# Weak Perspective Projection

- Approximation to Perspective projection, approximately valid when distance of the camera is much greater than the depth variation of the object

$$x = -\frac{fX}{\overline{Z}} \qquad y = -\frac{fY}{\overline{Z}} \qquad \text{or} \qquad x = mX \qquad y = mY$$

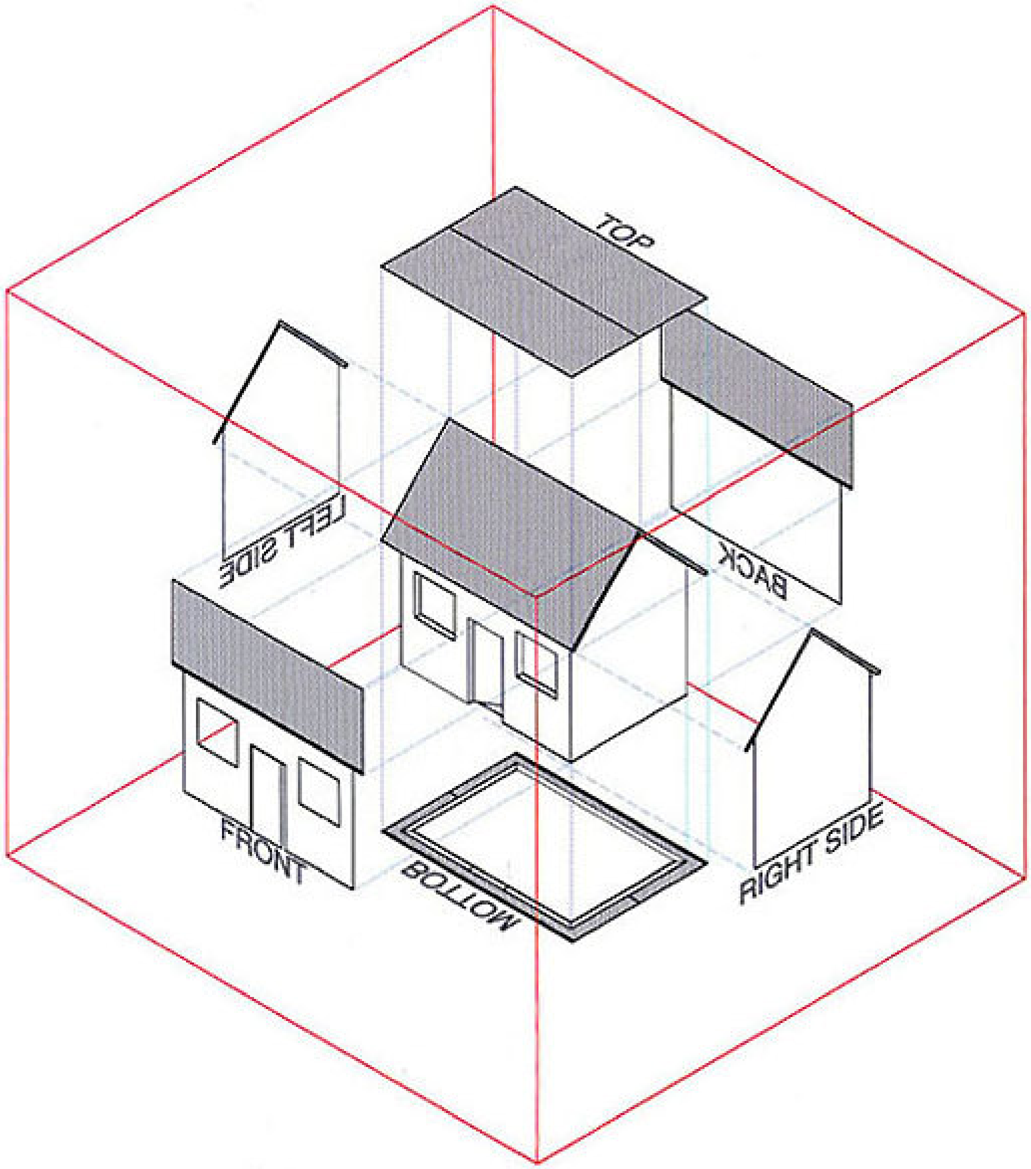
- Advantage: Computationally simpler [why?]
- Disadvantage: Not physically accurate

