模糊控制第三讲：

基于数据的模糊系统设计及在金融建模中的应用

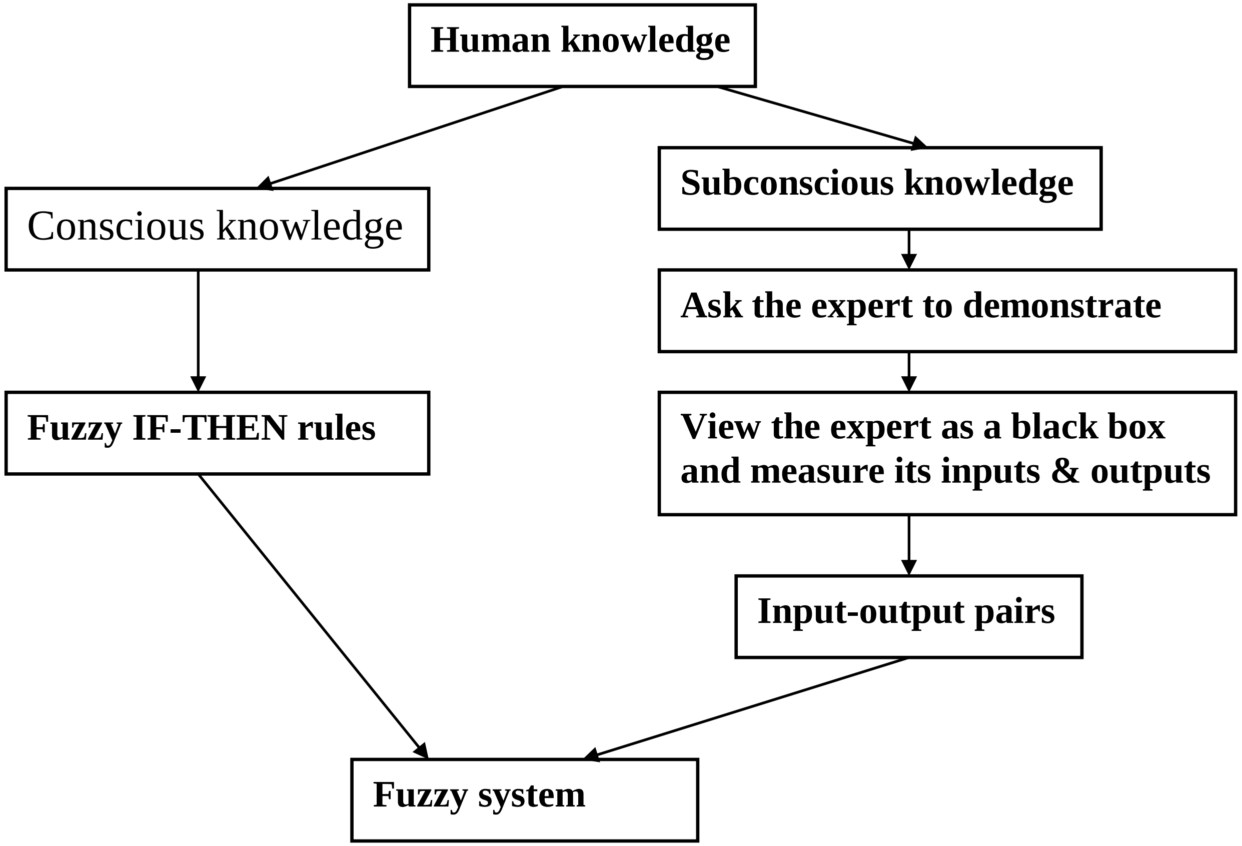
一、 由数据设计模糊系统的 WM 方法

二、 模糊神经网络

三、 递推最小二乘及聚类方法

四、 模糊系统在金融建模中的应用

一、由数据设计模糊系统的 WM 方法



12.1 Table WM Scheme for Designing Fuzzy Systems

The problem: Given input-output pairs:

$$(x_0^{\ p}; y_0^{\ p}), \quad \text{p} = 1,2,...\text{N}.$$

where $x_0^{\ p} \in U = [\alpha_1, \beta_1] \times \cdots \times [\alpha_n, \beta_n] \subset R^n$ and $y_0^{\ p} \in V = [\alpha_y, \beta_y] \subset R$. Design a fuzzy system $f : U \to V$ from these input-output pairs.

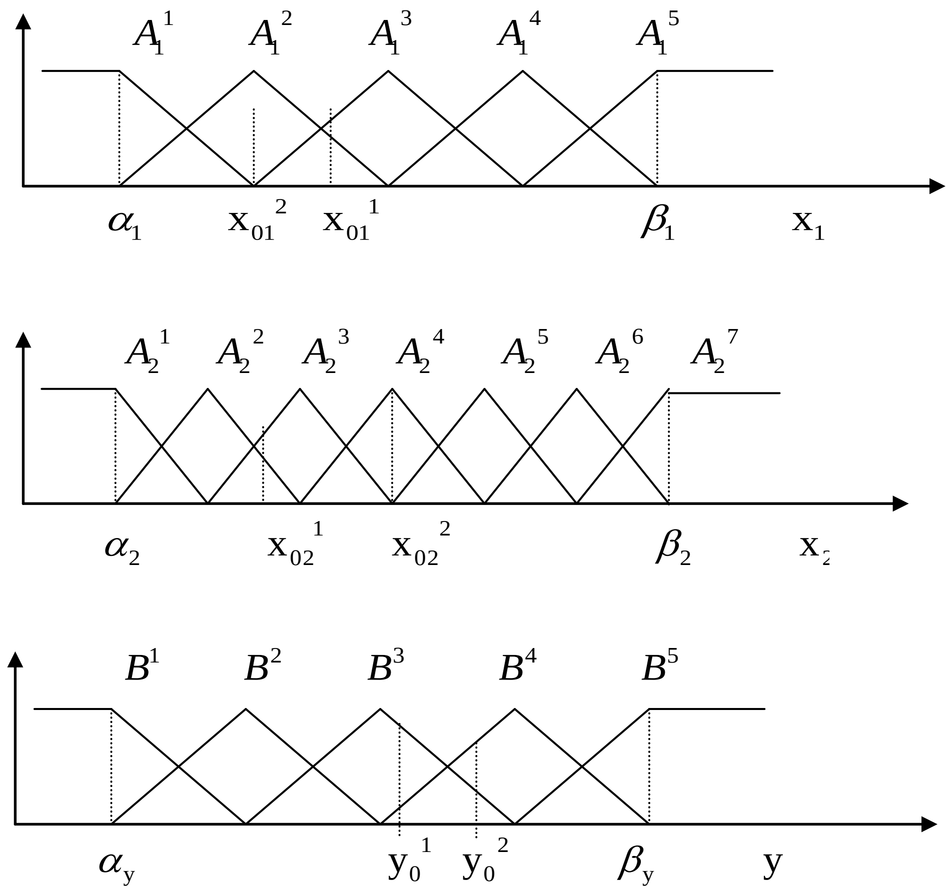
数据文件

x_1	x_2	...	x_n	y
2.1	3.5	...	1.1	0.3
3.3	3.1	...	2.4	1.2
2.2	2.7	...	1.5	1.9
$\vdots$				

$\rightarrow\rightarrow$ $$y = f(x)$$

The Design Procedure (5 steps):

- <u>Step 1</u>: Define fuzzy sets to cover the input and output spaces.



- Step 2: Generate one rule from one input-output pair, in the following way:
  - First: For each input-output pair $(x_{01}{}^p, \cdots, x_{0n}{}^p, y_0{}^p)$, compute

$$\mu_{A_i^j}(x_{0i}{}^p) \text{ for } j = 1,2,...,N_i$$

$$\mu_{B^l}(y_0{}^p) \text{ for } l = 1,2,..., N_y$$

  - Then: Determine j* and l* such that

$$\mu_{A_i^{j*}}(x_{0i}{}^p) \geq \mu_{A_i^j}(x_{0i}{}^p) \text{ for } j = 1,2,..., N_i$$

$$\mu_{B^{l*}}(y_0{}^p) \geq \mu_{B^l}(y_0{}^p) \text{ for } l = 1,2,..., N_y$$

  - Finally: Obtain a fuzzy IF-THEN rule as:

$$\text{IF } x_1 \text{ is } A_1^{j*} \text{ and } \ldots x_n \text{ is } A_n^{j*}, \text{ THEN } y \text{ is } B^{l*}.$$

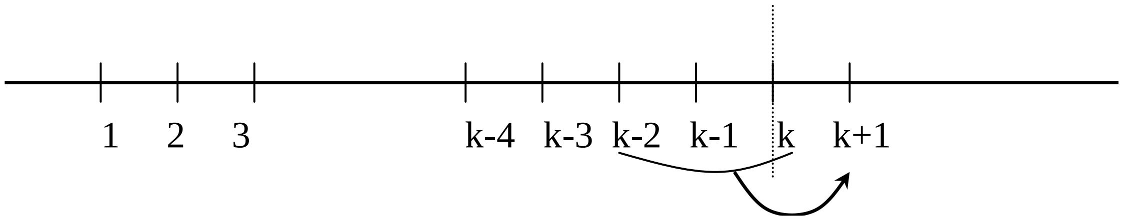
- Step 3: Assign a degree to each generated rule in Step 2.

$$D(rule) = \prod_{i=1}^{n} \mu_{A_i^{j*}}(x_{0i}{}^p)\mu_{B^{l*}}(y_0{}^p)$$

- Step 4: Create the fuzzy rule base.
  - Rules in Step 2 that do not conflict with other rules.
  - Rule in a conflicting group that has maximum degree.
  - Linguistic rule from human experts.
- Step 5: Construct the fuzzy system from the fuzzy rule base.

Example 1: Time-series prediction. Given a time series $x(1), x(2), x(3), \cdots, x(k)$ up to time point k, predict the value of $x(k+1)$.

