4.5 Problems

4.1. The ANOVA from a randomized complete block experiment output is shown below.

Source	DF	SS	MS	F	P
Treatment	4	1010.56	?	29.84	?
Block	?	?	64.765	?	?
Error	20	169.33	?		
Total	29	1503.71			

(a) Fill in the blanks. You may give bounds on the P-value.
(b) How many blocks were used in this experiment?
(c) What conclusions can you draw?

4.2. Consider the single-factor completely randomized single factor experiment shown in Problem 3.4. Suppose that this experiment had been conducted in a randomized complete block design, and that the sum of squares for blocks was 80.00. Modify the ANOVA for this experiment to show the correct analysis for the randomized complete block experiment.

4.3. A chemist wishes to test the effect of four chemical agents on the strength of a particular type of cloth. Because there might be variability from one bolt to another, the chemist decides to use a randomized block design, with the bolts of cloth considered as blocks. She selects five bolts and applies all four chemicals in random order to each bolt. The resulting tensile strengths follow. Analyze the data from this experiment (use $\alpha = 0.05$) and draw appropriate conclusions.

	Bolt				
Chemical	**1**	**2**	**3**	**4**	**5**
1	73	68	74	71	67
2	73	67	75	72	70
3	75	68	78	73	68
4	73	71	75	75	69

4.4. Three different washing solutions are being compared to study their effectiveness in retarding bacteria growth in 5-gallon milk containers. The analysis is done in a laboratory, and only three trials can be run on any day. Because days could represent a potential source of variability, the experimenter decides to use a randomized block design. Observations are taken for four days, and the data are shown here. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

	Days			
Solution	**1**	**2**	**3**	**4**
1	13	22	18	39
2	16	24	17	44
3	5	4	1	22

4.5. Plot the mean tensile strengths observed for each chemical type in Problem 4.3 and compare them to an appropriately scaled t distribution. What conclusions would you draw from this display?

4.6. Plot the average bacteria counts for each solution in Problem 4.4 and compare them to a scaled t distribution. What conclusions can you draw?

4.7. Consider the hardness testing experiment described in Section 4.1. Suppose that the experiment was conducted as described and that the following Rockwell C-scale data (coded by subtracting 40 units) obtained:

	Coupon			
Tip	**1**	**2**	**3**	**4**
1	9.3	9.4	9.6	10.0
2	9.4	9.3	9.8	9.9
3	9.2	9.4	9.5	9.7
4	9.7	9.6	10.0	10.2

(a) Analyze the data from this experiment.
(b) Use the Fisher LSD method to make comparisons among the four tips to determine specifically which tips differ in mean hardness readings.
(c) Analyze the residuals from this experiment.

4.8. A consumer products company relies on direct mail marketing pieces as a major component of its advertising campaigns. The company has three different designs for a new brochure and wants to evaluate their effectiveness, as there are substantial differences in costs between the three designs. The company decides to test the three designs by mailing 5000 samples of each to potential customers in four different regions of the country. Since there are known regional differences in the customer base, regions are considered as blocks. The number of responses to each mailing is as follows.

	Region			
Design	**NE**	**NW**	**SE**	**SW**
1	250	350	219	375
2	400	525	390	580
3	275	340	200	310

(a) Analyze the data from this experiment.
(b) Use the Fisher LSD method to make comparisons among the three designs to determine specifically which designs differ in the mean response rate.
(c) Analyze the residuals from this experiment.

4.9. The effect of three different lubricating oils on fuel economy in diesel truck engines is being studied. Fuel economy is measured using brake-specific fuel consumption after the engine has been running for 15 minutes. Five different truck engines are available for the study, and the experimenters conduct the following randomized complete block design.

	Truck				
Oil	**1**	**2**	**3**	**4**	**5**
1	0.500	0.634	0.487	0.329	0.512
2	0.535	0.675	0.520	0.435	0.540
3	0.513	0.595	0.488	0.400	0.510

(a) Analyze the data from this experiment.

(b) Use the Fisher LSD method to make comparisons among the three lubricating oils to determine specifically which oils differ in brake-specific fuel consumption.

(c) Analyze the residuals from this experiment.