# Orthographic Projection

- Scaling of weak perspective projection

$$x = X \quad y = Y$$

- Parallel lines remain parallel
- Useful for engineering drawings, scrolls, where the perspective shortening is not desired
- Computationally simpler




http://www2.arts.ubc.ca/TheatreDesign/crslib/drft_1/orthint.htm

# Plane + Perspective Model

- Assumptions:
  - Planar World
  - Rigid Motion of the World
  - Perspective Camera
- Qs: Relationship between two images taken under these conditions?

# Plane + Perspective Model

- Approach
  - Put planarity constraint in rigid model, then solve for the image-to-image relation
  - Assume that the camera is not moving but the world plane has moved in front of the camera

Step 1:
Relate the two sets 3D world points (before and after transformation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

# Plane + Perspective Model

Step 2:
Put in the planarity constraint

Since equation of a plane in 3D is

$$aX + bY + cZ = 1 \quad \text{or} \quad \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = 1$$

so

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \underbrace{\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix}}_{\text{3x3 matrix}} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

# Plane + Perspective Model

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Can be simplified to

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \underbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}}_{\mathbf{A} = \mathbf{R} + \mathbf{T}\,[a\ b\ c]} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

# Plane + Perspective Model

Step 3:
Take the perspective projection of image points and relate then in 2D

The image of point $[X, Y, Z]^T$ (**before** transformation) is formed at:

$$x = \frac{X}{Z}, \qquad y = \frac{Y}{Z}$$

Assume f = 1, ignore –ve sign

The image of point $[X', Y', Z']^T$ (**after** transformation) is formed at:

$$x' = \frac{X'}{Z'}, \qquad y' = \frac{Y'}{Z'}$$

We need to relate the two

# Plane + Perspective Model

Given $$x' = \frac{X'}{Z'}, \qquad y' = \frac{Y'}{Z'}$$

Substitute the values of *X’,Y’, Z”* :

$$x' = \frac{a_{11}X + a_{12}Y + a_{13}Z}{a_{31}X + a_{32}Y + a_{33}Z}$$

$$y' = \frac{a_{21}X + a_{22}Y + a_{23}Z}{a_{31}X + a_{32}Y + a_{33}Z}$$

# Plane + Perspective Model

Multiply and Divide by $a_{33}Z$

$$x' = \frac{a'_{11}\dfrac{X}{Z} + a'_{12}\dfrac{Y}{Z} + a'_{13}}{a'_{31}\dfrac{X}{Z} + a'_{32}\dfrac{Y}{Z} + 1}$$

$$y' = \frac{a'_{21}\dfrac{X}{Z} + a'_{22}\dfrac{Y}{Z} + a'_{23}}{a'_{31}\dfrac{X}{Z} + a'_{32}\dfrac{Y}{Z} + 1}$$

# Plane + Perspective Model

Substitute $X/Z = x$, $Y/Z = y$

$$x' = \frac{a'_{11}\,x + a'_{12}\,y + a'_{13}}{a'_{31}\,x + a'_{32}\,y + 1}$$

$$y' = \frac{a'_{21}\,x + a'_{22}\,y + a'_{23}}{a'_{31}\,x + a'_{32}\,y + 1}$$

This equation relates the two images captured before and after the transformation.

