Amirkabir University Of Technology

(Tehran Polytechnic)

Faculty of Electrical Engineering

EW Course Project

Non-Uniform Antenna Arraty (NULA)

W. Khan, S. Saeed and Atiq-ur-Rehman, "Linear Antenna-Array with Log-increasing Inter-element Spacing and Non-Uniform Weights," 2019 1st Global Power, Energy, and Communication Conference (GPECOM), 2019, pp. 5-9,

. DOI: 10.1109/GPECOM.2019.8778496

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2019 1st Global Power, Energy and Communication Conference (IEEE GPECOM2019), June 12-15, 2019, Cappadocia, Turkey

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***Abstract*—In this paper, linear antenna-array with logarithmically increasing inter-element spacing and non-uniform excitation amplitudes or weights has been investigated for beamwidth (BW), side lobe level (SLL) and directivity (D). This non-uniform linear array has been compared with uniform and non-uniform linear arrays, employing uniform and non-uniform weights, proposed in literature. Conventionally, non-uniform weighting techniques such as binomial, Dolph-Chebhychev, etc. have been applied to suppress side lobes at the cost of increased beamwidth. In this paper, we have shown that applying non-uniform weights to the proposed non-uniform linear array, SLL can be suppressed without compromising beamwidth, resulting in increased directivity. The proposed array has been compared with other linear array geometries and weighting schemes in terms of beamwidth, peak side lobe level (PSLL) and directivity.**

***Index Terms*—non uniform array, beampattern, directivity**

I. Introduction

In wireless communications and radars, antenna arrays have been used for synthesizing beampattern having BW and PSLL as small as possible. Thus, half-power beamwidth (HPBW) and PSLL are the key parameters while comparing the performance of different types of arrays. Directivity is also an important performance parameter which is a meausure of ratio of energy transmitted in the desired direction to the total energy transmitted. Conventionally, linear arrays with uniform inter-element spacing have been widely used[1]. Uniform linear arrays (ULA) with uniform excitation-current amplitudes or weights yield quite small BW but high PSLL. Employing non-uniform weights, such as triangular, binomial or Chebyshev[2], PSLL can be suppressed but the beamwidth increases drastically. Arrays with non-uniform spacing, termed as non-uniform linear array (NULA) in this paper, have also been investigated for the desired beampattern[3],[4]. Heuristic optimization techniques have been applied to find the optimum weights and inter-element distances. In particular, Particle Swarm Optimization (PSO)[5], [6], Genetic Algorithm (GA) [3], [7], Artificial Bee Colony (ABC) algorithm [8] and Firefly algorithm (FA) [4] have been successfully used to design ULAs and NULAs. However, in most of the reported works, e.g. [3], [4], [5], [8], reduced PSLL is accompanied with increased bandwidth (BW) and/or decreased directivity, when compared with ULA. Further, it may be noted that heuristic optimization algorithms are generally computationally expensive.

In this paper, we have proposed a NULA with non-uniform weights. For inter-element spacing, logarithmic function has been employed and the weights have been derived from a standard window function. In literature, a number of window functions are available, but for the proof of concept, only Bartlett-Hanning window function has been used here. The proposed array has been simulated for 9, 11, 15 and 19 elements and compared with various ULAs and NULAs in terms of HPBW, PSLL and directivity. The rest of the paper is organized as follows. In section II, the proposed system model is described. In section III, simulation results are presented and the performance metrics of the proposed and other arrays have been compared. Finally, section IV concludes the paper.

II. System Model

Consider a broadside linear array comprising of an odd number, L, of isotropic antenna elements placed along x-axis centered at origin, as shown in Fig. 1. Let the position of mth element is x_m; $m = -M/2, -M/2+1, ..., M/2$, where $M = L-1$. Array factor (AF) of non-uniform linear array is given by[9]

$$AF = \sum_{m=-M/2}^{M/2} a_m e^{j\psi_m} \qquad (1)$$

and the beampattern is calculated as $|AF|^2$. Here a_m is the weight applied to the signal transmitted by mth element, $\psi_m = 2\pi f_0 x_m \cos\theta/c$, f_0 is the operating frequency of the antenna array, θ is the angle of propagation of electromagnetic waves

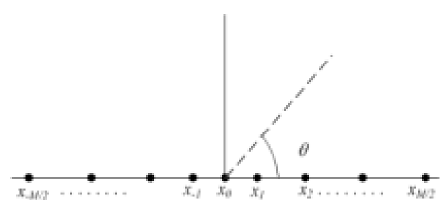


Fig. 1: System Model

I. INTRODUCTION

Antenna arrays have been employed in wireless communications and radars to synthesize beampatterns with the smallest BW and PSLL achievable. When evaluating the performance of different types of arrays, half-power beamwidth (HPBW) and PSLL are the main metrics to consider. Directivity, or the ratio of energy delivered in the intended direction to total energy communicated, is another key performance characteristic. Traditionally, linear arrays with consistent inter-element spacing have been commonly used. ULAs with uniform excitation-current amplitudes or weights have a short BW but a large PSLL.

PSLL can be decreased by using non-uniform weights such as triangular, binomial, or Chebyshev[2], however, the beamwidth rises dramatically. For the intended beampattern, arrays with non-uniform spacing referred to as non-uniform linear array (NULA) in this study, have also been investigated. The ideal weights and inter-element distances were determined using heuristic optimization approaches. Particle Swarm Optimization (PSO), Genetic Algorithm (GA), Artificial Bee Colony (ABC) algorithm, and Firefly algorithm (FA) have all been used to construct ULAs and NULAs effectively.

When compared to ULA, however, lower PSLL is accompanied by higher bandwidth (BW) and/or decreased directivity in the majority of the reported studies. It's also worth noting that heuristic optimization strategies are often expensive to compute.

