

EXAMINATIONS COUNCIL OF ZAMBIA

Examination for School Certificate Ordinary Level

Mathematics

4024/2

Paper 2

Thursday

2 NOVEMBER 2017

Additional materials:

Answer Booklet
Silent Electronic Calculator (non programmable)
Geometrical instruments
Graph paper (3 sheets)
Plain paper (1 sheet)

Time: 2 hours 30 minutes

Instructions to Candidates

Write your **name**, **centre number** and **candidate number** in the spaces provided on the Answer Booklet.

Write your answers and working in the **Answer Booklet provided**.

If you use more than one Answer Booklet, fasten the Answer Booklets together.

Omission of essential working will result in loss of marks.

There are **twelve (12)** questions in this paper.

Section A

Answer **all** questions.

Section B

Answer any **four** questions.

Silent non programmable Calculators may be used.

Cell phones are not allowed in the examination room.

Information for Candidates

The number of marks is given in brackets [] at the end of each question or part question.

The total marks for this paper is 100.

If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

Mathematical Formulae

1 ALGEBRA

Quadratic Equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 SERIES

Geometric Progression

$$S_n = \frac{a(1-r^n)}{1-r}, (r < 1)$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, (r > 1)$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

3 TRIGONOMETRY

Formula for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A.$$

$$\Delta = \frac{1}{2} bc \sin A$$

4 STATISTICS

Mean and standard deviation

Ungrouped data

$$\text{Mean } (\bar{x}) = \frac{\sum x}{n}, \text{SD} = \sqrt{\left\{ \frac{\sum (x - \bar{x})^2}{n} \right\}} = \sqrt{\left\{ \frac{\sum x^2}{n} - (\bar{x})^2 \right\}}$$

Grouped data

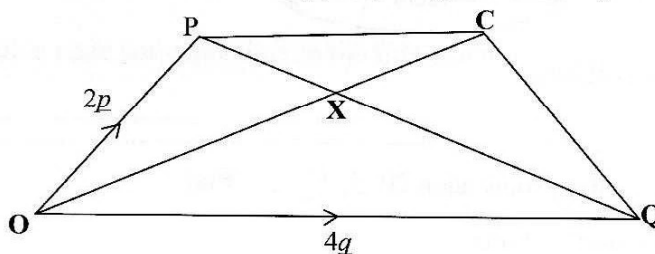
$$\text{Mean } (\bar{x}) = \frac{\sum fx}{\sum f}, \text{SD} = \sqrt{\left\{ \frac{\sum f(x - \bar{x})^2}{\sum f} \right\}} = \sqrt{\left\{ \frac{\sum fx^2}{\sum f} - (\bar{x})^2 \right\}}$$

Section A (52 Marks)

Answer all questions in this section

- 1 (a) Given that matrix $M = \begin{pmatrix} 3 & -2 \\ 5 & x \end{pmatrix}$,
- (i) find the value of x for which the determinant of M is 22, [2]
 - (ii) hence find the inverse of M . [2]
- (b) A survey carried out at Kamulima Farming Block showed that 44 farmers planted maize, 32 planted sweet potatoes, 37 planted cassava, 14 planted both maize and sweet potatoes, 24 planted both sweet potatoes and cassava, 20 planted both maize and cassava, 9 planted all the three crops and 6 did not plant any of these crops.
- (i) Illustrate this information on a Venn diagram. [2]
 - (ii) How many farmers
 - (a) were at this farming block, [1]
 - (b) planted maize only, [1]
 - (c) planted 2 different crops? [1]

- 2 (a) A box of chalk contains 5 white, 4 blue and 3 yellow pieces of chalk. A piece of chalk is selected at random from the box and not replaced. A second piece of chalk is then selected.
- (i) Draw a tree diagram to show all the possible outcomes. [2]
 - (ii) Find the probability of selecting pieces of chalk of the same colour. [3]
- (b) In the diagram below, $\vec{OP} = 2\mathbf{p}$, $\vec{OQ} = 4\mathbf{q}$ and $PX : XQ = 1 : 2$.

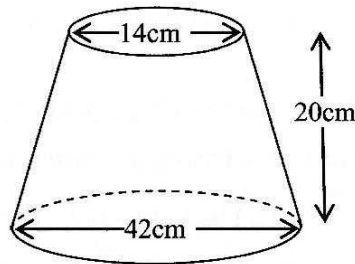


- (i) Express in terms of \mathbf{p} and/or \mathbf{q}
 - (a) \vec{PQ} , [1]
 - (b) \vec{PX} , [1]
 - (c) \vec{OX} . [1]
- (ii) Given that $\vec{OC} = h\vec{OX}$, show that

$$\vec{CQ} = 4\left(1 - \frac{h}{3}\right)\mathbf{q} - \frac{4h}{3}\mathbf{p}. \quad [2]$$

