#### **Homework 3 Solution**

### 1. Montgomery 4.1

4.1. The ANOVA from a randomized complete block experiment output is shown below.

| Source    | DF | SS      | MS     | F     | Р |
|-----------|----|---------|--------|-------|---|
| Treatment | 4  | 1010.56 | ?      | 29.84 | ? |
| Block     | ?  | ?       | 64.765 | ?     | ? |
| Епог      | 20 | 169.33  | ?      |       |   |
| Total     | 29 | 1503.71 |        |       |   |

(a) Fill in the blanks. You may give bounds on the P-value.

Completed table is:

| Source    | DF | SS      | MS      | F     | P         |
|-----------|----|---------|---------|-------|-----------|
| Treatment | 4  | 1010.56 | 252.640 | 29.84 | < 0.00001 |
| Block     | 5  | 323.82  | 64.765  |       |           |
| Error     | 20 | 169.33  | 8.467   |       |           |
| Total     | 29 | 1503.71 |         |       |           |

(b) How many blocks were used in this experiment?

Six blocks were used.

(c) What conclusions can you draw?

The treatment effect is significant; the means of the five treatments are not all equal.

#### 2. Montgomery 4.12

**4.12.** An article in *Nature Genetics* (2003, Vol. 34, pp. 85-90) "Treatment-Specific Changes in Gene Expression Discriminate in vivo Drug Response in Human Leukemia Cells" studied gene expressionas a function of different treatments for leukemia. Three treatment groups are: mercaptopurine (MP) only; low-dose methotrexate (LDMTX) and MP; and high-dose methotrexate (HDMTX) and MP. Each group contained ten subjects. The responses from a specific gene are shown in the table below:

|            |       |       |        |       | Proje | ect   |       |       |       |       |
|------------|-------|-------|--------|-------|-------|-------|-------|-------|-------|-------|
| MP ONLY    | 334.5 | 31.6  | 701    | 41.2  | 61.2  | 69.6  | 67.5  | 66.6  | 120.7 | 881.9 |
| MP + HDMTX | 919.4 | 404.2 | 1024.8 | 54.1  | 62.8  | 671.6 | 882.1 | 354.2 | 321.9 | 91.1  |
| MP + LDMTX | 108.4 | 26.1  | 240.8  | 191.1 | 69.7  | 242.8 | 62.7  | 396.9 | 23.6  | 290.4 |

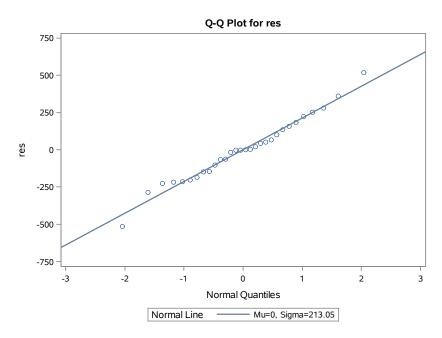
(a) Is there evidence to support the claim that the treatment means differ?

The ANOVA below identifies that the treatment means are different at  $\alpha$  = 0.05.

| Source       | DF | Type III SS | Mean Square | F Value | Pr > F |
|--------------|----|-------------|-------------|---------|--------|
| treatment    | 2  | 538442.2247 | 269221.1123 | 3.68    | 0.0457 |
| blck_subject | 9  | 920631.5550 | 102292.3950 | 1.40    | 0.2597 |

(b) Chec the normality assumption. Can we assume these samples are from normal populations?

The normal plot of residuals below shows a slightly non-normal distribution.



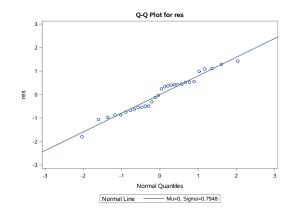
(c) Take the logarithm of the raw data. Is there evidence to support the claim that the treatment means differ for the transformed data?

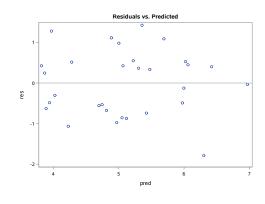
The ANOVA for the natural log transformed data identifies that the treatment means are not significantly different at  $\alpha$  = 0.05, and the pvalue of 0.07 is slightly greater than  $\alpha$ .

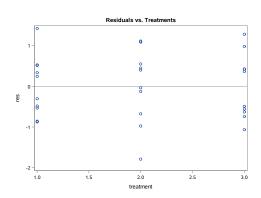
| Source       | DF | Type III SS | Mean Square | F Value | Pr > F |
|--------------|----|-------------|-------------|---------|--------|
| treatment    | 2  | 6.29704094  | 3.14852047  | 3.09    | 0.0700 |
| blck_subject | 9  | 14.74919627 | 1.63879959  | 1.61    | 0.1861 |

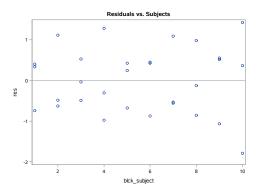
(d) Analyze the residuals from the transformed data and comment on model adequacy.

The residual plots below show no concerns with the model adequacy.









## 3. Montgomery 4.19

**4.19.** Consider the hardness testing experiment in Problem 4.7. Suppose that the observation for tip 2 in coupon 3 is missing. Analyze the problem by estimating the missing value.

$$y_{23}$$
 is missing.  $\hat{y}_{23} = \frac{ay_2 + by_3 - y_1}{(a-1)(b-1)} = \frac{4(28.6) + 4(29.1) - 144.2}{(3)(3)} = 9.62$ 

Therefore, 
$$y_2 = 38.22$$
,  $y_3 = 38.72$ , and  $y_4 = 153.82$ 

| Source | SS     | DF | MS       | $F_0$ |
|--------|--------|----|----------|-------|
| Tip    | 0.40   | 3  | 0.133333 | 19.29 |
| Coupon | 0.80   | 3  |          |       |
| Error  | 0.0622 | 9  | 0.006914 |       |
| Total  | 1.2622 | 15 |          |       |

 $F_{0.05,3,9}$ =3.86, Tips are significant.

# 4. Montgomery 4.23

**4.23.** An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design shown below. Analyze the data from this experiment  $(\alpha = 0.05)$  draw appropriate conclusions.

| Order of |      |        | Operator |      |
|----------|------|--------|----------|------|
|          |      |        |          |      |
| Assembly | 1    | 2      | 3        | 4    |
| 1        | C=10 | D=14   | A=7      | B=8  |
| 2        | B=7  | C=18   | D=11     | A=8  |
| 3        | A=5  | B = 10 | C=11     | D=9  |
| 4        | D=10 | A=10   | B = 12   | C=14 |

The analysis result below identifies assembly method as having a significant effect on assembly time.

| Source      | DF | Type III SS | Mean Square | F Value | Pr > F |
|-------------|----|-------------|-------------|---------|--------|
| assm_order  | 3  | 18.50000000 | 6.16666667  | 3.52    | 0.0885 |
| operator    | 3  | 51.50000000 | 17.16666667 | 9.81    | 0.0099 |
| assm_method | 3  | 72.50000000 | 24.16666667 | 13.81   | 0.0042 |

The residual plots below show no serious concerns with the model adequacy.

