

MATHEMATICS

(10-12/GCE)

PAPER ONE

SOLVED PAST PAPERS WITH EXPLANATIONS



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MATHEMATICS PAPER 1 (4024/1)

This paper comprises of **23 short answer questions** with spaces provided below each question for showing essential working where necessary. You will be expected to write your answers clearly in the spaces provided.

- * You have to show all your working clearly.
- * You are not allowed to use a calculator.

Study Tips

The only key to passing mathematics is **a lot of practice**. Make sure you go through some questions from different topics everyday.

The topics that appear in this paper are repeated every year as you will tell from this pamphlet. Therefore its important that you understand those topics and solve as many questions as possible.

Note:

- You will not use a calculator therefore revise basic mathematics like addition, subtraction, multiplication and division.
- Revise past papers from 2016 - present.
- Go through this pamphlet regularly after every topic to see how questions are asked in the exam and how to answer them.

Once you go through all the questions and answers in this pamphlet, I am certain that you will be ready to face the exam.

ALL THE BEST!!!!

Mr 6points



Mr 6points



@ mr_6points



Mr 6points

TOPIC 1 - BASIC ARITHMETIC

1. 2012 P1

Evaluate $7 + 21 \div 3 \times 7 - 7$

$$\begin{aligned} 7 + 21 \div 3 \times 7 - 7 & \quad \text{Step 1- Divide 21 by 3.} \\ = 7 + 7 \times 7 - 7 & \quad \text{Step 2- Multiply 7 by 7.} \\ = 7 + 49 - 7 & \quad \text{Step 3- You can either start with } 7 + 49 \text{ OR } 49 - 7. \\ = 56 - 7 & \quad \text{Step 4- Subtract 7 from 56.} \\ = 49 & \end{aligned}$$

2. 2014 P1

Evaluate $14 + 3(7 - 2) - 2 \times 5$

$$\begin{aligned} 14 + 3(7 - 2) - 2 \times 5 & \quad \text{Step 1- Multiply 3 with everything in the brackets.} \\ = 14 + 21 - 6 - 2 \times 5 & \quad \text{Step 2- Multiply -2 and 5.} \\ = 14 + 21 - 6 - 10 & \quad \text{Step 3- Add 14 and 21 and subtract -10 from -6.} \\ = 35 - 16 & \quad \text{Step 4- Subtract 16 from 35.} \\ = 19 & \end{aligned}$$

3. 2015 G.C.E P1

Evaluate $1 + 7 \times 3$

$$\begin{aligned} 1 + 7 \times 3 & \quad \text{Step 1- Multiply 7 with 3 and then add 1.} \\ = 1 + 21 & \quad \text{Step 2- Add 1 and 21.} \\ = 22 & \end{aligned}$$

4. 2016 G.C.E P1

Find the value of $3 - 3 \times 3 + 3$

$$\begin{aligned} 3 - 3 \times 3 + 3 & \quad \text{Step 1- Multiply -3 with 3.} \\ = 3 - 9 + 3 & \quad \text{Step 2- Subtract 9 from 3.} \\ = -6 + 3 & \quad \text{Step 3- Add -6 and 3.} \\ = -3 & \end{aligned}$$

TOPIC 2 - BASIC ALGEBRA

1. 2016 SPECIMEN P1

Simplify $5(2a - 3b) - (6a - 2)$

$$\begin{aligned} & \overline{5(2a - 3b)} \quad \overline{-(6a - 2)} \\ &= 10a - 15b - 6a + 2 \\ &= 10a - 6a - 15b + 2 \\ &= \mathbf{4a - 15b + 2} \end{aligned}$$

Step 1 - Start with the brackets and multiply 5 with $(2a - 3b)$ and $(-)$ with $(6a - 2)$.

Step 2 - Group the like terms.

Step 3 - Subtract 6a from 10a.

2. 2017 P1

Simplify $3x - (y - 2x) - 3y$

$$\begin{aligned} & 3x - \overline{(y - 2x)} - 3y \\ &= 3x - y + 2x - 3y \\ &= 3x + 2x - y - 3y \\ &= \mathbf{5x - 4y} \end{aligned}$$

Step 1 - Start with the brackets and multiply $(-)$ with $(y - 2x)$.

Step 2 - Group the like terms .

Step 3 - Add 3x and 2x and subtract $-3y$ from $-y$.

3. 2017 G.C.E P1

Simplify $2a - 7b - 2(a - 3b)$

$$\begin{aligned} & 2a - 7b - \overline{2(a - 3b)} \\ &= 2a - 7b - 2a + 6b \\ &= 2a - 2a - 7b + 6b \\ &= 0 - 1b \\ &= \mathbf{-b} \end{aligned}$$

Step 1 - Start with the brackets and multiply (-2) with $(a - 3b)$.

Step 2 - Group the like terms .

Step 3 - Subtract 2a from 2a and add 6b to $-7b$.

NOTE: $-1b = -b$

4. 2018 G.C.E P1

Simplify $4(x + 2y) - (3x - 8y)$

$$\begin{aligned} & \overline{4(x + 2y)} - \overline{(3x - 8y)} \\ &= 4x + 8y - 3x + 8y \\ &= 4x - 3x + 8y + 8y \\ &= x - 16y \end{aligned}$$

Step 1 - Start with the brackets and multiply 4 with $(x + 2y)$ and the $(-)$ with $(3x - 8y)$.

Step 2 - Group the like terms .

Step 3 - Subtract $3x$ from $4x$ and add $8y + 8y$.

NOTE: $1x = x$

5. 2019 GCE P1

Simplify $4 - 2(b - a) - 1$

$$\begin{aligned} & \overline{4} - \overline{2(b - a)} - 1 \\ &= 4 - 2b + 2a - 1 \\ &= 4 - 1 - 2b + 2a \\ &= 3 - 2b + 2a \\ &= 2a - 2b + 3 \end{aligned}$$

Step 1 - Start with the brackets and multiply (-2) with $(b - a)$.

Step 2 - Group the like terms.

Step 3 - Simplify

Step 4 - Rewrite the expression and put the constant at the end.

6. 2020 GCE P1

Simplify $2a + (b - a) - 2b$

$$\begin{aligned} & 2a + (b - a) - 2b \\ &= 2a + (-a + b) - 2b \\ &= 2a - a + b - 2b \\ &= a + b - 2b \\ &= a - b \end{aligned}$$

Step 1 - Rewrite the expression in the brackets from $(b - a)$ to $(-a + b)$.

Step 2 - Remove the domain brackets

Step 3 - Group the like terms

Step 3 - Subtract a from $2a$

Step 4 - Subtract $-2b$ from b

FACTORISATION

1. 2007 P2

Factorise Completely $3x - 12x^3$

$$\begin{aligned} & 3x - 12x^3 && \text{Step 1 - factorise } 3x \text{ out.} \\ & = 3x(1 - 4x^2) && \text{Step 2 - Find the square root of } 1, 4 \text{ and } x^2. \\ & = 3x(1 + 2x)(1 - 2x) && \text{Step 3 - Factorize using difference of two} \\ & && \text{squares. Remember } a^2 - b^2 = (a + b)(a - b) \end{aligned}$$

2. 2016 SPECIMEN P1

Factorise Completely $3x^2 - 27$

$$\begin{aligned} & 3x^2 - 27 && \text{Step 1 - factorise } 3 \text{ out.} \\ & = 3(x^2 - 9) && \text{Step 2 - Find the square root of } x^2 \text{ and } 9 \\ & = 3(x + 3)(x - 3) && \text{Step 3 - Factorize using difference of two} \\ & && \text{squares. Remember } a^2 - b^2 = (a + b)(a - b) \end{aligned}$$

3. 2017 P1

factorise completely $ax^2y - 4ay^3$

$$\begin{aligned} & ax^2y - 4ay^3 && \text{Step 1 - } a \text{ and } y \text{ are common (factorise).} \\ & = ay(x^2 - 4y^2) && \text{Step 2 - Find the square root of } x^2, 4 \text{ and } y^2 \\ & = ay(x + 2y)(x - 2y) && \text{Step 3 - Factorize using difference of two} \\ & && \text{squares. Remember } a^2 - b^2 = (a + b)(a - b) \end{aligned}$$

4. 2017 G.C.E P1

factorise competely $\frac{x^2}{4y^2} - \frac{1}{9}$

$$\frac{x^2}{4y^2} - \frac{1}{9}$$

Step 1 - find the sqaure root of each term
and then factorise using difference of squares .

$$= \left(\frac{x}{2y} + \frac{1}{3} \right) \left(\frac{x}{2y} - \frac{1}{3} \right)$$

5. 2018 GCE P1

factorise competely $5x^2 - 5$

$$5x^2 - 5$$

Step 1 - Factorise 5 out .

$$= 5(x^2 - 1)$$

Step 1 - Find the sqaure root of x^2 and $1 \cdot 1^2 = 1$

$$= 5(x + 1)(x - 1)$$

Step 3 - Factorize using difference of two
squares. Remember $a^2 - b^2 = (a + b)(a - b)$

6. 2019 GCE P1

factorise competely $32x^2 - 50$

$$32x^2 - 50$$

Step 1 - factorise 2 out.

$$= 2(16x^2 - 25)$$

Step 2 - Find the square root of $16x^2$ and 25

$$= 2(4x + 5)(4x - 5)$$

Step 3 - Factorize using difference of two
squares. Remember $a^2 - b^2 = (a + b)(a - b)$

TOPIC 3 - INDICES

1. 2016 P1 Evaluate $8^{\frac{2}{3}}$

$$\begin{aligned} & 8^{\frac{2}{3}} \\ &= (2^3)^{\frac{2}{3}} \\ &= 2^{3 \times \frac{2}{3}} \\ &= 2^2 \\ &= 4 \end{aligned}$$

Step 1 - Rewrite the 8 as 2^3

Step 2 - Multiply the powers and 3 will cancel 3 and you are left with 2.

Step 3 - multiply 2 twice.

2. 2016 GCE P1 Evaluate $3^2 + 2^3 + 2^0$

$$\begin{aligned} & 3^2 + 2^3 + 2^0 \\ &= 9 + 8 + 1 \\ &= 18 \end{aligned}$$

Step 1 - Any number to the power 0 is 1.

Step 2 - Add 9, 8 and 1.

3. 2017 G.C.E P1 Evaluate $25^x = 5$

$$\begin{aligned} & 25^x = 5 \\ &= 5^{2x} = 5^1 \\ &= 2x = 1 \\ &= \frac{2x}{2} = \frac{1}{2} \\ &x = \frac{1}{2} \end{aligned}$$

Step 1 - Rewrite the 25 as 5^2 .

Step 2 - When the base is the same just deal with the powers.

Step 3 - Divide both sides with 2 to find x.

4. 2018 GCE P1

Evaluate $27^{\frac{2}{3}}$

$$27^{\frac{2}{3}}$$

Step 1 - Rewrite the 27 as 3^3

$$= (3^3)^{\frac{2}{3}}$$

Step 2 - Multiply the powers and 3 will cancel 3 and you are left with 2

$$= 3^{3 \times \frac{2}{3}}$$

Step 3 - multiply 3 twice.

$$= 3^2$$

$$= 3 \times 3$$

$$= 9$$

5. 2019 P1

Evaluate $(-1)^0 \times 2$

$$(-1)^0 \times 2$$

Step 1 - Any number to the power 0 is 1.

$$= 1 \times 2$$

Step 2 - Multiply 1 and 2

$$= 2$$

6. 2019 GCE P1

Evaluate $\sqrt[4]{(81)}^3$

$$\sqrt[4]{(81)}^3$$

Step 1 - The 4th root of 81 is a number that when multiplied 4 times gives you 81. ie $3 \times 3 \times 3 \times 3 = 81$

$$= 3^3$$

Step 2 - Multiply 3 three times.

$$= 3 \times 3 \times 3$$

$$= 27$$

TOPIC 4 - COORDINATE GEOMETRY

1. 2017 P1

The gradient of the line joining the points $(-2, k)$ and $(k, -14)$ is 2.
Calculate the value of k .

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$2 = \frac{-14 - k}{k - (-2)}$$

$$-14 - k = 2(k+2)$$

$$-14 - k = 2k + 4$$

$$-14 - 4 = 2k + k$$

$$-18 = 3k$$

$$\frac{-18}{3} = \frac{3k}{3}$$

$$k = -6$$

Data

$$m = 2 \quad (-2, k) (k, -14)$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

Step 1 - Write the formula to find the gradient and place the values.

Step 2. Deal with $k - (-2)$ which will become $(k+2)$. Remember $(-)(-) = (+)$. Then cross multiply.

Step 3 - multiply 2 with $(k + 2)$

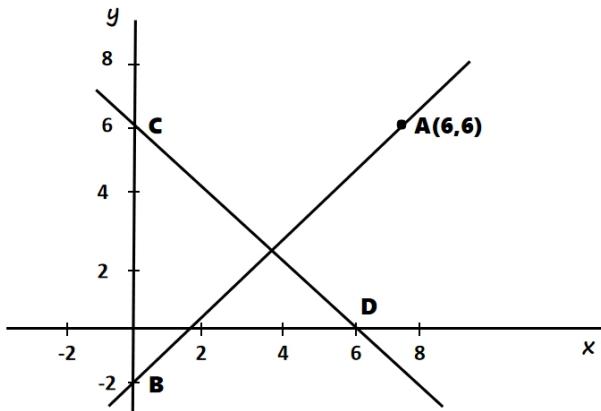
Step 4 - Group the like terms.

Step 5 - Subtract 4 from -14 and add k to 2k

Step 6 - Find k by dividing both sides with 3.

2. 2017 GCE P1

The diagram below shows a Cartesian plane with points A(6, 6), B(0, -2), C(0, 6) and D(6, 0).



Find the

- (a) equation of the line CD
- (b) distance AB

Data

$$m = ?$$

$$c = ?$$

$$\begin{matrix} C(0, 6) & D(6, 0) \\ x_1 & y_1 & x_2 & y_2 \end{matrix}$$

(a) line CD

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - 6}{6 - 0}$$

$$= \frac{-6}{6}$$

$$m = -1$$

$$y = mx + c$$

$$= 6 = -1(0) + c$$

$$= 6 = 0 + c$$

$$c = 6$$

equation

$$y = -x + 6$$

Step 1 - Write the formula to find the gradient and place the values.

Step 2 - Subtract -6 from 0 and 0 from 6

Step 3 - Find m by dividing -6 by 6

Step 4 - Write the formula to find the equation of a straight line and place the values.

Step 5 - Multiply -1 by 0

Step 6 - Group like terms

NOTE:

1. y is 6 because the line cuts the y axis at 6 when x = 0.

2. $-1x = -x$

(b) Distance AB

$$\begin{aligned}AB &= \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} \\&= \sqrt{(0 - 6)^2 + (-2 - 6)^2} \\&= \sqrt{(-6)^2 + (-8)^2} \\&= \sqrt{36 + 64} \\&= \sqrt{100}\end{aligned}$$

AB = 10 UNITS

Data

A (6 , 6) B(0 , -2)
 $x_1 \quad y_1 \quad x_2 \quad y_2$

Step 1 - Write the formula to find distance of a line and place the values.

Step 2 - Subtract 6 from 0 and subtract -6 from -2.

Step 3 - Evaluate - 6 squared and -8 squared.

Step 4 - Add 36 and 64

Step 5 - find the square root of 100

3. 2018 GCE P1

A line passing through A(3 , 2) and B(5, y) has a gradient -2. Find the value of y.

Data

$m = -2 \quad A(3 , 2) \quad B(5 , ?)$
 $x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 1 - Write the formula to find the gradient and place the values.

$$-2 = \frac{y - 2}{5 - 3}$$

Step 2 - Subtract 3 from 5.

$$-2 = \frac{y - 2}{2}$$

Step 3 - Cross multiply (y - 2) with 1 and -2 with 2.

$$y - 2 = -4$$

Step 4 - Group the like terms.

$$y = -4 + 2$$

Step 5 - Find y by adding -4 and 2

$$y = -2$$

4. 2019 GCE P1

Find the gradient of a line which through(-5 , 3) and (-4 , 1).

Data

$$m = ? \quad (-5, 3) (-4, 1)$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{1 - 3}{-4 - (-5)}$$

$$= \frac{-2}{1}$$

$$m = -2$$

Step 1 - Write the formula to find the gradient and place the values.

Step 2. Subtract 3 from 1 and add 5 to -4.
Remember $(-)(-) = (+)$

Step 3. Find m by dividing -2 over 1.

5. 2019 P1

A and B are coordinates (-3 , 3) and (5 , 9) respectively. Find the length AB.

Data

$$(-3, 3) (5, 9)$$
$$x_1 \quad y_1 \quad x_2 \quad y_2$$

$$AB = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2}$$

Step 1 - Write the formula to find distance of a line and place the values

$$= \sqrt{(5 - (-3))^2 + (9 - 3)^2}$$

Step 2 - Add 3 to 5 and subtract 3 from 9.

$$= \sqrt{8^2 + 6^2}$$

Step 3 - Evaluate 8 squared and 6 squared.

$$= \sqrt{64 + 36}$$

Step 4 - Add 64 and 36

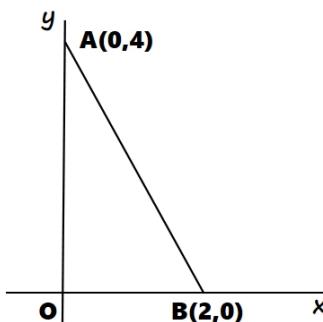
$$= \sqrt{100}$$

Step 5 - find the square root of 100

$$\text{AB} = 10 \text{ UNITS}$$

6. 2019 P1

In the diagram below, A is the point (0,4) and B is the point (2,0) and O is the origin.



Find the equation of a straight line passing through O parallel to line AB.

NOTE

-Parallel lines have the same gradient. Therefore find the gradient of line AB.

-The line is passing through O which is the origin. The coordinates are (0,0)

Data

$m = ?$

$c = ?$

$A(0, 4) \ B(2, 0)$

$x_1 \quad y_1 \quad x_2 \quad y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Step 1 - Write the formula to find the gradient and place the values.

$$= \frac{0 - 4}{2 - 0}$$

Step 2 - Subtract 4 from 0 and 0 from 2

$$= \frac{-4}{2}$$

Step 3 - Find m by dividing -4 by 2

$$m = -2$$

$$y = mx + c$$

Step 4 - Write the formula to find the equation of a straight line and place the values.

$$0 = -2(0) + c$$

Step 5 - Multiply -2 by 0

$$0 = 0 + c$$

Step 6 - Group like terms

$$c = 0$$

equation

$$y = -2x$$

Note x and y are 0 because the line passes through the origin (0,0) !!

TOPIC 5 - FUNCTIONS

1. 2007 P1

for the function $f(x) = \frac{2x - 3}{x}$

Find $f^{-1}(x)$

let $y = f(x)$

$$y = \frac{2x - 3}{x}$$

Step 1 - Represent $f(x)$ as y .

$$x = \frac{2y - 3}{y}$$

Step 2 - Swap each x with y . Make y the subject of the formula.

$$y \times x = \frac{2y - 3}{y} \times y$$

Step 3 - multiply both side with y

$$x y = 2y - 3$$

Step 4 - Group the like terms

$$x y - 2y = -3$$

Step 5 - Move the $2y$ to the other side.

$$y(x - 2) = -3$$

Step 6 - Factorise the y out.

$$\frac{y(x - 2)}{(x - 2)} = \frac{-3}{(x - 2)}$$

Step 7 - Divide both sides with $(x - 2)$.

$$y = \frac{-3}{x - 2}$$

$$f^{-1}(x) = \frac{-3}{x - 2}$$

Step 8 - Replace y with $f^{-1}(x)$.

2. 2017 P1

The functions f and g are defined by $f(x) = 2x + 1$ and $g(x) = 5x - 1$

Find

(a) $g^{-1}(x)$

(b) $fg(x)$

(c) $fg(-3)$

(a) let $y = g(x)$

$$y = 5x - 1$$

$$x = 5y - 1$$

$$x + 1 = 5y$$

$$\frac{x + 1}{5} = \frac{5y}{5}$$

$$y = \frac{x + 1}{5}$$

$$g^{-1}(x) = \frac{x + 1}{5}$$

Step 1 - Represent $g(x)$ as y .

Step 2 - Swap x and y . Make y the subject of the formula.

Step 3 - Group the like terms.

Step 4 - Divide both sides with 5

(b) $fg(x)$

$$f(x) = 2x + 1$$

$$fg(x) = 2(5x - 1) + 1$$

$$= 10x - 2 + 1$$

$$= 10x - 1$$

$$fg(x) = 10x - 1$$

$$f(x) = 2x + 1 \quad g(x) = 5x - 1$$



Step 1 - Replace the x in $2(x + 1)$ with the values of $g(x)$.

Step 2 - Multiply 2 with 5x and -1.

Step 3 - Subtract 1 from -2.

(c) $fg(-3)$

$$= 10(-3) - 1$$

$$= -30 - 1$$

$$fg(-3) = -31$$

Step 1 - Replace the x in $10(x - 1)$ with -3.

