Assignment # 01

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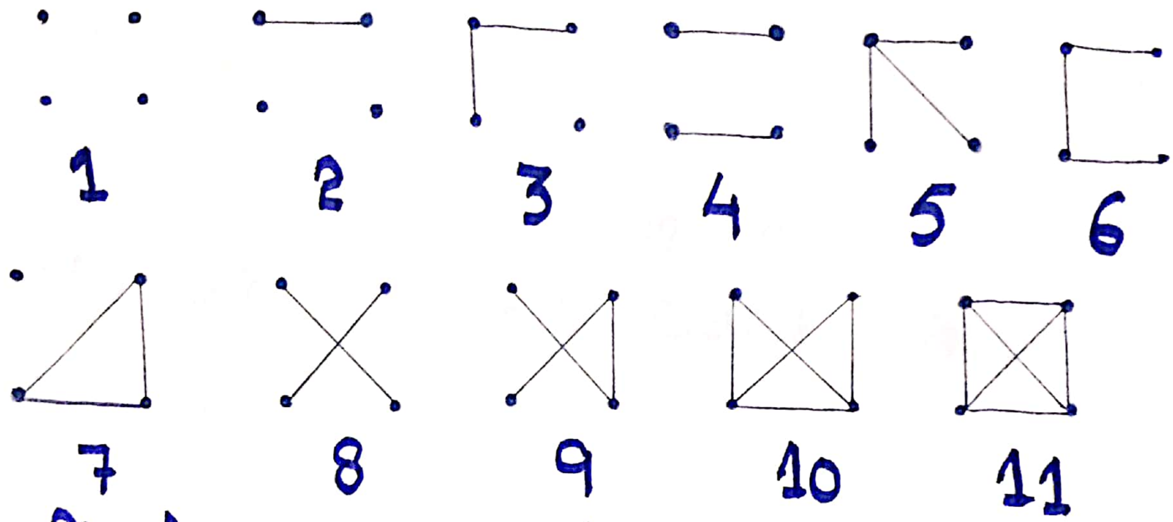
Assignment #1

·1 Draw the eleven unlabelled simple graphs with four vertices.

Solution:



·2 (a) If two graphs have the same degree sequence, must they be isomorphic?

No, two graphs with the same degree sequence are not necessarily isomorphic. This is known as the Havel-Hakimi theorem. While having the same degree sequence is a necessary condition for isomorphism, it is not a sufficient condition. There can be non-isomorphic graphs with identical degree sequence.

(b) If two graphs are isomorphic, must they have the same degree sequence?

Yes, if two graphs are isomorphic

ey must have the same degree sequence. Isomorphism implies that the structure of the graphs is the same, including the degrees of their vertices. Therefore, if two graphs are isomorphic, their degree sequence must be identical.

• 3 (2.5) Let G be a graph with degree sequence (1,2,3,4)...?

By Handshaking Lemma:

"The sum of the degrees of all vertices in a graph is equal to twice the number of edges."

No of vertices = 4

total degree sum = 1+2+3+4 = 10

No of edges = 10/2 = 5

So, the graph G with the degree sequence (1,2,3,4) has 4 vertices & 5 edges.

.4 (2.6) Prove that, If G is Simple graph (b)

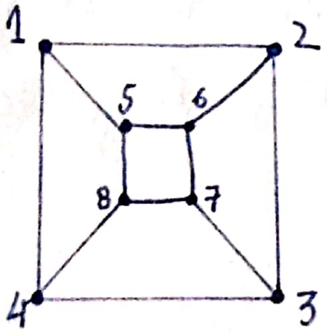
By Pigeonhole Principle:

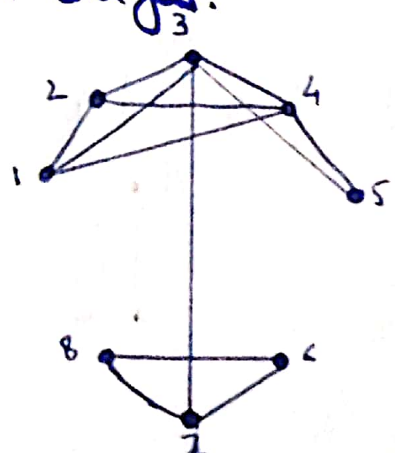
If G has exactly two vertices, they can either have the same degree or different degrees. If they have the same degree, We're done. If they have different degrees, then there are at least two vertices with different degrees.

