Lecture 5
Symbolic Logic

Renew CS Mentor Program

FREE Student-Run
Peer Mentor Program with
goal of supporting **Women
and Non-Binary** Students in
Computing!

We offer both in-person and remote tutoring sessions to help students with ENGR 101, EECS 183, EECS 203, and EECS 280.
For more information see: https://tinyurl.com/35vy26dt.

We also hold monthly sessions that cover special topics like Debugging and Recruiting

Interested? Complete our Interest form!

https://forms.gle/WmQTJ9Prk2W9S1Cq7

Announcements

- Reminder: HW1 is due **TONIGHT** at 10 pm
  - Make sure to **MATCH PAGES** when you upload to Gradescope
- There are 3 surveys out now that you need to complete:
  - FCI Beginning of the Term Survey   Due Wed, 1/26
  - Computing Cares Entry Survey   Due Wed, 1/26
  - Exam Date Confirmation Form   Due Fri 1/28
- These are for a (completion) grade, so make sure to fill them out

Learning Objectives: Lec 5

After today's lecture (and the associated readings, discussion, & homework), you should be able to:

- **Know Logic Symbols:** and, or, not, if-then, if-and-only-if (and equivalence), for all, there exists
- Translate propositions from English into logic, and logic into English
  - Correctly interpret variable scope
  - Correctly interpret domain restrictions in for-all/there-exists statements
- Solve logic puzzles by the method of translating them to truth tables (or similar)

Outline

- **Intro to Symbolic Logic**
- (Nested Quantifiers – Handout from Lec 4)
- Translation
  - English to Logic
  - Logic to English
  - Domain Restrictions
  - Quantifier Scoping
- Logic Puzzles

Where we're going

- **Last Time:**
  - We think of the words "or", "and", etc. as things that combine two propositions into one.
  - Truth value of compound proposition predictable from input truth values
  - Sometimes use p, q to stand for propositions

- **This Time:**
  - Moving towards **"Boolean algebra"**
  - We will associate **symbols** to words "or", "and", etc.
  - We think of the symbol as an **operator** that combines two **truth values** into one
    - Like how $+, -, \cdot$ combine two integers into one
  - We usually think of p, q as **variables that hold truth values** (T or F)

Symbol Example

- The symbol for "or" is ∨

p: "5 is even"

q: "5 is odd"

$$\boldsymbol{p} \vee \boldsymbol{q}$$

read: "p or q", this expression has the truth value "true"

Symbol Example

- The symbol for “and” is $\wedge$

p: "5 is even"

q: "5 is odd"

$$\boldsymbol{p \wedge q}$$

read: “p and q”, this expression has the truth value “false”

Symbols

We've been writing the **words** and, or, not, if/then, …
There are **symbols** to mean the same thing.

Symbol	"Main" Words
$\neg p$	"not p"
$p \wedge q$	"p and q"
$p \vee q$	"p or q"
$p \rightarrow q$	"if p, then q"
$p \leftrightarrow q$ $p \equiv q$	"p if and only if q" "p is equivalent to q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Handout

These two have the same truth table/logical meaning

Remembering OR vs AND symbols

$p \wedge q$

Sad face. Hard to satisfy it.
This one is **AND**

$p \vee q$

Happy face! Easy to
satisfy it. This one is **OR**

Which one is **p or q**, and which one is **p and q**?

Outline

- Intro to Symbolic Logic
- **(Nested Quantifiers – Handout from Lec 4)**
- Translation
  - English to Logic
  - Logic to English
  - Domain Restrictions
  - Quantifier Scoping
- Logic Puzzles

Multivariable Predicates

- Predicates can also have **several variables**.
  - $P(x, y) = "x + y = 2"$
  - $C(x, y) = "city\ x\ is\ the\ capital\ of\ state\ y"$
  - $T(a, b, c) = "a^2 + b^2 = c^2"$

- You can apply **different quantifiers** to these variables if you want.

Exercise

$$P(x, y) = "x + y = 2"$$

A

There exists an integer x such that **for all** integers y, $P(x, y)$

B

For all integers y, **there exists** an integer x such that $P(x, y)$

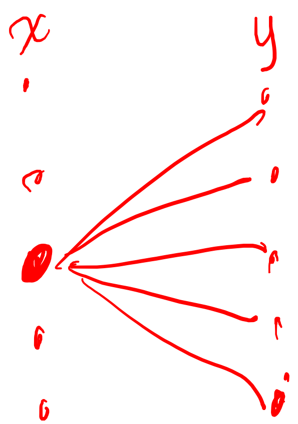
1. Both are true
2. A is true, B is false
3. A is false, B is true
4. Both are false
5. Not sure