- Determine the input-output pairs



$f : R^3 \rightarrow R$

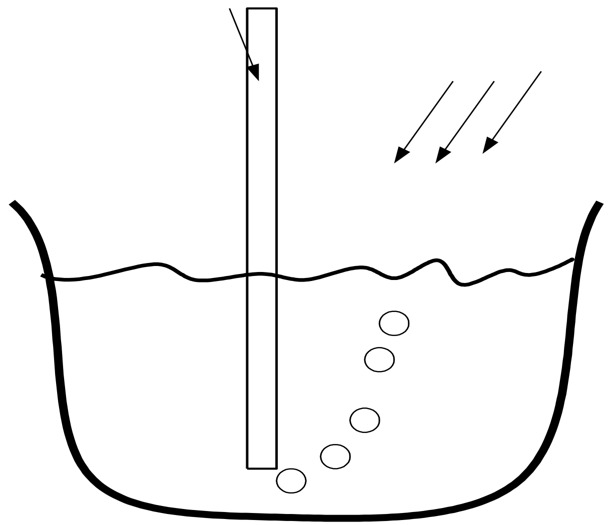
[x(k-3),x(k-2),x(k-1); x(k)]

[x(k-4),x(k-3),x(k-2); x(k-1)]

[x(k-5),x(k-4),x(k-3); x(k-2)]

⋮

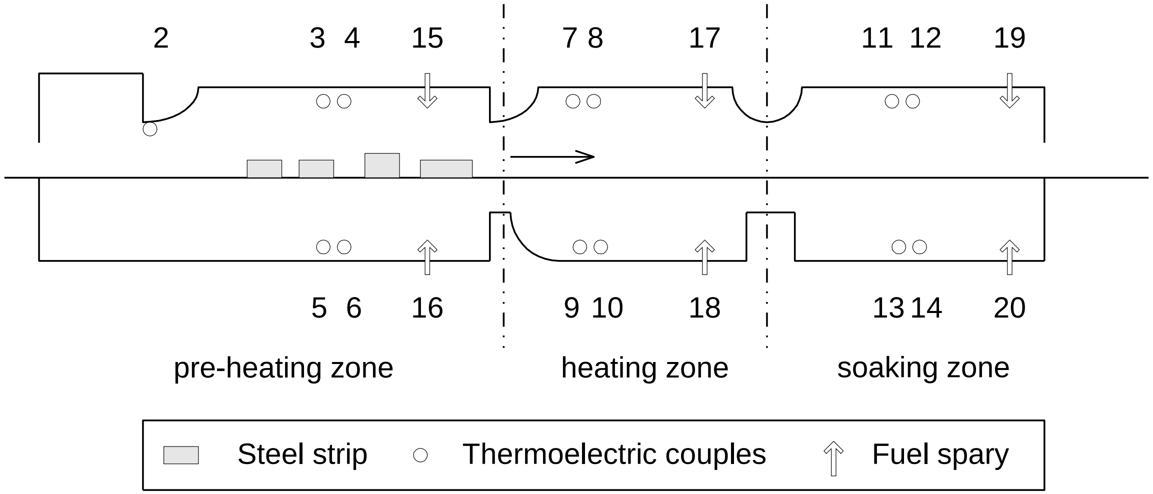
<u>例 2</u>：转炉吹氧炼钢数据建模。



输出变量： y：钢水质量指标

输入变量： x_1：钢水重量； x_2：钢水温度； x_3：吹氧量； x_4：辅材 1； x_5：辅材 2；…

例 3：钢坯加热炉数据建模。



$y_{ij}(t)$ = **i,j 探热点在 t 时刻的温度**

$u_p(t)$ = **p 点在 t 时刻的喷气量**

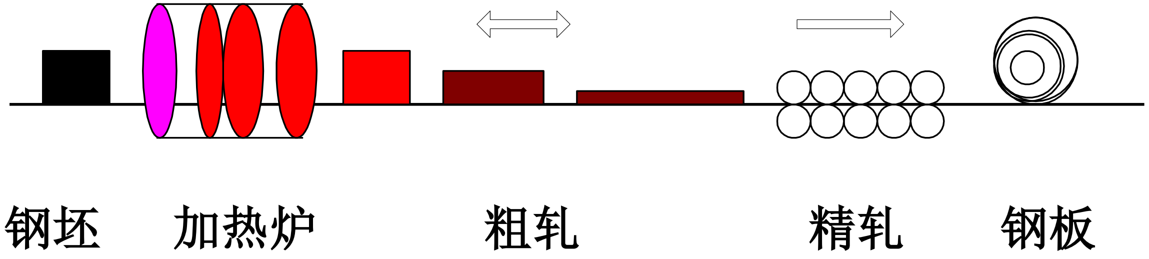
输出变量： $y_{78}(t+1)$

输入变量： $y_{78}(t)$ $y_{78}(t-1)$ $y_{34}(t-3)$ $y_{1112}(t-4)$ $u_{17}(t)$

预测模型：

$$y_{78}(t+1) = f[y_{78}(t), y_{78}(t-1), y_{34}(t-3), y_{1112}(t-4), u_{17}(t)]$$

例 4：钢板抗拉强度模型。



输出变量：钢板抗拉强度

输入变量：精轧机设定参数，粗轧输出厚度，粗轧温度，加热时间，加热温度，
钢坯成分，
…… ……

二、模糊神经网络

- **Approach in Chapter 12:**

  input-output pairs $\Longrightarrow$ fuzzy IF-THEN rules $\Longrightarrow$ fuzzy system

- **Approach in this Chapter:**

  structure of fuzzy system $\Longrightarrow$ parameters of fuzzy system

  $\Uparrow$

  input-output pairs

13.1 Network representation of the fuzzy system

$$f(x)=\frac{\sum_{l=1}^{M}\bar{y}^{l}[\prod_{i=1}^{n}\exp(-(\frac{x_i-\bar{x}_i^{\,l}}{\sigma_i^{\,l}})^2)]}{\sum_{l=1}^{M}[\prod_{i=1}^{n}\exp(-(\frac{x_i-\bar{x}_i^{\,l}}{\sigma_i^{\,l}})^2)]}$$

free paremeters: $\bar{y}^{l},\bar{x}_i^{\,l},\sigma_i^{\,l}$.

- *Task*: Determine the free parameters such that the fuzzy system matches the input-output pairs.
- Network representation of the fuzzy system

13.2 Design the Parameters by Gradient Descent Algorithm

- *Task*: Given input-output pairs: $(x_0{}^p; y_0{}^p), p = 1,2,...$, determine the free parameters $\bar{y}^l, \bar{x}_i{}^l, \sigma_i{}^l$ such that

$$e^p = \frac{1}{2}[f(x_0{}^p) - y_0{}^p]^2$$

is minimized.

- *The Method*: Gradient descent:

  - $y^l(q+1) = y^l(q) - \alpha \frac{\partial e^p}{\partial \bar{y}^l}|_q, \quad \mathrm{q} = 0,1,2,...$,

$$\boxed{y^l(q+1) = y^l(q) - \alpha \frac{f-y}{b} z^l}$$

where $z^l = \prod_{i=1}^{n} \exp(-(\frac{x_i - \bar{x}_i{}^l}{\sigma_i{}^l})^2)$, $b = \sum_{l=1}^{M} z^l$, $a = \sum_{l=1}^{M} \bar{y}^l z^l$, $f = \frac{a}{b}$

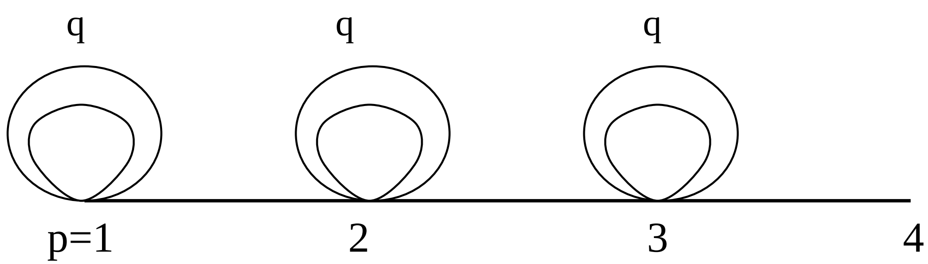
  - $\bar{x}_i{}^l(q+1) = \bar{x}_i{}^l(q) - \alpha \frac{\partial e^p}{\partial \bar{x}_i{}^l}|_q, \quad \mathrm{q} = 0,1,2,...$

$$\boxed{\bar{x}_i{}^l(q+1) = \bar{x}_i{}^l(q) - \alpha \frac{f-y}{b}[\bar{y}^l(q) - f]z^l \frac{2(x_{0i}{}^p - \bar{x}_i{}^l(q))}{(\sigma_i{}^l(q))^2}}$$

- $\sigma_i^l(q+1) = \sigma_i^l(q) - \alpha \frac{\partial e^p}{\partial \sigma_i^l}\Big|_q, \qquad \text{q} = 0,1,2,\ldots$

$$\boxed{\sigma_i^l(q+1) = \sigma_i^l(q) - \alpha \frac{f-y}{b}[\bar{y}^l(q) - f]z^l \frac{2(x_{0i}{}^p - \bar{x}_i^l(q))^2}{(\sigma_i^l(q))^3}}$$

- ***Detailed Steps:***
  - Step 1: Determine the structure and initial parameters.
  - Step 2: Present input $x_0{}^p$ and compute the values z^l, b, a, f along the network representation of the fuzzy system.
  - Step 3: Update the parameters using the gradient algorithm.
  - Step 4: Repeat from Step 2 with q=q+1 until e^p is small.
  - Step 5: Repeat with p=p+1.
  - Step 6: For finite number of input-output pairs, reset p=1.