4.10. An article in the *Fire Safety Journal* ("The Effect of Nozzle Design on the Stability and Performance of Turbulent Water Jets," Vol. 4, August 1981) describes an experiment in which a shape factor was determined for several different nozzle designs at six levels of jet efflux velocity. Interest focused on potential differences between nozzle designs, with velocity considered as a nuisance variable. The data are shown below:

Nozzle Design	Jet Efflux Velocity (m/s)					
	11.73	**14.37**	**16.59**	**20.43**	**23.46**	**28.74**
1	0.78	0.80	0.81	0.75	0.77	0.78
2	0.85	0.85	0.92	0.86	0.81	0.83
3	0.93	0.92	0.95	0.89	0.89	0.83
4	1.14	0.97	0.98	0.88	0.86	0.83
5	0.97	0.86	0.78	0.76	0.76	0.75

(a) Does nozzle design affect the shape factor? Compare the nozzles with a scatter plot and with an analysis of variance, using $\alpha = 0.05$.

(b) Analyze the residuals from this experiment.

(c) Which nozzle designs are different with respect to shape factor? Draw a graph of the average shape factor for each nozzle type and compare this to a scaled t distribution. Compare the conclusions that you draw from this plot to those from Duncan's multiple range test.

4.11. An article in *Communications of the ACM* (Vol. 30, No. 5, 1987) studied different algorithms for estimating software development costs. Six algorithms were applied to several different software development projects and the percent error in estimating the development cost was observed. Some of the data from this experiment is shown in the table below.

(a) Do the algorithms differ in their mean cost estimation accuracy?

(b) Analyze the residuals from this experiment.

(c) Which algorithm would you recommend for use in practice?

	Project					
Algorithm	**1**	**2**	**3**	**4**	**5**	**6**
1(SLIM	1244	21	82	2221	905	839
2(COCOMO-A)	281	129	396	1306	336	910
3(COCOMO-R)	220	84	458	543	300	794
4(COCONO-C)	225	83	425	552	291	826
5(FUNCTION POINTS)	19	11	−34	121	15	103
6(ESTIMALS)	−20	35	−53	170	104	199

4.12. An article in *Nature Genetics* (2003, Vol. 34, pp. 85–90) "Treatment-Specific Changes in Gene Expression Discriminate in vivo Drug Response in Human Leukemia Cells" studied gene expression as a function of different treatments for leukemia. Three treatment groups are: mercaptopurine (MP) only; low-dose methotrexate (LDMTX) and MP; and high-dose methotrexate (HDMTX) and MP. Each group contained ten subjects. The responses from a specific gene are shown in the table below.

(a) Is there evidence to support the claim that the treatment means differ?

(b) Check the normality assumption. Can we assume these samples are from normal populations?

(c) Take the logarithm of the raw data. Is there evidence to support the claim that the treatment means differ for the transformed data?

(d) Analyze the residuals from the transformed data and comment on model adequacy.

Treatments	Observations									
MP ONLY	334.5	31.6	701	41.2	61.2	69.6	67.5	66.6	120.7	881.9
MP + HDMTX	919.4	404.2	1024.8	54.1	62.8	671.6	882.1	354.2	321.9	91.1
MP + LDMTX	108.4	26.1	240.8	191.1	69.7	242.8	62.7	396.9	23.6	290.4

4.13. Consider the ratio control algorithm experiment described in Section 3.8. The experiment was actually conducted as a randomized block design, where six time periods were selected as the blocks, and all four ratio control algorithms were tested in each time period. The average cell voltage and the standard deviation of voltage (shown in parentheses) for each cell are as follows:

Ratio Control Algorithm	Time Period		
	1	2	3
1	4.93 (0.05)	4.86 (0.04)	4.75 (0.05)
2	4.85 (0.04)	4.91 (0.02)	4.79 (0.03)
3	4.83 (0.09)	4.88 (0.13)	4.90 (0.11)
4	4.89 (0.03)	4.77 (0.04)	4.94 (0.05)

Ratio Control Algorithm	Time Period		
	4	5	6
1	4.95 (0.06)	4.79 (0.03)	4.88 (0.05)
2	4.85 (0.05)	4.75 (0.03)	4.85 (0.02)
3	4.75 (0.15)	4.82 (0.08)	4.90 (0.12)
4	4.86 (0.05)	4.79 (0.03)	4.76 (0.02)

(a) Analyze the average cell voltage data. (Use $\alpha = 0.05$.) Does the choice of ratio control algorithm affect the average cell voltage?

(b) Perform an appropriate analysis on the standard deviation of voltage. (Recall that this is called "pot noise.") Does the choice of ratio control algorithm affect the pot noise?

(c) Conduct any residual analyses that seem appropriate.

(d) Which ratio control algorithm would you select if your objective is to reduce both the average cell voltage *and* the pot noise?