Exercise

$$P(x, y) = "x + y = 2"$$

A

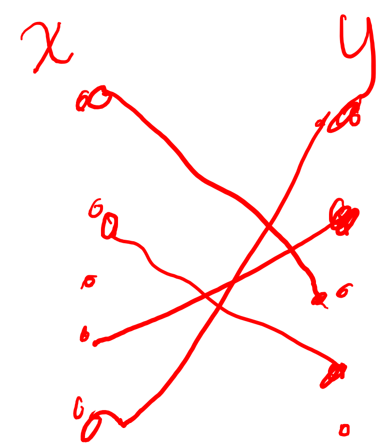
There exists an integer $\boldsymbol{x}$ such that **for all** integers $\boldsymbol{y}$, $P(x, y)$



for any
for each

B

For all integers $\boldsymbol{y}$, **there exists** an integer $\boldsymbol{x}$ such that $P(x, y)$



1. Both are true
2. A is true, B is false
3. **A is false, B is true**
4. Both are false
5. Not sure

Exercise

$$P(x, y) = "x + y = 2"$$

A

There exists an integer $\boldsymbol{x}$ such that **for all** integers $\boldsymbol{y}$, $P(x, y)$

There is no specific example of an integer x
(like $x = 1, x = 3.5, x = 12 \ldots$)
where, for any integer y you add to this x, you get 2.

False
(this is not a proof, just intuition)

B

For all integers $\boldsymbol{y}$, **there exists** an integer $\boldsymbol{x}$ such that $P(x, y)$

Exercise

$$P(x, y) = "x + y = 2"$$

A

There exists an integer $\boldsymbol{x}$ such that **for all** integers $\boldsymbol{y}$, $P(x, y)$

There is no specific example of an integer x

(like $x = 1, x = 3.5, x = 12 \ldots$)

where, for any integer y you add to this x, you get 2.

False
(this is not a proof, just intuition)

B

For all integers $\boldsymbol{y}$, **there exists** an integer $\boldsymbol{x}$ such that $P(x, y)$

Proof:

- Let y be an arbitrary integer

Exercise

$$P(x, y) = "x + y = 2"$$

A

There exists an integer $\boldsymbol{x}$ such that **for all** integers $\boldsymbol{y}$, $P(x, y)$

There is no specific example of an integer x
(like $x = 1, x = 3.5, x = 12...$)
where, for any integer y you add to this x, you get 2.

False
(this is not a proof, just intuition)

B

For all integers $\boldsymbol{y}$, **there exists** an integer $\boldsymbol{x}$ such that $P(x, y)$

Proof:

- Let y be an arbitrary integer
- Consider the specific integer $x = 2 - y$

Exercise

$$P(x, y) = "x + y = 2"$$

A

There exists an integer $\boldsymbol{x}$ such that **for all** integers $\boldsymbol{y}$, $P(x, y)$

There is no specific example of an integer x
(like $x = 1, x = 3.5, x = 12...$)
where, for any integer y you add to this x, you get 2.

False
(this is not a proof, just intuition)

B

For all integers $\boldsymbol{y}$, **there exists** an integer $\boldsymbol{x}$ such that $P(x, y)$

Proof:

- Let y be an arbitrary integer
- Consider the specific integer $x = 2 - y$
- We have $x + y = (2 - y) + y = 2$.

Exercise

$$P(x, y) = \text{"}x + y = 2\text{"}$$

A

There exists an integer $\boldsymbol{x}$ such that **for all** integers $\boldsymbol{y}$, $P(x, y)$

There is no specific example of an integer x

(like $x = 1, x = 3.5, x = 12$...)

where, for any integer y you add to this x, you get 2.

False
(this is not a proof, just intuition)

B

For all integers $\boldsymbol{y}$, **there exists** an integer $\boldsymbol{x}$ such that $P(x, y)$

Proof:

- Let y be an arbitrary integer
- Consider the specific integer $x = 2 - y$
- We have $x + y = (2 - y) + y = 2$.

True

Nested Quantifiers

How could we quantify a **2-var**

Same quantifier (both for-all or both there-exists):
You can swap the order without changing meaning.

Different quantifiers (one for-all, other there-exists):
Swapping the order changes the meaning!

