## A Surprisingly Effective Fix for Deep Latent Variable Modeling of Text

Bohan Li\*1, Junxian He\*1, Graham Neubig1, Taylor Berg-Kirkpatrick2, Yiming Yang1

#### Core idea:

- Pretraining encoder using AE objective
- Initializing VAE with pertained AE and training with FB objective

Yahoo							
LSTM-LM	60.75	-	-	-	-		
VAE	61.52	329.10	0	0.00	329.10		
+ anneal	61.21	328.80	0	0.00	328.80		
+ cyclic	66.93	333.80	4	2.83	336.63		
+ aggressive	59.77	322.70	15	5.70	328.40		
+ FBP ( $\lambda = 9$ )	62.59	322.91	6	9.08	331.99		
+ FBP ( $\lambda = 7$ )	62.76	324.66	5	7.03	331.69		
+ FBP ( $\lambda = 5$ )	62.78	326.26	3	5.07	331.32		
+ FBP ( $\lambda = 3$ )	62.88	328.13	2	3.06	331.19		
Ours $(\lambda = 6)$	59.23	317.39	$-\bar{3}\bar{2}$	12.09	$\bar{3}\bar{2}\bar{9}.\bar{4}\bar{8}$		
Ours $(\lambda = 8)$	<b>59.51</b>	315.31	<b>32</b>	15.02	330.33		
Ours $(\lambda = 9)$	59.60	315.09	32	15.49	330.58		

Le Fang<sup>†</sup>, Chunyuan Li<sup>§</sup>, Jianfeng Gao<sup>§</sup>, Wen Dong<sup>†</sup>, Changyou Chen<sup>†</sup>

#### **Core ideas:**

- Attributing posterior collapse to the restrictive Gaussian assumption, and advocate more flexible samplebased posterior representation.
- Proposed iVAE:
  - Instead of assuming posterior as Gaussian, use sampling mechanism for  $q_{\phi}(z|x)$ .  $z_{x,i} = G_{\phi}(x, \epsilon_i)$ ,  $\epsilon_i \sim q(\epsilon)$
  - How to evaluate KL on this? Use dual form of KL(q<sub>φ</sub>(z|x) | p(z)):

$$\mathsf{KL}\left(q_{\phi}(\boldsymbol{z}|\boldsymbol{x}) \parallel p(\boldsymbol{z})\right) \tag{7}$$

$$= \max_{\nu} \mathbb{E}_{\boldsymbol{z} \sim q_{\phi}(\boldsymbol{z}|\boldsymbol{x})} \nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z}) - \mathbb{E}_{\boldsymbol{z} \sim p(\boldsymbol{z})} \exp(\nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z})),$$

New iVAE objective:

$$\mathcal{L}_{\text{iVAE}} = \mathbb{E}_{\boldsymbol{x} \sim \mathcal{D}} \mathbb{E}_{\boldsymbol{z} \sim q_{\boldsymbol{\phi}}(\boldsymbol{z}|\boldsymbol{x})} \log p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{z})$$

$$- \mathbb{E}_{\boldsymbol{x} \sim \mathcal{D}} \mathbb{E}_{\boldsymbol{z} \sim q_{\boldsymbol{\phi}}(\boldsymbol{z}|\boldsymbol{x})} \nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z})$$

$$+ \mathbb{E}_{\boldsymbol{x} \sim \mathcal{D}} \mathbb{E}_{\boldsymbol{z} \sim p(\boldsymbol{z})} \exp(\nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z})), \quad (8)$$

• Here,  $V\psi$  is a MLP taking in (x, z).