But if we have more than two vertices then the maximum possible degree of a vertex in G is $|V|-1$, Where $|V|$ is the number of vertices in G. This is because in a simple graph, a vertex can be adjacent to all other vertices except itself.

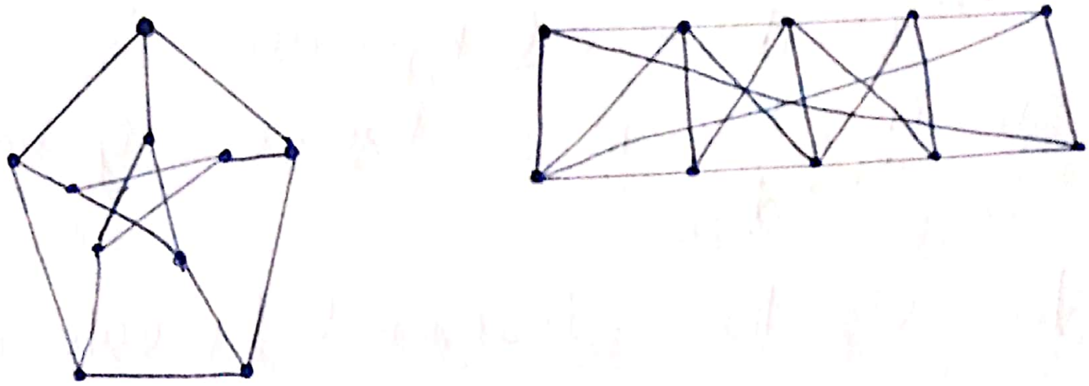
·5 (2.10) Draw:

(a) two non-isomorphic regular with 8 vertices and 12 edges.





(b) two non-isomorphic regular graphs...



6 (2.11) Determine the no of edges....

(a) C_{10}

Cycle with 10 Vertices.

↳ A cycle with n vertices has n edges.

So, C_{10} has 10 edges.

(b) $K_{9,10}$

Complete bipartite graph with parts of size 9 and 10.

↳ A complete bipartite graph with parts of size m and n has $m*n$ edges.

So, $K_{9,10}$ has $9*10 = 90$ edges.

(c) K_{10}

Complete graph with 10 Vertices.

A complete graph with n vertices has $C(n,2) = \frac{n(n-1)}{2}$ edges.

↳ So, K_{10} has $\frac{10(10-1)}{2} = 45$ edges.

(d) Q_5 (5-dimensional hypercube)

The no of edges in a hypercube of dimension d is $2^d * d/2$

For Q_5, the 5-dimensional hypercube, it has

$$\frac{2^5 \times 5}{2} = 80 \text{ edges.}$$

(e) The dodechedron (a polyhedron with 12 faces

By Euler's formula for polyhedra:

$$V - E + F = 2$$

Where

$V \rightarrow$ Vertices

$E \rightarrow$ Edges & $F \rightarrow$ Faces

For a dodechedron $V = 20$ & $F = 12$

So,

$$E: 20 - E + 12 = 2$$

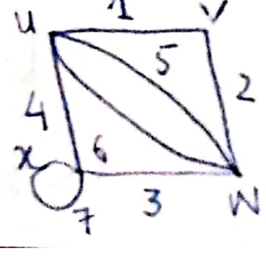
$$\Rightarrow E = 30$$

↳ So, dodechedron has 30 edges.

Q7 (2.1) Consider the graph G shown on....?

(a) Vertices V and x are adjacent

These vertices are not adjacent as they are not connected by an edge.



edge 6 is incident with vertex w

Yes, edge 6 is incident with vertex w as it is endpoint.

(c) Vertex x is incident with edge 4.