1. There exists **x** such that there exists **y** such that **P(x, y).**
2. There exists **y** such that there exists **x** such that **P(x, y).**

THE SAME

3. There exists **x** such that for all **y**, **P(x, y).**
4. For all **y**, there exists **x** such that **P(x, y).**

5. For all **x**, there exists **y** such that **P(x, y).**
6. There exists ~~**x**~~ y such that for all ~~**y**~~ x, **P(x, y).**

7. For all **x**, for all **y**, **P(x, y)**.
8. For all **y**, for all **x**, **P(x, y)**.

THE SAME

Nested Quantifiers – additional exercises

Handout

1. Let P(x,y) = "4x – y = 0", domain = integers

Determine whether each of the following propositions is true or false:

1. There exists **x** such that there exists **y** such that **P(x, y).** True / False
2. There exists **x** such that for all **y**, **P(x, y).** True / False
3. For all **y**, there exists **x** such that **P(x, y).** True / False
4. for each For all **x**, there exists **y** such that **P(x, y).** True / False

   works b/c value of x can depend on the value of y
5. There exists **y** such that for all **x**, **P(x, y).** True / False
6. For all **x**, for all **y**, **P(x, y).** True / False

Nested Quantifiers – additional exercises

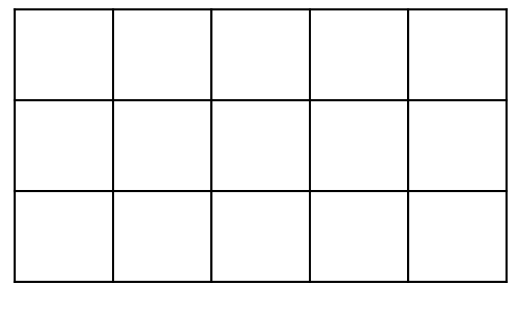
Handout

P(x,y) = "the square at row **x**, column **y** is shaded". Shade squares so that …

(a)

"*There exists* **x**, such that *there exists* **y**, such that **P(x,y)**" is **TRUE**

(b)



"*There exists* **x**, such that *there exists* **y**, such that **P(x,y)**" is **FALSE**

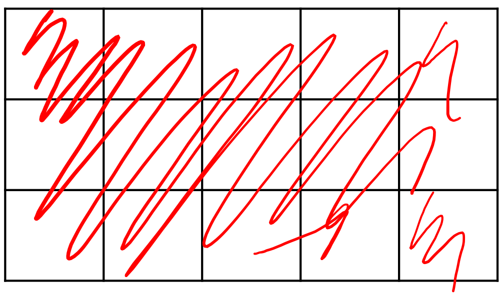
none

Nested Quantifiers – additional exercises

Handout

P(x,y) = "the square at row **x**, column **y** is shaded". Shade squares so that …

(c)

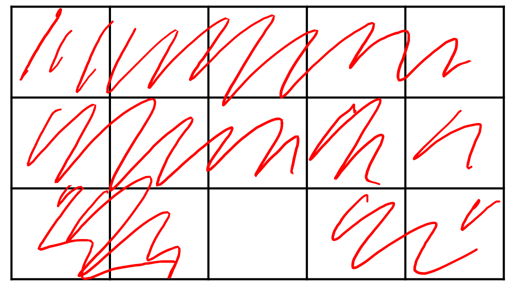


"*For all* **x**, *for all* **y**, **P(x,y)**"

is **TRUE**

Question: Does P(x,y) have to be **true** for *every* pair of (x,y)?

(d)



"*For all* **x**, *for all* **y**, **P(x,y)**"

is **FALSE**

Question: Does P(x,y) have to be **false** for *every* pair of (x,y)?

Nested Quantifiers – additional exercises

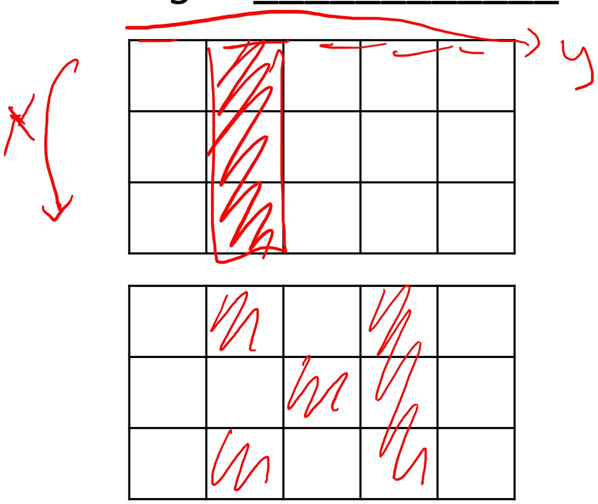
Handout

Select the logical expression that matches the English statement.

Then provide *two different shadings* that satisfy the statement.