4.14. An aluminum master alloy manufacturer produces grain refiners in ingot form. The company produces the product in four furnaces. Each furnace is known to have its own unique operating characteristics, so any experiment run in the foundry that involves more than one furnace will consider furnaces as a nuisance variable. The process engineers suspect that stirring rate affects the grain size of the product. Each furnace can be run at four different stirring rates. A randomized block design is run for a particular refiner, and the resulting grain size data is as follows.

	Furnace			
Stirring Rate (rpm)	1	2	3	4
5	8	4	5	6
10	14	5	6	9
15	14	6	9	2
20	17	9	3	6

(a) Is there any evidence that stirring rate affects grain size?

(b) Graph the residuals from this experiment on a normal probability plot. Interpret this plot.

(c) Plot the residuals versus furnace and stirring rate. Does this plot convey any useful information?

(d) What should the process engineers recommend concerning the choice of stirring rate and furnace for this particular grain refiner if small grain size is desirable?

4.15. Analyze the data in Problem 4.4 using the general regression significance test.

4.16. Assuming that chemical types and bolts are fixed, estimate the model parameters τ_i and β_j in Problem 4.3.

4.17. Draw an operating characteristic curve for the design in Problem 4.4. Does the test seem to be sensitive to small differences in the treatment effects?

4.18. Suppose that the observation for chemical type 2 and bolt 3 is missing in Problem 4.3. Analyze the problem by estimating the missing value. Perform the exact analysis and compare the results.

4.19. Consider the hardness testing experiment in Problem 4.7. Suppose that the observation for tip 2 in coupon 3 is missing. Analyze the problem by estimating the missing value.

4.20. ***Two missing values in a randomized block.*** Suppose that in Problem 4.3 the observations for chemical type 2 and bolt 3 and chemical type 4 and bolt 4 are missing.

(a) Analyze the design by iteratively estimating the missing values, as described in Section 4.1.3.

(b) Differentiate SS_E with respect to the two missing values, equate the results to zero, and solve for estimates of the missing values. Analyze the design using these two estimates of the missing values.

(c) Derive general formulas for estimating two missing values when the observations are in *different* blocks.

(d) Derive general formulas for estimating two missing values when the observations are in the *same* block.

4.21. An industrial engineer is conducting an experiment on eye focus time. He is interested in the effect of the distance of the object from the eye on the focus time. Four different distances are of interest. He has five subjects available for the experiment. Because there may be differences among individuals, he decides to conduct the experiment in a randomized block design. The data obtained follow. Analyze the data from this experiment (use $\alpha = 0.05$) and draw appropriate conclusions.

	Subject				
Distance (ft)	1	2	3	4	5
4	10	6	6	6	6
6	7	6	6	1	6
8	5	3	3	2	5
10	6	4	4	2	3

4.22. The effect of five different ingredients (*A, B, C, D, E*) on the reaction time of a chemical process is being studied. Each batch of new material is only large enough to permit five runs to be made. Furthermore, each run requires approximately $1\frac{1}{2}$ hours, so only five runs can be made in one day. The experimenter decides to run the experiment as a Latin square so that day and batch effects may be systematically controlled. She obtains the data that follow. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

	Day				
Batch	**1**	**2**	**3**	**4**	**5**
1	$A = 8$	$B = 7$	$D = 1$	$C = 7$	$E = 3$
2	$C = 11$	$E = 2$	$A = 7$	$D = 3$	$B = 8$
3	$B = 4$	$A = 9$	$C = 10$	$E = 1$	$D = 5$
4	$D = 6$	$C = 8$	$E = 6$	$B = 6$	$A = 10$
5	$E = 4$	$D = 2$	$B = 3$	$A = 8$	$C = 8$

4.23. An industrial engineer is investigating the effect of four assembly methods (*A, B, C, D*) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below. Analyze the data from this experiment ($\alpha = 0.05$) and draw appropriate conclusions.

	Operator			
Order of Assembly	**1**	**2**	**3**	**4**
1	$C = 10$	$D = 14$	$A = 7$	$B = 8$
2	$B = 7$	$C = 18$	$D = 11$	$A = 8$
3	$A = 5$	$B = 10$	$C = 11$	$D = 9$
4	$D = 10$	$A = 10$	$B = 12$	$C = 14$

4.24. Consider the randomized complete block design in Problem 4.4. Assume that the days are random. Estimate the block variance component.

4.25. Consider the randomized complete block design in Problem 4.7. Assume that the coupons are random. Estimate the block variance component.

