■ **TABLE 6.28**

Regression Analysis for the Circuit Experiment (Interaction Term Only)

```
The regression equation is
V = 1.00 IR

Predictor          Coef         Std. Dev.        T          P
Noconstant
IR               1.00073        0.00550       181.81      0.000

S = 0.1255
Analysis of Variance

Source           DF      SS        MS          F          P
Regression        3    71.267    23.756    1085.95      0.000
Residual Error    4     0.088     0.022
Total             7    71.354
```

that the estimate of the interaction term regression coefficient is now different from what it was in the previous engineering-units analysis because the design in engineering units is not orthogonal. The coefficient is also virtually unity.

Generally, the engineering units are not directly comparable, but they may have physical meaning as in the present example. This could lead to possible simplification based on the underlying mechanism. In almost all situations, the coded unit analysis is preferable. It is fairly unusual for a simplification based on some underlying mechanism (as in our example) to occur. The fact that coded variables let an experimenter see the relative importance of the design factors is useful in practice.

6.10 Problems

6.1. An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results are as follows:

			Treatment	Replicate		
A	*B*	*C*	Combination	I	II	III
−	−	−	(1)	22	31	25
+	−	−	*a*	32	43	29
−	+	−	*b*	35	34	50
+	+	−	*ab*	55	47	46
−	−	+	*c*	44	45	38
+	−	+	*ac*	40	37	36
−	+	+	*bc*	60	50	54
+	+	+	*abc*	39	41	47

(a) Estimate the factor effects. Which effects appear to be large?

(b) Use the analysis of variance to confirm your conclusions for part (a).

(c) Write down a regression model for predicting tool life (in hours) based on the results of this experiment.

(d) Analyze the residuals. Are there any obvious problems?

(e) On the basis of an analysis of main effect and interaction plots, what coded factor levels of *A*, *B*, and *C* would you recommend using?

6.2. Reconsider part (c) of Problem 6.1. Use the regression model to generate response surface and contour plots of the tool life response. Interpret these plots. Do they provide insight regarding the desirable operating conditions for this process?

6.3. Find the standard error of the factor effects and approximate 95 percent confidence limits for the factor effects in Problem 6.1. Do the results of this analysis agree with the conclusions from the analysis of variance?

6.4. Plot the factor effects from Problem 6.1 on a graph relative to an appropriately scaled t distribution. Does this graphical display adequately identify the important factors? Compare the conclusions from this plot with the results from the analysis of variance.

6.5. A router is used to cut locating notches on a printed circuit board. The vibration level at the surface of the board as it is cut is considered to be a major source of dimensional variation in the notches. Two factors are thought to influence vibration: bit size (A) and cutting speed (B). Two bit sizes ($\frac{1}{16}$ and $\frac{1}{8}$ in.) and two speeds (40 and 90 rpm) are selected, and four boards are cut at each set of conditions shown below. The response variable is vibration measured as the resultant vector of three accelerometers (x, y, and z) on each test circuit board.

			Replicate			
A	B	Treatment Combination	I	II	III	IV
−	−	(1)	18.2	18.9	12.9	14.4
+	−	a	27.2	24.0	22.4	22.5
−	+	b	15.9	14.5	15.1	14.2
+	+	ab	41.0	43.9	36.3	39.9

(a) Analyze the data from this experiment.

(b) Construct a normal probability plot of the residuals, and plot the residuals versus the predicted vibration level. Interpret these plots.

(c) Draw the AB interaction plot. Interpret this plot. What levels of bit size and speed would you recommend for routine operation?

6.6. Reconsider the experiment described in Problem 6.1. Suppose that the experimenter only performed the eight trials from replicate I. In addition, he ran four center points and obtained the following response values: 36, 40, 43, 45.

(a) Estimate the factor effects. Which effects are large?

(b) Perform an analysis of variance, including a check for pure quadratic curvature. What are your conclusions?

(c) Write down an appropriate model for predicting tool life, based on the results of this experiment. Does this model differ in any substantial way from the model in Problem 6.1, part (c)?

(d) Analyze the residuals.

(e) What conclusions would you draw about the appropriate operating conditions for this process?

6.7. An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table:

	Replicate			Replicate	
Treatment Combination	I	II	Treatment Combination	I	II
(1)	90	93	d	98	95
a	74	78	ad	72	76
b	81	85	bd	87	83
ab	83	80	abd	85	86
c	77	78	cd	99	90
ac	81	80	acd	79	75
bc	88	82	bcd	87	84
abc	73	70	$abcd$	80	80

(a) Estimate the factor effects.

(b) Prepare an analysis of variance table and determine which factors are important in explaining yield.

(c) Write down a regression model for predicting yield, assuming that all four factors were varied over the range from −1 to +1 (in coded units).

(d) Plot the residuals versus the predicted yield and on a normal probability scale. Does the residual analysis appear satisfactory?

(e) Two three-factor interactions, ABC and ABD, apparently have large effects. Draw a cube plot in the factors A, B, and C with the average yields shown at each corner. Repeat using the factors A, B, and D. Do these two plots aid in data interpretation? Where would you recommend that the process be run with respect to the four variables?

6.8. A bacteriologist is interested in the effects of two different culture media and two different times on the growth of a particular virus. He or she performs six replicates of a 2^2 design, making the runs in random order. Analyze the bacterial growth data that follow and draw appropriate conclusions. Analyze the residuals and comment on the model's adequacy.

