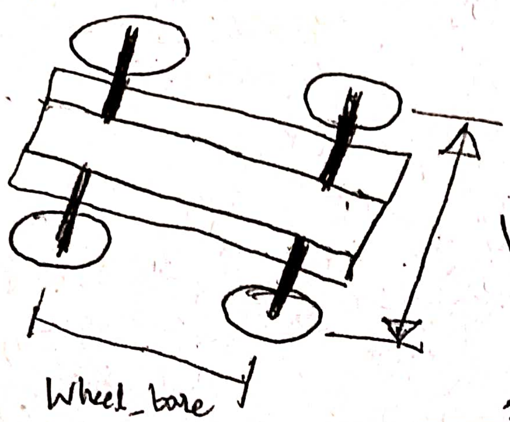
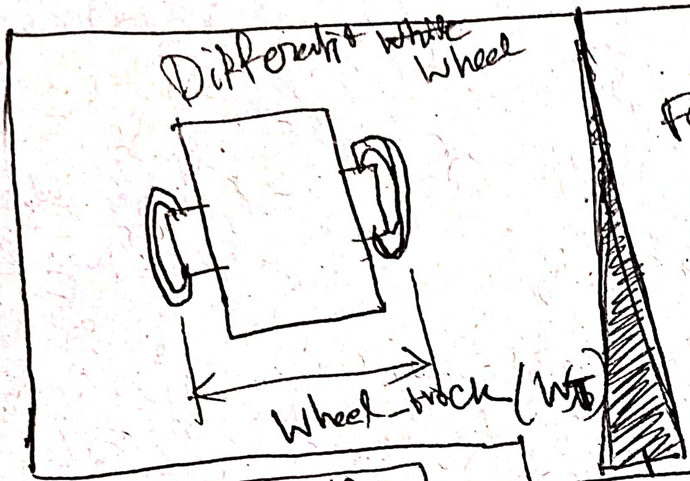
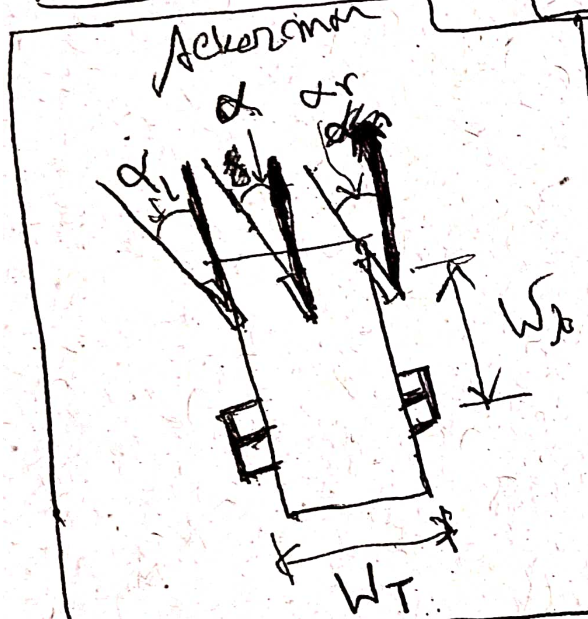
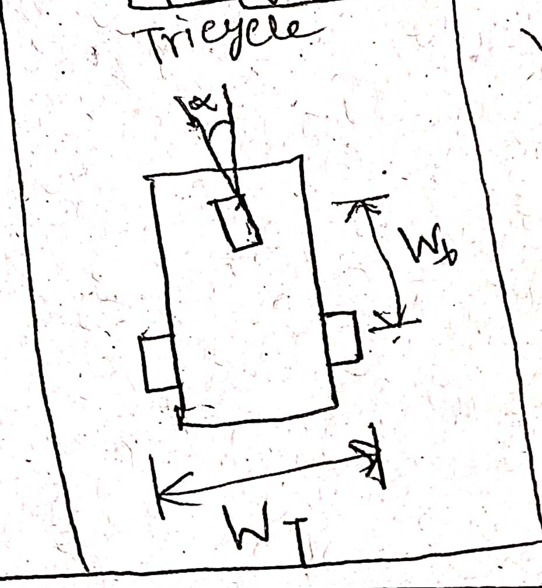
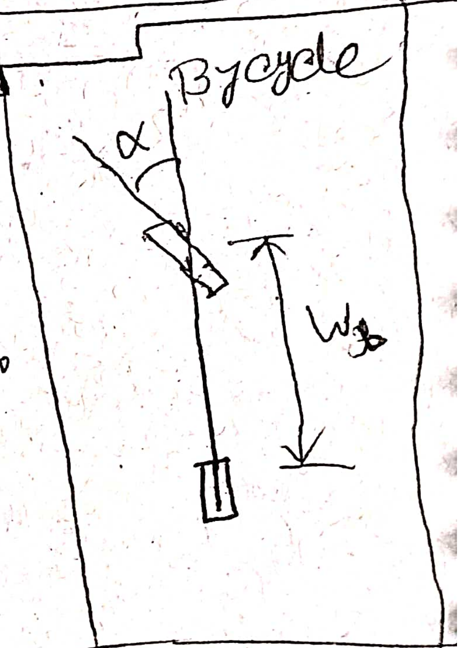


