模糊控制

模糊控制第一讲：模糊数学与模糊逻辑基础

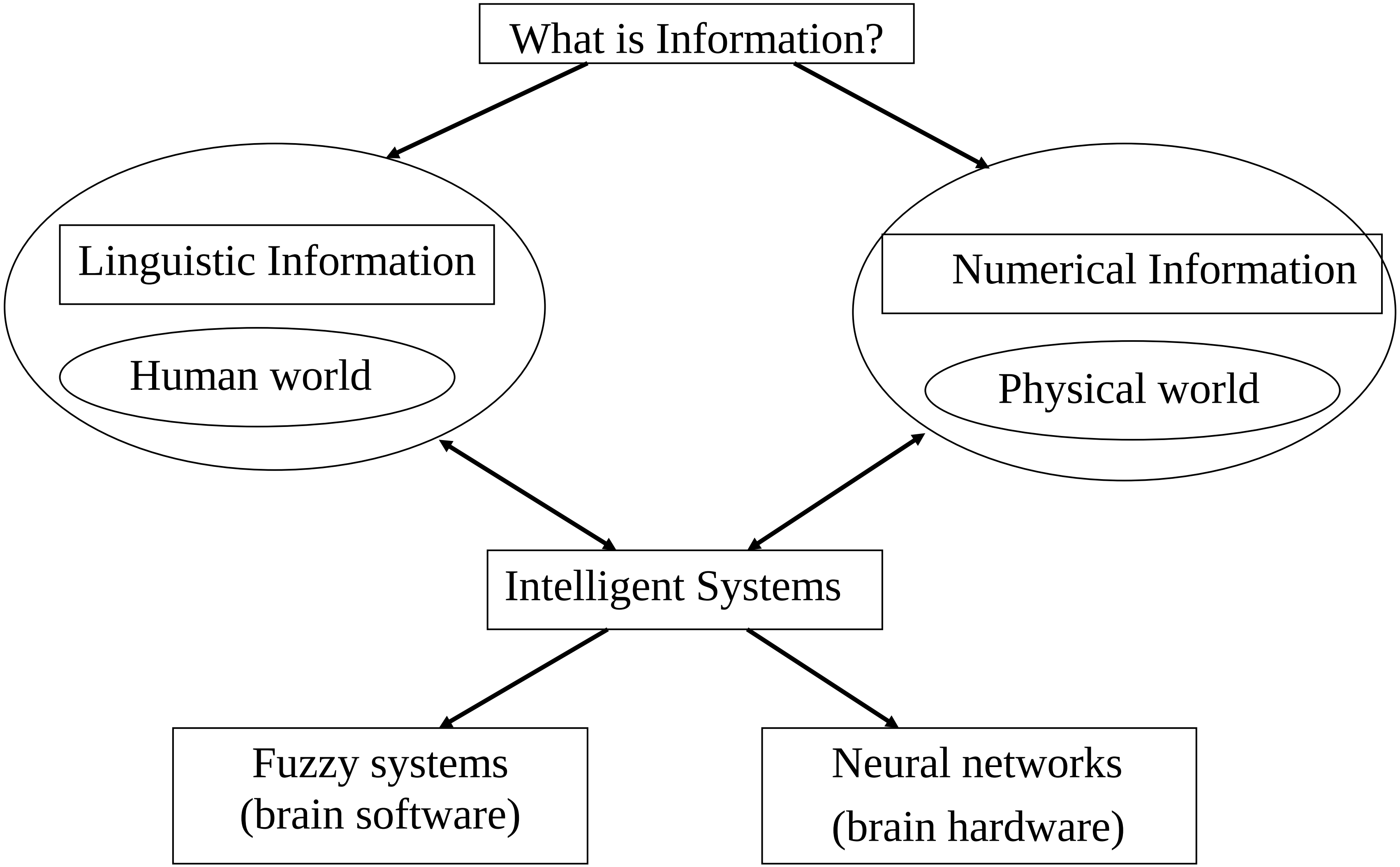
一、　模糊控制简介

二、　模糊集合及其基本运算

三、　模糊集合的广义运算

四、　模糊关系及推广原理

五、　语言变量及模糊规则


What is Information?
Linguistic Information
Human world
Numerical Information
Physical world
Intelligent Systems
Fuzzy systems
(brain software)
Neural networks
(brain hardware)

一、模糊控制简介

• *Why Fuzzy Systems*?

Reason 1: *The principle of Incompatibility* (*Zadeh,1972*)

As the complexity of a system increases, our ability to make precise and yet significant statement about its behaviour diminishes until a threshold is reached beyond which precision and significance became almost mutually exclusive characteristics.

Reason 2:

As we move into the information area, human knowledge becomes increasingly important. We need a theory to formulate human knowledge in a systematic manner and put it into engineering systems, together with other information like mathematical model and sensory measurements.

• *What are Fuzzy Systems*?

Fuzzy systems are *knowledge-based* or *rule-based* systems.

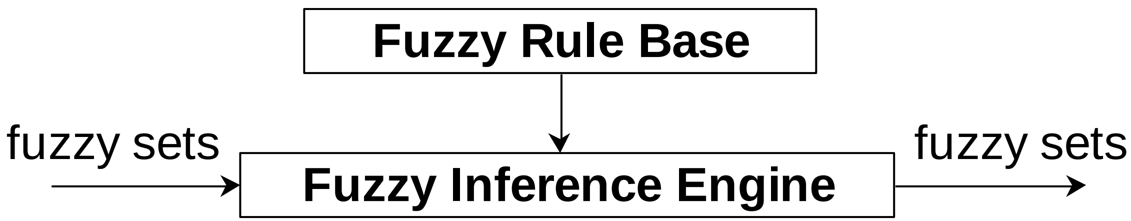
<u>*Example 1*</u>:

A fuzzy system used to control a car may contain the following rules:

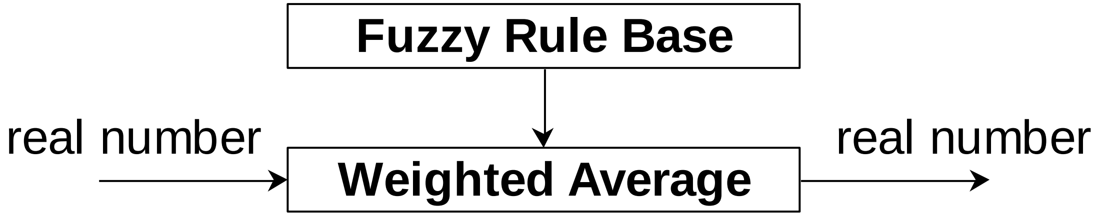
IF *the speed is* ***high***, THEN *apply* ***less*** *force to the accelerator.*
IF *the speed is* ***low***, THEN *...* ***more*** *...*
…

• *Three types of Fuzzy Systems*

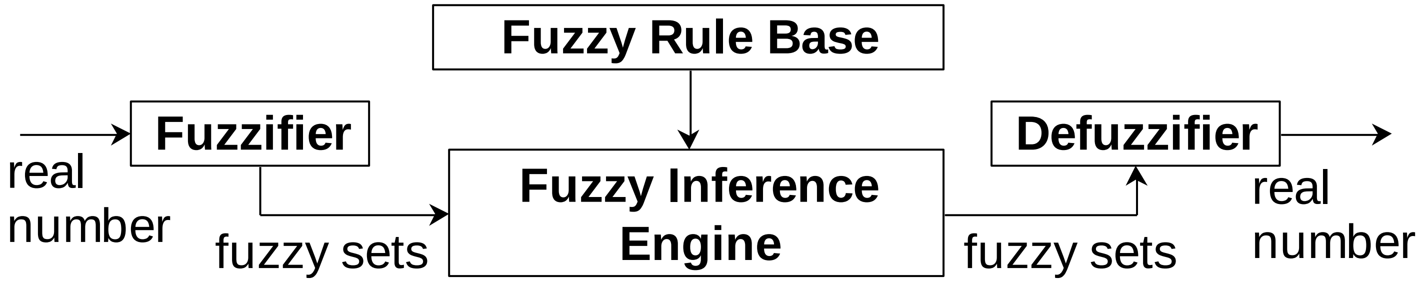
- *Pure fuzzy systems*



- *Takagi-Sugeno-Kang (TSK) fuzzy system*



- *Standard fuzzy systems*



• *What Are the Major Applications of Fuzzy Systems*?

Fuzzy systems are most suitable for problems where:

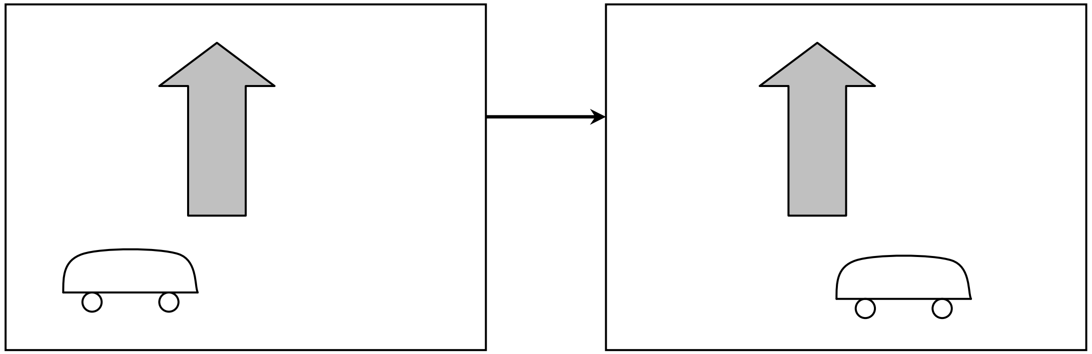
- no precise mathematical model, and
- there are human experts who can provide important information about the system.

Major application fields:

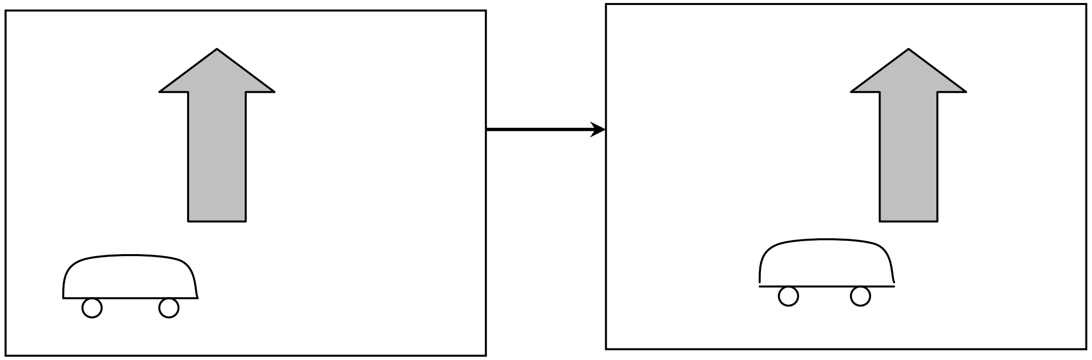
- Industrial process control, including cement kiln, steel plant, waste water treatment process, etc.
- Complex mechanical systems, including unmanned helicopter, subway train, etc.
- Home electronics, including camcorder, washing machine, air conditioner, etc.
- Financial and social systems

Example 2:

Digital Image Stabilizer



The hand is not shaking.



The hand is shaking.

