Chapter 1.1

1. Let,

A = {Patients visit physical therapist}

B = {Patients visit chiropractor}

Consider, $P(B) = x$

So, $P(A) = x + 16\% = x + 0.16$

Here, $P(A \cap B) = 28\% = 0.28$ and $P(A \cup B)' = 8\% = 0.08$

We know,

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Longrightarrow 1 - P(A \cup B)' = P(A) + P(B) - P(A \cap B)$

$\Longrightarrow 1 - 0.08 = x + 0.16 + x - 0.28$

$\Longrightarrow 0.92 = 2x - 0.12$

$\Longrightarrow 2x = 1.04$

$\Longrightarrow x = 0.52$

So, $P(A) = 0.52 + 0.16 = 0.68 = 68\%$

2. Let,

A = {Customers insure more than one car}

B = {Customers insure a sports car}

Given, $P(A) = 85\% = 0.85$

$P(B) = 23\% = 0.23$

And $P(A \cap B) = 17\% = 0.17$

Now,

$P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - (0.85 + 0.23 - 0.17)$

$= 0.09 = 9\%$. (answer)

3. Here, $n(S) = 52$

a) $n(A) = 12$ and $n(B) = 6$

so, P (A) = $\frac{12}{52}$ and P (B) = $\frac{6}{52}$

b) n (A $\cap$ B) = 2

so, P (A ∩ B) = $\frac{n\ (A \cap B)}{n\ (S)} = \frac{2}{52}$

c) P (A ∪ B) = P (A) + P (B) – P (A ∩ B)

$= \frac{12}{52} + \frac{6}{52} - \frac{2}{52}$

$= \frac{16}{52}$

d) n (C) = 13 and n (D) = 39

so, P (C) = $\frac{13}{52}$ and P (D) = $\frac{39}{52}$

n (C ∩ D) = 0

so, P (C ∩ D) = $\frac{n\ (C \cap D)}{n\ (S)}$ = 0

e) P (C ∪ D) = P (C) + P (D) – P (C ∩ D)

$= \frac{13}{52} + \frac{39}{52} - 0$

= 1 (answer)

4. a) The sample space, S =

$$\left\{\begin{array}{llll} HHHH & HHHT & HHTH & HHTT \\ HTHH & HTHT & HTTH & HTTT \\ THHH & THHT & THTH & THTT \\ TTHH & TTHT & TTTH & TTTT \end{array}\right\}$$

Here, n(S) = 16.

b) Let,

A = {At least 3 heads} = {HHHH, HHHT, HHTH, HTHH, THHH}

B = {At most 2 heads} = {HHTT, TTHH, HTHT, HTTH, HTTT, THHT, THTH, THTT, TTHT, TTTH, TTTT}

C = {Heads on the third toss} = {HHHH, HTHH, THHH, TTHH, HHHT, HTHT, THHT, TTHT}

D = {1 head and 3 tails} = {HTTT, THTT, TTHT, TTTH}

Now, n(A) = 5, n(B) = 11, n(C) = 8, n(D) = 4

(i) P (A) = $\frac{5}{16}$

(ii) n (A ∩ B) = 0;

So, $P(A \cap B) = 0$

(iii) $P(B) = \frac{11}{16}$

(iv) $n(A \cap C) = 4$;

So, $P(A \cap C) = \frac{4}{16}$

(v) $P(C) = \frac{n(C)}{n(S)} = \frac{8}{16}$

$P(D) = \frac{n(D)}{n(S)} = \frac{4}{16}$

(vi) $P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$$= \frac{5}{16} + \frac{8}{16} - \frac{4}{16}$$

$$= \frac{9}{16}$$

(vii) $n(B \cap D) = 4$;

So, $P(B \cap D) = \frac{4}{16}$

5. Given, $P(A) = \frac{1}{6}$.

So, $P(B) = 1 - \frac{1}{6} = \frac{5}{6}$ $[\because B = A']$

Now, $P(A \cap B) = 0$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$= \frac{1}{6} + \frac{5}{6} - 0$$

= 1. (answer)

6. Given,

$P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.5 + 0.4 - 0.3$

$= 0.6$

b) $P(A \cap B') = P(A) - P(A \cap B)$

$= 0.4 - 0.3$

$= 0.1$

c) $P(A' \cup B') = P(A \cap B)'$

$= 1 - P(A \cap B)$

$= 1 - 0.3$

$= 0.7$

7. Here, $P(A \cup B) = 0.76$ and $P(A \cup B') = 0.87$

We know, $P(A \cup B') = P(A) + P(A \cup B)'$

So, $P(A) = P(A \cup B') - P(A \cup B)'$

$= P(A \cup B') - [1 - P(A \cup B)]$

$= 0.87 - (1 - 0.76)$

$= 0.63$ (answer)