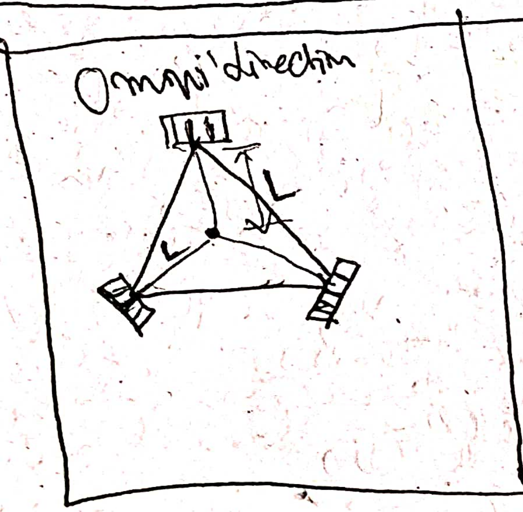
r = radius of wheel
W_b = wheel_bore
W_T = wheel_track

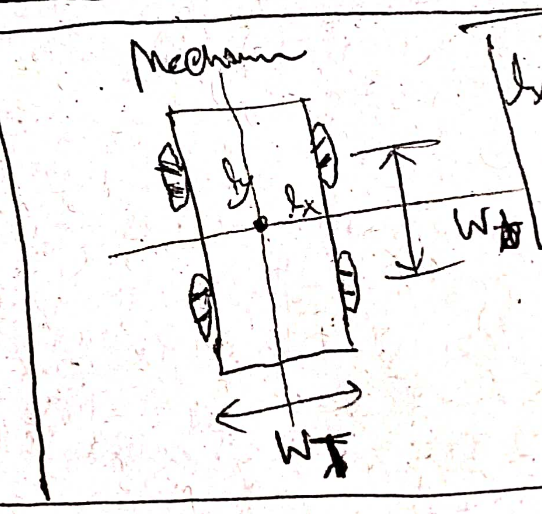












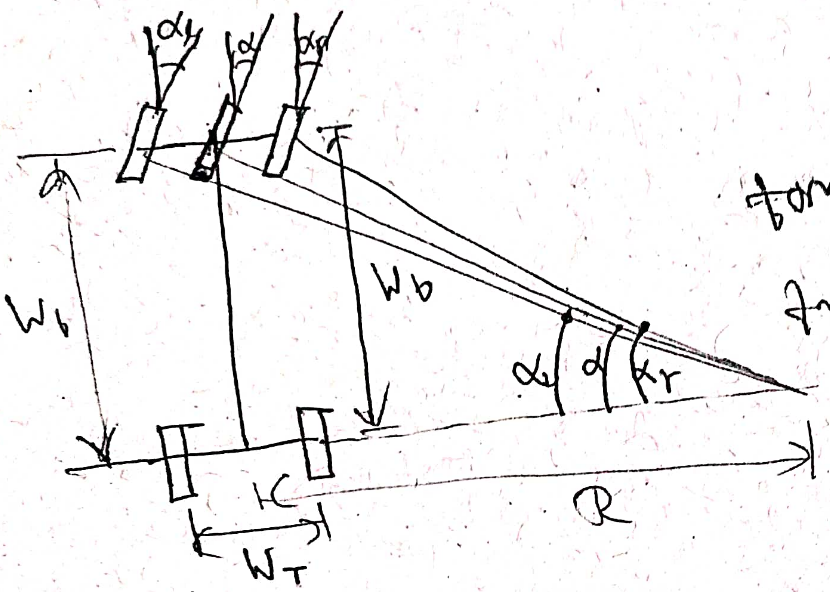
$l_x = \frac{W_T}{2}$

$l_y = \frac{W_b}{2}$

Steering Mechanism

Bycycle | Tricycle | Ackermann

Input $(V_x, \dot{\theta})$ or (V_x, ω_z)



