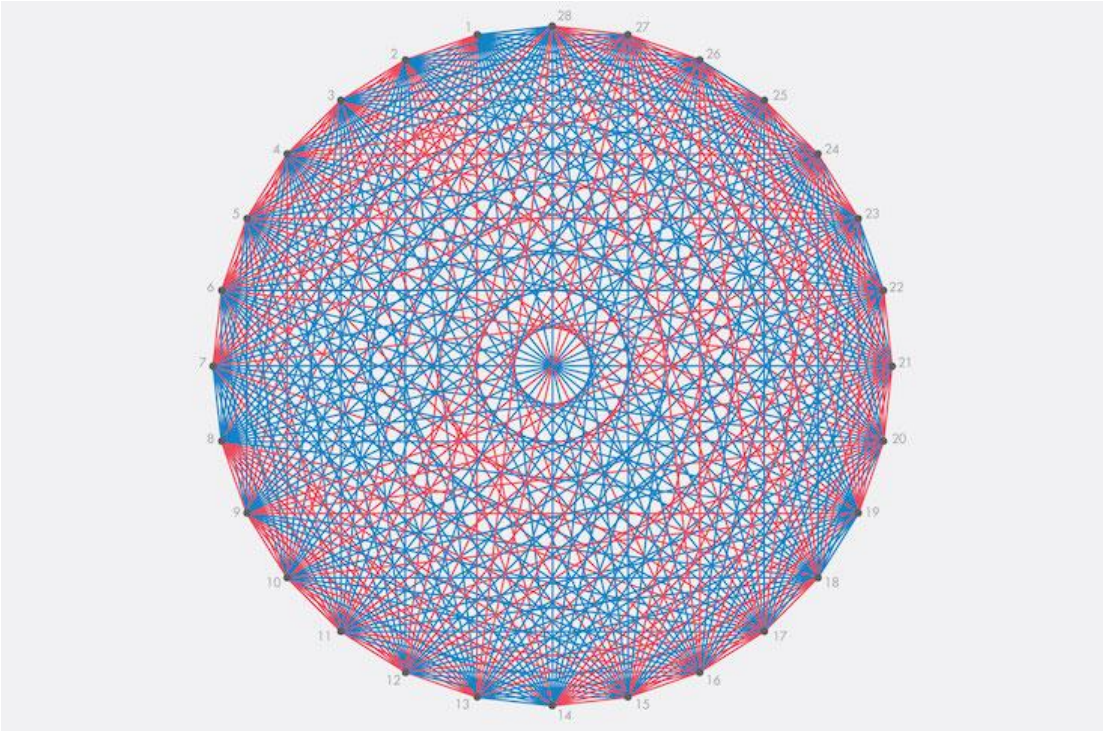
Pick up a handout as you come in

Welcome to EECS 203!



EECS 203: Discrete Mathematics

Lecture 1

Learning Objectives: Lec 1

After today's lecture (and the associated readings, discussion, & homework), you should be able to:

- Understand the meaning of "discrete math"
- Understand class logistics
  - Homework schedule
  - Lecture/discussion schedule
  - COVID policies
  - Exams and grading
  - Q&A resources: piazza, admin form
- **Know Technical Vocab:** proposition, negation, axiom, theorem, proof, refutation, counterexample, constructive proof
- Find the negation of simple propositions
- Prove or refute simple statements about numbers

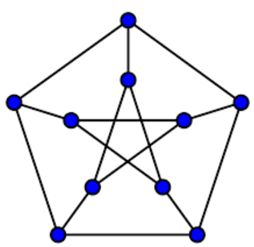
Outline

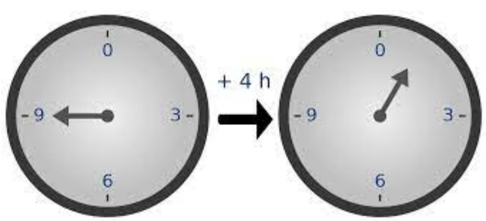
- **Intro to Discrete Structures**
- Intermission: Course Logistics
- Intro to Proofs
  - Propositions and negations
  - Proofs
  - Refutations

EECS 203 is two classes for the price of one! Wow!

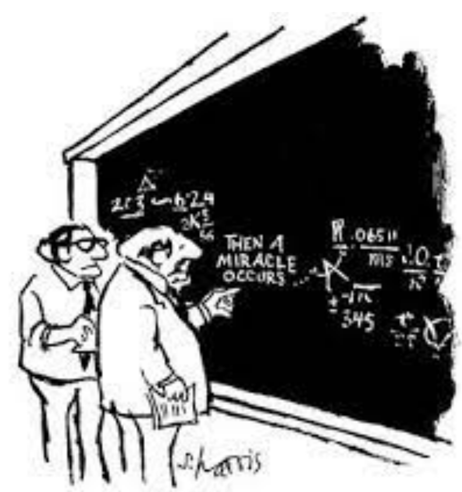
You will learn:

How to think about **discrete structures**





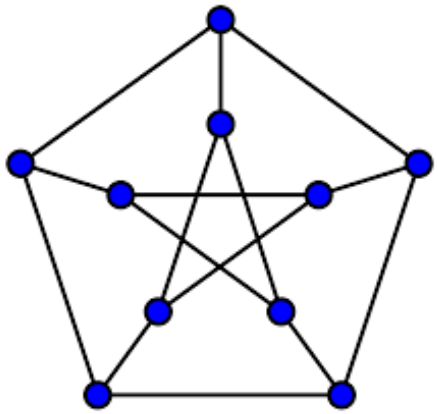
How to **argue ("prove") mathematical truths**

