CLASS 9th

Mathematics

FORMULA SHEET

Number System

⇒ Natural Numbers : All counting numbers are called natural numbers.

$$\{1, 2, 3, 4 - - - \}$$

⇒ Whole Numbers : Natural numbers along with 0 form the collection of whole numbers.

$$\{0, 1, 2, 3 - - - \}$$

(*)⇒ Every natural number is whole number but every whole number is not natural number.

As $0 \longrightarrow$ whole number but not natural no.

⇒ Integers : The natural numbers, zero and negative of all natural numbers form the collection of all integers.

$$\{ - - - -3, -2, -1, 0, 1, 2, 3, - - - \}$$

⇒ Rational Numbers : The numbers which can be expressed in the form of p/q, where p and q are integers and $q \neq 0$, are called rational numbers.

eg:- $\frac{3}{2}, \frac{-4}{3}, 0, 5$

$0 \rightarrow \frac{0}{1}$, $5 \rightarrow \frac{5}{1}$

(*)⇒ $\frac{1}{5} = \frac{2}{10} = \frac{3}{15} = \frac{4}{20}$ - - - are called equivalent rational nos.

⇒ Writing Rational numbers between two given numbers:

steps (i) A rational number between two rational number a and b is $\frac{a+b}{2}$

(ii) Then find rational numbers between a and $\frac{a+b}{2}$

i.e. $\frac{a+\left(\frac{a+b}{2}\right)}{2}$ and so on...

⇒ Decimal expansion of a rational number is either terminating or non-terminating.

(*)⇒ Terminating and non-terminating repeating are Rational numbers.

eg:- 3.14, $0.88888---$, $1.\overline{4}$, 0 } Rational Numbers

⇒ Rat. No. $\begin{smallmatrix}+\\-\\\times\end{smallmatrix}$ Rat. No = Rational No.

Rat. No. $\begin{smallmatrix}+\\-\\\times\end{smallmatrix}$ Irr. No = Irrational No. (except when rat. no. = 0)

Irr. No. $\begin{smallmatrix}+\\-\\\times\end{smallmatrix}$ Irr. No. = may be rat. maybe irr.

⇒ $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$; $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$\Rightarrow \quad (\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = (\sqrt{a})^2-(\sqrt{b})^2$
$\qquad\qquad\qquad\qquad\qquad\qquad = a-b$

$\Rightarrow \quad (a+\sqrt{b})(a-\sqrt{b}) = (a)^2-(\sqrt{b})^2$
$\qquad\qquad\qquad\qquad\qquad\qquad = a^2-b$

$\Rightarrow$ To rationalise the denominator of $\dfrac{1}{\sqrt{a}+b}$, we multiply this by $\dfrac{\sqrt{a}-b}{\sqrt{a}+b}$, where a and b are integers

$\Rightarrow$ Let a is a +ve real number. $(a>0)$
m and n are integers

**

$$a^m \cdot a^n = a^{m+n}$$

$$(a^m)^n = a^{mn}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^m b^m = (ab)^m \quad \{b>0\}$$

Polynomials

⇒ General form of a polynomial of degree 'n' is given by

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0, \quad a_n \neq 0$$

where n is a +ve integer and $a_n, a_{n-1}, \ldots, a_1, a_0$ are real no.

⇒ $\underbrace{ax^2}_{\text{terms}} + \underbrace{bx}_{\text{terms}} + \underbrace{c}_{\text{terms}}$; $a, b, c \rightarrow$ constants, $x \rightarrow$ variable

In any term, $\underbrace{a}_{\text{coefficient of } x^2}\ \underbrace{x^2}_{\text{variable part}}$

So, No. of terms in above polynomial ⇒ 3

coefficient of x^2 ⇒ a

coefficient of x ⇒ b

coefficient of x^0 ⇒ c

highest degree of x ⇒ 2

this is called **degree of polynomial**.

