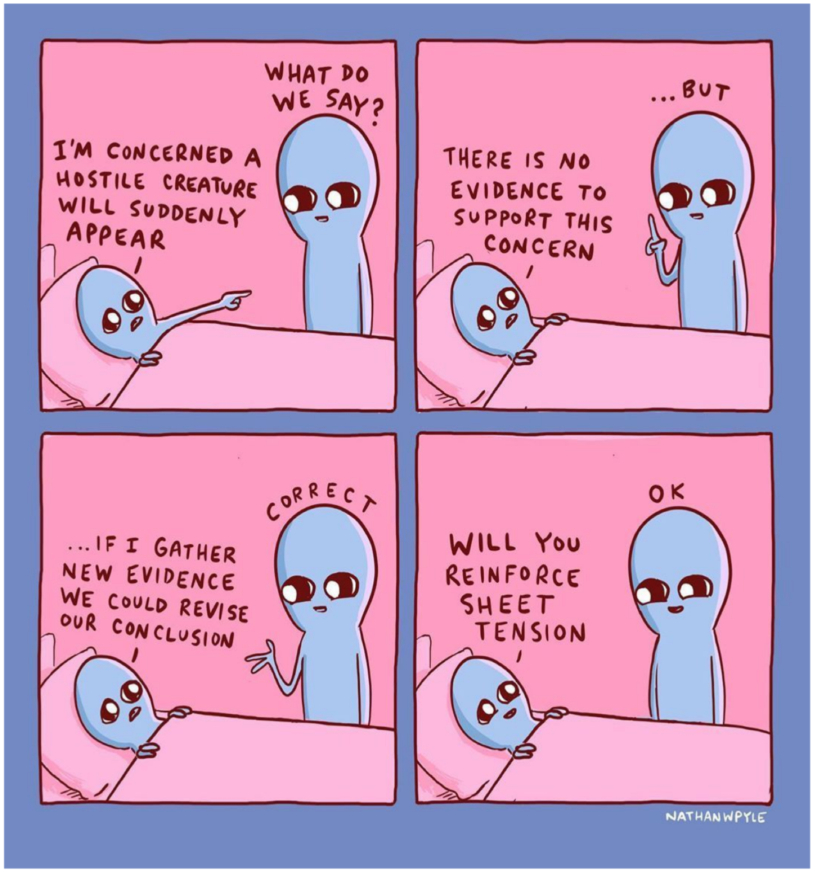
L26: Bayes' Theorem



3 End-of-Term Surveys

- Surveys due ***Wed. April 20th @11:59pm***:
  - Foundational Course Initiative (FCI) end of term survey.
  - Computing Cares Exit survey.
  - These are both assignments and part of your course grade

- Extra Credit
  - Complete your **Course Evaluations** for both lecture and discussion
  - Submit a screenshot of the confirmation page to Gradescope to earn extra credit added to your overall course grade.

Learning Objectives

After today's lecture (and associated readings, discussion & homework), you should be able to:

- State the relationship between Bernoulli Trials and Binomial Experiments, and between Bernoulli Trials and Geometric (aka "Waiting Time") Experiments
- Calculate probabilities and expected values for Binomial and Geometric random variables.

- Conditional probability and how to use it.
- Reasoning from inconclusive evidence.
- Using Bayes' theorem.
- Terminology: ***prior*** (distribution), ***likelihood***, ***posterior*** (distribution).

Overview

- **First part of lecture:**
  - Review on how to solve probability problems
  - Overview of two common "probability distributions"
- **Second part of lecture:**
  - How to use probability to *update your beliefs*
  - Bayes' Law and applications

Outline

- **Geometric Distribution**
- Binomial Distribution

- Bayes' Theorem
  - Ex: Diagnosing a Rare Disease

- Law of Total Probability with Bayes'
  - Total Probability
  - Ex: Soccer Team Drug Testing
  - Ex: Pocket full of Dice

- Monty Hall Problem

Geometric Random Variables

- Suppose you have a biased coin comes up Heads with probability $p \in (0,1)$ and Tails with probability $1 - p$.
- You flip repeatedly until you get the first Head. Let X be the total number of flips (including the last Head).
- What is the probability distribution, p(X = k)?
  - What is the sample space S?
  - What is p(s) and X(s) for each outcome?

Geometric Random Variables

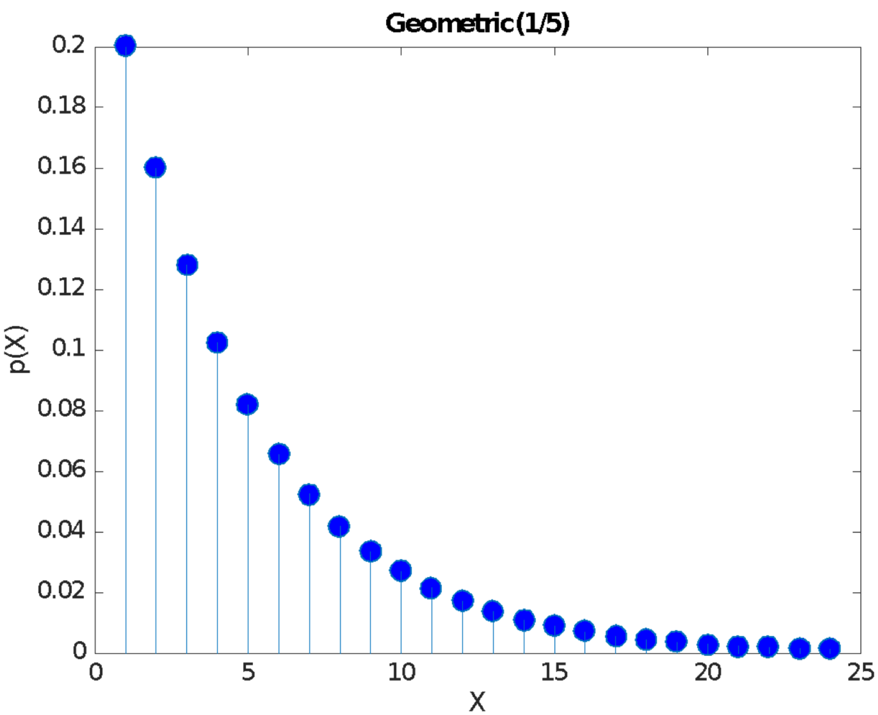
- Suppose you have a biased coin comes up Heads with probability $p \in (0,1)$ and Tails with probability $1 - p$.
- You flip repeatedly until you get the first Head. Let X be the total number of flips (including the last Head).
- What is the probability distribution, p(X = k)?
  - What is the sample space S?
  - What is p(s) and X(s) for each outcome?