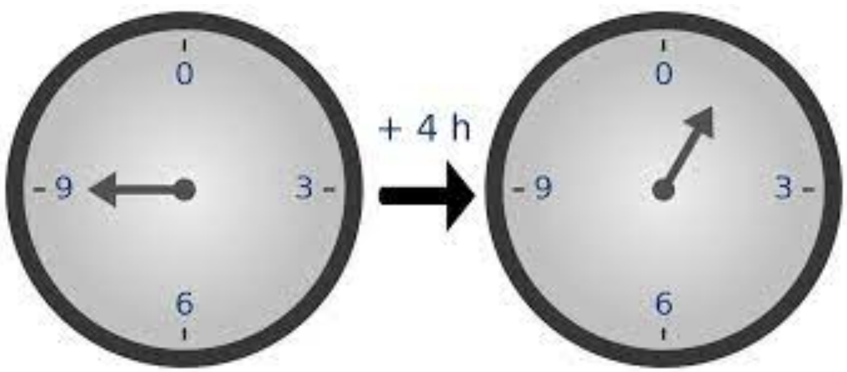


"I think you should be more explicit here in step two."

EECS 203: Discrete Mathematics

Part 1: What is a discrete structure?





EECS 203: Discrete Mathematics

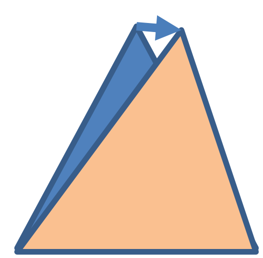
"**Discrete**" is the opposite of "**continuous**"

- **Continuous:** you can make an arbitrarily small change to the input, and it's still a valid input.

The real numbers are continuous:

x is a real number → $x + 0.000001$ is also a real number

Geometry is continuous:



Small change to coordinate → still a valid triangle

EECS 203: Discrete Mathematics

"**Discrete**" is the opposite of "**continuous**"

- **Discrete:** no "tiny" changes to the input are allowed: input must "jump" from one option to another

The integers are discrete:

x is an integer → $x + 0.000001$ is **not** an integer

Truth values are discrete:

- "$2 + 2 = 4$" is true
- "Ann Arbor is the capital of Michigan" is false
- Statements can't be "only kind of true"

Continuous vs. Discrete

- Let x be a **real number (continuous)**.
- For what value(s) of x is y smallest, given that

$$y = (x\text{-}0.25)^2?$$

– Solution: techniques from calculus

- *Find derivative* $\frac{dy}{dx}$, *and find value of* x *for which the derivative is 0.*

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Continuous vs. Discrete

- Let x be an **integer (discrete)**.
- For what value(s) of x is y smallest, given that

$$y = (x\text{-}0.25)^2?$$

- Attempt: *find derivative* $\frac{dy}{dx}$…
  - STOP!
  - Derivatives measure "tiny change in y / tiny change in x"
  - This is fundamentally **continuous** and doesn't work over the integers.
  - So what should we do instead?

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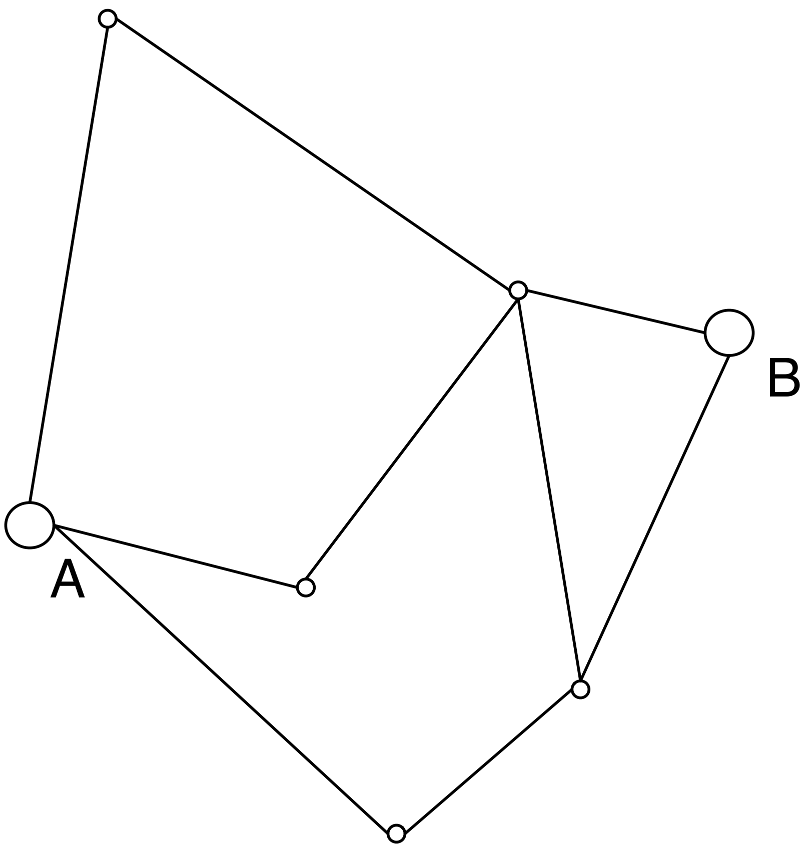
Continuous vs. Discrete