	Culture Medium			
Time (h)	1		2	
12	21	22	25	26
	23	28	24	25
	20	26	29	27
18	37	39	31	34
	38	38	29	33
	35	36	30	35

6.9. An industrial engineer employed by a beverage bottler is interested in the effects of two different types of 32-ounce bottles on the time to deliver 12-bottle cases of the product. The two bottle types are glass and plastic. Two workers are used to perform a task consisting of moving 40 cases of the product 50 feet on a standard type of hand truck and stacking the cases in a display. Four replicates of a 2^2 factorial design are performed, and the times observed are listed in the following table. Analyze the data and draw appropriate conclusions. Analyze the residuals and comment on the model's adequacy.

	Worker			
Bottle Type	1		2	
Glass	5.12	4.89	6.65	6.24
	4.98	5.00	5.49	5.55
Plastic	4.95	4.43	5.28	4.91
	4.27	4.25	4.75	4.71

6.10. In Problem 6.9, the engineer was also interested in potential fatigue differences resulting from the two types of bottles. As a measure of the amount of effort required, he measured the elevation of the heart rate (pulse) induced by the task. The results follow. Analyze the data and draw conclusions. Analyze the residuals and comment on the model's adequacy.

Bottle Type	Worker 1		Worker 2	
Glass	39	45	20	13
	58	35	16	11
Plastic	44	35	13	10
	42	21	16	15

6.11. Calculate approximate 95 percent confidence limits for the factor effects in Problem 6.10. Do the results of this analysis agree with the analysis of variance performed in Problem 6.10?

6.12. An article in the *AT&T Technical Journal* (March/April 1986, Vol. 65, pp. 39–50) describes the application of two-level factorial designs to integrated circuit manufacturing. A basic processing step is to grow an epitaxial layer on polished silicon wafers. The wafers mounted on a susceptor are positioned inside a bell jar, and chemical vapors are introduced. The susceptor is rotated, and heat is applied until the epitaxial layer is thick enough. An experiment was run using two factors: arsenic flow rate (A) and deposition time (B). Four replicates were run, and the epitaxial layer thickness was measured (μm). The data are shown in Table P6.1.

■ **TABLE P6.1**
The 2^2 Design for Problem 6.12

A	B	Replicate I	II	III	IV
−	−	14.037	16.165	13.972	13.907
+	−	13.880	13.860	14.032	13.914
−	+	14.821	14.757	14.843	14.878
+	+	14.888	14.921	14.415	14.932

Factor	Low (−)	High (+)
A	55%	59%
B	Short (10 min)	Long (15 min)

(a) Estimate the factor effects.

(b) Conduct an analysis of variance. Which factors are important?

(c) Write down a regression equation that could be used to predict epitaxial layer thickness over the region of arsenic flow rate and deposition time used in this experiment.

(d) Analyze the residuals. Are there any residuals that should cause concern?

(e) Discuss how you might deal with the potential outlier found in part (d).

6.13. ***Continuation of Problem 6.12.*** Use the regression model in part (c) of Problem 6.12 to generate a response surface contour plot for epitaxial layer thickness. Suppose it is critically important to obtain layer thickness of 14.5μm. What settings of arsenic flow rate and decomposition time would you recommend?

6.14. ***Continuation of Problem 6.13.*** How would your answer to Problem 6.13 change if arsenic flow rate was more difficult to control in the process than the deposition time?

6.15. A nickel–titanium alloy is used to make components for jet turbine aircraft engines. Cracking is a potentially serious problem in the final part because it can lead to nonrecoverable failure. A test is run at the parts producer to determine the effect of four factors on cracks. The four factors are pouring temperature (A), titanium content (B), heat treatment method (C), and amount of grain refiner used (D). Two replicates of a 2^4 design are run, and the length of crack (in mm $\times 10^{-2}$) induced in a sample coupon subjected to a standard test is measured. The data are shown in Table P6.2

■ **TABLE P6.2**
The Experiment for problem 6.15

A	B	C	D	Treatment Combination	Replicate I	II
−	−	−	−	(1)	7.037	6.376
+	−	−	−	*a*	14.707	15.219
−	+	−	−	*b*	11.635	12.089
+	+	−	−	*ab*	17.273	17.815
−	−	+	−	*c*	10.403	10.151
+	−	+	−	*ac*	4.368	4.098
−	+	+	−	*bc*	9.360	9.253
+	+	+	−	*abc*	13.440	12.923
−	−	−	+	*d*	8.561	8.951
+	−	−	+	*ad*	16.867	17.052
−	+	−	+	*bd*	13.876	13.658
+	+	−	+	*abd*	19.824	19.639
−	−	+	+	*cd*	11.846	12.337
+	−	+	+	*acd*	6.125	5.904
−	+	+	+	*bcd*	11.190	10.935
+	+	+	+	*abcd*	15.653	15.053

(a) Estimate the factor effects. Which factor effects appear to be large?

(b) Conduct an analysis of variance. Do any of the factors affect cracking? Use $\alpha = 0.05$.

(c) Write down a regression model that can be used to predict crack length as a function of the significant main effects and interactions you have identified in part (b).

(d) Analyze the residuals from this experiment.

(e) Is there an indication that any of the factors affect the variability in cracking?

(f) What recommendations would you make regarding process operations? Use interaction and/or main effect plots to assist in drawing conclusions.

6.16. ***Continuation of Problem 6.15.*** One of the variables in the experiment described in Problem 6.15, heat treatment method (C), is a categorical variable. Assume that the remaining factors are continuous.

(a) Write two regression models for predicting crack length, one for each level of the heat treatment method variable. What differences, if any, do you notice in these two equations?