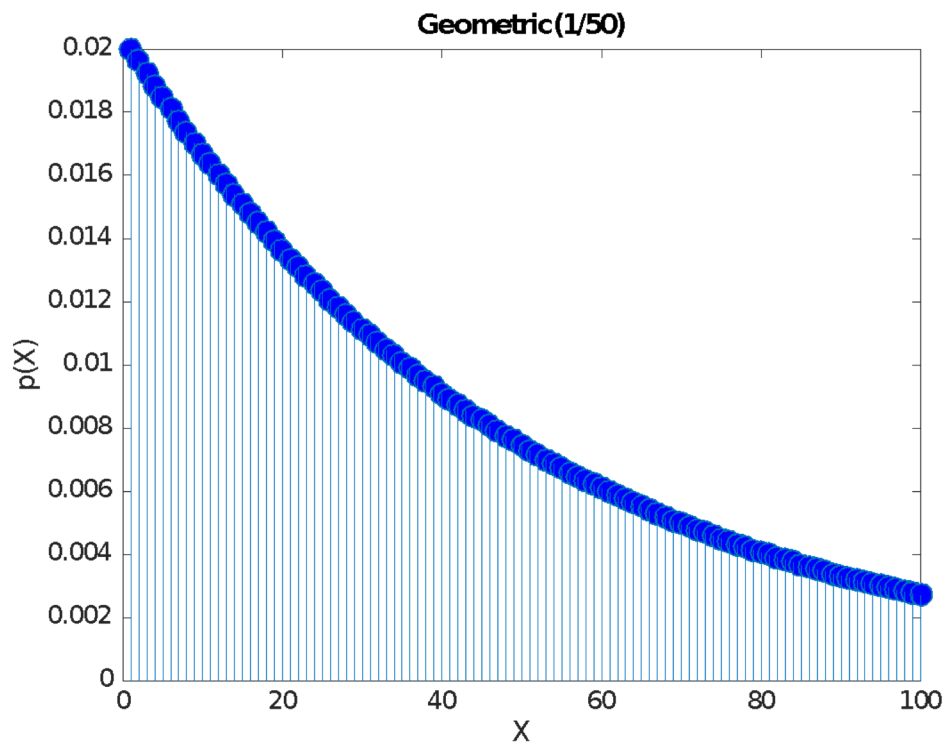
- We have a (countably) infinite sample space:

s	p(s)	X(s) = # of flips until first Head
– H	p	1
– TH	$(1-p)p$	2
– TTH	$(1-p)^2 p$	3
– …		
– $T^{k-1}H$	$(1-p)^{k-1}p$	k
– …		

$$\boxed{p(X = k) \ = (1-p)^{k-1}p}$$

Geometric Random Variables





Geometric Random Variables

- Suppose you have a biased coin comes up Heads with probability $p \in (0,1)$ and Tails with probability $1 - p$.
- You flip repeatedly until you get the first Head. Let X be the total number of flips (including the last Head).
- The probability distribution is $p(X = k) \; = (1 - p)^{k-1}p$

- What is E(X)?
- Consider the first flip. It's either heads or tails.
  - If it's tails, then X = 1.
  - If it's heads, then we add 1 to our total and repeat from scratch.

$$E(X) = \overbrace{p \cdot 1}^{\text{tails}} + \overbrace{(1 - p)(E(X) + 1)}^{\text{heads}}$$

Solve for E(X) and you get…

$$E(X) = 1/p$$

Geometric Random Variables

- Suppose you have a biased coin comes up Heads with probability $p \in (0,1)$ and Tails with probability $1 - p$.
- You flip repeatedly until you get the first Head. Let X be the total number of flips (including the last Head).
  - Geometrically Distributed

A r.v. X has a ***geometric distribution*** with parameter p if:

$$p(X = k) \ = (1 - p)^{k-1}p$$

If X is geometrically distributed with parameter p then:

$$E(X) = 1/p$$

Watching Seinfeld Reruns

- Every night a Seinfeld episode is drawn ***uniformly at random*** from the 180 shows and broadcast.
- What is the ***expected number of nights*** you need to watch to see all episodes?

- This is a tricky problem if you don't start right!
  - Use linearity of expectations.
  - You know the expectation of a geometric distribution.

Seinfeld Reruns

- Let $X_{i \to j}$ be the number of days you have to watch to go from having watched i distinct shows to having watched j distinct shows. Then,

$$X_{0\to180} = X_{0\to1} + X_{1\to2} + \cdots + X_{178\to179} + X_{179\to180}$$

By Linearity of Expectations...