Rule 1:

IF all points in the picture are moving in the same direction, THEN the hand is shaking;

Rule 2:

IF only some points in the picture are moving,
THEN the hand is not shaking

Example 3:

Fuzzy Control of A Subway Train

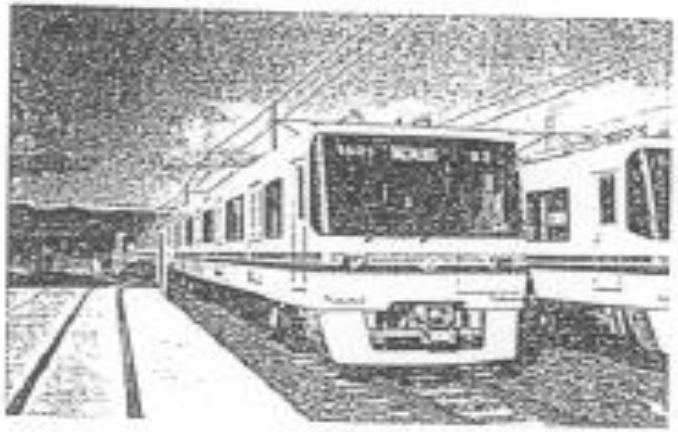


Fig. 3.45 Cars of the Sendai subway system operated using fuzzy control.

For safety:

IF the speed is approaching the limit, THEN select the maximum brake notch.

For riding comfort:

IF the train will stop in the allowed zone, THEN do not change the control notch.

For riding comfort and safety:

IF the train is in the allowed zone, THEN gradually change the control notch from acceleration to braking.

Example 4:

股票价格动态模型

Heuristic 1: A buy (sell) signal is generated if a shorter moving average of the price is crossing a longer moving average of the price from below (above). Usually, the larger the difference between the two moving averages, the stronger the buy (sell) signal.

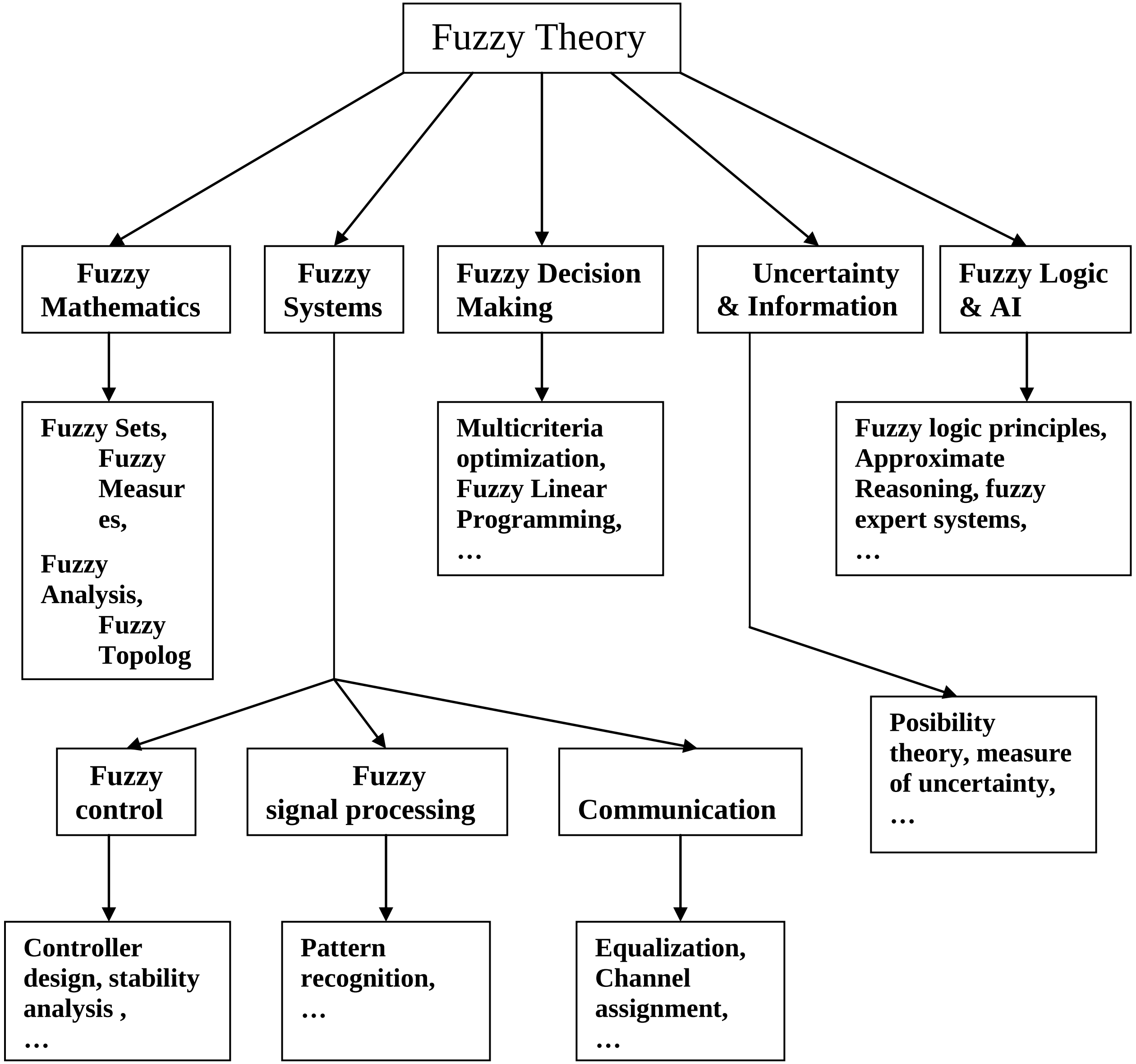
trend following

But, if the difference between the two moving averages is too large, the stock may be over-bought (over-sold), so a small sell (buy) order should be placed to safeguard the investment.

contrarian

What are the price dynamics when Heuristic 1 is used by technical traders?

Major Research Fields in Fuzzy Theory



The History of Fuzzy Systems

- *The 1960s: The Beginning of Fuzzy Theory*
  - Fuzzy set (Zadeh, 1965)
  - Fuzzy algorithms (Zadeh, 1968)
  - Fuzzy decision making (Zadeh, 1969)

- *The 1970s: Theory Continued to Grow and Real Applications Appeared*
  - Fuzzy IF-THEN rules (Zadeh, 1973)
  - Fuzzy control (Mamdani, 1975)
  - Fuzzy control of cement kiln (Holmblad, 1978)

- *The 1980s: Massive Applications Made a Difference*
  - In Japan: fuzzy control of water purification, fuzzy robots, fuzzy self-parking car, fuzzy control of subway systems, …

- *The 1990s: Recognized as One of the Three Major Fields in Computational Intelligence*
    - In 1992: First *IEEE* Fuzzy Conf.
    - In 1993: First *IEEE* Trans. on Fuzzy system.