(b) Generate appropriate response surface contour plots for the two regression models in part (a).

(c) What set of conditions would you recommend for the factors A, B, and D if you use heat treatment method $C = +$?

(d) Repeat part (c) assuming that you wish to use heat treatment method $C = -$.

6.17. An experimenter has run a single replicate of a 2^4 design. The following effect estimates have been calculated:

$A = 76.95$	$AB = -51.32$	$ABC = -2.82$
$B = -67.52$	$AC = 11.69$	$ABD = -6.50$
$C = -7.84$	$AD = 9.78$	$ACD = 10.20$
$D = -18.73$	$BC = 20.78$	$BCD = -7.98$
	$BD = 14.74$	$ABCD = -6.25$
	$CD = 1.27$	

(a) Construct a normal probability plot of these effects.

(b) Identify a tentative model, based on the plot of the effects in part (a).

6.18. The effect estimates from a 2^4 factorial design are as follows: ABCD = −1.5138, ABC = −1.2661, ABD = −0.9852, ACD = −0.7566, BCD = −0.4842, CD = −0.0795, BD = −0.0793, AD = 0.5988, BC = 0.9216, AC = 1.1616, AB = 1.3266, D = 4.6744, C = 5.1458, B = 8.2469, and A = 12.7151. Are you comfortable with the conclusions that all main effects are active?

6.19. The effect estimates from a 2^4 factorial experiment are listed here. Are any of the effects significant? ABCD = −2.5251, BCD = 4.4054, ACD = −0.4932, ABD = −5.0842, ABC = −5.7696, CD = 4.6707, BD = −4.6620, BC = −0.7982, AD = −1.6564, AC = 1.1109, AB = −10.5229, D = −6.0275, C = −8.2045, B = −6.5304, and A = −0.7914.

6.20. Consider a variation of the bottle filling experiment from Example 5.3. Suppose that only two levels of carbonation are used so that the experiment is a 2^3 factorial design with two replicates. The data are shown in Table P6.3.

TABLE P6.3
Fill Height Experiment from Problem 6.20

	Coded Factors			Fill Height Deviation			Factor Levels	
Run	A	B	C	Replicate 1	Replicate 2		Low (−1)	High (+1)
1	−	−	−	−3	−1	A (%)	10	12
2	+	−	−	0	1	B (psi)	25	30
3	−	+	−	−1	0	C (b/m)	200	250
4	+	+	−	2	3			
5	−	−	+	−1	0			
6	+	−	+	2	1			
7	−	+	+	1	1			
8	+	+	+	6	5			

(a) Analyze the data from this experiment. Which factors significantly affect fill height deviation?

(b) Analyze the residuals from this experiment. Are there any indications of model inadequacy?

(c) Obtain a model for predicting fill height deviation in terms of the important process variables. Use this model to construct contour plots to assist in interpreting the results of the experiment.

(d) In part (a), you probably noticed that there was an interaction term that was borderline significant. If you did not include the interaction term in your model, include it now and repeat the analysis. What difference did this make? If you elected to include the interaction term in part (a), remove it and repeat the analysis. What difference does the interaction term make?

6.21. I am always interested in improving my golf scores. Since a typical golfer uses the putter for about 35–45 percent of his or her strokes, it seems reasonable that improving one's putting is a logical and perhaps simple way to improve a golf score ("The man who can putt is a match for any man."—Willie Parks, 1864–1925, two time winner of the British Open). An experiment was conducted to study the effects of four factors on putting accuracy. The design factors are length of putt, type of putter, breaking putt versus straight putt, and level versus downhill putt. The response variable is distance from the ball to the center of the cup after the ball comes to rest. One golfer performs the experiment, a 2^4 factorial design with seven replicates was used, and all putts are made in random order. The results are shown in Table P6.4.

■ **TABLE P6.4**
The Putting Experiment from Problem 6.21

Design Factors				Distance from Cup (replicates)						
Length of putt (ft)	**Type of putter**	**Break of putt**	**Slope of putt**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
10	Mallet	Straight	Level	10.0	18.0	14.0	12.5	19.0	16.0	18.5
30	Mallet	Straight	Level	0.0	16.5	4.5	17.5	20.5	17.5	33.0
10	Cavity back	Straight	Level	4.0	6.0	1.0	14.5	12.0	14.0	5.0
30	Cavity back	Straight	Level	0.0	10.0	34.0	11.0	25.5	21.5	0.0
10	Mallet	Breaking	Level	0.0	0.0	18.5	19.5	16.0	15.0	11.0
30	Mallet	Breaking	Level	5.0	20.5	18.0	20.0	29.5	19.0	10.0
10	Cavity back	Breaking	Level	6.5	18.5	7.5	6.0	0.0	10.0	0.0
30	Cavity back	Breaking	Level	16.5	4.5	0.0	23.5	8.0	8.0	8.0
10	Mallet	Straight	Downhill	4.5	18.0	14.5	10.0	0.0	17.5	6.0
30	Mallet	Straight	Downhill	19.5	18.0	16.0	5.5	10.0	7.0	36.0
10	Cavity back	Straight	Downhill	15.0	16.0	8.5	0.0	0.5	9.0	3.0
30	Cavity back	Straight	Downhill	41.5	39.0	6.5	3.5	7.0	8.5	36.0
10	Mallet	Breaking	Downhill	8.0	4.5	6.5	10.0	13.0	41.0	14.0
30	Mallet	Breaking	Downhill	21.5	10.5	6.5	0.0	15.5	24.0	16.0
10	Cavity back	Breaking	Downhill	0.0	0.0	0.0	4.5	1.0	4.0	6.5
30	Cavity back	Breaking	Downhill	18.0	5.0	7.0	10.0	32.5	18.5	8.0

(a) Analyze the data from this experiment. Which factors significantly affect putting performance?