P =

$X_{k\to k+1}$ is a geometric r.v. with success probability $\dfrac{180-k}{180}$

1/p

$$E[X_{k\to k+1}] = \frac{180}{180-k}$$

$$E[X_{0\to180}] = \frac{180}{180} + \frac{180}{179} + \frac{180}{178} + \cdots + \frac{180}{2} + \frac{180}{1}$$

$$\approx 180 \cdot 5.77 < 1039$$

Outline

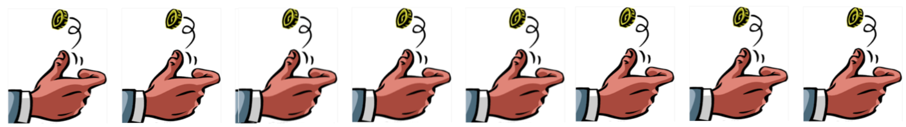
- Geometric Distribution
- **Binomial Distribution**
- Bayes' Theorem
  - Ex: Diagnosing a Rare Disease
- Law of Total Probability with Bayes'
  - Total Probability
  - Ex: Soccer Team Drug Testing
  - Ex: Pocket full of Dice
- Monty Hall Problem

Coin-toss Example

$p(\text{heads}) = 2/3$

A coin whose probability of getting heads is 2/3 is tossed 8 times.

What is the probability of exactly 3 heads in the 8 tosses?



“success” = heads

$p(\text{success}) = 2/2$

One sequence with 3 heads: T H H T T H T T

Probability of this sequence: $\frac{1}{3}\ \frac{2}{3}\ \frac{2}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{2}{3}\ \frac{1}{3}\ \frac{1}{3} = (2/3)^3 * (1/3)^5$

- So, what is the probability of any particular sequence with 3 heads? $(2/3)^3 * (1/3)^5$
- How many sequences with 3 heads are there? $\binom{8}{3}$

Probability of getting 3 heads in the 8 tosses:

$$P(3\ heads) = \binom{8}{3}\left({}^{2}/_{3}\right)^{3}\left({}^{1}/_{3}\right)^{5}$$

what if $p(\text{success}) = p$

$$p(k\ \text{heads}) = \binom{n}{k} p^{k} (1-p)^{n-k}$$

Coin-toss Example

A coin whose probability of getting heads is 2/3 is tossed 8 times.

What is the probability of exactly 3 heads in the 8 tosses?

One sequence with 3 heads: T H H T T H T T

Probability of this sequence: $\frac{1}{3}\ \frac{2}{3}\ \frac{2}{3}\ \frac{1}{3}\ \frac{1}{3}\ \frac{2}{3}\ \frac{1}{3}\ \frac{1}{3} = (2/3)^3 * (1/3)^5$

- So, what is the probability of any particular sequence with 3 heads? $(2/3)^3 * (1/3)^5$
- How many of sequences wit

Probability of getting 3 head

$$P(3\ heads) = \binom{8}{3}(2$$

Take-aways from the example:

- The *Experiment* consists of repeating a simpler experiment *n* times.
- This *simpler experiment* is "simpler" because it's sample space consists of two outcomes.
- Events of interest may consist of the *number of times* one of the two simpler experimental outcomes occurs.

Bernoulli Trials, Binomial Experiment, and Binomial Distribution

- Bernoulli Trial
  - An experiment that has exactly 2 outcomes, S = {success, failure}
    - p(success) = p
    - p(failure) = q = (1-p)

Success/failure can represent any two outcomes, e.g. heads/tails

- Binomial Experiment
  - Repeat the Bernoulli trial *n* times where each trial has the same probability of success and all trials are mutally independent
- Binomial Distribution
  - Let event be that *k* successes occur in *n* Bernoulli trials. Then

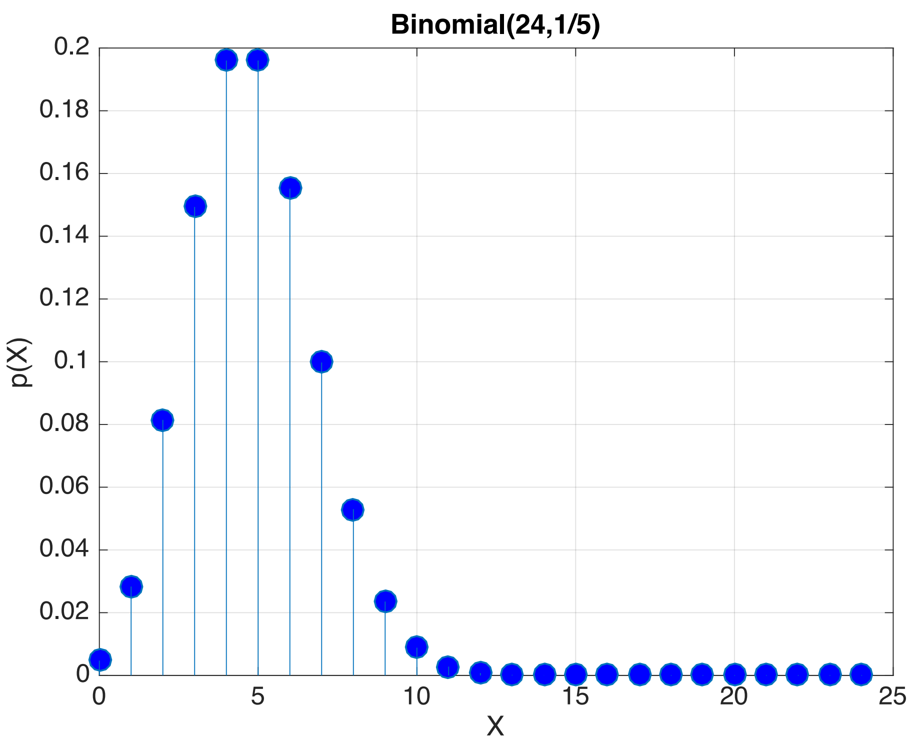
$$\boxed{\text{p(exactly } k \text{ successes in } n \text{ trials)} = \binom{n}{k} p^k q^{n-k}}$$