- *The 2000s to now: Need New Breakthroughs*

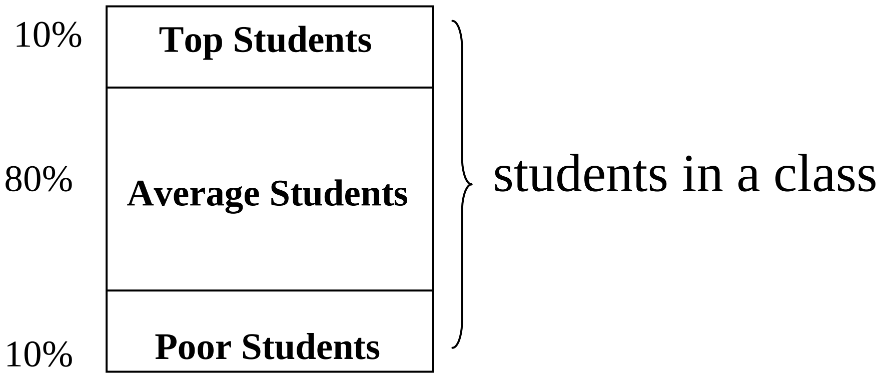
二、 模糊集合及其基本运算

2.1 From Classical Sets to Fuzzy Sets

- *Classical Set*
  - Rule method: $A=\{ x \in U \mid x$ meets some conditions$\}$
  - Membership method:

$$\mu_A(x) = \begin{cases} 1, & if \quad x \in A; \\ 0, & if \quad x \notin A. \end{cases}$$

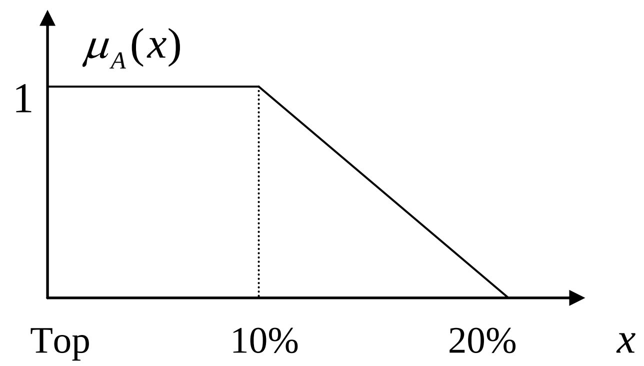
Example: 1



• *Fuzzy Set:*

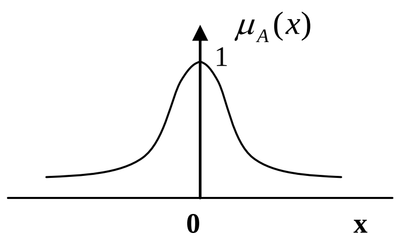
A fuzzy set in a universe of discourse U is characterized by a membership function $\mu_A(x)$ that take values in the interval $[0,1]$.

Example: 2 A ="***top students***"

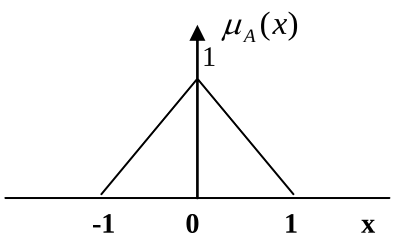


Example: 3 A = "***numbers close to zero***"

$$\mu_A(x) = e^{-x^2}$$



or



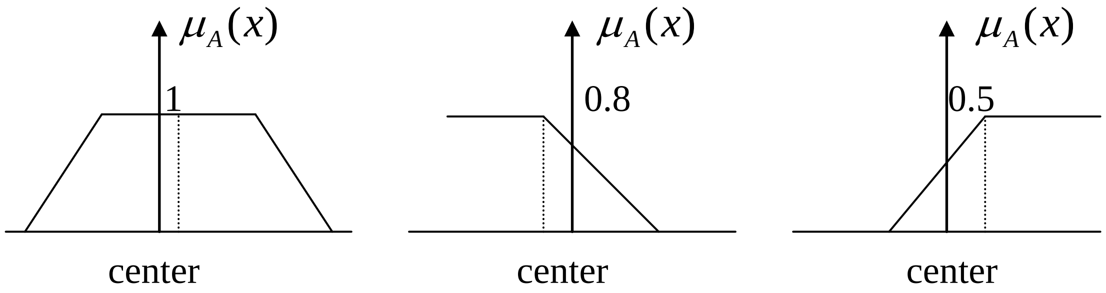
2.2 Basic Concepts Associated with Fuzzy Set

- *Support of a fuzzy set*: $Supp(A) = \{x \in U \mid \mu_A(x) > 0\}$

- *Fuzzy singletion*: A fuzzy set whose support is a single point in U.

- *Center of a fuzzy set*:

The mean of all points in U at which $\mu_A(x)$ achieves its maximum values; if the mean equals infinite, center is defined as follows:



- *Height of a fuzzy set*: The largest membership value attained by any point.

- *Normal fuzzy set*: The height equals one.

- $\alpha - cut$: $A_\alpha = \{x \in U \mid \mu_A(x) \geq \alpha\}$, $0 \leq \alpha \leq 1$.

- *Convex fuzzy set*: A is convex iff its α – cut A_α is a convex set for any $\alpha \in (0,1]$.

LEMMA 2.1

A is a fuzzy set.
A is convex $\Leftrightarrow \mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2)), \quad \forall x_1, x_2 \in R^n, \lambda \in [0,1].$

2.3 Operations on Fuzzy Sets

- *Equality* : $A = B$ *iff* $\mu_A(x) = \mu_B(x)$, for all $x \in U$.
- *Containment:* $A \subset B$ *iff* $\mu_A(x) \leq \mu_B(x)$, for all $x \in U$.
- *Complement:* $\mu_{\overline{A}}(x) = 1 - \mu_A(x)$.
- *Union:* $\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$.
- *Intersection:* $\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$.