Le Fang<sup>†</sup>, Chunyuan Li<sup>§</sup>, Jianfeng Gao<sup>§</sup>, Wen Dong<sup>†</sup>, Changyou Chen<sup>†</sup>

#### **Core ideas:**

- Training Scheme:
  - Sample a mini-batch of  $x_i \sim \mathcal{D}$ ,  $\epsilon_i \sim q(\epsilon)$ , and generate  $z_{x_i,\epsilon_i} = G(x_i,\epsilon_i;\phi)$ ; Sample a mini-batch of  $z_i \sim p(z)$ .
  - Update  $\psi$  in  $\nu_{\psi}(\boldsymbol{x}, \boldsymbol{z})$  to maximize

$$\sum_{i} \nu_{\psi}(\boldsymbol{x}_{i}, \boldsymbol{z}_{\boldsymbol{x}_{i}, \epsilon_{i}}) - \sum_{i} \exp(\nu_{\psi}(\boldsymbol{x}_{i}, \boldsymbol{z}_{i})) \quad (9)$$

• Update parameters  $\{\phi, \theta\}$  to maximize

$$\sum_{i} \log p_{\boldsymbol{\theta}}(\boldsymbol{x}_{i}|\boldsymbol{z}_{\boldsymbol{x}_{i},\epsilon_{i}}) - \sum_{i} \nu_{\boldsymbol{\psi}}(\boldsymbol{x}_{i},\boldsymbol{z}_{\boldsymbol{x}_{i},\epsilon_{i}})$$
(10)

Le Fang<sup>†</sup>, Chunyuan Li<sup>§</sup>, Jianfeng Gao<sup>§</sup>, Wen Dong<sup>†</sup>, Changyou Chen<sup>†</sup>

#### **Core ideas:**

- Proposed Mutual Information Regularized iVAE
  - Replace –KL  $(q_{\phi}(z|x) \parallel p(z))$  with –KL  $(q_{\phi}(z) \parallel p(z))$ , where  $q_{\phi}(z) = \int q(x)q_{\phi}(z|x)dx$ , estimated by ancestral sampling in practice.
  - This objective also maximize the mutual information I(x,z), as they claimed.
  - The new objective:

$$\mathsf{KL}\left(q_{\phi}(\boldsymbol{z}) \parallel p(\boldsymbol{z})\right)$$

$$= \max_{\nu} \mathbb{E}_{\boldsymbol{z} \sim q_{\phi}(\boldsymbol{z})} \nu_{\boldsymbol{\psi}}(\boldsymbol{z}) - \mathbb{E}_{\boldsymbol{z} \sim p(\boldsymbol{z})} \exp(\nu_{\boldsymbol{\psi}}(\boldsymbol{z})).$$

$$(13)$$

$$\mathcal{L}_{\text{iVAE}_{\text{MI}}} = \mathbb{E}_{\boldsymbol{x} \sim D} \mathbb{E}_{\boldsymbol{z} \sim q_{\boldsymbol{\phi}}(\boldsymbol{z}|\boldsymbol{x})} \log p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{z}) \qquad (14)$$
$$- \mathbb{E}_{\boldsymbol{z} \sim q_{\boldsymbol{\phi}}(\boldsymbol{z})} \nu_{\boldsymbol{\psi}}(\boldsymbol{z}) + \mathbb{E}_{\boldsymbol{z} \sim p(\boldsymbol{z})} \exp(\nu_{\boldsymbol{\psi}}(\boldsymbol{z})),$$

• The only difference with iVAE is that  $\nu \psi$  is a MLP taking in only z.

Le Fang<sup>†</sup>, Chunyuan Li<sup>§</sup>, Jianfeng Gao<sup>§</sup>, Wen Dong<sup>†</sup>, Changyou Chen<sup>†</sup>