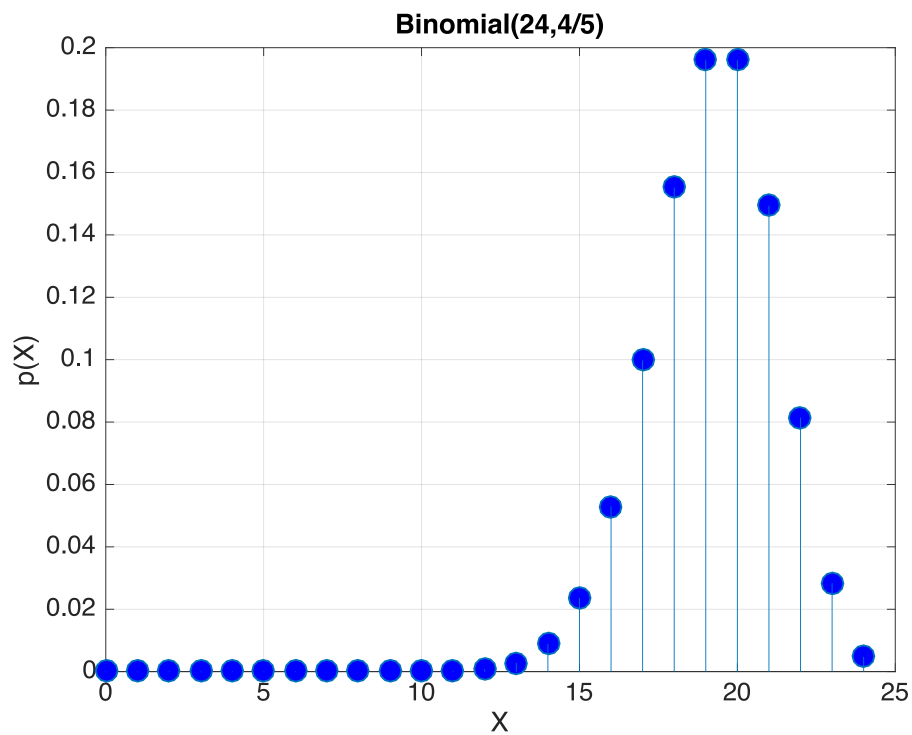
$(1-p)^{n-k}$

Random Variable: Binomial Distribution

- X is the number of successes in *n* Bernoulli trials where probability of a success is *p*

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$





Lecture 26 Handout: Bernoulli Trials, Binomial Distribution & Bayes' Theorem

Bernoulli Trial

Experiment that has

exactly 2 outcomes,

- S = {success, failure}
- p(success) = p
- p(failure) = q = 1 - p

Binomial Experiment

Repeat the Bernoulli trial n times, where

- Each trial has the same probability of success
- All trials are mutually independent

Binomial Distribution:

The probability of exactly $\boldsymbol{k}$ successes in $\boldsymbol{n}$ independent (and identically distributed) Bernoulli trials is

$$\binom{n}{k} p^{k} (1-p)^{n-k}$$

Outline

- Geometric Distribution
- Binomial Distribution

- **Bayes' Theorem**
  - **Ex: Diagnosing a Rare Disease**

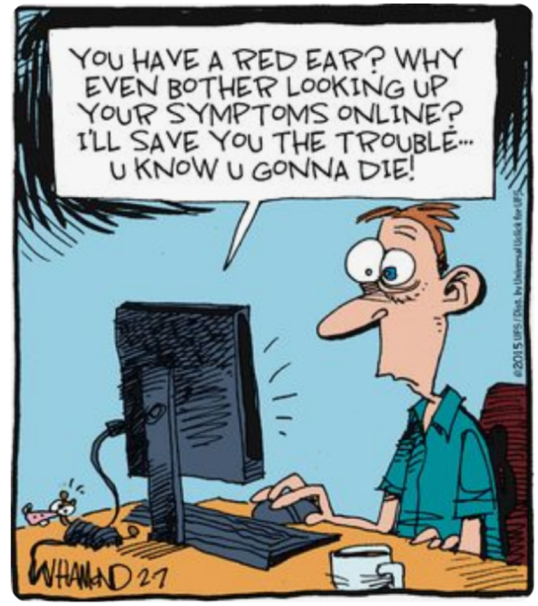
- Law of Total Probability with Bayes'
  - Total Probability
  - Ex: Soccer Team Drug Testing
  - Ex: Pocket full of Dice

- Monty Hall Problem

Reasoning & Probability

- Applications of probability:
  - Reasoning about ***games*** and their value. If X is your lottery winnings, is $E(X) >$ the cost of the ticket?
  - Reasoning about ***randomized algorithms*** (more in EECS 281, 376, 477, etc.)
  - Reasoning about partial/inconclusive evidence. Using statistical evidence to make more accurate guesses about reality.
    - ***Bayes' Theorem*** is the most basic tool.
    - Critical in modern ML (more in EECS 492, 445)

Conditional Probabilities – Diagnosing a Rare Disease



- Meningitis is rare: $p(m) = \frac{1}{50{,}000} = 0.00002$
- Meningitis causes stiff neck: $p(s|m) = 0.5$
- Stiff neck is not so rare: $p(s) = \frac{1}{20} = 0.05$
- You have a stiff neck.