8. Let,

$A = \{\text{Having lab work}\}$

$B = \{\text{Having a referral}\}$

Given, $P(A) = 0.41$ and $P(B) = 0.53$

Here, $P(A \cup B)' = 0.21$

Now,

$P(A \cup B)' = 0.21$

$\Rightarrow 1 - P(A \cup B) = 0.21$

$\Rightarrow P(A \cup B) = 0.79$

So, $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.41 + 0.53 - 0.79$

$= 0.15$ (answer)

Chapter 1.3

1. a) $P(B_1) = \frac{5{,}000}{1{,}000{,}000}$

b) $P(A_1) = \frac{78{,}515}{1{,}000{,}000}$

c) $P(A_1 | B_2) = \frac{n(A1 \cap B2)}{n(B2)} = \frac{73{,}630}{995{,}000}$

d) $P(B_1 | A_1) = \frac{n(A1 \cap B1)}{n(A1)} = \frac{4{,}885}{78{,}515}$

2. a) $P(A_1) = \frac{1041}{1456}$

b) $P(A_1 | S_1) = \frac{n(A1 \cap S1)}{n(S1)} = \frac{392}{633}$

c) $P(A_1 | S_2) = \frac{n(A1 \cap S2)}{n(S2)} = \frac{649}{823}$

3. a) $P(A_1 \cap B_1) = \frac{n(A1 \cap B1)}{n(S)} = \frac{5}{35}$

b) $P(A_1 \cup B_1) = P(A_1) + P(B_1) - P(A_1 \cap B_1)$

$$= \frac{n(A1)}{n(S)} + \frac{n(B1)}{n(S)} - \frac{5}{35}$$

$$= \frac{12}{35} + \frac{19}{35} - \frac{5}{35} = \frac{26}{35}$$

c) $P(A_1 | B_1) = \frac{n(A1 \cap B1)}{n(B1)} = \frac{5}{19}$

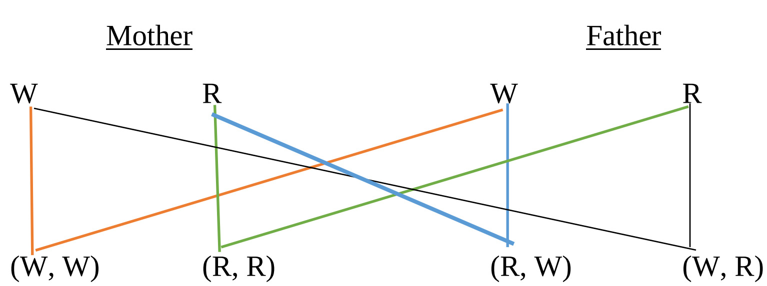
d) $P(B_2 | A_2) = \frac{n(A2 \cap B2)}{n(A2)} = \frac{9}{23}$

4. a) P (two hearts) $= \frac{13}{52} \times \frac{12}{51} = \frac{1}{17}$

b) P (a heart on the first and club on second) $= \frac{13}{52} \times \frac{13}{51} = \frac{13}{204}$

c) P (Non-Ace heart, Ace) + P (Ace of heart, non-heart Ace) $= \frac{12}{52} \times \frac{4}{51} + \frac{1}{52} \times \frac{3}{51} = \frac{1}{52}$

5.



a) Sample space, S= {(W, W), (W, R), (R, W), (R, R)}

b) P (WW | White) = $\frac{1}{3}$

6.

	Heart disease	Non-heart disease	Total
Parental	111	223	334
Non-parental	110	538	648
Total	221	761	982

P (Heart disease | Non-parental) = $\frac{\text{n (Heart disease} \cap \text{Non}-\text{parental)}}{\text{n (Non}-\text{parental)}} = \frac{110}{648}$

7. P (At least one orange) = P ($O_1 \cap O_2$) + P ($O_1 \cap B_2$) + P ($B_1 \cap O_2$)

$$= \frac{2}{4} \times \frac{1}{3} + \frac{2}{4} \times \frac{2}{3} + \frac{2}{4} \times \frac{2}{3} = \frac{5}{6}$$