$$\tan\alpha = \frac{W_b}{R} \Rightarrow \cot\alpha = \frac{R}{W_b}$$

$$\tan\alpha_l = \frac{W_b}{R + \frac{W_T}{2}}$$

$$\tan\alpha_r = \frac{W_b}{R - \frac{W_T}{2}}$$

$$\frac{1}{\tan\alpha_l} - \frac{1}{\tan\alpha_r} = \cot\alpha_l - \cot\alpha_r$$

$$= \frac{R + W_T/2}{W_b} - \frac{R - W_T/2}{W_b}$$

$$\boxed{\cot\alpha_l - \cot\alpha_r = \frac{W_T}{W_b}} \Rightarrow \boxed{\cot\alpha_r = \cot\alpha_l - \frac{W_T}{W_b}}$$

$$\cot\alpha_r - \cot\alpha = \frac{R - W_T/2}{W_b} - \frac{R}{W_b} = -\frac{W_T}{2W_b}$$

So, $\Rightarrow \boxed{\cot\alpha_r = \cot\alpha - \frac{W_T}{2W_b}}$

$$\cot\alpha_l - \cot\alpha = \frac{R + W_T/2}{W_b} - \frac{R}{W_b} = \frac{W_T}{2W_b}$$

$$\Rightarrow \boxed{\cot\alpha_l = \cot\alpha + \frac{W_T}{2W_b}}$$

$$\cot \alpha_r = \cot \alpha - \frac{W_T}{2W_b}$$

$$\cot \alpha_l = \cot \alpha + \frac{W_T}{2W_b}$$

$$\Rightarrow \cancel{\alpha_r} \quad \tan \alpha_r = \frac{1}{\cot \alpha_r}$$

$$\Rightarrow \tan \alpha_r = \frac{1}{\cot \alpha - \frac{W_T}{2W_b}}$$

$$= \frac{1}{\frac{\cos \alpha}{\sin \alpha} - \frac{W_T}{2W_b}}$$

$$\boxed{\tan \alpha_r = \frac{2W_b \sin \alpha}{2W_b \cos \alpha - W_T \sin \alpha}}$$

$$\Rightarrow \tan \alpha_l = \frac{1}{\cot \alpha_l} = \frac{1}{\cos \alpha + \frac{W_T}{2W_b}}$$

$$\Rightarrow \boxed{\tan \alpha_l = \frac{2W_b \sin \alpha}{2W_b \cos \alpha + W_T \sin \alpha}}$$

$$\alpha_r = \operatorname{atan}\left(\frac{2W_b \sin\alpha}{2W_b \cos\alpha - W_T \sin\alpha}\right)$$

$$\alpha_l = \operatorname{atan}\left(\frac{2W_b \sin\alpha}{2W_b \cos\alpha + W_T \sin\alpha}\right)$$

$$\alpha = \operatorname{atan}\left(\frac{\omega_z * W_b}{V_x}\right)$$

$$\alpha = \operatorname{atan}\left(\frac{\dot{\theta} * W_b}{V_x}\right)$$

if $\dot{\theta} = 0$, $\alpha = 0$

if $V_x = 0$, $\alpha \Rightarrow \infty$

wheel_radius = r

Control I/P

$$\begin{bmatrix} V_x \\ V_y \\ \omega_z \end{bmatrix}$$

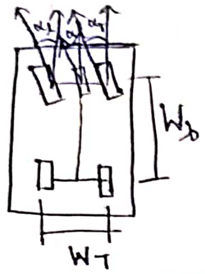
$V_y \rightarrow 0$ (no lateral slip $V_y = 0$)