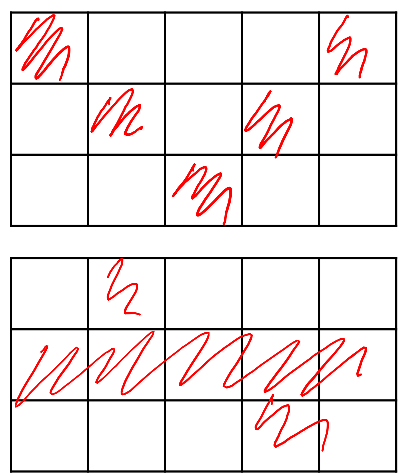
(e) *English*: **The grid has an entire column that is shaded.**

Logic: D



(f) *English*: **Every column has at least one shaded square.**

Logic: B



Answer options:

A. There exists an **x**, such that for all **y**, **P(x, y).**

B. For all **y**, there exists an **x**, such that **P(x, y).**

C. For all **x**, there exists a **y**, such that **P(x, y).**

D. There exists a **y**, such that for all **x**, **P(x, y).**

Outline

- Intro to Symbolic Logic
- **Translation**
    - **English to Logic**
    - Logic to English
    - Domain Restrictions
    - Quantifier Scoping
- Logic Puzzles

fails because you must eat choc. or popcorn it says if it rains

$\neg(p \vee c) \rightarrow r$

$\neg r \rightarrow \neg(p \vee c)$

contrapositives

$\cancel{r} \rightarrow c \wedge p$

Translating English to Logic

r = "it rains"
w = "I'll watch a movie"
p = "I'll eat popcorn"
c = "I'll eat chocolate"

Translate:

If I don't eat popcorn, I'll eat chocolate.

$\neg p \rightarrow c$

If it rains, I'll watch a movie and eat popcorn.

$r \rightarrow (w \wedge p)$

I won't eat chocolate or popcorn unless it rains.

$r \leftrightarrow (p \vee c)$

$\neg r \rightarrow \neg c \wedge \neg p$

Symbol	Words
¬p	"not p"
p∧q	"p and q"
p∨q	"p or q (or both)"
p→q	"if p then q"

Handout

Use logical symbols to translate each sentence into logic

r = “it rains” p = “I’ll eat popcorn”
w = “I’ll watch a movie” c = “I’ll eat chocolate”

Symbol	Words
¬p	“not p”
p∧q	“p and q”
p∨q	“p or q (or both)”
p→q	“if p then q”

English **Logic**

- If I don’t eat popcorn, I’ll eat chocolate. ____________________
- If it rains, I’ll watch a movie and eat popcorn. ____________________
- I won’t eat chocolate or popcorn unless it rains. ____________________

Translating English to Logic

r = "it rains"
w = "I'll watch a movie"
p = "I'll eat popcorn"
c = "I'll eat chocolate"

These are "if-then" propositions, even though the "then" is sometimes omitted in English.

English does not always put propositions in exactly the "right" form!

Translate:

- If I don't eat popcorn, I'll eat chocolate.

  $\neg p \rightarrow c$

- If it rains, I'll watch a movie and eat popcorn.

  $r \rightarrow (w \land p)$

Symbol	Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q (or both)"
$p \rightarrow q$	"if p then q"

Translating English to Logic

r = "it rains"
w = "I'll watch a movie"
p = "I'll eat popcorn"
c = "I'll eat chocolate"

"unless" is a connector

But not one of our "main" logical words!

What does it **mean?**

Translate:

I won't eat chocolate or popcorn unless it rains.

Symbol	Words
$\neg p$	"not p"
$p \wedge q$	"p and q"
$p \vee q$	"p or q (or both)"
$p \rightarrow q$	"if p then q"

Translating English to Logic

r = "it rains"
w = "I'll watch a movie"
p = "I'll eat popcorn"
c = "I'll eat chocolate"

"unless" is a connector

But not one of our "main" logical words!

What does it **mean?**

Translate:

I won't eat chocolate or popcorn unless it rains.

"If it doesn't rain, then I won't eat chocolate or popcorn."

Symbol	Words
$\neg p$	"not p"
$p \wedge q$	"p and q"
$p \vee q$	"p or q (or both)"
$p \rightarrow q$	"if p then q"

Translating English to Logic

r = "it rains"
w = "I'll watch a movie"
p = "I'll eat popcorn"
c = "I'll eat chocolate"

"unless" is a connector

But not one of our "main" logical words!

What does it **mean?**

Translate:

I won't eat chocolate or popcorn unless it rains.

"If it doesn't rain, then I won't eat chocolate or popcorn."

$$\neg r \rightarrow \neg(c \vee p)$$

Some other interpretations possible, with the same logical meaning, such as

$$\neg(c \vee p) \vee r$$

Symbol	Words
$\neg p$	"not p"
$p \wedge q$	"p and q"
$p \vee q$	"p or q (or both)"
$p \rightarrow q$	"if p then q"

Translating English to Logic

slido.com
#203203

p: you get an A on the final

q: you do every exercise in the textbook

r: you get an A in the class

"You get an A in the class, but you do not do every exercise in the book."

