PROGRAM 1:

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Import numpy as np
States = ['Rainy', 'Sunny']
Observations = ['Playing', 'Studying']
Initial_prob = [0.6, 0.4]
Transition_prob = np.array([
  [0.7, 0.3],
 [0.4, 0.6]
])
Emission_prob = np.array([
 [0.1, 0.9],
 [0.8, 0.2]
])
Obs_seq = ['Playing', 'Studying', 'Playing', 'Playing']
Obs_map = {obs: idx for idx, obs in enumerate(observations)}
State_map = {state: idx for idx, state in enumerate(states)}
Obs seq indices = [obs map[obs] for obs in obs seq]
Def forward_algorithm(initial_prob, transition_prob, emission_prob, obs_seq_indices):
  Num_states = len(states)
  Num_obs = len(obs_seq_indices)
  Alpha = np.zeros((num_obs, num_states))
  For state in range(num_states):
    Alpha[0, state] = initial_prob[state] * emission_prob[state, obs_seq_indices[0]]
  For t in range(1, num_obs):
    For state in range(num_states):
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Alpha[t, state] = np.sum(alpha[t-1] * transition_prob[:, state]) * emission_prob[state,
obs_seq_indices[t]]
  Final_prob = np.sum(alpha[num_obs-1])
  Return final_prob, alpha
Def viterbi_algorithm(initial_prob, transition_prob, emission_prob, obs_seq_indices):
  Num_states = len(states)
  Num obs = len(obs seg indices)
  Delta = np.zeros((num_obs, num_states))
  Psi = np.zeros((num obs, num states), dtype=int)
  For state in range(num_states):
   Delta[0, state] = initial_prob[state] * emission_prob[state, obs_seq_indices[0]]
   Psi[0, state] = 0
  For t in range(1, num_obs):
   For state in range(num_states):
     Max_prob = -1
     Max state = -1
     For prev_state in range(num_states):
       Prob = delta[t-1, prev_state] * transition_prob[prev_state, state]
       If prob > max_prob:
         Max prob = prob
         Max_state = prev_state
     Delta[t, state] = max_prob * emission_prob[state, obs_seq_indices[t]]
     Psi[t, state] = max_state
  Best_final_state = np.argmax(delta[num_obs-1])
  Best_path = [best_final_state]
  For t in range(num_obs-2, -1, -1):
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Best_state = psi[t+1, best_path[-1]]

Best_path.append(best_state)

Best_path.reverse()

Return best_path, delta

Final_prob, alpha = forward_algorithm(initial_prob, transition_prob, emission_prob, obs_seq_indices)

Print("Forward Algorithm Probability of Observation Sequence: ", final_prob)

Best_path, delta = viterbi_algorithm(initial_prob, transition_prob, emission_prob, obs_seq_indices)

Print("Most Likely Sequence of States (Viterbi):")

For t, state_idx in enumerate(best_path):

Print(f"Time {t}: {states[state_idx]}")
```

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Output

Forward Algorithm Probability of Observation Sequence: 0.03341850000000002

Most Likely Sequence of States (Viterbi):

Time 0: Sunny

Time 1: Rainy

Time 2: Sunny

Time 3: Sunny
```

PROGRAM 2:

Import numpy as np

 $N_states = 2$

N_observations = 3

Transition_probs = np.array([[0.7, 0.3],

[0.4, 0.6]

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Emission_probs = np.array([[0.5, 0.4, 0.1],
              [0.1, 0.3, 0.6]]
Initial_probs = np.array([0.6, 0.4])
Observations = [0, 1, 2, 1, 0, 1, 2, 0, 1, 2]
Def forward_algorithm(obs, A, B, pi):
  N_{states} = A.shape[0]
  N_{obs} = len(obs)
  Alpha = np.zeros((n_states, n_obs))
   For s in range(n_states):
   Alpha[s, 0] = pi[s] * B[s, obs[0]]
   For t in range(1, n_obs):
    For s in range(n_states):
      Alpha[s, t] = np.sum(alpha[:, t-1] * A[:, s]) * B[s, obs[t]]
    Return alpha
Def viterbi_algorithm(obs, A, B, pi):
  N_{states} = A.shape[0]
  N obs = len(obs)
    Viterbi = np.zeros((n_states, n_obs))
  Path = np.zeros((n_states, n_obs), dtype=int)
    For s in range(n_states):
    Viterbi[s, 0] = pi[s] * B[s, obs[0]]
    Path[s, 0] = 0
   For t in range(1, n_obs):
    For s in range(n_states):
      Max_prob = -1
      Max_state = -1
```

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For s_prev in range(n_states):
       Prob = viterbi[s_prev, t-1] * A[s_prev, s] * B[s, obs[t]]
       If prob > max_prob:
         Max_prob = prob
         Max_state = s_prev
     Viterbi[s, t] = max_prob
     Path[s, t] = max_state
    Best_path = np.zeros(n_obs, dtype=int)
  Best_path[-1] = np.argmax(viterbi[:, -1])
    For t in range(n_{obs} - 2, -1, -1):
    Best_path[t] = path[best_path[t + 1], t + 1]
    Return best_path
Alpha = forward_algorithm(observations, transition_probs, emission_probs, initial_probs)
Print("Forward Algorithm Result (Alpha):")
Print(alpha)
Best_path = viterbi_algorithm(observations, transition_probs, emission_probs,
initial_probs)
Print("Most Likely Hidden States (Viterbi):")
Print(best_path)
```

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Output
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```
5.52115200e-04 4.16402496e-04 3.43338002e-04 2.33566826e-05
    1.32785096e-05 1.05345477e-05]]
 Most Likely Hidden States (Viterbi):
 [0 0 1 0 0 0 1 0 0 1]
PROGRAM 3:
Import numpy as np
States = ["Sunny", "Rainy"]
Observations = ["Playing", "Studying"]
Transition_probs = np.array([[0.7, 0.3],
            [0.4, 0.6]
Emission_probs = np.array([[0.6, 0.4],
           [0.1, 0.9]]
Initial_probs = np.array([0.8, 0.2])
Obs_sequence = [0, 1, 0, 0, 1]
N_states = len(states)
N_observations = len(obs_sequence)
Forward = np.zeros((n_states, n_observations))
```

Forward Algorithm Result (Alpha):

3.19682456e-05 2.76891758e-06]

[[3.0000000e-01 9.0400000e-02 7.69600000e-03 6.72832000e-03 3.52246400e-03 1.07462835e-03 9.18800845e-05 1.00825630e-04

[4.00000000e-02 3.42000000e-02 2.85840000e-02 5.83776000e-03

```
For s in range(n_states):

Forward[s, 0] = initial_probs[s] * emission_probs[s, obs_sequence[0]]

For t in range(1, n_observations):

For s in range(n_states):

Forward[s, t] = np.sum(forward[:, t-1] * transition_probs[:, s]) * emission_probs[s, obs_sequence[t]]

Total_prob = np.sum(forward[:, n_observations - 1])

Print("Forward Matrix:")

Print(forward)

Print("\nTotal Probability of the Observation Sequence:", total_prob)
```

```
Output

Forward Matrix:
[[0.48     0.1376     0.091488     0.04143744     0.01216212]
[0.02     0.1404     0.012552     0.00349776     0.0130769 ]]

Total Probability of the Observation Sequence: 0.025239024
```