13.3 Application to Nonlinear Dynamic System Identification

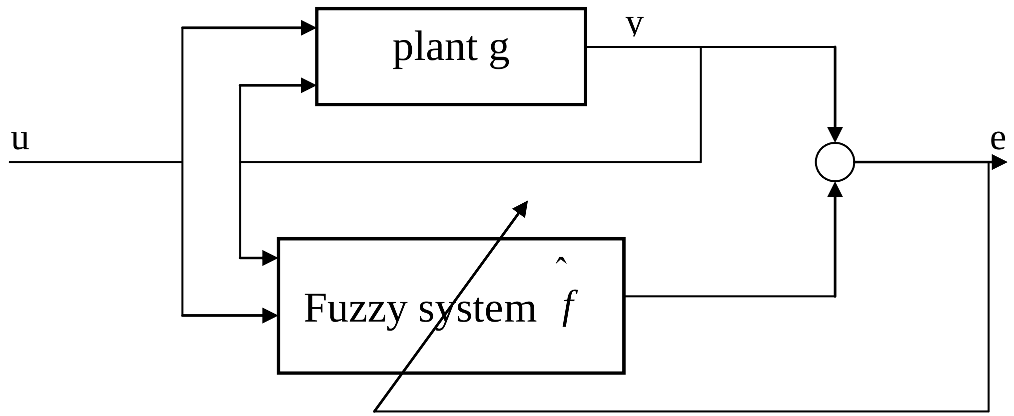
- *Task*: Consider the nonlinear dynamic system

$$y(k+1) = g[y(k),...,y(k-n+1),u(k),...,u(k-m+1)]$$

where g is unknown. Design the fuzzy system f in the following identification model

$$\hat{y}(k+1) = f_k[y(k),...,y(k-n+1),u(k),...,u(k-m+1)]$$

such that $\hat{y}(k+1)$ converges to $y(k+1)$ as $k \to \infty$.



- **Input-output Pairs**: (x_0^{k+1}, y_0^{k+1}), when

$$x_0^{k+1} = (y(k),...,y(k-n+1),u(k),...,u(k-m+1)) \quad , \qquad y_0^{k+1} = y(k+1)$$

三、递推最小二乘及聚类方法

- Chapter 12: WM, one-pass
- Chapter 13: Minimize $e^{p}=\frac{1}{2}[f(x_0{}^{p})-y_0{}^{p}]$
- Chapter 14: Minimize $J_p=\sum_{j=1}^{p}[f(x_0{}^{j})-y_0{}^{j}]^2$

Let

$$f(x)=\frac{\sum_{l=1}^{M}\bar{y}^{l}[\prod_{i=1}^{n}\mu_{A_i^l}(x_i)]}{\sum_{l=1}^{M}[\prod_{i=1}^{n}\mu_{A_i^l}(x_i)]}$$

Fix $\mu_{A_i^l}(x_i)$, view $\bar{y}^{l}$ as parameters. Let

$$b^{l}(x)=\frac{\prod_{i=1}^{n}\mu_{A_i^l}(x_i)}{\sum_{l=1}^{M}[\prod_{i=1}^{n}\mu_{A_i^l}(x_i)]}\qquad\qquad \theta^{l}=\bar{y}^{l}$$

$$b(x)=[b^{1}(x),b^{2}(x),\cdots,b^{M}(x)]^{T},\quad \theta=[\bar{y}^{1},\bar{y}^{2},\cdots,\bar{y}^{M}]^{T}$$

then

$$f(x) = b^T(x)\theta$$

so

$$J_p = \sum_{j=1}^{p}[b^T(x_0{}^j)\theta - y_0{}^j]^2$$

$$= [b^T(x_0{}^1)\theta - y_0{}^1, \cdots, b^T(x_0{}^p)\theta - y_0{}^p] \cdot \begin{bmatrix} b^T(x_0{}^1)\theta - y_0{}^1 \\ \vdots \\ b^T(x_0{}^p)\theta - y_0{}^p \end{bmatrix}$$

$$= \left(B_p\theta - Y_0{}^p\right)^T\left(B_p\theta - Y_0{}^p\right)$$

where

$$B_p = [b(x_0{}^1), \cdots, b(x_0{}^p)]^T, \quad Y_0{}^p = [y_0{}^1, \cdots, y_0{}^p]^T.$$

The θ that minimize J_P is obtained from:

$$\frac{\partial J_P}{\partial \theta} = 0 \Rightarrow B_P{}^T(B_P\theta - Y_0{}^P) = 0$$

$$\Rightarrow \boxed{\theta(p) = (B_p{}^T B_p)^{-1} B_p{}^T Y_0{}^p}$$

This is the Least-Squares solution.

- ***Our Task***: Find a recursive formula to compute $\theta(p)$, that is , represent $\theta(p)$ as a function of $\theta(p-1)$. This is called *recursive least squares.*

- **Derivation of the Recursive Least Squares Algorithm:**

  - Matrix Inversion Lemma:

$$(P^{-1}+bb^T)^{-1} = P - Pb(b^T Pb+1)^{-1} b^T P$$

- ***Design of Fuzzy System by Recursive Least Squares***

  - Step 1: Define fuzzy sets to cover the input domain $U = [\alpha_1, \beta_1] \times ... \times [\alpha_n, \beta_n]$

  - Step 2: Construct the fuzzy system. (determine the structure, and let $\bar{y}^l$ be free parameters)

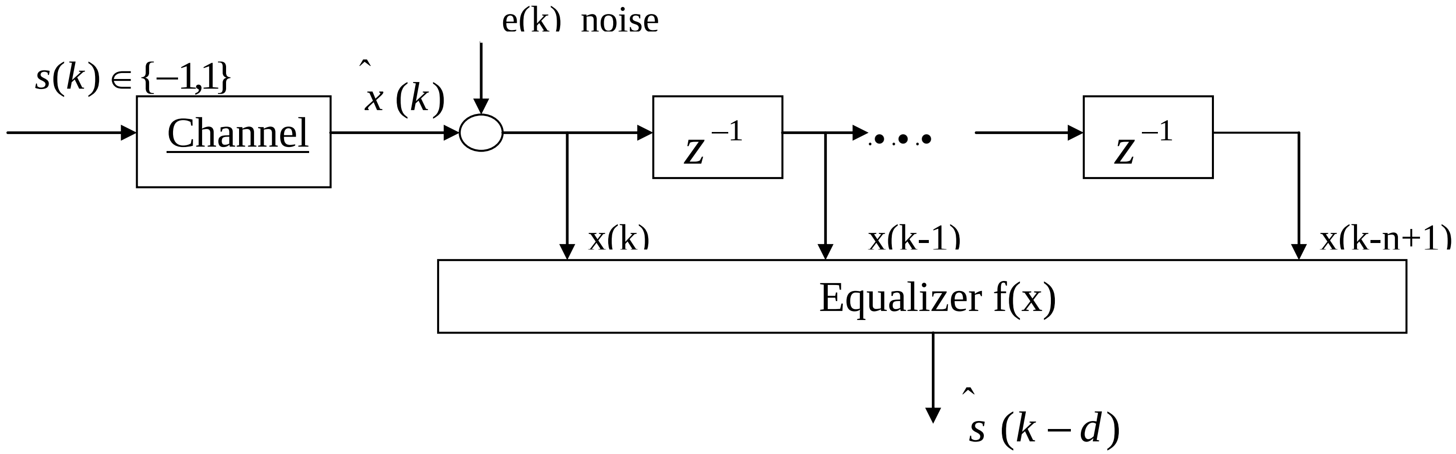
  - Step 3: Fix the initial values of $\theta^{\,l}(0) = y^l(0)$

  - Step 4: Use the recursive least squares algorithm to update the free parameters $\theta(p)$.

$$\{\text{steps 1-3}\} \Rightarrow \text{offline} \qquad \{\text{step 4}\} \Rightarrow \text{online}$$

14.3 Application to Equalization of Nonlinear Communication Channels

problem $\Rightarrow$ input-output pairs $\Rightarrow$ design f(x) $\Rightarrow$ use the designed f(x)



n: order of the equalizer, d: delay, $f(x): R^n \to \{-1,1\}$.

- **The problem**: Design a fuzzy system f(x) (equalizer) such that $\hat{s}(k-d) = \mathrm{sgn}(f(x))$ is a good estimate of s(k-d).
- **Input-Output Pairs**: Send a test sequence {s(1), s(2), …, s(k-d)}. The input-output pairs are:

  [x(k),x(k-1),…,x(k-n+1); s(k-d)]
  [x(k-1),x(k-2),…,x(k-n); s(k-d-1)]
  .
  [… … s(1)]

- **Design fuzzy system using input-output pairs**. (training phase)
- **Use the designed f(x) as equalizer for unknown s(k)**. (application phase)

- **Example**: Channel: $\hat{x}(k) = s(k) + 0.5s(k-1) - 0.9[s(k) + 0.5s(k-1)]^3$, n=2, d=0, Fig 14.3-14.5

  The Optimal Equalizer:

  Let $P_{n,d}(1) = \{\hat{x}(k) \in R^n \mid s(k-d) = 1\}$

  $$P_{n,d}(-1) = \{\hat{x}(k) \in R^n \mid s(k-d) = -1\}$$

then the optimal equalizer is

$$f_{opt}(x(k)) = \mathrm{sgn}[P_1(x(k) \mid \hat{x}(k) \in P_{n,d}(1)) - P_{-1}(x(k) \mid \hat{x}(k) \in P_{n,d}(-1)) \geq 0]$$

where P_1, P_{-1} represent probability density function. Fig 14.2, n=2,d=1, Figs 14.6-14.8.

15.1 An Optimal Fuzzy System

- ***Task:*** Given input-output pairs $(x_0^l; y_0^l), l = 1,2,..., N$ .Design a fuzzy system f(x) such that

$$| f(x_0^l) - y_0^l | < \varepsilon$$

for all $l = 1,2,....N$ and arbitrary small $\varepsilon > 0$.