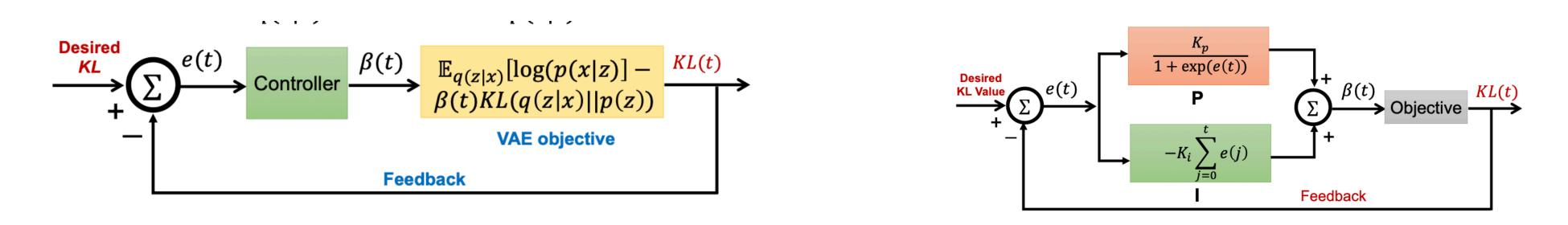
Methods	-ELBO↓	PPL↓	KL↑	MI↑	AU↑			
Dataset: PTB								
VAE	102.6	108.26	1.08	0.8	2			
$\beta$ (0.5)-VAE	104.5	117.92	7.50	3.1	5			
SA-VAE	102.6	107.71	1.23	0.7	2			
Cyc-VAE	103.1	110.50	3.48	1.8	5			
iVAE	<b>87.6</b>	54.46	6.32	3.5	32			
$iVAE_{MI}$	87.2	53.44	12.51	12.2	32			
	Data	set: Yaho	0	l				
VAE	328.6	61.21	0.0	0.0	0			
$\beta$ (0.4)-VAE	328.7	61.29	6.3	2.8	8			
SA-VAE	327.2	60.15	5.2	2.7	10			
Lag-VAE	326.7	59.77	5.7	2.9	15			
iVAE	309.5	48.22	8.0	4.4	32			
$iVAE_{MI}$	309.1	47.93	11.4	10.7	32			
Dataset: Yelp								
VAE	357.9	40.56	0.0	0.0	0			
$\beta$ (0.4)-VAE	358.2	40.69	4.2	2.0	4			
SA-VAE	355.9	39.73	2.8	1.7	8			
Lag-VAE	355.9	39.73	3.8	2.4	11			
iVAE	348.2	36.70	7.6	4.6	32			
$iVAE_{MI}$	348.7	36.88	11.6	11.0	32			

### ControlVAE: Controllable Variational Autoencoder (ICML 2020)

Huajie Shao et al

#### **Core ideas:**

 Enhance β-VAE by controlling β with a new non-linear PI controller, β(t). The controller is not learnable, with Kp, Ki, Desired KL being hyperparameters.



• Intuition: Comparing error between Desired KL and Sampled KL (from inference output); if Sampled KL is too small, β(t) will be small so KL-divergence is encouraged to grow, and vice versa.