(b) Analyze the residuals from this experiment. Are there any indications of model inadequacy?

6.22. Semiconductor manufacturing processes have long and complex assembly flows, so matrix marks and automated 2d-matrix readers are used at several process steps throughout factories. Unreadable matrix marks negatively affect factory run rates because manual entry of part data is required before manufacturing can resume. A 2^4 factorial experiment was conducted to develop a 2d-matrix laser mark on a metal cover that protects a substrate-mounted die. The design factors are A = laser power (9 and 13 W), B = laser pulse frequency (4000 and 12,000 Hz), C = matrix cell size (0.07 and 0.12 in.), and D = writing speed (10 and 20 in./sec), and the response variable is the unused error correction (UEC). This is a measure of the unused portion of the redundant information embedded in the 2d-matrix. A UEC of 0 represents the lowest reading that still results in a decodable matrix, while a value of 1 is the highest reading. A DMX Verifier was used to measure UEC. The data from this experiment are shown in Table P6.5.

(a) Analyze the data from this experiment. Which factors significantly affect UEC?

(b) Analyze the residuals from this experiment. Are there any indications of model inadequacy?

6.23. Reconsider the experiment described in Problem 6.20. Suppose that four center points are available and that the UEC response at these four runs is 0.98, 0.95, 0.93, and 0.96, respectively. Reanalyze the experiment incorporating a test for curvature into the analysis. What conclusions can you draw? What recommendations would you make to the experimenters?

6.24. A company markets its products by direct mail. An experiment was conducted to study the effects of three factors on the customer response rate for a particular product. The three factors are A = type of mail used (3rd class, 1st class), B = type of descriptive brochure (color, black-and-white), and C = offered price (\$19.95, \$24.95). The mailings are made to two groups of 8000 randomly selected customers, with 1000 customers in each group receiving each treatment combination. Each group of customers is considered as a replicate. The response variable is the number of orders placed. The experimental data are shown in Table P6.6.

■ **TABLE P6.5**
The 2^4 Experiment for Problem 6.22

Standard Order	Run Order	Laser Power	Pulse Frequency	Cell Size	Writing Speed	UEC
8	1	1.00	1.00	1.00	−1.00	0.8
10	2	1.00	−1.00	−1.00	1.00	0.81
12	3	1.00	1.00	−1.00	1.00	0.79
9	4	−1.00	−1.00	−1.00	1.00	0.6
7	5	−1.00	1.00	1.00	−1.00	0.65
15	6	−1.00	1.00	1.00	1.00	0.55
2	7	1.00	−1.00	−1.00	−1.00	0.98
6	8	1.00	−1.00	1.00	−1.00	0.67
16	9	1.00	1.00	1.00	1.00	0.69
13	10	−1.00	−1.00	1.00	1.00	0.56
5	11	−1.00	−1.00	1.00	−1.00	0.63
14	12	1.00	−1.00	1.00	1.00	0.65
1	13	−1.00	−1.00	−1.00	−1.00	0.75
3	14	−1.00	1.00	−1.00	−1.00	0.72
4	15	1.00	1.00	−1.00	−1.00	0.98
11	16	−1.00	1.00	−1.00	1.00	0.63

■ **TABLE P6.6**
The Direct Mail Experiment from Problem 6.24

	Coded Factors			Number of Orders			Factor Levels	
Run	A	B	C	Replicate 1	Replicate 2		Low (−1)	High (+1)
1	−	−	−	50	54	A (class)	3rd	1st
2	+	−	−	44	42	B (type)	BW	Color
3	−	+	−	46	48	C ($)	$19.95	$24.95
4	+	+	−	42	43			
5	−	−	+	49	46			
6	+	−	+	48	45			
7	−	+	+	47	48			
8	+	+	+	56	54			

(a) Analyze the data from this experiment. Which factors significantly affect the customer response rate?

(b) Analyze the residuals from this experiment. Are there any indications of model inadequacy?

(c) What would you recommend to the company?

6.25. Consider the single replicate of the 2^4 design in Example 6.2. Suppose that we had arbitrarily decided to analyze the data assuming that all three- and four-factor interactions were negligible. Conduct this analysis and compare your results with those obtained in the example. Do you think that it is a good idea to arbitrarily assume interactions to be negligible even if they are relatively high-order ones?

6.26. An experiment was run in a semiconductor fabrication plant in an effort to increase yield. Five factors, each at two levels, were studied. The factors (and levels) were A = aperture setting (small, large), B = exposure time (20% below nominal, 20% above nominal), C = development time (30 and 45 s), D = mask dimension (small, large), and E = etch time (14.5 and 15.5 min). The unreplicated 2^5 design shown below was run.

(1) = 7	d = 8	e = 8	de = 6
a = 9	ad = 10	ae = 12	ade = 10
b = 34	bd = 32	be = 35	bde = 30
ab = 55	abd = 50	abe = 52	$abde$ = 53
c = 16	cd = 18	ce = 15	cde = 15
ac = 20	acd = 21	ace = 22	$acde$ = 20
bc = 40	bcd = 44	bce = 45	$bcde$ = 41
abc = 60	$abcd$ = 61	$abce$ = 65	$abcde$ = 63

(a) Construct a normal probability plot of the effect estimates. Which effects appear to be large?

(b) Conduct an analysis of variance to confirm your findings for part (a).

