The $100(1-\alpha)$ confidence interval for the ratio of the population variances σ_1^2/σ_2^2 is

$$\frac{S_1^2}{S_2^2} F_{1-\alpha/2, n_2-1, n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{S_1^2}{S_2^2} F_{\alpha/2, n_2-1, n_1-1} \tag{2.50}$$

To illustrate the use of Equation 2.50, the 95 percent confidence interval for the ratio of variances σ_1^2/σ_2^2 in Example 2.2 is, using $F_{0.025,9,11} = 3.59$ and $F_{0.975,9,11} = 1/F_{0.025,11,9} = 1/3.92 = 0.255$,

$$\frac{14.5}{10.8}(0.255) \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{14.5}{10.8}(3.59)$$

$$0.34 \le \frac{\sigma_1^2}{\sigma_2^2} \le 4.82$$

2.7 Problems

2.1. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Variance	Minimum	Maximum
Y	9	19.96	?	3.12	?	15.94	27.16

2.2. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Sum
Y	16	?	0.159	?	399.851

2.3. Suppose that we are testing $H_0: \mu = \mu_0$ versus $H_1: \mu \neq \mu_0$. Calculate the P-value for the following observed values of the test statistic:

(a) $Z_0 = 2.25$ **(b)** $Z_0 = 1.55$ **(c)** $Z_0 = 2.10$
(d) $Z_0 = 1.95$ **(e)** $Z_0 = -0.10$

2.4. Suppose that we are testing $H_0: \mu = \mu_0$ versus $H_1: \mu > \mu_0$. Calculate the P-value for the following observed values of the test statistic:

(a) $Z_0 = 2.45$ **(b)** $Z_0 = -1.53$ **(c)** $Z_0 = 2.15$
(d) $Z_0 = 1.95$ **(e)** $Z_0 = -0.25$

2.5. Consider the computer output shown below.

```
One-Sample Z
Test of mu = 30 vs not = 30
The assumed standard deviation = 1.2
N    Mean     SE Mean    95% CI               Z   P
16   31.2000  0.3000     (30.6120, 31.7880)   ?   ?
```

(a) Fill in the missing values in the output. What conclusion would you draw?

(b) Is this a one-sided or two-sided test?

(c) Use the output and the normal table to find a 99 percent CI on the mean.

(d) What is the P-value if the alternative hypothesis is $H_1: \mu > 30$?

2.6. Suppose that we are testing $H_0: \mu_1 = \mu_2$ versus $H_0: \mu_1 \neq \mu_2$ where the two sample sizes are $n_1 = n_2 = 12$. Both sample variances are unknown but assumed equal. Find bounds on the P-value for the following observed values of the test statistic.

(a) $t_0 = 2.30$ **(b)** $t_0 = 3.41$ **(c)** $t_0 = 1.95$ **(d)** $t_0 = -2.45$

2.7. Suppose that we are testing $H_0: \mu_1 = \mu_2$ versus $H_0: \mu_1 > \mu_2$ where the two sample sizes are $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the P-value for the following observed values of the test statistic.

(a) $t_0 = 2.31$ **(b)** $t_0 = 3.60$ **(c)** $t_0 = 1.95$ **(d)** $t_0 = 2.19$

2.8. Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is a sample from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?

2.9. A computer program has produced the following output for a hypothesis-testing problem:

```
Difference in sample means:  2.35
Degrees of freedom: 18
Standard error of the difference in sample means: ?
Test statistic:  t0 = 2.01
P-value: 0.0298
```

(a) What is the missing value for the standard error?

(b) Is this a two-sided or a one-sided test?

(c) If $\alpha = 0.05$, what are your conclusions?

(d) Find a 90% two-sided CI on the difference in means.

2.10. A computer program has produced the following output for a hypothesis-testing problem:

```
Difference in sample means:  11.5
Degrees of freedom: 24
Standard error of the difference in sample means: ?
Test statistic:   t0 = -1.88
P-value: 0.0723
```

(a) What is the missing value for the standard error?

(b) Is this a two-sided or a one-sided test?

(c) If $\alpha = 0.05$, what are your conclusions?

(d) Find a 95% two-sided CI on the difference in means.