(A) $\neg q \rightarrow r$

(B) $r \rightarrow \neg q$

(C) $(r \wedge \neg q) \rightarrow q$

(D) $r \wedge \neg q$

(E) $r \vee \neg q$

"But" is the connector - which logical symbol does it mean?

Translating English to Logic

p: you get an A on the final

q: you do every exercise in the textbook

r: you get an A in the class

"You get an A in the class, but you do not do every exercise in the book."

(A) $\neg q \rightarrow r$

(B) $r \rightarrow \neg q$

(C) $(r \wedge \neg q) \rightarrow q$

(D) $r \wedge \neg q$

(E) $r \vee \neg q$

"But" is the connector - which logical symbol does it mean?

but = and

(often used in English when RHS seems unlikely)

Translating English to Logic

slido.com
#203203

p: you get an A on the final

q: you do every exercise in the textbook

r: you get an A in the class

"Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class."

(A) $\neg r \rightarrow (p \wedge q)$

(B) $\neg r \rightarrow (p \vee q)$

(C) $(p \vee q) \rightarrow r$

(D) $(p \wedge q) \rightarrow r$

(E) $r \leftrightarrow (p \wedge q)$

"Is sufficient for" is the connector - which logical symbol does it mean?

Translating English to Logic

p: you get an A on the final

q: you do every exercise in the textbook

r: you get an A in the class

"Getting an A on the final and doing every exercise in the book is sufficient for getting an A in this class."

(A) $\neg r \rightarrow (p \land q)$

(B) $\neg r \rightarrow (p \lor q)$

(C) $(p \lor q) \rightarrow r$

(D) $\boldsymbol{(p \land q) \rightarrow r}$

(E) $r \leftrightarrow (p \land q)$

"Is sufficient for" is the connector - which logical symbol does it mean?

slido.com
#203203

Translating English to Logic

q p

"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

$p \rightarrow q$

(A) $\neg(F \lor L) \rightarrow (A \land Q)$ $\longrightarrow$ $q \rightarrow p$

(B) $(A \land Q) \rightarrow (\neg F \land \neg L)$

(C) $\neg F \land \neg L \land A \land Q$

(D) $(\neg A \lor \neg Q) \rightarrow (F \lor L)$

Translating English to Logic

This is an if-then statement, **but presented in the other order.**

"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

(A) $\neg(F \vee L) \rightarrow (A \wedge Q)$

(B) $(A \wedge Q) \rightarrow (\neg F \wedge \neg L)$

(C) $\neg F \wedge \neg L \wedge A \wedge Q$

(D) $(\neg A \vee \neg Q) \rightarrow (F \vee L)$

"Neither the fox nor the lynx can catch the hare"

"The hare is alert and quick"

Translating English to Logic

Handout

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

Which of these is:
"Neither the fox nor the lynx can catch the hare if the hare is alert and quick"?

Translate:

(A) $\neg(F \lor L) \rightarrow (A \land Q)$ ____________________________________

(B) $(A \land Q) \rightarrow (\neg F \land \neg L)$ ____________________________________

(C) $\neg F \land \neg L \land A \land Q$ ____________________________________

(D) $(\neg A \lor \neg Q) \rightarrow (F \lor L)$ ____________________________________

Translating English to Logic

"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

Translation of wrong option (A):

(A) $\neg(F \vee L) \rightarrow (A \wedge Q)$

(B) $(A \wedge Q) \rightarrow (\neg F \wedge \neg L)$

(C) $\neg F \wedge \neg L \wedge A \wedge Q$

(D) $(\neg A \vee \neg Q) \rightarrow (F \vee L)$

Then the hare is alert and quick

If neither the fox nor the lynx can catch the hare

Translating English to Logic

"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

Translation of wrong option (C):

(A) $\neg(F \vee L) \rightarrow (A \wedge Q)$

(B) $(A \wedge Q) \rightarrow (\neg F \wedge \neg L)$

(C) $\neg F \wedge \neg L \wedge A \wedge Q$

(D) $(\neg A \vee \neg Q) \rightarrow (F \vee L)$

The fox and lynx both can't catch the hare, and the hare is alert and quick.

Translating English to Logic

"Neither the fox nor the lynx can catch the hare if the hare is alert and quick."