So,

$$\begin{bmatrix} V_x \\ \omega_z \end{bmatrix}$$

or,

$$\begin{bmatrix} V_x \\ \dot{\theta} \end{bmatrix}$$



if ($V_x == 0$)
or,
$\alpha = 0$
if ($\dot{\theta} == 0$)
$\alpha = 0$

if ($V_x == 0$ && $\dot{\theta}$!= 0)
if ($\dot{\theta} > 0$) {
$\alpha = \pi$
}
else
{
$\alpha = -\pi$
}

$$\boxed{W_s = \frac{\text{abs}(\dot{\theta}) * W_b}{r}}$$

else {

$$\alpha = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

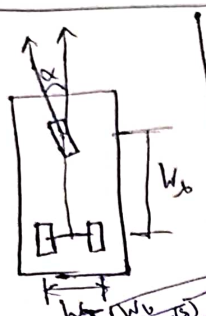
$$W_s = \frac{V_x}{r \times \cos(\delta)}$$

}

$$R = \frac{W_b}{\tan(\delta)}$$

$$\begin{cases} W_r = W_s\left(1 + \frac{W_T}{2R}\right) \\ W_l = W_s\left(1 - \frac{W_T}{2R}\right) \end{cases}$$

$$\begin{cases} \alpha_r = \text{atan2}\left(\frac{2W_b \sin\alpha}{2W_b\cos\alpha - W_T\sin\alpha}\right) \\ \alpha_l = \text{atan2}\left(\frac{2W_b \sin\alpha}{2W_b\cos\alpha + W_T\sin\alpha}\right) \end{cases}$$



$R = \frac{W_b}{\tan(\delta)}$

$$\alpha = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$V_r = V\left(1 + \frac{W_T}{2R}\right)$$

$$V_l = V\left(1 - \frac{W_T}{2R}\right)$$

if ($V_x == 0$)
$\alpha = (\dot{\theta} > 0\ ?\ \pi : -\pi)$

$$W_s = \frac{\dot{\theta} W_b}{r}$$

else

$$W_r = W_s\left(1 + \frac{W_T}{2R}\right)$$

$$W_l = W_s\left(1 - \frac{W_T}{2R}\right)$$

$$\alpha = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$W_s = \frac{V_x}{r\cos(\delta)}$$

$$R = \frac{W_b}{\tan(\delta)}$$

$$W_s = \frac{V_x}{r} \frac{\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}}{V_x}$$

$$W_s = \frac{1}{r}\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}$$

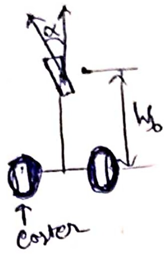
$$\Rightarrow \boxed{V_s = \sqrt{V_x^2 + \dot{\theta}^2 W_b^2}}$$

So,

$$W_r = \frac{1}{r}\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}\left(1 + \frac{W_T}{2R}\right)$$

$$W_l = \frac{1}{r}\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}\left(1 - \frac{W_T}{2R}\right)$$

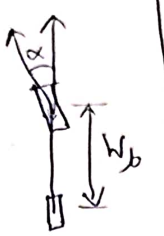
$$\Rightarrow \boxed{\begin{aligned} V_r &= V\left(1 + \frac{W_T}{2R}\right) \\ V_l &= V\left(1 - \frac{W_T}{2R}\right) \end{aligned}}$$



$$\alpha = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$V = \sqrt{(\dot{\theta} W_b)^2 + V_x^2}$$

Because $W_T = 0$



$$\alpha = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$W_s = \frac{V_x}{r\cos(\delta)}$$

if ($V_x == 0$)
{

$$W_s = \frac{\dot{\theta} W_b}{r}$$

}

~~turning radius~~ = R

steer_pos = δ

$$\delta = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$\delta = \text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$\cos\left(\text{atan}\left(\frac{\dot{\theta} W_b}{V_x}\right)\right) = \frac{1}{\sqrt{1 + \frac{\dot{\theta}^2 W_b^2}{V_x^2}}}$$

$$\because \cos(\text{atan}\,\theta) = \frac{1}{\sqrt{1 + \theta^2}}$$

wheel_radius = r

Control I/P

$$\begin{bmatrix} V_x \\ V_y \\ \omega_z \end{bmatrix} \quad V_y \to 0 \ (\text{no lateral slip } V_y = 0)$$