- 3 (a) Construct a quadrilateral ABCD in which $AB = 10\text{cm}$, $\angle ABC = 120^\circ$, $\angle BAD = 60^\circ$, $BC = 7\text{cm}$ and $AD = 11\text{cm}$. [1]
- (b) Measure and write the length of CD. [1]
- (c) Within the quadrilateral ABCD, draw the locus of points which are
- (i) 8cm from A, [1]
- (ii) equidistant from BC and CD. [2]
- (d) A point P, within the quadrilateral ABCD, is such that it is 8cm from A and equidistant from BC and CD. Label point P. [1]
- (e) Another point Q, within the quadrilateral ABCD, is such that it is nearer to CD than BC and greater than or equal to 8cm from A. Indicate, by shading, the region in which Q must lie. [2]
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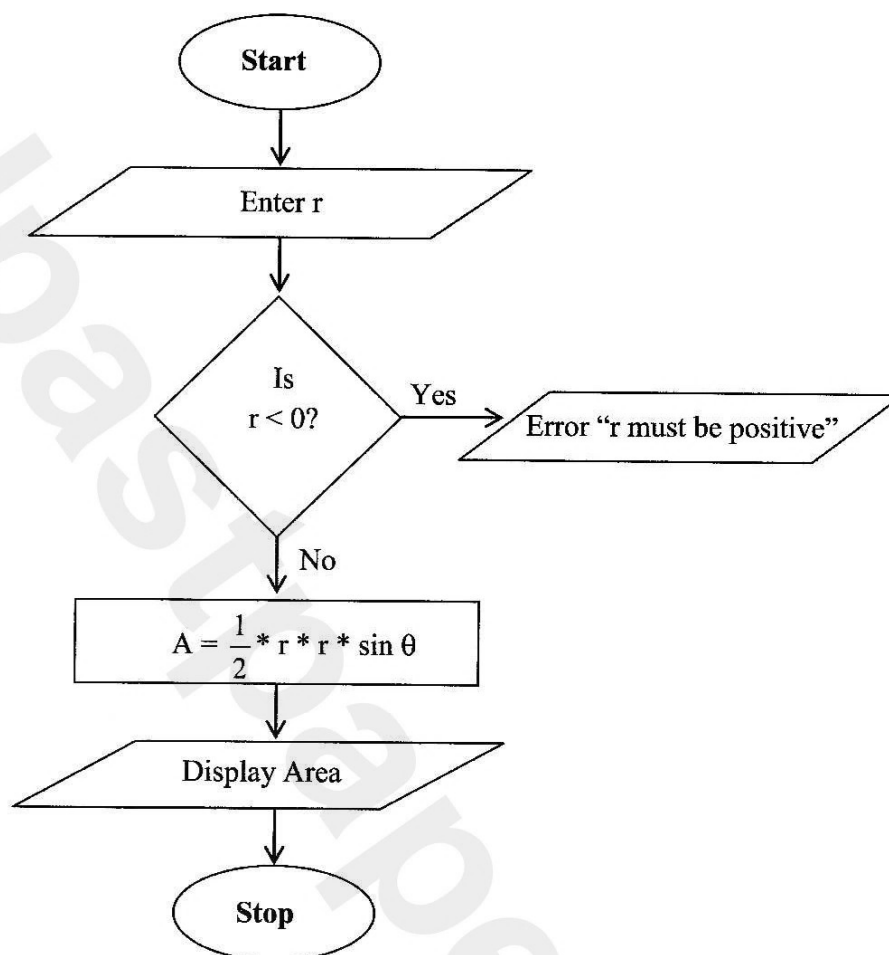
- 4 (a) Solve the equation $2x^2 = 6x + 3$, giving your answers correct to 2 decimal places. [5]
- (b) The figure below is a frustum of a cone. The base diameter and top diameter are 42cm and 14cm respectively, while the height is 20cm. (Take π as 3.142)



Calculate its volume. [6]

- 5 (a) For the geometric progression $20, 5, 1\frac{1}{4}, \dots$, find
- (i) the common ratio, [2]
- (ii) the n^{th} term, [2]
- (iii) the sum of the first 8 terms. [3]
- (b) Simplify $\frac{14x^3}{9y^2} \div \frac{7x^4}{18y^3}$. [2]
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6 Study the flow chart below.



Write a pseudo code corresponding to the flow chart program above.

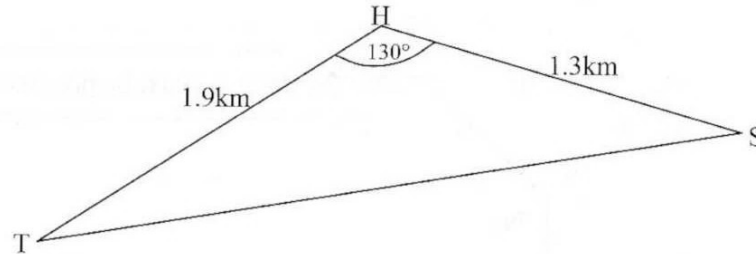
[5]

Section B [48 marks]

Answer any four questions in this section.