- What is $p(m|s)$, the probability of meningitis if you have a stiff neck?

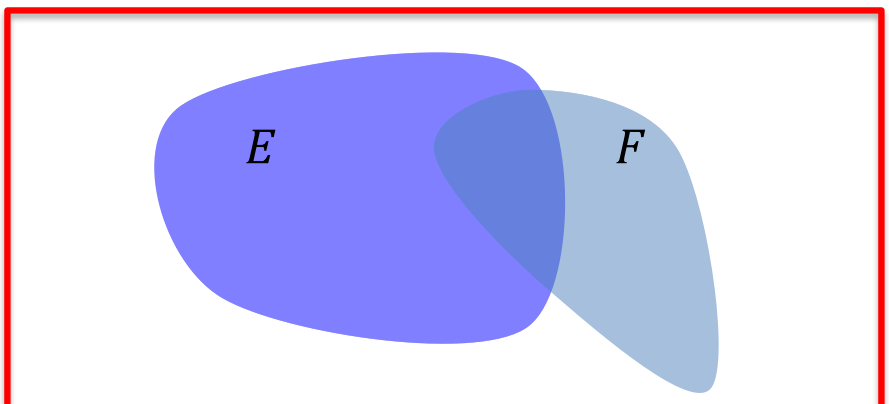
$$p(m|s) = \frac{p(m \cap s)}{p(s)}$$

??

$$p(s|m) = \frac{p(s \cap m)}{p(m)}$$

Conditional Probability Review

- E, F are **events**.
- $p(F|E) = \frac{p(F \cap E)}{p(E)}$ is the ***conditional probability*** of F holding, given that E holds. (Only sensible if $p(E) > 0$.)
- Symmetrically: $p(E|F) = \frac{p(E \cap F)}{p(F)}$.



Bayes' Theorem

- **Bayes' Theorem:** *(is just rearranging terms in conditional probability)*

$$p(F|E) = \frac{p(E|F)\,p(F)}{p(E)}$$

Bayes' Theorem

- **Bayes' Theorem:** *(is just rearranging terms in conditional probability)*

$\Pr(E|F)$ is the "likelihood" of seeing evidence E, if F holds

$\Pr(F)$ is the "prior" probability of F, i.e., before learning about evidence E

F : is some event that you care about

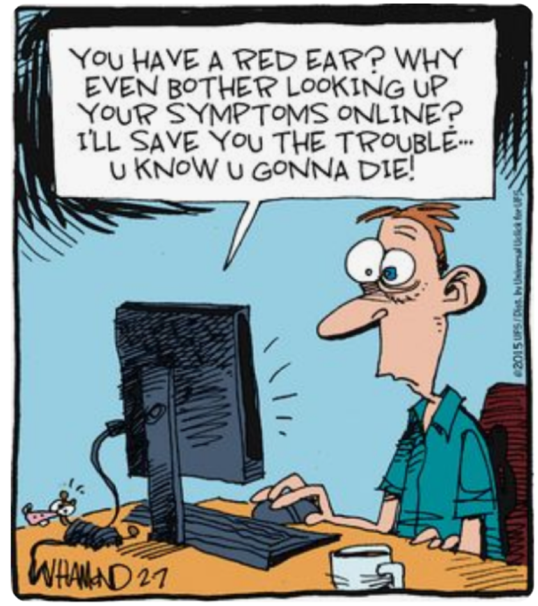
$\Pr(F|E)$ is the "posterior" probability of F (after seeing evidence E)

$$\Pr(F|E) = \frac{\Pr(E|F)\Pr(F)}{\Pr(E)}$$

E : is some ***evidence*** that you get

$\Pr(E)$ is the probability of seeing evidence E.

Conditional Probabilities – Diagnosing a Rare Disease



- Meningitis is rare: $p(m) = \frac{1}{50{,}000} = 0.00002$
- Meningitis causes stiff neck: $p(s|m) = 0.5$
- Stiff neck is not so rare: $p(s) = \frac{1}{20} = 0.05$
- You have a stiff neck.

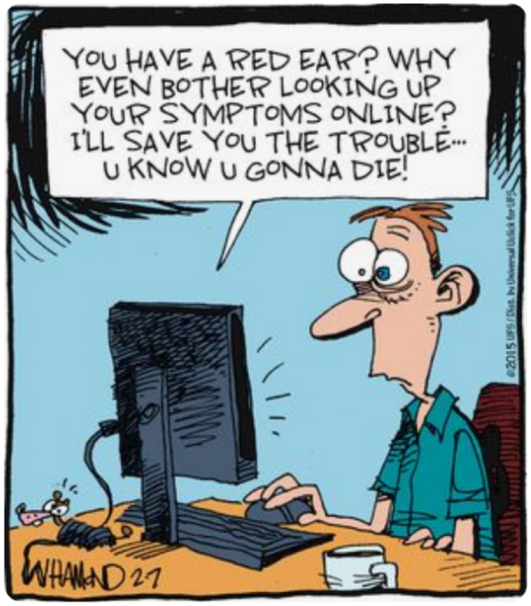
- What is $p(m|s)$, the probability of meningitis if you have a stiff neck?

$$p(m|s) = \frac{p(s|m)\,p(m)}{p(s)}$$

$$= \frac{(0.5)(1/50{,}000)}{1/20} = \frac{1}{5{,}000}$$

Conditional Probabilities – Diagnosing a Rare Disease

- Meningitis is rare: $p(m) = \frac{1}{50{,}000} = 0.00002$
- Meningitis causes stiff neck: $p(s|m) = 0.5$
- Stiff neck is not so rare: $p(s) = \frac{1}{20} = 0.05$
- You have a stiff neck.