Step 2 - Multiply 10 with -3 and -1.

Step 3 - Subtract -1 from -30.

3. 2017 GCE P1

The functions g and f are defined as $g : x \rightarrow \frac{x-1}{2}$ and $f : x \rightarrow 3x - 5$. Find

- (a) $g^{-1}(x)$,
- (b) x , if $f(x) = g(x)$,
- (c) $g^{-1}f(x)$.

(a) let $y = g(x)$

$$y = \frac{x-1}{2}$$

Step 1 - Represent $g(x)$ as y .

$$x = \frac{y-1}{2}$$

Step 2 - Swap x and y . Make y the subject of the formula.

$$2x = y - 1$$

Step 3 - Group the like terms. -1 will cross the equal sign.

$$y = 2x + 1$$

$$g^{-1}(x) = 2x + 1$$

Step 4 - Replace y with $g^{-1}(x)$.

(b) x , if $f(x) = g(x)$

$$f(x) = g(x)$$

$$3x - 5 = \frac{x-1}{2}$$

Step 1 - Write down the values of $f(x) = g(x)$.

$$2(3x - 5) = x - 1$$

Step 2 - Cross multiply 2 with $(3x - 5)$ and $(x - 1)$ with 1.

$$6x - 10 = x - 1$$

Step 3 - Group the like terms.

$$6x - x = 10 - 1$$

Step 4 - Subtract x from $6x$ and 1 from 10.

$$5x = 9$$

\downarrow

$$\frac{5x}{5} = \frac{9}{5}$$

Step 5 - Divide both sides with 5 to have x alone.

$$x = \frac{9}{5}$$

$$(c) g^{-1}f(x).$$

$$g^{-1}(x) = 2x + 1$$

$$f(x) = 3x - 5$$

$$\downarrow$$

$$2(3x - 5) + 1$$

$$= 6x - 10 + 1$$

$$= 6x - 9$$

Step 1 - Replace the x in $2(x + 1)$ with the values of $f(x) = 3x - 5$.
 Step 2 - Multiply 2 with $3x$ and -5 .

4. 2018 GCE P1

Given that $f(x) = 8x$ and $g(x) = \frac{3x - 2}{4}$, find

- (a) $f^{-1}(x)$
- (b) an expression for $fg(x)$
- (c) the value of x for which $fg(x) = 20$.

(a) let $y = f(x)$

$$y = 8x \quad \text{Step 1 - Represent } g(x) \text{ as } y.$$

$$x = 8y \quad \text{Step 2 - Swap } x \text{ and } y. \text{ Make } y \text{ the subject of the formula.}$$

$$\frac{x}{8} = \frac{8y}{8} \quad \text{Step 3 - Divide both sides with 8.}$$

$$y = \frac{x}{8} \quad \text{Step 4 - Replace } y \text{ with } f^{-1}(x).$$

(b) $fg(x)$

$$fg(x) = \frac{8(3x - 2)}{4} \quad \begin{matrix} f(x) = 8x \\ g(x) = \frac{3x - 2}{4} \end{matrix}$$

$$= \frac{8(3x - 2)}{4} \quad \text{Step 2 - Divide with 2 to get rid of the bottom.}$$

$$= 2(3x - 2) \quad \text{Step 3 - Multiply 2 with } 3x \text{ and } -2.$$

$$fg(x) = 6x - 4$$

- (c) the value of x for which $fg(x) = 20$.

$$fg(x) = 6x - 4$$

$$6x - 4 = 20$$

Step 1 - Write the value of $fg(x)$ and equate it to 20.

$$6x = 20 + 4$$

Step 2 - Add 20 and 4.

$$6x = 24$$

Step 3 - Make x the subject of the formula.

$$\frac{6x}{6} = \frac{24}{6}$$

Step 4 - Divide both sides by 6 to find x .

$$x = 4$$

5. 2019 P1

The functions f and g are defined by $f(x) = 2x - 3$ and $g(x) = 3x$

Find

(a) $f^{-1}(x)$

(b) $gf(x)$

(c) $gf(2)$

(a) let $y = f(x)$

$$y = 2x - 3$$

Step 1 - Represent $f(x)$ as y .

$$x = 2y - 3$$

Step 2 - Swap x and y . Make y the subject of the formula.

$$x + 3 = 2y$$

Step 3 - Make y the subject of the formula.

$$\frac{x + 3}{2} = \frac{2y}{2}$$

Step 4 - Divide both sides by 2.

$$y = \frac{x + 3}{2}$$

$$f^{-1}(x) = \frac{x + 3}{2}$$

Step 4 - Replace y with $f^{-1}(x)$.

(b) $gf(x)$

$$g(x) = 3x \quad f(x) = 2x - 3$$

$$= 3(2x - 3)$$

Step 1 - Replace the x in $3x$ with the values of $f(x)$.

$$= 6x - 9$$

Step 2 - Multiply 3 with 2x and -3.

(c) $gf(2)$

$$gf(x) = 6x - 9$$

Step 1 - Replace the x in $6x - 9$ with 2.

$$gf(2) = 6(2) - 9$$

Step 2 - Multiply 6 with 2.

$$= 12 - 9$$

Step 3 - Subtract 9 from 12

$$= 3$$

6. 2019 GCE P1

The functions f and g are defined as $f(x) = 2x + 1$ and $g(x) = \frac{3x - 5}{2}$, Find

(a) $f^{-1}(x)$,

(b) $fg(x)$

(c) $fg(4)$.

(a) let $y = f(x)$

$$y = 2x + 1$$

Step 1 - Represent $f(x)$ as y .

$$x = 2y + 1$$

Step 2 - Swap x and y . Make y the subject of the formula.

$$x - 1 = 2y$$

Step 3 - Make y the subject of the formula.

$$\frac{x - 1}{2} = \frac{2y}{2}$$

Step 4 - Divide both sides with 2.

$$y = \frac{x - 1}{2}$$

$$f^{-1}(x) = \frac{x - 1}{2}$$

Step 4 - Replace y with $f^{-1}(x)$.

(b) $fg(x)$

$$\begin{aligned} f(x) &= 2x + 1 & g(x) &= \frac{3x - 5}{2} \\ = 2\frac{(3x - 5)}{2} + 1 & & \downarrow & \text{Step 1 - Replace the } x \text{ in } 2x + 1 \text{ with the values of } g(x). \text{ Remember} \\ & & & \text{to add the 1 at end.} \\ = 2\frac{(3x - 5)}{2} + 1 & & & \text{Step 2 - Cancel out the 2's.} \\ = 3x - 5 + 1 & & & \text{Step 3 - Add 1 to -5} \\ = 3x - 4 & & & \end{aligned}$$

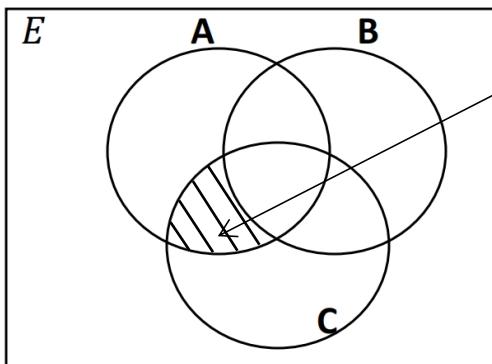
(c) $fg(4)$

$$\begin{aligned} fg(x) &= 3x - 4 \\ fg(4) &= 3(4) - 4 & \text{Step 1 - Replace the } x \text{ in } 3x - 4 \text{ with 4.} \\ &= 3(4) - 4 & \text{Step 2 - Multiply 3 with 4.} \\ &= 12 - 4 & \text{Step 3 - Subtract 4 from 12} \\ &= 8 & \end{aligned}$$

TOPIC 6 - SETS

1. 2017 P1

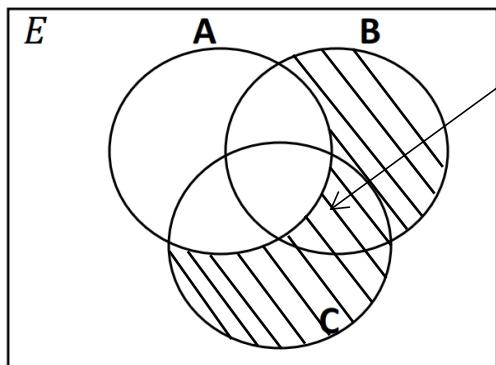
Shade $B' \cap (A \cap C)$ in the Venn diagram in the answer space.



Step 1 - $B' \cap (A \cap C)$ means everything outside B and then shade $A \cap C$.

2. 2017 GCE P1

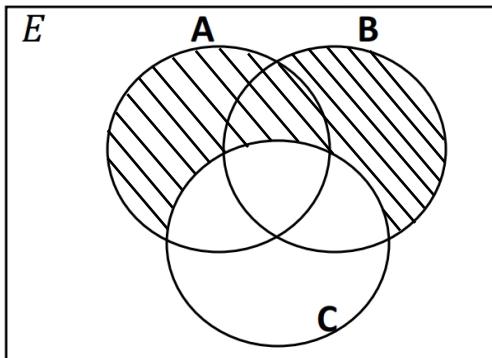
Shade $A' \cap (A \cup C)$ in the Venn diagram in the answer space.



Step 1 - $A' \cap (B \cup C)$ means everything outside A and then shade $B \cup C$.

3 2018 GCE P1

Use set notation to describe the region shaded in the diagram below.



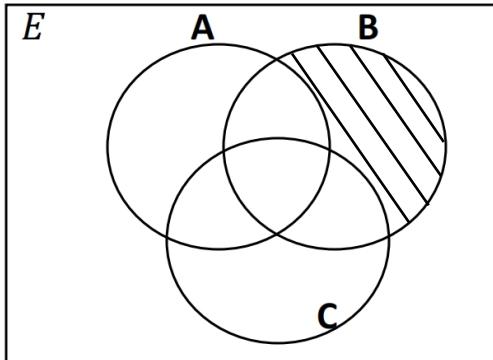
Step 1 - C has not been shaded so we get C' .

Step 2 - Everything else and in A and B that do not also belong to C have been shaded.

$$= C' \cap (A \cup B)$$

4. 2019 P1

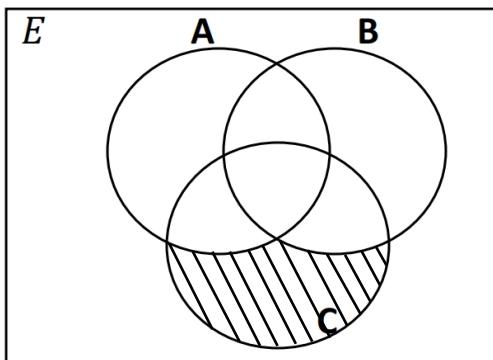
Shade $(A \cup C)'$ $\cap B$ in the Venn diagram in the answer space.



Step 1 - $(A \cup C)'$ $\cap B$ means everything outside $A \cup C$ and then shade B.

5. 2019 GCE P1

The venn diagram below shows three sets A,B and C. Use set notation to describe the region shaded



Step 1 - A and B have not been shaded so we get $(A \cup B)'$.

Step 2 - Everything else in C that does not also belong to A or B has been shaded.

$$= (A \cup B)' \cap C$$

6. Given that Set A has 5 elements,
Find the number of subsets of A

$$\begin{aligned}(\text{a}) \text{ Number of subsets} &= 2^n \\&= 2^5 \\&= 2 \times 2 \times 2 \times 2 \times 2 \\&= 32\end{aligned}$$

Step 1 - To find subsets use the formula 2^n

Step 2 - n is the number of elements.

Step 3 - Multiply 2 five times.

A has 32 subsets

TOPIC 7 - ESTIMATION

1. 2017 P1

Misozi and Filamba estimated the length of a line to be 9cm and 10cm respectively. If the true length of the line was 9.6cm, find

- (a) Misozi's absolute error
- (b) Filamba's percentage error

(a) Absolute error = True measurement - Estimated value

$$\begin{aligned} &= 9.6 - 9 && \text{Step 1 - Subtract 9 from 9.6.} \\ &= \mathbf{0.6\text{cm}} \end{aligned}$$

(b) percentage error = $\frac{\text{Estimated value}}{\text{True measurement}} \times 100$

$$= \frac{10 - 9.6}{9.6} \times 100 \% \quad \text{Step 1 - Subtract 9.6 from 10.}$$

$$= \frac{0.4}{9.6} \times 100 \% \quad \text{Step 2 - Multiply 0.4 with 100.}$$

$$= \frac{40}{96}$$

$$= \frac{40 \times 10}{9.6 \times 10} \quad \text{Step 3 - Multiply both sides with 10 to get rid of the decimal point.}$$

$$= \frac{400}{96} \quad \text{Step 5 - Divide both sides with 16.}$$

$$= \frac{25}{6} \%$$

2. 2017 GCE P1

True measurement Absolute error



A bag of potatoes has mass (15.4 ± 0.05) kg. Find the

(a) Find the tolerance of this mass,

(b) Write down the relative error of the mass, as a fraction, in its simplest form.

$$(a) \text{Tolerance} = \text{Absolute error} \times 2$$

$$= 0.05 \times 2$$

$$= 0.05$$

$$\begin{array}{r} \times \quad 2 \\ \hline 0.10 \end{array}$$

$$= 0.1$$

Step 1 - Multiply 0.05 with 2.

$$(b) \text{Relative error} = \frac{\text{Absolute error}}{\text{True measurement}}$$

$$= \frac{0.05}{15.4}$$

$$= \frac{0.05}{15.4} \times \frac{100}{100}$$

$$= \frac{5}{1540}$$

Step 1 - Multiply both sides with 100.

$$= \frac{1}{308}$$

Step 2 - Divide both sides with 5.

3. 2018 GCE P1

A piece of timber measures 5.25cm long. Calculate the relative error of the measurement.

Absolute error = half the least unit of measurement

$$\begin{aligned} &= \frac{1}{2} \times 0.01 \\ &= 0.5 \times 0.01 \\ &= 0.005 \end{aligned}$$

Step 1 - First find the absolute error by finding the half of the least unit of measurement

Step 2 - Multiply 0.5 with 0.01

Note.

The least unit of measurement of number

- A whole number eg $4 = 1$
 - 1 decimal place eg $4.1 = 0.1$
 - 2 decimal places eg $4.22 = 0.01$
 - 3 decimal places eg $4.345 = 0.001$
- etc

Relative error = $\frac{\text{Absolute error}}{\text{True measurement}}$

$$\begin{aligned} &= \frac{0.005}{5.25} \\ &= \frac{0.005}{5.25} \times \frac{1000}{1000} \\ &= \frac{5}{5250} \\ &= \frac{1}{1050} \end{aligned}$$

Step 3 - Find the relative error

Step 4 - Multiply both sides with 1000.

Step 5 - Divide both sides with 5.

4. 2019 P1

The length of a piece of wire is measured as 4.5 cm. Calculate

(a) the tolerance.

(b) the relative error.

$$\text{Tolerance} = \text{Absolute error} \times 2$$

Absolute error = half the least unit of measurement

$$\begin{aligned} &= \frac{1}{2} \times 0.1 \\ &= 0.5 \times 0.1 \\ &= 0.5 \\ &\quad \times \underline{0.1} \\ &\quad \quad 05 \\ &+ \underline{\quad 0\quad} \\ &\quad \quad \underline{0.05} \end{aligned}$$

Step 1 - First find the absolute error by finding the half of the least unit of measurement.

Step 2 - Multiply 0.5 with 0.1

$$\text{Tolerance} = \text{Absolute error} \times 2$$

$$\begin{aligned} &= 0.05 \times 2 && \text{Step 3 - Multiply 0.05 with 2} \\ &= 0.05 \\ &\quad \times \underline{2} \\ &\quad \quad 0.10 \\ &= \mathbf{0.1} \end{aligned}$$

$$\text{(b) Relative error} = \frac{\text{Absolute error}}{\text{True measurement}}$$

$$\begin{aligned} &= \frac{0.05}{4.5} \\ &= \frac{0.05}{4.5} \times \frac{100}{100} && \text{Step 1 - Multiply both sides with 100} \\ &= \frac{5}{450} && \text{Step 2 - Divide both sides with 5} \\ &= \frac{1}{90} \end{aligned}$$

5. 2019 GCE P1

(a) The mass, m, of a block of wood is 876.4 g, correct to 1 decimal place. Complete the statement in the answer space below.

(b) The length of a piece of wire is measured as 15.2 cm, correct to 1 decimal place. What is the relative error of the length of the piece of wire.

(a) $876.35 \leq m \leq 876.44$

Step 1 - Find the highest and lowest possible number you can have from 876.4 without changing the number completely when rounded off.

(b) Absolute error = half the least unit of measurement

$$\begin{aligned} &= \frac{1}{2} \times 0.1 && \text{Step 1 - First find the absolute error by finding the half of the least unit of measurement.} \\ &= 0.5 \times 0.1 \\ &= 0.5 \\ &\quad \times 0.1 \\ &\quad \underline{\quad\quad\quad} \\ &+ 0 \quad 0 \\ &\underline{\quad\quad\quad} \\ &\quad \underline{\quad\quad\quad} \end{aligned}$$

Step 2 - Multiply 0.5 with 0.1

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True measurement}}$$

$$\begin{aligned} &= \frac{0.05}{15.2} \\ &= \frac{0.05}{15.2} \times \frac{100}{100} && \text{Step 3 - Multiply both sides with 100.} \\ &= \frac{5}{1520} && \text{Step 5 - Divide both sides with 5.} \\ &= \frac{1}{304} \end{aligned}$$

TOPIC 8 - MATRICES

1. 2017 P1

Given that $A = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 3 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

find,

(a) A^T

(b) AB as a single matrix

$$(a) A^T = \begin{pmatrix} 3 & 4 \\ 2 & 3 \\ 1 & 0 \end{pmatrix}$$

Step 1 - Let the rows become columns and columns become rows.

$$(b) AB = \begin{pmatrix} 3 & 2 & 1 \\ 4 & 3 & 0 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

Step 1 - multiply each row of the first matrix against each column of the second matrix, and adding the results together to form a single matrix.

$$= \begin{pmatrix} (3 \times 1) + (2 \times 3) + (1 \times 0) & (3 \times 2) + (2 \times 2) + (1 \times 1) & (3 \times 3) + (2 \times 1) + (1 \times 0) \\ (4 \times 1) + (3 \times 3) + (0 \times 0) & (4 \times 2) + (3 \times 2) + (0 \times 1) & (4 \times 3) + (3 \times 1) + (0 \times 0) \end{pmatrix}$$

$$= \begin{pmatrix} 3+6+0 & 6+4+1 & 9+2+0 \\ 4+9+0 & 8+6+0 & 12+3+0 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 11 & 11 \\ 13 & 14 & 15 \end{pmatrix}$$

2. 2017 GCE P1

(a) Given that $\begin{pmatrix} 2 & x \\ -5 & 1 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$, find the value of x.

(b) If $A^T = (1 \ -2 \ 3 \ -4 \ 5)$, write the matrix A.

$$(a) \begin{pmatrix} (2 \times 4) + (x \times 3) \\ (-5 \times 4) + (1 \times 3) \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$$

Step 1 - Start by multiplying the matrices

$$\begin{pmatrix} 8 + 3x \\ -20 + 3 \end{pmatrix} = \begin{pmatrix} 14 \\ -17 \end{pmatrix}$$

Step 2 - Solve $8 + 3x = 14$ to find x.
 $-20 + 3$ is already equal to -17.

$$8 + 3x = 14$$

$$3x = 14 - 8$$

Step 3- Group the like terms

$$3x = 6$$

$$\frac{3x}{3} = \frac{6}{3}$$

Step 5- Divide both sides with 3.

$$x = 2$$

$$(b) A = \begin{pmatrix} 1 \\ -2 \\ 3 \\ -4 \\ 5 \end{pmatrix}$$

Step 1 - Let the rows become columns and columns become rows.

3. 2018 GCE P1

The transpose of a matrix P is $\begin{pmatrix} 1 & 3 & 4 \\ 2 & 5 & 1 \\ 3 & 0 & 1 \end{pmatrix}$. Write down the matrix P .

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 0 \\ 4 & 1 & 1 \end{pmatrix}$$

Step 1 - Let the rows become columns and columns becomes rows.

4. 2019 P1

(a) The transpose of a matrix A is $(-1 \ 4 \ 5)$. Find the matrix A .

