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FA18-BSE-072

Linear Algebra

Assignment #2

Determinants

Q: What is Determinant?

A scalar value that is calculated from the elements of a square matrix is called Determinant.

It is also an element that identifies and determines nature of a matrix.

For 2x2 Matrix:

$$A = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$|A| = ad - bc$$

For 3x3 matrix:-

$$A = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$|A| = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$|A| = a(ei - hf) - b(di - gf) + c(dh - eg)$$

Q: Properties of Determinants with Examples

1. **Determinant of Matrix with Row or Column of Zeros:**

If all elements of a column or row are zero then determinant is also 0.

Example:

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 9 & 7 & 4 \end{vmatrix} = 0$$

2. Negative Sign with interchanged Rows and Columns:

If any two rows/columns of a matrix are interchanged then sign of determinant also changes.

Example:

$$\begin{vmatrix} 7 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 8 & 9 \end{vmatrix}$$

Interchanging R_1 and R_3

$$\begin{vmatrix} 1 & 8 & 9 \\ 4 & 5 & 6 \\ 7 & 2 & 3 \end{vmatrix}$$

$$1\begin{vmatrix} 5 & 6 \\ 2 & 3 \end{vmatrix} - 8\begin{vmatrix} 4 & 6 \\ 7 & 3 \end{vmatrix} + 9\begin{vmatrix} 4 & 5 \\ 7 & 2 \end{vmatrix}$$

$$= 1(15-12) - 8(12-42) + 9(8-35)$$

$$= 3 + 240 - 244$$

$$= -1$$

3. **Multiplication by Scalar**

If row/column of matrix is multiplied by any non-zero value then determinant also gets multiplied by same constant.

Example:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$\begin{vmatrix} 5(1) & 5(2) & 5(3) \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = 5\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}.$$

4. **Determinant of Transpose**

Let A be a matrix where A^T is the transpose of A, then

$$\det(A^T) = \det(A)$$

$A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$ and $A^T = \begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix}$

Now.

$|A| = [(1)(4) - (2)(3)]$
$= [4-5]$

$|A| = -1$

$|A^T| = [(1)(4) - (2)(3)]$
$= [4-5]$

$|A^T| = -1$

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5. **Redundant Rows/Columns**

If any two rows or two columns are same in matrix, then determinant will be 0.

Example:

$$A = \begin{vmatrix} 1 & 2 & 1 \\ 5 & 6 & 0 \\ 1 & 2 & 1 \end{vmatrix}$$

$|A| = 0$

6. **Triangle Property:**

If in a matrix, above or below the values a zero then determinant is equal to product of diagonal elements.

Example:

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 5 & 6 \\ 0 & 0 & 9 \end{vmatrix} = (1)(5)(9)$$

$|A| = 45.$

7. **Identity Matrix:**

Determinant of identity matrix is always 1.

$A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

$|A| = 1$

8. **Sum Property**

If entry of row or column consists of two terms as a sum then its determinant can be expressed as.

Example:

$$A = \begin{vmatrix} a_{11}+b_{11} & a_{12} & a_{13} \\ a_{21}+b_{21} & a_{22} & a_{23} \\ a_{31}+b_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} b_{11} & a_{12} & a_{13} \\ b_{21} & a_{22} & a_{23} \\ b_{31} & a_{23} & a_{33} \end{vmatrix}$$