- What is $p(m|s)$, the probability of meningitis if you have a stiff neck?
- Solution: Apply Bayes' Theorem

$$p(m|s) = \frac{p(s|m)p(m)}{p(s)} = \frac{0.5 \times 0.00002}{0.05} = \frac{1}{5{,}000} = 0.0002$$

Having the additional information that you have a stiff neck increases the chances you also have meningitis by a factor of 10, but it's still really unlikely.

Bayes' Theorem

Suppose that $\boldsymbol{E}$ and $\boldsymbol{F}$ are events from a sample space $\boldsymbol{S}$ such that $\boldsymbol{p}(\boldsymbol{E}) > 0$ and $\boldsymbol{p}(\boldsymbol{F}) > 0$. Then

$$\boldsymbol{p}(\boldsymbol{F}|\boldsymbol{E}) = \frac{p(E|F)\,p(F)}{p(E)}$$

Alternative form of Bayes', with expanded denominator:

$$\boldsymbol{p}(\boldsymbol{F}|\boldsymbol{E}) = \frac{p(E|F)p(F)}{\qquad\qquad\qquad\qquad}$$

Helpful tips for Bayes':

- $p(E|F) + p(\_\_\_\_\_\_) = 1$
- $p(E|\bar{F}) + p(\bar{E}|\bar{F}) = \_\_\_\_$
- $p(E) + p(\_\_\_\_) = 1$
- $p(\_\_\_\_) + p(\bar{F}) = 1$

Why is Bayes' Rule so Useful?

- Bayes' Rule = **How to Update Your Beliefs, Given New Evidence**

- In many situations
  - Diagnostic evidence $P(disease \mid symptom)$ is often hard to get.
    - But it's usually what you really want.
    - This is the "posterior" probability of having the disease (after seeing the evidence of the symptom
  - Causal evidence $P(symptom \mid disease)$ is often easier to get.
    - This is the "likelihood" of seeing the evidence (the symptom), if someone has the disease

Outline

- Geometric Distribution
- Binomial Distribution

- Bayes' Theorem
  - Ex: Diagnosing a Rare Disease

- **Law of Total Probability with Bayes'**
  - **Total Probability**
  - **Ex: Soccer Team Drug Testing**
  - **Ex: Pocket full of Dice** ***(time permitting)***

- Monty Hall Problem

Soccer Team Drug Testing

- When a test for steroids is given to soccer players
  - 98% of the players taking steroids test positive
  - 12% of the players not taking steroids test positive
- Suppose 5% of soccer players take steroids.
- What is the probability that a soccer player who tests positive takes steroids?
  - Let S = event that a player takes Steroids,
    Let T = event that a player Tested positive

$p(T|S) = .98$

$p(T|\bar{S}) = .12$

$p(S) = .05$

Find $p(S|T)$

$$p(S|T) = \frac{p(T|S)\,p(S)}{p(T)}$$

$$= \frac{p(T|S)\,p(S)}{p(T|S)\,p(S) + p(T|\bar{S})\,\underbrace{p(\bar{S})}_{1-p(S)}}$$

$$= \frac{(0.98)(.05)}{(.98)(.05)+(.12)(.95)}$$

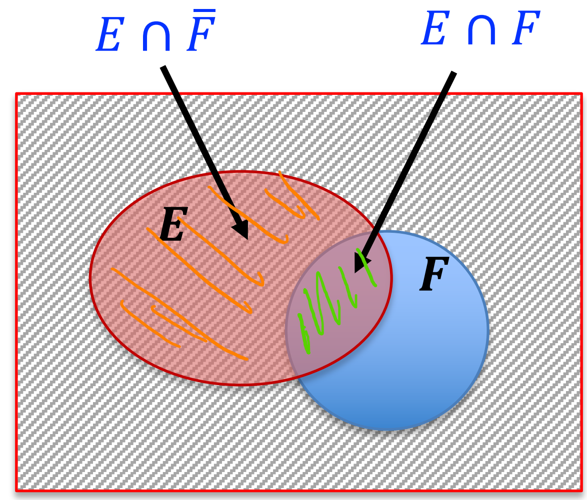
$$\approx .301$$

Law of Total Probability

For two events E and F, the **total probability** of E is

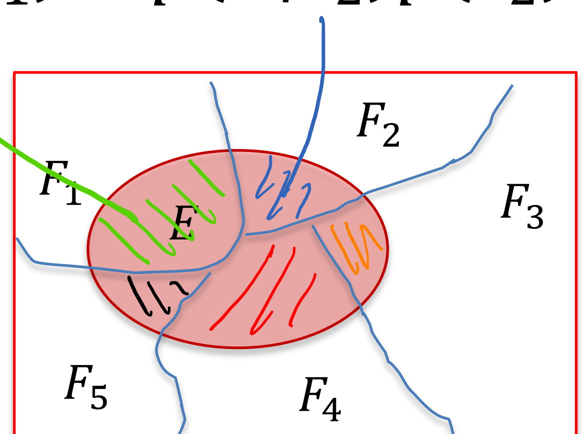
$$p(E) = p(E \cap F) + p(E \cap \bar{F})$$

$$p(E) = p(E|F)p(F) + p(E|\bar{F})p(\bar{F})$$



If events $F_1, F_2, \ldots, F_n$ partition the sample space, then the **total probability** of event E is

$$p(E) = p(E|F_1)p(F_1) + p(E|F_2)p(F_2) + \cdots + p(E|F_n)p(F_n)$$



Bayes' Theorem

- **Bayes' Theorem:** *(is just rearranging terms in conditional probability)*

$$p(F|E) = \frac{p(E|F)\,p(F)}{p(E)}$$

- Sometimes you don't have direct access to the probability in the denominator.
  - Either F or $\bar{F}$ holds. $E = E \cap (F \cup \bar{F}) = (E \cap F) \cup (E \cap \bar{F})$
  - $p(E) = p(E \cap F) + p(E \cap \bar{F}) = p(E|F)p(F) + p(E|\bar{F})p(\bar{F})$

$$p(F|E) = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})}$$

p(E)