- **Theorem 15.1**: The fuzzy system

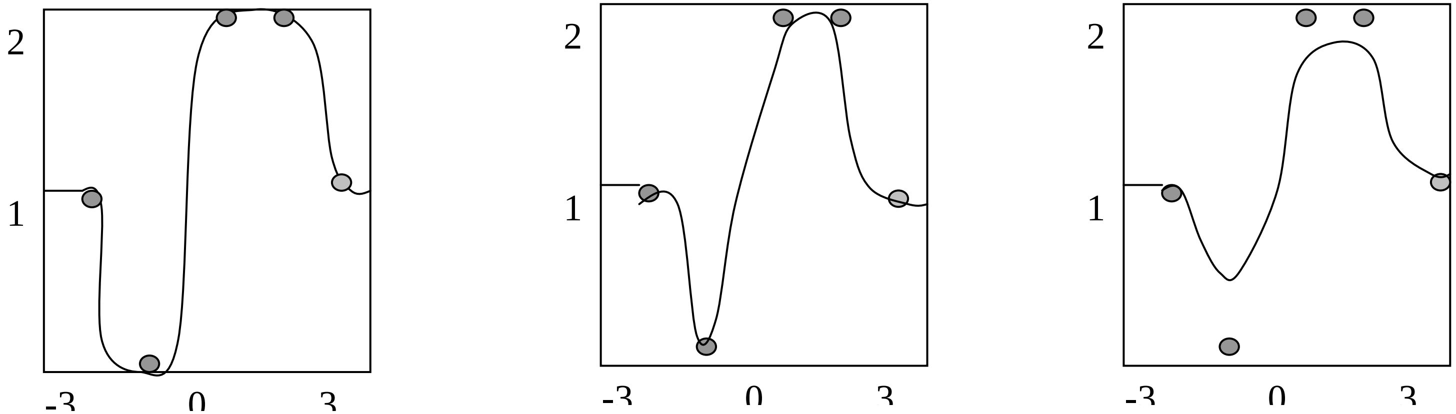
$$f(x) = \frac{\sum_{l=1}^{N} y_0^l \exp(-\frac{|x - x_0^l|^2}{\sigma^2})}{\sum_{l=1}^{N} \exp(-\frac{|x - x_0^l|^2}{\sigma^2})}$$

gives

$$| f(x_0^l) - y_0^l | < \varepsilon$$

for all $l = 1,2,....N$ and arbitrary small $\varepsilon > 0$.

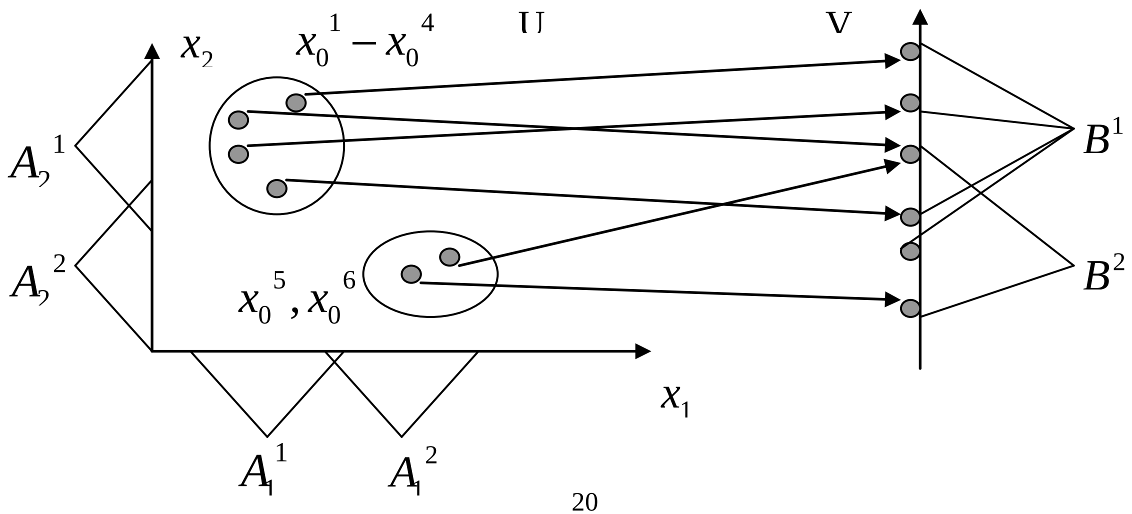
- **Example**: (The importance of σ) Given five input-output pairs: (-2,1), (-1,0), (0,2), (1,2), (2,1). Design a fuzzy system to match them.



$\sigma = 0.1$ $\qquad$ $\sigma = 0.3$ $\qquad$ $\sigma = 0.5$

15.2 Design of Fuzzy Systems By Clustering

- **The idea**:



- **The Detailed Algorithm:**
  - **Step 1**: State with $(x_0^{\ 1}; y_0^{\ 1})$, let cluster center $x_c^{\ 1} = x_0^{\ 1}$, and set $A^1(1) = y_0^{\ 1}, B^1(1) = 1$. Select a radius r.
  - **Step 2**: Suppose when considering $(x_0^{\ k}; y_0^{\ k})$, k=2,3,…, there are M clusters with centers $x_c^{\ 1}, x_c^{\ 2}, \ldots, x_c^{\ M}$. Compute

$$|x_0^{\ k} - x_c^{\ l}|, \qquad l = 1,2,\ldots M$$

and l_k be the nearest to $x_0^{\ k}$. Then:

a) If $|x_0^{\ k} - x_c^{\ l_k}| \geq r$, establish a new cluster $x_c^{\ M+1} = x_0^{\ k}$ and $A^{M+1}(k) = y_0^{\ k}, B^{M+1}(k) = 1$. Keep $A^l(k) = A^l(k-1)$ and $B^l(k) = B^l(k-1)$ for $l = 1,2,\ldots M$.

b) If $|x_0^{\ k} - x_c^{\ l_k}| \leq r$, do

$$A^{l_k}(k) = A^{l_k}(k-1) + y_0^{\ k},$$

$$B^{l_k}(k) = B^{l_k}(k-1) + 1$$

and

$$A^l(k) = A^l(k-1),\ B^l(k) = B^l(k-1)$$

for $l \neq l_k$.

Physical meaning of

$A^l(k)$**:** the summation of $y_0{}'s$ in l'the cluster after k input-output pairs being considered.

$B^l(k)$**:** the number of pairs in cluster l at k.

- **Step 3**: The fuzzy system is designed as

$$f(x)=\frac{\sum_{l=1}^{M}\left(\frac{A^l(k)}{B^l(k)}\right)\exp\left(-\frac{|x-x_c{}^l|^2}{\sigma^2}\right)}{\sum_{l=1}^{M}\exp\left(-\frac{|x-x_c{}^l|^2}{\sigma^2}\right)}$$

- **Step 4**: Go to Step 2 with k=k+1.

- **Example**: Five input-output pairs: (-2,1),(-1,0), (0,2), (1,2), (2,1), r=1.5.
- **Example**: Consider the nonlinear system

$$y(k+1)=g[y(k),y(k-1)]+u(k)$$

where

$$g[y(k),y(k-1)]=\frac{y(k)y(k-1)[y(k)+2.5]}{1+y^2(k)+y^2(k-1)}$$

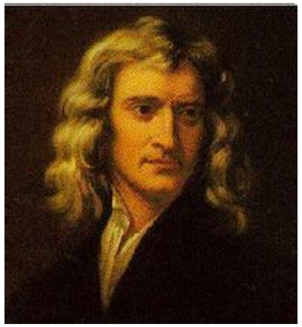
is assumed unknown. *Our objective* is to determine control u(k) such that y(k+1) will follow $y_m(k+1)$ of the reference model:

$$y_m(k+1)=0.6y_m(k)+0.2y_m(k-1)+\sin(2\pi k/25)$$

That is, we want $e(k)=y(k)-y_m(k)$ converges to zero as $k\to\infty$.

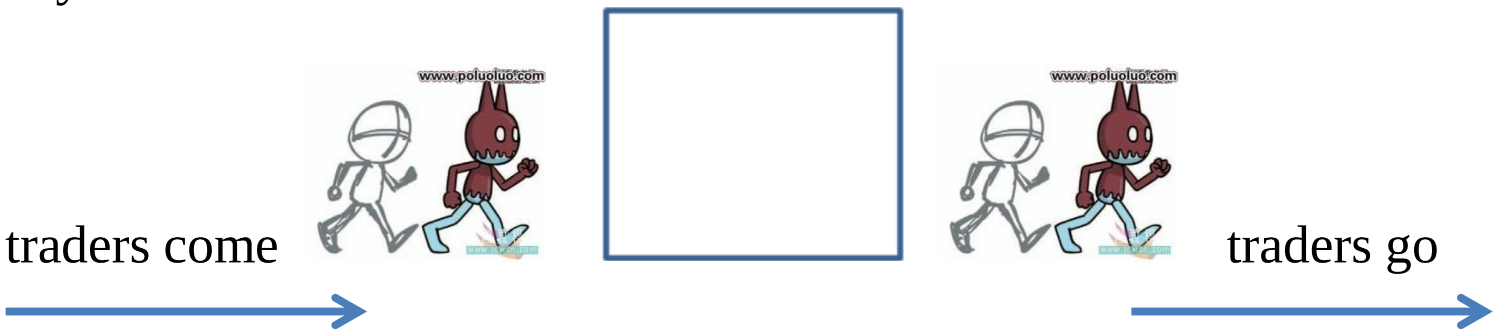
四、<u>模糊系统在金融建模中的应用</u>

- Understanding the dynamics of stock prices is one of the hardest challenges to human intelligence.



"I can predict the motions of the heavenly bodies,
but not the madness of people." --- Isaac Newton

- Why is it so hard?



Different traders use different trading strategies, result:

<u>**Prices are inherently Non-Stationary**</u>

- Prices are determined by the actions of traders.

According to a survey of 692 fund managers in five countries (Menkhoff, L., "The Use of Technical Analysis by Fund Managers: International Evidence," *Journal of Banking & Finance* 34: 2573-2586, 2010)

For prediction horizon of weeks:

relative importance	technical analysis	fundamental	order flow
US fund managers	29.4	28.4	1.4
German	60	22.6	6.7
Swiss	56.8	20	7.5
Italian	45.7	31.5	0.9
Thai	55.8	35.1	2.4

Let's model the technical traders!