⇒ CLASSIFICATION OF POLYNOMIALS

- On basis of "No. of terms"
  - 1 term ⇓ monomial
  - 2 term ⇓ Binomial
  - 3 term ⇓ trinomial
- On basis of "Degree of Polynomial"
  - DOP = 0 ⇓ Constant Polynomial
  - DOP = 1 ⇓ Linear
  - DOP = 2 ⇓ Quadratic
  - DOP = 3 ⇓ Cubic

⇒ Constant Polynomial → { 1, 2, 3, 4, 5 - - - }

$1x^0$ $2x^0$ $3x^0$ $4x^0$

so, a polynomial with degree 0 is constant polynomial.

⇒ Zero polynomial {0}

↳ $0\,x^{(\text{positive integer})}$

so, degree → not defined

⇒ Remainder theorem: let p(x) be any polynomial of degree greater than or equal to one and 'a' be any real no. If p(x) is divided by (x−a), then remainder is equal to p(a)

⇒ Factor theorem: If p(x) is a polynomial of degree $n \geq 1$ and 'a' is any real number then,

(i) (x−a) is a factor of p(x), if p(a) = 0

(ii) p(a) = 0, if (x−a) is a factor of p(x).

⇒ Algebraic Identities:

$$(a+b)^2 = a^2 + b^2 + 2ab$$
$$(a-b)^2 = a^2 + b^2 - 2ab$$
$$a^2 - b^2 = (a+b)(a-b)$$
$$(x+a)(x+b) = x^2 + (a+b)x + ab$$
$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$(a+b)^3 = a^3+b^3+3ab(a+b)$$

$$(a-b)^3 = a^3-b^3-3ab(a-b)$$

$$a^3+b^3 = (a+b)(a^2+b^2-ab)$$

$$a^3-b^3 = (a-b)(a^2+b^2+ab)$$

$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

if $a+b+c=0$, then $a^3+b^3+c^3 = 3abc$

Lines and Angles

⇒ Line with two end points is called a line segment.

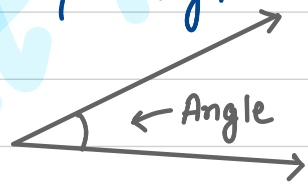
⇒ Part of a line with one end point is called Ray.

⇒ Line segment is denoted by $\overline{}$, like line segment AB is denoted by $\overline{AB}$.

⇒ If three or more points lie on the same line, they are called collinear points.

otherwise it is called non-collinear points.

⇒ Angle is formed when two rays originate from the same end points.



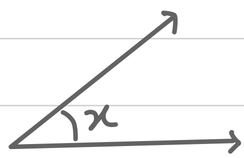
⇒ Rays making an angle are called arms of the angle and the end points are called vertex of the angle.