Each question in this section carries 12 marks.

- 7 (a) The diagram below shows the location of houses for a village Headman (H), his Secretary (S) and a Trustee (T). H is 1.3km from S, T is 1.9km from H and $\widehat{THS} = 130^\circ$.



Calculate

- (i) the area of triangle THS, [2]
 (ii) the distance TS, [5]
 (iii) the shortest distance from H to TS. [2]
- (b) Find the angle between 0° and 90° which satisfies the equation $\cos \theta = \frac{2}{3}$. [1]
- (c) Simplify $\frac{2x^2 - 8}{x + 2}$. [2]

- 8 The table below shows the amount of money spent by 100 learners at school on a particular day.

Amount in Kwacha	$0 < x \leq 5$	$5 < x \leq 10$	$10 < x \leq 15$	$15 < x \leq 20$	$20 < x \leq 25$	$25 < x \leq 30$
Frequency	13	27	35	16	7	2

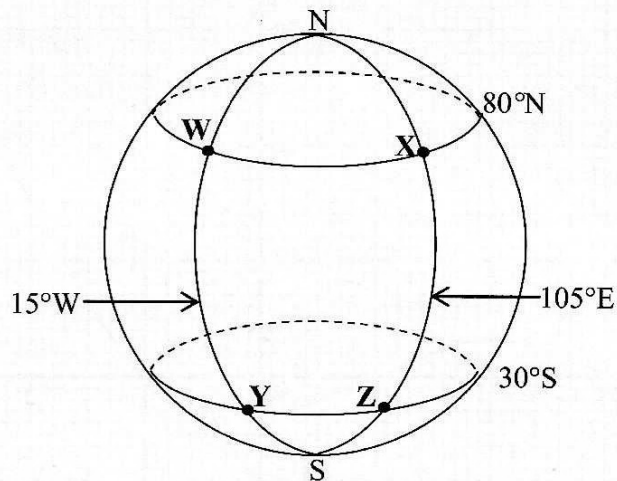
- (a) Calculate the standard deviation. [6]
- (b) Answer this part of the question on a sheet of graph paper.

- (i) Using the table above, copy and complete the cumulative frequency table below.

Amount in Kwacha	≤ 0	≤ 5	≤ 10	≤ 15	≤ 20	≤ 25	≤ 30
Frequency	0	13	40				100

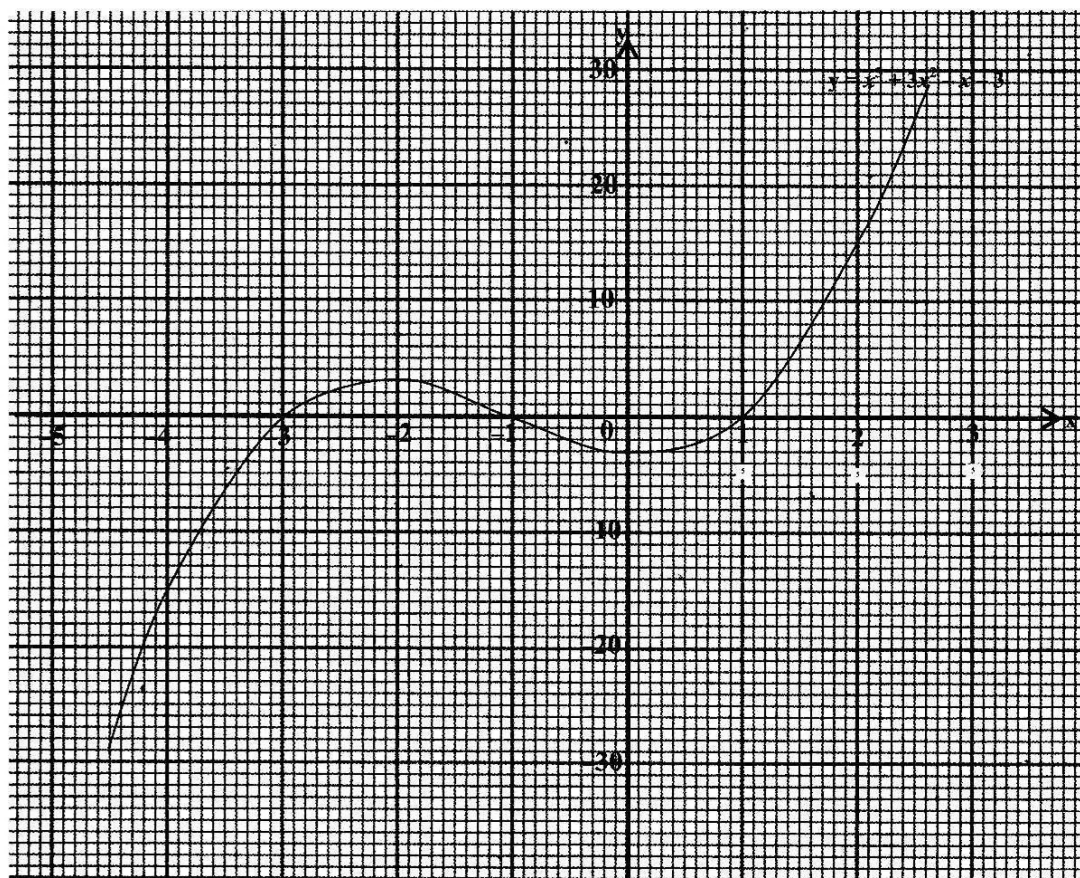
- (ii) Using a scale of 2cm to represent 5 units on the horizontal axis and 2cm to represent 10 units on the vertical axis, draw a smooth cumulative frequency curve. [1]
 [3]
- (iii) Showing your method clearly, use your graph to estimate the semi-interquartile range. [2]