- **Moving Average Rules**

Heuristic 1: A buy (sell) signal is generated if a shorter moving average of the price is crossing a longer moving average of the price from below (above). Usually, the larger the difference between the two moving averages, the stronger the buy (sell) signal.

trend following

But, if the difference between the two moving averages is too large, the stock may be over-bought (over-sold), so a small sell (buy) order should be placed to safeguard the investment. contrarian

- **<u>Price dynamical model with Heuristic 1</u>**

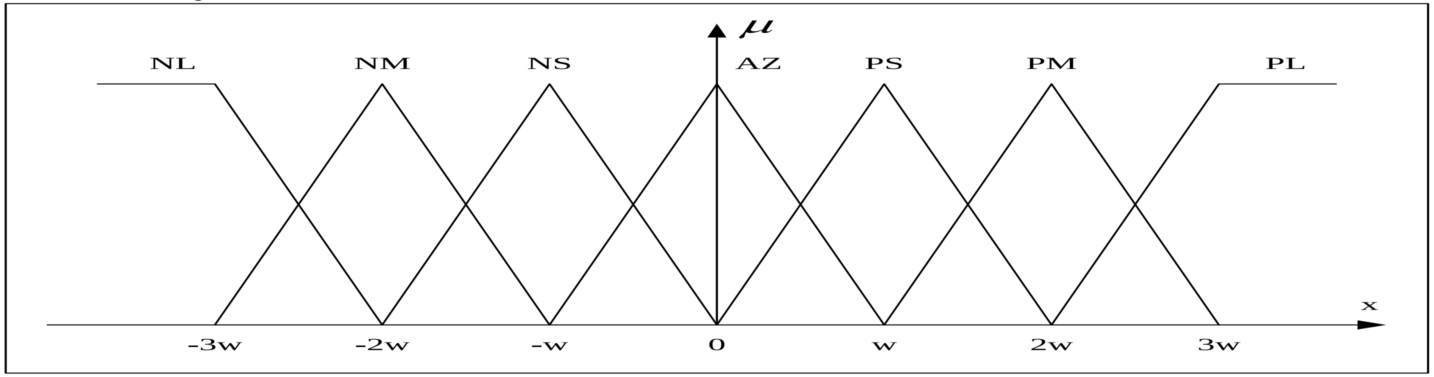
-- Moving averages:

$$\bar{p}_{t,n} = \frac{1}{n}\sum_{i=0}^{n-1} p_{t-i}$$

-- Difference between moving averages, the key variable

$$x_{1,t}^{(m,n)} = \ln\left(\bar{p}_{t,m} / \bar{p}_{t,n} \right)$$

-- Define fuzzy sets to characterize



-- Heuristic 1 becomes Rule-1-Group:

Rule 1_1: IF $x_{1,t}^{(m,n)}$ is Positive Small (PS), THEN ed_1 is Buy Small (BS) -- trend-follow
Rule 1_2: IF $x_{1,t}^{(m,n)}$ is Positive Medium (PM), THEN ed_1 is Buy Big (BB) -- trend-follow
Rule 1_3: IF $x_{1,t}^{(m,n)}$ is Positive Large (PL), THEN ed_1 is Sell Medium (SM) -- contrarian
Rule 1_4: IF $x_{1,t}^{(m,n)}$ is Negative Small (NS), THEN ed_1 is Sell Small (SS) -- trend-follow
Rule 1_5: IF $x_{1,t}^{(m,n)}$ is Negative Medium (NM), THEN ed_1 is Sell Big (SB) -- trend-follow
Rule 1_6: IF $x_{1,t}^{(m,n)}$ is Negative Large (NL), THEN ed_1 is Buy Medium (BM) -- contrarian
Rule 1_7: IF $x_{1,t}^{(m,n)}$ is Around Zero (AZ), THEN ed_1 is Neutral (N) -- trend-follow

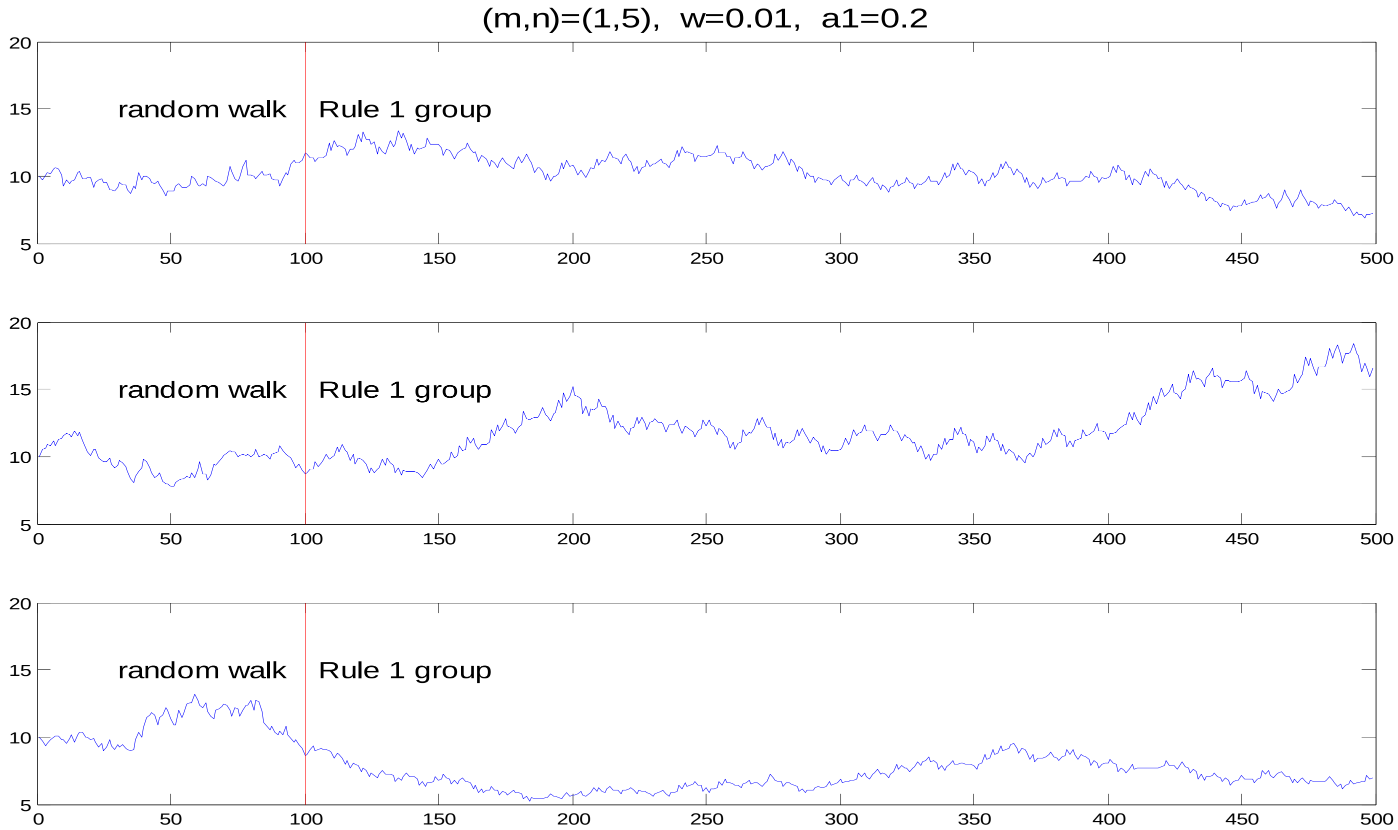
Combining Rule-1-Group into a fuzzy system:

$$ed_1\left(x_{1,t}^{(m,n)}\right) = \frac{\sum_{i=1}^{7} c_i \mu_{A_i}\left(x_{1,t}^{(m,n)}\right)}{\sum_{i=1}^{7} \mu_{A_i}\left(x_{1,t}^{(m,n)}\right)}$$

The price dynamical model is

$$\ln(p_{t+1}) = \ln(p_t) + a_1(t)\, ed_1\left(x_{1,t}^{(m,n)}\right)$$

-- Simulations



Random walk (random)

$$\ln(p_{t+1}) = \ln(p_t) + \sigma\ \varepsilon_t$$

Rule-1-Group dynamics (deterministic)

$$\ln(p_{t+1}) = \ln(p_t) + a_1(t)\ ed_1\left(x_{1,t}^{(m,n)}\right)$$

- Recalled the model:

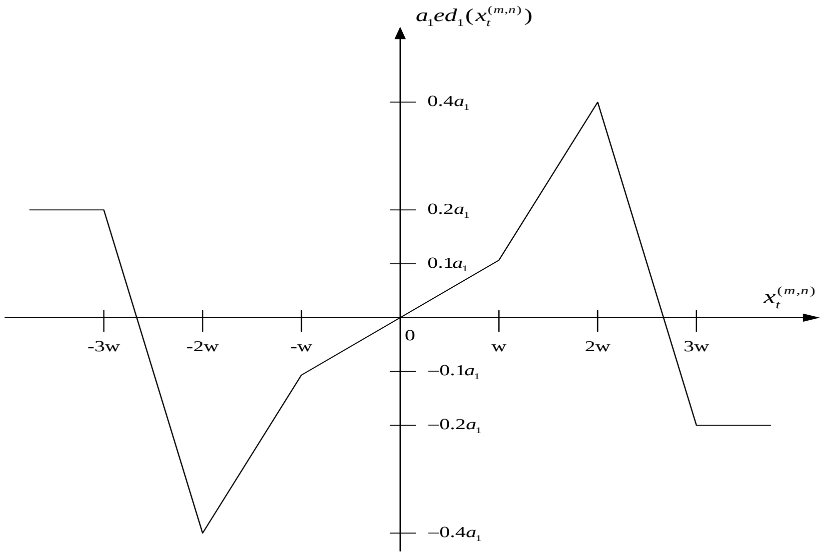
$$\ln(p_{t+1}) = \ln(p_t) + a_1(t)\ ed_1\left(x_{1,t}^{(m,n)}\right)$$

$$x_{1,t}^{(m,n)} = \ln\left(\bar{p}_{t,m}/\bar{p}_{t,n}\right) \qquad \bar{p}_{t,n} = \frac{1}{n}\sum_{i=0}^{n-1} p_{t-i} \qquad ed_1\left(x_{1,t}^{(m,n)}\right) = \frac{\sum_{i=1}^{7} c_i \mu_{A_i}\left(x_{1,t}^{(m,n)}\right)}{\sum_{i=1}^{7} \mu_{A_i}\left(x_{1,t}^{(m,n)}\right)}$$

Three key parameters:

1) Strength parameter $a_1(t)$; 2) "Frequency" parameter m,n ;

3) "Phase" parameter w (characterizes the switching between trend followers and contrarians)