⇒ Acute angle

$$0° < x < 90°$$

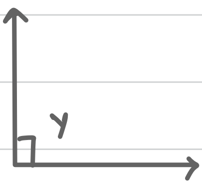
or

less than 90° and more than 0°



⇒ Right Angle

$$y = 90°$$



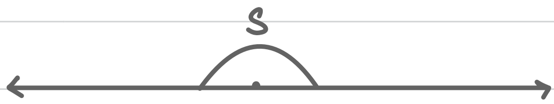
⇒ Obtuse Angle

$$90° < z < 180°$$

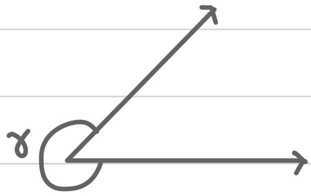
or

more than 90° and less than 180°

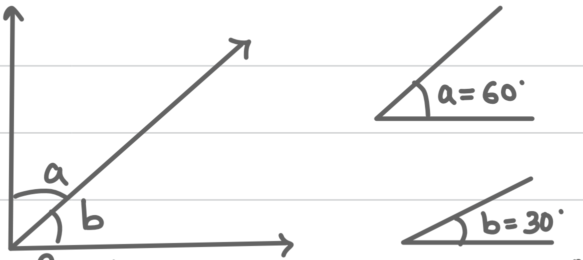
⇒ Straight Angle $s = 180°$



⇒ Reflex Angle $180° < r < 360°$

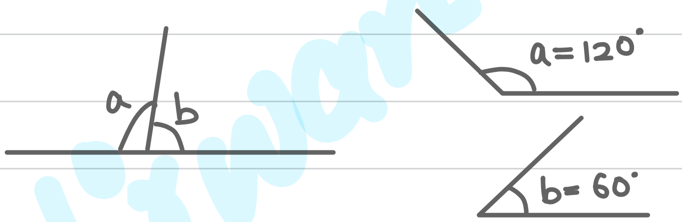


⇒ Complementary Angle
Two angles whose sum is 90° are called complementary angles.



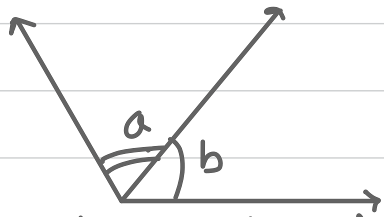
in both cases → $a + b = 90°$

⇒ Supplementary angles
Two angles whose sum is 180° are called supplementary angles



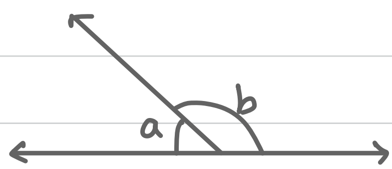
in both cases → $a + b = 180°$

⇒ Adjacent Angles
Two angles are adjacent if they have a common arm and their non-common arms are on different sides of common arm.

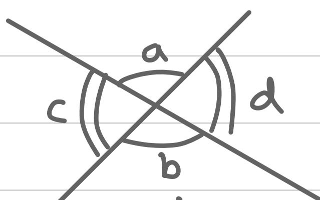


a & b are adjacent angles

⇒ Linear Pair of angles
When a ray stands on a line and form an adjacent angle of sum 180°, is called linear pair angles.

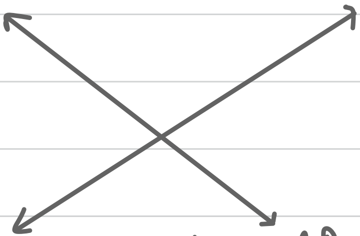


⇒ Vertically opposite angles (V.O.A)
When two lines intersect each other at a point, then the opposite angle formed are called V.O.A.
⊛ ⇒ they are always equal.



$a = b$
$c = d$

⇒ Intersecting and Non-intersecting lines



intersecting lines

non-intersecting lines

⇒ the lengths of the common perpendicular at different points on these parallel lines are same. This equal lengths is called the distance between parallel lines

Important Axioms :

Axiom 1) If a ray stands on a line, then the sum of two adjacent angles so formed is 180°.

Axiom 2) If the sum of two adjacent angles is 180°, then the non-common arms of the angles form a line.

Important Theorems :

Theorem 1) If two lines intersect each other, then the vertically opposite angles are equal.

Theorem 2) Lines which are parallel to the same line are parallel to each other.

Triangles

⇒ Two figures are said to be congruent, if they have same shape and size.

⇒ Two line segments are equal when their lengths are equal.

⇒ Two circles are congruent when radii are equal.

⇒ SAS congruence rule : Two triangles are congruent if two sides and the included angle of one triangle are equal to the sides and the included angles of other triangles.

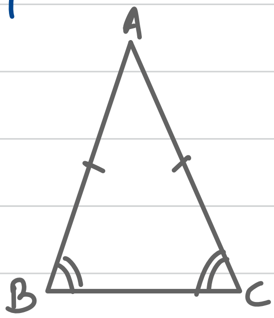
⇒ ASA congruence rule : Two Δ are congruent if two angles and the included side of one triangle are equal to the angles and the included side of other Δ.

⇒ AAS congruence rule : Two Δ are congruent if any two pairs of angles and one pair of corresponding sides are equal.

⇒ SSS congruence rule : If three sides of one Δ are equal to the three sides of another Δ, then the two Δ are congruent.