So,

$$\begin{bmatrix} V_x \\ \omega_z \end{bmatrix}$$

or,

$$\begin{bmatrix} V_x \\ \dot{\theta} \end{bmatrix}$$

~~turning radius = R~~

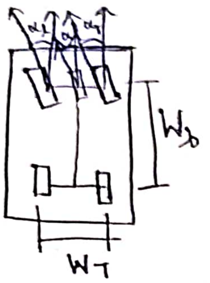
Steer_pos = δ

$$\delta = \tan^{-1}\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$\delta = \arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$\cos\left(\arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)\right) = \left(\frac{1}{\sqrt{1 + \frac{\dot{\theta}^2 W_b^2}{V_x^2}}}\right)$$

$$\because \cos(\arctan\theta) = \frac{1}{\sqrt{1+\theta^2}}$$



if $(V_x == 0)$
or, $\alpha = 0$
if $(\dot{\theta} == 0)$
$\alpha = 0$

if $(V_x == 0$ && $\dot{\theta}\ != 0)$
if $(\dot{\theta} > 0)$ {
$\alpha = \pi$
}
else
{
$\alpha = -\pi$
}

$$\boxed{W_s = \frac{\text{abs}(\dot{\theta}) * W_b}{r}}$$

else {

$$\alpha = \arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

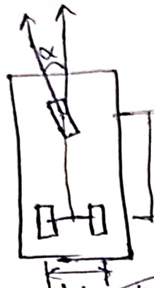
$$W_s = \frac{V_x}{r \times \cos(\theta)}$$

}

$$R = \frac{W_b}{\tan(\delta)}$$

$$\begin{cases} W_r = W_s\left(1 + \frac{W_T}{2R}\right) \\ W_l = W_s\left(1 - \frac{W_T}{2R}\right) \end{cases}$$

$$\begin{cases} \alpha_r = \text{atan2}\left(\frac{2W_b \sin\alpha}{2W_b\cos\alpha - W_T\sin\alpha}\right) \\ \alpha_l = \text{atan2}\left(\frac{2W_b \sin\alpha}{2W_b\cos\alpha + W_T\sin\alpha}\right) \end{cases}$$



W_T / W_b / $\tan(\delta)$

R =

$$\alpha = \arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$V_r = V\left(1 + \frac{W_T}{2R}\right)$$

$$V_l = V\left(1 - \frac{W_T}{2R}\right)$$

if $(V_x == 0)$
$\alpha = (\dot{\theta} > 0\ ?\ \pi : -\pi)$

$$W_s = \frac{\dot{\theta} W_b}{r}$$

else

$$W_r = W_s\left(1 + \frac{W_T}{2R}\right)$$

$$W_l = W_s\left(1 - \frac{W_T}{2R}\right)$$

$$\alpha = \arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$W_s = \frac{V_x}{r\cos(\delta)}$$

$$R = \frac{W_b}{\tan(\delta)}$$

$$W_s = \frac{V_x}{r} \frac{\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}}{V_x}$$

$$W_s = \frac{1}{r}\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}$$

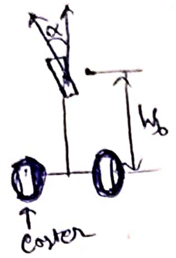
$$\Rightarrow \boxed{V_s = \sqrt{V_x^2 + \dot{\theta}^2 W_b^2}}$$

So,

$$W_r = \frac{1}{r}\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}\left(1 + \frac{W_T}{2R}\right)$$

$$W_l = \frac{1}{r}\sqrt{V_x^2 + \dot{\theta}^2 W_b^2}\left(1 - \frac{W_T}{2R}\right)$$