### ControlVAE: Controllable Variational Autoencoder (ICML 2020)

Huajie Shao et al

#### **Core ideas:**

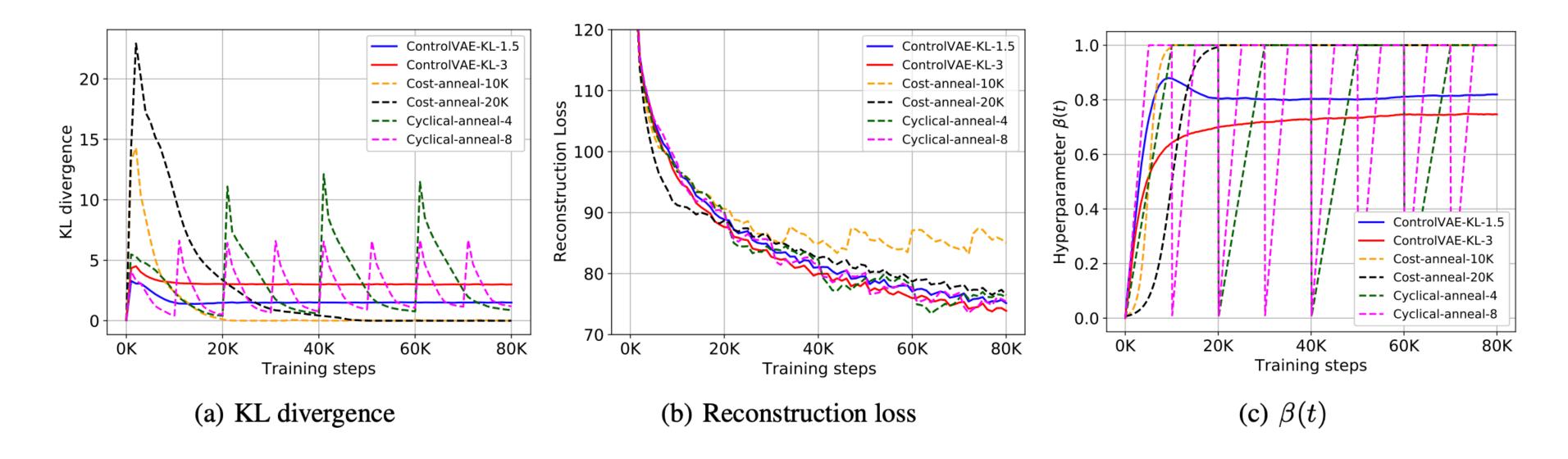
PI Algorithm

#### Algorithm 1 PI algorithm.

```
1: Input: desired KL v_{kl}, coefficients K_p, K_i, max/min value
     \beta_{max}, \beta_{min}, iterations N
 2: Output: hyperparameter \beta(t) at training step t
 3: Initialization: I(0) = 0, \beta(0) = 0
 4: for t = 1 to N do
       Sample KL-divergence, \hat{v}_{kl}(t)
 6: e(t) \leftarrow v_{kl} - \hat{v}_{kl}(t)
     P(t) \leftarrow \frac{K_p}{1 + \exp(e(t))}
      if \beta_{min} \leq \beta(t-1) \leq \beta_{max} then I(t) \leftarrow I(t-1) - K_i e(t)
       else
       I(t) = I(t-1) // Anti-windup
       end if
     \beta(t) = P(t) + I(t) + \beta_{min}
14: if \beta(t) > \beta_{max} then
        eta(t) = eta_{max}
        end if
17: if \beta(t) < \beta_{min} then
       eta(t)=eta_{min}
19: end if
20: Return \beta(t)
21: end for
```

### ControlVAE: Controllable Variational Autoencoder (ICML 2020)

Huajie Shao et al



Methods/metric	Dis-1	Dis-2	self-BLEU-2	self-BLEU-3	PPL
ControlVAE-KL-35	<b>6.27K</b> $\pm$ 41	$95.86K \pm 1.02K$	$0.663 \pm 0.012$	$0.447 \pm 0.013$	$8.81 \pm 0.05$
ControlVAE-KL-25	$6.10K \pm 60$	$83.15K \pm 4.00K$	$0.698 \pm 0.006$	$0.495 \pm 0.014$	$12.47 \pm 0.07$
Cost anneal-KL-17	$5.71K \pm 87$	$69.60K \pm 1.53K$	$0.721 \pm 0.010$	$0.536 \pm 0.008$	$16.82 \pm 0.11$
Cyclical ( $KL = 21.5$ )	$5.79K \pm 81$	$71.63K \pm 2.04K$	$0.710 \pm 0.007$	$0.524 \pm 0.008$	$17.81 \pm 0.33$

Xiaoan Ding, Kevin Gimpel

#### **Core ideas:**

- Using normalizing flows (NF) for the prior distribution. In this paper, using real-valued non-volume preserving (real NVP) transformations (Dinh et al., 2016)
  - $z_L = f_L \circ f_{L-1} \circ ... \circ f_1(z_0), z_0 \sim N(0, I), z_L$  is the sentence latent variable.