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

Translation of wrong option (D):

(A) $\neg(F \lor L) \rightarrow (A \land Q)$

(B) $(A \land Q) \rightarrow (\neg F \land \neg L)$

(C) $\neg F \land \neg L \land A \land Q$

(D) $(\neg A \lor \neg Q) \rightarrow (F \lor L)$

If the hare isn't alert or the hare isn't quick

Then the fox or the lynx can catch the hare

Symbol	"Main" Words	Some alternate words you might see (incomplete list)
$\neg p$	"not p"	
$p \wedge q$	"p and q"	"p but q"
$p \vee q$	"p or q"	
$p \rightarrow q$	"if p, then q"	"p implies q" "p only if q" "q if p" "p is sufficient for q" "q is necessary for p"
$p \leftrightarrow q$ $p \equiv q$	"p if and only if q"	"p is equivalent to q" "p iff q" "p is necessary and sufficient for q"
$\forall x\ P(x)$	"for all x, P(x)"	"for each x, P(x)" "for any x, P(x)" "for every x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"	"for some x, P(x)" "there is an x such that P(x)"

Outline

- Intro to Symbolic Logic
- Translation
  - English to Logic
  - **Logic to English**
  - Domain Restrictions
  - Quantifier Scoping
- Logic Puzzles

Translating Logic to English #1

Handout

Domain of:
m: all movies
x, y: people in this room

$V(x, m)$: "person x has seen movie m"

a) $\exists x \, \exists y \, [x \neq y \; \wedge \; \exists m \left(V(x, m) \wedge V(y, m)\right)]$

b) $\exists x \, \exists y \, [x \neq y \; \wedge \forall m \left(V(x, m) \leftrightarrow V(y, m)\right)]$

Symbol	"Main" Words
¬p	"not p"
p∧q	"p and q"
p∨q	"p or q"
p→q	"if p, then q"
∀x P(x)	"for all x, P(x)"
∃x P(x)	"there exists x such that P(x)"

Translating Logic to English #1

Domain of:	
m:	all movies
x, y:	people in this room

$V(x, m)$: "person x has seen movie m"

a) $\exists x \, \exists y \, [x \neq y \, \wedge \, \exists m \, (V(x, m) \wedge V(y, m))]$

Two people have seen the same movie.

b) $\exists x \, \exists y \, \boxed{[x \neq y \, \wedge \forall m \, (V(x, m) \leftrightarrow V(y, m))]}$

There are two people who have seen the exact same set of movies.

Symbol	"Main" Words
¬p	"not p"
p∧q	"p and q"
p∨q	"p or q"
p→q	"if p, then q"
∀x P(x)	"for all x, P(x)"
∃x P(x)	"there exists x such that P(x)"

Handout

slido.com
#203203

Translating Logic to English

K(x,y) : "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

(B) "Someone knows two people who do not know each other"

(C) "Among the group of people that someone knows, each pair in the group do not know each other"

(D) "Everyone doesn't know two people who know each other"

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

K(x,y) : "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

(B) "Someone knows two people who do not know each other"

(C) "Among the group of people that someone knows, each pair in the group do not know each other"

(D) "Everyone doesn't know two people who know each other"

If y,z are two distinct people that **x** both knows

Then they don't know each other

Symbol	"Main" Words
¬p	"not p"
p∧q	"p and q"
p∨q	"p or q"
∃x P(x)	"there exists x such that P(x)"

Translating Logic to English

why not $\land \neg K(y,z)$

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z \, (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

(B) "Someone knows two people who do not know each other"

(C) "Among the group of people that someone knows, each pair in the group do not know each other"

(D) "Everyone doesn't know two people who know each other"

If y,z are two distinct people that **x** both knows

Then they don't know each other

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z \, (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

$\forall x$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x \, P(x)$	"for all x, P(x)"
$\exists x \, P(x)$	"there exists x such that P(x)"

Translating Logic to English

K(x,y) : "x knows y" *(not the same as "y knows x")*

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

$$\forall x\ \exists y\ \exists z\ (K(x,y) \land K(x,z) \land y \neq z)$$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(A) "Everyone knows two people who do not know each other"

$$\forall x\ \exists y\ \exists z\ (K(x,y) \land K(x,z) \land y \neq z \land \neg K(y,z) \land \neg K(z,y))$$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\, (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(B) "Someone knows two people who do not know each other"

$\exists x$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\, P(x)$	"for all x, P(x)"
$\exists x\, P(x)$	"there exists x such that P(x)"

Translating Logic to English

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z \, (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(B) "Someone knows two people who do not know each other"

$$\exists x \; \exists y \; \exists z \; (K(x,y) \land K(x,z) \land y \neq z)$$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x \, P(x)$	"for all x, P(x)"
$\exists x \, P(x)$	"there exists x such that P(x)"

Translating Logic to English

K(x,y) : "x knows y" *(not the same as "y knows x")*

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(B) "Someone knows two people who do not know each other"

$$\exists x\ \exists y\ \exists z\ (K(x,y) \land K(x,z) \land y \neq z \land \neg K(y,z) \land \neg K(z,y))$$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

K(x,y) : "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(D) "Everyone doesn't know two people who know each other"

