EXAMPLE 7.3 A 2^3 Design with Partial Confounding

Consider Example 6.1, in which an experiment was conducted to develop a plasma etching process. There were three factors, A = gap, B = gas flow, and C = RF power, and the response variable is the etch rate. Suppose that only four treatment combinations can be tested during a shift, and because there could be shift-to-shift differences in etching tool performance, the experimenters decide to use shifts as a blocking factor. Thus, each replicate of the 2^3 design must be run in two blocks. Two replicates are run, with ABC confounded in replicate I and AB confounded in replicate II. The data are as follows:

Replicate I, *ABC* Confounded

(1) = 550	a = 669
ab = 642	b = 633
ac = 749	c = 1037
bc = 1075	abc = 729

Replicate II, *AB* Confounded

(1) = 604	a = 650
c = 1052	b = 601
ab = 635	ac = 868
abc = 860	bc = 1063

The sums of squares for A, B, C, AC, and BC may be calculated in the usual manner, using all 16 observations. However, we must find SS_{ABC} using only the data in replicate II and SS_{AB} using only the data in replicate I as follows:

$$SS_{ABC} = \frac{[a + b + c + abc - ab - ac - bc - (1)]^2}{n2^k} = \frac{[650 + 601 + 1052 + 860 - 635 - 868 - 1063 - 604]^2}{(1)(8)} = 6.1250$$

$$SS_{AB} = \frac{[(1) + abc - ac + c - a - b + ab - bc]^2}{n2^k} = \frac{[550 + 729 - 749 + 1037 - 669 - 633 + 642 - 1075]^2}{(1)(8)} = 3528.0$$

The sum of squares for the replicates is, in general,

$$SS_{\text{Rep}} = \sum_{h=1}^{n} \frac{R_h^2}{2^k} - \frac{y_{\dots}^2}{N} = \frac{(6084)^2 + (6333)^2}{8} - \frac{(12{,}417)^2}{16} = 3875.0625$$

where R_h is the total of the observations in the hth replicate. The block sum of squares is the sum of SS_{ABC} from replicate I and SS_{AB} from replicate II, or $SS_{\text{Blocks}} = 458.1250$.

The analysis of variance is summarized in Table 7.11. The main effects of A and C and the AC interaction are important.

■ **TABLE 7.11**
Analysis of Variance for Example 7.3

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0	P-Value
Replicates	3875.0625	1	3875.0625	—	
Blocks within replicates	458.1250	2	229.0625	—	
A	41,310.5625	1	41,310.5625	16.20	0.01
B	217.5625	1	217.5625	0.08	0.78
C	374,850.5625	1	374,850.5625	146.97	<0.001
AB (rep. I only)	3528.0000	1	3528.0000	1.38	0.29
AC	94,404.5625	1	94,404.5625	37.01	<0.001
BC	18.0625	1	18.0625	0.007	0.94
ABC (rep. II only)	6.1250	1	6.1250	0.002	0.96
Error	12,752.3125	5	2550.4625		
Total	531,420.9375	15			

7.9 Problems

7.1. Consider the experiment described in Problem 6.1. Analyze this experiment assuming that each replicate represents a block of a single production shift.

7.2. Consider the experiment described in Problem 6.5. Analyze this experiment assuming that each one of the four replicates represents a block.

7.3. Consider the alloy cracking experiment described in Problem 6.15. Suppose that only 16 runs could be made on a single day, so each replicate was treated as a block. Analyze the experiment and draw conclusions.

7.4. Consider the data from the first replicate of Problem 6.1. Suppose that these observations could not all be run using the same bar stock. Set up a design to run these observations in two blocks of four observations each with ABC confounded. Analyze the data.

7.5. Consider the data from the first replicate of Problem 6.7. Construct a design with two blocks of eight observations each with $ABCD$ confounded. Analyze the data.

7.6. Repeat Problem 7.5 assuming that four blocks are required. Confound ABD and ABC (and consequently CD) with blocks.

7.7. Using the data from the 2^5 design in Problem 6.26, construct and analyze a design in two blocks with $ABCDE$ confounded with blocks.

7.8. Repeat Problem 7.7 assuming that four blocks are necessary. Suggest a reasonable confounding scheme.

7.9. Consider the data from the 2^5 design in Problem 6.26. Suppose that it was necessary to run this design in four blocks with $ACDE$ and BCD (and consequently ABE) confounded. Analyze the data from this design.

7.10. Consider the fill height deviation experiment in Problem 6.18. Suppose that each replicate was run on a separate day. Analyze the data assuming that days are blocks.

7.11. Consider the fill height deviation experiment in Problem 6.20. Suppose that only four runs could be made on each shift. Set up a design with ABC confounded in replicate 1 and AC confounded in replicate 2. Analyze the data and comment on your findings.

7.12. Consider the potting experiment in Problem 6.21. Analyze the data considering each replicate as a block.

7.13. Using the data from the 2^4 design in Problem 6.22, construct and analyze a design in two blocks with $ABCD$ confounded with blocks.

7.14. Consider the direct mail experiment in Problem 6.24. Suppose that each group of customers is in a different part of the country. Suggest an appropriate analysis for the experiment.

7.15. Consider the isatin yield experiment in Problem 6.38. Set up the 2^4 experiment in this problem in two blocks with ABCD confounded. Analyze the data from this design. Is the block effect large?

7.16. The experiment in Problem 6.39 is a 2^5 factorial. Suppose that this design had been run in four blocks of eight runs each.

(a) Recommend a blocking scheme and set up the design.

(b) Analyze the data from this blocked design. Is blocking important?

7.17. Repeat Problem 6.16 using a design in two blocks.

7.18. The design in Problem 6.40 is a 2^4 factorial. Set up this experiment in two blocks with ABCD confounded. Analyze the data from this design. Is the block effect large?

7.19. The design in Problem 6.42 is a 2^3 factorial replicated twice. Suppose that each replicate was a block. Analyze all of the responses from this blocked design. Are the results comparable to those from Problem 6.42? Is the block effect large?

7.20. Design an experiment for confounding a 2^6 factorial in four blocks. Suggest an appropriate confounding scheme, different from the one shown in Table 7.8.

7.21. Consider the 2^6 design in eight blocks of eight runs each with $ABCD$, ACE, and $ABEF$ as the independent effects chosen to be confounded with blocks. Generate the design. Find the other effects confounded with blocks.

7.22. Consider the 2^2 design in two blocks with AB confounded. Prove algebraically that $SS_{AB} = SS_{\text{Blocks}}$.

7.23. Consider the data in Example 7.2. Suppose that all the observations in block 2 are increased by 20. Analyze the data that would result. Estimate the block effect. Can you explain its magnitude? Do blocks now appear to be an important factor? Are any other effect estimates impacted by the change you made to the data?

7.24. Suppose that in Problem 6.1 we had confounded ABC in replicate I, AB in replicate II, and BC in replicate III. Calculate the factor effect estimates. Construct the analysis of variance table.

7.25. Repeat the analysis of Problem 6.1 assuming that ABC was confounded with blocks in each replicate.

7.26. Suppose that in Problem 6.7 $ABCD$ was confounded in replicate I and ABC was confounded in replicate II. Perform the statistical analysis of this design.

7.27. Construct a 2^3 design with ABC confounded in the first two replicates and BC confounded in the third. Outline the analysis of variance and comment on the information obtained.