LEMMA 2.2: The De Morgan's Laws are true for fuzzy sets:

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$
$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

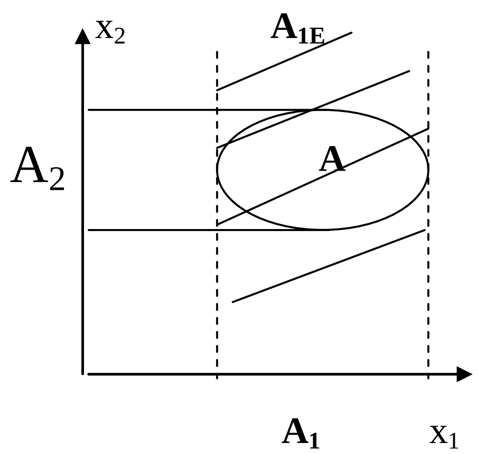
● *Projection of a fuzzy set*

Let A be a fuzzy set in $U = R^2$, the projection of A on x_1 is a fuzzy set A_1 with

$$\mu_{A_1}(x_1) = \max_{x_2 \in R}[\mu_A(x_1, x_2)].$$

Similarly,

$$\mu_{A_2}(x_2) = \max_{x_1 \in R}[\mu_A(x_1, x_2)].$$



三、模糊集合的广义运算

Basic Operations:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x).$$
$$\mu_{A\cup B}(x) = \max[\mu_A(x), \mu_B(x)].$$
$$\mu_{A\cap B}(x) = \min[\mu_A(x), \mu_B(x)].$$

3.1 Fuzzy Complement

Let $c(\mu_A(x)) = \mu_{\bar{A}}(x)$, c is a ***fuzzy complement*** if:

Axiom c1. $c(0)=1$ and $c(1)=0$ (*boundary condition*)

Axiom c2. For all $a,b \in (0,1]$, $a<b$ implies $c(a) > c(b)$. (*nonincreasing condition*)

Example 1：*Sugeno class* of fuzzy complements

$$C_\lambda(\alpha) = \frac{1-\alpha}{1+\lambda\alpha}, \qquad \lambda \in (-1, \infty).$$

Example 2：*Yager class* of fuzzy complements

$$C_w(\alpha) = \left(1-\alpha^w\right)^{1/w}, \qquad w \in (0, \infty)$$

3.2 Fuzzy union—The S-Norms

Let $S[\mu_A(x), \mu_B(x)] = \mu_{A\cup B}(x)$. S is a fuzzy union or S-norm if:

Axiom s1. $s(1,1)=1$, $s(0,a)=s(a,0)=a$. (*boundary condition*)
Axiom s2. $s(a,b)=s(b,a)$. (*commutative condition*)
Axiom s3. If $a \le a', b \le b'$, then $s(a,b) \le s(a',b')$. (*nonincreasing condition*)
Axiom s4. $s(s(a,b),c)=s(a,s(b,c))$. (*associative condition*)

Examples:

• *Dombi class*

$$s_\lambda(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{-1/\lambda}}, \quad \lambda \in (0,\infty).$$

• *Dubios-Prade class*

$$s_\alpha(a,b) = \frac{a+b-ab-\min(a,b,1-\alpha)}{\max(1-a,1-b,\alpha)}, \quad \alpha \in [0,1].$$

- *Yager class*

$$s_w(a,b) = \min\left[1, \left(a^w + b^w\right)^{1/w}\right],\ w \in (0,\infty).$$

- *Drastic Sum*

$$s_{ds}(a,b) = \begin{cases} a, & if\ b = 0; \\ b, & if\ a = 0; \\ 1, & otherwise. \end{cases}$$

- *Algebraic Sum* $\quad s_{as}(a,b) = a + b - ab.$

- *Maximum* $\quad s_{\max}(a,b) = \max(a,b).$

Theorem 3.1 $\quad \max(a,b) \le s(a,b) \le s_{ds}(a,b).$

3.3 Fuzzy Intersection – The T-Norms

Let $t[\mu_A(x), \mu_B(x)] = \mu_{A\cap B}(x)$, t is a ***fuzzy intersection*** or t-norm if:

Axiom s1. $t(0,0) = 0$, $t(1,a) = t(a,1) = a$. (*boundary condition*)

Axiom s2. $t(a,b) = t(b,a)$. (commutative condition)

Axiom s3. If $a \le a', b \le b'$, then $t(a,b) \le t(a',b')$. (*nonincreasing condition*)

Axiom s4. $t(t(a,b),c) = t(a,t(b,c))$. (*associative condition*)

Examples:

• *Dombi class*

$$t_\lambda(a,b) = \frac{1}{1 + \left[\left(\frac{1}{a} - 1\right)^\lambda + \left(\frac{1}{b} - 1\right)^\lambda\right]^{-1/\lambda}}, \quad \lambda \in (0,\infty).$$

• *Dubios-Prade class*

$$t_\alpha(a,b) = \frac{ab}{\max(a,b,\alpha)}, \quad \alpha \in [0,1].$$

• *Yager class*

$$t_w(a,b) = 1 - \min\left[1, \left((1-a)^w + (1-b)^w\right)^{1/w}\right],\ w \in (0,\infty).$$

• *Drastic product*

$$t_{dp}(a,b) = \begin{cases} a, & if\ b = 1, \\ b, & if\ a = 1, \\ 0, & otherwise. \end{cases}$$

