# Homework 1 Solutions

## 1) Montgomery 2.5

**2.5.** Consider the computer output shown below.

| One-Sample Z | | | | | |
|---|---|---|---|---|---|
| Test of mu = 30 vs not = 30 | | | | | |
| The assumed standard deviation = 1.2 | | | | | |
| N | Mean | SE Mean | 95% CI | Z | P |
| 16 | 31.2000 | 0.3000 | (30.6120, 31.7880) | ? | ? |

(a) Fill in the missing values in the output. What conclusion would you draw?

$Z = 4$ $\quad$ $P = 0.00006$; therefore, the mean is not equal to 30.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) Use the output and the normal table to find a 99 percent CI on the mean.

CI = 30.42725, 31.97275

(d) What is the *P*-value if the alternative hypothesis is $H_1$: $\mu > 30$

*P*-value = 0.00003

## 2) Montgomery 2.6

**2.6.** Suppose that we are testing $H_0$: $\mu_1 = \mu_2$ versus $H_1$: $\mu_1 = \mu_2$ with a sample size of $n_1 = n_2 = 12$. Both sample variances are unknown but assumed equal. Find bounds on the *P*-value for the following observed values of the test statistic:

(a) $t_0 = 2.30$ $\quad$ Table *P*-value = 0.02, 0.05 $\quad$ Computer *P*-value = 0.0313

(b) $t_0 = 3.41$ $\quad$ Table *P*-value = 0.002, 0.005 $\quad$ Computer *P*-value = 0.0025

(c) $t_0 = 1.95$ $\quad$ Table *P*-value = 0.1, 0.05 $\quad$ Computer *P*-value = 0.0640

(d) $t_0 = -2.45$ $\quad$ Table *P*-value = 0.05, 0.02 $\quad$ Computer *P*-value = 0.0227

Note that the degrees of freedom is (12 +12) – 2 = 22. This is a two-sided test

## 3) Montgomery 2.22

**2.22.** The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

| Hours | | | |
|---|---|---|---|
| 159 | 280 | 101 | 212 |
| 224 | 379 | 179 | 264 |
| 222 | 362 | 168 | 250 |
| 149 | 260 | 485 | 170 |

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

$$H_0\text{: } \mu = 225 \qquad H_1\text{: } \mu > 225$$

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

$$\bar{y} = 241.50$$
$$S^2 = 146202 / (16 - 1) = 9746.80$$

$$S = \sqrt{9746.8} = 98.73$$

$$t_o = \frac{\bar{y} - \mu_o}{\frac{S}{\sqrt{n}}} = \frac{241.50 - 225}{\frac{98.73}{\sqrt{16}}} = 0.67$$

since $t_{0.05,15} = 1.753$; do not reject $H_0$

(c) Find the $P$-value for this test. $P$=0.26

(d) Construct a 95 percent confidence interval on mean repair time.

The 95% confidence interval is $\bar{y} - t_{\alpha/2, n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2, n-1}\frac{S}{\sqrt{n}}$

$$241.50 - (2.131)\left(\frac{98.73}{\sqrt{16}}\right) \le \mu \le 241.50 + (2.131)\left(\frac{98.73}{\sqrt{16}}\right)$$

$$188.9 \le \mu \le 294.1$$

## 4) Montgomery 2.29

**2.29.** Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kÅ) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

| 95 °C | 100 °C |
|---|---|
| 11.176 | 5.623 |
| 7.089 | 6.748 |
| 8.097 | 7.461 |
| 11.739 | 7.015 |
| 11.291 | 8.133 |
| 10.759 | 7.418 |
| 6.467 | 3.772 |
| 8.315 | 8.963 |

(a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use $\alpha = 0.05$.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$
$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(8-1)(4.41) + (8-1)(2.54)}{8+8-2} = 3.48$$
$$S_p = 1.86$$
$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{9.37 - 6.89}{1.86\sqrt{\frac{1}{8} + \frac{1}{8}}} = 2.65$$
$$t_{0.05,14} = 1.761$$

Since $t_{0.05,14} = 1.761$, reject $H_0$. There appears to be a lower mean thickness at the higher temperature. This is also seen in the computer output.

