Assignment # 2

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Course Title:

Sampling & Design Analysis.

Assignment #2

QNO1: A Sample of Size n=3 is to be.....??

Solution:

We have population values 0, 3, 6, 9 and 12, Population Size N=5 and sample Size n=2

Thus the number of possible samples Which can be drawn Without replacement is:

$$\binom{N}{n} = \binom{5}{3} = 10$$

Sample No	Sample Values	Sample Mean($\bar{x}$)	Sample No	Sample Values	Sample Mean ($\bar{X}$)
1	0, 3	1.5	6	3, 9	6
2	0, 6	3	7	3, 12	7.5
3	0, 9	4.5	8	6, 9	7.5
4	0, 12	6	9	6, 12	9
5	3, 6	4.5	10	9, 12	10.5

The sampling distribution of the sample mean $\bar{X}$ and its mean and S.D are:

$\bar{X}$	f	$f(\bar{X})$	$\bar{X} f(\bar{X})$	$\bar{X}^2 f(\bar{X})$
1.5	1	1/10	1.5/10	2.25/10
3	1	1/10	3/10	9/10
4.5	2	2/10	9/10	40.5/10
6	2	2/10	12/10	72/10
7.5	2	2/10	15/10	112.5/10
9	1	1/10	9/10	81/10
10.5	1	1/10	10.5/10	110.25/10
	10	1	60/10	427.5/10

$$E(\bar{X}) = \Sigma \bar{X} f(\bar{X}) = \frac{60}{10} = 6$$

$$Var(\bar{X}) = \Sigma \bar{X}^2 f(\bar{X}) - [\Sigma \bar{X} f(\bar{X})]^2$$

$$= \frac{427.5}{10} - \left(\frac{60}{10}\right)^2 = 42.75 - 36 = 6.75$$

The mean and variance of Population are.

X	0	3	6	9	12	$\Sigma X = 30$
X^2	0	9	36	81	144	$\Sigma X^2 = 270$

$$\mu = \frac{\Sigma X}{N} = \frac{30}{5} = 6$$

$$\sigma^2 = \frac{\Sigma X^2}{N} - \left(\frac{\Sigma X}{N}\right)^2 = \frac{270}{5} - \left(\frac{30}{5}\right)^2 = 54 - 36$$

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To Verify

$$E(\bar{X}) = \mu = 6$$

$$Var(\bar{X}) = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{18}{3}\left(\frac{5-3}{5-1}\right) = 3$$

Hence Proved.

Q.2 The population values $\{5, 10, 15, 20\}$ are written on slips of paper and put in a hat. Two slips are randomly selected with replacement.

(a) Find the mean, standard deviation and variance of the population.

(b) List all possible sample size $n=2$ and calculate the mean of each.

Solution:

(a)

population
5
10
15
20

$\mu = 12.5$

$\sigma = 5.59$

$\sigma^2 = 31.25$

(b) These means from the sampling distribution of the sample mean.

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sample	Sample mean $\bar{X}$	Sample	Sample mean
5,5	5	15,5	10
5,10	7.5	15,10	12.5
5,15	10	15,15	15
5,20	12.5	15,20	17.5
10,5	7.5	20,5	12.5
10,10	10	20,10	15
10,15	12.5	20,20	20
10,20	15		

QNO3 We are given the following data:

Population A: $\mu_1 = 4.3$ years, $\sigma_1 = 0.6$ years, $n_1 = 49$

opulation B: $\mu_2 = 4.0$ years, $\sigma_2 = 0.4$ years, $n_2 = 36$

Solution: .... ??

Both sample sizes ($n_1 = 49$, $n_2 = 36$) are arge enough to assume that the sampling listribution of the differene $\bar{X}_1 - \bar{X}_2$ is approximately a normal with mean

$$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 = 4.3 - 4.0 = 0.3 \text{ years.}$$

and

S.D ⇒ $$\sigma_{\bar{x}_1} - \sigma_{\bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{0.36}{49} + \frac{0.16}{36}}$$

$= 0.1086$ years.