(b) Given that $(1 \ 2 \ 3) \begin{pmatrix} 1 \\ x \\ 5 \end{pmatrix} = (24)$, find the value of x .

$$(a) \ A = \begin{pmatrix} -1 \\ 4 \\ 5 \end{pmatrix}$$

Step 1 - Let the rows become columns and columns become rows.

$$(b) \ ((1 \times 1) + (2 \times x) + (3 \times 5)) = (24)$$

Step 1 - Start by multiplying the matrices

$$= 1 + 2x + 15 = 24$$

Step 2 - Add 1 and 16

$$= 2x + 16 = 24$$

Step 4 - Group the like terms

$$= 2x = 24 - 16$$

Step 5 - Divide both sides with 2 to find x .

$$= 2x = 8$$

Step 5 - Divide both sides with 2 to find x .

$$= \frac{2x}{2} = \frac{8}{2}$$

$$x = 4$$

5. 2019 GCE P1

Given that $R = \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix}$ and $S = \begin{pmatrix} 1 & 0 & -4 \\ -2 & 1 & 1 \end{pmatrix}$

find,

(a) S^T

(b) RS

$$(a) \quad S^T = \begin{pmatrix} 1 & -2 \\ 0 & 1 \\ -4 & 1 \end{pmatrix}$$

Step 1 - Let the rows become columns and columns become rows.

$$(b) \quad \begin{pmatrix} 2 & -1 \\ 1 & 3 \end{pmatrix} \times \begin{pmatrix} 1 & 0 & -4 \\ -2 & 1 & 1 \end{pmatrix}$$

Step 1 - multiply each row of the first matrix against each column of the second matrix, and adding the results together to form a single matrix.

$$= \begin{pmatrix} (2 \times 1) + (-1 \times -2) & (2 \times 0) + (-1 \times 1) & (2 \times -4) + (-1 \times 1) \\ (1 \times 1) + (3 \times -2) & (1 \times 0) + (3 \times 1) & (1 \times -4) + (3 \times 1) \end{pmatrix}$$

$$= \begin{pmatrix} 2 + 2 & 0 + (-1) & -8 + (-1) \\ 1 + (-6) & 0 + 3 & -4 + 3 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & -1 & -9 \\ -5 & 3 & -1 \end{pmatrix}$$

TOPIC 9 - SEQUENCES

1. 2017 P1

-3 -3

For the sequence 25, 22, 19, 16... find

- (a) formula for the n^{th} term
(b) sum of the first 20 terms

Step 1 - Check if it is an Arithmetic or geometric progression. Difference between terms is equal hence It is an AP .

$$\text{Common difference} = T_2 - T_1 = 22 - 25 = -3$$

Step 2 - Find the common difference.

(a) $T_n = a + (n - 1)d$

Step 3 - Write the formula and place the values.

$$T_n = 25 + (n - 1) \cdot -3$$

Step 4 - Multiply -3 with n and -1.

$$T_n = 25 + (-3n + 3)$$

Step 5 - Group the like terms and add 25 and 3.

$$T_n = 25 + 3 - 3n$$

$$T_n = 28 - 3n$$

(b) $S_n = \frac{n}{2}[2a + (n - 1)d]$

Step 1 - Write the formula and place the values.

$$\begin{aligned} S_{20} &= \frac{20}{2}[2 \times 25 + (20 - 1) \cdot -3] \\ &= 10[50 + (19) \cdot -3] \\ &= 10[50 - 57] \\ &= 10[-7] \end{aligned}$$

Step 2 - Divide 20 by 2, multiply 2 with 25 and subtract 1 from 20.

$$S_{20} = -70$$

Step 3 - Multiply -3 with 19

Step 4 - Subtract 57 from 50 and then multiply 10 with -7.

2. 2017 GCE P1

2 2

= It is an AP

(a) For the sequence 11, 13, 15, 17... find the 13th term.

(b) If the arithmetic mean of 5 and c is 11, what is the value of c

(a) Common difference = $T_2 - T_1 = 13 - 11 = 2$

Step 1 - Find the common difference.

$$T_{13} = a + (n - 1)d$$

Step 2 - Write the formula and place the values.

$$= 11 + (13 - 1) \cdot 2$$

Step 3 - Subtract 1 from 13

$$= 11 + (12) \cdot 2$$

Step 4 - Multiply 2 with 12 and then add 11 and 24.

$$= 11 + 24$$

$$T_{13} = 35$$

$$(b) \frac{5+c}{2} = 11$$

$$5+c = 22$$

$$c = 22 - 5$$

$$c = 17$$

Step 1 - To find the average of two values we add both values and then divide by 2.

Step 2 - Group the like terms.

Step 3 - Subtract 5 from 22 to find c.

3. 2018 GCE P1

The first three terms in an arithmetic progression are 5 ,7 and 9.Find the

(a) common difference .

(b) sum of the first 12 terms.

$\frac{2}{2}$ $\frac{2}{2}$ = It is an AP

(a) Common difference

$$= T_2 - T_1$$

$$= 7 - 5$$

$$= 2$$

Step 1 - Subtract 5 from 7 to find the common difference.

$$(b) S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{12} = \frac{12}{2}[2 \times 5 + (12-1)2]$$

$$= 6[10 + (11)2]$$

$$= 6[10 + 22]$$

$$= 6[32]$$

$$S_{12} = 192$$

Step 1 - Write the formula and place the values.

Step 2 - Divide 12 by 2, multiply 2 with 5 and subtract 1 from 12.

Step 3 - Multiply 2 with 11

Step 4 - Add 10 and 22 and then multiply 6 with 32.

4. 2019 P1

(a) Given that $17 + m + 27 + \dots$ are consecutive terms of an Arithmetic Progression, find the arithmetic mean, m

(b) For the sequence 11,13,15,..., find the formula for the n^{th} term

$\frac{2}{2} = \frac{2}{2}$ = It is an AP

$$(a) m = \frac{T_1 + T_3}{2}$$

$$= \frac{17 + 27}{2}$$

$$= \frac{44}{2}$$

$$= 22$$

Step 1 - To find the average of two values we add both values and then divide by 2.

Step 2 - Add 17 and 27.

Step 3 - Divide 44 with 2.

$$\text{Common difference} = T_2 - T_1 = 13 - 11 = 2$$

Step 1 - Find the common difference.

$$(b) T_n = a + (n - 1)d$$

Step 2 - Write the formula and place the values.

$$T_n = 11 + (n - 1)2$$

Step 3 - Multiply 2 with n and -1.

$$T_n = 11 + 2n - 2$$

Step 4 - Group like terms and subtract 2 from 11.

$$T_n = 11 - 2 + 2n$$

$$T_n = 9 + 2n$$

5. 2019 GCE P1

$\frac{3}{3} = \frac{3}{3}$ = It is an AP

For the sequence $-10, -7, -4, -1, \dots$, find the

(a) 17^{th} term.

(b) sum of the first 20 terms.

$$\text{Common difference} = T_2 - T_1 = -7 - (-10) = 3$$

Step 1 - Find the common difference.

$$(a) T_n = a + (n - 1)d$$

Step 2 - Write the formula and place the values.

$$T_{17} = -10 + (17 - 1)3$$

Step 3 - Subtract 1 from 17 and the

$$= -10 + (16)3$$

Step 4 - Multiply 3 with 16 and then add -10 with 48.

$$= -10 + 48$$

$$T_{17} = 38$$

$$(b) S_n = \frac{n}{2}[2a + (n - 1)d]$$

Step 1 - Write the formula and place the values.

$$S_{20} = \frac{20}{2}[2 \times (-10) + (20 - 1)3]$$

Step 2 - Divide 20 by 2, multiply 2 with -10 and subtract 1 from 20.

$$= -10[-20 + (19)3]$$

Step 3 - Multiply 3 with 19

$$= 10[-20 + 57]$$

Step 4 - Add -20 and 57 and then multiply 10 with 37.

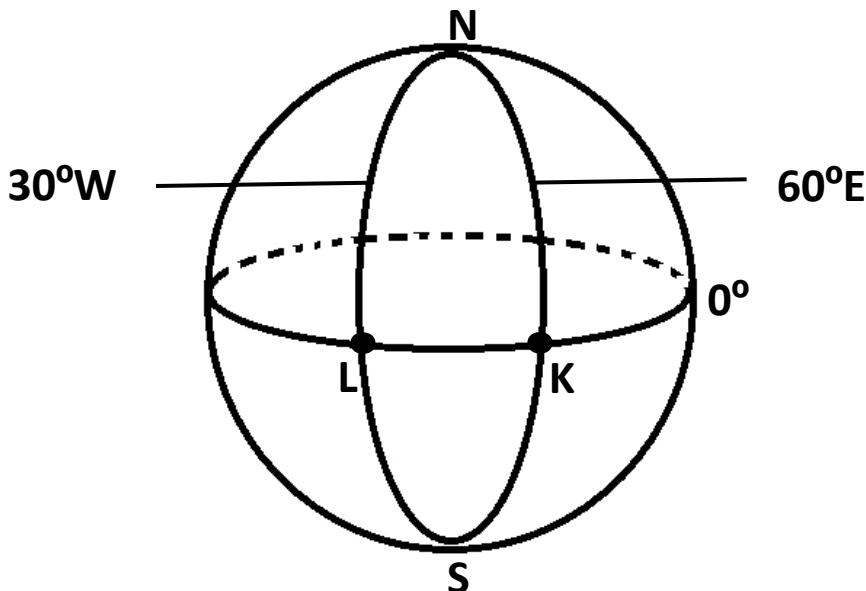
$$= 10[37]$$

$$S_{20} = 370$$

TOPIC 10 - EARTH GEOMETRY

1. 2017 P1

Town L is on $(0^\circ, 30^\circ \text{W})$ and town K is on $(0^\circ, 60^\circ \text{E})$ as shown below.



- (a) If a radio quiz is scheduled to start at 12:00 hours at L, find the time at which the people at K will be listening to the quiz.
(b) What is the difference between L and K in kilometers?

$$[R = 6370 \text{ km}, \pi = \frac{22}{7}]$$

(a) L($0^\circ, 30^\circ \text{W}$) K($0^\circ, 60^\circ \text{E}$)
 $30^\circ + 60^\circ = 90^\circ$

Step 1 - Only deal with the longitudes to find time.

Step 2 - Find the difference in longitudes.

$$\frac{90^\circ}{15^\circ} = 6 \text{ hrs}$$

Step 3 - Find the difference in time

Time at K = 12:00+6hrs
=18:00hrs

Step 4 - Add 6 to 12 hours because time increases when moving from West to East.

$$\begin{aligned}
 (b) &= \frac{\theta}{360} \times 2\lambda R \cos x \\
 &= \frac{90}{360} \times \frac{2 \times 22}{7} \times 6370 \times \cos x 0^\circ \\
 &= \frac{1}{4} \times \frac{44}{7} \times 6370 \times 1 \\
 &= \frac{11}{7} \times 6370 \\
 &= \frac{70070}{7} \\
 &= \mathbf{10010 \text{ km}}
 \end{aligned}$$

2. 2017 GCE P1

- (a) A soccer match kicked off at 14:00 hours at A(20°N , 30°E). What would the kick off time of the soccer match at B($20^\circ\text{N}, 15^\circ\text{W}$)?
- (b) Two towns P and Q are on the same longitude. Given that P is ($40^\circ\text{ N}, 15^\circ\text{ W}$) and PQ is 7200nm, find the position of Q.

$$\begin{aligned}
 (\text{a}) \quad &A(20^\circ, 30^\circ\text{E}) \quad B(20^\circ, 15^\circ\text{W}) \\
 &= 30^\circ + 15^\circ = 45^\circ
 \end{aligned}$$

$$\frac{45^\circ}{15^\circ} = 3 \text{ hrs}$$

$$\begin{aligned}
 \text{Time at B} &= 14:00 - 3 \text{ hrs} \\
 &= \mathbf{11:00\text{hrs}}
 \end{aligned}$$

Step 1 - Only deal with the longitudes.

Step 2 - Find the difference in longitude. Add because the longitudes are different i.e East West.

Step 3 - Find the difference in time

Step 4 - Subtract 3 hours from 14 because time decreases when moving from East to West

$$\begin{aligned}
 (b) \quad & PQ = \theta \times 60 \\
 = & 7200 = \theta \times 60 \\
 = & \frac{7200}{60} = \frac{\theta \times 60}{60} \\
 = & \frac{7200}{60} = \theta \\
 = & \theta = 120^\circ \\
 = & 120^\circ - 40^\circ = 80^\circ \\
 = & Q(80^\circ S, 15^\circ W)
 \end{aligned}$$

Step 1 - Write the formula to find distance in longitudes and place the values to find theta.

Step 2 - Divide both sides by 60

Step 3 - Divide 7200 by 60.

Step 4 - Divide 7200 by 60.

Step 5 - Subtract 40° from 120° to find the position on Q.

3. 2018 GCE P1

(a) The difference in longitude between town A and town B is 105° . B is west of A. A family at A was watching a football match at 16:00 hours. At what time did a family at B watch the same match?

(b) The distance between P and Q is 3600nm. If an aeroplane flies from P to Q at 600 knots, how long will it take?

$$(a) \frac{105^\circ}{15^\circ} = 7 \text{ hrs}$$

Step 1 - Find the difference in time

$$\begin{aligned}
 \text{Time at B} &= 16:00 - 7 \text{ hrs} \\
 &= 09:00 \text{ hrs}
 \end{aligned}$$

Step 2 - Subtract 7 hours from 16 because time decreases when moving from East to West

$$(b) S = \frac{D}{T}$$

Step 1 - Write the formula to find speed and place the values to find the time taken.

$$600 = \frac{3600}{T}$$

Step 2 - cross multiply 600 with T and 3600 with 1.

$$600T = 3600$$

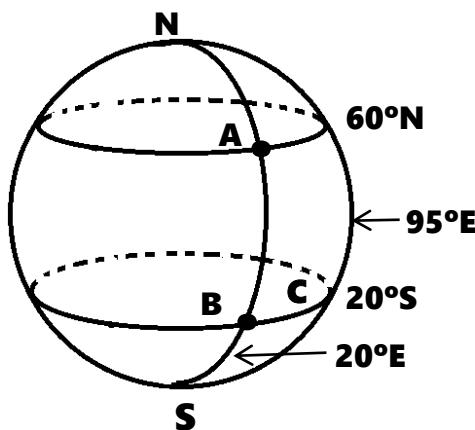
$$\frac{600T}{600} = \frac{3600}{600}$$

Step 3 - Divide both sides with 600 to find the time.

$$T = 6\text{hrs}$$

4. 2019 P1

(a) The diagram below shows the position of towns A,B and C on the earth's surface.



(a) If it is 08:20 at A, what time is it at C?

(b) A plane flies from A to B at a speed of 400 knots. How long does the journey take if AB = 4800nm?

(a) A(60°N, 20°E) C(20°S, 95°E)

$$95^\circ - 20^\circ = 75^\circ$$

Step 1 - Only deal with the longitudes.

Step 2 - Find the difference in longitude. Subtract because the longitudes are the same.i.e East East.

$$\frac{75^\circ}{15^\circ} 5\text{hrs}$$

Step 3 - Find the difference in time

Time at C = 08:20 + 5hrs
= 13:20hrs

Step 4 - Add 5 hours to 08:20 because C is ahead of A.

$$(b) S = \frac{D}{T}$$

Step 1 - Write the formula to find speed and place the values to find the time taken.

$$400 = \frac{4800}{T}$$

Step 2 - cross multiply 400 with T and 4800 with 1.

$$400T = 4800$$

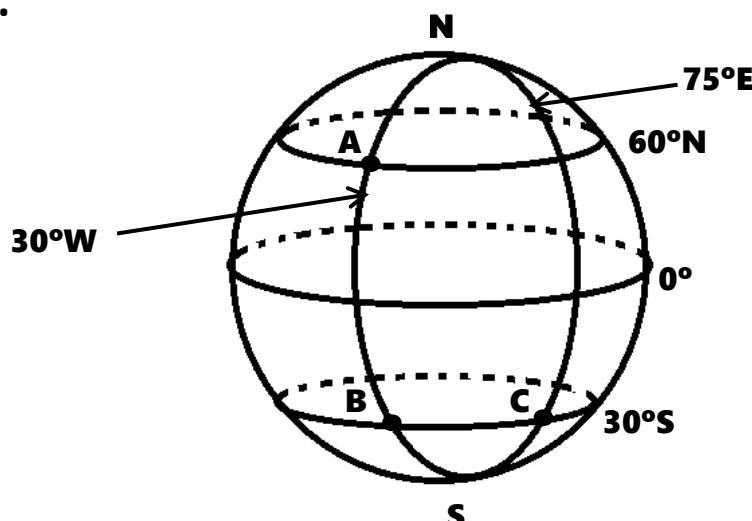
$$\frac{400T}{400} = \frac{4800}{400}$$

Step 3 - Divide both sides with 400 to find the time.

$$T = 12\text{hrs}$$

5. 2019 GCE P1

(a) The diagram below shows points A($60^{\circ}\text{N}, 30^{\circ}\text{W}$) ,B($30^{\circ}\text{S}, 30^{\circ}\text{W}$) and C($30^{\circ}\text{S}, 75^{\circ}\text{E}$).



(a) If the local time at B is 15:00, what is the local time at C.

(b) It takes a plane 6 hours to fly from A to B. What is its speed in knots?

(a) A(60°N,30°W) B(30°S 30° W) C(30°S,75°E) Step 1 - Only deal with the longitudes BC.
30° + 75° = 105°

$$\frac{105^{\circ}}{15^{\circ}} = 7 \text{hrs}$$

Step 2 - Find the difference in longitude.
Add because the longitudes are different
i.e East West.

Step 3 - Find the difference in time

Time at C = 15:00 + 7hrs
=22:00hrs

Step 4 - Add 7 hours to 15:00 because
C is ahead of B.

(b) $S = \frac{D}{T}$

Step 1 - Write the formula to find speed.

Distance between A and B
 $D = \Phi \times 60$
 $= 90 \times 60$
 $= 5400 \text{ nm}$

Step 1 - Write the formula to find distance in
longitudes and place the values to find theta.

$$S = \frac{5400}{6}$$

Step 2 - theta is 90 because we added the
latitudes of A and B.

Step 3 - Multiply 90 and 60 to find the distance.

Step 4 - Find the speed by dividing 5400 by 6.

S = 900 knots

TOPIC 11 - VARIATION

1. 2017 P1

It is given that $t = k^2$, where k is the constant of the variation.

v	1	b	5
t	4	36	a

Use the information given in the table to find the

- (a) value of k
- (b) value of a
- (c) values of b

$$(a) t = kv^2$$

$$4 = k \times 1^2$$

$$k = 4$$

Step 1 - Substitute the values. From the table when the value of $v = 1$ then $t = 4$.

$$(b) t = 4v^2$$

$$a = 4 \times 5^2$$

$$a = 4 \times 25$$

$$a = 100$$

Step 1 - Rewrite the equation and place 4 on k.

Step 1 - Substitute the values. From the table when the value of $v = 5$ then $t = a$.

Step 3 - Multiply 4 and 25

$$(c) t = 4v^2$$

$$36 = 4v^2$$

$$\frac{36}{4} = \frac{4v^2}{4}$$

$$v^2 = 9$$

$$\sqrt{v^2} = \sqrt{9}$$

Step 1 - Substitute the values. From the table when the value of $v = b$ then $t = 36$.

Step 2 - Divide both sides by 4 to find v

Step 3 - Find the square roots of v^2 and 9

$$v = -3 \text{ or } 3$$

2. 2017 GCE P1

The table below shows the relationship between two variables x and y . It is given that y varies inversely as the square root of x , where x is positive.

y	2	8	$\frac{8}{9}$
x	16	1	a

- (a) Write an expression for y in terms of x and the constant of variation k
 (b) Find the value of

- (i) k ,
 (ii) a .

$$(a) \quad y = \frac{k}{\sqrt{x}}$$

Step 1 - Represent y varies inversely as the square root of x .
 Remember the constant k .

$$(b)(i) \quad y = \frac{k}{\sqrt{x}}$$

Step 1 - Get 2 corresponding values and substitute those values. Lets get 8 and 1.

$$8 = \frac{k}{\sqrt{1}}$$

Step 2 - Cross multiply.

$$k = 8 \times 1$$

$$\mathbf{k = 8}$$

$$(b)(ii) \quad y = \frac{8}{\sqrt{x}}$$

Step 1 - Rewrite the formula and place 8 on k .

$$= \frac{8}{9} = \frac{8}{\sqrt{x}}$$

Step 2 - Get the corresponding value for a .

$$8 \times \sqrt{a} = 8 \times 9$$

Step 3 - Cross multiply.

$$\frac{8\sqrt{a}}{8} = \frac{8 \times 9}{8}$$

Step 4 - Divide both sides with 8 to find a .

$$\sqrt{a} = 9$$

Step 5 - Square both sides so that you are left with only a .

$$\sqrt{a^2} = 9^2$$

$$\mathbf{a = 81}$$

3. 2018 GCE P1

Two variables x and y have corresponding values as shown in the table below.

x	2	3	a
y	20	40	104

Given that y varies directly as $(x^2 + 1)$, find the

- (a) constant of variation, k .
- (b) equation connecting y and x .
- (c) values of a .