4.26. Consider the randomized complete block design in Problem 4.9. Assume that the trucks are random. Estimate the block variance component.

4.27. Consider the randomized complete block design in Problem 4.11. Assume that the software projects that were used as blocks are random. Estimate the block variance component.

4.28. Consider the gene expression experiment in Problem 4.12. Assume that the subjects used in this experiment are random. Estimate the block variance component.

4.29. Suppose that in Problem 4.20 the observation from batch 3 on day 4 is missing. Estimate the missing value and perform the analysis using the value.

4.30. Consider a $p \times p$ Latin square with rows (α_i), columns (β_k), and treatments (τ_j) fixed. Obtain least squares estimates of the model parameters α_i, β_k, and τ_j.

4.31. Derive the missing value formula (Equation 4.27) for the Latin square design.

4.32. ***Designs involving several Latin squares.*** [See Cochran and Cox (1957), John (1971).] The $p \times p$ Latin square contains only p observations for each treatment. To obtain more replications the experimenter may use several squares, say n. It is immaterial whether the squares used are the same or different. The appropriate model is

$$y_{ijkh} = \begin{array}{l} \mu + \rho_h + \alpha_{i(h)} \\ + \ \tau_j + \beta_{k(h)} \\ + \ (\tau\rho)_{jh} + \epsilon_{ijkh} \end{array} \begin{cases} i = 1, 2, \ldots, p \\ j = 1, 2, \ldots, p \\ k = 1, 2, \ldots, p \\ h = 1, 2, \ldots, n \end{cases}$$

where y_{ijkh} is the observation on treatment j in row i and column k of the hth square. Note that $\alpha_{i(h)}$ and $\beta_{k(h)}$ are the row and column effects in the hth square, ρ_h is the effect of the hth square, and $(\tau\rho)_{jh}$ is the interaction between treatments and squares.

(a) Set up the normal equations for this model, and solve for estimates of the model parameters. Assume that appropriate side conditions on the parameters are $\Sigma_h \hat{\rho}_h = 0$, $\Sigma_i \hat{\alpha}_{i(h)} = 0$, and $\Sigma_k \hat{\beta}_{k(h)} = 0$ for each h, $\Sigma_j \hat{\tau}_j = 0$, $\Sigma_j (\widehat{\tau\rho})_{jh} = 0$ for each h, and $\Sigma_h (\widehat{\tau\rho})_{jh} = 0$ for each j.

(b) Write down the analysis of variance table for this design.

4.33. Discuss how the operating characteristics curves in the Appendix may be used with the Latin square design.

4.34. Suppose that in Problem 4.22 the data taken on day 5 were incorrectly analyzed and had to be discarded. Develop an appropriate analysis for the remaining data.

4.35. The yield of a chemical process was measured using five batches of raw material, five acid concentrations, five standing times (*A, B, C, D, E*), and five catalyst concentrations ($\alpha, \beta, \gamma, \delta, \epsilon$). The Graeco-Latin square that follows was used. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Batch	Acid Concentration 1	2	3	4	5
1	$A\alpha = 26$	$B\beta = 16$	$C\gamma = 19$	$D\delta = 16$	$E\epsilon = 13$
2	$B\gamma = 18$	$C\delta = 21$	$D\epsilon = 18$	$E\alpha = 11$	$A\beta = 21$
3	$C\epsilon = 20$	$D\alpha = 12$	$E\beta = 16$	$A\gamma = 25$	$B\delta = 13$
4	$D\beta = 15$	$E\gamma = 15$	$A\delta = 22$	$B\epsilon = 14$	$C\alpha = 17$
5	$E\delta = 10$	$A\epsilon = 24$	$B\alpha = 17$	$C\beta = 17$	$D\gamma = 14$

4.36. Suppose that in Problem 4.23 the engineer suspects that the workplaces used by the four operators may represent an additional source of variation. A fourth factor, workplace (α, β, γ, δ) may be introduced and another experiment conducted, yielding the Graeco-Latin square that follows. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Order of Assembly	Operator 1	2	3	4
1	$C\beta = 11$	$B\gamma = 10$	$D\delta = 14$	$A\alpha = 8$
2	$B\alpha = 8$	$C\delta = 12$	$A\gamma = 10$	$D\beta = 12$
3	$A\delta = 9$	$D\alpha = 11$	$B\beta = 7$	$C\gamma = 15$
4	$D\gamma = 9$	$A\beta = 8$	$C\alpha = 18$	$B\delta = 6$

4.37. Construct a 5×5 hypersquare for studying the effects of five factors. Exhibit the analysis of variance table for this design.