- Find the shortest path between A and B
  - **Continuous** problem: tiny change to a valid path is still valid
  - You know the answer is a straight line. Can you prove it?
    - *Yes, you can…with a lot of **continuous** work using line integrals and the Pythagorean differential.*
    - *Discussion: http://www.lecture-notes.co.uk/susskind/classical-mechanics/lecture-2/shortest-distance-between-two-points/*

Continuous vs. Discrete



- Find the shortest path between A and B **among those made of shorter lines**.
  - *This is a* ***discrete problem:***
    - *tiny change to a valid path ≠ valid path*
  - *Geometry is not helpful anymore!*
  - We need fundamentally new definitions and techniques
    - **Graphs, edges, vertices** – coming later in this course

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Take away

- Two familiar problems where we need new solutions when part of the problem is discrete.
  - Function minimization: *integers* instead of *reals*
  - Shortest path: paths made from shorter, allowable paths instead of *any path*

- In both cases, ***calculus*** won’t help. We need **Discrete Math**.

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Why study discrete problems?

- **Computers are fundamentally discrete**
  - Everything stored as 0/1s
  - Even if they work hard to give the **illusion** of continuity! (Floating point numbers…)
- Communication: error-detecting and error-correcting codes
- Genomics (DNA alphabet {A,C,G,T} only)
- Physics – parts of an atom, electron states
- Data science
- …

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Outline

- Intro to Discrete Structures
- **Intermission: Course Logistics**
- Intro to Proofs
  - Propositions and negations
  - Proofs
  - Refutations

Prof. Diaz

- Pronouns: she/her/hers
- You can address me as:
  - Prof. Diaz
  - Dr. Diaz
  - ~~Ms. Diaz~~

- I believe in an open, interactive, collaborative classroom (yes, even in a huge lecture, with masks)
  - Ask questions!
  - Get to know each other

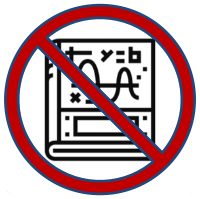
Lecture 1 Handout: Welcome to EECS 203!

Handouts are not submitted or graded. They're just a supplement to lecture.

- Meet your classmates
  - Name & uniqname for 3 classmates?
    - ______________________________
    - ______________________________
    - ______________________________
  - Something you all have in common? ____________________________________

- Something you'll do for self-care this semester ______________________________

Important things to know about EECS 203



Discrete Math is a different kind of math than calculus

The material generally takes time to digest

- Come to discussion & office hours
- Start assignments early!

THIS IS A NEW KIND OF MATH. It might not come easily - AND THAT'S OK!

We encourage you to adopt a **Growth Mindset**

- The belief that your abilities can be developed through dedication and hard work

Our Goals for the Course

- Help you learn the material
- Allow for collaborative learning
- We want 203 to be a roadmap, not an obstacle, for your academic & professional career

- We've made a lot of changes this year
  - E.g., streamlined course content
  - We will be asking for your feedback periodically to see how things are working for you.

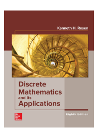
Course Logistics Cheat Sheet

canvas — Home base for the course – announcements, calendar, files, links

gradescope — Submit homeworks here (Most Thursdays, 10:00pm)

piazza — Ask clarifying questions about course content or logistics

Recordings and some livestreams available if you don't want to attend in person



Textbook ("Discrete Mathematics and its Applications," 8th edition, Rosen)

Admin Form — Request accommodations for special circumstances (e.g., extended illness, etc.)

Course Resources

Canvas is our central site for the course

- Syllabus, schedule, readings, homework, grades, announcements
- Lecture slides, lecture/discussion recordings

Canvas has links to all other resources you'll need:

Course Calendars

Find Lecture & Discussion calendar
Find Office Hours calendar

Piazza

Online Q&A forum

Gradescope

Submit your homework here

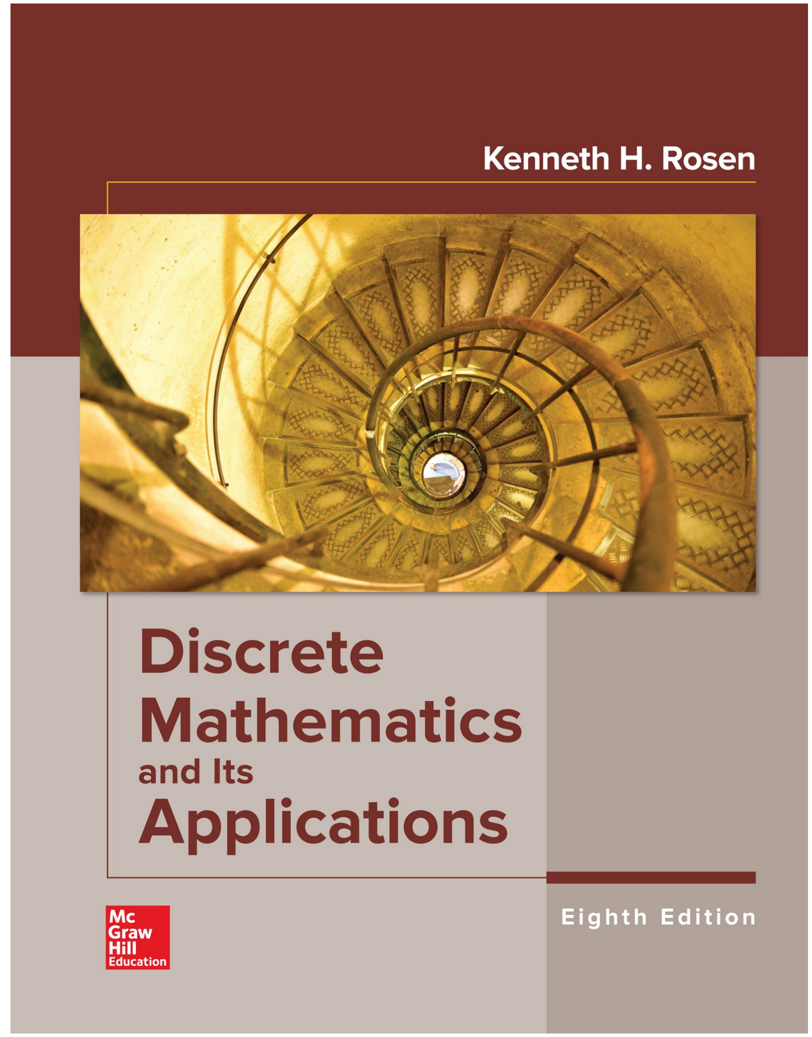
Textbook

- "Discrete Mathematics and its Applications" 8th Edition, by Ken Rosen

- 7th Edition is fine, but the page numbers/problem numbers/etc. might be slightly different

- Online version available from UM library:

https://ebookcentral-proquest-com.proxy.lib.umich.edu/lib/umichigan/detail.action?docID=547126 9#



Lectures & Discussion

You can attend any lecture and discussion*

- Get off the waitlist ASAP
- Attendance is **not required***
- *except Focus on Fundamentals discussions
- Fri 12:30-2:30pm (Sec 17)
- Mon 5:30-7:30pm (Sec 16)

M	T	W	Th	F

Discussion cycle:

- Thursday → Wednesday cover the same discussion material

Discussions begin today!

No discussions on MLK Day (Mon, Jan 17)

If you are in a Monday Section, you can:

- Attend a different discussion on Thurs/Fri/Tues/Weds
- Watch one of the discussion recordings

Homework

Homework 0 (optional) posts tonight

Homework 1 posts in 1 week (1/13)

Weekly Homework

Typically due Thursdays (10:00 pm)

Grace period for late submissions until 11:59pm

Submitting Homework

- Submit homework to **Gradescope**
- Submit as **PDF**
- **Refer to the Syllabus for detailed instructions on submission requirements and formats**
- Additional homework policies (including drop policy and honor code) are also in the Syllabus

Exams

EECS 203 has 3 exams

Exams are equally weighted

- Exam 1: Weds Feb 16, 7-9pm
- Exam 2: Weds Mar 23, 7-9pm
- Exam 3: Tues Apr 26, 7-9pm

Exams will be given **remotely** (more details TBA)
We will book a room that you can optionally use
if you need a quiet place to take the test.

Exam Accommodations and Conflicts

SSD Accommodations

- Instructors now get SSD VISA form directly through the University
- For questions about your accommodations, contact us via the **Admin Form**

If you have a conflict with one of the exam dates...

Note it on the Exam Date Confirmation Form (link and deadline on Canvas)

! If your conflict is with Exam 3 (Apr 26)

- You need to clear with us by **Jan 25** (add/drop deadline)

- **We will not be able to provide an alternate Exam 3 due to travel or different time zones**
- **We recommend you do not schedule travel before Exam 3.**

Everyone needs to fill out the Exam Date Confirmation Form

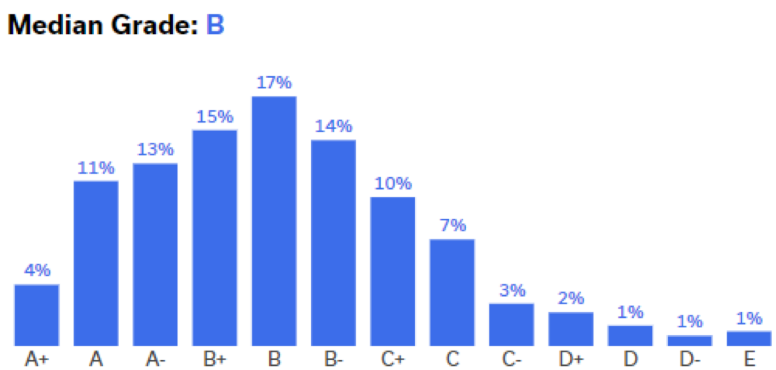
(even if you don't have any conflicts).

Grades

- Grade Breakdown in table at right
- Exams are curved, other assignments are not
- All grades will be posted to Canvas

Component	% of grade
Homework	20%
Exams	78%
Surveys	2%

- Historical grade distribution for EECS 203 is in the graph at right



https://atlas.ai.umich.edu/

When You Have Questions

Content Questions, General Logistical Questions

Attend **Office Hours**

Visit **Piazza**

- Ask questions
  - **Search first** to see if your question has already been asked and answered
- Answer other students' questions

Individual Administrative Questions

Use **Admin Form**

- Absences
- VISA accommodations
- Other personal, administrative questions

COVID-Related Policies

We want to provide an environment where all students can feel that they can **learn safely**. We also want to do our part to **maintain in-person learning** for the whole semester and follow **university guidelines**.