- **What is the array Half Power Beam Width (HPBW)?**

For a given array the HPBW is a function of the direction of the main beam. The HPBW is minimum for a broadside direction and maximum for the end-fire direction. The beamwidth monotonically increases as the main beam tilts towards the axis of the array.

The HPBW for the broad-side array and the end-fire array is approximately given as:

$$\varphi_{BS} = \frac{\lambda}{dN} = \frac{\lambda}{\text{Length of the array}}$$

$$\varphi_{EF} = \sqrt{\frac{2\lambda}{dN}} = \sqrt{\frac{2\lambda}{\text{Length of the array}}}$$

The HPBW is inversely related to the array length. The larger the array narrower is the beam i.e., the smaller HPBW. For large arrays, the HPBW is approximately taken as half of the BWFN.

- **What is the array Directivity?**

The directivity of the uniform array is given by:

$$D = \frac{4\pi}{\iint |AF|^2 \, d\Omega}$$

For a large array, the integral can be approximated by the solid angle of the main beam and the directivities for the broadside and end-fire arrays are given as:

$$D_{BS} = \frac{4\pi}{2\pi\phi_{BS}} = \frac{2dN}{\lambda}$$

$$D_{EF} = \frac{4\pi}{\pi\left(\frac{\phi_{EF}}{2}\right)^2} = \frac{16}{\left(\frac{2\lambda}{dN}\right)} = \frac{8dN}{\lambda}$$

We propose a NULA with non-uniform weights in this work. The weights were calculated using a normal window function and the logarithmic function was used for inter-element spacing. A variety of window functions are available in the literature, but only the Bartlett-Hanning window function was employed for this proof of concept.

II. MODEL OF THE SYSTEM

As illustrated in **Fig. 1**, a broadside linear array with an odd number, L, of isotropic antenna elements positioned along the x-axis centered at the origin. Let the mth element's location be x_m; m = -M/2; -M/2 + 1, …, M/2, where M = L- 1. The array factor (AF) of a non-uniform linear array is calculated as follows:

$$AF = \sum_{m=-M/2}^{M/2} a_m e^{j\psi_m} \quad (1)$$

$am\ =\ weight\ applied\ to\ the\ signal\ transmitted\ by\ mth\ element;$
$\Psi m\ =\ 2\pi f0xm\ cos\ \theta/c$
$f0\ =\ is\ the\ operating\ frequency\ of\ the\ antenna$
$array;$
$\theta\ =\ is\ the\ angle\ of\ propagation\ of\ electromagnetic\ waves\ for\ the\ x - axis$
$c\ =\ is\ the\ speed\ of\ light.$

and the beampattern is calculated as $|AF|^2$.

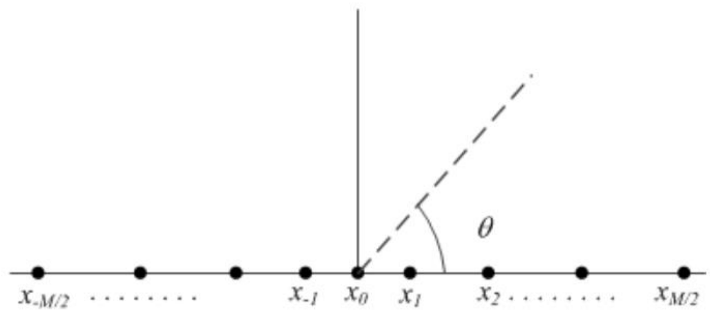


Figure 1. System Model

Consider the fact that the placements of the components are determined by:

$$x_m = \begin{cases} 0, & \text{m} = 0 \\ \text{sign}(m)\frac{\lambda}{2}(|m| + \log_M|m|), & m = \pm1, \pm2, ..., \pm M/2 \end{cases} \tag{2}$$

Here $\log_M(\ )$ represents log base M and λ denotes the wavelength. In this scenario, the positions of antenna elements are;

$$\begin{aligned} &x_0 = 0 \\ &x_{\pm1} = \pm\frac{\lambda}{2} \\ &x_{\pm2} = \pm\frac{\lambda}{2}\{2 + \log_M(2)\} \\ &\vdots \\ &x_{\pm M/2} = \pm\frac{\lambda}{2}\{M + \log_M(M/2)\} \end{aligned} \tag{3}$$

It's worth noting that the minimum inter-element distance is $\lambda/2$ and grows by $\log_M(m)$ times $\lambda/2$ as we move out from the array's center. It should also be noticed that omitting the log term from (2) and (3) results in a uniform linear array with $\lambda/2$ spacing.

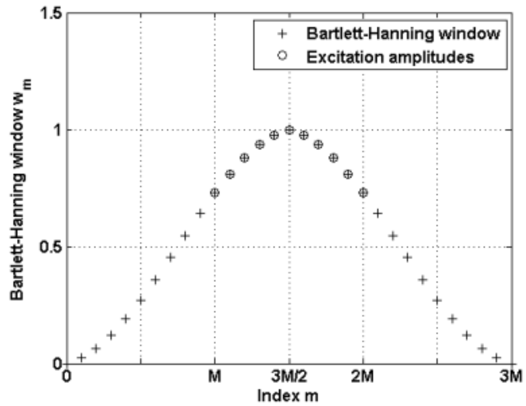
In this paper, non-uniform weights obtained from a typical window function, the Bartlett-Hanning window, were used to verify the suggested geometry. The window function:

$$w_n = 0.62 - 0.48\left|\frac{n}{N} - 0.5\right| + 0.38cos\left[2\pi\left(\frac{n}{N} - 0.5\right)\right]; \quad n = 0, ..., N-1 \tag{4}$$