P (Both orange | At least one orange) = $\frac{\text{P (Both orange} \cap \text{At least one orange)}}{\text{P (At least one orange)}}$

$$= \frac{1/6}{5/6} = \frac{1}{5}$$

8. a) P (WWW) = $\frac{3}{20} \times \frac{2}{19} \times \frac{1}{18} = \frac{1}{1140}$

b) P (WLWW) + P (LWWW) + P (WWLW)

$= \frac{3}{20} \times \frac{17}{19} \times \frac{2}{18} \times \frac{1}{17} + \frac{17}{20} \times \frac{3}{19} \times \frac{2}{18} \times \frac{1}{17} + \frac{3}{20} \times \frac{2}{19} \times \frac{17}{18} \times \frac{1}{17}$

$= \frac{1}{380}$

14. a) P (A_1) = $\frac{\text{n(A1)}}{\text{n(S)}} = \frac{30}{100}$

b) P (A_3) = $\frac{\text{n(A1)}}{\text{n(S)}} = \frac{29}{100}$

P (B_2) = $\frac{\text{n(B2)}}{\text{n(S)}} = \frac{41}{100}$

P ($A_3 \cap B_2$) = $\frac{\text{n (A3} \cap \text{B2)}}{\text{n (S)}} = \frac{9}{100}$

c) P ($A_2 \cup B_3$) = P (A_2) + P (B_3) – P ($A_2 \cap B_3$)

$= \frac{41}{100} + \frac{28}{100} - \frac{9}{100} = \frac{60}{100}$

d) Probability of A_1 if it is B_2,

$$P(A_1 \mid B_2) = \frac{P(A1 \cap B2)}{P(B2)} = \frac{n(A1 \cap B2)/n(S)}{n(B2)/n(S)} = \frac{11/100}{41/100} = \frac{11}{41}$$

e) Probability of B_1 if it is A_3,

$$P(B_1 \mid A_3) = \frac{P(B1 \cap A3)}{P(A3)} = \frac{n(B1 \cap A3)/n(S)}{n(A3)/n(S)} = \frac{13/100}{29/100} = \frac{13}{29}$$

15.

A	B
Red ball = 8	Red ball = n
Blue ball = 7	Blue ball = 9

P (Two balls of same color) = P (RR) + P (BB)

$\Longrightarrow \frac{151}{300} = \frac{8}{15} \times \frac{n}{(n+9)} + \frac{7}{15} \times \frac{9}{(9+n)}$

$\Longrightarrow \frac{151}{300} = \frac{8n+63}{15(n+9)}$

$\Longrightarrow 300\,(8n+63) = 151\,(15n+135)$

$\Longrightarrow 2400n + 18900 = 2265n + 20385$

$\Longrightarrow 135n = 1485$

$\therefore n = 11$

So, there are 11 red balls. (answer)

16.

A	B
Red ball = 4	Red ball = 4
White ball = 2	White ball = 3

P (RB) = P (RA ∩ RB) + P (WA ∩ RB)

$= P(RA)\,P(RB \mid RA) + P(WA)\,P(RB \mid WA)$

$= \frac{3}{5} \times \frac{5}{8} + \frac{2}{5} \times \frac{4}{8}$

$= \frac{23}{40}$ (answer)

Chapter 1.4

1. Given, P (A) = 0.7, P (B) = 0.2 and both A and B are independent.

a) $P(A \cap B) = P(A) \times P(B)$

$= (0.7) \times (0.2)$

$= 0.14$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.2 - 0.14$

$= 0.76$

c) $P(A' \cup B') = P(A \cap B)'$

$= 1 - P(A \cap B)$

$= 1 - 0.14$

$= 0.86$ (answer)

2. Given, P(A) = 0.3 & P(B) = 0.6

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A) \times P(B)$ [as A and B are independent]

$= 0.3 + 0.6 - 0.3 \times 0.6$

$= 0.72$

b) $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$= 0$ [as A and B are mutually exclusive $P(A \cap B) = 0$]

3. Given, $P(A) = \frac{1}{4}$ & $P(B) = \frac{2}{3}$

$P(A)' = 1 - P(A)$

$= 1 - \frac{1}{4}$

$= \frac{3}{4}$

$P(B)' = 1 - P(B)$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$

a) $P(A \cap B) = P(A) \times P(B)$

$= \frac{1}{4} \times \frac{2}{3}$

$= \frac{1}{6}$

b) $P(A \cap B') = P(A) \times P(B')$

$= \frac{1}{4} \times \frac{1}{3}$

$= \frac{1}{12}$

c) $P(A' \cap B') = P(A \cup B)'$

$= 1 - P(A \cup B)$

$= 1 - [P(A) + P(B) - P(A \cap B)]$

$= 1 - \frac{1}{4} - \frac{2}{3} + \frac{1}{6}$

$= \frac{1}{4}$

d) $P[(A \cup B)'] = P[A' \cap B']$

$= P(A)' \times P(B)'$

$= \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

e) $P(A' \cap B) = P(B') - P(A \cap B)$

$= \frac{2}{3} - \frac{1}{6} = \frac{1}{2}$

5. Give, $P(A) = 0.8$, $P(B) = 0.5$ & $P(A \cup B) = 0.9$

$P(A \cap B) = P(A) \times P(B)$

$= 0.8 \times 0.5$

$= 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Longrightarrow P(A \cap B) = 0.8 + 0.5 - 0.9$