4.38. Consider the data in Problems 4.23 and 4.36. Suppressing the Greek letters in problem 4.36, analyze the data using the method developed in Problem 4.32.

4.39. Consider the randomized block design with one missing value in Problem 4.19. Analyze this data by using the exact analysis of the missing value problem discussed in Section 4.1.4. Compare your results to the approximate analysis of these data given from Problem 4.19.

4.40. An engineer is studying the mileage performance characteristics of five types of gasoline additives. In the road test he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Additive	Car 1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	12		13	12	9
4	13	11	11	12	
5	11	12	10		8

4.41. Construct a set of orthogonal contrasts for the data in Problem 4.33. Compute the sum of squares for each contrast.

4.42. Seven different hardwood concentrations are being studied to determine their effect on the strength of the paper produced. However, the pilot plant can only produce three runs each day. As days may differ, the analyst uses the balanced incomplete block design that follows. Analyze the data from this experiment (use $\alpha = 0.05$) and draw conclusions.

Hardwood Concentration (%)	Days 1	2	3	4	5	6	7
2	114				120		117
4	126	120				119	
6		137	117				134
8	141		129	149			
10		145		150	143		
12			120		118	123	
14				136		130	127

4.43. Analyze the data in Example 4.5 using the general regression significance test.

4.44. Prove that $k\,(\sum_{i=1}^{a} Q_i^2/(\lambda a)$ is the adjusted sum of squares for treatments in a BIBD.

4.45. An experimenter wishes to compare four treatments in blocks of two runs. Find a BIBD for this experiment with six blocks.

4.46. An experimenter wishes to compare eight treatments in blocks of four runs. Find a BIBD with 14 blocks and $\lambda = 3$.

4.47. Perform the interblock analysis for the design in Problem 4.40.

4.48. Perform the interblock analysis for the design in Problem 4.42.

4.49. Verify that a BIBD with the parameters $a = 8$, $r = 8$, $k = 4$, and $b = 16$ does not exist.

4.50. Show that the variance of the intrablock estimators $\{\hat{\tau}_i\}$ is $k(a - 1)\sigma^2/(\lambda a^2)$.

4.51. ***Extended incomplete block designs.*** Occasionally, the block size obeys the relationship $a < k < 2a$. An extended incomplete block design consists of a single replicate of each treatment in each block along with an incomplete block design with $k^* = k - a$. In the balanced case, the incomplete block design will have parameters $k^* = k - a$, $r^* = r - b$, and λ^*. Write out the statistical analysis. (*Hint:* In the extended incomplete block design, we have $\lambda = 2r - b + \lambda^*$.)

4.52. Suppose that a single-factor experiment with five levels of the factor has been conducted. There are three replicates and the experiment has been conducted as a complete randomized design. If the experiment had been conducted in blocks, the pure error degrees of freedom would be reduced by (choose the correct answer):

(a) 3

(b) 5

(c) 2

(d) 4

(e) None of the above

4.53. Physics graduate student Laura Van Ertia has conducted a complete randomized design with a single factor, hoping to solve the mystery of the unified theory and complete her dissertation. The results of this experiment are summarized in the following ANOVA display:

Source	DF	SS	MS	F
Factor	-	-	14.18	-
Error	-	37.75	-	
Total	23	108.63		

Answer the following questions about this experiment.

(a) The sum of squares for the factor is ______.

(b). The number of degrees of freedom for the single factor in the experiment is______.

(c) The number of degrees of freedom for error is ______.

(d) The mean square for error is______.

(e) The value of the test statistic is______.

(f) If the significance level is 0.05, your conclusions are not to reject the null hypothesis. (Yes or No)

(g) An upper bound on the P-value for the test statistic is______.

(h) A lower bound on the P-value for the test statistic is______.

(i) Laura used ______ levels of the factor in this experiment.

(j) Laura replicated this experiment ______ times.

(k) Suppose that Laura had actually conducted this experiment as a randomized complete block design and the sum of squares for blocks was 12. Reconstruct the ANOVA display above to reflect this new situation. How much has blocking reduced the estimate of experimental error?

4.54. Consider the direct mail marketing experiment in Problem 4.8. Suppose that this experiment had been run as a complete randomized design, ignoring potential regional differences, but that exactly the same data was obtained. Reanalyze the experiment under this new assumption. What difference would ignoring blocking have on the results and conclusions?