- Attendance Policies
- Mask Policies
- Self-Care and Accommodations

COVID-Related Policies

Attendance Policies

- Attendance is **not** required for lecture or discussion
  - Recordings will be available
  - Some lectures are livestreamed (see Canvas announcement)
    - If you aren't feeling well, please stay home
    - If you just aren't comfortable coming to class in-person, please stay home
- Virtual office hours are also available

COVID-Related Policies

Mask Policies

- **Masks are required in the classroom**
- If you come without a mask, we'll ask you to put one on
    - You also have the option to leave
- **If you don't want to wear a mask, please use the online options for lecture & discussion**

COVID-Related Policies

Self-Care and Accommodations

- This is a tough time… as has been the last two years
  - Please be sure to take time to attend to your **physical and mental health**
  - We recognize that **you have lives outside of this course** (so do we!), and that sometimes things happen that impact your learning
  - We've tried to adopt policies that **allow for flexibility** when you need it

- If something is going on that is affecting you, please let us know.
  - We will look into possible **accommodations** within the course
  - We can connect you with **other resources on campus**
  - Contact us via the **Admin Form** (link on Canvas) or at eecs203-admin@umich.edu

Other Course Policies

- See the **Syllabus** for full policies
- Your class work might be used for research purposes and to help us improve the course. You can opt out anytime using the **Admin Form**.

Outline

- Intro to Discrete Structures
- Intermission: Course Logistics
- **Intro to Proofs**
  - Propositions and negations
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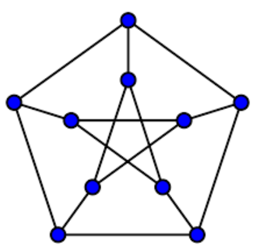
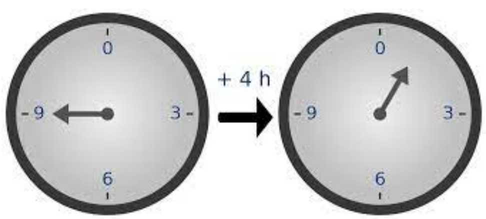
EECS 203 is two classes for the price of one! Wow!

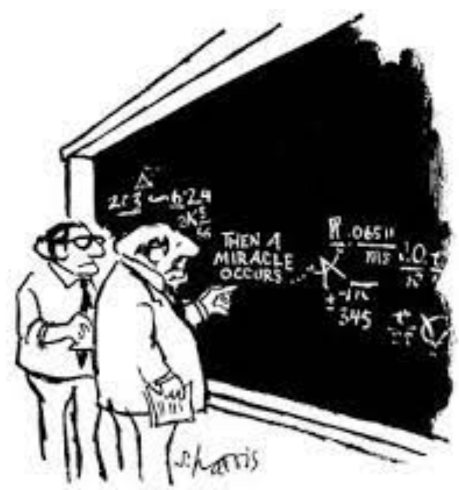
You will learn:

How to think about **discrete structures**





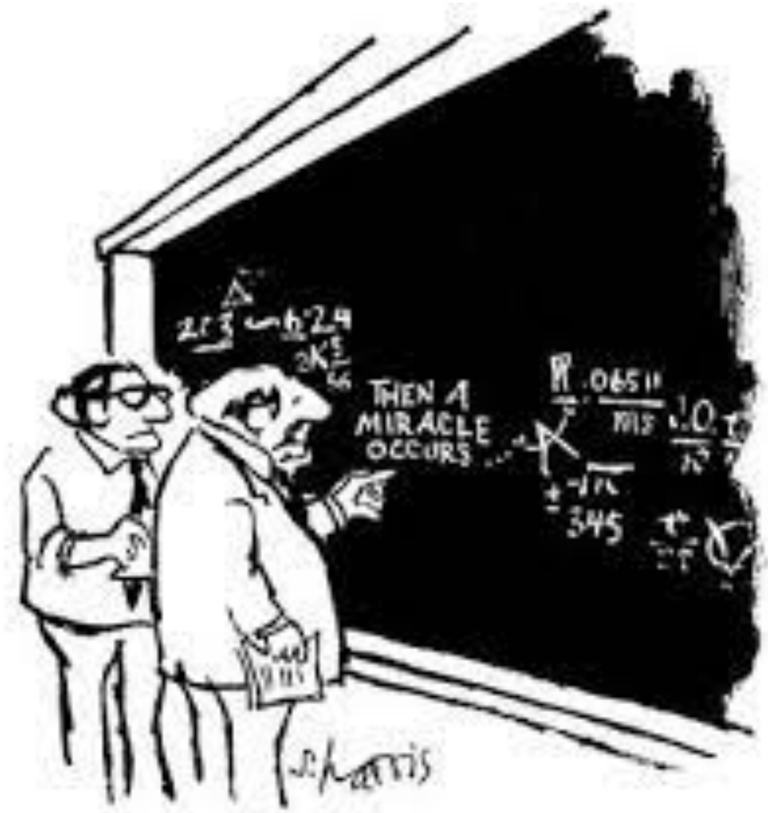
How to **argue ("prove") mathematical truths**



"I think you should be more explicit here in step two."