$= 0.4$

$= P(A) \times P(B)$

As they are same, so A and B are independent (answer)

7. Given, $P(A_1) = 0.5$, $P(A_2) = 0.7$, $P(A3) = 0.6$

a) P (Exactly one player is successful)

$= P(A_1)\, P(A_2)'\, P(A_3)' + P(A_1)'\, P(A_2)\, P(A_3)' + P(A_1)'\, P(A_2)'\, P(A_3)$

$= 0.5 \times (1 – 0.7) \times (1 – 0.6) + (1 – 0.5) \times 0.7\times (1 – 0.6) + (1 – 0.5) \times (1 – 0.7) \times 0.6$

$= 0.29$

b) P (Exactly two players make a goal)

$= P(A_1)\, P(A_2)\, P(A_3)' + P(A_1)\, P(A_2)'\, P(A_3) + P(A_1)'\, P(A_2)\, P(A_3)$

$= 0.5 \times 0.7 \times (1 – 0.6) + 0.5 \times (1 – 0.7) \times 0.6 + (1 – 0.5) \times 0.7 \times 0.6$

$= 0.47$ (answer)

8. Let, A = {Orange comes up die on A};

B = {Orange comes up die on B} &

B = {Orange comes up die on C}

$P(A) = \frac{1}{6}$; $P(A') = \frac{5}{6}$

$P(B)= \frac{2}{6}$; $P(B') = \frac{4}{6}$

$P(C)= \frac{3}{6}$; $P(C') = \frac{3}{6}$

P (exactly two players make a goal)

$= P(A)\, P(B)\, P(C') + P(A)\, P(B)'\, P(C) + P(A')\, P(B)\, P(C)$

$= \frac{1}{6} \times \frac{2}{6} \times \frac{3}{6} + \frac{1}{6} \times \frac{4}{6} \times \frac{3}{6} + \frac{5}{6} \times \frac{2}{6} \times \frac{3}{6}$

$= \frac{2}{9}$ (answer)

9. Given,

$P(A) =0.5$; $P(A') = 0.5$

$P(B)= 0.8$; $P(B') = 0.2$

$P(C)= 0.9$; $P(C') = 0.1$

a) P (All three events occur) $= P(A) \times P(B) \times P(C)$

$= 0.5\times0.8\times0.9$

$= 0.36$

b) P (Exactly two events occur) = $P(A)\,P(B)\,P(C)' + P(A)\,P(B)'\,P(C) + P(A)'\,P(B)\,P(C)$

$= 0.5 \times 0.8 \times 0.1 + 0.5 \times 0.2 \times 0.9 + 0.5 \times 0.8 \times 0.9$

$= 0.49$

c) P (None of the events occur) = $P(A)'\,P(B)'\,P(C)'$

$= 0.5 \times 0.2 \times 0.1$

$= 0.01$ (answer)

Chapter 1.5

1.

Red ball = 0 White ball = 4	Red ball = 2 White ball = 0	Red ball = 2 White ball = 2	Red ball = 1 White ball = 3
$\mathbf{B_1}$	$\mathbf{B_2}$	$\mathbf{B_3}$	$\mathbf{B_4}$

Given,

$P(B_1) = \frac{1}{2}$, $P(B_2) = \frac{1}{4}$, $P(B_3) = \frac{1}{8}$, $P(B_4) = \frac{1}{8}$

a) $P(W) = P(W \cap B_1) + P(W \cap B_2) + P(W \cap B_3) + P(W \cap B_4)$

$= P(B_1)\,P(W|B_1) + P(B_2)\,P(W|B_2) + P(B_3)\,P(W|B_3) + P(B_4)\,P(W|B_4)$

$= \frac{1}{2} \times 1 + \frac{1}{4} \times 0 + \frac{1}{8} \times \frac{2}{4} + \frac{1}{8} \times \frac{3}{4} = \frac{21}{32}$

b) $P(B_1 | W) = \frac{P(W \cap B1)}{P(W)}$

$= \frac{1/2}{21/32}$

$= \frac{16}{21}$ (answer)