(a) $y = k(x^2 + 1)$

Step 1 - Represent y varies directly as $(x^2 + 1)$ with k .

$$20 = k(2^2 + 1)$$

Step 2 - replace y with 20 and find k .

$$20 = k(5)$$

Step 3 - Divide both sides with 5 to find k .

$$\frac{20}{5} = \frac{5k}{5}$$

$$k = 4$$

(b) $y = k(x^2 + 1)$

Step 1 - Write the equation again.

$$y = 4(x^2 + 1)$$

Step 2 - Substitute k with 4 and then simplify.

$$y = 4x^2 + 4$$

(c) $y = 4x^2 + 4$

Step 1 - Write the equation again.

$$104 = 4a^2 + 4$$

Step 2 - Substitute y with 104 and find a .

$$104 - 4 = 4a^2$$

Step 3 - Group like terms

$$100 = 4a^2$$

Step 4 - Divide both sides with 4.

$$\frac{100}{4} = \frac{4a^2}{4}$$

Step 4 - Divide both sides with 4.

$$a^2 = 25$$

Step 5 - Find the square root of a^2 and 25.

$$a = 5 \text{ or } -5$$

4. 2019 P1

a varies directly as b and as the square of c and a = 30 when b = 2.5 and c = 2,
Find the

(a) value of k, the constant of variation.

(b) values of a when b = 2 and c = 3

(c) values of c when a = 300 and b = 4

(a) $a = kbc^2$

Step 1 - Represent a varies directly as b and as the square of c and a and add the constant.

$$30 = k \times 2.5 \times 2^2$$

Step 2 - Substitute a with 30, b with 2.5 to find k.

$$30 = k \times 2.5 \times 4$$

Step 3 - Divide both sides with 5 to find k.

$$30 = k \times 10$$

$$30 = 10k$$

Step 4 - Divide both sides with 10 to find k.

$$k = 3$$

(b) $a = 3bc^2$

Step 1 - Write the equation again and add 3 the constant.

$$a = 3 \times 2 \times 3^2$$

Step 2 - Substitute b with 2 and c with 3.

$$a = 6 \times 9$$

Step 3 - Multiply 6 with 9

$$a = 54$$

(c) $a = 3bc^2$

Step 1 - Write the new equation .

$$300 = 3 \times 4 \times c^2$$

Step 2 - Substitute a with 300 and b with 4.

$$300 = 12 \times c^2$$

$$\frac{300}{12} = \frac{12}{12}c^2$$

Step 3 - Divide both sides with 12 to find c.

$$25 = c^2$$

$$\sqrt{25} = \sqrt{c^2}$$

Step 4 - Find the square roots of 25 and c^2 .

$$c = 5 \text{ or } -5$$

5. 2019 GCE P1

Given that y varies directly as x and inversely as the square of z , and that $y = 10$ when $x = 32$ and $z = 4$,

find

(a) the value of k , the constant of the variation.

(b) y when $x = 20$ and $z = 5$.

(c) z when $x = 9$ and $y = 5$

$$(a) \quad y = \frac{kx}{z^2}$$

$$10 = \frac{32k}{16}$$

$$\frac{160}{32} = k$$

$$k = 5$$

Step 1 - Represent y varies directly as x and inversely as the square of z

Step 2 - Substitute y with 10 and x with 32 to find k and then cross multiply.

Step 3 - Divide 160 by 32.

$$(b) \quad y = \frac{5x}{z^2}$$

$$= \frac{5 \times 20}{5^2}$$

$$= \frac{100}{25}$$

$$y = 4$$

Step 1 - Write the equation again and add 5 the constant.

Step 2 - Substitute x with 20 and z with 5.

Step 3 - Divide 100 by 25.

$$(b) \quad y = \frac{5x}{z^2}$$

$$5 = \frac{5 \times 9}{z^2}$$

$$5z^2 = 5 \times 9$$

$$\frac{5z^2}{5} = \frac{5 \times 9}{5}$$

$$z^2 = 9$$

$$\sqrt{z^2} = \sqrt{9}$$

$$z = 3 \text{ or } -3$$

Step 1 - Write the equation .

Step 2 - Substitute y with 5 and x with 9 and cross multiply.

Step 3 - Divide both sides with 5 to find z .

Step 4 - Find the square roots of 9 and z^2 .

TOPIC 12 - VECTORS

1. 2017 P1

The points P and Q have the coordinates (2,4) and (-3,1) respectively.
Express \overrightarrow{PQ} as a column vector

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$= \begin{pmatrix} -3 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} -5 \\ -3 \end{pmatrix}$$

Step 1 - When moving from P to Q, we start from O, the origin.

Step 2 - Represent the values in column vector form and the subtract 2 from -3 and 4 from 1.

2. 2017 GCE P1

Given that $\underline{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$, find $|\underline{a}|$.

$$|\underline{a}| = \sqrt{x^2 + y^2}$$

$$= \sqrt{3^2 + (-4)^2}$$

$$= \sqrt{9+16}$$

$$= \sqrt{25}$$

$$= 5$$

Step 1 - Write the formula to find the magnitude of a vector.

Step 2 - Substitute x and y with 3 and -4 and then evaluate.

Step 3 - Add 9 and 16

Step 4 - Find the square root of 25.

3. 2018 GCE P1

The coordinates of B and C are $(2, 5)$ and $(4, -3)$ respectively. If M is the midpoint of BC, what is the position vector of M?

$$M.P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Step 1 - Write the formula to find the midpoint of a vector.

$$= \left(\frac{2 + 4}{2}, \frac{5 + (-3)}{2} \right)$$

Step 2 - Substitute the values and then evaluate.

$$= \left(\frac{6}{2}, \frac{2}{2} \right)$$

Step 3 - Divide 6 with 2 and 2 with 2.

$$= (3, 1)$$

$$\overrightarrow{OM} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

Step 4 - Express the answer in column vector form.

4. 2019 P1

M is the point $(0, 5)$ and $\overrightarrow{MN} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, Find \overrightarrow{ON}

$$\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM}$$

Step 1 - Represent M moving N with the origin.

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \end{pmatrix}$$

Step 2 - Substitute the values and represent \overrightarrow{ON} with x and y .

$$\begin{pmatrix} -3 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 5 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Step 3 - Move the numbers to the other side and evaluate.

$$\begin{pmatrix} -3 \\ 9 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\overrightarrow{ON} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$$

5. 2019 GCE P1

The vector $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, Given that the point p is (1 , 4), find the coordinates of point Q.

$$\overrightarrow{OP} = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \overrightarrow{OQ} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Step 1 - Represent P moving Q with the origin.

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$$

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix}$$

Step 2 - Substitute the values and represent \overrightarrow{OQ} with x and y .

$$\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Step 3 - Move the numbers to the other side and evaluate.

$$\begin{pmatrix} -2 \\ 6 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

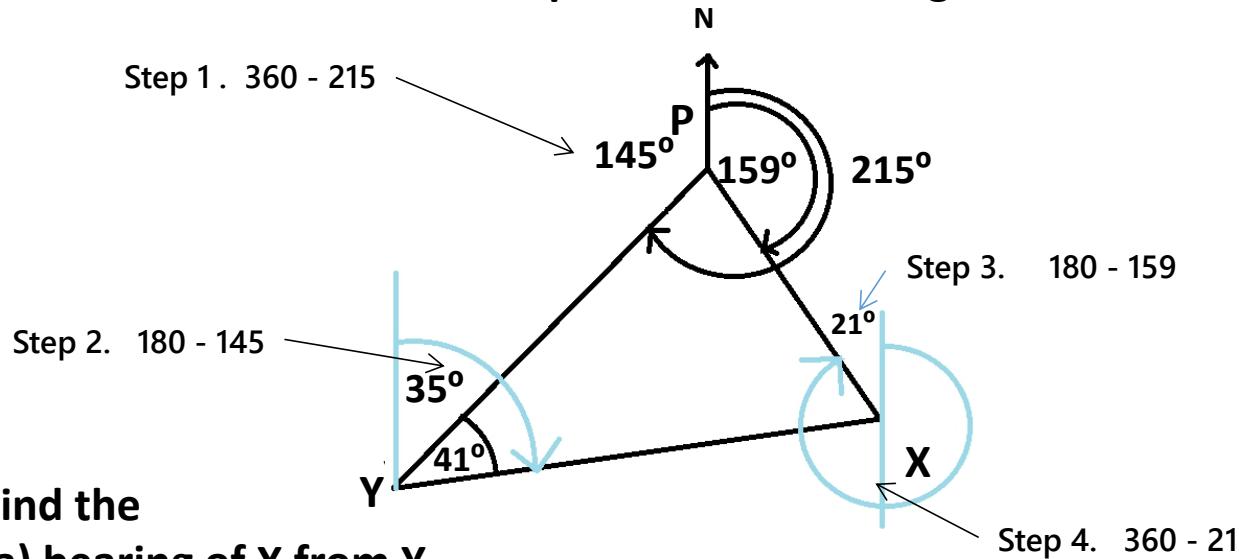
$$Q = (-2, 6)$$

Step 4 - Represent the answer in coordinate form.

TOPIC 13 - BEARINGS

1. 2017 P1

Two boats X and Y leave port P at the same time. X travels on a bearing of 159° and Y travels on a bearing of 215° as shown in the diagram below. After sometime, X and Y are at points such that angle PYX = 41° .



Find the

- (a) bearing of X from Y.
- (b) Bearing of P from X.

$$\begin{aligned}(a) \quad & 360 - 215 \\&= 145\end{aligned}$$

$$\begin{aligned}180 - 145 \\&= 35 \\X \text{ from } Y &= 35 + 41 \\&= 076^\circ\end{aligned}$$

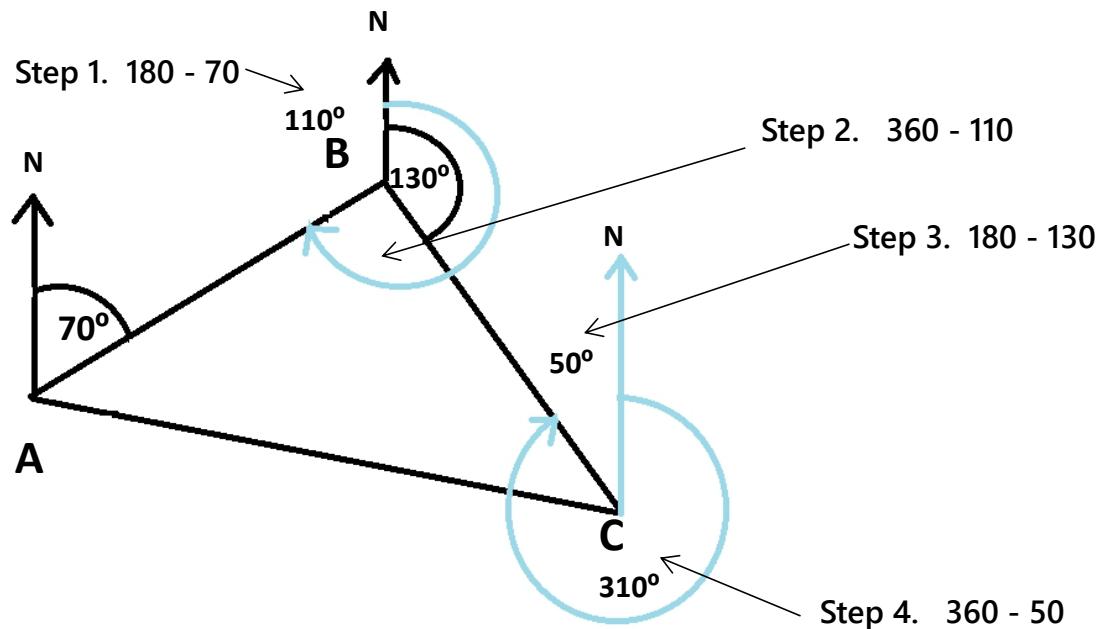
$$\begin{aligned}(b) \quad & 180 - 159 \\&= 21 \\P \text{ from } X &= 360 - 21 \\&= 339^\circ\end{aligned}$$

NOTE

- Angles on a straight line = 180°
- A complete circle = 360° .

2. 2017 GCE P1

A, B and C are three points on level ground. B is on a bearing of 070 from A and C is on a bearing from B.



Calculate the bearing of

- (a) A from B.
- (b) B from C.

$$(a) 180 - 70$$

$$= 110$$

$$\text{A from B} = 360 - 110$$

$$= 250^\circ$$

$$(b) 180 - 130$$

$$= 50$$

$$\text{B from C} = 360 - 50$$

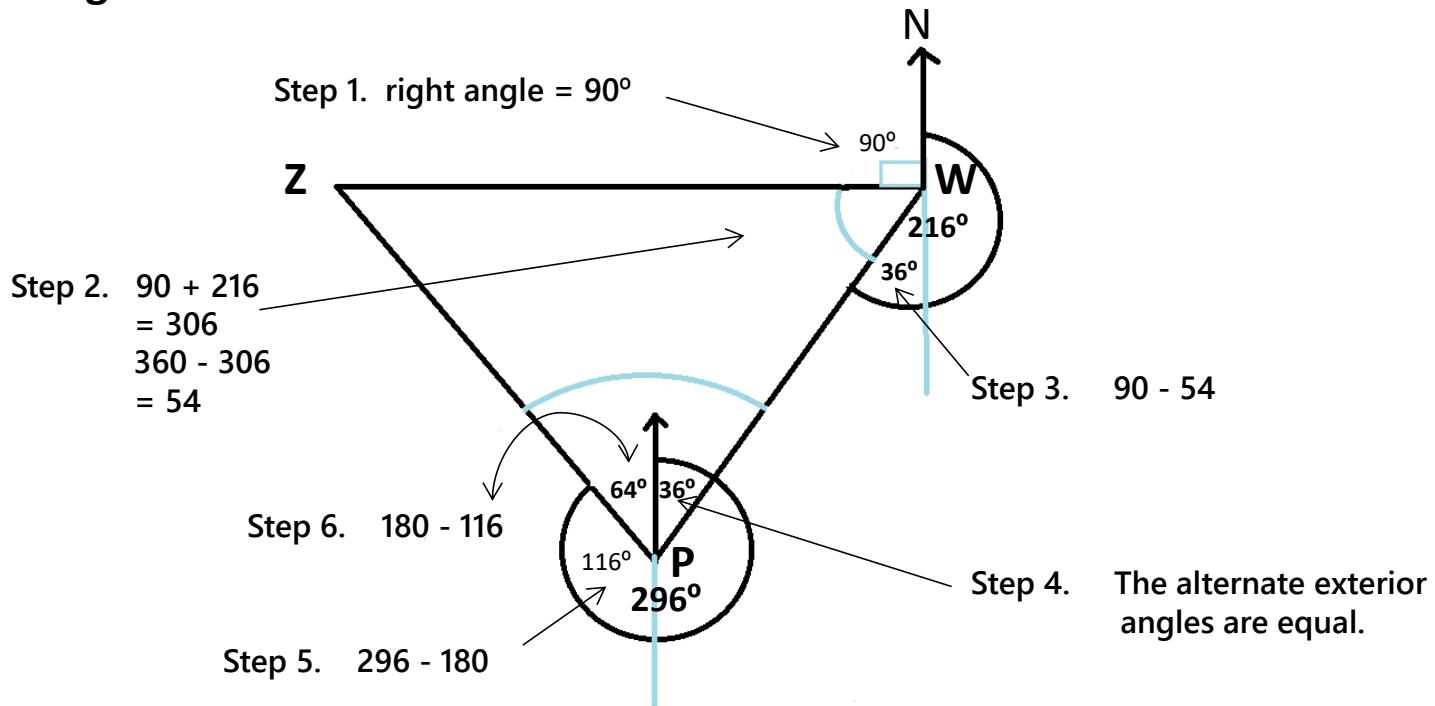
$$= 310^\circ$$

NOTE

- Angles on a straight line = 180°
- A complete circle = 360° ..

3. 2018 GCE P1

In the diagram below, a cyclist starts from town W and cycles on the bearing of 216° to town P. She then leaves town P and cycles on a bearing of 296° to Z. Z is west of W



Calculate

(a) $\hat{P}WZ$

(b) \hat{WPZ}

$$\begin{aligned}(a) 90 + 216 &= 306 \\ P \hat{W} Z &= 360 - 306 \\ &= 054^\circ\end{aligned}$$

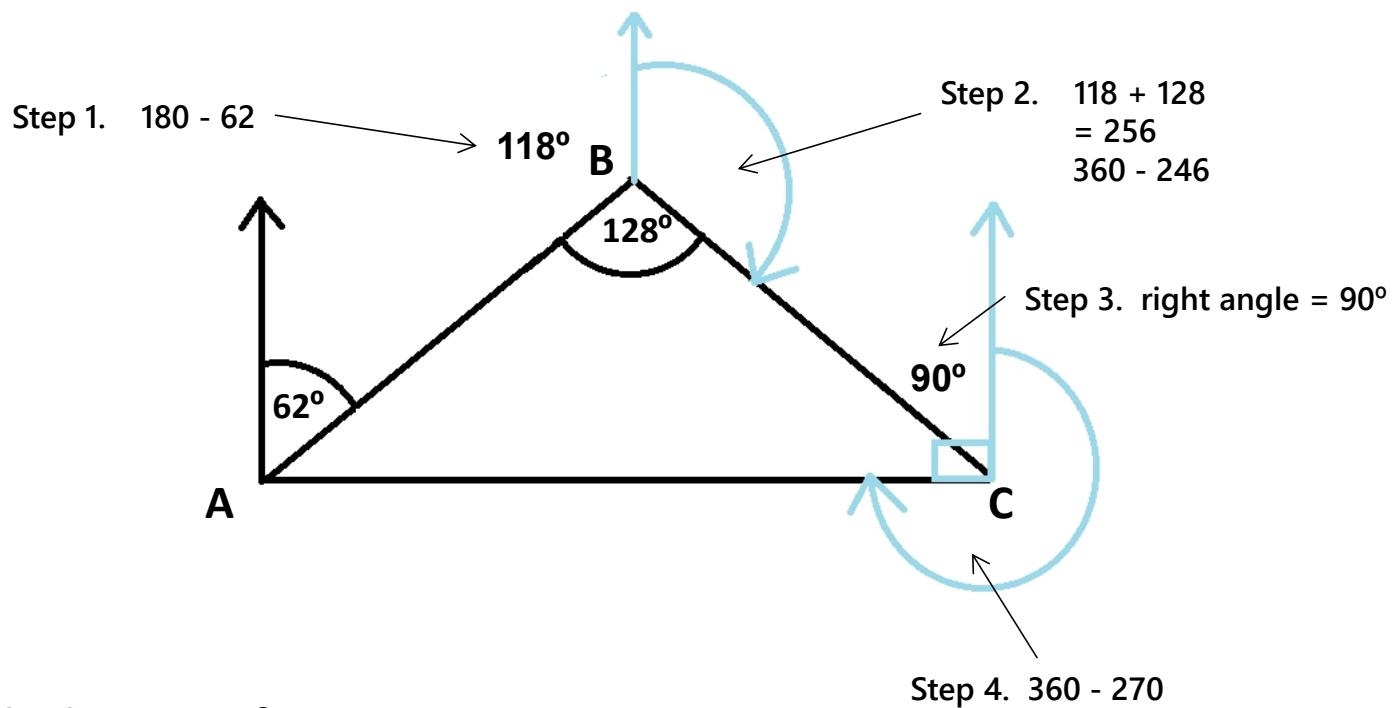
$$\begin{aligned}(b) W \hat{P} Z &= 64 + 36 \\ &= 100^\circ\end{aligned}$$

NOTE

- Right angle = 90°
- Angles on a straight line = 180°
- A complete circle = 360°
- The alternate exterior angles are equal.

4. 2019 P1

In the diagram below, A, B and C are three points on level ground. The bearing of B from A is 062° and angle ABC = 128° . C is due east of A.



Find the bearing of

- (a) C from B.
- (b) A from C.