EECS 203: Discrete Mathematics

Part 2: How do we argue (“prove”) mathematical truths?



"I think you should be more explicit here in step two."

Propositions

- Our goal in this class is to **make statements**, and then show that they are either true or false.

- A **proposition** is a statement about the world that has a truth value (either true or false).

Proposition: $\boxed{2 + 2 \ = \ 4}$ (true)

Proposition: Ann Arbor is the capital of Michigan (false)

Proposition: Every even number ≥ 4 is the sum of two prime numbers (either true or false, but nobody knows)

Propositions

A **proposition** is a statement about the world that has a **truth value** (either true or false).

Why might something **not** be a proposition?

- It's not a declaration about the world
  - "Why does my knee hurt?"
  - "Go Blue!"

- It's paradoxical or not well defined
  - "Colorless green ideas sleep furiously"
  - "This statement is false"

- Some part is left unspecified (when specified, it would become a proposition)
  - $x + 5 \neq 10$

Negating Propositions

- Every proposition also has a **negation**: opposite statement with opposite truth value.
- In English, we often just add or remove the word **not** to get the negation.

Proposition: I am 107 years old.

Negated Proposition: I am **not** 107 years old.

Proposition: $2 + 2 = 4$

Negated Proposition: $2 + 2 \neq 4$

Propositions & Negations, Proofs & Disproofs

- **Proposition**: statement about the world that has a truth value
- **Negation** of a proposition is an opposite statement with the opposite truth value

- **Proof**: step-by-step process showing that a proposition is __________
- **Disproof/Refutation**: step-by-step process showing a proposition is ______
- Disproving a proposition is the same as ________________________

Disproving a "For all" statement is like proving a __________________ statement

Disproving a "There exists" statement is like proving a ______________ statement

The Toolkit

- Given a proposition, how can we show that it's true?
- Algebra, geometry, calculus are **tools** that can show that certain propositions are true

Proposition:

$$2(x+1)^2 = 2x^2 + 4x + 2$$

(how do you know it's true?)

Algebra:

$$2(x+1)^2 = 2(x^2 + 2x + 1) = 2x^2 + 4x + 2$$

The Toolkit

- Given a proposition, how can we show that it's true?
- Algebra, geometry, calculus are **tools** that can show that certain propositions are true

Proposition
For all integers **x**, if **x** is even, then **x+2** is even.

This proposition is true – but we will need to **mix algebra and logic** (using a "proof") in order to show it.

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Proposition

For all integers **x**, if **x** is even, then **x+2** is even.

goes with "for all"

Proof: …

- Let x be an arbitrary integer
- Assume x is even. Then there is an integer k, with x = 2k.
- x + 2 = 2k + 2 = 2 (k+1)
- Note: k+1 is an integer

So x+2 is twice an integer, so it's even

Thus for all integers x, if x is even, then x+2 is even.

scope of my assumption

Not ready to start proving yet! What does "**even**" mean, formally?

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition

For all integers **x**, if **x** is even, then **x+2** is even.

Proof: …

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition

For all integers **x**, if **x** is even, then **x+2** is even.

Proof:

- Let x be an arbitrary integer.

Our goal here is to show a statement for **all** integers x – so we shouldn't make any assumptions about x.

- **Important Keyphrase:** "For all"

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition
For all integers **x**, if **x** is even, then **x+2** is even.

Proof:
- Let x be an arbitrary integer.
- Assume x is even; there is an integer k with $x = 2k$.

Use the formal definition! No hope of a correct proof without it.

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition
For all integers **x**, if **x** is even, then **x+2** is even.

Proof:

- Let x be an arbitrary integer.
- Assume x is even; there is an integer k with $x = 2k$.
- So $x + 2 = 2k + 2$

Use the rules of algebra to add $+2$ to both sides

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition
For all integers **x**, if **x** is even, then **x+2** is even.

Proof:

- Let x be an arbitrary integer.
- Assume x is even; there is an integer k with $x = 2k$.
- So $x + 2 = 2k + 2$
- So $x + 2 = 2(k + 1)$

Use the rules of algebra to factor right-hand side

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition

For all integers **x**, if **x** is even, then **x+2** is even.

Proof:

- Let x be an arbitrary integer.
- Assume x is even; there is an integer k with $x = 2k$.
- So $x + 2 = 2k + 2$
- So $x + 2 = 2(k + 1)$
- Since k is an integer, $k + 1$ is also an integer

Uses: Integers are **closed** under addition (int + int = int)

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition
For all integers **x**, if **x** is even, then **x+2** is even.

Proof:
- Let x be an arbitrary integer.
- Assume x is even; there is an integer k with $x = 2k$.
- So $x + 2 = 2k + 2$
- So $x + 2 = 2(k + 1)$
- Since k is an integer, $k + 1$ is also an integer
- So $x + 2$ is even (it's 2 times an integer)
- **If x is even, then x+2 is even**

Use formal definition again. (And we're done!)

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

Proposition

For all integers **x**, if **x** is even, then **x+2** is even.

Proof:

- Let x be an arbitrary integer.
- […]
- **If x is even, then x+2 is even**

Proofs are often written in **paragraph** rather than **bullet point** format

- (We're starting with bullet points to emphasize the steps)

Proofs assume certain basic facts to be true, called "**axioms**"

- Rules of algebra
- Integers are closed under addition, subtraction, and multiplication

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

~~**Proposition**~~ **Theorem**

For all integers **x**, if **x** is even, then **x+2** is even.