(c) Write down the regression model relating yield to the significant process variables.

(d) Plot the residuals on normal probability paper. Is the plot satisfactory?

(e) Plot the residuals versus the predicted yields and versus each of the five factors. Comment on the plots.

(f) Interpret any significant interactions.

(g) What are your recommendations regarding process operating conditions?

(h) Project the 2^5 design in this problem into a 2^k design in the important factors. Sketch the design and show the average and range of yields at each run. Does this sketch aid in interpreting the results of this experiment?

6.27. ***Continuation of Problem 6.26.*** Suppose that the experimenter had run four center points in addition to the 32 trials in the original experiment. The yields obtained at the center point runs were 68, 74, 76, and 70.

(a) Reanalyze the experiment, including a test for pure quadratic curvature.

(b) Discuss what your next step would be.

6.28. In a process development study on yield, four factors were studied, each at two levels: time (A), concentration (B), pressure (C), and temperature (D). A single replicate of a 2^4 design was run, and the resulting data are shown in Table P6.7.

■ **TABLE P6.7**
Process Development Experiment from Problem 6.28

Run Number	Actual Run Order	A	B	C	D	Yield (lbs)
1	5	−	−	−	−	12
2	9	+	−	−	−	18
3	8	−	+	−	−	13
4	13	+	+	−	−	16
5	3	−	−	+	−	17
6	7	+	−	+	−	15
7	14	−	+	+	−	20
8	1	+	+	+	−	15
9	6	−	−	−	+	10
10	11	+	−	−	+	25
11	2	−	+	−	+	13
12	15	+	+	−	+	24
13	4	−	−	+	+	19
14	16	+	−	+	+	21
15	10	−	+	+	+	17
16	12	+	+	+	+	23

Factor Levels	Low (−)	High (+)
A (h)	2.5	3
B (%)	14	18
C (psi)	60	80
D (°C)	225	250

(a) Construct a normal probability plot of the effect estimates. Which factors appear to have large effects?

(b) Conduct an analysis of variance using the normal probability plot in part (a) for guidance in forming an error term. What are your conclusions?

(c) Write down a regression model relating yield to the important process variables.

(d) Analyze the residuals from this experiment. Does your analysis indicate any potential problems?

(e) Can this design be collapsed into a 2^3 design with two replicates? If so, sketch the design with the average and range of yield shown at each point in the cube. Interpret the results.

6.29. ***Continuation of Problem 6.28.*** Use the regression model in part (c) of Problem 6.28 to generate a response surface contour plot of yield. Discuss the practical value of this response surface plot.

6.30. ***The scrumptious brownie experiment.*** The author is an engineer by training and a firm believer in learning by doing. I have taught experimental design for many years to a wide variety of audiences and have always assigned the planning, conduct, and analysis of an actual experiment to the class participants. The participants seem to enjoy this practical experience and always learn a great deal from it. This problem uses the results of an experiment performed by Gretchen Krueger at Arizona State University.

There are many different ways to bake brownies. The purpose of this experiment was to determine how the pan material, the brand of brownie mix, and the stirring method affect the scrumptiousness of brownies. The factor levels were

Factor	Low (−)	High (+)
A = pan material	Glass	Aluminum
B = stirring method	Spoon	Mixer
C = brand of mix	Expensive	Cheap

The response variable was scrumptiousness, a subjective measure derived from a questionnaire given to the subjects who sampled each batch of brownies. (The questionnaire dealt with such issues as taste, appearance, consistency, aroma, and so forth.) An eight-person test panel sampled each batch and filled out the questionnaire. The design matrix and the response data are as follows.

(a) Analyze the data from this experiment as if there were eight replicates of a 2^3 design. Comment on the results.

(b) Is the analysis in part (a) the correct approach? There are only eight batches; do we really have eight replicates of a 2^3 factorial design?

(c) Analyze the average and standard deviation of the scrumptiousness ratings. Comment on the results. Is this analysis more appropriate than the one in part (a)? Why or why not?

Brownie				Test Panel Results							
Batch	A	B	C	1	2	3	4	5	6	7	8
1	−	−	−	11	9	10	10	11	10	8	9
2	+	−	−	15	10	16	14	12	9	6	15
3	−	+	−	9	12	11	11	11	11	11	12
4	+	+	−	16	17	15	12	13	13	11	11
5	−	−	+	10	11	15	8	6	8	9	14
6	+	−	+	12	13	14	13	9	13	14	9
7	−	+	+	10	12	13	10	7	7	17	13
9	+	+	+	15	12	15	13	12	12	9	14

6.31. An experiment was conducted on a chemical process that produces a polymer. The four factors studied were temperature (A), catalyst concentration (B), time (C), and pressure (D). Two responses, molecular weight and viscosity, were observed. The design matrix and response data are shown in Table P6.8.

■ TABLE P6.8
The 2^4 Experiment for Problem 6.31

Run Number	Actual Run Order	A	B	C	D	Molecular Weight	Viscosity
1	18	−	−	−	−	2400	1400
2	9	+	−	−	−	2410	1500
3	13	−	+	−	−	2315	1520
4	8	+	+	−	−	2510	1630
5	3	−	−	+	−	2615	1380
6	11	+	−	+	−	2625	1525
7	14	−	+	+	−	2400	1500
8	17	+	+	+	−	2750	1620
9	6	−	−	−	+	2400	1400
10	7	+	−	−	+	2390	1525
11	2	−	+	−	+	2300	1500
12	10	+	+	−	+	2520	1500
13	4	−	−	+	+	2625	1420
14	19	+	−	+	+	2630	1490
15	15	−	+	+	+	2500	1500
16	20	+	+	+	+	2710	1600
17	1	0	0	0	0	2515	1500
18	5	0	0	0	0	2500	1460
19	16	0	0	0	0	2400	1525
20	12	0	0	0	0	2475	1500

Factor Levels	Low (−)	High (+)
A (°C)	100	120
B (%)	4	8
C (min)	20	30
D (psi)	60	75

(a) Consider only the molecular weight response. Plot the effect estimates on a normal probability scale. What effects appear important?