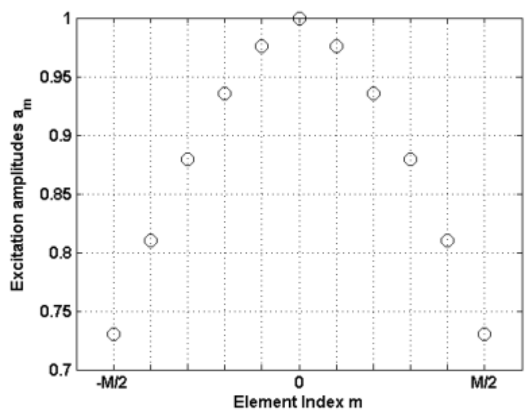
and the array weights a_m are given by;

$$a_m = w_{m+3M/2};\ m = -M/2, ..., M/2 \tag{5}$$

The window length is N = 3M + 1 in this case. As a result, the length of the window is approximately three times the number of array components; however, only the middle area of the window is used for weighting, as shown in **Fig. 2**.



(a) Bartlett-Hanning Window and array weights combined



(b) Weights derived from Bartlett-Hanning window

Figure 2. Bartlett-Hanning Window w_m(N = 3M + 1 = 31) and the array weights a_m

We analyze this article result by solving some problems.

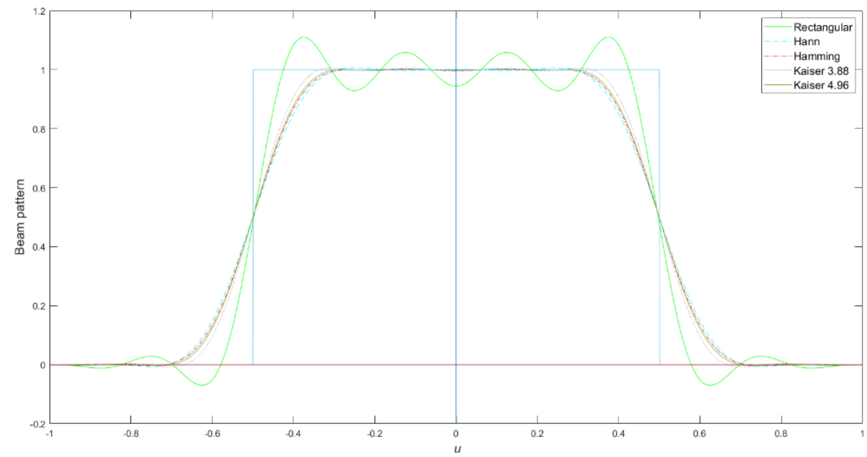
Problem 1:

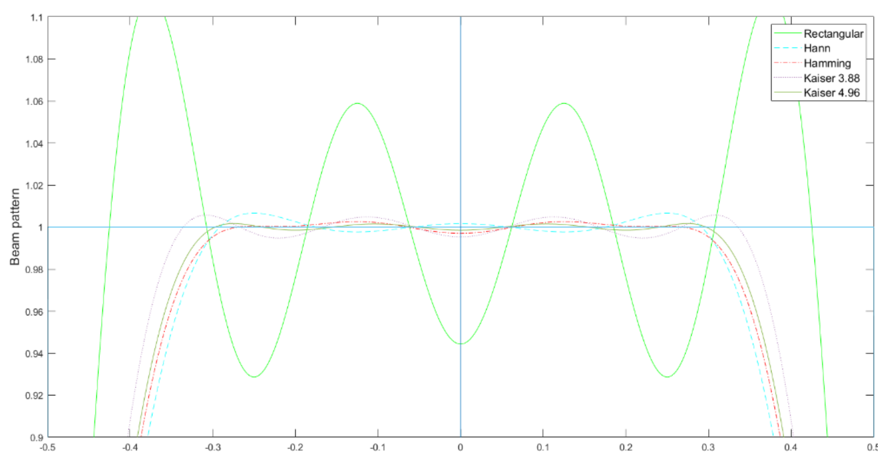
First, regarding the window that is used for weighting we can see this resulting figure that describes the Beam patterns for various windows for N = 16;

$$B_{HANN}(\psi) = c_1 \sum_{m=-\frac{N-1}{2}}^{\frac{N-1}{2}} \left[\frac{\sin(0.5m\pi)}{m\pi}\right] \left[0.5 + 0.5\cos\left(2\pi\frac{m}{N}\right)\right] e^{jm\psi},$$

$$B_{HAMMING}(\psi) = c_2 \sum_{m=-\frac{N-1}{2}}^{\frac{N-1}{2}} \left[\frac{\sin(0.5m\pi)}{m\pi}\right] \left[0.54 + 0.46\cos\left(2\pi\frac{m}{N}\right)\right] e^{jm\psi},$$

$$B_{KAISER}(\psi) = c_3 \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} \left[\frac{\sin(0.5m\pi)}{m\pi}\right] \left[I_0\left(\beta\sqrt{1-\left(\frac{2m}{N}\right)^2}\right)\right] e^{jm\psi},$$





In the Figure above, we show an expanded view of the plateau of the beam pattern. The effect of the windows is to reduce the overshoot and to widen the main lobe (which causes the transition region to widen).