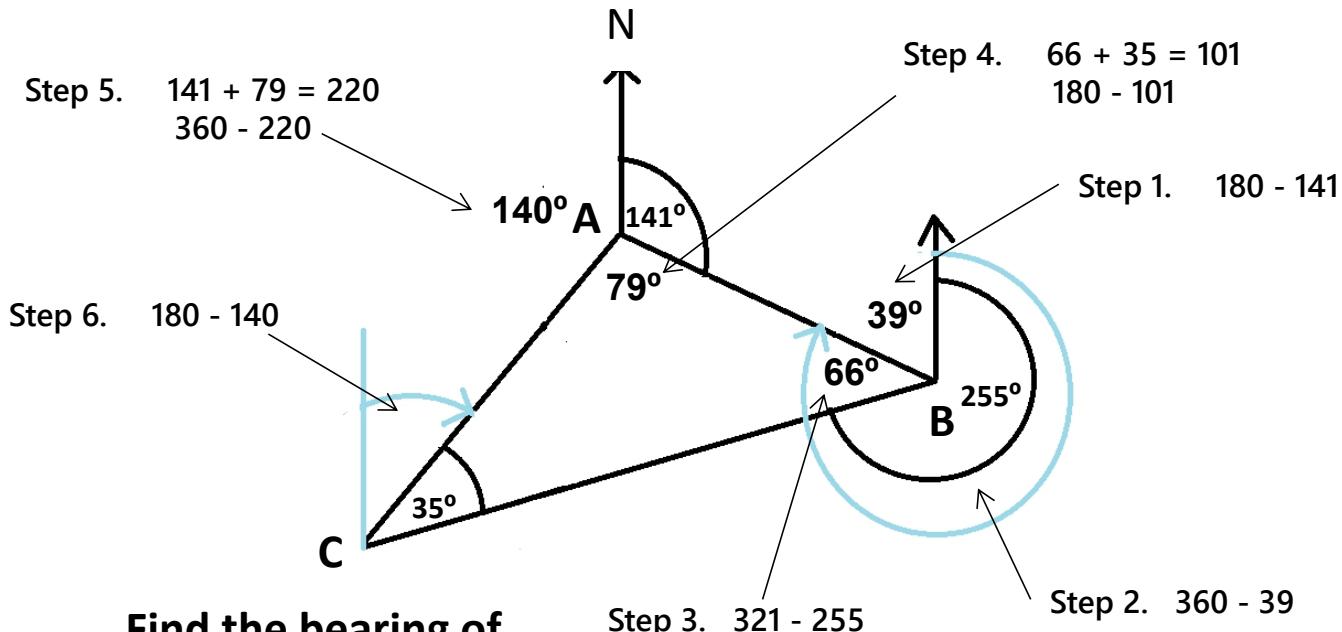
$$\begin{aligned}(a) \quad & 118 + 128 \\ & = 246\end{aligned}$$

$$\begin{aligned}C \text{ from } B &= 360 - 246 \\ &= 114^\circ\end{aligned}$$

$$\begin{aligned}(b) \quad A \text{ from } C &= 360 - 90 \\ &= 270^\circ\end{aligned}$$

5. 2019 GCE P1

The diagram below shows Mr Monda's trip. He travels on a bearing of 141° from A to B. He then decides to continue with his trip from B on a bearing of 255° to C. The angle $BCA = 35^\circ$.



Find the bearing of
(a) A from B.
(b) A from C.

$$\begin{aligned} \text{(a) } 180 - 141 \\ = 39 \end{aligned}$$

$$\begin{aligned} \text{A from B} &= 360 - 39 \\ &= 321^\circ \end{aligned}$$

NOTE

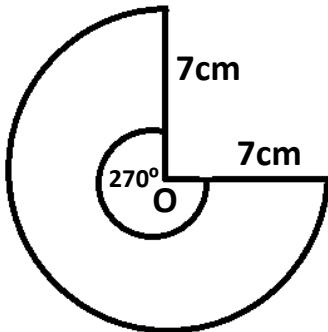
- All angles in a triangle = 180°
- Angles on a straight line = 180°
- A complete circle = 360° .

$$\begin{aligned} \text{(b) A from C} &= 180 - 140 \\ &= 040^\circ \end{aligned}$$

TOPIC 14 - MENSURATION

1. 2017 P1

The diagram below is a sector with center O and radius 7cm. Angle at O is 270° .



Calculate the area of the sector. $[\lambda = \frac{22}{7}]$

$$A = \frac{\theta}{360} \times \pi r^2$$

Step 1 - Write the equation to calculate area of sector.

$$= \frac{270}{360} \times \frac{22}{7} \times 7 \times 7$$

Step 2 - Substitute the values and cancel the 7's

$$= \frac{270}{360} \times 22 \times 7$$

Step 3 - Divide 270 and 360 with 90.

$$= \frac{3}{4} \times 22 \times 7$$

Step 4 - Divide 4 and 22 with 2.

$$= \frac{3}{2} \times 11 \times 7$$

Step 5 - Multiply 11 and 7.

$$= \frac{3}{2} \times 77$$

Step 6 - Multiply 3 and 11 and then divide by 2

$$= \frac{231}{2}$$

$$= 115.5 \text{ cm}^2$$

2. 2017 GCE P1

The curved surface area of a cone is 88cm^2 . Given that the base area is 4cm , calculate the slant height of the cone. $[\lambda = \frac{22}{7}]$, $A = \lambda rl$

$$A = \pi rl$$

Step 1 - Write the equation given.

$$88 = \frac{22}{7} \times 4 \times l$$

Step 2 - Substitute the values and multiply 22 and 4.

$$88 = \frac{88l}{7}$$

Step 3 - Cross multiply.

$$88l = 88 \times 7$$

$$\frac{88l}{88} = \frac{88 \times 7}{88}$$

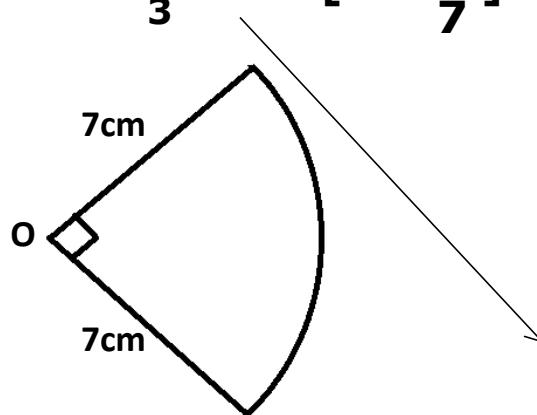
Step 4- Divide both sides with 88.

$$l = 7\text{cm}$$

3. 2018 P1

The diagram below shows a sector AOB of a circle with centre O and radius 7cm.

The area of the sector is $25 \frac{2}{3}\text{ cm}^2$, $[\lambda = \frac{22}{7}]$



make it a proper fraction
 $= 25 \times 3 + 2$
 $= \frac{77}{3}$

Calculate \hat{AOB} .

$$A = \frac{\theta}{360} \times \lambda r^2$$

Step 1 - Write the equation to calculate area of sector.

$$\frac{77}{3} = \frac{\theta}{360} \times \frac{22}{7} \times 7 \times 7$$

Step 2 - Substitute the values, cancel the 7's and divide 22 and 360 with 2.

$$\frac{77}{3} = \frac{\theta}{180} \times 11 \times 7$$

Step 3 - Multiply 11 and 7.

$$\frac{77}{3} = \frac{77\theta}{180}$$

Step 4 - Cross multiply.

$$\frac{77\theta \times 3}{77 \times 3} = \frac{77 \times 180}{77 \times 3}$$

Step 5 - Cancel out 77 and 3.

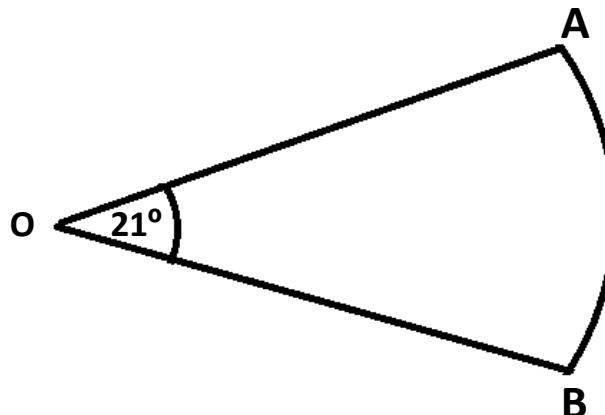
$$\theta = \frac{180}{3}$$

Step 6 - Divide 180 by 3 to find theta.

$$\theta = 60^\circ$$

4. 2019 P1

The diagram below shows a sector AOB. Arc AB subtends an angle of 21° at the center O.



Given that the area of the sector is 14.85 cm^2 , calculate the radius. [$\lambda = \frac{22}{7}$]

$$A = \frac{\theta}{360} \times \lambda r^2$$

$$14.85 = \frac{21}{360} \times \frac{22}{7} \times r^2$$

$$14.85 = \frac{3}{180} \times \frac{11}{1} \times r^2$$

$$14.85 = \frac{1}{60} \times 11 \times r^2$$

$$14.85 = \frac{11r^2}{60}$$

$$\frac{11r^2}{11} = \frac{14.85 \times 60}{11}$$

$$r^2 = \frac{891}{11}$$

$$r^2 = 81$$

$$\sqrt{r^2} = \sqrt{81}$$

$$r = 9$$

Step 1 - Write the equation to calculate area of sector.

Step 2 - Substitute the values and divide 360 and 22 with
and 2 then 7 and 21 with 7.

Step 3. Divide 3 and 180 with 3.

Step 4 - Multiply 1 with 11 r^2

Step 5 - Cross multiply

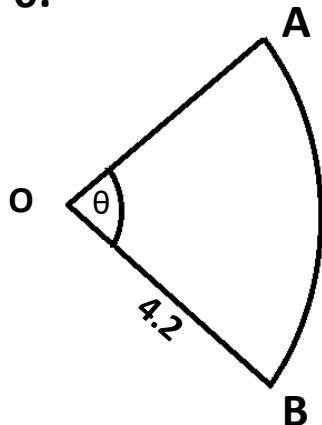
Step 6 - Cross the 11's out and multiply 14.85 with
60.

Step 7 - Cross the 11's out and multiply 14.85 with
60.

Step 8 - Find the square root of 81 to find r.

5. 2019 GCE P1

The diagram below shows a sector of a circle with centre O and radius 4.2cm. Angle AOB = θ .



Given that the area of the sector AOB is 9.24 cm², find the value

$$\text{of } \theta. \quad [\lambda = \frac{22}{7}]$$

$$A = \frac{\theta}{360} \times \pi r^2$$

$$9.24 = \frac{\theta}{360} \times \frac{22}{7} \times 4.2 \times 4.2$$

$$9.24 = \frac{\theta}{180} \times 11 \times 0.6 \times 4.2$$

$$9.24 = \frac{\theta}{180} \times 6.6 \times 4.2$$

$$9.24 = \frac{\theta}{180} \times 27.72$$

$$\frac{9.24}{1} = \frac{27.72\theta}{180}$$

$$\frac{27.72\theta}{27.72} = \frac{9.24 \times 180}{27.72}$$

$$\theta = \frac{180}{3}$$

$$\theta = 60^\circ$$

Step 1 - Write the equation to calculate area of sector.

Step 2 - Substitute the values and divide 360 and 22 with and 2 then 7 and 4.2 with 7.

Step 3 - Multiply 11 and 0.6

Step 4 - Multiply 6.6 and 4.2

Step 5 - Multiply 27.72 with theta.

Step 6 - Cross multiply.

Step 7 Cross out the 27.72's and then divide 9.24 and 27.72 with 9.24.

Step 8 - Divide 180 by 3 to find theta.

TOPIC 15 - RATIOS

1. 2017 P1

The ratio of the surface area of two cubes is 16:25.What is the volume of the smaller cube if the volume of the bigger cube is 500cm³

$$\text{Ratio of height} = \sqrt{16} : \sqrt{25} \\ = 4 : 5$$

Step 1 - Find the ratio of their heights by finding their square roots.

$$\text{Ratio of volume} = 4^3 : 5^3 \\ = 64 : 125$$

Step 2 - Find the ratio of their volume by cubing their heights.

$$64 : 125 \\ x = 500 \\ 125x = 500 \times 64 \\ \frac{125x}{125} = \frac{500 \times 64}{125} \\ x = 4 \times 64 \\ x = 256\text{cm}^3$$

Step 3 - Cross multiply

Step 4 - Cross out the 125's and then divide 500 and 125 with 125.

Step 5 - Multiply 4 and 64

2. 2017 GCE P1

Two tins are geometrically similar.If the ratio of their volumes is 27:64 , find the ratio of thier curved surface areas.

$$\text{Ratio of height} = \sqrt[3]{27} : \sqrt[3]{64} \\ = 3 : 4$$

Step 1 - Find the ratio of their heights by finding the third root of each

ratio of thelr curved surface areas

$$= 3^2 : 4^2 \\ = 9 : 16$$

Step 2 - Find the ratio of their curved surface area by squaring their heights.

3. 2018 GCE P1

The ratio of the heights of two containers that are geometrically similar is 2:3. If the surface area of the smaller container is 80cm^2 , find the surface area of the larger container.

Ratio of height = 2 : 3

Ratio of surface area

$$= 2^2 : 3^2$$

$$= 4 : 9$$

$$4 : 9$$

$$80 : x$$

$$4x = 80 \times 9$$

$$\frac{4x}{4} = \frac{720}{4}$$

$$x = 180\text{cm}^2$$

Step 1 - Find the ratio of their surface area by squaring their heights.

Step 2 - Cross multiply and then multiply 80 with 9.

Step 3 - Cross out the 4's and then divide 720 by 4.

4. 2019 P1

The heights of two similar cylinders are 4 cm and 6 cm. If the volume of the smaller cylinder is 48cm^3 , find the volume of the larger cylinder.

Ratio of height = 4 : 6

Ratio of volume

$$= 2^3 : 3^3$$

$$= 8 : 27$$

$$8 : 27$$

$$48 : x$$

$$8x = 27 \times 48$$

$$\frac{8x}{8} = \frac{27 \times 48}{8}$$

$$x = 27 \times 6$$

$$x = 162^3$$

Step 1 - Find the ratio of their volume by cubing their heights.

Step 2 - cross multiply

Step 3 - cross out the 8's and then divide 38 by 8

Step 4 - Multily 27 and 6.

5. 2019 GCE P1

The ratio of the volumes of two similar solids is 64:27. The surface area of the smaller solid is 180cm³. What is the surface area of the bigger solid.

$$\begin{aligned}\text{Ratio of height} &= \sqrt[3]{64} : \sqrt[3]{27} \\ &= 4 : 3\end{aligned}$$

Step 1 - Find the ratio of their heights by finding the third root of each.

$$\begin{aligned}\text{Ratio of area} &= 4^2 : 3^2 \\ &= 16 : 9\end{aligned}$$

Step 2 - Find the ratio of their surface area by squaring their heights.

$$\begin{aligned}16 : 9 \\ x : 180 \\ 9x = 180 \times 16 \\ \frac{9x}{9} = \frac{180 \times 16}{9} \\ x = 20 \times 16\end{aligned}$$

Step 3 - Cross multiply.

Step 4 - Cross out the 9's and then divide 180 and 9 by 9.

Step 5 - Multiply 20 and 16.

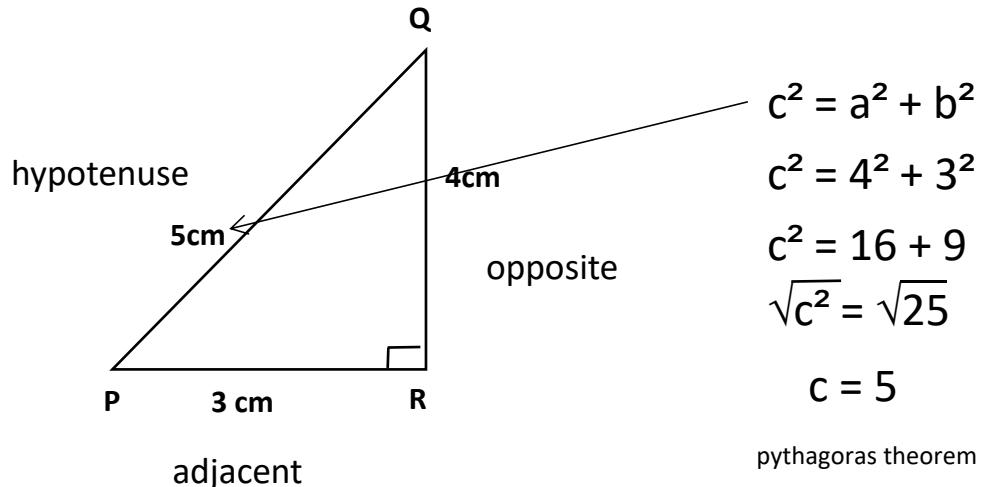
$$x = 320\text{cm}^2$$

TOPIC 16 - TRIGONOMETRY

1. 2017 P1

It is given that ΔPQR below is right angled at R. QR = 4cm and

$$\tan \hat{QPR} = \frac{4}{3}$$



Find $\sin \hat{PQR}$

(a) SOH CAH TOA

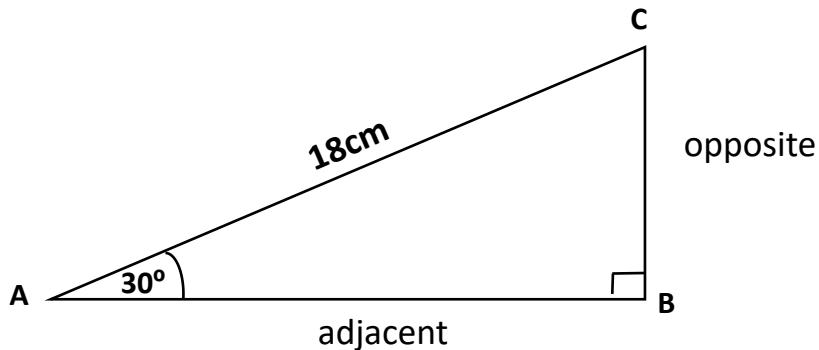
$$\tan \hat{QPR} = \frac{O}{A} = \frac{4}{3}$$

$$\sin \hat{PQR} = \frac{O}{H} = \frac{3}{5}$$

Step 1 - Write the formula to find sin and substitute the values

2. 2017 GCE P1

The diagram below shows triangle ABC in which AC = 18cm, $\angle CAB = 30^\circ$ and $\angle ABC = 90^\circ$.



Calculate the length of BC.

SOH CAH TOA

Only sin has both opposite and hypotenuse.

$$\sin \theta = \frac{O}{H}$$

$$\sin 30^\circ = \frac{BC}{18}$$

Step 1 - Write the formula to find sin and substitute the values.

$$0.5 = \frac{BC}{18}$$

Step 2 - Cross multiply.

$$\frac{0.5BC}{0.5} = \frac{18}{0.5}$$

Step 3 - Cross out the 0.5's and divide 18 by 0.5.

$$BC = 9\text{cm}$$

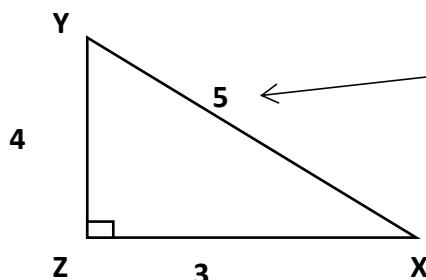
3. 2018 GCE P1

Given a right angled triangle XYZ where $\angle Z$ is 90 and $\sin X = \frac{4}{3}$ find the value of $\cos X$.

SOH CAH TOA

$$\cos X = \frac{A}{H}$$

$$= \frac{3}{5}$$



imaginary triangle

$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 3^2$$

$$c^2 = 16 + 9$$

$$\sqrt{c^2} = \sqrt{25}$$

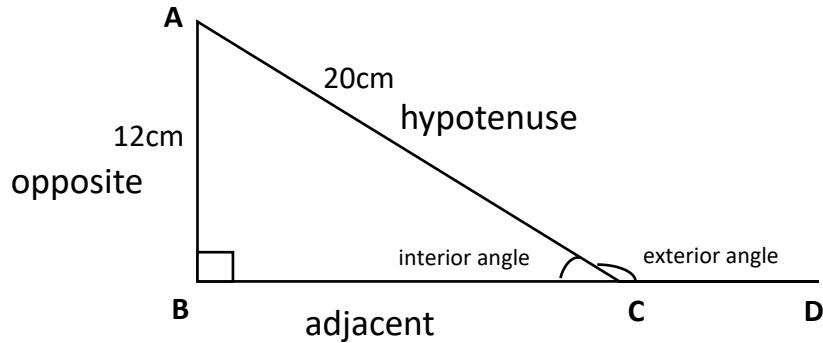
$$c = 5$$

pythagoras theorem

Step 1 - Write the formula to find cos and substitute the values

4. 2019 P1

In the diagram below BCD is a straight line $AB = 12\text{cm}$, $AC = 20\text{cm}$ and angle $ABC = 90^\circ$



Find the value of $\cos \hat{A}CD$.

$$\cos \hat{A}CD = - \cos \text{interior}$$

$$\cos \hat{A}CD = - \frac{A}{H}$$

$$c^2 = a^2 + b^2$$

Step 1 - Use pythagoras theorem to find the adjacent b .

$$20^2 = 12^2 + b^2$$

Step 2 - Substitute the values.

$$400 = 144 + b^2$$

Step 3 - Group the like terms.

$$400 - 144 = b^2$$

Step 4 - Subtract 144 from 100.

$$\sqrt{256} = \sqrt{b^2}$$

Step 5 - Find the square root of 256.

$$b = 16$$

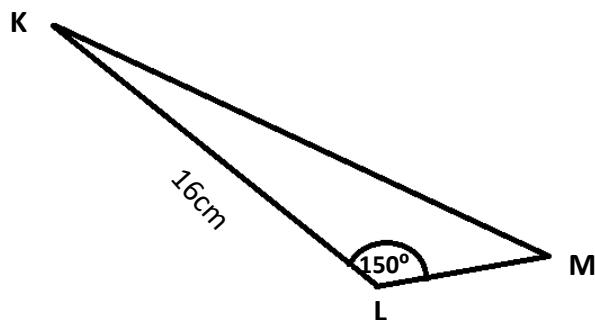
$$\cos \hat{A}CD = - \frac{16}{20}$$

Step 6 - Place the values and reduce to the lowest terms.

$$= - \frac{4}{5}$$

5. 2019 GCE P1

The diagram below shows triangle KLM in which $KL = 16\text{cm}$, $\angle KLM = 150^\circ$ and its area is 32cm^2



Calculate the length of LM

$$A = \frac{1}{2}ab \times \sin C$$

Step 1 - Write the formula to find the area of a triangle and substitute the values.

$$32 = \frac{1}{2} \times 16 \times LM \times \sin 150$$

Step 2 - Divide 2 and 16 with 2 and find $\sin 150$.

$$32 = 8 \times LM \times 0.5$$

Step 3 - Multiply 8 and 0.5.

$$32 = 4LM$$

$$\frac{32}{4} = \frac{4LM}{4}$$

Step 3 - Divide both sides with 4 to find LM.