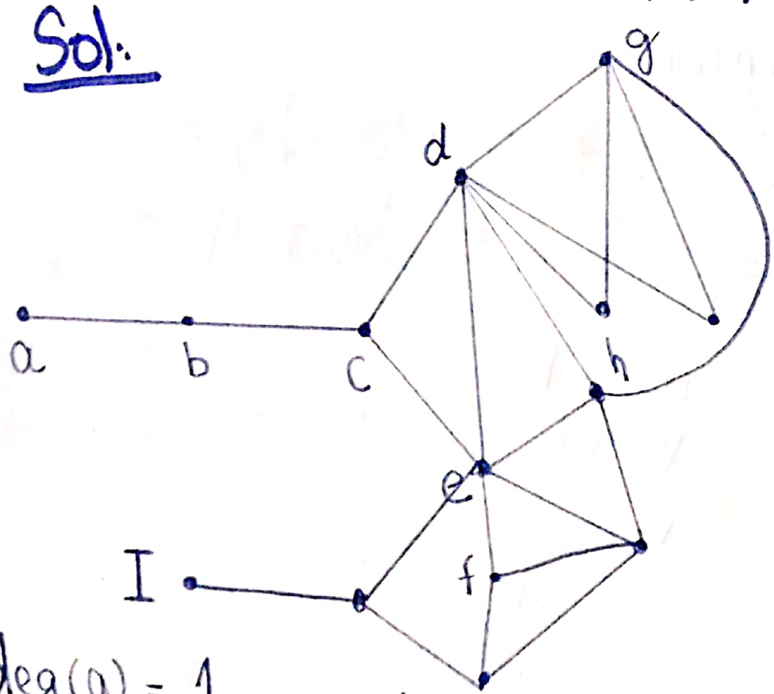
Yes, vertex x is incident with edge 4.

(d) Vertex w and edge 5 & 6 form a subgraph of G.

No, vertex w and edge 5 & 6 form a subgraph.

Q No 8 Draw simple connected graph with degree sequence (1,1,2,3,3,4,4,6).

Sol:



deg(a) = 1

deg(b) = 2

deg(c) = 3

deg(d) = 6

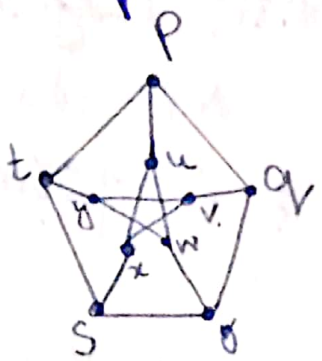
deg(f) = 3

deg(g) = 4

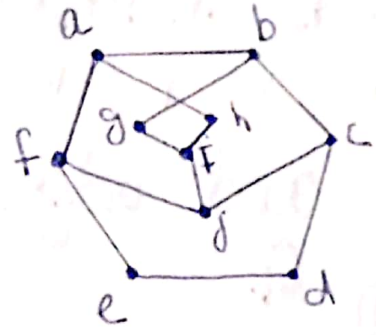
deg(h) = 4

deg(I) = 1

QNO9 By Suitably labelling the vert 8
Show that the following graphs are isomorphic.



(G_1)



(G_2)

No of Edges $= E(G_1) = 15 =$ No of edges of E_2

No of Vertices $= V(G_1) = 10 = V(G_2)$

Degree of Sequence:

$G_1: \{3, 3, 3, 3, 3, 3, 3, 3, 3, 3\}$

$G_2: \{3, 3, 3, 3, 3, 3, 3, 3, 3, 3\}$

Now,

$p \leftrightarrow a \qquad u \leftrightarrow h$

$q \leftrightarrow b \qquad v \leftrightarrow c$

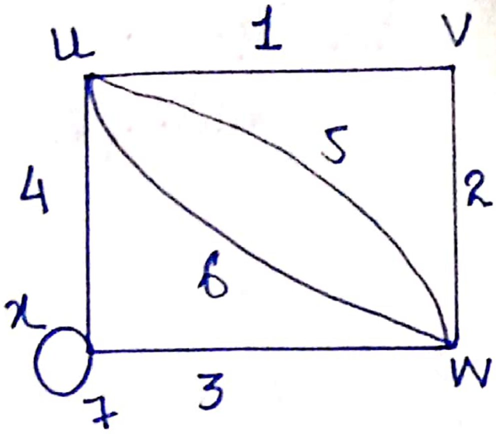
$r \leftrightarrow g \qquad w \leftrightarrow j$

$s \leftrightarrow e \qquad x \leftrightarrow d$

$t \leftrightarrow f \qquad y \leftrightarrow i$

QNO 10 For a graph shown on the right, write down:

(b) A walk of length 6 between 'u' & 'w'.



uvuxuvvw

All the cycles of length 1, 2, 3, 4.

length 1: the loop xx

length 2: the multiple edge uxu

length 3: the triangle uwxu.