# Plane + Perspective Model

- Conclusion:
- Planar world and perspective camera yields **projective** relationship between the images
- Similarly, it can be shown that planar world and orthographic camera yields _________ relationship between images

# Outline of Alternate Proof

- Consider Z=0 Plane in the world
- Then,

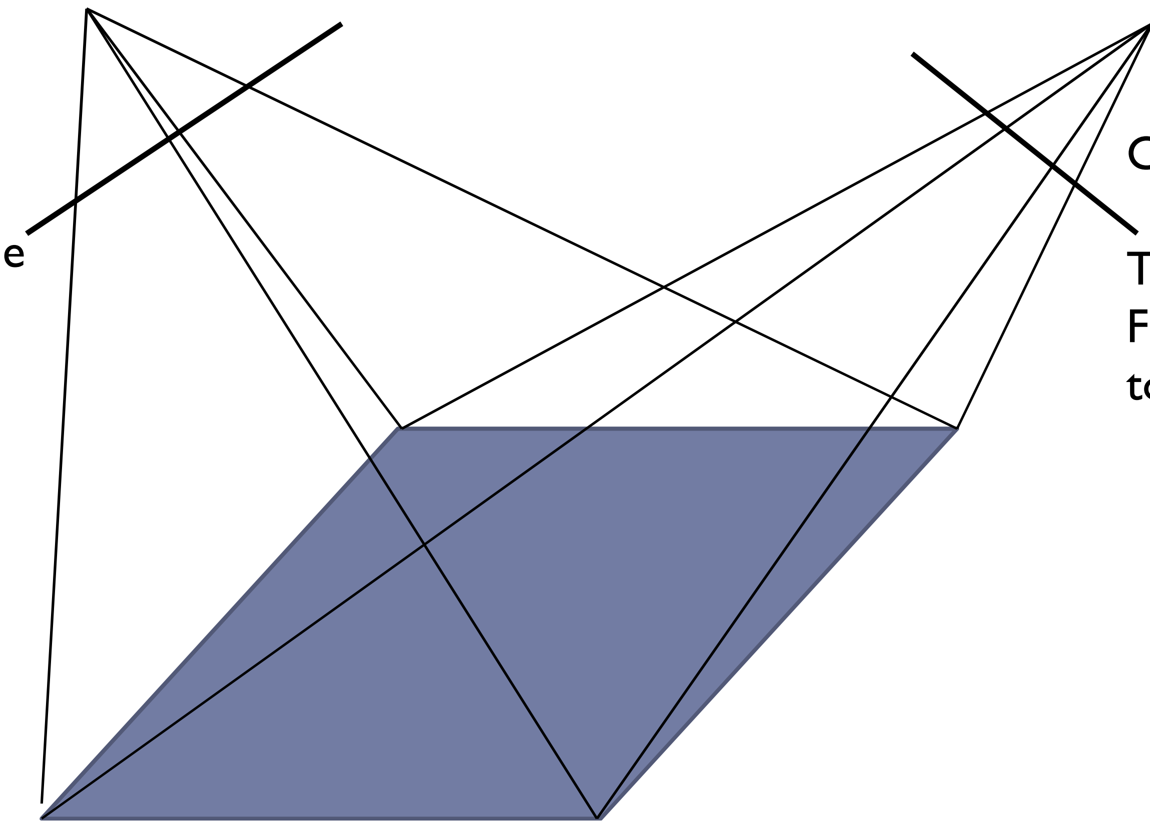
$$\begin{bmatrix} hx \\ hy \\ h \end{bmatrix} = \begin{bmatrix} . & . & . & . \\ . & . & . & . \\ . & . & . & . \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z=0 \\ 1 \end{bmatrix} = \begin{bmatrix} . & . & . \\ . & . & . \\ . & . & . \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

3x4 perpective transform matrix
Note: 3rd column does not matter
because of Z=0

- Since in homogeneous coordinates, scale factor does not matter, the 3x3 matrix is a projective transform between the world plane and the camera image plane.

Continued…

# Outline of Alternate Proof




Transformation From Camera 1 to Camera 2 will be $H_1 H_2^{-1}$ (Why?)

It will be a projective transform if projective transformation operation forms a group (Prove)

# Examples of Projective Transformations