divided into 16 smaller squares, each of which is 32′ by 32′, and therefore comparable in size to the tolerance range for which the eye performs most efficiently. The numbers of dots in each small square were counted and then χ^2 was calculated as

$$\sum_j (N_j - N_j')^2/2\bar{N};$$

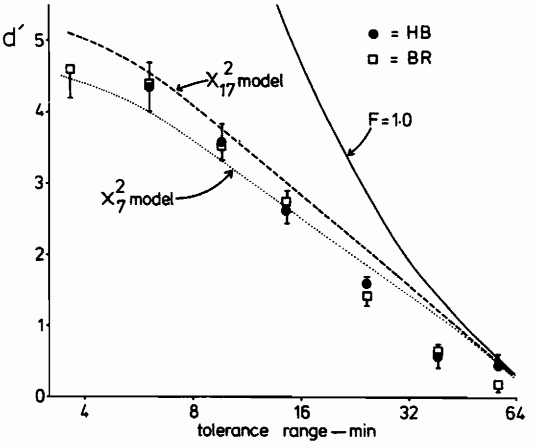


Fig. 12. The same experimental points as in Fig. 11 fitted by the dotted curve that describes the performance of a model, determined by Monte Carlo simulation. This model bases its discrimination of symmetry on the value of χ^2 computed for the hypothesis that there is no symmetry about the vertical midline axis. For the lower curve the large square was divided into 16 small squares each 31′ by 32′, and the test was applied to the number of dots in symmetrically placed squares. If there is no symmetry the expected value is 7, the number of degrees of freedom for the 8 pairs. For the upper curve the large square was subdivided into 36 small squares each 21′ by 21′, with expected χ^2 of 17. With symmetry, the value of χ^2 is lowered and a criterion for deciding if it is present is set to equalize errors of the two kinds. The results of the fully efficient process of exhaustive pair-searching (see text) is also shown as a continuous curve marked $F = 1$.

where N_j, N_j' are the numbers of dots in one pair of the 8 paired symmetric squares and $\bar{N}$ is the mean. This corresponds to the hypothesis that there was no symmetrical disposition of dots about the vertical midline and if the hypothesis is correct χ^2 should have an average value equal to the number of degrees of freedom, which is 7 in this case. If the hypothesis is incorrect, the numbers in the symmetric squares will be more nearly equal than expected and χ^2 will be lower than 7, dropping to zero for perfect symmetry. The model takes a criterion value of χ^2 that causes approximately equal numbers of detection failures and false positives, and the dotted line in Fig. 12 shows its performance as determined from Monte Carlo trials. It is of course much worse than the process in which all pairs of dots are tested to see if they qualify as symmetric, which is shown as the continuous line marked $F = 1$ in Fig. 12, but the χ^2 model gives a reasonable fit to the experimental points. The dashed curve shows that results for a similar model using 36 squares each 21′ by 21′; this improves performance at small tolerances. At large tolerances both models out-perform the subjects.

We do not attach much importance to the details of the model, but the adequacy of the fit does suggest that symmetry detection on our tasks requires nothing more than the comparison of dot densities measured over quite large areas symmetrically placed about the putative axis of symmetry. In the present case this reduces the number of comparisons that must be made from the total number of dot pairs (4950 for 100 dots) to the number of paired regions