(b) Use an analysis of variance to confirm the results from part (a). Is there indication of curvature?

(c) Write down a regression model to predict molecular weight as a function of the important variables.

(d) Analyze the residuals and comment on model adequacy.

(e) Repeat parts (a)–(d) using the viscosity response.

6.32. ***Continuation of Problem 6.31.*** Use the regression models for molecular weight and viscosity to answer the following questions.

(a) Construct a response surface contour plot for molecular weight. In what direction would you adjust the process variables to increase molecular weight?

(b) Construct a response surface contour plot for viscosity. In what direction would you adjust the process variables to decrease viscosity?

(c) What operating conditions would you recommend if it was necessary to produce a product with molecular weight between 2400 and 2500 and the lowest possible viscosity?

6.33. Consider the single replicate of the 2^4 design in Example 6.2. Suppose that we ran five points at the center (0, 0, 0, 0) and observed the responses 93, 95, 91, 89, and 96. Test for curvature in this experiment. Interpret the results.

6.34. ***A missing value in a 2^k* factorial.** It is not unusual to find that one of the observations in a 2^k design is missing due to faulty measuring equipment, a spoiled test, or some other reason. If the design is replicated n times ($n > 1$), some of the techniques discussed in Chapter 5 can be employed. However, for an unreplicated factorial ($n = 1$) some other method must be used. One logical approach is to estimate the missing value with a number that makes the highest order interaction contrast zero. Apply this technique to the experiment in Example 6.2 assuming that run ab is missing. Compare the results with the results of Example 6.2.

6.35. An engineer has performed an experiment to study the effect of four factors on the surface roughness of a machined part. The factors (and their levels) are A = tool angle (12, 15°), B = cutting fluid viscosity (300, 400), C = feed rate (10 and 15 in./min), and D = cutting fluid cooler used (no, yes). The data from this experiment (with the factors coded to the usual −1, +1 levels) are shown in Table P6.9.

(a) Estimate the factor effects. Plot the effect estimates on a normal probability plot and select a tentative model.

(b) Fit the model identified in part (a) and analyze the residuals. Is there any indication of model inadequacy?

(c) Repeat the analysis from parts (a) and (b) using $1/y$ as the response variable. Is there an indication that the transformation has been useful?

(d) Fit a model in terms of the coded variables that can be used to predict the surface roughness. Convert this prediction equation into a model in the natural variables.

■ **TABLE P6.9**
The Surface Roughness Experiment from Problem 6.35

Run	A	B	C	D	Surface Roughness
1	−	−	−	−	0.00340
2	+	−	−	−	0.00362
3	−	+	−	−	0.00301
4	+	+	−	−	0.00182
5	−	−	+	−	0.00280
6	+	−	+	−	0.00290
7	−	+	+	−	0.00252
8	+	+	+	−	0.00160
9	−	−	−	+	0.00336
10	+	−	−	+	0.00344
11	−	+	−	+	0.00308
12	+	+	−	+	0.00184
13	−	−	+	+	0.00269
14	+	−	+	+	0.00284
15	−	+	+	+	0.00253
16	+	+	+	+	0.00163

6.36. Resistivity on a silicon wafer is influenced by several factors. The results of a 2^4 factorial experiment performed during a critical processing step is shown in Table P6.10.

■ **TABLE P6.10**
The Resistivity Experiment from Problem 6.36

Run	A	B	C	D	Resistivity
1	−	−	−	−	1.92
2	+	−	−	−	11.28
3	−	+	−	−	1.09
4	+	+	−	−	5.75
5	−	−	+	−	2.13
6	+	−	+	−	9.53
7	−	+	+	−	1.03
8	+	+	+	−	5.35
9	−	−	−	+	1.60
10	+	−	−	+	11.73
11	−	+	−	+	1.16
12	+	+	−	+	4.68
13	−	−	+	+	2.16
14	+	−	+	+	9.11
15	−	+	+	+	1.07
16	+	+	+	+	5.30

(a) Estimate the factor effects. Plot the effect estimates on a normal probability plot and select a tentative model.

(b) Fit the model identified in part (a) and analyze the residuals. Is there any indication of model inadequacy?

(c) Repeat the analysis from parts (a) and (b) using ln (y) as the response variable. Is there an indication that the transformation has been useful?

(d) Fit a model in terms of the coded variables that can be used to predict the resistivity.

6.37. ***Continuation of Problem 6.36.*** Suppose that the experimenter had also run four center points along with the 16 runs in Problem 6.36. The resistivity measurements at the center points are 8.15, 7.63, 8.95, and 6.48. Analyze the experiment again incorporating the center points. What conclusions can you draw now?

6.38. The book by Davies (*Design and Analysis of Industrial Experiments*) describes an experiment to study the yield of isatin. The factors studied and their levels are as follows:

Factor	Low (−)	High (+)
A: Acid strength (%)	87	93
B: Reaction time (min)	15	30
C: Amount of acid (mL)	35	45
D: Reaction temperature (°C)	60	70

The data from the 2^4 factorial is shown in Table P6.11.