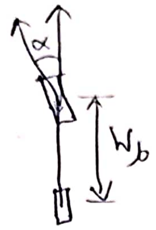
$$\Rightarrow \boxed{\begin{aligned} V_r &= V\left(1 + \frac{W_T}{2R}\right) \\ V_l &= V\left(1 - \frac{W_T}{2R}\right) \end{aligned}}$$



$$\alpha = \arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$V = \sqrt{(\dot{\theta} W_b)^2 + V_x^2}$$

because $W_T = 0$



$$\alpha = \arctan\left(\frac{\dot{\theta} W_b}{V_x}\right)$$

$$W_s = \frac{V_x}{r\cos(\delta)}$$

if $(V_x == 0)$ {

$$W_s = \frac{\dot{\theta} W_b}{r}$$

}

Ackerman_config | Without encoder |

$$\boxed{\alpha = \text{atan}\left(\frac{\dot\theta W_b}{V_x}\right)}$$

$$W_S = \frac{V_x}{r\cos(\alpha)} = \frac{1}{r}\left(\sqrt{V_x^2 + \dot\theta^2 W_b^2}\right)$$

$$\therefore \quad V = r W_S$$

$$\Rightarrow \boxed{V = \sqrt{V_x^2 + \dot\theta^2 W_b^2}}$$

if ($V_x == 0$ ~~&&~~ or $\dot\theta == 0$)
{
$\alpha = 0$
$V = 0$
}

~~else~~ if ($V_x == 0$ && $\dot\theta != 0$)
{

$$\boxed{\text{atan}\left(\frac{\dot\theta W_1}{0}\right) \Rightarrow \text{atan}(\infty) \Rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$$

$$\alpha = \dot\theta > 0 \,?\, \pi : -\pi$$

$$\boxed{V = W_S r} \Rightarrow V = \dot\theta W_b \quad \left(\because \sqrt{0^2 + \dot\theta^2 W_b^2} \Uparrow V = 0\right)$$

}

else {

if (α < 1e-6) $\boxed{\text{means } \alpha \to 0}$ {

$$\alpha = \text{atan}\left(\frac{\dot\theta W_b}{V_x}\right) \}$$

$$V = \sqrt{V_x^2 + \dot\theta^2 W_b^2} \}$$

$\boxed{V = V_r = V_l \quad \alpha = \alpha_r = \alpha_l}$
}

else {

$$R = \frac{W_b}{\tan(\delta)}$$

$$V_r = V\left(1 + \frac{W_T}{2R}\right)$$

$$V_l = V\left(1 - \frac{W_T}{2R}\right)$$

$$\alpha_r = \text{atan2}\left(\frac{2W_b \sin\alpha}{2W_b\cos\alpha - W_T\sin\alpha}\right)$$

$$\alpha_l = \text{atan2}\left(\frac{2W_b \sin\alpha}{2W_b\cos\alpha + W_T\sin\alpha}\right)$$

}
}

| With encoder |

angular

if (encoder == 1)
{

$$\boxed{W_S = \frac{V_x}{r\cos(\delta)}} \quad \boxed{R = \frac{W_b}{\tan(\delta)}}$$

δ = encoder given steering angle

if two steering angle is there

δ_l δ_r

$$\boxed{\delta = \left(\frac{\delta_l + \delta_r}{2}\right)}$$

else $(\delta) \leftarrow$ one encoder val
}

position | $S_{cur} = S * r$ = traction_pos * radius

If Two wheel (traction 2 wheel)

$$\Delta S_r = S_{cur}^{(r)} - S_{prev}^{(r)}$$

$$\Delta S_l = S_{cur}^{(l)} - S_{prev}^{(l)}$$

$$\Delta S = \left(\frac{\Delta S_l + \Delta S_r}{2}\right)$$

Linear velocity

$$\to V = \frac{\Delta S}{\Delta t}$$

angular

$$\to \omega = \frac{\tan(\delta) * V}{W_b}$$

if one traction wheel

$\boxed{S_{cur} = S * r = \text{traction\_pos} * \text{radius}}$

$$\Delta S = S_{cur} - S_{prev}$$

$$\boxed{V = \frac{\Delta S}{\Delta t} \qquad \omega = \left(\frac{\tan\delta * V}{W_b}\right)}$$

Tricycle config

↳ Tricycle with Three power wheel (Two wheels for drive and one for steer)

Type(I)

if ($V_x == 0$ && $\dot{\theta} == 0$)
{
$\alpha = 0.0$;
$V = 0.0$
}
else if ($V_x == 0$ && $\dot{\theta} != 0$)
{
$\alpha = \dot{\theta} > 0 \, ? \, \pi : -\pi$;
$V = \dot{\theta} W_b$
}
else if ($\alpha < 1e-6$)
{
$\alpha = \text{atan}\left(\dfrac{\dot{\theta} W_b}{V_x}\right)$