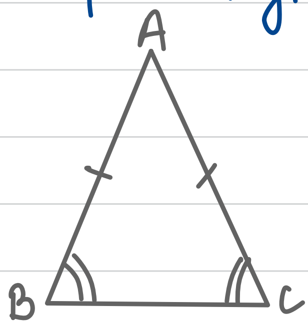
⇒ RHS congruence rule : If in two right Δ the hypotenuse and one side of one Δ are equal to the hypotenuse and one side of other Δ, then two Δ are congruent.

⇒ Angles opposite to equal sides of an isosceles Δ are equal.



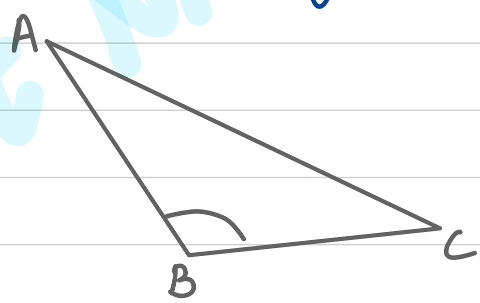
if $AB = AC$
then, $\angle B = \angle C$

⇒ The sides opposite to equal angles of a Δ are equal.



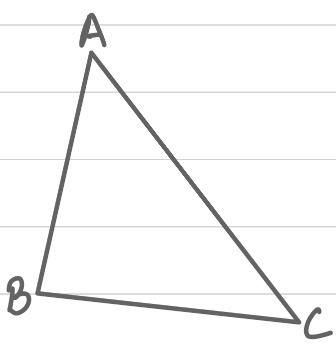
if $\angle B = \angle C$
then, $AB = AC$

⇒ In a Δ, the angle opposite to longer side is larger.



⇒ Vice versa, the side opposite to larger angle is larger.

⇒ The sum of any two sides of a Δ is greater than the third side.



$AB + AC > BC$
$AB + BC > AC$
$BC + AC > AB$

Heron's Formula

The area of Δ when its height is given $= \frac{1}{2} \times \text{base} \times \text{height}$

but if there is no height ?
for example A triangular park with three sides 40 m, 30 m and 20 m. How will we calculate ?

For that we use Heron's formula :-

$$\boxed{\text{Area of } \Delta = \sqrt{s(s-a)(s-b)(s-c)}}$$

where a, b and c are the sides of the triangle and s is the semi perimeter i.e. $s = \frac{a+b+c}{2}$