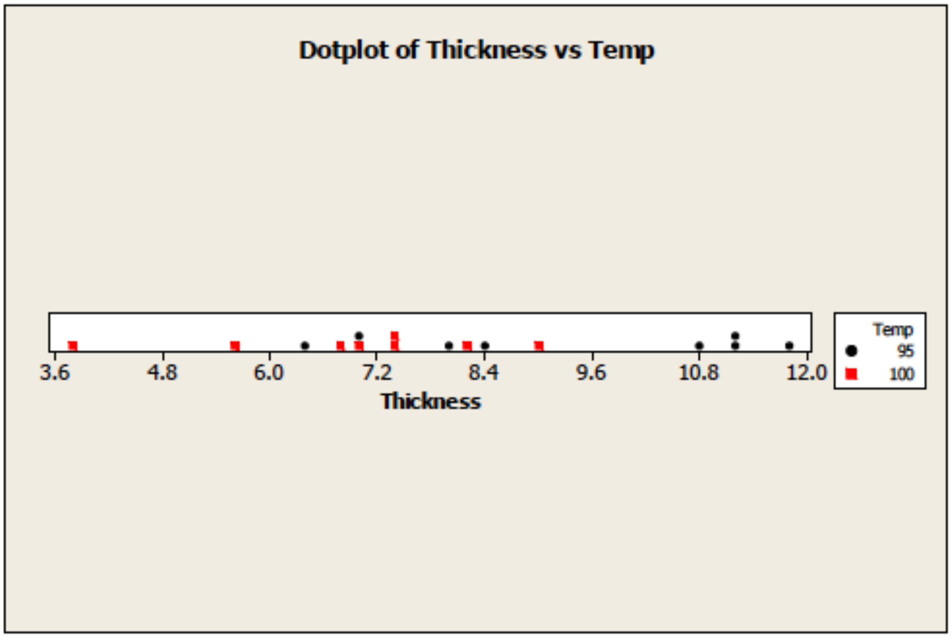
(b) What is the $P$-value for the test conducted in part (a)? $P = 0.009$

(c) Find a 95% CI on the difference in means. Provide a practical interpretation of this interval.

Two sided 95% CI for the difference in means ranges from 0.485 to 4.475. Since the lower confidence bound is greater than zero, the mean thicknesses cannot be the same for the different temperatures.

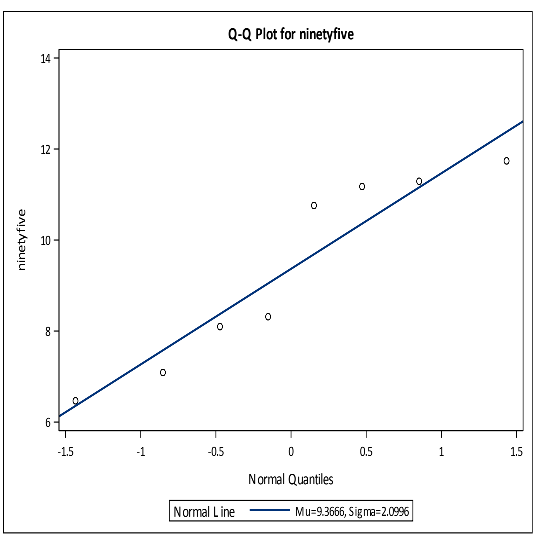
One-sided confidence interval is [0.8609, infinity]

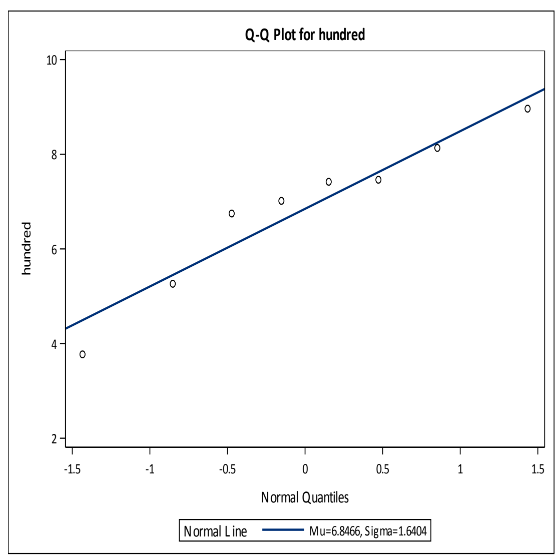
(d) Draw dot diagrams to assist in interpreting the results from this experiment.




(e) Check the assumption of normality of the photoresist thickness.

The QQ-plots for both the 95 and 100 thicknesses are shown below. No deviations from normality are evident.




From the Normal Probability Plots and Anderson-Darling Normality tests, the assumption of normal distribution appears to hold in both data sets.

(f) Find the power of this test for detecting an actual difference in means of 2.5 kÅ.

The power of this test for detecting a difference of 2.5 kA is 0.706.

Using pooled standard deviation as an estimate of sigma (=1.884), the power is 0.695:

The POWER Procedure
Two-Sample t Test for Mean Difference

| Fixed Scenario Elements | |
|---|---|
| Distribution | Normal |
| Method | Exact |
| Mean Difference | 2.5 |
| Standard Deviation | 1.884 |
| Sample Size per Group | 8 |
| Number of Sides | 2 |
| Null Difference | 0 |
| Alpha | 0.05 |

| Computed Power |
|---|
| Power |
| 0.695 |

(g) What sample size would be necessary to detect an actual difference in means of 1.5 kÅ with a power of at least 0.9?

Using pooled standard deviation as an estimate of sigma (=1.884), the number of subjects required in each group should be 35 which can give a power of at least 0.90.

The POWER Procedure
Two-Sample t Test for Mean Difference

| Fixed Scenario Elements | |
|---|---|
| Distribution | Normal |
| Method | Exact |
| Mean Difference | 1.5 |
| Standard Deviation | 1.884 |
| Nominal Power | 0.9 |
| Number of Sides | 2 |
| Null Difference | 0 |
| Alpha | 0.05 |

| Computed N per Group | |
|---|---|
| Actual Power | N per Group |
| 0.907 | 35 |