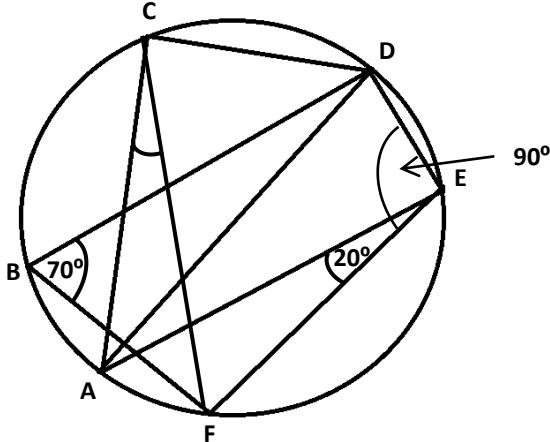
$$LM = 8 \text{ cm}$$

TOPIC 17 - CIRCLE THEOREMS

1. 2017 P1

In the diagram below, A, B, C, D, E and F are points on the circumference of a circle.

$$\hat{FBD} = 70^\circ \text{ and } \hat{AEF} = 20^\circ$$



(a) Explain why AD is the diameter

(b) Find

(i) \hat{ACF} (ii) \hat{DEF}

(a) AD is the diameter of the circle because it cuts the circle into two equal semi-circles.

(b)(i) $\hat{ACF} = 20^\circ$

Step 1 - Angles in the same segment are equal.

(b)(ii) $70 + 20 + x = 180^\circ$

$$90 + x = 180^\circ$$

$$x = 180 - 90$$

$$x = 90$$

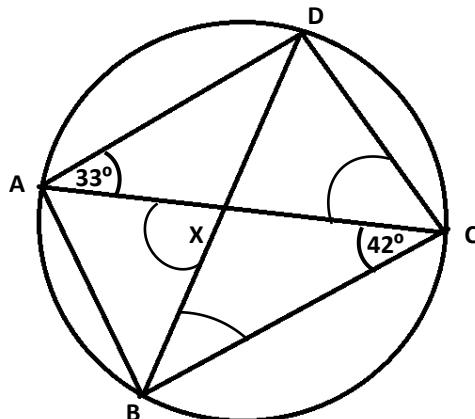
$$\hat{DEF} = 90 + 20$$

$$= 110^\circ$$

Step 1 - Opposite angles in a cyclic quadrilateral sum to 180°

2. 2017 GCE P1

In the diagram below, points A, B, C and D are on a circle, BD is the diameter of the circle. $\hat{ACB} = 42^\circ$, $\hat{CAD} = 33^\circ$ and the lines AC and BD intersect at X.



Find

- (a) \hat{CBD}
- (b) \hat{ACD}
- (c) \hat{AXB}

(a) $\hat{CBD} = 33^\circ$

Step 1 - Angles in the same segment are equal.

(b) $\hat{ACD} = 90 - 42 = 48^\circ$

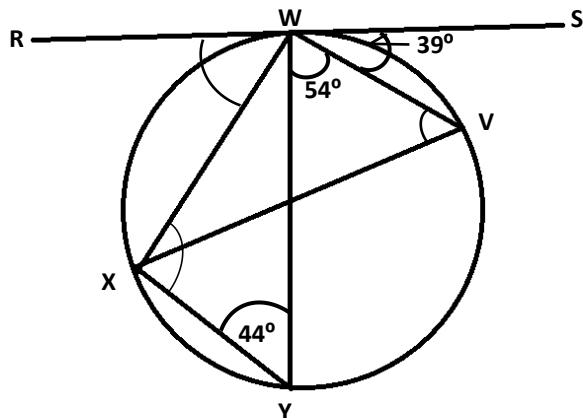
Step 1 - Angles in a semi circle is always 90°

(c) $\hat{ACD} = 42 + 33 = 75^\circ$

Step 1 - The exterior angle is equal to the sum of the two opposite interior angles

3. 2018 GCE P1

The diagram below shows a circle with a tangent $\overset{\wedge}{RWS}$. The point V, W , and Y are on the circle such that $\overset{\wedge}{XYW} = 44^\circ$, $\overset{\wedge}{VWY} = 54^\circ$ and $\overset{\wedge}{SWV} = 39^\circ$.



Calculate

- (a) $\overset{\wedge}{RWX}$
- (b) $\overset{\wedge}{XVW}$
- (c) $\overset{\wedge}{YXW}$

(a) $\overset{\wedge}{RWX} = 44^\circ$

Step 1 - The angle which lies between a chord and a tangent through any of the end points of the chord is equal to the angle in the alternate segment.

(b) $\overset{\wedge}{XVW} = 44^\circ$

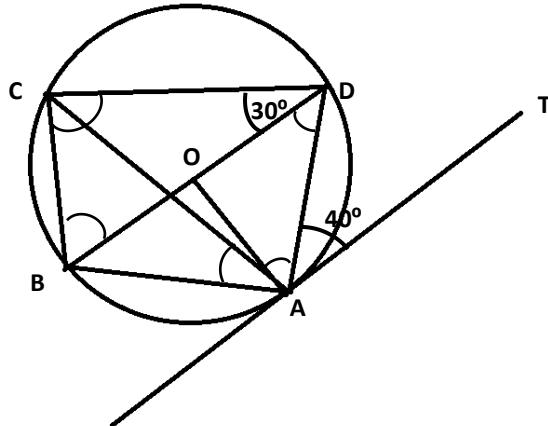
Step 1 - Angles in the same segment are equal.

(c) $\overset{\wedge}{YXW} = 43 + 44 = 87$
 $= 180 - 87$
 $= 93^\circ$

Step 1 - Angles in a triangle equal 180° .

4. 2019 P1

In the diagram below, A, B, C and D are points on the circumference of a circle, center O. $\hat{DAT} = 40^\circ$, $\hat{BDC} = 30^\circ$ and AT is a tangent to the circle at A.



Calculate

- (a) \hat{CBD}
- (b) \hat{BAC}
- (c) \hat{AOB}

$$\begin{aligned}(a) \hat{CBD} &= 90 + 30 = 120 \\ &= 180 - 120 \\ &= 60^\circ\end{aligned}$$

Step 1 - Angles in a semi circle is always 90°

Step 2 - Angles in a triangle equal 180° .

$$(b) \hat{BAC} = 30^\circ$$

Step 1 - Angles in the same segment are equal.

$$\begin{aligned}(c) \hat{AOB} &= 180 - 80 \\ &= 100^\circ\end{aligned}$$

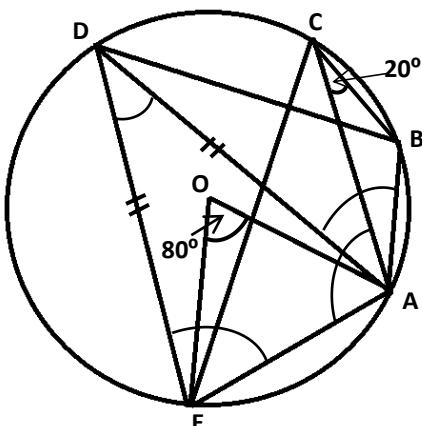
Step 1 - The point where the radius and tangent meet make and angle of 90° .

Step 2 - Angles in a triangle equal 180° .

Step 3 - Angles on a straight line equal 180°

5. 2019 GCE P1

In the diagram below, A, B, C, D, E and F are points on the circumference of the circle with centre O. $\hat{DE} = AD$, $\hat{ACB} = 20^\circ$ and $\hat{AOE} = 80^\circ$.



Find

- (a) \hat{ADE}
- (b) \hat{DAE}
- (c) \hat{BAD}

$$(a) \text{angle } \hat{ADE} = \frac{1}{2} \times 80 \\ = 40^\circ$$

Step 1 - The angle at the center is twice the angle at the circumference.

$$(b) \text{angle } \hat{DAC} = 180 - 40 \\ = 140 \\ = \frac{140}{2} \\ = 70^\circ$$

Step 1 - Two sides in an isosceles triangle are equal

$$(c) \text{angle } \hat{BAD} = 180 - 110 - 20 \\ = 50^\circ$$

Step 1 - Angles in the same segment are equal.

Step 2 - Opposite angles in a cyclic quadrilateral sum to 180°

Step 3 - Angles in a triangle equal 180° .

TOPIC 18 - COMMERCIAL ARITHMETIC

1. 2017 P1

A businessman bought 300 company shares at K60.00. The nominal price was K30.00. How much does he pay for the shares?

$$\begin{aligned} \text{K60} - \text{K30} \\ = \text{K}30 \times 300 \\ = \text{K}9000 \end{aligned}$$

- Step 1 - Subtract the nominal cost from the actual cost.
Step 2 - Multiply the difference with the number of shares.

2. 2017 GCE P1

Maphone Manufacturing Company paid a total dividend of K12 600.00 at the end of 2015 on 6 000 shares. If Magula owned 200 shares in the company, how much was paid out in dividends to her?

$$\frac{200}{6000} \times 12600$$

Step 1 - Divide her number of shares over the total number of shares times the dividend paid out.

$$\frac{200}{60} \times 126$$

Step 2 - Divide 6 and 126 by 6.

$$\frac{20}{6} \times 126$$

Step 3 - Multiply 20 and 21.

$$20 \times 21$$

K420

3. 2018 GCE P1

Ngiwezi invested K14 500.00 in a business firm. The condition was that if she left shares in the firm for 12 months, a profit of 5% would be added to her shares. How much will she get at the end of 12 months?

$$I = \frac{PRT}{100}$$

Step 1 - Write the formula to calculate interest and substitute the values.

$$I = \frac{14500 \times 5 \times 1}{100}$$

$$I = 145 \times 5$$

Step 2 - Multiply 145 and 5 to find the interest.

$$I = 725$$

Amount she will receive = Interest + principle

$$= 725 + 14500$$

$$= \text{K15,225}$$

4. 2019 P1

A company declared a dividend of K1.50 per share. Musalala has 600 shares in company. How much will she get?

$$600 \times 1.50$$

$$= 600 \times \frac{3}{2}$$

$$= 300 \times 3$$

$$= \text{K900}$$

Step 1 - Multiply the share per cost with the number of shares

Step 2 - Convert 1.50 into a fraction

Step 3 - Multiply 300 and 3

5. 2019 GCE P1

A company's working capital consists of 450 10% preference shares of K50.00 each and 700 ordinary shares of K10.00 each. After 6 months, the company declared a dividend of K5 750.00. How much dividend will be paid to each ordinary shareholder?

$$\frac{700}{450+700} \times 5750$$

Step 1 - First find the amount for the ordinary shares by dividing the ordinary shares over the total number of shares times the dividend.

$$= \frac{700}{1150} \times 5750$$

Step 2 - Divide 575 and 115 with 115.

$$= \frac{700}{115} \times 575$$

Step 3 - Multiply 700 and 5 to find the amount for the ordinary shares.

$$= 700 \times 5$$

$$= \text{K3500}$$

$$= \frac{3500}{700}$$

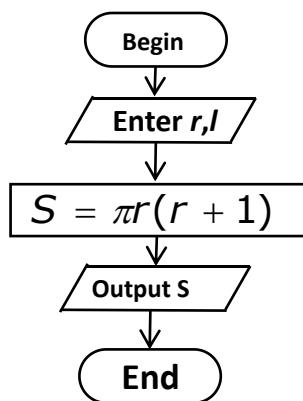
Step 4 - Divide the amount for the ordinary shares by the number of ordinary shares to find the dividend paid to each ordinary shareholder.

$$= \text{K5}$$

TOPIC 19 - COMPUTERS

1. 2017 P1

The diagram below is an incomplete program flow chart to calculate the curved surface area , S ,of a cone with base radius r and slant height l . Complete the flow chart below by writing appropriate statements in the blank symbols.

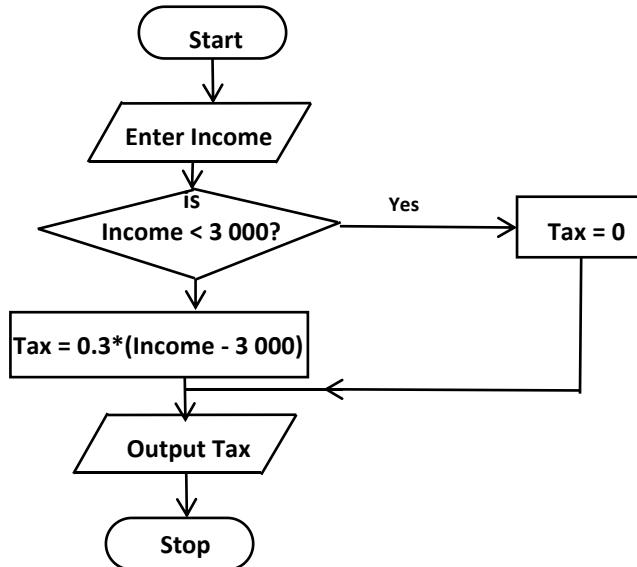


Step 1 - Enter the radius and height

Step 2 -Enter formula used to find curved surface area.

2. 2017 GCE P1

The diagram below shows a flow chart for the program to calculate tax on an income.



* = ×

$$5000 - 3000 = 2000$$

$$2000 \times 0.3$$

$$= 600$$

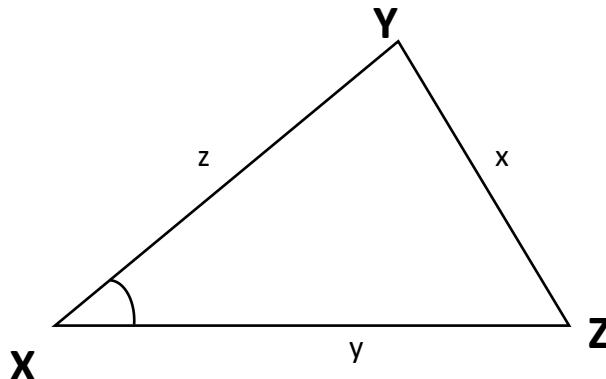
Complete the table below

Income	Tax
K2 900.00	0
K5 000.00	K600

Step 1 - K2 900 is less than K3000 so tax = 0
Step 2 - K5 000 is greater than K3000 so tax = $0.3 \times (\text{income} - 3000)$

3. 2018 GCE P1

The diagram in the answer space below is an incomplete flow chart to calculate $\cos X$ for the triangle shown below.



cosine rule

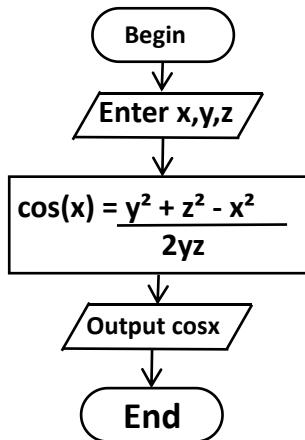
$$c^2 = a^2 + b^2 - 2ab \cos(c)$$

$$x^2 = y^2 + z^2 - 2yz \cos(x)$$

$$\frac{2yz \cos(x)}{2yz} = \frac{y^2 + z^2 - x^2}{2yz}$$

$$\cos(x) = \frac{y^2 + z^2 - x^2}{2yz}$$

Complete the flow chart below by writing the appropriate statements in the blank symbols.



Step 1 -Enter sides x,y and z.

Step 2 - Write the formula to calculate $\cos x$.

4. 2019 P1

In the answer space below is an incomplete program written in pseudocode for calculating the volume V of a cuboid,given the length, l ,base, b , and the height, h .Complete the program by filling in the blank spaces with appropriate statements.

Begin

Enter.....

l,b,h

$V = l * b * h$

Output V

End

Step 1 -Enter sides l, b and h.

Step 2 - write formula to calculate volume.

5. 2019 GCE P1

In the answer space below is an incomplete program written in pseudocode for calculating the mean(m) of 10 numbers whose sum is S . Complete the program filling in the blank spaces with appropriate statements.

Start

Enter..... S

Step 1 -Enter sum S .

$$m = \frac{S}{10}$$

Step 2 -Enter formula to calculate the mean of numbers.

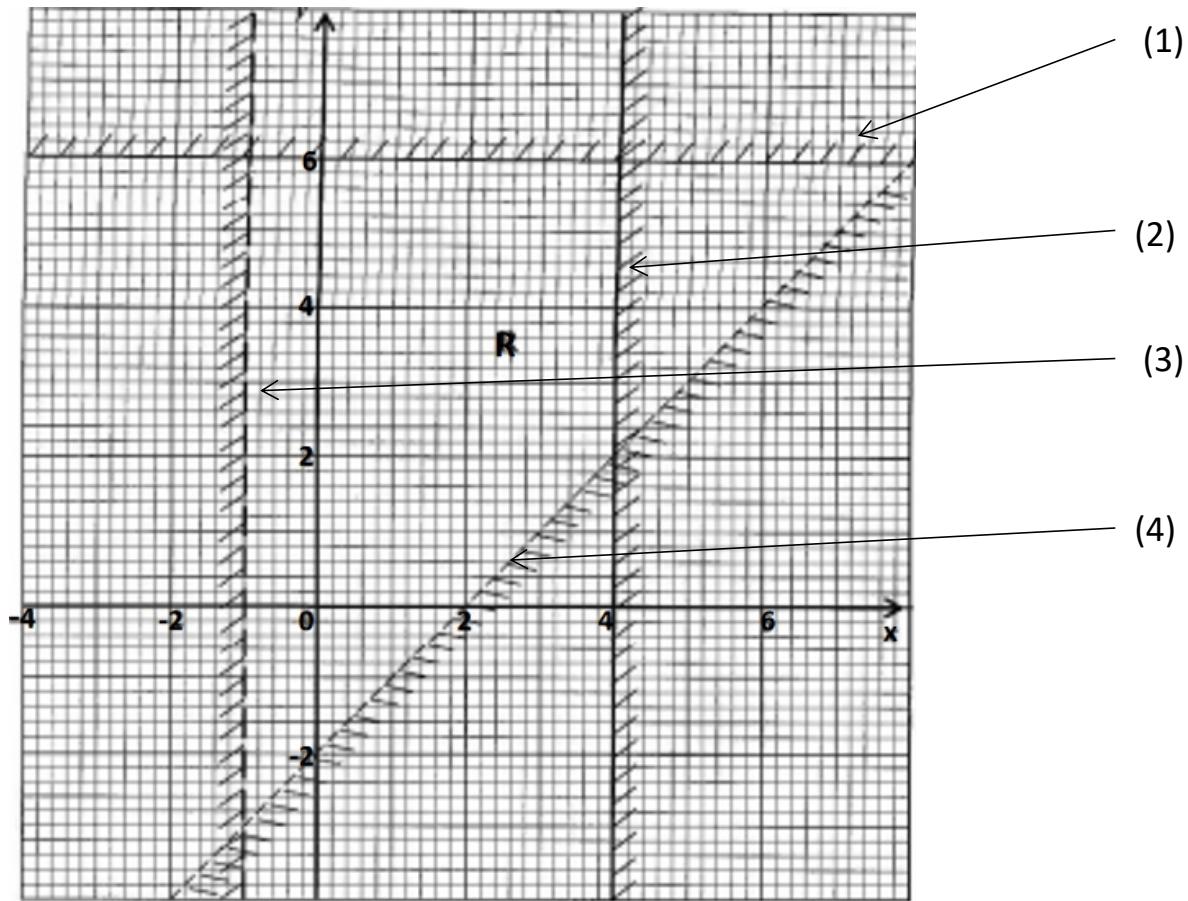
Output m

Stop

TOPIC 20 - LINEAR PROGRAMMING

2017 P1

Write down the four inequalities that define the unshaded region R, on the diagram below.



(1) $y = 6$
 $y \leq 6$

Step 1 - The line is passing on $y = 6$
- The line is bold
- The shaded area is greater than R therefore use \leq .

(2) $x = 4$
 $x \leq 4$

Step 2 - The line is passing on $x = 4$
- The line is bold
- The shaded area is greater than R therefore use \leq .