- <u>Stability analysis of price dynamical model</u>

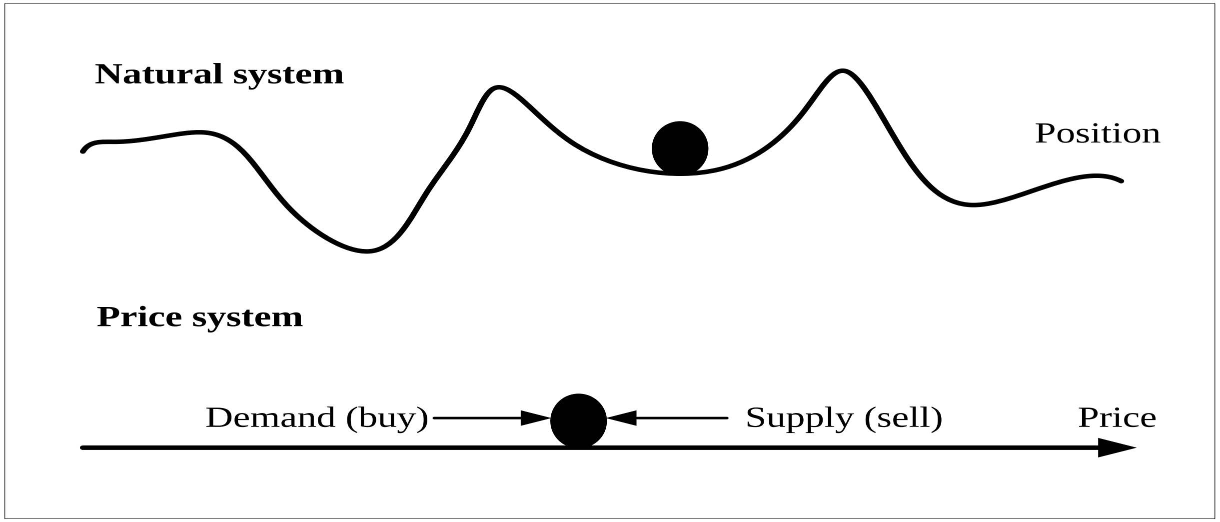
$$\ln(p_{t+1}) = \ln(p_t) + a_1(t)\, ed_1\left(x_{1,t}^{(m,n)}\right)$$

<u>Theorem 1</u>: For any positive number $p^* \in R_+$, $y^* = (p^*, \ldots, p^*)^T_{n\times 1}$ is an equilibrium of the price dynamical model.

<u>Theorem 2</u>: All equilibriums of the price dynamical model are unstable.

These results show that the classical concept of equilibrium and stability developed for Natural Systems may not be suitable for Price Systems.

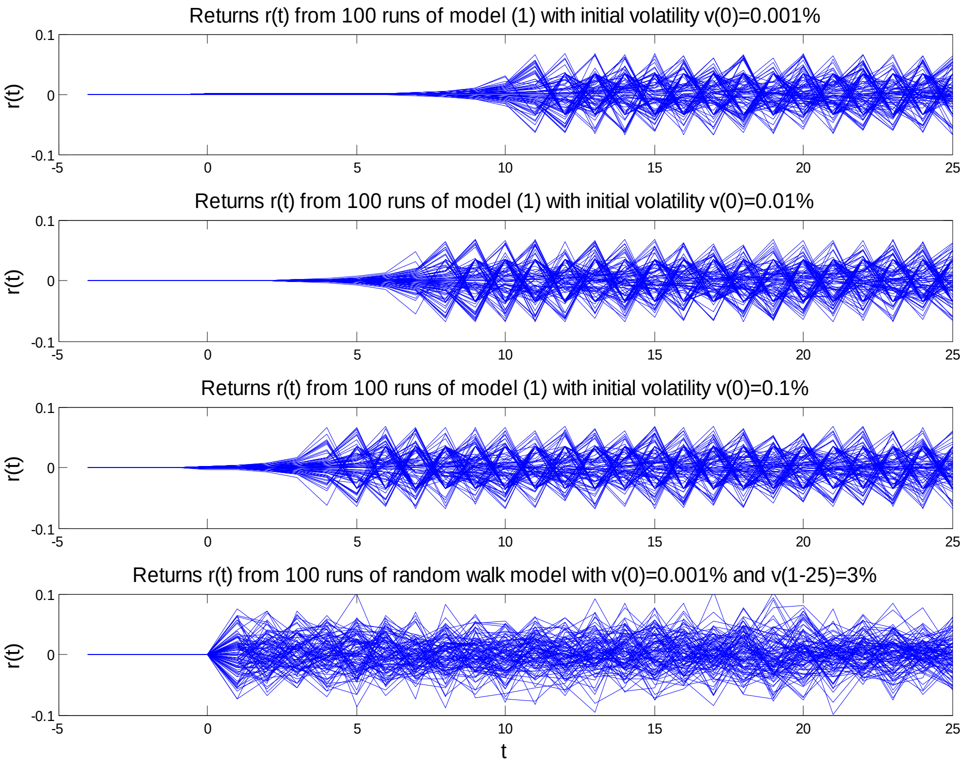
We need new concepts of stability for Social Systems.



● <u>Short-term prediction: the difference between chaos and random</u>

Chaos: Sensitive dependence on initial conditions.

Monte-Carlo simulations with different initial conditions:



● Big Buyer and Big Seller Rules

Big buyers and big sellers are institutional traders who usually want to buy or sell a large amount of stocks.

Since the amount of stocks offered or asked around the trading price is very small, the large buy or sell order has to be cut into small pieces and implemented incrementally over a long period of time.

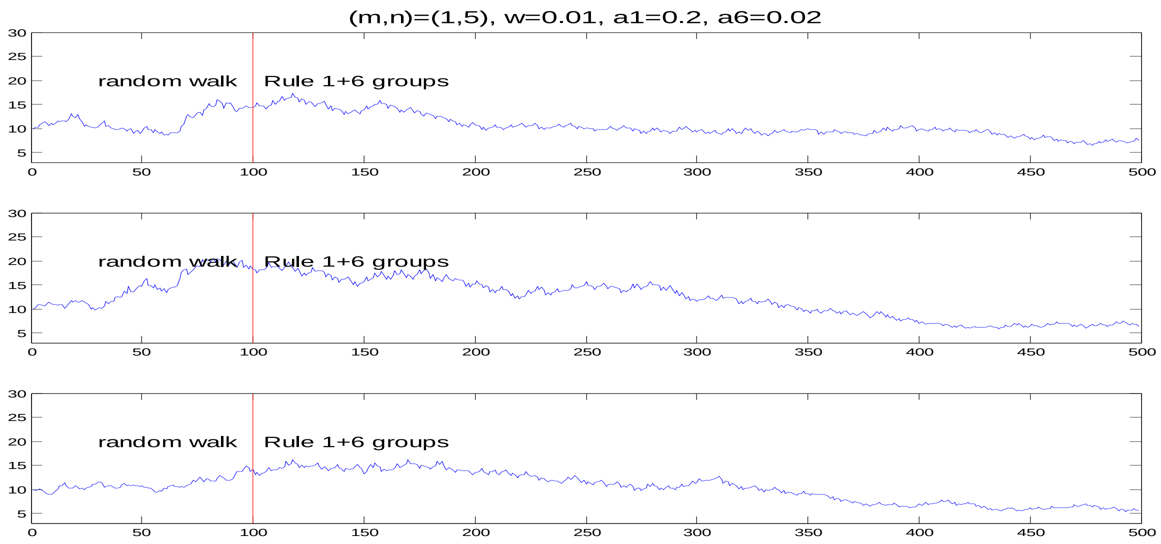
Heuristic 6 (Big seller rule): For a big seller, sell if the price of the stock is increasing: the stronger the increase, the larger the sell order. Keep neutral if the price is decreasing or moving horizontally.

Heuristic 7 (Big buyer rule): For a big buyer, buy if the price of the stock is decreasing: the larger the decrease, the larger the buy order. Keep neutral if the price is increasing or moving horizontally.

- Heuristic 6 (Big seller rule) = Rule-6-Group:

Rule 6_1: IF $x_{1,t}^{(m,n)}$ is Positive Small (PS), THEN ed_6 is Sell Small (SS)
Rule 6_2: IF $x_{1,t}^{(m,n)}$ is Positive Medium (PM), THEN ed_6 is Sell Medium (SM)
Rule 6_3: IF $x_{1,t}^{(m,n)}$ is Positive Large (PL), THEN ed_6 is Sell Big (SB)
Rule 6_4: IF $x_{1,t}^{(m,n)}$ is Around Zero (AZ), THEN ed_6 is Neutral (N)

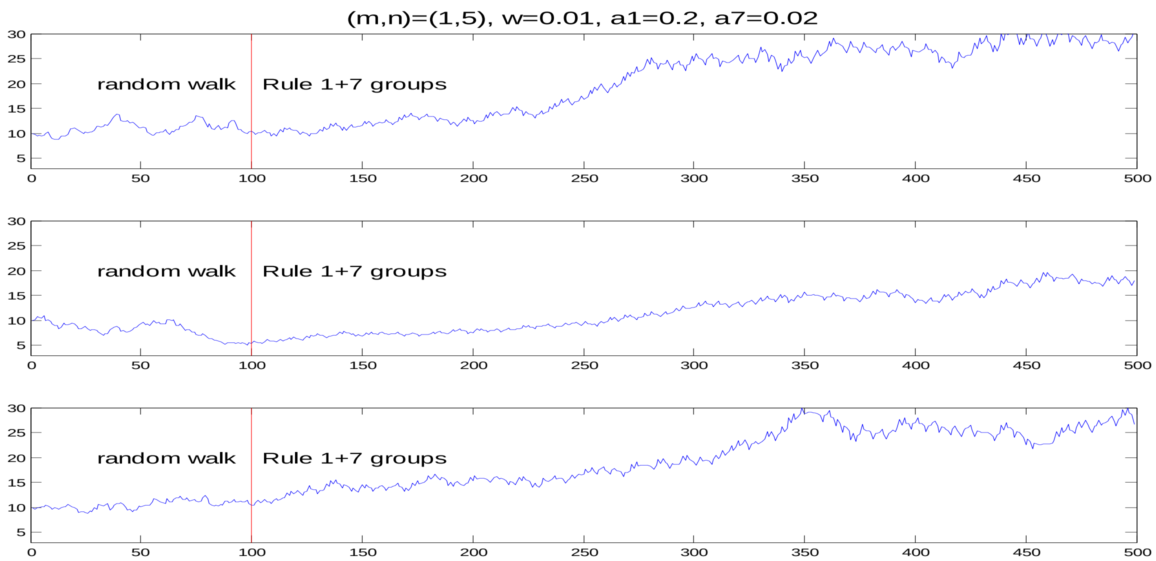
$$ed_6\left(x_{1,t}^{(1,n)}\right) = \frac{-0.1\,\mu_{PS}\left(x_{1,t}^{(1,n)}\right) - 0.2\,\mu_{PM}\left(x_{1,t}^{(1,n)}\right) - 0.4\,\mu_{PL}\left(x_{1,t}^{(1,n)}\right)}{\mu_{PS}\left(x_{1,t}^{(1,n)}\right) + \mu_{PM}\left(x_{1,t}^{(1,n)}\right) + \mu_{PL}\left(x_{1,t}^{(1,n)}\right) + \mu_{AZ}\left(x_{1,t}^{(1,n)}\right)}$$