$\cancel{\alpha}$

$V = \sqrt{V_x^2 + \dot{\theta}^2 W_b^2}$ $(\because V = V_r = V_l)$
}
else
{
$R = \dfrac{W_b}{\tan(\alpha)}$

$V_r = V\left(1 + \dfrac{W_T}{2R}\right)$

$V_l = V\left(1 - \dfrac{W_T}{2R}\right)$
}
}

Bycycle case

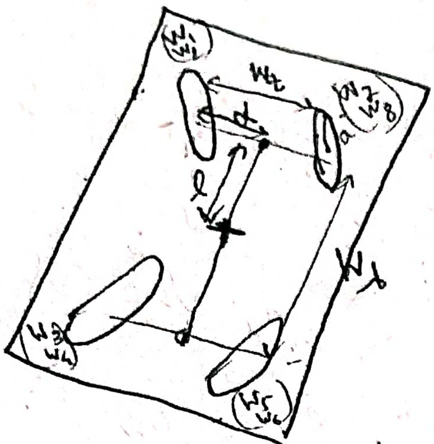
Same like (II)

Type II (One power wheel and one steering)

if ($V_x == 0$ && $\dot{\theta} == 0$)
{ $\alpha = 0$; $V = 0.0$;
}
if ($V_x == 0$ && $\dot{\theta} != 0$) { $\alpha = \dot{\theta} > 0 \, ? \, \pi : -\pi$; $V = \dot{\theta} W_b$ }
else ~~if (α~~
{
$\alpha = \text{atan}\left(\dfrac{\dot{\theta} W_b}{V_x}\right)$

$V = \sqrt{V_x^2 + \dot{\theta}^2 W_b^2}$
}

4 powered steering and driving wheel



$\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ \theta_1 \end{pmatrix}$ $\begin{pmatrix} w_3 \\ w_4 \end{pmatrix} = \begin{pmatrix} V_2 \\ \theta_2 \end{pmatrix}$ $\begin{pmatrix} w_5 \\ w_6 \end{pmatrix} = \begin{pmatrix} V_3 \\ \theta_3 \end{pmatrix}$ $\begin{pmatrix} w_7 \\ w_8 \end{pmatrix} = \begin{pmatrix} V_4 \\ \theta_4 \end{pmatrix}$

$V_{1x} = V_1 \cos\theta_1$ $V_{1y} = V_1 \sin\theta_1$	$V_{3x} = V_3 \cos\theta_3$ $V_{3y} = V_3 \sin\theta_3$
$V_{2x} = V_2 \cos\theta_2$ $V_{2y} = V_2 \sin\theta_2$	$V_{4x} = V_4 \cos\theta_4$ $V_{4y} = V_4 \sin\theta_4$

$$l = \frac{w_l}{2}$$
$$d = \frac{w_t}{2}$$

$$\begin{bmatrix} V_{1x} \\ V_{1y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -d \\ 0 & 1 & l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} u - dr \\ v + lr \end{bmatrix}$$

$$\begin{bmatrix} V_{2x} \\ V_{2y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -d \\ 0 & 1 & -l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} u - dr \\ v - lr \end{bmatrix}$$

$$\begin{bmatrix} V_{3x} \\ V_{3y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & +d \\ 0 & 1 & -l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} u + dr \\ v - lr \end{bmatrix}$$

$$\begin{bmatrix} V_{4x} \\ V_{4y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & d \\ 0 & 1 & l \end{bmatrix} \begin{bmatrix} u \\ v \\ r \end{bmatrix} = \begin{bmatrix} u + dr \\ v + lr \end{bmatrix}$$

$$\underbrace{\begin{bmatrix} V_{1x} \\ V_{1y} \\ V_{2x} \\ V_{2y} \\ V_{3x} \\ V_{3y} \\ V_{4x} \\ V_{4y} \end{bmatrix}}_{8\times 1}^{V} = \underbrace{\begin{bmatrix} 1 & 0 & -d \\ 0 & 1 & l \\ 1 & 0 & -d \\ 0 & 1 & -l \\ 1 & 0 & d \\ 0 & 1 & -l \\ 1 & 0 & d \\ 0 & 1 & l \end{bmatrix}}_{8\times 3}^{W} \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{3\times 1}$$