(3) $x = -1$
 $x > -1$

Step 3 - The line is passing on $x = -1$
- The line is dotted
- The shaded area is less than R therefore use $>$.

$$(4) \begin{matrix} x_1 & y_1 \\ 0 & -2 \end{matrix} \quad \begin{matrix} x_2 & y_2 \\ 2 & 0 \end{matrix}$$

Step 4 - The line cuts through both x and y so find the gradient of the line and then the equation of the line.

$$\begin{aligned} m &= \frac{Y_2 - Y_1}{X_2 - X_1} \\ &= \frac{0 - (-2)}{2 - 0} \\ &= \frac{2}{2} \\ &= 1 \end{aligned}$$

$$y = mx + c$$

$$y = x - 2$$

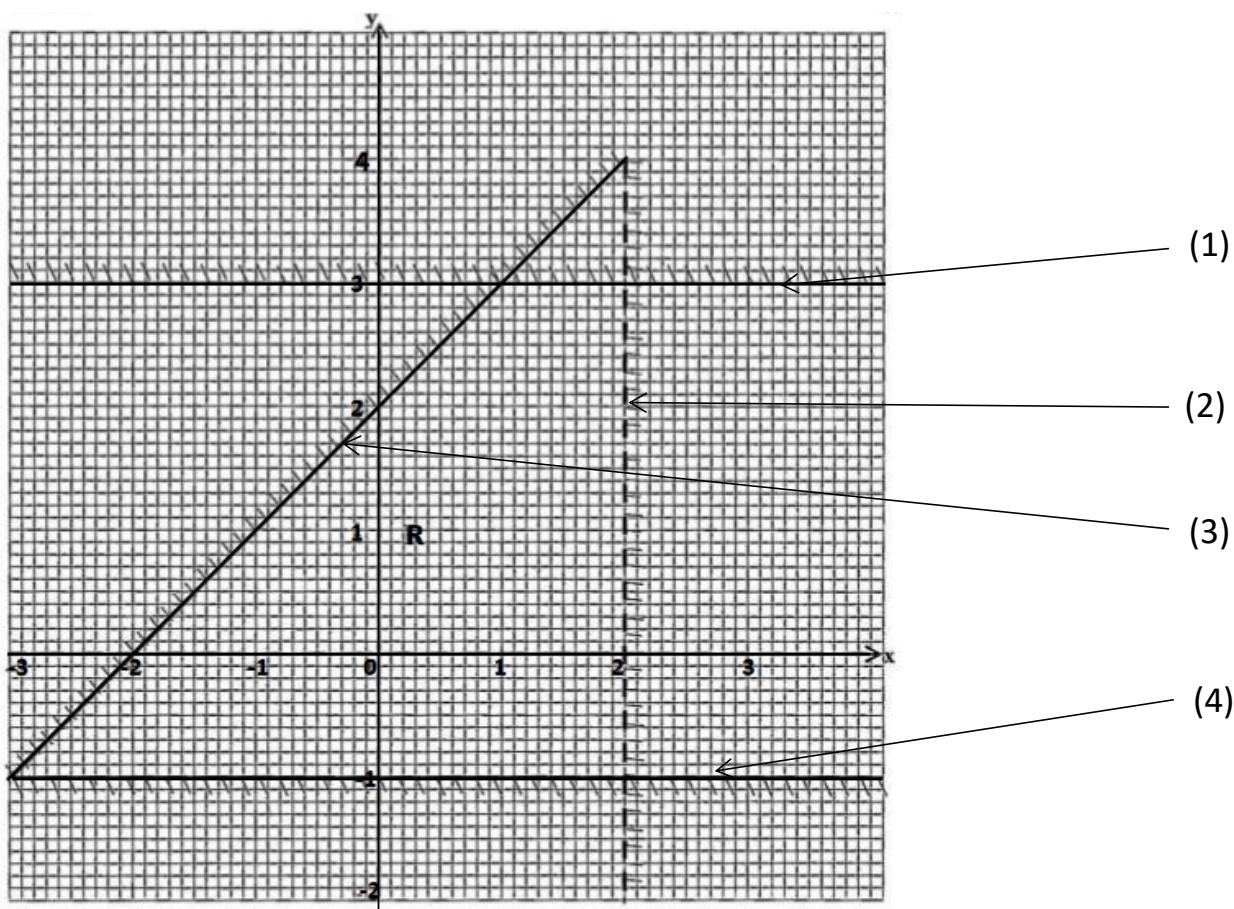
$$y > x - 2$$

Step 5 - The line is dotted

- The shaded area is less than R therefore use $>$.

2. 2017 GCE P1

Write down the four inequalities that define the unshaded region R, on the XOY plane below.



$$(1) y = 3$$
$$y \leq 3$$

Step 1 - The line is passing on $y = 3$
- The line is bold
- The shaded area is greater than R therefore use \leq .

$$(2) x = 2$$
$$x < 2$$

Step 3 - The line is passing on $x = 2$
- The line is dotted
- The shaded area is greater than R therefore use $<$.

$$(3) (-2, 0) (0, 2)$$

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{2 - 0}{0 - (-2)}$$

$$= \frac{2}{2}$$

$$= 1$$

$$y = mx + c$$

$$y = x + 2$$

$$y \leq x + 2$$

Step 4 - The line cuts through both x and y so find the gradient of the line and then the equation of the line.

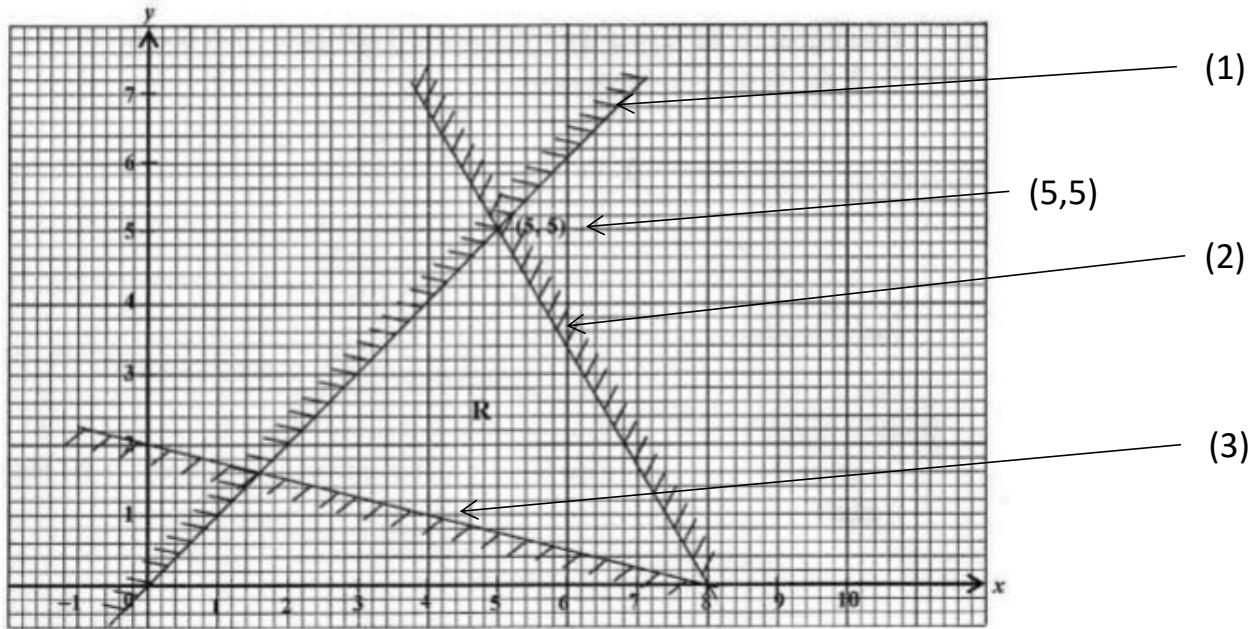
Step 5 - The line is bold
- The shaded area is greater than R therefore use \leq .

$$(4) x = -1$$
$$x \geq -1$$

Step 2 - The line is passing on $x = -1$
- The line is bold
- The shaded area is less than R therefore use \geq .

3. 2018 GCE P1

In the diagram below, R is the unshaded region. Write three inequalities which describe the region R.



$$(1) y = x$$

$$y \leq x$$

$$(2) (5, 5) (8, 0)$$

Step 1 - The line is passing on $y = 0$ or $y = x$

- The line is bold

- The shaded area is greater than R therefore use \leq .

Step 2 - Find the equation of the line by finding the gradient and the value of c.

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{0 - 5}{8 - 5}$$

$$= -\frac{5}{3}$$

$$y = mx + c$$

$$5 = -\frac{5}{3}(5) + c$$

$$5 = -\frac{25}{3} + c$$

$$\frac{5}{1} + \frac{25}{3} = c$$

$$c = \frac{40}{3}$$

$$y = -\frac{5}{3}x + \frac{40}{3}$$

$$\mathbf{y} \leq -\frac{5}{3}\mathbf{x} + \frac{40}{3}$$

Step 3 - The line is bold

- The shaded area is greater than R therefore use \leq .

$$(3) (0, 2) (8, 0)$$

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{0 - 2}{8 - 0}$$

$$= \frac{-2}{8}$$

$$= -\frac{1}{4}$$

$$y = mx + c$$

$$y = -\frac{1}{4}x + 2$$

$$\mathbf{y} \geq -\frac{1}{4}\mathbf{x} + 2$$

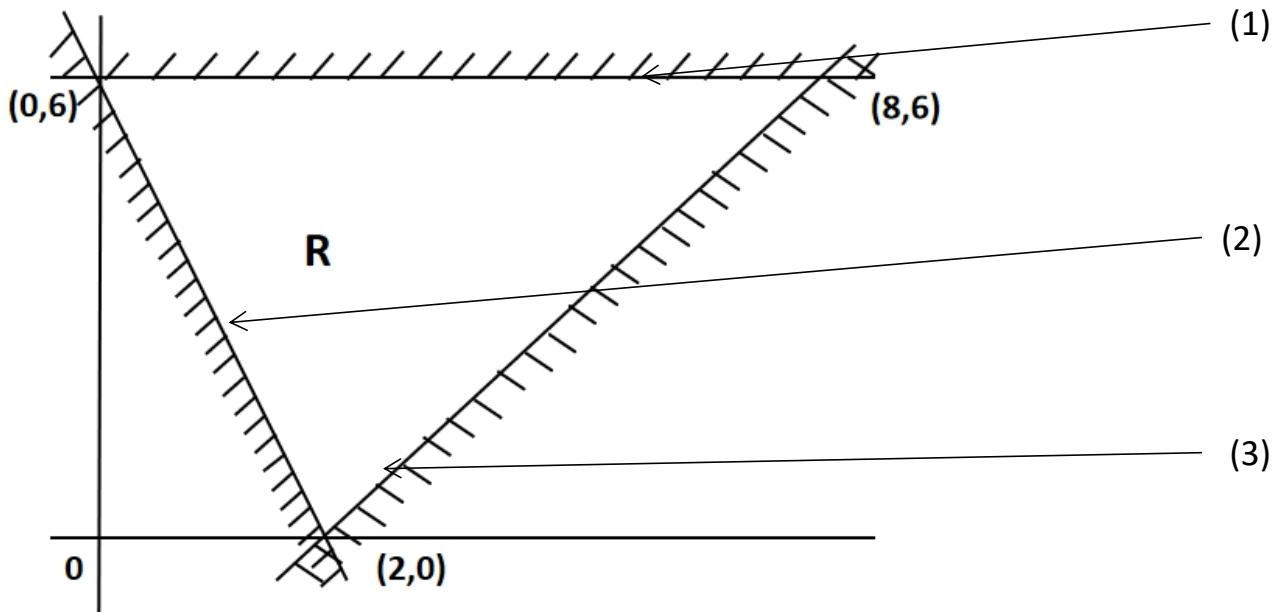
Step 4 - The line cuts through both x and y so find the gradient of the line and then the equation of the line.

Step 5 - The line is bold

- The shaded area is less than R therefore use \geq .

4. 2019 P1

Write three inequalities which describe the unshaded region R, on the diagram below.



$$(1) y = 6$$

$$y \leq 6$$

Step 1 - The line is passing on $y = 6$

- The line is bold

- The shaded area is greater than R therefore use \leq .

$$(2) (0, 6) (2, 0)$$

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{0 - 6}{2 - 0}$$

$$= \frac{-6}{2}$$

$$= -3$$

Step 2 - The line cuts through both x and y so find the gradient of the line and then the equation of the line.

$$y = mx + c$$

$$y = -3x + 6$$

$$y \geq -3x + 6$$

Step 3 - The line is bold

- The shaded area is less than R therefore use \geq .

$$(3) \begin{array}{cccc} X_1 & Y_1 & X_2 & Y_2 \\ (2, 0) & (8, 6) \end{array}$$

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}$$

$$= \frac{6 - 0}{8 - 2}$$

$$= \frac{6}{6}$$

$$= 1$$

$$y = mx + c$$

$$0 = 1(2) + c$$

$$0 - 2 = c$$

$$c = -2$$

$$y = x - 2$$

$$y > x - 2$$

Step 4 - Find the equation of the line by finding the gradient and the value of c.

Step 5 - The line is dotted

- The shaded area is less than R therefore use >.

TOPIC 21 - CALCULUS

1. 2017 P1

Differentiate $y = \frac{1}{3}x^3 - 5x^2 - 2x$ with respect to x.

$$y = \frac{1}{3}x^3 - 5x^2 - 2x$$

$$\frac{dy}{dx} = (\frac{1}{3} \times 3)x^{3-1} - (5 \times 2)x^{2-1} - 2x$$

$$\frac{dy}{dx} = x^2 - 10x - 2$$

$$\frac{d}{dx}x^n = n \cdot x^{n-1}$$

2. 2017 GCE P1

Find the derivative of $y = 2x^3 - 2x^2 - 3x + 1$, with respect to x.

$$y = 2x^3 - 2x^2 - 3x + 1$$

$$\frac{dy}{dx} = (3 \times 2)x^{3-1} - (2 \times 2)x^{2-1} - 3x$$

$$\frac{dy}{dx} = 6x^2 - 4x - 3$$

$$\frac{d}{dx}x^n = n \cdot x^{n-1}$$

3. 2018 GCE P1

Integrate $3x^4 - 4x^{-3}$ with respect to x.

$$\begin{aligned} & \int (3x^4 - 4x^{-3}) \\ &= \frac{3x^{4+1}}{4+1} - \frac{4x^{-3+1}}{-3+1} + C \\ &= \frac{3x^5}{5} - \frac{4x^{-2}}{-2} + C \\ &= \frac{3x^5}{5} + 2x^{-2} + C \\ &= \frac{3x^5}{5} + \frac{2}{x^2} + C \end{aligned}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

4. 2019 P1

Find $\int (3x^2 + 8x - 5) dx$

$$\int (3x^2 + 8x - 5) dx$$

$$= \frac{3x^{2+1}}{2+1} + \frac{8x^{1+1}}{1+1} - 5x + C$$

$$= \frac{3x^3}{3} + \frac{8x^2}{2} - 5x + C$$

$$= \mathbf{x^3} + \mathbf{4x^2} - \mathbf{5x} + \mathbf{C}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

5. 2019 GCE P1

Find the integral of $\int \frac{3x^3}{2} - 5x + \frac{1}{x^2}$ with respect to x.

$$\int \frac{3x^3}{2} - 5x + \frac{1}{x^2}$$

$$\int \frac{\frac{3x^{3+1}}{2}}{3+1} - \frac{5x^{1+1}}{1+1} + \frac{x^{-2+1}}{-2+1} + C$$

$$\frac{3+1}{3+1}$$

$$= \frac{3x^4}{2} \div 4 - \frac{5x^2}{2} - \frac{x^{-1}}{-1} + C$$

$$= \frac{3x^4}{2} \times \frac{1}{4} - \frac{5x^2}{2} - x^{-1} + C$$

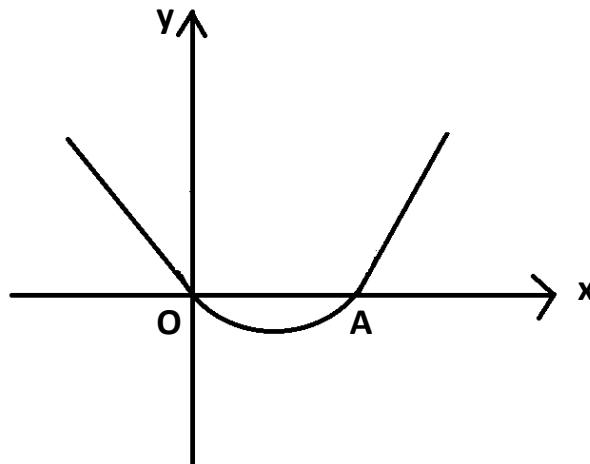
$$= \frac{3x^4}{8} - \frac{5}{2}x^2 - \frac{1}{x} + \mathbf{C}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

TOPIC 22 - GRAPHS OF FUNCTIONS

1. 2017 P1

The sketch below represents a section of the curve $y = x(x - 2)$



- (i) Find the coordinates of the points where the curve cuts the x-axis.
(ii) What is the minimum value of the function?

(i) $y = x(x - 2)$
 $0 = x(x - 2)$
 $x(x - 2) = 0$
 $x = 0 \text{ or } x - 2 = 0$
 $x = 2$
 $(0,0) \text{ & } (2,0)$

Step 1 - Write the equation of the curve when $y = 0$ to find the values of x
Step 2 - Equate x to 0 and $(x - 2)$ to 0 to find the values of x .
Move the -2 to the other side to value of the other x

(ii) $y = x(x - 2)$
 $= x^2 - 2x$
 $b = -2$

Step 1 - Find the value of b expanding the equation.

$$x = \frac{-b}{2a} = \frac{-(-2)}{2 \times 1} = 1$$

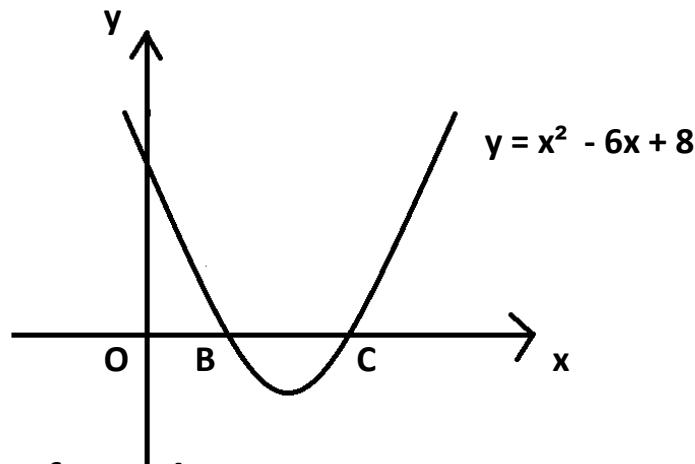
Step 2 - Find the turning point of x .

$$\begin{aligned}y &= x(x - 2) \\y &= 1(1 - 2) \\y &= 1(-1) \\y &= -1\end{aligned}$$

Step 3 - Replace x with 1 in the formula to find the minimum value of the function.

2. 2017 GCE P1

The diagram below shows a sketch of the graph of $y = x^2 - 6x + 8$, cutting the x-axis at B and C



(a) Find the coordinates of B and C

(b) Find the coordinates of the turning point of the graph.

$$(a) \quad \begin{array}{ccc} a & b & c \\ y = x^2 - 6x + 8 & & \\ x^2 - 6x + 8 = 0 & & \end{array}$$

$$\begin{array}{l} P = 8 \\ S = -6 \\ F = -2, -4 \end{array}$$

$$\begin{array}{l} (x - 2)(x - 4) = 0 \\ x = 2 \text{ or } x = 4 \\ (2,0) (4,0) \end{array}$$

$$B = (2,0) \quad C = (4,0)$$

Step 1 - Write the equation of the curve when $y = 0$ to find the values of x

Step 2 - Find the product, sum and factors.

- Product = value of $a \times c$

- Sum = value of b.

- factors = 2 numbers which when multiplied give you the value of c and when added give you b.

Step 3 - Equate $(x-2)$ and $(x-4)$ to 0 to find the values of x.
Move the -2 and the -4 to the other side.

(b) $x = 3$
 $y = x^2 - 6x + 8$
 $y = 3^2 - 6(3) + 8$
 $y = 9 - 18 + 8$
 $y = -1$
 $(3, -1)$

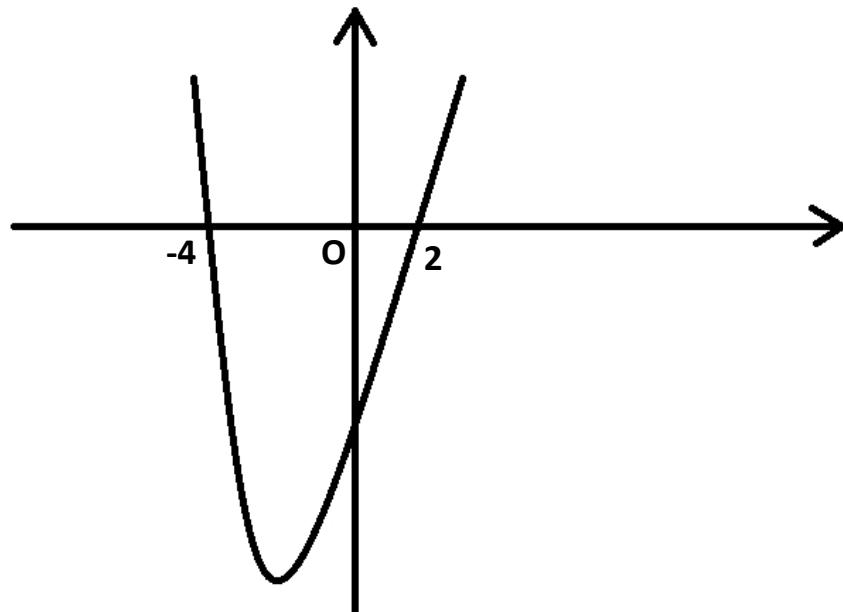
Step 1 - Find the number between 2 and 4 = 3 or use the formula to find the turning point..