• *Algebraic product*

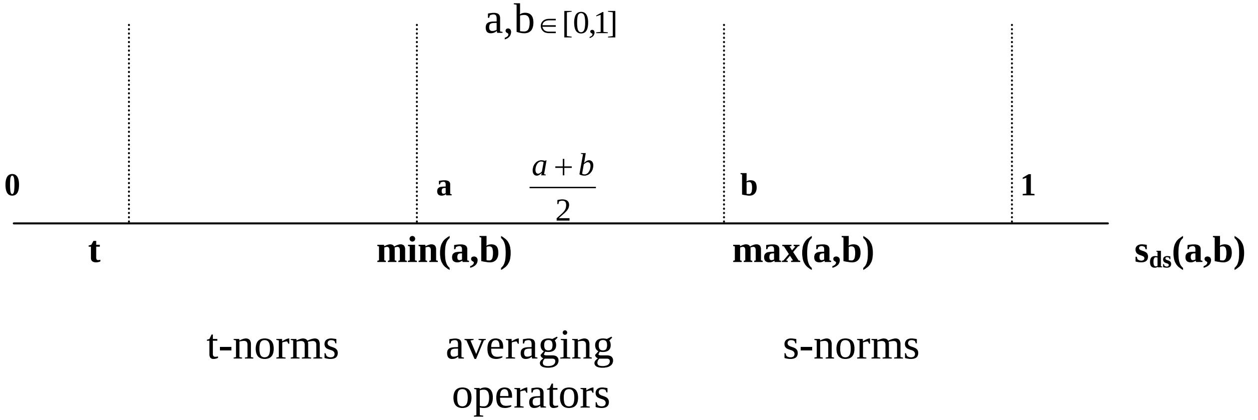
$$t_{ap}(a,b) = ab.$$

• *Minimum*

$$t_{\min}(a,b) = \min(a,b).$$

Theorem 3.2 $$t_{dp}(a,b) \le t(a,b) \le \min(a,b).$$

3.4 Averaging Operators



- *Max-min averages*:

$$V_\lambda(a,b) = \lambda \min(a,b) + (1-\lambda)\max(a,b), \quad \lambda \in [0,1].$$

- *Generalized averages*:

$$V_\alpha(a,b) = \left(\frac{a^\alpha + b^\alpha}{2}\right)^{1/\alpha}, \quad \alpha \in R, (\alpha \neq 0).$$

- *“Fuzzy and’’*:

$$V_p(a,b) = p\min(a,b) + (1-p)\frac{a+b}{2}, \quad p \in [0,1].$$

- *“Fuzzy or’’*:

$$V_\gamma(a,b) = \gamma \max(a,b) + (1-\gamma)\frac{a+b}{2}, \quad \gamma \in [0,1].$$

四、模糊关系及推广原理

4.1 From Classical Relations to Fuzzy Relations

• *Classical Relation*:

Let $U_1, U_2, \ldots, U_n$ be universe of discourse and $U_1 \times U_2 \times \cdots \times U_n$ be the ***cartesian product***, i.e.,

$$U_1 \times U_2 \times \cdots \times U_n = \{(u_1, \ldots u_n) \,|\, u_1 \in U_1, \ldots u_n \in U_n\}$$

then a ***relation*** among U_1, U_2, …, U_n is a subset of the cartesian product $U_1 \times U_2 \times \cdots \times U_n$, i.e.,

$$Q(U_1, U_2, \cdots, U_n) \subset U_1 \times U_2 \times \cdots \times U_n.$$

<u>*Example*:</u> $U = \{1,2,3\}$, $V = \{2,3,4\}$. $Q(U,V) = \{(2,2),(3,2),(3,3)\}$

In matrix form:

$$\begin{array}{cc} & V \\ & \begin{array}{cccc} & 2 & 3 & 4 \\ 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 \end{array} \end{array} \quad U \qquad U \times V = \{(1,2),(1,3),(1,4),(2,2),\cdots\}.$$

• *Fuzzy Relation*:

A ***fuzzy relation*** is a fuzzy set defined in $U_1 \times U_2 \times \cdots \times U_n$.

<u>*Example*</u>:

U ={*Hong Kong, Tokyo*}, V ={*New York, Hong Kong*}, Q=“*very far*”.

		V	
		NY	*HK*
U	*HK*	1	0
	Tokyo	0.9	0.2

• *Projections*:

Let Q be a fuzzy relation in $U_1 \times U_2 \times \cdots \times U_n$ and $\{i_1, \cdots, i_k\}$ be a subsequence of $\{1,2,\ldots,n\}$. Then the projection of Q on $U_{1k} \times \cdots \times U_{ik}$ is a fuzzy relation Q_P in $U_{1k} \times \cdots \times U_{ik}$ with

$$\mu_{Q_p}\left(u_{i_1},\ldots,u_{i_k}\right) = \max_{u_{j_1} \in U_{j_1},\ldots,u_{j_{(n-k)}} \in U_{j(n-k)}} \mu_Q\left(u_1,\ldots,u_n\right)$$

where $\left\{u_{j_1},\ldots,u_{j_{(n-k)}}\right\} \oplus \left\{u_{i_1},\ldots,u_{i_k}\right\} = \left\{u_1,\ldots,u_n\right\}$.

• *Cylindric extension*:

Let Q_P be a fuzzy relation in $U_{1k} \times \cdots \times U_{ik}$ and $\{i_1, \cdots, i_k\}$ is a subsequence of $\{1,2,\ldots,n\}$. Then the ***cylindric extension*** of Q_P to $U_1 \times U_2 \times \cdots \times U_n$ is a fuzzy relation Q_{PE} in $U_1 \times U_2 \times \cdots \times U_n$ with