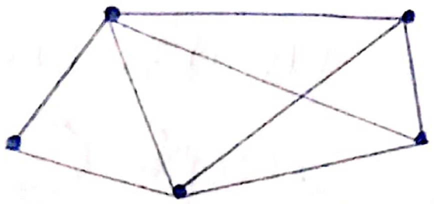
length 4: the quadrilaterals uvwxu.

(c) A path of maximum length.

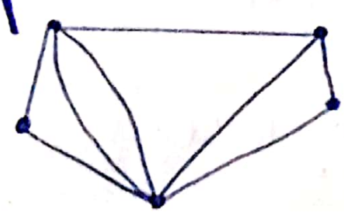
uvwx

Q No 12 (2.8) Draw four connected graphs G_1, G_2, G_3 and G_4 with 5 vertices & 8 edges satisfying:

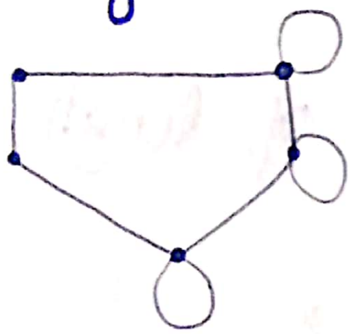
↳ G_1 is simple graph.



↳ G_2 is non-simple graph with no loops.



↳ G_3 is non-Simple graph with no multiple edges.



↳ G_4 is a graph with both loops and multiple edgss.

QNo13 (2.13) The girth of a graph 'G' is the length of Shortest cycle and circumference is length of longest cycle. Find both for:

(a) Peterson graph.

girth = 4 , Circumference = 9

(b) the 9-Cube graph Q_4

girth = 4

Circumference = 7

no
.No14 (2.14) Prove that, if every cycle of a graph has an even number of edges then graph is bipartite?

Sol: If 'G' is bipartite with vertex set V_1 and V_2. Every step along a walk takes you either from V_1 to V_2 or V_2 to V_1 to end up where you started. Therefore, even step take, But in these situation, suppose every cycle of 'G' is even and V_0 any vertex. For each vertex V, the same component Shortest Path from 'V' to 'V_0' Do same for 'G' component it would even cycle and thus 'G' is bipartite.

Q 2.15 The Line graph L(G) of a Simple graph G is the graph obtained by taking....?

The line graph L(G) is a simple graph 'G' is defined as mentioned. To find an expression for the number of edges in L(G) in terms of degree, we use Hand Shake lemma. let G and E and number of edges, and vertices $d_1 \dots d_n$.

No of edges in $L(G) = \frac{1}{2} \Sigma (d_i \cdot d_i - 1)$

(a) $L(C_n)$ is isomorphic to C_n;

For a cycle graph 'c' with n-vertices, each vertex has degree '2'

No of edges in $L(C) = \frac{1}{2} \Sigma\, 2(2-1)n = \frac{1}{2} 2n = n$

$L(C)$ is isomorphic to 'C'.

(b) $L(K_n)$ has $\frac{1}{2n}(n-1)$ vertices and regular of degree $2n-4$.

For a complete graph 'k' with n-vertices each vertex has a degree of $n-1$ and the number of edges in k is $(n(n-1))/2$. Using the expression

No of edges in $L(K) = \frac{1}{2} \Sigma (n-1)(n-1-1)$

$$= \frac{1}{2} \Sigma (n-1)(n-2)$$

$$= \frac{2n^3 - 3n^2 + n}{2}$$

(c) L(tetrahedron graph) = octahedron graph

The line graph of tetrahedron graph is isomorphic to an octahedron graph. This can be verified visually.

(d) the complement $L(K_5)$ is the Petersen graph.

The complement of a line graph $L(G)$ is a graph where two vertices are adjacent, iff they are not adjacent in $L(G)$. The complement of $L(K_5)$ is isomorphic to the Petersen graph.