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$$f(y) = \frac{3}{8}y^2, 0 < y < 2$$

1.

a. Mean  $\mu = E[Y]$

$$\mu = E[Y] = \int_0^2 y f(y) dy$$

$$= \int_0^2 y \left( \frac{3}{8} y^2 \right) dy$$

$$= \frac{3}{8} \int_0^2 y^3 dy$$

$$= \frac{3}{8} \left[ \frac{y^4}{4} \right]_0^2$$

$$= \frac{3}{8} \left( \frac{2^4}{4} - 0 \right)$$

$$= \frac{12}{8}$$

$$\mu = 1.5$$

$$E[Y]^2 = \int_0^2 y^2 f(y) dy$$

$$= \int_0^2 y^2 \left( \frac{3}{8} y^2 \right) dy$$

$$= \frac{3}{8} \int_0^2 y^4 dy$$

$$= \frac{3}{8} \left[ \frac{y^5}{5} \right]_0^2$$

$$= \frac{3}{8} \left( \frac{2^5}{5} - 0 \right)$$

$$= \frac{3}{8} \cdot \frac{32}{5}$$

$$= \frac{96}{40}$$

$$= 2.4$$

variance  $\sigma^2$  is:

$$\begin{aligned}\sigma^2 &= E[y^2] - (E[y])^2 \\ &= 2.4 - (1.5)^2 \\ &= 0.15\end{aligned}$$

b. Probability of  $Y > 2$

$$P(y > 2) = \int_2^{\infty} f(y) dy$$

Since the function is only for  $0 < y < 2$

$$P(y > 2) = 0$$

$$c. P(0.5 < Y < 1.5) = \int_{0.5}^{1.5} f(y) dy$$

$$\begin{aligned}&= \int_{0.5}^{1.5} \frac{3}{8} y^2 dy \\ &= \frac{3}{8} \left[ \frac{y^3}{3} \right]_{0.5}^{1.5} \\ &= \frac{3}{8} \left( \frac{(1.5)^3}{3} - \frac{(0.5)^3}{3} \right) \\ &= \frac{3}{8} \left( \frac{3.375}{3} - \frac{0.125}{3} \right) \\ &= \frac{3}{8} \cdot 1.08333 \\ &= 0.40625\end{aligned}$$

$$P(0.5 < Y < 1.5) = 0.406$$

2.

a.

$$P(6-8 \text{ hours}) = 4/20$$

$$P(C|6-8 \text{ hours}) = 2/4$$

$$P(6-8 \text{ hours}, C) = P(6-8 \text{ hours}) \cdot P(C|6-8 \text{ hours})$$

$$\begin{aligned}&= \frac{4}{20} \cdot \frac{2}{4} \\ &= 0.10\end{aligned}$$

b.

$$P(3-5 \text{ hours} | B) = 1/5$$

$$P(B) = 5/20$$

$$P(3-5 \text{ hours}) = 6/20$$

$$P(B|3-5 \text{ hours}) = \frac{P(3-5 \text{ hours}|B)P(B)}{P(3-5 \text{ hours})}$$

$$= \frac{\frac{1}{5} \cdot \frac{5}{20}}{\frac{6}{20}}$$

$$\begin{aligned}
 &= \frac{1}{6} \\
 &= 0.167
 \end{aligned}$$

3.

$$\begin{aligned}
 P(A) &= 0.7 \\
 P(B) &= 0.3 \\
 P(\text{Def} \mid A) &= 0.1 \\
 P(\text{Def} \mid B) &= 0.2
 \end{aligned}$$

$$\begin{aligned}
 P(B, \text{Def}) &= P(B) * P(\text{Def} \mid B) \\
 &= 0.3 * 0.2 \\
 &= 0.06
 \end{aligned}$$

4.

$$\begin{aligned}
 P(M) &= 0.6 \text{ (Probability of being Male)} \\
 P(A) &= 0.5 \text{ (Probability of being over 30)}
 \end{aligned}$$

$$\begin{aligned}
 P(M, A) &= P(M) * P(A) \\
 &= 0.5 * 0.6 \\
 &= 0.3
 \end{aligned}$$

5.

$$\begin{aligned}
 P(M) &= 0.7 \\
 P(E) &= 0.5 \\
 P(M, E) &= 0.3
 \end{aligned}$$

$$P(M \mid E) = ?$$

$$P(M, E) = P(E) * P(M \mid E)$$

$$\begin{aligned}
 P(M \mid E) &= \frac{P(M, E)}{P(E)} \\
 &= \frac{0.3}{0.5} \\
 &= 0.6
 \end{aligned}$$