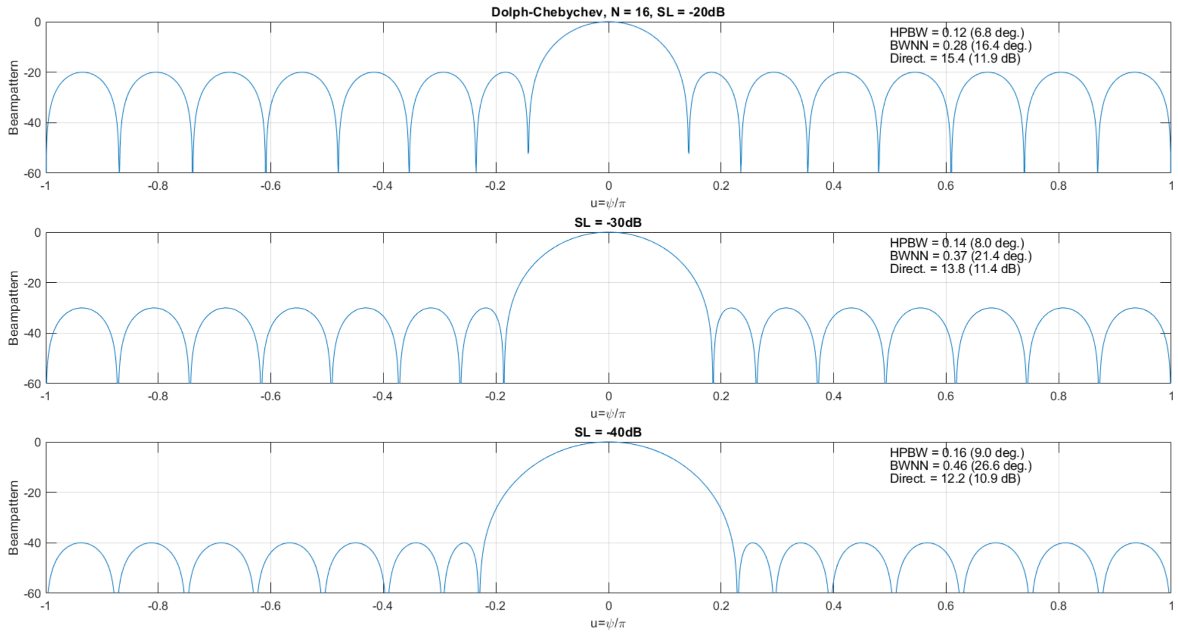
Problem 2:

Second, to clarify the relationship between HPBW, BW, and directivity as we mentioned before we consider a standard 16-element linear array pointed at the broadside.

Fist we find the Dolph-Chebychev weightings for sidelobes of -20dB, -30dB, and -40dB.

After that, we plot the resulting beam pattern and we compute the HPBW. BW, and the directivity.

As we mentioned before we can see that with SL increasing the HPBW and BW get greater but directivity gets smaller.



Problem 3: <u>(Talking about Spatially Non-uniform Linear Arrays)</u>

One application in which we encounter non-uniform arrays is the thinned or sparse array problem. In this case, we start with an iV-element uniform linear array or linear aperture of length *L* that has the desired weighting and associated beam pattern. We then construct a linear array with fewer elements that retain the desirable features of the beam pattern.

By using the technique introduced in the reference [2] "Optimum Array Processing" by *Harry L. Van Trees* we design a beam pattern that maximizes the directivity subject to a -35 dB constraint of the sidelobe; considering a standard linear array with 16 elements.

After that, we compare the resulting BW with that of a 16 element SLA using Dolph-Chebychev weighting.

i. Algorithm review:

Concerning the new algorithm, we should explain it briefly. At first, is due to Bell et al. [3] and is developed previously by Olen and Compton [4] and …

The objective is to find weights that maximize the directivity of the array subject to a set of constraints on the beam pattern, which limits the sidelobe levels. We develop the algorithm in the context of linear arrays of isotropic elements, although it applies to arrays of arbitrary geometry, and with non-isotropic elements.

We assume a linear array of isotropic elements on the 2-axis with *N x 1* array response vector v(u). When the pattern response at the main response axis or pointing direction is equal to one, the directivity is given by:

$$\begin{aligned} D &= \left\{\frac{1}{2}\int_{-1}^{1} |B(u)|^2 du\right\}^{-1} \\ &= \left\{\frac{1}{2}\int_{-1}^{1} |\mathbf{w}^H \mathbf{v}(u)|^2 du\right\}^{-1} \\ &= \left\{\mathbf{w}^H \mathbf{A} \mathbf{w}\right\}^{-1}, \end{aligned}$$

where

$$\mathbf{A} = \frac{1}{2}\int_{-1}^{1} \mathbf{v}(u)\mathbf{v}^H(u)du.$$

The entries in $\mathbf{A}$ are:

$$[\mathbf{A}]_{mn} = \operatorname{sinc}\left(\frac{2\pi}{\lambda}|p_m - p_n|\right),$$

where p_n is the position of the nth element.

Let $v_T = v(u_T)$ be the array response vector for the steering direction. The basic problem is to maximize the directivity (or equivalently minimize the inverse of the directivity), subject to the unity response constraint at the main response axis, that is,

$$\min\ \mathbf{w}^H \mathbf{A} \mathbf{w} \quad \text{s.t. } \mathbf{w}^H \mathbf{v}_T = 1.$$

The solution is

$$\mathbf{w} = \mathbf{A}^{-1}\mathbf{v}_T\left(\mathbf{v}_T^H \mathbf{A}^{-1}\mathbf{v}_T\right)^{-1}.$$

In the special case of a uniform linear array, A = I, and the maximum directivity weight vector is the uniform weight vector steered to the desired direction, $w = \frac{v_T}{N}$. For both uniformly and non-uniformly spaced arrays, we wish to obtain lower sidelobes by sacrificing some directivity. This can be done by partitioning u-space into r sectors, Ω_1, ..., Ω_r, and defining a desired (although not necessarily realizable) beam pattern in each sector,