- Heuristic 7 (Big buyer rule) = Rule-7-Group:

Rule 7_1: IF $x_{1,t}^{(m,n)}$ is Negative Small (NS), THEN ed_7 is Buy Small (BS)
Rule 7_2: IF $x_{1,t}^{(m,n)}$ is Negative Medium (NM), THEN ed_7 is Buy Medium (BM)
Rule 7_3: IF $x_{1,t}^{(m,n)}$ is Negative Large (NL), THEN ed_7 is Buy Big (BB)
Rule 7_4: IF $x_{1,t}^{(m,n)}$ is Around Zero (AZ), THEN ed_7 is Neutral (N)

$$ed_7\left(x_{1,t}^{(1,n)}\right) = \frac{0.1\,\mu_{NS}\left(x_{1,t}^{(1,n)}\right) + 0.2\,\mu_{NM}\left(x_{1,t}^{(1,n)}\right) + 0.4\,\mu_{NL}\left(x_{1,t}^{(1,n)}\right)}{\mu_{NS}\left(x_{1,t}^{(1,n)}\right) + \mu_{NM}\left(x_{1,t}^{(1,n)}\right) + \mu_{NL}\left(x_{1,t}^{(1,n)}\right) + \mu_{AZ}\left(x_{1,t}^{(1,n)}\right)}$$



● Application to Hong Kong Stocks

Basic Idea: Follow the big buyer. But how?

Main problem facing the big buyer: Liquidity is very limited.

Conclusion: "when you're dealing with major institutions, managing large sums of money, you need to tell them that the only time they can really buy in quantity is during the decline, and the only time they can really sell in quantity is in a rally." --- Walter Deemer

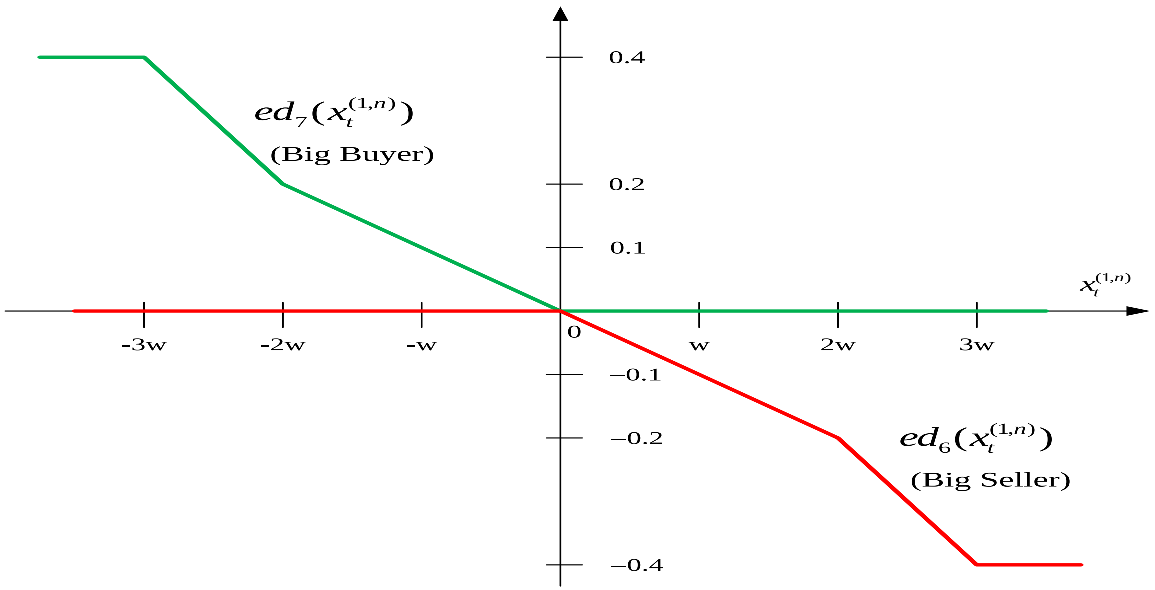
Assumption 1: The stock prices are mainly determined by the actions of the institutional investors; we call them big buyers and big sellers.

Assumption 2: The big buyers (big sellers) use the Big Buyer Heuristic (Big Seller Heuristic) in their real trading.

Assumption 3: For some stocks around some time periods, there is a dominant big buyer trading the stock.

● **Price dynamic model and basic ideas of trading**

$$\ln(p_{t+1}) = \ln(p_t) + a_6(t)\ ed_6\left(x_t^{(1,n)}\right) + a_7(t)\ ed_7\left(x_t^{(1,n)}\right) + \varepsilon(t)$$



■ Positive $a_6(t)$ implies the existence of big sellers;

■ Positive $a_7(t)$ implies the existence of big buyers;

■ $a_7(t) > a_6(t)$ indicates rising mode, $a_7(t) < a_6(t)$ declining mode.

Parameter estimation algorithm

Recursive lease squares with exponential forgetting:

$$\ln(p_{t+1}) = \ln(p_t) + a_6(t)\ ed_6\left(x_t^{(1,n)}\right) + a_7(t)\ ed_7\left(x_t^{(1,n)}\right) + \varepsilon(t)$$

$$r_{t+1} = ed_t^T a_t + \varepsilon(t)$$

$$a_t = \left(a_6(t), a_7(t)\right)^T$$

$$ed_t = \left(ed_6\left(x_t^{(1,n)}\right), ed_7\left(x_t^{(1,n)}\right)\right)^T$$

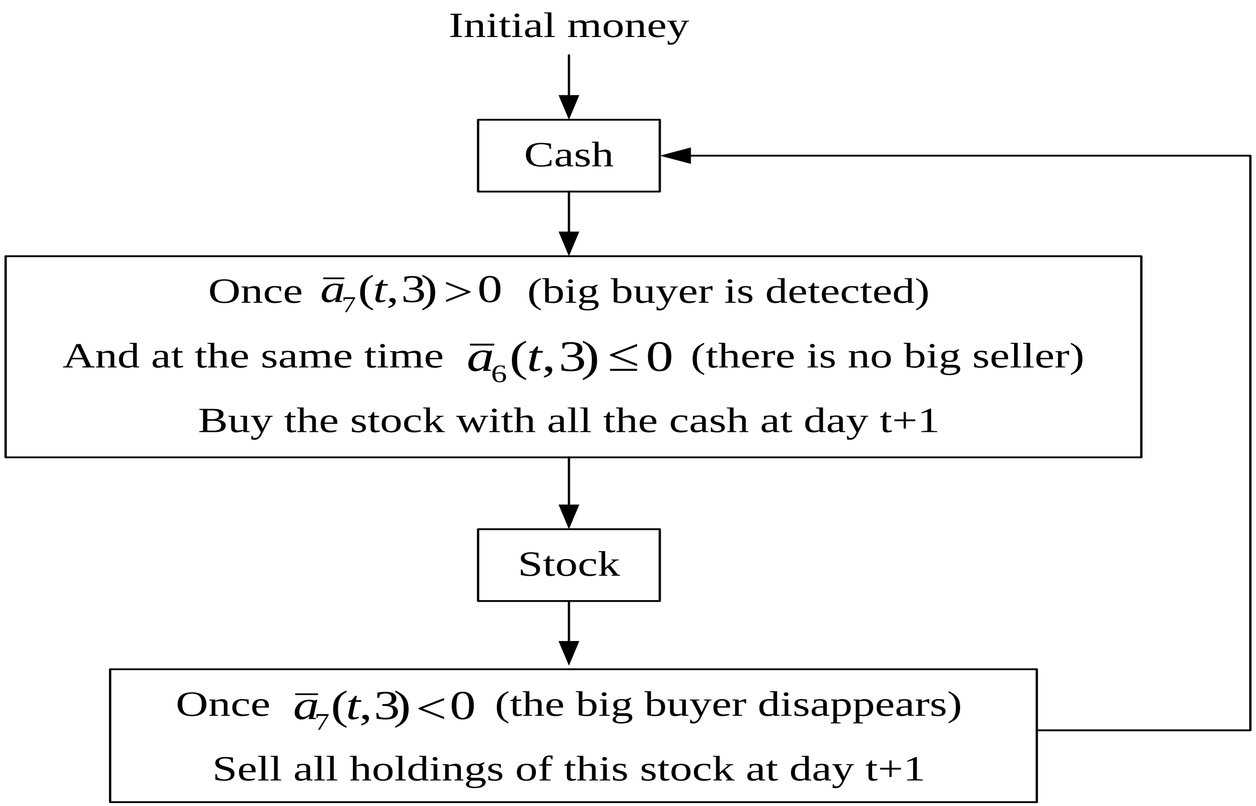
$$\hat{a}_t = \hat{a}_{t-1} + K_t(r_{t+1} - ed_t^T \hat{a}_{t-1})$$