$\forall x$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

$K(x,y)$: "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(D) "Everyone doesn't know two people who know each other"

$$\forall x\ \neg(\exists y\ \exists z\ (K(x,y) \land K(x,z) \land y \neq z))$$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Translating Logic to English

K(x,y) : "x knows y" ***(not the same as "y knows x")***

$$\exists x \forall y \forall z\ (K(x,y) \land K(x,z) \land y \neq z) \rightarrow \neg K(y,z)$$

(D) "Everyone doesn't know two people who know each other"

$$\forall x\ \neg(\exists y\ \exists z\ (K(x,y) \land K(x,z) \land y \neq z) \land K(y,z) \land K(z,y))$$

Symbol	"Main" Words
$\neg p$	"not p"
$p \land q$	"p and q"
$p \lor q$	"p or q"
$p \rightarrow q$	"if p, then q"
$\forall x\ P(x)$	"for all x, P(x)"
$\exists x\ P(x)$	"there exists x such that P(x)"

Outline

- Intro to Symbolic Logic
- Translation
  - English to Logic
  - Logic to English
  - **Domain Restrictions**
  - Quantifier Scoping
- Logic Puzzles

Domain Restrictions

$S(x)$ = "x is a student in this class"

$C(x)$ = "x has studied calculus"

"Every student in this class has studied calculus."

Is this translation correct? $\forall x\ [\ S(x) \land C(x)\ ]$ ***(domain = all people)***

Domain Restrictions

S(x) = "x is a student in this class"

C(x) = "x has studied calculus"

"Every student in this class has studied calculus."

Is this translation correct? $\forall x\ [\ S(x) \land C(x)\ ]$ ***(domain = all people)***

NO! This says: "Everyone is a student in this class who has studied Calculus."

Use **if/then** with **universal quantifiers** to make a proposition about only **some** of the domain.

$$\forall x\ [\ S(x) \rightarrow C(x)\ ]$$

Domain Restrictions

$S(x)$ = "x is a student in this class"

$C(x)$ = "x has studied calculus"

"**Some** student in this class has studied calculus."

Is this translation correct? $\exists x \; [\; S(x) \rightarrow C(x) \;]$ ***(domain = all people)***

Domain Restrictions

S(x) = "x is a student in this class"

C(x) = "x has studied calculus"

"**Some** student in this class has studied calculus."

Is this translation correct? $\exists x\ [\ S(x) \rightarrow C(x)\ ]$ ***(domain = all people)***

NO! Remember: if-then with existential quantifiers is unintuitive

(This is satisfied by any x with $\neg S(x)$, because $F \rightarrow T$ and $F \rightarrow F$)

Use **and** with **existential quantifiers** to make a proposition about only **some** of the domain.

$$\exists x\ [\ S(x) \land C(x)\ ]$$

Outline

- Intro to Symbolic Logic
- Translation
  - English to Logic
  - Logic to English
  - Domain Restrictions
  - **Quantifier Scoping**
- Logic Puzzles

Handout

Scoping

Translate the following logical statements into English.

Do the two statements have the same meaning? ___________

B(x, y) = "x buys a y"

Logic	**English**
$\forall x\,[\,B(x, \text{umbrella}) \lor B(x, \text{raincoat})\,]$	
$[\forall x\, B(x, \text{umbrella})] \lor [\forall x\, B(x, \text{raincoat})]$	

Scoping

$B(x, y)$ = "x buys a y"

Translate: ***(domain = all people)***

$\forall x\, [B(x,\text{umbrella}) \vee B(x,\text{raincoat})]$

$[\forall x\, B(x,\text{umbrella})] \vee [\forall x\, B(x,\text{raincoat})]$

Notice: the **scope** of the variable **x** ends at the parentheses, so we can reuse the name!

Scoping

$B(x, y)$ = "x buys a y"

Translate: ***(domain = all people)***

$\forall x\, [B(x,\text{umbrella}) \vee B(x,\text{raincoat})]$

$[\forall x\, B(x,\text{umbrella})] \vee [\forall y\, B(y,\text{raincoat})]$

More common: use different variable names, just to avoid confusion

Scoping

B(x, y) = "x buys a y"

Translate: ***(domain = all people)***

$\forall x\ [B(x,\text{umbrella}) \vee B(x,\text{raincoat})]$

$[\forall x\ B(x,\text{umbrella})] \vee [\forall y\ B(y,\text{raincoat})]$

"Everyone buys an umbrella or a raincoat."

"Everyone buys an umbrella, or everyone buys a raincoat."

"Variable scope:" the part of the expression in which a variable created by a for-all/there-exists exists is still active.

This matters! It causes these two propositions to have different meanings.