(a) Fit a main-effects-only model to the data from this experiment. Are any of the main effects significant?

(b) Analyze the residuals. Are there any indications of model inadequacy or violation of the assumptions?

(c) Find an equation for predicting the yield of isatin over the design space. Express the equation in both coded and engineering units.

(d) Is there any indication that adding interactions to the model would improve the results that you have obtained?

■ TABLE P6.11
The 2^4 Factorial Experiment in Problem 6.38

A	B	C	D	Yield
−1	−1	−1	−1	6.08
1	−1	−1	−1	6.04
−1	1	−1	−1	6.53
1	1	−1	−1	6.43
−1	−1	1	−1	6.31
1	−1	1	−1	6.09
−1	1	1	−1	6.12
1	1	1	−1	6.36
−1	−1	−1	1	6.79
1	−1	−1	1	6.68
−1	1	−1	1	6.73
1	1	−1	1	6.08
−1	−1	1	1	6.77
1	−1	1	1	6.38
−1	1	1	1	6.49
1	1	1	1	6.23

6.39. An article in *Quality and Reliability Engineering International* (2010, Vol. 26, pp. 223–233) presents a 2^5 factorial design. The experiment is shown in Table P6.12.

■ TABLE P6.12
The 2^5 Design in Problem 6.39

A	B	C	D	E	y
−1.00	−1.00	−1.00	−1.00	−1.00	8.11
1.00	−1.00	−1.00	−1.00	−1.00	5.56
−1.00	1.00	−1.00	−1.00	−1.00	5.77
1.00	1.00	−1.00	−1.00	−1.00	5.82
−1.00	−1.00	1.00	−1.00	−1.00	9.17
1.00	−1.00	1.00	−1.00	−1.00	7.8
−1.00	1.00	1.00	−1.00	−1.00	3.23
1.00	1.00	1.00	−1.00	−1.00	5.69
−1.00	−1.00	−1.00	1.00	−1.00	8.82
1.00	−1.00	−1.00	1.00	−1.00	14.23
−1.00	1.00	−1.00	1.00	−1.00	9.2
1.00	1.00	−1.00	1.00	−1.00	8.94
−1.00	−1.00	1.00	1.00	−1.00	8.68
1.00	−1.00	1.00	1.00	−1.00	11.49
−1.00	1.00	1.00	1.00	−1.00	6.25
1.00	1.00	1.00	1.00	−1.00	9.12
−1.00	−1.00	−1.00	−1.00	1.00	7.93
1.00	−1.00	−1.00	−1.00	1.00	5
−1.00	1.00	−1.00	−1.00	1.00	7.47
1.00	1.00	−1.00	−1.00	1.00	12
−1.00	−1.00	1.00	−1.00	1.00	9.86
1.00	−1.00	1.00	−1.00	1.00	3.65
−1.00	1.00	1.00	−1.00	1.00	6.4
1.00	1.00	1.00	−1.00	1.00	11.61
−1.00	−1.00	−1.00	1.00	1.00	12.43
1.00	−1.00	−1.00	1.00	1.00	17.55
−1.00	1.00	−1.00	1.00	1.00	8.87
1.00	1.00	−1.00	1.00	1.00	25.38
−1.00	−1.00	1.00	1.00	1.00	13.06
1.00	−1.00	1.00	1.00	1.00	18.85
−1.00	1.00	1.00	1.00	1.00	11.78
1.00	1.00	1.00	1.00	1.00	26.05

(a) Analyze the data from this experiment. Identify the significant factors and interactions.

(b) Analyze the residuals from this experiment. Are there any indications of model inadequacy or violations of the assumptions?

(c) One of the factors from this experiment does not seem to be important. If you drop this factor, what type of design remains? Analyze the data using the full factorial model for only the four active factors. Compare your results with those obtained in part (a).

(d) Find settings of the active factors that maximize the predicted response.

6.40. A paper in the *Journal of Chemical Technology and Biotechnology* ("Response Surface Optimization of the Critical Media Components for the Production of Surfactin," 1997, Vol. 68, pp. 263–270) describes the use of a designed experiment to maximize surfactin production. A portion of the data from this experiment is shown in Table P6.13. Surfactin was assayed by an indirect method, which involves measurement of surface tensions of the diluted broth samples. Relative surfactin concentrations were determined by serially diluting the broth until the critical micelle concentration (CMC) was reached. The dilution at which the surface tension starts rising abruptly was denoted by CMC^{-1} and was considered proportional to the amount of surfactant present in the original sample.

■ **TABLE P6.13**
The Factorial Experiment in Problem 6.40

Run	Glucose ($g\ dm^{-3}$)	NH_4NO_3 ($g\ dm^{-3}$)	$FeSO_4$ ($g\ dm^{-3} \times 10^{-4}$)	$MnSO_4$ ($g\ dm^{-3} \times 10^{-2}$)	$\dot{y}$ $(CMC)^{-1}$
1	20.00	2.00	6.00	4.00	23
2	60.00	2.00	6.00	4.00	15
3	20.00	6.00	6.00	4.00	16
4	60.00	6.00	6.00	4.00	18
5	20.00	2.00	30.00	4.00	25
6	60.00	2.00	30.00	4.00	16
7	20.00	6.00	30.00	4.00	17
8	60.00	6.00	30.00	4.00	26
9	20.00	2.00	6.00	20.00	28
10	60.00	2.00	6.00	20.00	16
11	20.00	6.00	6.00	20.00	18
12	60.00	6.00	6.00	20.00	21
13	20.00	2.00	30.00	20.00	36
14	60.00	2.00	30.00	20.00	24
15	20.00	6.00	30.00	20.00	33
16	60.00	6.00	30.00	20.00	34

(a) Analyze the data from this experiment. Identify the significant factors and interactions.