$$\mu_{Q_{pE}}(u_1,\ldots,u_n) = \mu_{Q_p}\left(u_{i_1},\ldots,u_{i_k}\right).$$

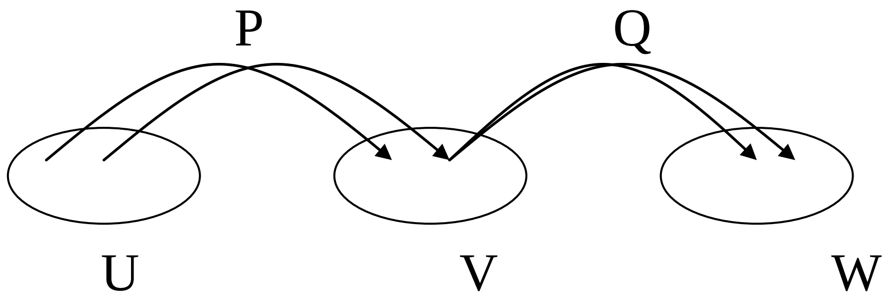
• *Cartesian Product of Fuzzy Sets*

Let A_1, $A_2,\ldots$, A_n be fuzzy sets in U_1, U_2, …, U_n, respectively. The ***cartesian product of*** A_1**,** A_2, **…**, A_n is a fuzzy relation in $U_1 \times U_2 \times \cdots \times U_n$ with

$$\mu_{\mathrm{A}_1 \times \ldots \times \mathrm{A}_n}(u_1,\ldots,u_n) = \mu_{\mathrm{A}_1}(u_1) * \cdots * \mu_{\mathrm{A}_1}(u_1)$$

where $*$ denotes any *t*-norm.

4.2 Composition of Fuzzy Relations



For nonfuzzy P and Q, the ***composition*** $P \circ Q$ is a relation in $U \times W$ such that $(x, z) \in P \circ Q$ iff $\exists y \in V$ s.t. $(x, y) \in P$, $(y, z) \in Q$.

Lemma 4.2

$P \circ Q$ is a composition of $P(U, V)$ and $Q(V, W)$ iff

$$\mu_{P \circ Q}(x, z) = \sup_{y \in V} t[\mu_P(x, y), \mu_Q(y, z)] \qquad (4.28)$$

for any $(x, z) \in U \times W$.

Let $P(U, V)$ and $Q(V, W)$ be fuzzy relations. Their ***composition*** $P \circ Q$ is a fuzzy relation in $U \times W$ with

$$\mu_{P \circ Q}(x, z) = \max_{y \in V} t[\mu_P(x, y), \mu_Q(y, z)].$$

• *max-min composition:*

$$\mu_{P\circ Q}(x,z) = \max_{y\in V} \min[\mu_P(x,y), \mu_Q(y,z)]$$

• *max-product composition:*

$$\mu_{P\circ Q}(x,z) = \max_{y\in V} [\mu_P(x,y)\mu_Q(y,z)]$$

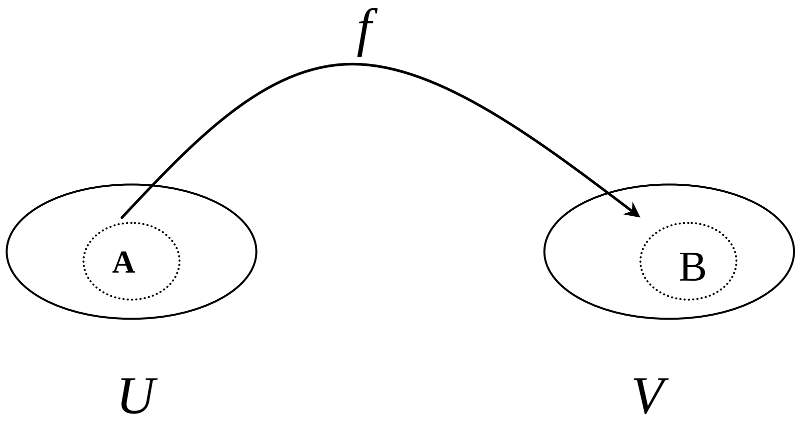
<u>*Example*:</u>

Let

$$P:\quad U \begin{array}{c|cc} & \multicolumn{2}{c}{V} \\ & v_1 & v_2 \\ \hline u_1 & 0.3 & 0.9 \\ u_2 & 1 & 0 \\ u_3 & 0.9 & 0.2 \end{array} \qquad Q:\quad V \begin{array}{c|cc} & \multicolumn{2}{c}{W} \\ & w_1 & w_2 \\ \hline v_1 & 0.8 & 0.1 \\ v_2 & 0.1 & 0.9 \end{array}$$

Compute $P \circ Q$ using max-min and max-product compositions.

4.3 The Extension Principle



Given $f: U \to V$ and fuzzy set A in U, how to determine $B=f(A)$?

If f is one-to-one, then

$$\mu_B(y) = \mu_A(f^{-1}(y)), \qquad y \in V.$$

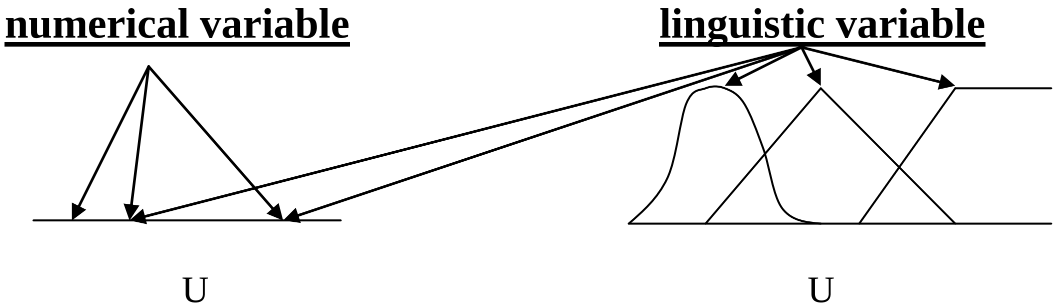
If f is not one-to-one, then

$$\mu_B(y) = \max_{x \in f^{-1}(y)} \mu_A(x), \qquad y \in V$$

where $f^{-1}(y)$ is the set of all points x in U with $f(x) = y$. This is the ***extension principle***.

五、语言变量及模糊规则

5.1 From Numerical Variables to Linguistic Variables



If a variable can take words as its values, it is a linguistic variable, where the words are represented by fuzzy sets.