$$z_{0} = f_{1}^{-1} \circ \dots \circ f_{L-1}^{-1} \circ f_{L}^{-1}(z_{L})$$

$$\log p_{\psi}(z_{L}) = \log p_{0}(z_{0}) - \sum_{l=1}^{L} \log |\det(\frac{\partial f_{l}(z_{l-1})}{\partial z_{l-1}})|$$

$$(6)$$

• Inspired by Real-NVP, the choice of f is an affine transformation, which is f:  $Z_{i-1} \rightarrow Z_i$ 

$$\begin{split} z_i^{(1:d)} &= z_{i-1}^{(1:d)} \\ z_i^{(d+1:D)} &= z_i^{(d+1:D)} \odot \exp(s(z_{i-1}^{(1:d)})) + t(z_{i-1}^{(1:d)}) \end{split}$$

- 1) easily invertible
- 2) its Jacobian determinant is easy to compute. (Do not require inverse/Jacobian of s and t)
- So we can model s and t using NNs, denoted by pψ(z)

$$\mathbf{J} = \begin{bmatrix} \mathbb{I}_d & \mathbf{0}_{d \times (D-d)} \\ \frac{\partial \mathbf{y}_{d+1:D}}{\partial \mathbf{x}_{1:d}} & \text{diag}(\exp(s(\mathbf{x}_{1:d}))) \end{bmatrix}$$

Xiaoan Ding, Kevin Gimpel

#### **Core ideas:**

New Objective: Using importance weighting + Monte Carlo for KL

$$\mathcal{L}(\theta, \phi, \psi; x) = \log \frac{1}{N} \sum_{i=1}^{N} \frac{p_{\theta}(x|z^{(i)}) p_{\psi}(z^{(i)})}{q_{\phi}(z^{(i)}|x)} + \frac{1}{N} \sum_{i=1}^{N} \log p_{\theta}(x|z^{(i)}) - \text{KL}_{\phi,\psi}(x, \{z^{(i)}\}_{i=1}^{N})$$
s.t.  $z^{(i)} \sim q_{\phi}(z|x)$  (7)

Xiaoan Ding, Kevin Gimpel

#### **Core ideas:**

Training Scheme

- 1. Draw N samples  $z_L^{(1)}, z_L^{(2)}, ..., z_L^{(N)}$  from the inference network using the reparameterization trick.
- 2. Perform the inverse transformation to get the image of each point under the base distribution:  $z_0^{(1)}, z_0^{(2)}, ..., z_0^{(N)}$ .
- 3. Compute the exact log likelihood of the sample prior with change of variable theorem (Eq. 6).
- 4. Compute and backpropagate the loss (Eq. 7).

Xiaoan Ding, Kevin Gimpel

Model	PPL(↓)	Recon(↓)	KL	AU(†)	MI(†)
VAE	101.40	101.28	0.00	0	0.00
Cyc-VAE	107.73	101.17	2.01	5	1.24
Lag-VAE	100.25	100.41	1.04	3	0.79
VAE + FB	101.56	99.84	4.46	32	0.90
Pre-VAE + FB	96.35	94.52	8.15	32	6.30
MoG-VAE	98.22	100.54	0.00	0	0.00
MoG-VAE + FB	97.50	99.44	2.35	32	0.68
Vamp-VAE	98.27	100.56	0.00	0	0.00
Vamp-VAE + FB	97.83	99.53	2.31	32	0.72
FlowPrior	94.72	98.46	3.28	2	2.25
FlowPrior + FB	93.58	99.20	7.21	31	2.83
110 111101 1 1 1 1	75.50	77.20	7.21	<u> </u>	2.03

Table 1: Language modeling results on PTB dataset.