$$K_t = \frac{P_{t-1} ed_t}{(ed_t^T P_{t-1} ed_t + \lambda)}$$

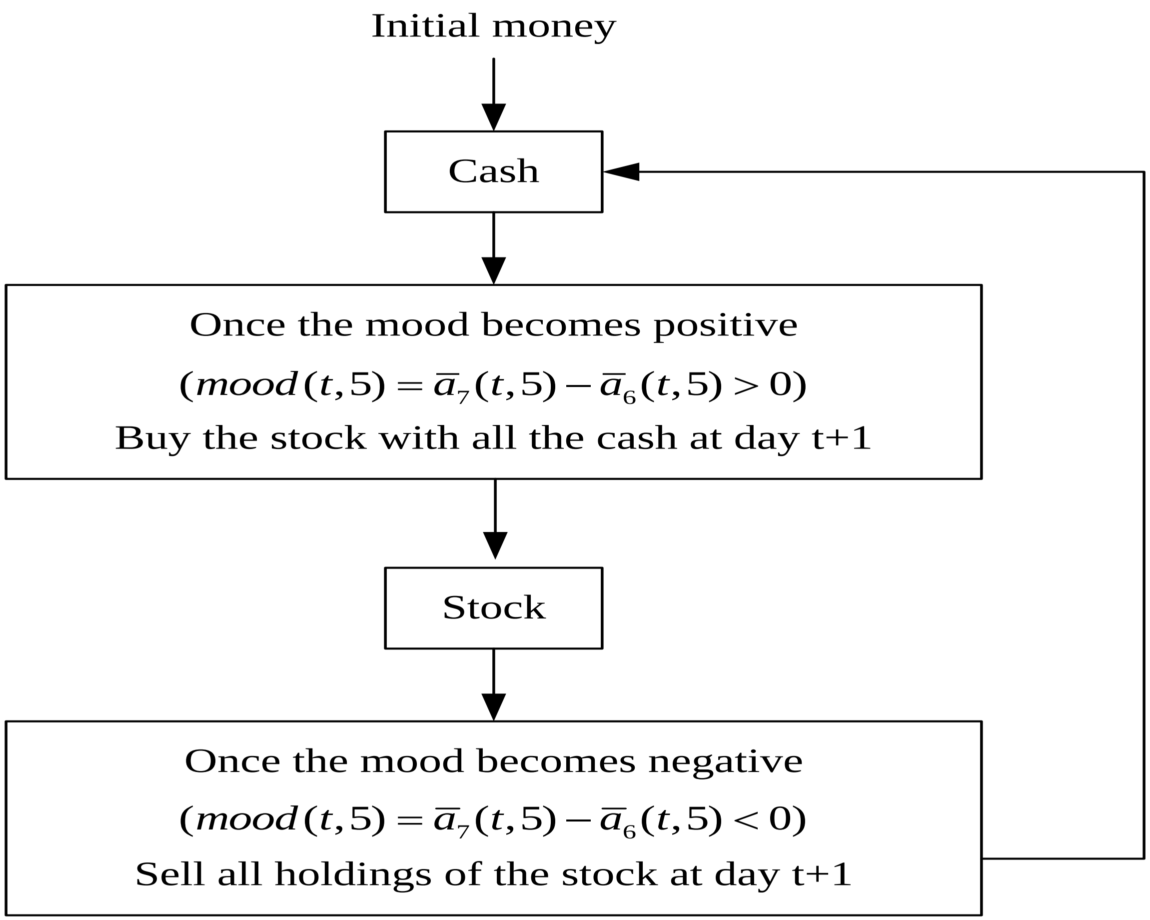
$$P_t = (I - K_t ed_t^T) P_{t-1}/\lambda$$

● **Trading strategies**

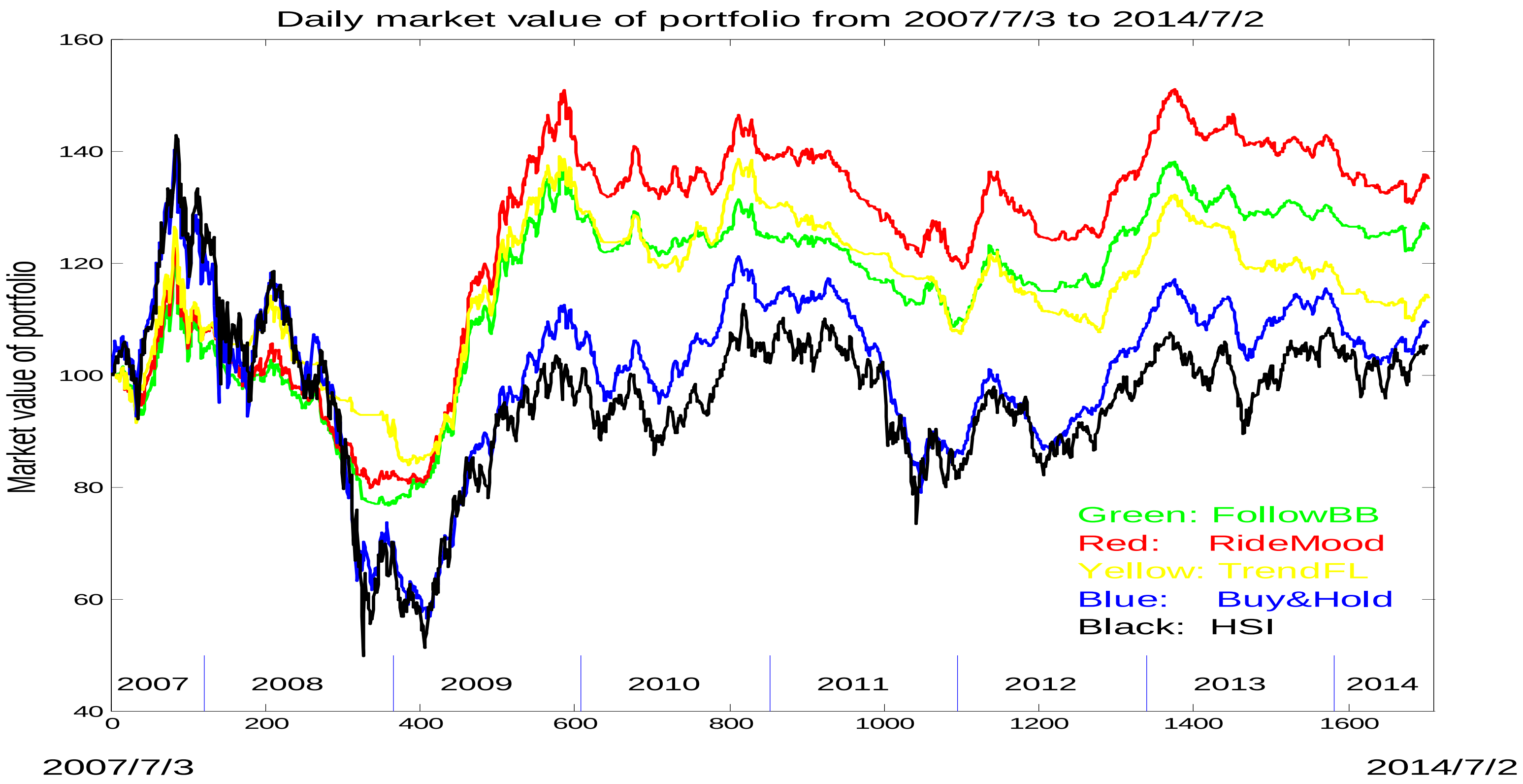
Follow-the-Big-Buyer (FollowBB)



Ride-the-Mood (RideMood)



Performance



- **Support and Resistance Rules**

-- A *resistance point* is the highest peak in the time interval [t-n,t-1]:

$$resi_t^{(n)} = \max_{t-n \le k \le t-1} \{ p_k | p_k > p_{k-1}, p_k > p_{k+1} \}$$

A *support point* is the lowest trough in the interval [t-n,t-1]:

$$supp_t^{(n)} = \min_{t-n \le k \le t-1} \{ p_k | p_k < p_{k-1}, p_k < p_{k+1} \}$$

Heuristic 2: A buy (sell) signal is generated if the current price breaks the resistance (support) point $resi_t^{(n)}$ ($supp_t^{(n)}$) from below (above). The buy (sell) signal is weak if the breakup (breakdown) is small because a small breakup (breakdown) often ends up with a throwback (pullback). When the breakup (breakdown) becomes reasonably large, the trend is more or less confirmed so that a strong buy (sell) is recommended. If the breakup (breakdown) is too large, the stock may be over-bought (over-sold), so a small sell (buy) order should be placed to prepare for a possible reversal.

-- Define:

$$x_{2,t}^{(n)} = \ln\left(p_t \Big/ resi_t^{(n)} \right) \qquad x_{3,t}^{(n)} = \ln\left(p_t \Big/ supp_t^{(n)} \right)$$

Heuristic 2 becomes the Rule-2-Group:

Rule 2_1: IF $x_{2,t}^{(n)}$ is Positive Small (PS), THEN ed_2 is Buy Small (BS)
Rule 2_2: IF $x_{2,t}^{(n)}$ is Positive Medium (PM), THEN ed_2 is Buy Big (BB)
Rule 2_3: IF $x_{2,t}^{(n)}$ is Positive Large (PL), THEN ed_2 is Sell Medium (SM)
Rule 2_4: IF $x_{2,t}^{(n)}$ is Negative Small (NS), THEN ed_2 is Sell Small (SS)
Rule 2_5: IF $x_{2,t}^{(n)}$ is Negative Medium (NM), THEN ed_2 is Sell Big (SB)
Rule 2_6: IF $x_{2,t}^{(n)}$ is Negative Large (NL), THEN ed_2 is Buy Medium (BM)

Combining them into a fuzzy system:

$$ed_2\left(x_{2,t}^{(n)}, x_{3,t}^{(n)}\right) = \frac{\sum_{i=1}^{3} c_i \mu_{A_i}\left(x_{2,t}^{(n)}\right) + \sum_{i=4}^{6} c_i \mu_{A_i}\left(x_{3,t}^{(n)}\right)}{\sum_{i=1}^{3} \mu_{A_i}\left(x_{2,t}^{(n)}\right) + \sum_{i=4}^{6} \mu_{A_i}\left(x_{3,t}^{(n)}\right)}$$

The price dynamical model is

$$\ln(p_{t+1}) = \ln(p_t) + a_2(t) ed_2(x_{2,t}^{(n)}, x_{3,t}^{(n)})$$

-- Simulations (jumps occur when the prices cross the support or resistance lines)