Bayes' Theorem

Suppose that $\boldsymbol{E}$ and $\boldsymbol{F}$ are events from a sample space $\boldsymbol{S}$ such that $\boldsymbol{p(E)} > 0$ and $\boldsymbol{p(F)} > 0$. Then

$$\boldsymbol{p(F|E)} = \frac{p(E|F)\,p(F)}{p(E)}$$

Alternative form of Bayes', with expanded denominator:

$$\boldsymbol{p(F|E)} = \frac{p(E|F)p(F)}{p(E|F)p(F) + p(E|\bar{F})p(\bar{F})}$$

Helpful tips for Bayes':

- $p(E|F) + p(\_\_\_\_\_\_) = 1$
- $p(E|\bar{F}) + p(\bar{E}|\bar{F}) = \_\_\_\_$
- $p(E) + p(\underline{\ \bar{E}\ }) = 1$
- $p(\underline{\ F\ }) + p(\bar{F}) = 1$

Soccer Team Drug Testing

- When a test for steroids is given to soccer players
  - 98% of the players taking steroids test positive
  - 12% of the players not taking steroids test positive
- Suppose 5% of soccer players take steroids.
- What is the probability that a soccer player who tests positive takes steroids?
  - Let S = event that a player takes Steroids,
    Let T = event that a player Tested positive

Soccer Team Drug Testing

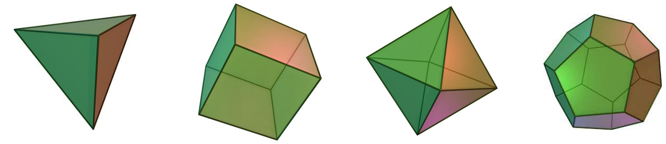
- When a test for steroids is given to soccer players
  - 98% of the players taking steroids test positive
  - 12% of the players not taking steroids test positive
- Suppose 5% of soccer players take steroids.
- What is the probability that a soccer player who tests positive takes steroids?
  - Let S = event that a player takes Steroids,
    Let T = event that a player Tested positive

$$p(S|T) = \frac{p(T|S)p(S)}{p(T|S)p(S) + p(T|\bar{S})p(\bar{S})}$$

$$= \frac{p(T|S)p(S)}{p(T|S)p(S) + p(T|\bar{S})(1 - p(S))}$$

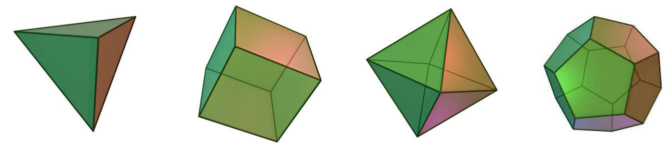
$$= \frac{(0.98)(0.05)}{(0.98)(0.05)+(0.12)(0.95)} \approx 0.301$$

A Pocket Full of Dice



- In my pocket I have a 4-sided, 6-sided, 8-sided, and 12-sided die. I pick one and roll it twice and tell you that it showed the same number ($\boldsymbol{S}$) both times. What's the probability that it is a $\boldsymbol{T}$(etrahedron), $\boldsymbol{C}$(ube), $\boldsymbol{O}$(ctahedron), $\boldsymbol{D}$(odecahedron)?

A Pocket Full of Dice



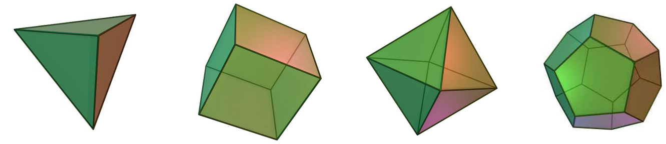
- In my pocket I have a 4-sided, 6-sided, 8-sided, and 12-sided die. I pick one and roll it twice and tell you that it showed the same number (***S***) both times. What's the probability that it is a ***T***(etrahedron), ***C***(ube), ***O***(ctahedron), ***D***(odecahedron)?
- What are the priors?
  - $\Pr(T), \Pr(C), \Pr(O), \Pr(D) = \frac{1}{4}$
- What are
  - $\Pr(S|T) = 1/4$
  - $\Pr(S|C) = 1/6$
  - $\Pr(S|O) = 1/8$
  - $\Pr(S|D) = 1/12$
  - $\Pr(S) = \Pr(S|T)\Pr(T) + \Pr(S|C)\Pr(C) + \Pr(S|O)\Pr(O) + \Pr(S|D)\Pr(D)$

$$= \frac{1}{4} \cdot \left(\frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{12}\right) = \frac{1}{4} \cdot \frac{15}{24} = \frac{5}{32}$$

$$\boxed{\Pr(T|S) = \frac{\Pr(S|T)\Pr(T)}{\Pr(S)} = \frac{\frac{1}{4} \cdot \frac{1}{4}}{\frac{5}{32}} = \frac{2}{5}}$$

$$\boxed{\Pr(C|S) = \frac{\Pr(S|C)\Pr(C)}{\Pr(S)} = \frac{\frac{1}{6} \cdot \frac{1}{4}}{\frac{5}{32}} = \frac{4}{15}}$$