Proof:

- Let x be an arbitrary integer.
- […]
- **If x is even, then x+2 is even**

A true proposition that has been proved is sometimes called a **theorem**.

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Theorem:
There exists an integer x for which x^2 is not positive.

Make sure you have a formal definition!

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"positive"** if $\boldsymbol{x} > \mathbf{0}$.

Theorem:
There exists an integer x for which x^2 is not positive.

"There exists" is the other important keyphrase
(the first was "for all")

You can prove "there exists" statements **just by naming one example, and showing that your example works**.
(called a "constructive proof")

Proofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"positive"** if $\boldsymbol{x} > \mathbf{0}$.

Theorem:
There exists an integer x for which x^2 is not positive.

Proof:
- Consider $x = 0$.
- So $x^2 = 0$
- $0 > 0$ is false, so 0 is not positive.

Done!

Disproofs/Refutations

- You can sometimes show that a proposition is true using a **proof**.
- You can sometimes show that a proposition is false using a **disproof** or **refutation**.
- A **disproof** or **refutation** is exactly the same kind of step-by-step process as a **proof**, but arguing that the proposition is **false** instead of **true**.

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

(False) Proposition:
There exists an integer x for which $4x$ is not even.

To **disprove** a "there exists" statement, you have to show that **no possible choice** of x will work. So you shouldn't make any assumptions about x…

Prove: for all integers x, $4x$ is even

the negation

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

(False) Proposition:
There exists an integer x for which $4x$ is not even.

Disproof:

- Let x be an arbitrary integer

Disproving a "there exists" statement is kind of like **proving** a "for all" statement!

(More on this shortly)

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

(False) Proposition:
There exists an integer x for which $4x$ is not even.

Disproof:
- Let x be an arbitrary integer
- Notice that $4x = 2(2x)$

Uses rules of algebra

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

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Disproof:
- Let x be an arbitrary integer
- Notice that $4x = 2(2x)$
- Since x is an integer, $2x$ is also an integer

Uses: Integers are **closed** under multiplication
(integer · integer = integer)

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"even"** if there exists an integer **k** with $\boldsymbol{x = 2k}$.

(False) Proposition:
There exists an integer x for which $4x$ is not even.

Proving: For all x, ~~4x~~ $4x$ is even

Disproof:

- Let x be an arbitrary integer
- Notice that $4x = 2(2x)$
- Since x is an integer, $2x$ is also an integer
- **So $4x$ is 2 times an integer, so it is even.**

Done!

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"a multiple of"** another integer **y** if there exists an integer **k** with $\boldsymbol{x = ky}$.

(False) Proposition:
For all integers x, $3x$ is a multiple of 9.

To **disprove** a "for all" statement, you only have to show one value of x where the statement doesn't hold…

*(Called a **counterexample**)*

Prove the negation!
There exists an integer x, where 3x is not a multiple of 9.

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"a multiple of"** another integer **y** if there exists an integer **k** with $\boldsymbol{x} = \boldsymbol{ky}$.

(False) Proposition:
For all integers x, $3x$ is a multiple of 9.

Disproof:

- Consider $x = 1$.

disproving a "for all" statement is kind of like **proving** a "there exists" statement!

(More on this in a few lectures)

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"a multiple of"** another integer **y** if there exists an integer **k** with $\boldsymbol{x = ky}$.

(False) Proposition:
For all integers x, $3x$ is a multiple of 9.

Disproof:
- Consider $x = 1$.
- So $3x = 3$.

Uses laws of algebra

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"a multiple of"** another integer **y** if there exists an integer **k** with $\boldsymbol{x = ky}$.

(False) Proposition:
For all integers x, $3x$ is a multiple of 9.

Disproof:

- Consider $x = 1$.
- So $3x = 3$.
- The unique real number k with $3x = 9k$ is $k = \frac{1}{3}$.

Uses laws of algebra

Disproofs

Integers: {... , -3, -2, -1, 0, 1, 2, 3, ...}

Definition: An integer **x** is **"a multiple of"** another integer **y** if there exists an integer **k** with $\boldsymbol{x = ky}$.

(False) Proposition:
For all integers x, $3x$ is a multiple of 9.

Disproof:
- Consider $x = 1$.
- So $3x = 3$.
- The unique real number k with $3x = 9k$ is $k = \frac{1}{3}$.
- $\frac{1}{3}$ is not an integer, so $3x$ is not a multiple of 9.

Done!

Disproofs

Proposition:
For all integers x, $3x$ is a multiple of 9.

Negated Proposition:
There exists an integer x for which $3x$ is **not** a multiple of 9.

Disproving a proposition is the same as **proving its negation**.

- We showed an example $x = 1$ where $3x$ is not a multiple of 9.

We'll talk more about how to negate complicated propositions in the next few lectures.

Disproofs

Proposition:
There exists an integer x for which $4x$ is not even.

Negated Proposition:
For all integers x, $4x$ is even.

Disproving a proposition is the same as **proving its negation**.

- We showed an example $x = 1$ where $3x$ is not a multiple of 9.

We'll talk more about how to negate complicated propositions in the next few lectures.

So what's a proof, anyways?

- Is a proof when you take a statement
  - ~~about algebra~~
  - ~~that you already knew~~
  - and explain it in step-by-step detail?