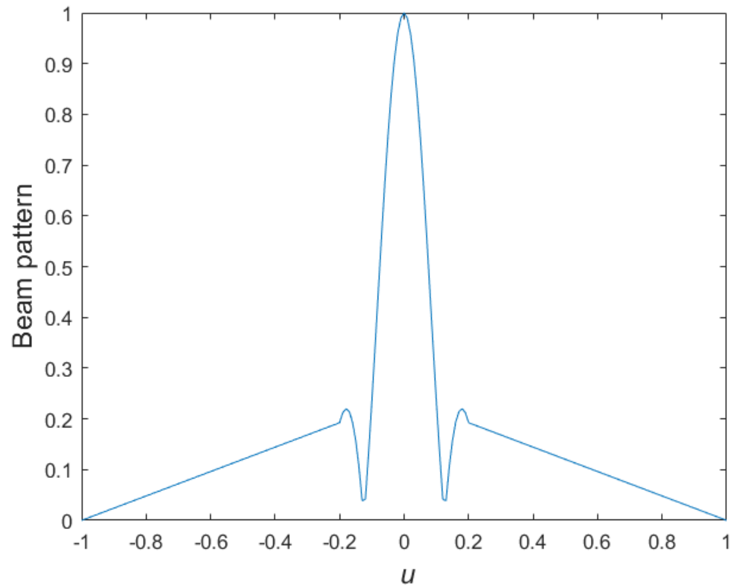


Figure 3. Desired three-sector beam pattern

then limiting deviations between the synthesized and desired beam pattern. A typical desired beam pattern defined in three sectors is shown in **Figure 3**, in which the main beam sector has a beam pattern with some desired main beam shape, and the sidelobe sectors are ideally zero. We assume their sector is a weight vector $w_{d,i}$ that generates the desired beam pattern in the ith sector. Let vector $B_d, i(u) = w_{d,I}^h * V(U)$ be the corresponding beam pattern. The square error between the beam pattern generated by the synthesized weight vector and the desired beam pattern over the region Ω_i is given by;

$$\begin{aligned} \epsilon_i^2 &= \int_{\Omega_i} |B(u) - B_{d,i}(u)|^2 \, du \\ &= \int_{\Omega_i} \left| \mathbf{w}^H \mathbf{v}(u) - \mathbf{w}_{d,i}^H \mathbf{v}(u) \right|^2 du \\ &= (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}), \end{aligned}$$

where

$$\mathbf{Q}_i = \int_{\Omega_i} \mathbf{v}(u) \mathbf{v}(u)^H du.$$

Let Ω_i be the region $(u_i - \Delta_i, u_i + \Delta_i)$. The entries in Q_i are:

$$[\mathbf{Q}_i]_{mn} = e^{j\frac{2\pi}{\lambda}(p_m - p_n)u_i} 2\Delta_i \ \operatorname{sinc}\left(\frac{2\pi\Delta_i}{\lambda} |p_m - p_n| \right).$$

Now we can maximize directivity subject to constraints on the pattern error as follows:

$$
\begin{aligned}
\min \quad & \mathbf{w}^H \mathbf{A} \mathbf{w} \quad \text{s.t. } \mathbf{w}^H \mathbf{v}_T = 1 \\
\text{s.t.} \quad & (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}) \leq L_i \quad\quad i = 1 \ldots r.
\end{aligned}
$$

We define;

$$
\begin{aligned}
F = \quad & \mathbf{w}^H \mathbf{A} \mathbf{w} + \lambda_0 (\mathbf{w}^H \mathbf{v}_T - 1) + \lambda_0^* (\mathbf{v}_T^H \mathbf{w} - 1) \\
& + \sum_{i-1}^{r} \lambda_i (\mathbf{w} - \mathbf{w}_{d,i})^H \mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i}).
\end{aligned}
$$

Differentiating with respect to $\boldsymbol{w}^{\boldsymbol{H}}$ and setting the result equal to zero gives

$$
\mathbf{A}\mathbf{w} + \lambda_0 \mathbf{v}_T^H + \sum_{i=1}^{r} \lambda_i [\mathbf{Q}_i (\mathbf{w} - \mathbf{w}_{d,i})] = \mathbf{0}. \tag{4}
$$

Defining

$$
\mathbf{A}_Q = \mathbf{A} + \sum_{i=1}^{r} \lambda_i \mathbf{Q}_i,
$$

and

$$
\mathbf{w}_Q = \sum_{i=1}^{r} \lambda_i \mathbf{Q}_i \mathbf{w}_{d,i},
$$

we can write (4) as

$$
\mathbf{w} = -\lambda_0 \mathbf{A}_Q^{-1} \mathbf{v}_T + \mathbf{A}_Q^{-1} \mathbf{w}_Q. \tag{5}
$$

Solving for $\boldsymbol{\lambda}_0$ and substituting the result into (5) gives:

$$
\begin{aligned}
\mathbf{w} = \quad & \mathbf{A}_Q^{-1} \mathbf{v}_T \left(\mathbf{v}_T^H \mathbf{A}_Q^{-1} \mathbf{v}_T \right)^{-1} \\
& + \left[\mathbf{A}_Q^{-1} - \mathbf{A}_Q^{-1} \mathbf{v}_T \left(\mathbf{v}_T^H \mathbf{A}_Q^{-1} \mathbf{v}_T \right)^{-1} \mathbf{v}_T^H \mathbf{A}_Q^{-1} \right] \mathbf{w}_Q.
\end{aligned} \tag{6}
$$

We can obtain tight sidelobe control by defining a set of small sectors in the sidelobe region, as shown in **Figure 4**, and setting the desired beam pattern to zero in these regions. The desired weight vector in each sector is just the all-zero vector.