(b) Analyze the residuals from this experiment. Are there any indications of model inadequacy or violations of the assumptions?

(c) What conditions would optimize the surfactin production?

6.41. **Continuation of Problem 6.40.** The experiment in Problem 6.40 actually included six center points. The responses at these conditions were 35, 35, 35, 36, 36, and 34. Is there any indication of curvature in the response function? Are additional experiments necessary? What would you recommend doing now?

6.42. An article in the *Journal of Hazardous Materials* ("Feasibility of Using Natural Fishbone Apatite as a Substitute for Hydroxyapatite in Remediating Aqueous Heavy Metals," Vol. 69, Issue 2, 1999, pp. 187–196) describes an experiment to study the suitability of fishbone, a natural, apatite rich substance, as a substitute for hydroxyapatite in the sequestering of aqueous divalent heavy metal ions. Direct comparison of hydroxyapatite and fishbone apatite was performed using a three-factor two-level full factorial design. Apatite (30 or 60 mg) was added to 100 mL deionized water and gently agitated overnight in a shaker. The pH was then adjusted to 5 or 7 using nitric acid. Sufficient concentration of lead nitrate solution was added to each flask to result in a final volume of 200 mL and a lead concentration of 0.483 or 2.41 mM, respectively. The experiment was a 2^3 replicated twice and it was performed for both fishbone and synthetic apatite. Results are shown in Table P6.14.

■ **TABLE P6.14**
The Experiment for Problem 6.42. For apatite, + is 60 mg and − is 30 mg per 200 mL metal solution. For initial pH, + is 7 and − is 4. For Pb + is 2.41 mM (500 ppm) and − is 0.483 mM (100 ppm)

			Fishbone		Hydroxyapatite	
Apatite	pH	Pb	Pb, mM	pH	Pb, mM	pH
+	+	+	1.82	5.22	0.11	3.49
+	+	+	1.81	5.12	0.12	3.46
+	+	−	0.01	6.84	0.00	5.84
+	+	−	0.00	6.61	0.00	5.90
+	−	+	1.11	3.35	0.80	2.70
+	−	+	1.04	3.34	0.76	2.74
+	−	−	0.00	5.77	0.03	3.36
+	−	−	0.01	6.25	0.05	3.24
−	+	+	2.11	5.29	1.03	3.22
−	+	+	2.18	5.06	1.05	3.22
−	+	−	0.03	5.93	0.00	5.53
−	+	−	0.05	6.02	0.00	5.43
−	−	+	1.70	3.39	1.34	2.82
−	−	+	1.69	3.34	1.26	2.79
−	−	−	0.05	4.50	0.06	3.28
−	−	−	0.05	4.74	0.07	3.28

(a) Analyze the lead response for fishbone apatite. What factors are important?

(b) Analyze the residuals from this response and comment on model adequacy.

(c) Analyze the pH response for fishbone apatite. What factors are important?

(d) Analyze the residuals from this response and comment on model adequacy.

(e) Analyze the lead response for hydroxyapatite apatite. What factors are important?

(f) Analyze the residuals from this response and comment on model adequacy.

(g) Analyze the pH response for hydroxyapatite apatite. What factors are important?

(h) Analyze the residuals from this response and comment on model adequacy.

(i) What differences do you see between fishbone and hydroxyapatite apatite? The authors of this paper concluded that that fishbone apatite was comparable to hydroxyapatite apatite. Because the fishbone apatite is cheaper, it was recommended for adoption. Do you agree with these conclusions?

6.43. Often the fitted regression model from a 2^k factorial design is used to make predictions at points of interest in the design space. Assume that the model contains all main effects and two-factor interactions.

(a) Find the variance of the predicted response $\hat{y}$ at a point $x_1, x_2, \ldots, x_k$ in the design space. *Hint:* Remember that the x's are coded variables and assume a 2^k design with an equal number of replicates n at each design point so that the variance of a regression coefficient $\hat{\beta}$ is $\sigma^2/(n2^k)$ and that the covariance between any pair of regression coefficients is zero.

(b) Use the result in part (a) to find an equation for a $100(1-\alpha)$ percent confidence interval on the true mean response at the point $x_1, x_2, \ldots, x_k$ in design space.

6.44. ***Hierarchical models.*** Several times we have used the hierarchy principle in selecting a model; that is, we have included nonsignificant lower order terms in a model because they were factors involved in significant higher order terms. Hierarchy is certainly not an absolute principle that must be followed in all cases. To illustrate, consider the model resulting from Problem 6.1, which required that a nonsignificant main effect be included to achieve hierarchy. Using the data from Problem 6.1.

(a) Fit both the hierarchical and the nonhierarchical models.

(b) Calculate the PRESS statistic, the adjusted R^2, and the mean square error for both models.

(c) Find a 95 percent confidence interval on the estimate of the mean response at a cube corner ($x_1 = x_2 = x_3 = \pm 1$). *Hint:* Use the results of Problem 6.36.

(d) Based on the analyses you have conducted, which model do you prefer?

6.45. Suppose that you want to run a 2^3 factorial design. The variance of an individual observation is expected to be about 4. Suppose that you want the length of a 95 percent confidence interval on any effect to be less than or equal to 1.5. How many replicates of the design do you need to run?