A Pocket Full of Dice



- In my pocket I have a 4-sided, 6-sided, 8-sided, and 12-sided die. I pick one and roll it twice and tell you that it showed the same number (***S***) both times. What's the probability that it is a ***T***(etrahedron), ***C***(ube), ***O***(ctahedron), ***D***(odecahedron)?
- What are the priors?
  - $\Pr(T), \Pr(C), \Pr(O), \Pr(D) = \frac{1}{4}$
- What are
  - $\Pr(S|T) = 1/4$
  - $\Pr(S|C) = 1/6$
  - $\Pr(S|O) = 1/8$
  - $\Pr(S|D) = 1/12$
  - $\Pr(S) = \Pr(S|T)\Pr(T) + \Pr(S|C)\Pr(C) + \Pr(S|O)\Pr(O) + \Pr(S|D)\Pr(D)$

$$= \frac{1}{4}\cdot\left(\frac{1}{4}+\frac{1}{6}+\frac{1}{8}+\frac{1}{12}\right) = \frac{1}{4}\cdot\frac{15}{24} = \frac{5}{32}$$

$$\boxed{\Pr(O|S) = \frac{\Pr(S|O)\Pr(O)}{\Pr(S)} = \frac{\frac{1}{8}\cdot\frac{1}{4}}{\frac{5}{32}} = \frac{1}{5}}$$

$$\boxed{\Pr(D|S) = \frac{\Pr(S|D)\Pr(D)}{\Pr(S)} = \frac{\frac{1}{12}\cdot\frac{1}{4}}{\frac{5}{32}} = \frac{2}{15}}$$

Outline

- Geometric Distribution
- Binomial Distribution

- Bayes' Theorem
  - Ex: Diagnosing a Rare Disease

- Law of Total Probability with Bayes'
  - Total Probability
  - Ex: Soccer Team Drug Testing
  - Ex: Pocket full of Dice

- **Monty Hall Problem**
  - **Let's Make a Deal**

Monty Hall Problem

From an old game show, “Let’s Make a Deal”

- You pick a door: A, B, C.
- Behind one is a new car. Behind the other two are goats.
  - Goats are cool but assume for this problem you want the car

- Monty opens one of the other two doors that ***does not*** have the car and shows you a goat.
- He then asks whether you’d like to switch doors.

  - (a) Doesn’t matter, it’s 50-50 either way.
  - (b) Always switch.
  - (c) Never switch.

Monty Hall Problem

From an old game show, "Let's Make a Deal"

- You pick a door: A, B, C.
- Behind one is a new car. Behind the other two are goats.
  - Goats are cool but assume for this problem you want the car

- Monty opens one of the other two doors that ***does not*** have the car and shows you a goat.
- He then asks whether you'd like to switch doors.
  - (a) Doesn't matter, it's 50-50 either way.
  - **(b) Always switch.**
  - (c) Never switch.

Monty Hall Problem

Suppose you initially pick curtain “A,” and Monty reveals a goat behind curtain “B”. (Using *without loss of generality*, this covers all cases)

You have **new evidence** (goat behind door B), so how should you **update your beliefs**?

Bayes’ will help us
We’re going to set up a grid to better visualize the problem

Four Possibilities:	**Car behind A**	**Car not behind A**
Monte reveals B		
~~Monte reveals C~~		

Monty Hall Problem

Suppose you initially pick curtain "A," and Monty reveals a goat behind curtain "B"

Four Possibilities:	**Car behind A**	**Car not behind A**
Monte reveals B		
~~Monte reveals C~~		

$$\Pr[\text{Car A}] = \frac{1}{3}$$

$$\Pr[\text{Car A and Reveal B}] = \Pr[\text{Reveal B} \mid \text{Car A}] \cdot \Pr[\text{Car A}]$$

$$= \frac{1}{2} \cdot \frac{1}{3} = \mathbf{\frac{1}{6}}$$

$$\Pr[\text{Reveal B} \mid \text{Car A}] = \frac{1}{2}$$

Monty Hall Problem

Suppose you initially pick curtain “A,” and Monte reveals a goat behind curtain “B”

Four Possibilities:	**Car behind A**	**Car not behind A**
Monte reveals B	$\frac{1}{6}$	
~~Monte reveals C~~		

$$\Pr[\text{Car not A}] = \frac{2}{3}$$

$$\Pr[\text{Car not A and Reveal B}]$$
$$= \Pr[\text{Reveal B} \mid \text{Car not A}] \cdot \Pr[\text{Car not A}]$$
$$= \frac{1}{2} \cdot \frac{2}{3} = \mathbf{\frac{1}{3}}$$

- Car in B or C
- So Monte reveals B iff car in C

$$\Pr[\text{Reveal B} \mid \text{Car not A}] = \frac{1}{2}$$

Monty Hall Problem

Suppose you initially pick curtain "A," and Monte reveals a goat behind curtain "B"

Four Possibilities:	**Car behind A**	**Car not behind A**
Monte reveals B	$\mathbf{\frac{1}{6}}$	$\mathbf{\frac{1}{3}}$
~~Monte reveals C~~		

$$\frac{P(\text{behind A} \cap \text{reveal B})}{P(\text{reveal B})}$$

$$\Pr[\text{Car behind A} \mid \text{Monte reveals B}] = \frac{\frac{1}{6}}{\frac{1}{6}+\frac{1}{3}} = \mathbf{\frac{1}{3}}$$

$\Rightarrow P(\text{behind C} \mid \text{reveal B})$

$= 1 - 1/3 = 2/3$

That’s a wrap on Probability

- Final two lectures will be on algorithms and how fast they run