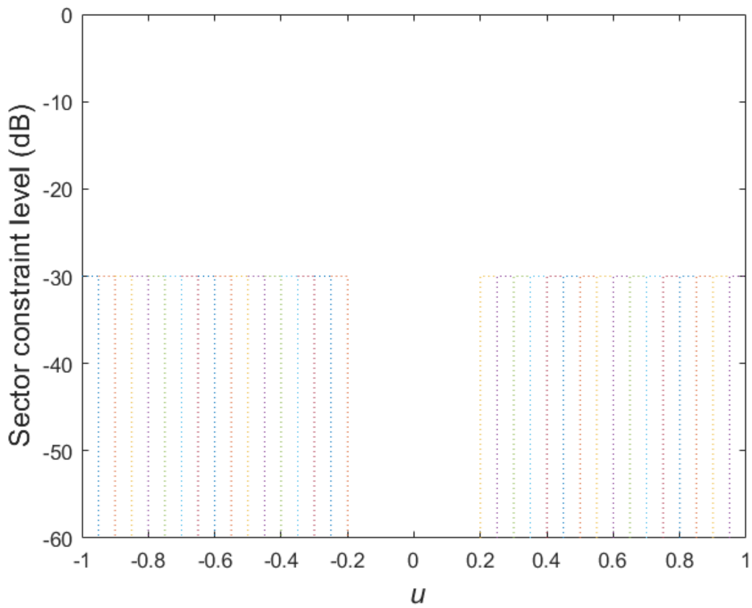


Figure 4. Sidelobe sectors.

In the limit of infinitesimally small sectors, the pattern error constraints become constraints on the magnitude squared of the beam pattern at every point in the sidelobe region. The allowed deviation can be set to the maximum allowable sidelobe level, and the sidelobe levels can be controlled directly. By choosing wider but relatively small sectors, we can still control sidelobe levels fairly accurately. Furthermore, if we choose to constrain pattern "error" only in the sidelobe region and not in the main beam, the desired weight vector in each constrained sector will be zero, and the second term in (6) drops out, so the weight vector becomes;

$$\mathbf{w} = \mathbf{A}_Q^{-1}\mathbf{v}_T \left(\mathbf{v}_T^H \mathbf{A}_Q^{-1}\mathbf{v}_T\right)^{-1}.$$

In this expression, a weighted sum of loading matrices $Q_i, i = 1 \ldots r$ is added to $\mathbf{A}$. The loading factors balance the maximum directivity pattern with the desired low sidelobe level pattern. There is generally a set of optimum loading levels $\lambda_i, i = 1 \ldots r$ that satisfies the constraints; however, there is no closed-form solution for the loading levels, even when $r = 1$. It can be shown that the mean-square pattern error decreases with increasing $\boldsymbol{\lambda}$i, but at the expense of decreased directivity. An iterative procedure can be used to adjust the loading levels to achieve the sidelobe level constraints. At each iteration, the pattern errors are computed and checked against the constraints. If a constraint is exceeded, the loading for that sector is increased, and the weights are updated.

One way to achieve fast convergence is to let the loading increment at the p_{th} iteration, $\delta_i^{(p)}$ be a fraction of the current loading value, that is, $\delta_i^{(p)} = \alpha\lambda_i^{(p)}$. This requires that the initial loading level be non-zero. One possibility is to initialize all of the loading levels to some small value, such as, $\lambda_i^{(0)} = \boldsymbol{\lambda_0}, \boldsymbol{i} = \mathbf{1} \ldots \boldsymbol{r}$. If the initial loading is small weight enough, the initial weight vector is essentially the maximum directivity weight vector. The update procedure is:

$$
\begin{array}{rcl}
\text{if} & & \mathbf{w}^{(p-1)H}\mathbf{Q}_i\mathbf{w}^{(p-1)} > L_i, \\
\text{then} & & \delta_i^{(p)} = \alpha\lambda_i^{(p-1)}, \\
\text{else} & & \delta_i^{(p)} = 0. \\
\lambda_i^{(p)} & = & \lambda_i^{(p)} + \delta_i^{(p)}. \\
\mathbf{A}_Q^{(p)} & = & \mathbf{A}_Q^{(p-1)} + \sum_{i=1}^{r} \delta_i^{(p)}\mathbf{Q}_i. \\
\mathbf{w}^{(p)} & = & \left(\mathbf{A}_Q^{(p)}\right)^{-1}\mathbf{v}_T\left\{\mathbf{v}_T^H\left(\mathbf{A}_Q^{(p)}\right)^{-1}\mathbf{v}_T\right\}^{-1}.
\end{array}
$$

The iteration is repeated until a convergence criterion is satisfied. It is usually necessary to adjust the sectors included in the sidelobe region at each iteration. As the sidelobes are pushed down, the main beam widens, and some sectors previously in the sidelobe region fall in the main beam. The constraints on these sectors must then be dropped.

ii. <u>Hello World!</u>

Now let's move on to the question we asked at the beginning of this section;
After applying this algorithm to the data of our question we have the result below, with $BW_{NNdc} = 0.4156$.