Shuyang Dai 'Zhe Gan Yu Cheng Chenyang Tao Lawrence Carin Jingjing Liu

#### **Core ideas:**

- Natural languages have latent hierarchy, but prior distribution from Euclidean space couldn't capture it, so resorting to Riemannian Geometry.
- Proposing a prior based on Poincaré Ball Model in Hyperbolic space, with the following advantages:

$$\mathbb{B}^n_c := \left\{ oldsymbol{z} \in \mathbb{R}^n \, | \, c \| oldsymbol{z} \|^2 < 1 
ight\}$$

It implies significant representation ability.

 $oldsymbol{z} \oplus_c oldsymbol{z}' :=$ 

• It's a well defined compact metric space, with well defined algebraic operations that allows back-

propagation.

$$\frac{(1+2c\langle \boldsymbol{z},\boldsymbol{z}'\rangle+c\|\boldsymbol{z}'\|^{2})\boldsymbol{z}+(1-c\|\boldsymbol{z}\|^{2})\boldsymbol{z}'}{1+2c\langle \boldsymbol{z},\boldsymbol{z}'\rangle+c^{2}\|\boldsymbol{z}\|^{2}\|\boldsymbol{z}'\|^{2}}.$$

$$\exp_{\boldsymbol{\mu}}^{c}(\boldsymbol{u}):=\boldsymbol{\mu}\oplus_{c}(\tanh(\sqrt{c}\frac{\lambda_{\boldsymbol{\mu}}^{c}\|\boldsymbol{u}\|}{2})\frac{\boldsymbol{u}}{\sqrt{c}\|\boldsymbol{u}\|}),$$

$$\log_{\boldsymbol{\mu}}^{c}(\boldsymbol{y}):=\frac{2}{\sqrt{c}\lambda_{\boldsymbol{\mu}}^{c}}\tanh^{-1}(\sqrt{c}\|\boldsymbol{\kappa}_{\boldsymbol{\mu},\boldsymbol{y}}\|)\frac{\boldsymbol{\kappa}_{\boldsymbol{\mu},\boldsymbol{y}}}{\|\boldsymbol{\kappa}_{\boldsymbol{\mu},\boldsymbol{y}}\|},$$
(5)
$$P_{0\to\boldsymbol{\mu}}^{c}(\boldsymbol{v})=\log_{\boldsymbol{\mu}}^{c}(\boldsymbol{\mu}\oplus_{c}\exp_{0}^{c}(\boldsymbol{v}))=\frac{\lambda_{0}^{c}}{\lambda_{\boldsymbol{\mu}}^{c}}\boldsymbol{v}.$$
(6)
$$\text{where } \boldsymbol{\kappa}_{\boldsymbol{\mu},\boldsymbol{y}}:=(-\boldsymbol{\mu})\oplus_{c}\boldsymbol{y}.$$

Shuyang Dai Zhe Gan Yu Cheng Chenyang Tao Lawrence Carin Jingjing Liu

#### **Core ideas:**

- How does the defined compact metric space contribute to the formulation of a prior?
  - Poincaré ball based normal prior  $z \sim N_{Bn_c}(\mu, \Sigma)$ :

$$z = \exp_{\mu}^{c} \left( \frac{\lambda_{0}^{c}}{\lambda_{\mu}^{c}} v \right), v \sim \mathcal{N}(0, \Sigma).$$
 (7)

- Inspired by iVAE, we can enhance  $v := G(x, \xi; \phi_1)$  and  $\mu := F(x; \phi_2)$ , as outputs of encoder ( $\phi$ ). Then we get z from the above formula.
- Same as iVAE, they use dual form to evaluate KL(q<sub>Φ</sub>(z|x) | p(z)):

$$\mathbb{D}_{KL}(q_{\phi}(\boldsymbol{z}|\boldsymbol{x}) \parallel p(\boldsymbol{z})) = \max_{\boldsymbol{\psi}}$$

$$\left\{ \mathbb{E}_{\boldsymbol{z} \sim q_{\phi}(\boldsymbol{z}|\boldsymbol{x})} \nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z}) - \mathbb{E}_{\boldsymbol{z} \sim p(\boldsymbol{z})} \exp \nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z}) \right\},$$
(9)