Example: x = speed of a car ， x $\in$ {slow, medium, fast}, x $\in$ $[0, V_{max}]$

5.2 Linguistic Hedges

Hedges: “very”, “more or less”, $\mu_{veryA}(x) = [\mu_A(x)]^2$ $\mu_{\text{more or less}A}(x) = [\mu_A(x)]^{1/2}$

5.3 Fuzzy IF-THEN Rules

- **Fuzzy Propositions**
  - ***Atomic fuzzy proposition***:

    x is A $\leftrightarrow \mu_A(x)$, where x: linguistic variable, A: fuzzy set
  - ***Compound fuzzy proposition***:

    Combination of atomatic fuzzy propositions with "and","or","not".

    x is S and x is not M or x is F

    x is F and y is L.
    - *"and" = fuzzy intersection*

$$\underbrace{\underline{\text{x is A}}}_{\mu_A}\ \underbrace{\underline{\text{and}}}_{t}\ \underbrace{\underline{\text{y is B}}}_{\mu_B} \leftrightarrow \mu_{A\cap B}(x,y) = t[\mu_A(x), \mu_B(y)]$$

- *"or" = fuzzy union*

x is A or y is B $\leftrightarrow$ $\mu_{A\cup B}(x,y) = s[\mu_A(x), \mu_B(y)]$

- *"not" = fuzzy complement*

x is not A$\leftrightarrow$ $\mu_{\bar{A}}(x) = c[\mu_A(x)]$

• Fuzzy IF-THEN Rules

IF <fuzzy proposition>, THEN <fuzzy proposition>

In classical logic

p	q	$p \to q$	$\bar{p} \vee q$	$(p \wedge q) \vee \bar{p}$
T	T	T	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$$p \to q \equiv \bar{p} \vee q \equiv \bar{p} \vee q$$

Consider **IF <FP1>, THEN <FP2> = Q**

where FP1 is a fuzzy proposition (relation) in $U = U_1 \times \cdots \times U_n$ and FP2 is a fuzzy proposition in $V = V_1 \times \cdots \times V_n$, so Q is a fuzzy relation in $U \times V$.

- *Dienes-Rescher Implication*

$$\mu_{Q_D}(x,y) = \max[1-\mu_{FP_1}(x), \mu_{FP_2}(y)]$$

- *Lukasiewicz Implication*

$$\mu_{Q_L}(x,y) = \min[1-\mu_{FP_1}(x) + \mu_{FP_2}(y)]$$

- *Zadeh Implication*

$$\mu_{Qz}(x,y) = \max[\min(\mu_{FP_1}(x), \mu_{FP_2}(y)), 1-\mu_{FP_1}(x)]$$

Lemma 5.1: For all $(x,y) \in U \times V$, it is true that

$$\mu_{Qz}(x,y) \leq \mu_{Q_D}(x,y) \leq \mu_{Q_L}(x,y)$$

IF <FP1>, THEN <FP2>

$$\Updownarrow$$

IF <FP1>, THEN <FP2> ELSE <Nothing>

$$p \to q \equiv \bar{p} \vee q$$

- *Mamdani Implication*

$$\mu_{Q_M}(x, y) = \min[\mu_{FP_1}(x), \mu_{FP_2}(y)]$$

or

$$\mu_{Q_M}(x, y) = \mu_{FP_1}(x)\mu_{FP_2}(y)$$

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主要研究方向：资产价格动态模型，市场微观结构，交易策略，模糊系统，舆情网络。在以下四个方面为模糊系统及模糊控制领域作出开拓性贡献：

1. 提出第一个由数据产生语言规则的方法:Wang-Mendel(WM)方法，将语言信息和数据信息统一在同一个数学框架之下，从而促成并引领**模糊神经网络**领域的形成与发展。作为模糊神经网络领域的经典算法，WM 方法是后续算法性能比较的标杆，在广泛领域取得成功应用。WM 方法的原始论文（L.X. Wang and J.M. Mendel, “Generating fuzzy rules by learning from examples,” IEEE Trans. on Systems, Man, and Cybern., Vol.22, pp.1414-1427, 1992）在 Google Scholar 上单篇他引 2756 次（至 2016 年 11 月）。

2. 通过引入模糊基函数，将由语言规则构成的模糊系统转化成数学表达式，进而利用泛函分析的经典定理证明模糊系统是万能逼近器，开创**模糊逼近**领域。二十年来模糊逼近领域得到了深入的发展，为模糊系统在广泛领域内的应用提供坚实的理论支撑。该项成果的原始论文（L.X. Wang and

J.M. Mendel, “Fuzzy basis functions, universal approximation, and orthogonal least squares learning,” IEEE Trans. on Neural Networks, Vol.3, pp.807-814, 1992）在 Google Scholar 上单篇他引 **2107** 次 （至 2016 年 11 月）。

3. 设计出第一个确保稳定性的**自适应模糊控制**器，开启模糊控制从经验学科向严格理论升级的大门，引领模糊控制理论二十年来蓬勃的发展。该项成果的原始论文（L.X. Wang, “Stable adaptive fuzzy control of nonlinear systems,” IEEE Trans. on Fuzzy Systems, Vol.1, pp.146-155, 1993）在 Google Scholar 上单篇他引 **1487** 次（至 2016 年 11 月）；而更加全面深入总结该项成果的研究专著（L.X.Wang, *Adaptive Fuzzy Systems and Control: Design and Stability Analysis*, Prentice-Hall: Englewood Cliffs, NJ, 1994）在 Google Scholar 上单篇他引 **4143** 次（至 2016 年 11 月）。

4. 出版模糊系统与模糊控制领域的经典教材（L.X. Wang, A Course in Fuzzy Systems and Control, Prentice-Hall: Englewood Cliffs, NJ, 1997），引领一代又一代学者在这个领域成长、发展、走向成功，为领域的进步作出基础性贡献。该书在 Google Scholar 上单篇他引 **3734** 次（至 2016 年 11 月）。

近年来，从计算智能视角研究金融系统与社交网络，开创 “投机动态系统理论”和“模糊舆情网络理论”。相关论文参见科学网博文：《王飞跃 PK 王立新：14 年前的故事》：

http://blog.sciencenet.cn/blog-2999994-951407.html 的论文链接。