Math109 Week2 Lecture 1

April 8, 2019

1 Proof by contradiction

Example 1:

Prove that

$$78m + 102n = 11 \tag{1}$$

doesn't have an integer solution.

Solution:

Assume by contradiction that there is an integer solution, i.e. that for some $m, n \in \mathbb{Z}, 78m + 102n = 1$. Since 2 divides 78 and 102, $78m + 12n$ is an even number, but 11 is an odd number, a contradiction.

Example 2: Prove that

$$\sqrt{2}\ \textit{is irrational.} \tag{2}$$

x is rational if there exists $p, q \in \mathbb{Z}, q \neq 0$, such that $x = \frac{p}{q}$. x is called irrational if it is not rational.

Solution:

Assume by contradiction that there exists $p, q \in \mathbb{Z}, q \neq 0$, so that $\sqrt{2} = \frac{p}{q}$ and $\frac{p}{q}$ is an irreducible fraction.

Then:

$$\sqrt{2}q = p \tag{3}$$

$$2q^2 = p^2 \;\Rightarrow\; p^2 \textit{ is even} \;\Rightarrow\; p \textit{ is even}. \tag{4}$$

(to be proved). Then there exists $k \in \mathbb{Z}$ so that

$$p = 2k \Rightarrow 2q^2 = 4k^2 \Rightarrow q \textit{ is even}. \tag{5}$$

Thus, since q, p are both even, they are reducible, which is an contradiction.

Now let's prove p^2 *is even* $\Rightarrow$ p *is even*:

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