2. Here,

$P(A) = 40\% = 0.4$; $P(G | A) = 85\% = 0.85$

$P(B) = 60\% = 0.6$; $P(G | B) = 75\% = 0.75$

a) $P(G) = P(A)\,P(G | A) + P(B)\,P(G | B)$

$= 0.4 \times 0.85 + 0.6 \times 0.75$

$= 0.79$

$= 79\%$

b) $P(A \mid G) = \frac{P(A)\,P(\mathbf{G}|\mathbf{A})}{P(G)}$

$= \frac{0.4 \times 0.85}{0.79}$

$= 0.43$

$= 43\%$ (answer)

5. Let, $A = \{\text{Patients are critical}\}$

$B = \{\text{Patients are serious}\}$

$C = \{\text{Patients are stable}\}$

$P(A) = 20\% = 0.2$

$P(B) = 30\% = 0.3$

$P(C) = 50\% = 0.5$

$P(D \mid A) = 30\% = 0.3$

$P(D \mid B) = 10\% = 0.1$

$P(D \mid C) = 1\% = 0.01$

Now, $P(D) = P(D \cap A) + P(D \cap B) + P(D \cap C)$

$= P(A)\,P(D|A) + P(B)\,P(D|B) + P(C)\,P(D|C)$

$= 0.2 \times 0.3 + 0.3\times0.1 + 0.5\times0.01$

$= 0.095$

Now, $P(A \mid D) = \frac{P(A \cap D)}{P(D)}$

$= \frac{0.2\times0.3}{0.095}$

$= 63\%.$ (answer)

7. Given,

$P(I^+) = 20\% = 0.2$

$P(I^-) = 80\% = 0.8$

$P(D^+ \mid I^+) = 0.9$; $P(D^- \mid I^+) = 0.1$

$P(D^+ \mid I^-) = 0.05$; $P(D^- \mid I^-) = 0.95$

Now, $P(D^+) = P(D^+ \cap I^+) + P(D^+ \cap I^-)$

$= P(I^+)\, P(D^+ \mid I^+) + P(I^-)\, P(D^+ \mid I^-)$

$= 0.2 \times 0.9 + 0.8 \times 0.05 = 0.22$

Then, $P(I^+ \mid D^+) = \frac{P(I+ \cap D+)}{P(D+)}$

$= \frac{0.2 \times 0.9}{0.22}$

= 81%. (answer)

9. Here, P (disease) = 0.05% = 0.0005

And P (non-disease) = 0.9995

P (detect | disease) = 99% = 0.99

P (not detect | disease) = 0.01

P (detect | non-disease) = 3% = 0.03

P (not detect | non-disease) = 0.97

Then, $P(\text{disease} \mid \text{detect}) = \frac{P(\text{disease} \cap \text{detect})}{P(\text{detect})}$

$$= \frac{P(\text{disease})\, P(\text{detect} \mid \text{disease})}{P(\text{disease})\, P(\text{detect} \mid \text{disease}) + P(\text{non}-\text{disease})\, P(\text{detect} \mid \text{non}-\text{disease})}$$

$$= \frac{0.0005 \times 0.99}{0.0005 \times 0.99 + 0.9995 \times 0.03}$$

= 0.016

Now, P (non-disease | detect) = 1 – P (disease | detect)

= 1 – 0.016

= 0.984 (answer)

10. Given, $P(A^+) = 0.02$; $P(A^-) = 0.98$

$P(D^- \mid A^+) = 0.08$; $P(D^+ \mid A^+) = 0.92$

$P(D^- \mid A^-) = 0.95$; $P(D^+ \mid A^-) = 0.05$

a) $P(D^+) = P(D^+ \cap A^+) + P(D^+ \cap A^-)$

$= P(A^+)\, P(D^+ \mid A^+) + P(A^-)\, P(D^+ \mid A^-)$

$= 0.02 \times 0.92 + 0.98 \times 0.05 = 0.0674$

b) $P(A^{-} \mid D^{+}) = \frac{P(D+ \cap A-)}{P(D+)}$

$= \frac{P(A-)\,P(D+ \mid A-)}{P(D+)}$

$= \frac{0.98 \times 0.05}{0.0674}$

$= 0.727$

So, $P(A^{+} \mid D^{+}) = 1 - P(A^{-} \mid D^{+})$

$= 1 - 0.727$

$= 0.273$ (answer)