Thus the variable

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - 0.3}{0.1086}$$ is approximately normal.

$$Z \sim N(0,1)$$

We want $P(\bar{X}_1 - \bar{X}_2 \geq 0.5)$

Transforming $(\bar{X}_1 - \bar{X}_2)$ to Z-value, we find that

$$Z = \frac{0.5 - 0.3}{0.1086} = 1.84$$

Hence using table of areas under Normal curve, we find

$$P(\bar{X}_1 - \bar{X}_2 \geq 0.5) = \left(\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \geq \frac{0.5 - 0.3}{0.1086} \right)$$

$$= (Z \geq 1.84) = 1 - P(Z = 1.84)$$

$$= 0.0329$$

QNO4

Let a population-1 having values 4, 6 and 8 and population-2 have the values 2 & 4. Sample of size two with replacement are drawn from both populations. Construct sampling distribution of difference between sample means.

Pop-I

4, 6 & 8 Pop size $N_1 = 3$

Sample size ; $n_1 = 2$ (WR)

All possible samples $N_1^{n_1} = 3^2 = 9$

Sample	Mean $\bar{x}_1$	Sample	Mean $\bar{x}_2$
4,4	4	6,8	7
4,6	5	8,4	6
4,8	6	8,6	7
6,4	5	8,8	8
6,6	6		

Pop II

2 & 4 Pop Size $N_2 = 2$

Sample Size ; $n_2 = 2$ (WR)

All possible samples $N_2^{n_2} = 2^2 = 4$

Sample	Mean $\bar{x}_2$	Sample	Mean $\bar{x}_2$
2,2	2	4,2	3
2,4	3	4,4	4

All possible Difference $(\bar{X}_1 - \bar{X}_2)$

	X_1								
X_2	4	5	6	5	6	7	6	7	8
2	2	3	4	3	4	5	4	5	6
3	1	2	3	2	3	4	3	4	5
3	1	2	3	2	3	4	3	4	5
4	0	1	2	1	2	3	2	3	4

Difference $d = x_1 - x_2$	f	$P(d)$	$d\,P(d)$	$d^2 P(d)$
0	1	1/36	0	0
1	4	4/36	4/36	4/36
2	8	8/36	16/36	32/36
3	10	10/36	30/36	90/36
4	8	8/36	32/36	128/36
5	4	4/36	20/36	100/36
6	1	1/36	6/36	36/36
	$\Sigma f = 36$	$36/36 = 1$	$108/36 = E(d)$	$390/36$ $E(d^2)$

Mean: $\mu_{\bar{x}_1 - \bar{x}_2} = \mu_d = E(d) = 108/36 = 3$

Variance: $\sigma^2_{\bar{x}_1 - x_2} = \sigma_d^2 = E(d^2) - [E(d)]^2 = \frac{390}{36} - \left(\frac{108}{36}\right)^2$

$= 10.83 - 9 = 1.83$

Mean & Variance of Pop I & Pop II

Pop – I

X	X^2
4	16
6	36
8	64
$\Sigma X = 18$	$\Sigma X^2 = 116$

Pop mean : $\mu_1 = \Sigma X / N = 18/3 = 6$

Pop variance : $\sigma_1^2 = \dfrac{\Sigma X^2}{N} - \left(\dfrac{\Sigma X}{N}\right)^2$

$= 20/2 - (6/2)^2$

$= 1.0$

Pop – II

X	X^2
2	4
4	16
$\Sigma X = 6$	$\Sigma X^2 = 20$

Pop mean $= \mu_2 = \dfrac{\Sigma X}{N} = \dfrac{6}{2} = 3$

Pop variance $= \sigma_2^2 = \dfrac{\Sigma X^2}{N} - \left(\dfrac{\Sigma X}{N}\right)^2$

$= 116/3 - \left(\dfrac{18}{3}\right)^2$

$= 2.66$

$\mu_{\bar{x}_1 - \bar{x}_2} = \mu_1 - \mu_2 \Rightarrow$

$3 = 6 - 3 = 3$

$\sigma^2_{\bar{x}_1 - \bar{x}_2} = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$