2.11. Suppose that we are testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu > \mu_0$ with a sample size of $n = 15$. Calculate bounds on the P-value for the following observed values of the test statistic:

(a) $t_0 = 2.35$ **(b)** $t_0 = 3.55$ **(c)** $t_0 = 2.00$ **(d)** $t_0 = 1.55$

2.12. Suppose that we are testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$ with a sample size of $n = 10$. Calculate bounds on the P-value for the following observed values of the test statistic:

(a) $t_0 = 2.48$ **(b)** $t_0 = -3.95$ **(c)** $t_0 = 2.69$
(d) $t_0 = 1.88$ **(e)** $t_0 = -1.25$

2.13. Consider the computer output shown below.

```
One-Sample T: Y

Test of mu = 91 vs. not = 91

Variable N  Mean     Std. Dev. SE Mean    95% CI          T    P
   Y     25 92.5805     ?      0.4673  (91.6160, ?) 3.38 0.002
```

(a) Fill in the missing values in the output. Can the null hypothesis be rejected at the 0.05 level? Why?

(b) Is this a one-sided or a two-sided test?

(c) If the hypotheses had been $H_0 : \mu = 90$ versus $H_1: \mu \neq 90$ would you reject the null hypothesis at the 0.05 level?

(d) Use the output and the t table to find a 99 percent two-sided CI on the mean.

(e) What is the P-value if the alternative hypothesis is $H_1: \mu > 91$?

2.14. Consider the computer output shown below.

```
One-Sample T: Y

Test of mu = 25 vs > 25

                                             95% Lower
Variable    N   Mean      Std. Dev. SE Mean   Bound    T    P
Y          12  25.6818       ?      0.3360      ?      ?  0.034
```

(a) How many degrees of freedom are there on the t-test statistic?

(b) Fill in the missing information.

2.15. Consider the computer output shown below.

```
Two-Sample T-Test and CI: Y1, Y2

Two-sample T for Y1 vs Y2

      N    Mean     Std. Dev.    SE Mean
Y1   20   50.19       1.71        0.38
Y2   20   52.52       2.48        0.55

Difference = mu (X1) - mu (X2)
Estimate for difference: -2.33341
95% CI for difference: (-3.69547, -0.97135)
T-Test of difference = 0 (vs not =) : T-Value = -3.47
P-Value = 0.001 DF = 38
Both use Pooled Std. Dev. = 2.1277
```

(a) Can the null hypothesis be rejected at the 0.05 level? Why?

(b) Is this a one-sided or a two-sided test?

(c) If the hypotheses had been $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 \neq 2$ would you reject the null hypothesis at the 0.05 level?

(d) If the hypotheses had been $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1 : \mu_1 - \mu_2 < 2$ would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

(e) Use the output and the t table to find a 95 percent upper confidence bound on the difference in means.

(f) What is the P-value if the hypotheses are $H_0 : \mu_1 - \mu_2 = 2$ versus $H_1: \mu_1 - \mu_2 \neq 2$?

2.16. The breaking strength of a fiber is required to be at least 150 psi. Past experience has indicated that the standard deviation of breaking strength is $\sigma = 3$ psi. A random sample of four specimens is tested, and the results are $y_1 = 145$, $y_2 = 153$, $y_3 = 150$, and $y_4 = 147$.

(a) State the hypotheses that you think should be tested in this experiment.

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

(c) Find the P-value for the test in part (b).

(d) Construct a 95 percent confidence interval on the mean breaking strength.

2.17. The viscosity of a liquid detergent is supposed to average 800 centistokes at 25°C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is $\sigma = 25$ centistokes.

(a) State the hypotheses that should be tested.

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

(c) What is the P-value for the test?

(d) Find a 95 percent confidence interval on the mean.

2.18. The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of $\sigma = 0.0001$ inch. A random sample of 10 shafts has an average diameter of 0.2545 inch.

(a) Set up appropriate hypotheses on the mean μ.

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

(c) Find the P-value for this test.

(d) Construct a 95 percent confidence interval on the mean shaft diameter.

2.19. A normally distributed random variable has an unknown mean μ and a known variance $\sigma^2 = 9$. Find the sample size required to construct a 95 percent confidence interval on the mean that has total length of 1.0.