for example: sides of Δ are $a = 40\text{m}$
$b = 24\text{m}$
$c = 32\text{m}$

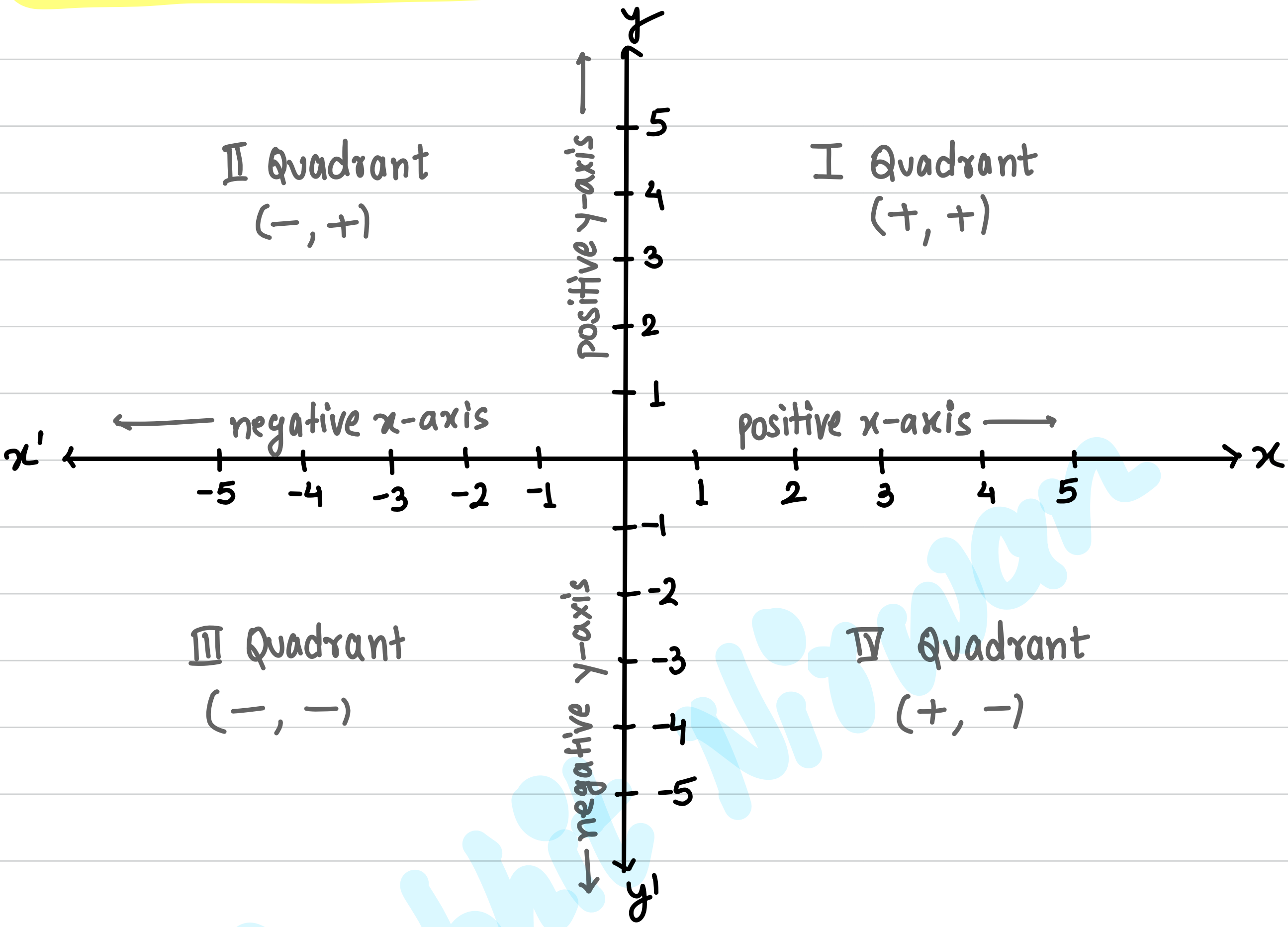
solution: $s = \frac{40+24+32}{2}$

$$\boxed{s = 48\text{m}}$$

Now, $\left\{ \begin{array}{l} s-a = (48-40)\text{m} = 8\text{m} \\ s-b = (48-24)\text{m} = 24\text{m} \\ s-c = (48-32)\text{m} = 16\text{m} \end{array} \right\}$

$\therefore$ Area of park $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{48 \times 8 \times 24 \times 16} \Rightarrow 384\ \text{m}^2$

Coordinate Geometry



- Horizontal line is called x-axis.
- Vertical line is called y-axis.
- Point of intersection of x-axis & y-axis is called origin.
- Quadrant → both the axes divide the plain in four equal parts and each part is called quadrant.
- Coordinate (Ordered pair) –
  A coordinate is the mathematical expression for denoting the value of x-axis and y-axis and denoted by (x, y)
  where value of x is called as abcissa
  and, value of y is ordinate

Linear Eqs. in 2 Var.

⇒ A linear equation in two variables has infinite solutions

⇒ An equation of the form $ax+by+c=0$, where a, b and c are real numbers, such that a and b are not both zero, is called a linear equation in 2 variables.

⇒ Every point on the graph of a linear equation in 2 variables is a solution of the linear equation.

⇒ And, every solution of the linear equation is a point on the graph of the linear equation.

⇒ Equation of y-axis → $x=0$
Equation of x-axis → $y=0$

⇒ The graph $x=a$ is a line parallel to y-axis
The graph $y=b$ is a line parallel to x-axis.