Proofs don't have to be about algebra! The **first unit** of this class will focus on numbers, but proofs can be about all kinds of things.

The best proofs convince you of a proposition that wasn't at all clearly true ahead of time!

We'll see proofs like this pretty soon.

This one is real.

Propositions & Negations, Proofs & Disproofs

- **Proposition**: statement about the world that ___________________________
- **Negation** of a proposition is an _________________ statement with the ______________ truth value

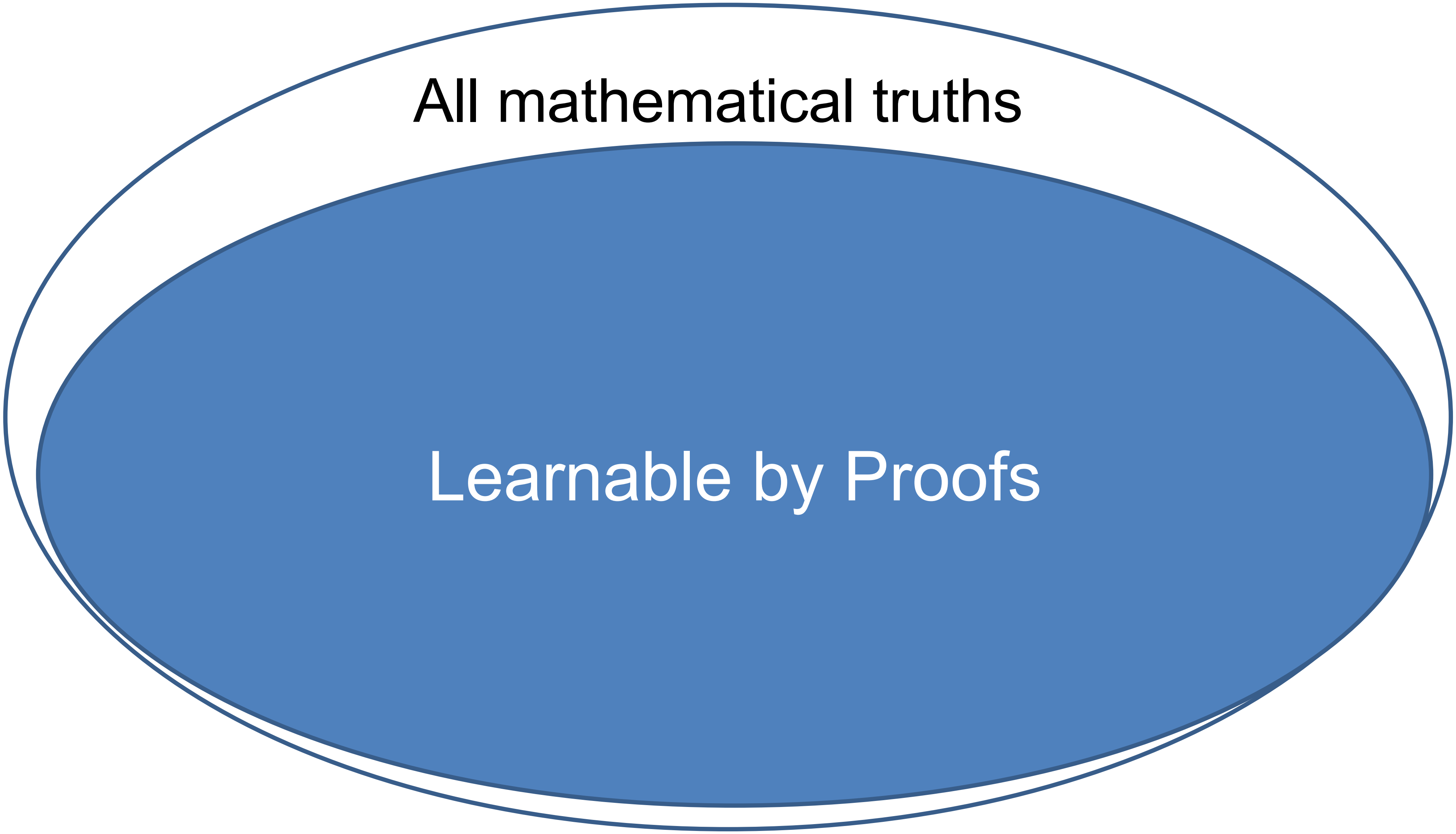
- **Proof**: step-by-step process showing that a proposition is true
- **Disproof/Refutation**: step-by-step process showing a proposition is false
- Disproving a proposition is the same as proving its negation

Disproving a "For all" statement is like proving a there exists statement

→ giving a counterexample

Disproving a "There exists" statement is like proving a for all statement

Your Toolkit



Next Lecture

- More practice with proofs

- **Intro to Modular Arithmetic:** A discrete structure that helps us think about even/odd, multiples, divisibility, …

Exercise: Decide whether or not these are propositions. If so, find their negation.

- Paris has the largest population of any city in France.

- There exists an integer x such that $x + y = 7$

- For all integers x, $(x + 2)^2 = \mathrm{x}^2 + 4$

Additional Exercise: Prove or disprove this proposition

Proposition: For all integers **x**, $\boldsymbol{x^2 > x}$.

Additional Exercise: Prove or disprove this proposition

Proposition: For all integers **x**, $4x + 3$ is odd.