2.20. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

(b) Test these hypotheses using $\alpha = 0.01$. What are your conclusions?

(c) Find the P-value for the test in part (b).

(d) Construct a 99 percent confidence interval on the mean shelf life.

2.21. Consider the shelf life data in Problem 2.20. Can shelf life be described or modeled adequately by a normal distribution? What effect would the violation of this assumption have on the test procedure you used in solving Problem 2.15?

2.22. The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair times for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

(c) Find the P-value for the test.

(d) Construct a 95 percent confidence interval on mean repair time.

2.23. Reconsider the repair time data in Problem 2.22. Can repair time, in your opinion, be adequately modeled by a normal distribution?

2.24. Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviations of $\sigma_1 = 0.015$ and $\sigma_2 = 0.018$. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

Machine 1		Machine 2	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

(a) State the hypotheses that should be tested in this experiment.

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

(c) Find the P-value for this test.

(d) Find a 95 percent confidence interval on the difference in mean fill volume for the two machines.

2.25. Two types of plastic are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known that $\sigma_1 = \sigma_2 = 1.0$ psi. From random samples of $n_1 = 10$ and $n_2 = 12$ we obtain $\bar{y}_1 = 162.5$ and $\bar{y}_2 = 155.0$. The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should they use plastic 1? In answering this question, set up and test appropriate hypotheses using $\alpha = 0.01$. Construct a 99 percent confidence interval on the true mean difference in breaking strength.

2.26. The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the mean and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypothesis that the two variances are equal. Use $\alpha = 0.05$.

(b) Using the results of (a), test the hypothesis that the mean burning times are equal. Use $\alpha = 0.05$. What is the P-value for this test?

(c) Discuss the role of the normality assumption in this problem. Check the assumption of normality for both types of flares.

2.27. An article in *Solid State Technology*, "Orthogonal Design for Process Optimization and Its Application to Plasma Etching" by G. Z. Yin and D. W. Jillie (May 1987) describes an experiment to determine the effect of the C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. All of the runs were made in random order. Data for two flow rates are as follows:

C_2F_6 Flow	Uniformity Observation					
(SCCM)	1	2	3	4	5	6
125	2.7	4.6	2.6	3.0	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

(a) Does the C_2F_6 flow rate affect average etch uniformity? Use $\alpha = 0.05$.

(b) What is the P-value for the test in part (a)?

(c) Does the C_2F_6 flow rate affect the wafer-to-wafer variability in etch uniformity? Use $\alpha = 0.05$.

(d) Draw box plots to assist in the interpretation of the data from this experiment.

2.28. A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\bar{y}_1 = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\bar{y}_2 = 10.2$, $S_2^2 = 94.73$, $n_2 = 9$.

(a) Can you conclude that the two variances are equal? Use $\alpha = 0.05$.

(b) Has the filtering device reduced the percentage of impurity significantly? Use $\alpha = 0.05$.

2.29. Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kA) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

95 °C	100 °C
11.176	5.263
7.089	6.748
8.097	7.461
11.739	7.015
11.291	8.133
10.759	7.418
6.467	3.772
8.315	8.963

(a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use $\alpha = 0.05$.

(b) What is the P-value for the test conducted in part (a)?

(c) Find a 95 percent confidence interval on the difference in means. Provide a practical interpretation of this interval.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.

(e) Check the assumption of normality of the photoresist thickness.

(f) Find the power of this test for detecting an actual difference in means of 2.5 kA.

(g) What sample size would be necessary to detect an actual difference in means of 1.5 kA with a power of at least 0.9?

2.30. Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using two cool-down times, 10 and 20 seconds, and 20 housings were evaluated at each level of cool-down time. All 40 observations in this experiment were run in random order. The data are as follows.

10 seconds		20 seconds	
1	3	7	6
2	6	8	9
1	5	5	5
3	3	9	7
5	2	5	4
1	1	8	6
5	6	6	8
2	8	4	5
3	2	6	8
5	3	7	7

(a) Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use $\alpha = 0.05$.

(b) What is the P-value for the test conducted in part (a)?

(c) Find a 95 percent confidence interval on the difference in means. Provide a practical interpretation of this interval.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.

(e) Check the assumption of normality for the data from this experiment.