Same as iVAE, νψ is optimized by the following:

$$\mathcal{L}_{1} = \mathbb{E}_{\boldsymbol{x} \sim \boldsymbol{X}} \left[ \mathbb{E}_{\boldsymbol{z} \sim q_{\boldsymbol{\phi}}(\boldsymbol{z}|\boldsymbol{x})} \nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z}) - \mathbb{E}_{\boldsymbol{z} \sim p(\boldsymbol{z})} \exp \nu_{\boldsymbol{\psi}}(\boldsymbol{x}, \boldsymbol{z}) \right], \quad (10)$$

Shuyang Dai Zhe Gan Yu Cheng Chenyang Tao Lawrence Carin Jingjing Liu

#### **Core ideas:**

• Different from iVAE, here the real prior is not assumed Gaussian. It's estimated by sampling scheme used in VampPrior (Tomczak and Welling, 2018)

$$p_{\delta}(\boldsymbol{z}) = \frac{1}{K} \sum_{k=1}^{K} q_{\phi}(\boldsymbol{z}|\boldsymbol{s}_{k}), \quad (12)$$

•  $\delta := \{s_k\}_{k=1}^K$  is now learnable pseudo inputs. Replacing p(z) with  $p_{\delta}(z)$  seeks to match the aggregated posterior  $q(z) = \sum q_{\phi}(z|x) / N$ .

Shuyang Dai Zhe Gan Yu Cheng Chenyang Tao Lawrence Carin Jingjing Liu

#### **Core ideas:**

 For the geometry-aware decoder, they use a deterministic (and learnable) hyperbolic linear function f to extract feature, then pass to LSTM decoder. (a, b, together with the LSTM, are trainable parameters of decoder θ)

$$f_{\boldsymbol{a},\boldsymbol{b}}^{c}(\boldsymbol{z}) = \operatorname{sign}(\langle \boldsymbol{a}, \log_{\boldsymbol{b}}^{c}(\boldsymbol{z}) \rangle_{\boldsymbol{b}}) \|\boldsymbol{a}\|_{\boldsymbol{b}} d_{c}^{\mathbb{B}}(\boldsymbol{z}, H_{\boldsymbol{a},\boldsymbol{b}}^{c}),$$

$$(8)$$
where  $H_{\boldsymbol{a},\boldsymbol{b}}^{c} = \{\boldsymbol{z} \in \mathbb{B}_{c}^{n} | \langle \boldsymbol{a}, \log_{\boldsymbol{b}}^{c}(\boldsymbol{z}) \rangle_{\boldsymbol{b}} = 0\},$ 

$$d_{c}^{\mathbb{B}}(\boldsymbol{z}, H_{\boldsymbol{a},\boldsymbol{b}}^{c}) = \frac{1}{\sqrt{c}} \sinh^{-1}\left(\frac{2\sqrt{c}|\langle \boldsymbol{\kappa}_{\boldsymbol{b},\boldsymbol{z}}, \boldsymbol{a} \rangle|}{(1-c||\boldsymbol{\kappa}_{\boldsymbol{b},\boldsymbol{z}}||^{2})||\boldsymbol{a}||}\right)$$

• θ and φ are optimized by the following objective

$$\mathcal{L}_{2} = \mathbb{E}_{\boldsymbol{x} \sim \boldsymbol{X}} \mathbb{E}_{\boldsymbol{z} \sim q_{\boldsymbol{\phi}}(\boldsymbol{z}|\boldsymbol{x})} [\log p_{\boldsymbol{\theta}}(\boldsymbol{x}|\boldsymbol{z}) - \nu_{\boldsymbol{\psi}}(\boldsymbol{x},\boldsymbol{z})]. \tag{11}$$