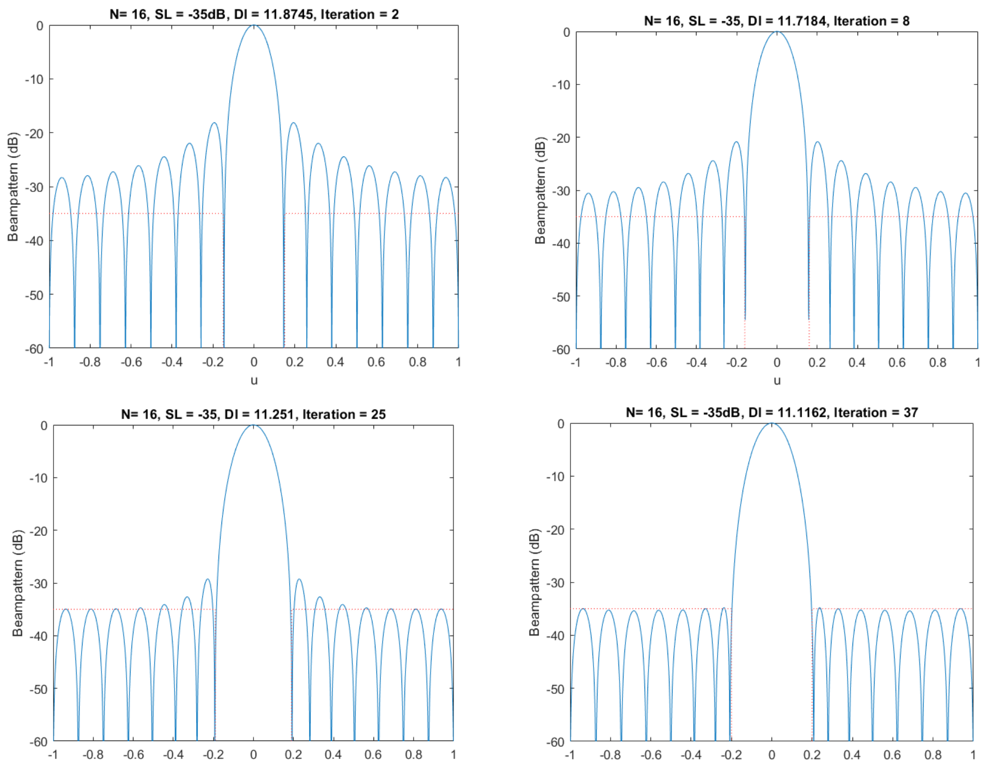
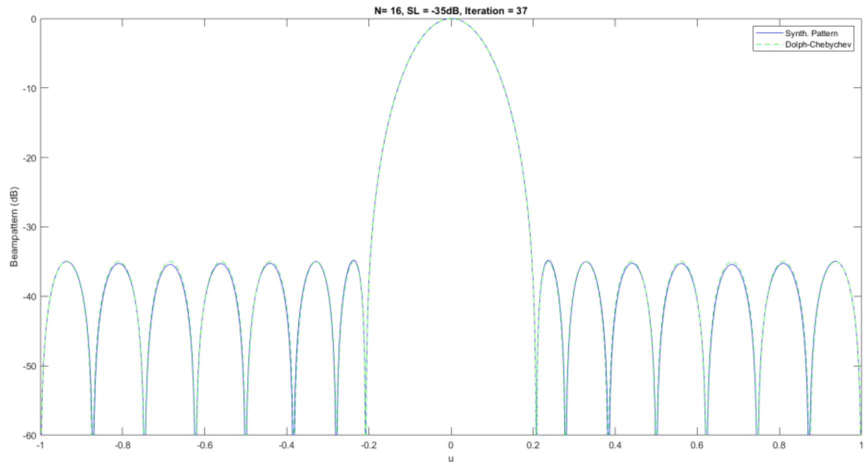


Figure 5. Beam pattern evolution for 16-element linear array and -35 dB sidelobes. (a) 2nd iteration; (b) 8th iteration; (c) 25th iteration; (d) 37th iteration;



- ➔ Fine SL control can be achieved by choosing small sectors, and small loading increments α.
- ➔ Faster convergence and lower complexity are donated with large sectors and α.
- ➔ Pattern essentially same as Dolph-Chebyshev.

III. SUGGESTED METHOD

As for the method I suggested, I explain it as follows.

A greater number of antenna elements can be used to produce a higher angular resolution. A linear array with λ /2 spacing between elements, where is the free-space wavelength, is a common technique. The design's purpose is to strike a balance between aperture size and antenna channel count while retaining the appropriate beam width and side-lobe level. 16-element λ /2 spaced array, 16-element λ spaced array, 16-element non-uniformly spaced array, and 26-element λ /2 spaced array are all compared.

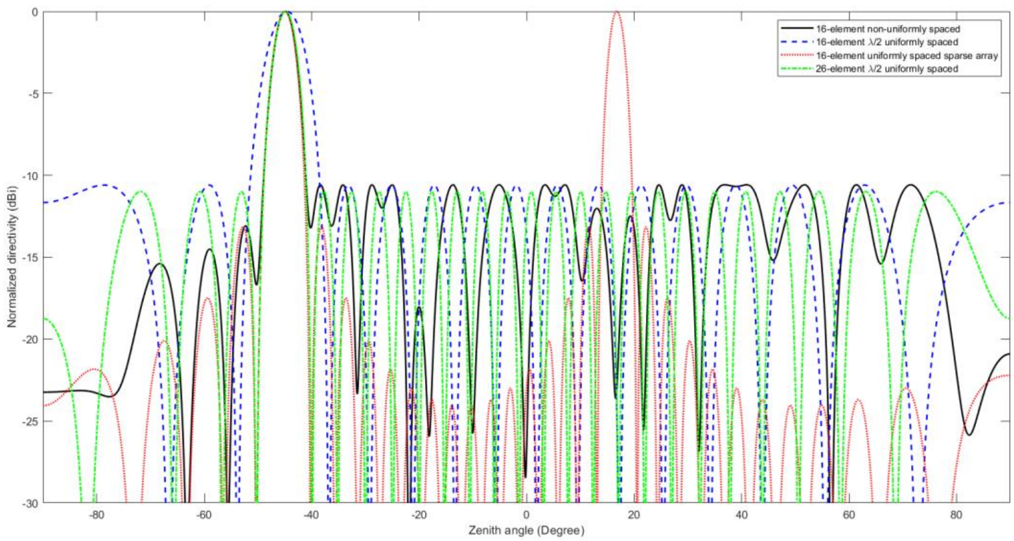


Figure 6. Comparison of the radiation patterns for four different linear array distributions with the main lobe steered to -45º

Their far-field radiation patterns when the main beam is steered to -450 are shown in Fig. 6. The 16-element $\boldsymbol{\lambda}$/2 spaced array is found to have the widest beamwidth. The 16-element non-uniformly spaced array has a beam width that is similar to the 26-element $\boldsymbol{\lambda}$/2 spaced array but with a 33% percent reduction in antenna elements. The 16-element $\boldsymbol{\lambda}$ spaced array produces equal beamwidth and sidelobe level as the non-uniformly spaced array with the same aperture size. The $\boldsymbol{\lambda}$- spaced array, on the other hand, has a grating lobe problem that may be solved by employing a non-uniformly spaced array.