Step 2 - Write the original equation and place 3 where x is to find the value of y.

Step 3 - Write the coordinates of the turning point.

3. 2018 GCE P1

The diagram below shows a sketch of a graph which meets the x axis at 4 and 2.



Find the

- (i) The equation of the graph
- (ii) Coordinates of the turning point.

$$(i) \quad x = -4, x = 2$$

$$x + 4, x - 2$$

$$= (x + 4)(x - 2)$$

$$= x^2 - 2x + 4x - 8$$

$$= x^2 + 2x - 8$$

$$y = x^2 + 2x - 8$$

Step 1 - Write the values of x as appears on the graph and then move them to the other side of the equals sign.

Step 2 - Expand and simplify

Step 3 - Write the equation of the graph.

$$(ii) \quad x = \frac{-b}{2a} = \frac{-2}{2 \times 1} = -1$$

Step 1 - Find the turning point of x.

$$x = -1$$

$$y = x^2 + 2x - 8$$

$$y = (-1)^2 + 2(-1) - 8$$

$$y = 1 - 2 - 8$$

$$y = -9$$

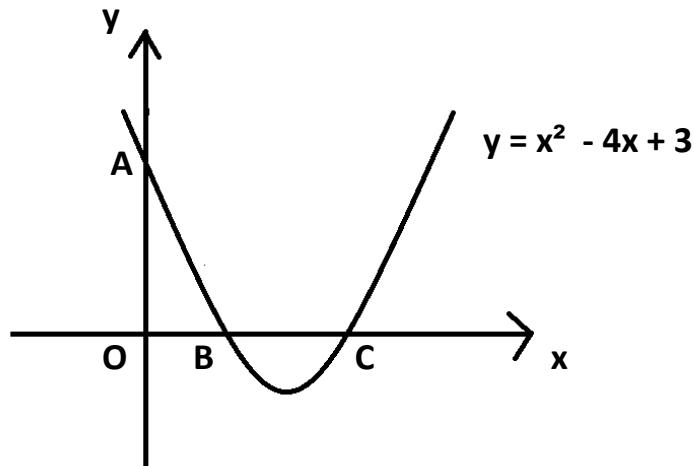
$$(-1, -9)$$

Step 2 - Write the equation and place -1 where x is to find the value of y.

Step 3 - Write the coordinates of the turning point.

4. 2019 P1

The diagram below represents the graph of the curve of $y = x^2 - 4x + 3$, passing through the points A, B and C.



Find

- (a) coordinates of the points B and C
- (b) minimum value of y.

$$\begin{array}{ll}
 \text{(a)} & \begin{array}{ccc} a & b & c \\ y = x^2 - 4x + 3 & & \\ x^2 - 4x + 3 = 0 & & \\ P = 3 & & \\ S = -4 & & \\ F = -1, -3 & & \end{array}
 \end{array}$$

$$\begin{aligned}
 (x - 1)(x - 3) &= 0 \\
 x = 1 \text{ or } x &= 3 \\
 (1, 0) (3, 0) &
 \end{aligned}$$

$$B = (1, 0) \quad C = (3, 0)$$

$$(b) \quad x = \frac{-b}{2a} = \frac{-(-4)}{2 \times 1} = 2$$

$$\begin{aligned}
 x &= 2 \\
 y &= x^2 - 4x + 3 \\
 y &= 2^2 - 4(2) + 3
 \end{aligned}$$

Step 1 - Write the equation of the curve when $y = 0$ to find the values of x

Step 2 - Find the product, sum and factors.

- Product = value of $a \times c$

- Sum = value of b.

- factors = 2 numbers which when multiplied give you the value of c and when added give you b.

Step 3 - Equate $(x-1)$ and $(x-3)$ to 0 to find the values of x.
Move the -1 and the -3 to the other side.

Step 4 - Write down the coordinates of b and c keeping in mind that c is ahead b therefore $c = (3, 0)$ and $b = (1, 0)$

Step 1 - Find the turning point of x.

Step 2 - Write the equation and place 2 where x is to find the value of y.

$$y = 4 - 8 + 3$$

$$y = -4 + 3$$

$$y = -1$$

Step 3 - Write the minimum value of y.

5. 2019 GCE P1

A function $y = (1 + x)(x - 2)$

- (i) Sketch the graph of the function in the answer space below
- (ii) Find the minimum value of y.

Step 1 - Write the equation and then find the equation in terms of a,b and c,

(i) $y = (1 + x)(x - 2)$

$$y = x + (-2) + x^2 - 2x$$

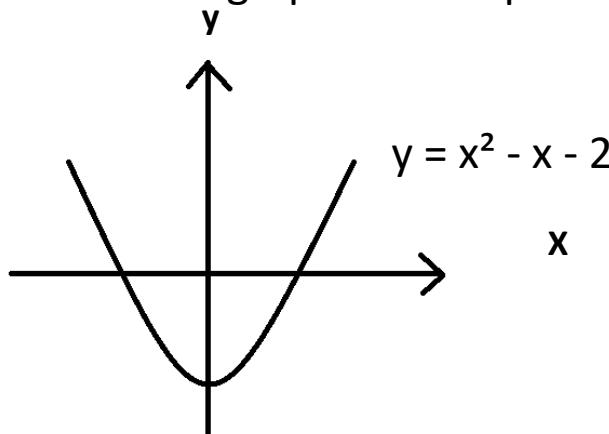
$$y = x - 2 + x^2 - 2x$$

$$y = x^2 - 2x - x - 2$$

$$y = x^2 - x - 2$$

Step 2 - Expand and simplify

$a > 0$ therefore graph is u shaped



- A sketch does not have any coordinates.
- The sketch should be u shaped because the value of a is greater than 0.
- If the value of a is less than zero then the graph would be n shaped.

(ii) $x = \frac{-b}{2a} = \frac{-(1)}{2 \times 1} = 0.5$

Step 1 - Find the turning point of x.

$$y = x^2 - x - 2$$

$$y = 0.5^2 - 0.5 - 2$$

$$y = 0.25 - 0.5 - 2$$

$$y = -2.25$$

$$y = -2$$

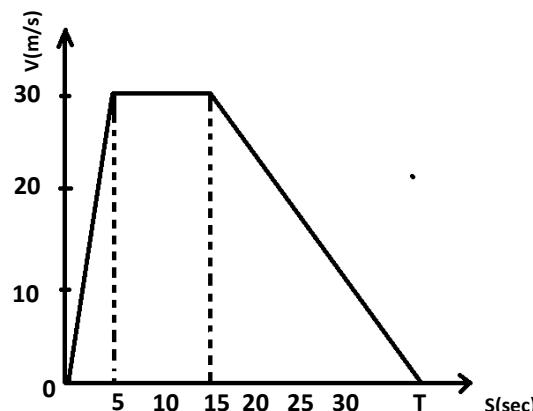
Step 2 - Write the equation and place 0.5 where x is to find the value of y.

Step 3 - Write the minimum value of y.

TOPIC 23 - TIME GRAPHS

1. 2017 P1

The diagram below shows a speed-time graph of a car journey.



- (a) Find the acceleration during the first 5 seconds
(b) If the total distance travelled was 825cm, find the value of T
(c) Find the average speed for the whole journey.

$$\begin{aligned} \text{(a)} \quad a &= \frac{v - u}{t} \\ &= \frac{30 - 0}{5} \\ &= \frac{30}{5} \\ &= 6 \text{m/s}^2 \end{aligned}$$

Step 1 - Write the formula to find acceleration and substitute the values.

$$\begin{aligned} \text{(b)} \quad A &= \frac{1}{2}(a + b)h \\ 825 &= \frac{1}{2}(10 + T)30 \end{aligned}$$

Step 1 - Use the formula to find area of a trapezium and substitute the values.

Step 2 - Divide 30 by 2 and then multiply 15 with 10 and T.

$$\begin{aligned} 825 &= 150 + 15T \\ 825 - 150 &= 15T \\ 675 &= 15T \\ \frac{675}{15} &= \frac{15T}{15} \end{aligned}$$

Step 3 - Group the like terms and subtract 150 from 825.

Step 3 - Divide both sides with 15 to find T.

$$T = 45 \text{ sec}$$

$$\begin{aligned}
 (c) \quad S &= \frac{D}{T} \\
 &= \frac{825}{45} \\
 &= \frac{165}{9} \\
 &= \frac{55}{3} \text{ m/s}
 \end{aligned}$$

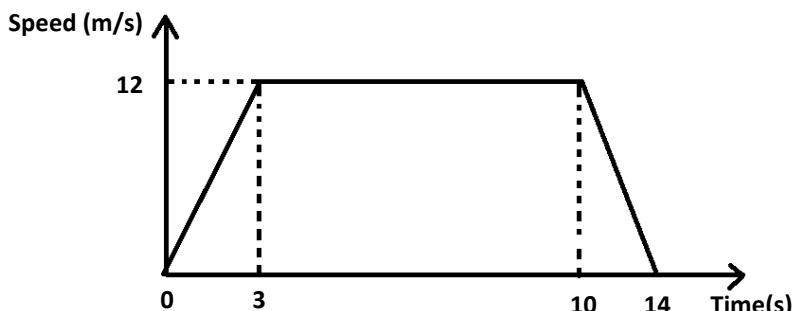
Step 1 - Write the formula to find speed and substitute the values.

Step 2 - divide 825 and 45 with 5.

Step 3 - divide 165 and 9 with 3.

2. 2017 GCE P1

The diagram below shows a speed-time graph of a 100m sprinter who accelerates uniformly for 3 seconds until he reaches a speed of 12m/s. He maintains the speed for 7 seconds and then uniformly retards for a further 4 seconds and comes to a stop.



Calculate the

- (a) acceleration during the first 3 seconds.
- (b) retardation at the end of his race.
- (c) distance he covered in the first 10 seconds.

$$\begin{aligned}
 (a) \quad a &= \frac{V - U}{t} \\
 &= \frac{12 - 0}{3} \\
 &= \frac{12}{3} \\
 &= 4 \text{ m/s}^2
 \end{aligned}$$

Step 1 - Write the formula to find acceleration and substitute the values.

Step 2 - divide 12 by 3

$$(b) -a = \frac{v - u}{t}$$

Step 1 - retardation is negative acceleration.

$$-a = \frac{0 - 12}{4}$$

Step 2 - Cross multiply.

$$-4a = -12$$

$$\frac{-4a}{-4} = \frac{-12}{-4}$$

Step 3 - divide both sides with -4.

$$\text{retardation} = 3 \text{ m/s}$$

$$(c) D = \frac{1}{2}(a + b)h$$

Step 1 - Use the formula to find area of a trapezium and substitute the values.

$$= \frac{1}{2}(7 + 10)12$$

Step 2 - Divide 12 by 2 and then add 7 with 10.

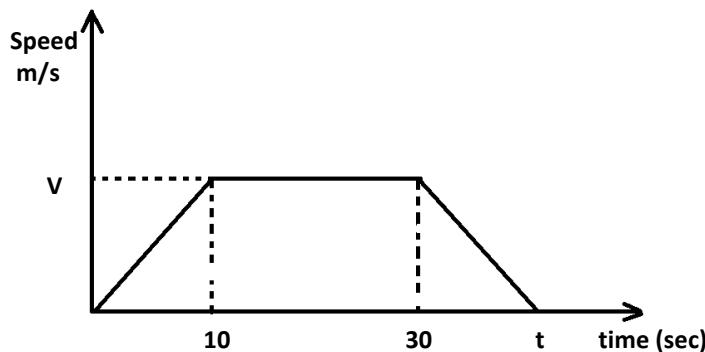
$$= 6(17)$$

Step 3 - Multiply 6 with 17.

$$= 102 \text{ m}$$

3. 2018 GCE P1

The diagram below shows the speed-time graph of a particle. The particle started off from rest and accelerated uniformly for 10 sec. It then travelled at a constant speed for 20 seconds and then decelerated to rest.



(a) Find the speed V the particle reached if its acceleration was 2 m/s^2 in the first 10 seconds.

(b) Given that the total distance covered was 750m, find the value of t in the diagram

(c) What was the speed at 40 seconds.

$$(a) \quad a = \frac{v - u}{t}$$

Step 1 - Write the formula to find acceleration and substitute the values.

$$2 = \frac{v - 0}{10}$$

Step 2 - Cross multiply

$$v = 2 \times 10$$

v = 20m/s

$$(b) \quad D = \frac{1}{2}(a + b)h$$

Step 1 - Use the formula to find area of a trapezium and substitute the values.

$$750 = \frac{1}{2}(20 + t)20$$

Step 2 - Divide 20 by 2.

$$750 = 10(20 + t)$$

Step 3 - Multiply 10 with 20 and t.

$$750 = 200 + 10t$$

Step 4 - Group the like terms and subtract 200 from 750

$$750 - 200 = 10t$$

$$550 = 10t$$

$$\frac{550}{10} = \frac{10t}{10}$$

Step 5 - Divide both sides with 10 to find t.

t = 55 sec

$$(c) \quad S = \frac{D}{T}$$

Step 1 - Write the formula to find speed and substitute the values.

$$= \frac{750}{40}$$

Step 2 - Cross out the zeros.

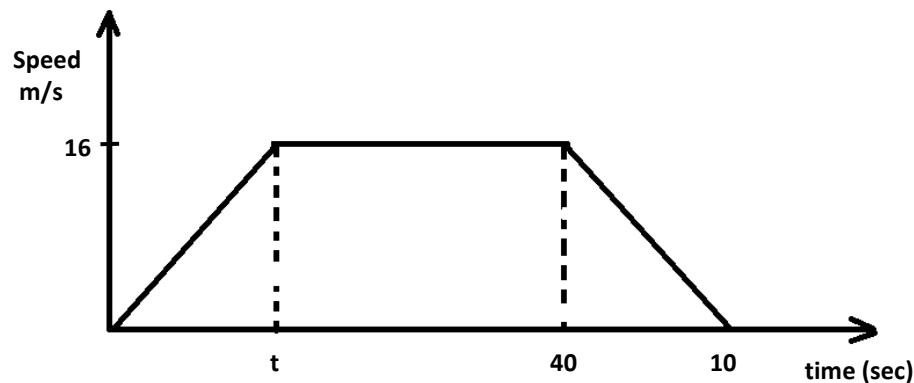
$$= \frac{75}{4}$$

Step 3 - Divide 75 by 4.

$$= 18.75 \text{ m/s}$$

4. 2019 P1

The diagram below is the speed time graph of a car. The car starts from rest and accelerates uniformly at 2 m/s^2 for t seconds until it reaches a speed of 16 m/s . It then travels at 16 m/s for 40 seconds, after which it comes to rest in a further 10 seconds



Find the

- (a) value of t ,
- (b) distance travelled in the last 50 seconds
- (c) speed of the car when $t = 53$ seconds

$$(a) a = \frac{V - U}{t}$$

Step 1 - Write the formula to find acceleration and substitute the values.

$$2 = \frac{16 - 0}{t}$$

Step 2 - Cross multiply.

$$2t = 16$$

Step 3 - Divide both sides with 2 to find t .

$$t = 8\text{s}$$

$$(b) D = \frac{1}{2}(a + b)h$$

Step 1 - Use the formula to find area of a trapezium and substitute the values.

$$= \frac{1}{2}(40 + 50)16$$

Step 2 - Divide 16 by 2 and add 40 and 50.

$$= 8(90)$$

Step 3 - Multiply 8 with 90.

$$= 720 \text{ m}$$

$$(c) S = \frac{D}{T}$$

Step 1 - Write the formula to find speed and then find the distance first.

$$D = \frac{1}{2}(a + b)h$$

Step 2 - Use the formula to find area of a trapezium and substitute the values.

$$\begin{aligned} &= \frac{1}{2}(40 + 58)16 \\ &= 8(98) \\ &= 784 \end{aligned}$$

Step 2 - Divide 16 by 2 and then add 40 and 58.

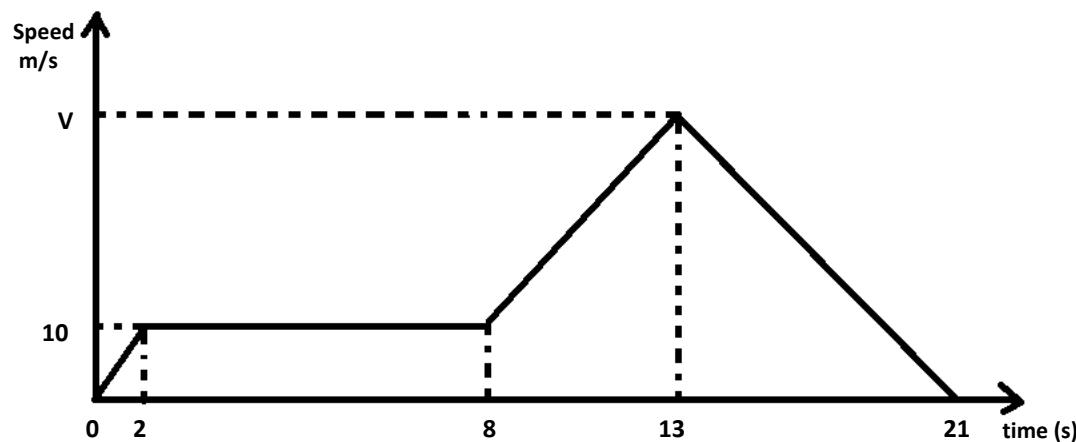
$$= \frac{784}{53} \text{ m/s}$$

Step 3 - Multiply 8 with 98.

Step 3 - Divide the distance by the time

5. 2019 GCE P1

The diagram below shows the speed-time graph of an object. It starts from rest and accelerates uniformly for 2 seconds until it reaches a speed of 10m/s. It moves at this constant speed for 6 seconds then accelerates until it reaches a speed of V m/s after 5 seconds. Finally it retards for the next 8 seconds until it comes to a halt.



Calculate the

- (a) acceleration during the first 2 seconds.
- (b) value of V if the retardation in the last 8 seconds is 3 m/s^2
- (c) average speed for the whole journey.

$$\begin{aligned}
 (a) \quad a &= \frac{v - u}{t} \\
 &= \frac{10 - 0}{2} \\
 &= \frac{10}{2} \\
 &= 5 \text{ m/s}^2
 \end{aligned}$$

Step 1 - Write the formula to find acceleration and substitute the values.

Step 2 - Divide 10 with 2.

$$\begin{aligned}
 (b) \quad a &= \frac{v - u}{t} \\
 -3 &= \frac{0 - v}{8} \\
 -v &= -24 \\
 &= \frac{-v}{-1} = \frac{-24}{-1}
 \end{aligned}$$

Step 1 - retardation is negative acceleration.

Step 2 - Cross multiply.

Step 3 - divide both sides with -1.

$$v = 24 \text{ m/s}$$

$$\begin{aligned}
 (c) \quad S &= \frac{D}{T} \\
 D_A &= \frac{1}{2}(a + b)h \\
 &= \frac{1}{2}(6 + 8)10 \\
 &= 5(14) \\
 &= 70 \text{ m} \\
 D_B &= \frac{1}{2}(a + b)h \\
 &= \frac{1}{2}(10 + 24)5 \\
 &= \frac{1}{2}(34)5 \\
 &= 85 \text{ m}
 \end{aligned}$$

Step 1 - Write the formula to find speed and then find the different distances.

Step 2 - Find distance 1 by using the formula to find the area of trapezium.

Step 3 - Divide 10 by 2 and then add 6 and 8.

Step 4 - Multiply 5 and 14.

Step 5 - Find distance 2 by using the formula to find the area of trapezium.

Step 6 - Add 10 and 24

Step 7 - Divide 34 with 2 and multiply 17 with 5.

$$D_c = \frac{1}{2} bh$$

$$= \frac{1}{2} \times 8 \times 24$$

$$= 96m$$

$$T.D = 96m + 85m + 70m = 251$$

$$S = \frac{251}{21}$$

$$= 11.95$$

Average speed= 12 m/s

Step 8 - Find distance 3 by using the formula to find the area of triangle.

Step 9 - Divide 8 with 2 and the multiply 4 with 24.

Step 10 - Add all the distances.

Step 11 - Divide the total distance over the total time.

Step 12 - Round it off to 12.



ALL THE BEST!!



Mr 6points



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