$1.83 = \dfrac{2.66}{2} + \dfrac{1}{2} = 1.83$

Mean: $\mu_{\bar{x}_1 - \bar{x}_2} = E(d) = \dfrac{108}{36} = 3$

Variance: $\sigma^2_{\bar{x}_1 - \bar{x}_2} = E(d^2) - [E(d)]^2 = \dfrac{390}{36} - \left(\dfrac{108}{36}\right)^2$

$= 10.83 - 9$

$= 1.83$

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(Top partially cut) ... population having elements 1, 2, 3, 4 ...ampling distribution Sample size 2 with replacement $\hat{p}$ proportion of odd number. Compute mean & variance of proportion. Verify that of Sampling distribution

Sol:

$$\mu_{\hat{p}} = P, \quad \sigma^2_{\hat{p}} = \frac{Pq}{n}$$

total sample $= N^n = 4^2 = 16$ $\{1, 2, 3, 4\}$

Sr.No	Sample	Proportion $\hat{p}$	Sr.No	Sample	Proportion $\hat{p}$
1	1,1	1	9	3,1	1
2	1,2	1/2	10	3,2	1/2
3	1,3	1	11	3,3	1
4	1,4	1/2	12	3,4	1/2
5	2,1	1/2	13	4,1	1/2
6	2,2	0	14	4,2	0
7	2,3	1/2	15	4,3	1/2
8	2,4	0	16	4,4	0

$\hat{p}$	f	$f(\hat{p})$	$\hat{p} f(\hat{p})$	$\hat{p}^2 f(\hat{p})$
0	4	4/16	0	0
1/2	8	8/16	4/16	2/16
1	4	4/16	4/16	4/16
	$\Sigma f = 16$	1	8/16	6/16

Mean & variance

$\mu_{\hat{p}} = \Sigma \hat{p} \cdot f(\hat{p}) = 8/16 = 1/2$

$\sigma_{\hat{p}}^2 = E(\hat{p}^2)^2 - [\Sigma(\hat{p})]^2$

$= 6/16 - (1/2)^2 = 6/16 - 4/16$

$= 2/16 = 1/8$

Verification:

$\mu_{\hat{p}} = P \Rightarrow 1/2 = 1/2$

$\sigma_{\hat{p}}^2 = \frac{Pq}{n}$

$1/8 = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

WR

QNo6

Let $\hat{P}_1$ represents the proportion of even number in a random sample $n_1 = 2$ without replacement from a finite population consisting of values 4, 6, 9

Similarly, ... ??

Sol: (a) $E(\hat{P}_1 - \hat{P}_2) = P_1 - P_2$.

Pop 1

We have pop 1: 4, 6, 9

$N_1 = 3$

$n_1 = 2$

$\binom{N_1}{n_1} = 3$

Pop 2

2, 3, 5

$N_2 = 3$

$n_2 = 2$

$\binom{N_2}{n_2} = 3$

Pop 1 | Pop 2

Sample No	Sample value	Sample Proportion ($\hat{P}_1$)	Sample No	Sample value	Sample Proportion $\hat{P}_2$
1	4, 6	$2/2$	1	2, 3	$1/2$
2	4, 9	$1/2$	2	2, 5	$1/2$
3	6, 9	$1/2$	3	3, 5	0

Here $3 \times 3 = 9$ possible difference are $\hat{P}_1 - \hat{P}_2$ are

$\hat{P}_2$ \ $\hat{P}_1$	1.0	0.5	0.5
0.5	0.5	0	0
0.5	0.5	0	0
0	1.0	0.5	0.5

Mean & Variance $(\hat{P}_1 - \hat{P}_2)$

$P_1 - P_2 = d$	f	$f(d)$	$d f(d)$	$d^2 f(d)$
0	4	$4/9$	0	0
0.5	4	$4/9$	$2/9$	$1/9$
1.0	1	$1/9$	$1/9$	$1/9$
	9	1	$3/9$	$2/9$

$E(\hat{P}_1 - \hat{P}_2) = E(d) = \Sigma d f(d) = 3/9 = 1/3$

$Var(\hat{P}_1 - \hat{P}_2) = Var(d) = 2/9 - (1/3)^2 = \frac{2}{9} - \frac{1}{9} = 1/9$

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Verification:

$$E(\hat{P}_1 - \hat{P}_2) = P_1 - P_2 = \frac{1}{3}$$