Scoping

L(x) = "x laughs at bad jokes"

P(x, y) = "x is a parent of y"

"Everyone who is the parent of someone laughs at bad jokes"

(A) $\forall x[\ (\exists y\ P(x,y)) \rightarrow L(x)\ ]$

(B) $\forall x \exists y\ [\ P(x,y) \rightarrow L(x)\ ]$

Scoping

L(x) = "x laughs at bad jokes"
P(x, y) = "x is a parent of y"
"Everyone who is the parent of someone laughs at bad jokes"

For all people, if they are the parent of someone, then they laugh at bad jokes.

(A) $\forall x[\,(\exists y\, P(x,y)) \rightarrow L(x)\,]$

(B) $\forall x \exists y\, [\, P(x,y) \rightarrow L(x)\,]$

If/Then directly inside an existential quantifier is a red flag! Probably non-meaningful behavior

Outline

- Intro to Symbolic Logic
- Translation
  - English to Logic
  - Logic to English
  - Domain Restrictions
  - Quantifier Scoping
- **Logic Puzzles**

A Puzzle

You are on an island where there are only two kinds of people:

- **Liars**, who always lie,
- **Truth-tellers**, who always tell the truth.

You meet an native, and ask: “are you a truth-teller?” A clap of thunder masks their response, so you say: “Excuse me, I couldn't hear you: did you say you were a truth-teller?” They answer “No, I said I was a liar.”

Is the native a liar or a truth-teller?

Logic Puzzle

You: Are you a truth-teller?

Answer 1: ???

You: Did you say you were a truth-teller?

Answer 2: No, I said I was a liar

In the beginning, we aren't sure whether the native is a truth teller or a liar

N	A1	A2
TT		
L		

Logic Puzzle

You: Are you a truth-teller?

Answer 1: ???

You: Did you say you were a truth-teller?

Answer 2: No, I said I was a liar

In either case, their first answer is yes

N	A1	A2
TT	*Yes*	
L	*Yes*	

Logic Puzzle

You: Are you a truth-teller?

Answer 1: ???

You: Did you say you were a truth-teller?

Answer 2: No, I said I was a liar

But they didn't say Yes in A2

N	A1	A2
~~TT~~	~~*Yes*~~	~~*Yes*~~
L	*Yes*	*No*

Native must be a liar

Logic Puzzle

You meet natives Anand, Blanca and Carol.

- You: Anand, are you a liar?
- Anand: [drowned out by a clap of thunder]
- You: Blanca, what did Anand say?
- Blanca: Anand said he's a liar.
- Carol: Don't believe Blanca, she's lying!
- Carol: Also, Anand is a liar.
- **Which one(s) are liars?**

Logic Puzzle

Handout

Q1: “Anand, are you a liar?”
Anand: “????”

Q2: “Blanca, what did Anand say?”
Blanca: “He said yes.”

Q3: “Is Blanca a liar?”
Carol: “Yes.”

Q4: “Is Anand a liar?”
Carol: “Yes.”

Anand	Blanca	Carol	Q1 (Anand)	Q2 (Blanca)	Q3 (Carol 1)	Q4 (Carol 2)
TT	TT	TT				
TT	TT	L				
TT	L	TT				
TT	L	L				
L	TT	TT				
L	TT	L				
L	L	TT				
L	L	L				

Logic Puzzle

Q1: “Anand, are you a liar?”
Anand: “No.” ***(Infer answer)***

Q2: “Blanca, what did Anand say?”
Blanca: “He said yes.” ***(Blanca is a liar.)***

Q3: “Is Blanca a liar?”
Carol: “Yes.” ***(Carol is a truth teller.)***

Q4: “Is Anand a liar?”
Carol: “Yes.” ***(Anand is a liar.)***

Anand	Blanca	Carol	Q1 (Anand)	Q2 (Blanca)	Q3 (Carol 1)	Q4 (Carol 2)
~~TT~~	~~TT~~	~~TT~~	~~**No**~~	~~*No*~~		
~~TT~~	~~TT~~	~~L~~	~~**No**~~	~~*No*~~		
~~TT~~	~~L~~	~~TT~~	~~**No**~~	~~**Yes**~~	~~**Yes**~~	~~*No*~~
~~TT~~	~~L~~	~~L~~	~~**No**~~	~~**Yes**~~	~~*No*~~	
~~L~~	~~TT~~	~~TT~~	~~**No**~~	~~*No*~~		
~~L~~	~~TT~~	~~L~~	~~**No**~~	~~*No*~~		
L	L	TT	**No**	**Yes**	**Yes**	**Yes**
~~L~~	~~L~~	~~L~~	~~**No**~~	~~**Yes**~~	~~*No*~~	

Wrapup

- Can turn English/word problems into “logic equations”

- Next time: manipulating these equations **algebraically**, instead of having to write out truth tables.