Shuyang Dai 'Zhe Gan Yu Cheng Chenyang Tao Lawrence Carin Jingjing Liu

#### **Core ideas:**

Training Procedure

#### Algorithm 1 Training procedure of APo-VAE.

- 1: **Input**: Data samples  $X = \{x_i\}_{i=1}^N$ , Poincaré curvature c, and number of pseudo-input K.
- 2: Initialize  $\theta$ ,  $\phi$ ,  $\psi$ , and  $\delta$ .
- 3: **for** iter from 1 to  $max\_iter$  **do**
- 4: Sample a mini-batch  $\{\boldsymbol{x}_m\}_{m=1}^M$  from  $\boldsymbol{X}$  of size M.
- 5: # Sampling in the Hyperbolic Space.
- 6: Obtain  $\mu_m$  and  $v_m$  from EncNet $_{\phi}(x_m)$ .
- 7: Move  $\boldsymbol{v}_m$  to  $\boldsymbol{u}_m = P_{\boldsymbol{0} \rightarrow \boldsymbol{\mu}_m}^c(\boldsymbol{v}_m)$  by (6).
- 8: Map  $\boldsymbol{u}_m$  to  $\boldsymbol{z}_m = \exp_{\boldsymbol{\mu}_m}^c(\boldsymbol{u}_m)$  by (5).
- 9: # Update the dual function and the pseudo-input.
- 10: Sample  $\tilde{z}_m$  by (12).
- 11: Update  $\psi$  and  $\delta$  by gradient ascent on (10)
- 12: # Update the encoder and decoder networks.
- 13: Update  $\theta$  and  $\phi$  by gradient ascent on (11).
- 14: **end for**

Shuyang Dai Zhe Gan Yu Cheng Chenyang Tao Lawrence Carin Jingjing Liu

Model	-ELBO	PPL	KL	MI	AU	
	PTB					
VAE	102.6	108.26	1.1	0.8	2	
$\beta$ -VAE	104.5	117.92	7.5	3.1	5	
SA-VAE	102.6	107.71	1.2	0.7	2	
vMF-VAE	95.8	93.70	2.9	3.2	21	
$\mathcal{P} ext{-VAE}$	91.4	76.13	4.5	2.9	23	
iVAE	87.2	53.44	12.5	12.2	<b>32</b>	
APo-VAE	87.2	53.32	8.4	4.8	<b>32</b>	
APo-VAE+VP	<b>87.0</b>	53.02	8.9	4.5	<b>32</b>	
		Yahoo	)			
VAE	328.6	61.21	0.0	0.0	0	
$\beta$ -VAE	328.7	61.29	6.3	2.8	8	
SA-VAE	327.2	60.15	5.2	2.9	10	
LAG-VAE	326.7	59.77	5.7	2.9	15	
vMF-VAE	318.5	53.92	6.3	3.7	23	
$\mathcal{P} ext{-VAE}$	313.4	50.57	7.2	3.3	27	
iVAE	309.1	47.93	11.4	<b>10.7</b>	<b>32</b>	
APo-VAE	286.2	47.00	6.9	4.1	<b>32</b>	
APo-VAE+VP	285.6	46.61	8.1	4.9	<b>32</b>	
	Yelp					
VAE	357.9	40.56	0.0	0.0	0	
$\beta$ -VAE	358.2	40.69	4.2	2.0	4	
SA-VAE	357.8	40.51	2.8	1.7	8	
LAG-VAE	355.9	39.73	3.8	2.4	11	
vMF-VAE	356.2	51.03	4.1	3.9	13	
$\mathcal{P} ext{-VAE}$	355.4	50.64	4.3	4.8	19	
iVAE	348.7	36.88	11.6	11.0	<b>32</b>	
APo-VAE	319.7	34.10	12.1	7.5	<b>32</b>	
APo-VAE+VP	316.4	32.91	<b>12.7</b>	6.2	<b>32</b>	