- 9 (a) W, X, Y and Z are four points on the surface of the earth as shown in the diagram below. (Take π as 3.142 and $R = 3437\text{nm}$)



- (i) Calculate the difference in latitude between W and Y. [2]
- (ii) Calculate the distance in nautical miles between
 - (a) X and Z along the longitude 105°E , [2]
 - (b) Y and Z along the circle of latitude 30°S , [2]
- (b) Find the coordinates of the points on the curve $y = 2x^3 - 3x^2 - 36x - 3$ where the gradient is zero. [4]
- (c) Evaluate $\int_{-1}^3 (3x^2 - 2x) dx$. [2]

- 10 (a) The diagram below shows the graph of $y = x^3 + 3x^2 - x - 3$.



- (i) Use the graph to find the solutions of the equations
- (a) $x^3 + 3x^2 - x - 3 = 0$, [2]
- (b) $x^3 + 3x^2 - x = 5$. [2]
- (ii) Calculate an estimate of
- (a) the gradient of the curve at the point $(-3, 0)$, [2]
- (b) the area bounded by the curve, $x = 0$, $y = 0$ and $y = 20$. [3]
- (b) Express $\frac{1}{x-4} - \frac{2}{5x-1}$ as a single fraction in its lowest terms. [3]

11 Answer the whole of this question on a sheet of graph paper.

Himakwebo orders maize and groundnuts for sale. The order price of a bag of maize is K75.00 and that of a bag of groundnuts is K150.00. He is prepared to spend up to K7 500.00 altogether. He intends to order at least 5 bags of maize and at least 10 bags of groundnuts. He does not want to order more than 70 bags altogether.

- (a) If x and y are the number of bags of maize and groundnuts respectively, write four inequalities which represent these conditions. [4]
- (b) Using a scale of 2cm to represent 10 bags on each axis, draw the x and y axes for $0 \leq x \leq 70$ and $0 \leq y \leq 70$ respectively and shade the unwanted region to show clearly the region where the solution of the inequalities lie. [4]
- (c) Given that the profit on a bag of maize is K25.00 and on a bag of groundnuts is K50.00, how many bags of each type should he order to have maximum profit? [2]
- (d) What is this estimate of the maximum profit? [2]

12 Answer the whole of this question on a sheet of graph paper.

Using a scale of 1cm to represent 1 unit on each axis, draw x and y axes for $-6 \leq x \leq 10$ and $-10 \leq y \leq 8$.

- (a) A quadrilateral ABCD has vertices $A(-5, 7)$, $B(-4, 8)$, $C(-3, 7)$ and $D(-4, 4)$ while its image has vertices $A_1(-5, -3)$, $B_1(-6, -2)$, $C_1(-5, -1)$ and $D_1(-2, -2)$.
 - (i) Draw and label the quadrilateral ABCD and its image $A_1B_1C_1D_1$. [2]
 - (ii) Describe fully the transformation which maps quadrilateral ABCD onto quadrilateral $A_1B_1C_1D_1$. [3]
- (b) The matrix $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$ maps the quadrilateral ABCD onto the quadrilateral $A_2B_2C_2D_2$.
 - (i) Find the coordinates of the vertices of the quadrilateral $A_2B_2C_2D_2$. [2]
 - (ii) Draw and label the quadrilateral $A_2B_2C_2D_2$. [1]
- (c) The quadrilateral ABCD is mapped onto the quadrilateral $A_3B_3C_3D_3$ where A_3 is $(4, -8)$, B_3 is $(2, -10)$, C_3 is $(0, -8)$ and D_3 is $(2, -2)$. Describe fully this transformation. [4]