$$\boxed{W^{*} = W^{-1}}$$

$$\Rightarrow W_v = \underbrace{\begin{bmatrix} 1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/4 & 0 \\ 0 & 1/4 & 0 & 1/4 & 0 & 1/4 & 0 & 1/4 \\ -\sigma_2 & \sigma_1 & -\sigma_2 & -\sigma_1 & \sigma_2 & -\sigma_1 & \sigma_2 & \sigma_1 \end{bmatrix}}_{3\times 8}$$

$$\boxed{\begin{bmatrix} u \\ v \\ r \end{bmatrix}_{3\times 1} = W^{*}_{3\times 8} \begin{bmatrix} \vec{V} \end{bmatrix}_{8\times 1}}$$

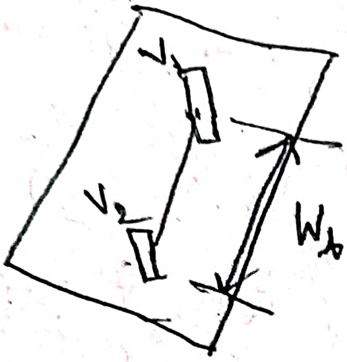
$$\boxed{\sigma_1 = \frac{l}{4(d^2+l^2)} \qquad \sigma_2 = \frac{d}{4(d^2+l^2)}}$$

$$\begin{cases} V_1 = \sqrt{V_{1x}^2 + V_{1y}^2} = \sqrt{(u-dr)^2 + (v+lr)^2} \\ \theta_1 = \operatorname{atan}\left(\frac{V_{1y}}{V_{1x}}\right) = \operatorname{atan}\left(\frac{v+lr}{u-dr}\right) \end{cases}$$

$$\begin{cases} V_2 = \sqrt{(u-dr)^2 + (v-lr)^2} \\ \theta_2 = \operatorname{atan}\left(\frac{v-lr}{u-dr}\right) \end{cases}$$

$$\begin{cases} V_3 = \sqrt{(u+dr)^2 + (v-lr)^2} \\ \theta_3 = \operatorname{atan}\left(\frac{v-lr}{u+dr}\right) \end{cases}$$

$$\begin{cases} V_4 = \sqrt{(u+dr)^2 + (v+lr)^2} \\ \theta_4 = \operatorname{atan}\left(\frac{v+lr}{u+dr}\right) \end{cases}$$



$$\boxed{\begin{aligned} W_r &= 0; \\ l &= \frac{W_b}{2} \end{aligned}}$$

$$\begin{bmatrix} V_{1x} \\ V_{1y} \end{bmatrix} = \begin{bmatrix} V_{1x} \\ V_{1y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ r \end{bmatrix}$$

$$\begin{bmatrix} V_{2x} \\ V_{2y} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -l \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ r \end{bmatrix}$$

So
$$\begin{aligned} V_{1x} &= u \\ V_{1y} &= v + l r \end{aligned} \qquad \begin{aligned} V_{2x} &= u \\ V_{2y} &= v - l r \end{aligned}$$

$$\begin{aligned} V_1 &= \sqrt{V_{1x}^2 + V_{1y}^2} = \sqrt{u^2 + (v + l r)^2} \\ \theta_1 &= \operatorname{atan}\left(\frac{V_{1y}}{V_{1x}}\right) = \operatorname{atan}\left(\frac{v + l r}{u}\right) \end{aligned}$$

$$\begin{aligned} V_2 &= \sqrt{V_{2x}^2 + V_{2y}^2} \\ \theta_2 &= \operatorname{ant}\left(\frac{V_{2y}}{V_{2x}}\right) = \operatorname{ant}\left(\frac{v - l r}{u}\right) \end{aligned}$$

$$\underbrace{\begin{bmatrix} V_{1x} \\ V_{1y} \\ V_{2x} \\ V_{2y} \end{bmatrix}}_{4 \times 1} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & l \\ 1 & 0 & 0 \\ 0 & 1 & -l \end{bmatrix}}_{4 \times 3} \overset{w}{} \underbrace{\begin{bmatrix} u \\ v \\ r \end{bmatrix}}_{3 \times 1}$$

$$\boxed{W^{*} = W^{-1}}$$