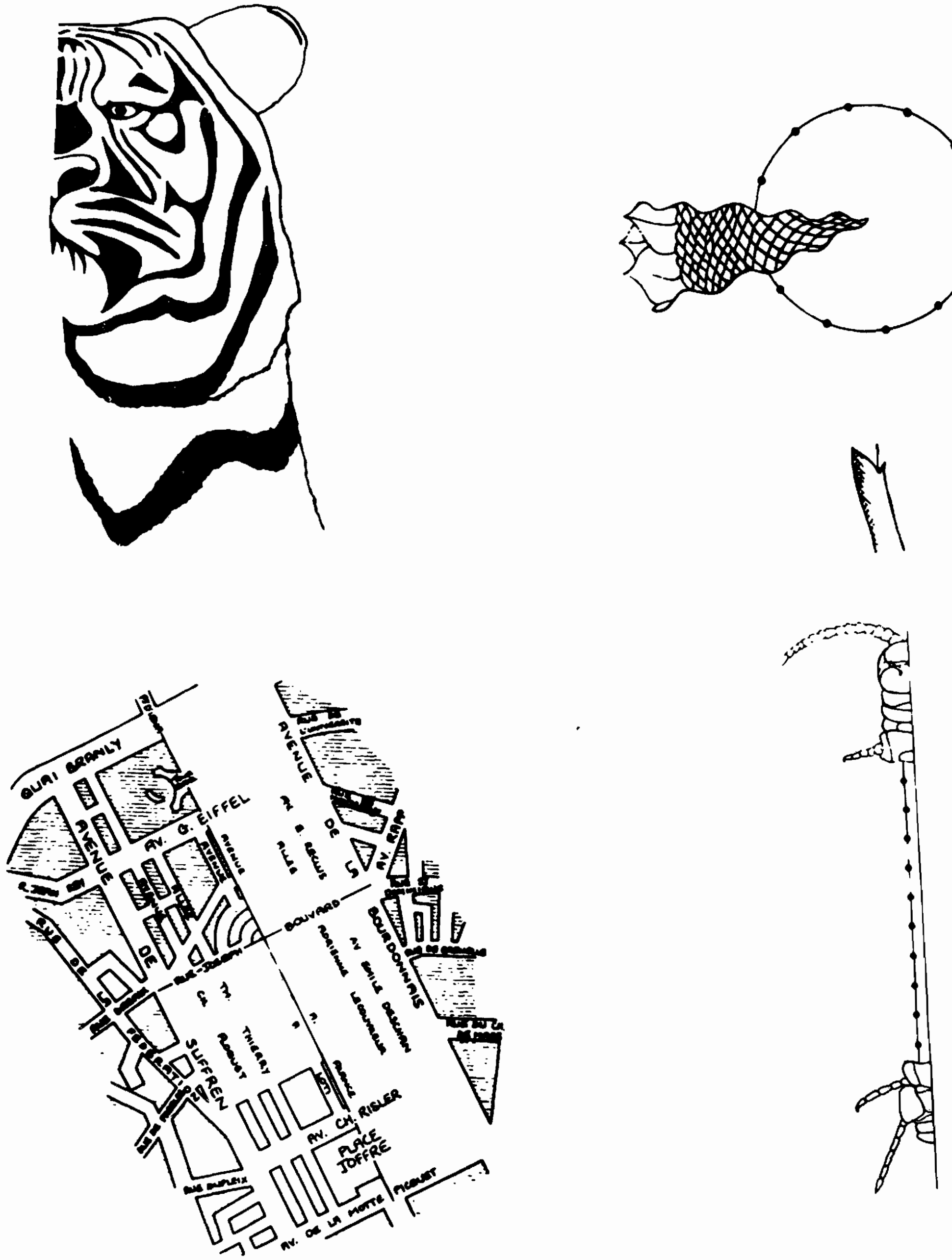


Fig. 13. The pictures of Fig. 1 have been blanked out in the regions which can be "filled in" by referring to other regions, once the symmetry is recognized. The recognition of symmetry allows considerable economy of representation.

(8 in the model of Fig. 12). Furthermore, making an assessment of the number of dots in a fixed area is the sort of operation one can visualize a neuron with a fixed receptive field performing. But although the large tolerance simplifies the task in the way suggested by the model, it is important to remember that the mechanism is versatile and can detect symmetry about axes that are not central and not vertical. Each different position of the axis requires a different set of comparisons, and the means by which these sets of comparisons are brought about is not easy to imagine.

Survival value of symmetry detection

Why is symmetry so important in perception and so universal in art? What is its survival value? One possibility is that living organisms and man-made artefacts are often symmetrical, and these are important to detect. Computer surveys of pictures transmitted from earth-satellites are sometimes programmed to detect symmetry because it is found to be a useful guide to areas of potential military importance (Stockman and Kanal, 1976; note also Fig. 1). Symmetry may also be important for shape recognition, since it helps to establish an object-centred coordinate frame (see Marr and Nishihara, 1978). There is, however, a more general and abstract aspect of symmetry that could lie behind the artist's repetitiveness, the regularity of living forms and man-made symmetrical features, and the visual system's apparent eagerness to detect it. This is the fact that symmetry in an image allows it to be described economically. For instance if one half of an object is the mirror image of the other half, then one half need not be described at all. Similarly if a surface is filled with a repeating pattern, the description of the unit cell and extent of the pattern is a complete description. Thus the detection of symmetry is potentially a powerful means of generating a more compact representation of the sensory input. Figure 13 shows what remains of Fig. 1 if the redundant, symmetric portions are removed.

Symmetry detection may thus be an important step in the formation of a less redundant representation of the visual image in the manner suggested by Attneave (1954) and Barlow (1961). Undoubtedly it is not possible for a visual image to be represented completely, and any regularity such as symmetry is valuable for the very reason that it represents more of the image than an arbitrary or irregular feature (Barlow, 1974). For the hard-worked brain the "Tyger's... fearful symmetry" (Blake, 1794) must come as welcome regularity among a chaotic jumble of sensory messages; but we can still wonder at the versatility and efficiency by which it is framed.

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