The non-uniform distribution of the 16-element linear array was found using a random search approach in the work of an investigator developing a portable radar[5]. MATLAB is used to analyze 100,000 random non-uniform distributions using the convex optimization software CVX [6], which can take more than 3 hours. But in our case, we just want to explain this approach for this we just show how its work.

```
------------------------------------------------------------
Status: Solved
Optimal value (cvx_optval): +0.512872
```

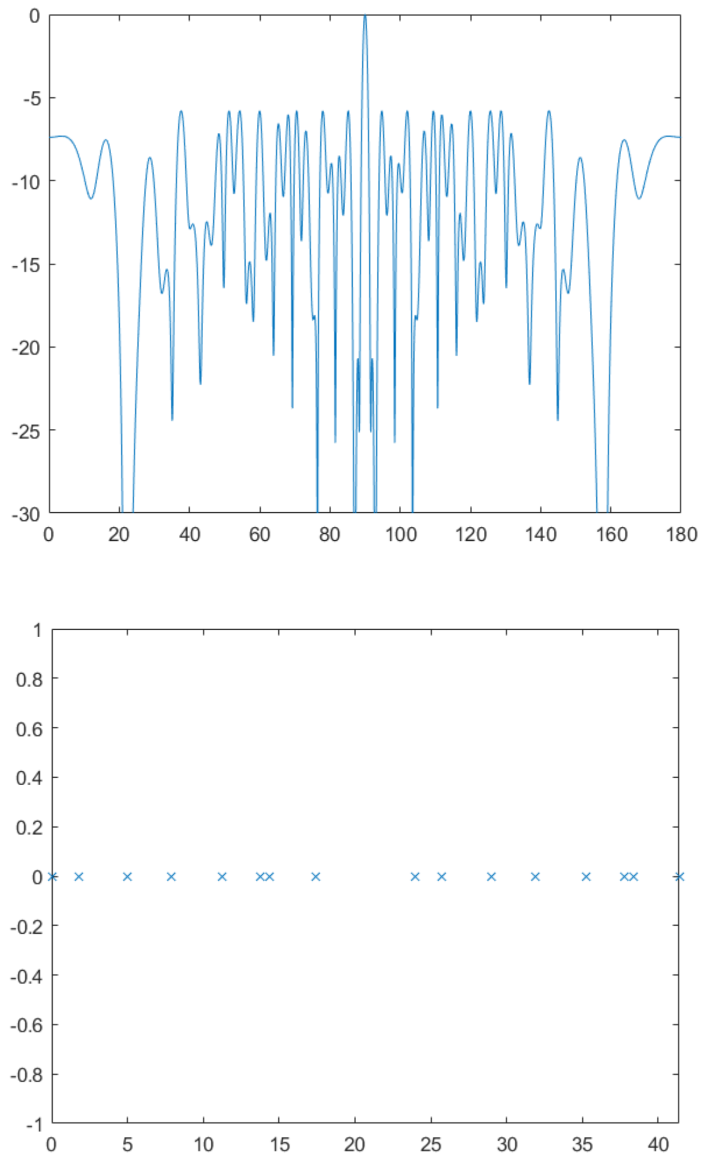


Figure 7. Results from code [*LA_random_spacing.m*]

To acquire the smallest main lobe width and side-lobe level, a stochastic search is performed to identify the array's distribution and weighting values. Among 100,000 random distributions, Fig. 7 illustrates the best distribution for the non-uniform array, where [x_1, x_2, x_3, .., x_{16}] are the locations of the array members.

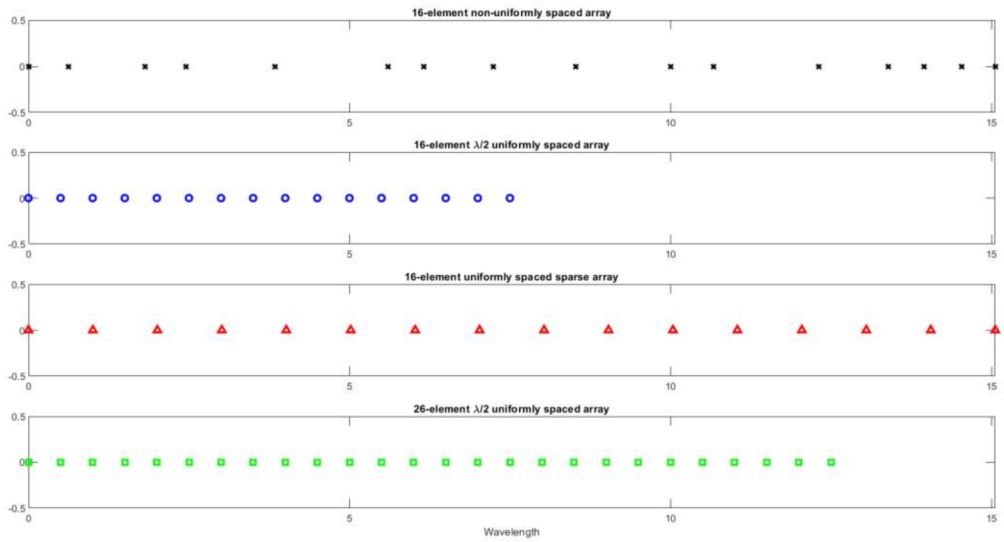


Figure 8. Distribution of the array elements (related to Fig.6).

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