模糊控制第四讲：

模糊自适应控制及模糊舆情网络

一、 间接模糊自适应控制

二、 直接模糊自适应控制

三、 鲁棒模糊自适应控制

四、 模糊舆情网络及其在金融预测中的应用

一、间接模糊自适应控制

考虑非线性系统

$$\boldsymbol{x}^{(n)} = \boldsymbol{f}(\vec{\boldsymbol{x}}) + \boldsymbol{u}$$

其中 $\vec{\boldsymbol{x}} = \left(\boldsymbol{x}, \dot{\boldsymbol{x}}, \dots, \boldsymbol{x}^{(n-1)}\right)^{T}$ 为状态向量，$\boldsymbol{u}$ 为控制，而非线性函数 $\boldsymbol{f}(\vec{\boldsymbol{x}})$ 未知。我们的目的是设计控制器

$$\boldsymbol{u} = \boldsymbol{u}(\vec{\boldsymbol{x}}|\boldsymbol{\theta})$$

使得 $\boldsymbol{x} \to \boldsymbol{x}_{m}$，其中 $\boldsymbol{x}_{m}$ 为任意给定的跟踪目标，$\boldsymbol{\theta}$ 为控制器参数向量。

如果 $\boldsymbol{f}(\vec{\boldsymbol{x}})$ 已知，则控制器

$$\boldsymbol{u} = -\boldsymbol{f}(\vec{\boldsymbol{x}}) + \boldsymbol{x}_{m}^{(n)} + \boldsymbol{k}_{1}\boldsymbol{e}^{(n-1)} + \cdots + \boldsymbol{k}_{n}\boldsymbol{e}$$

其中 $\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_{m}$ 为跟踪误差，使得

$$\boldsymbol{e}^{(n)} + \boldsymbol{k}_{1}\boldsymbol{e}^{(n-1)} + \cdots + \boldsymbol{k}_{n}\boldsymbol{e} = \boldsymbol{0}$$

我们可以设计 $\vec{\boldsymbol{k}} = \left(\boldsymbol{k}_{1}, \dots, \boldsymbol{k}_{n}\right)^{\mathrm{T}}$ 使得 $\boldsymbol{e} \to \boldsymbol{0}$。

由于 $\boldsymbol{f}(\vec{\boldsymbol{x}})$ 未知，我们用模糊系统 $\hat{\boldsymbol{f}}(\vec{\boldsymbol{x}}|\boldsymbol{\theta})$ 代替 $\boldsymbol{f}(\vec{\boldsymbol{x}})$，得到控制器

$$\boldsymbol{u}(\vec{\boldsymbol{x}}|\boldsymbol{\theta}) = -\hat{\boldsymbol{f}}(\vec{\boldsymbol{x}}|\boldsymbol{\theta}) + \boldsymbol{x}_{m}^{(n)} + \vec{\boldsymbol{k}}^{T}\vec{\boldsymbol{e}}$$

其中 $\vec{\boldsymbol{k}} = (\boldsymbol{k_1}, \dots, \boldsymbol{k_n})^{\mathbf{T}}$，$\vec{\boldsymbol{e}} = \left(\boldsymbol{e}^{(\boldsymbol{n-1})}, \dots, \boldsymbol{e}\right)^{\mathbf{T}}$。取模糊系统为

$$\hat{f}(\vec{x}) = \frac{\sum_{l=1}^{M} \bar{y}^l [\prod_{i=1}^{n} \mu_{A_i^l}(x_i)]}{\sum_{l=1}^{M} [\prod_{i=1}^{n} \mu_{A_i^l}(x_i)]}$$

Fix $\mu_{A_i^l}(x_i)$, view $\bar{y}^l$ as parameters $\boldsymbol{\theta}$. Let

$$b^l(\vec{x}) = \frac{\prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum_{l=1}^{M} [\prod_{i=1}^{n} \mu_{A_i^l}(x_i)]} \qquad \theta^l = \bar{y}^l$$

$b(x) = [b^1(x), b^2(x), \cdots, b^M(x)]^T$, $\quad \theta = [\bar{y}^1, \bar{y}^2, \cdots, \bar{y}^M]^T$

then

$$\hat{f}(\vec{x}|\theta) = b^T(\vec{x})\theta$$

其中 $\boldsymbol{b}(\vec{\boldsymbol{x}})$ 为已知模糊基函数，我们的任务是：如何确定模糊系统参数 $\boldsymbol{\theta}$ 的动态变化律使得 $\boldsymbol{e} \to \mathbf{0}$。

将控制器

$$\boldsymbol{u}(\vec{\boldsymbol{x}}|\boldsymbol{\theta}) = -\boldsymbol{b}^{\boldsymbol{T}}(\vec{\boldsymbol{x}})\boldsymbol{\theta} + \boldsymbol{x}_{\boldsymbol{m}}^{(\boldsymbol{n})} + \vec{\boldsymbol{k}}^{\boldsymbol{T}}\vec{\boldsymbol{e}}$$

带入系统方程，得

$$\boldsymbol{x}^{(\boldsymbol{n})} = \boldsymbol{f}(\vec{\boldsymbol{x}}) - \boldsymbol{b}^{\boldsymbol{T}}(\vec{\boldsymbol{x}})\boldsymbol{\theta} + \boldsymbol{x}_{\boldsymbol{m}}^{(\boldsymbol{n})} + \vec{\boldsymbol{k}}^{\boldsymbol{T}}\vec{\boldsymbol{e}}$$

用模糊系统 $\hat{f}(\vec{x}|\theta) = b^T(\vec{x})\theta$ 逼近未知函数 $\boldsymbol{f}(\vec{\boldsymbol{x}})$，设其最优参数为

$$\boldsymbol{\theta}^* = \mathbf{arg\,min_\theta}[\boldsymbol{sup}_{\vec{\boldsymbol{x}}\in\boldsymbol{R}^{\boldsymbol{n}}}(\boldsymbol{f}(\vec{\boldsymbol{x}}) - \boldsymbol{b}^{\boldsymbol{T}}(\vec{\boldsymbol{x}})\boldsymbol{\theta})]$$

则

$$\boldsymbol{f}(\vec{\boldsymbol{x}}) = \boldsymbol{b}^{\boldsymbol{T}}(\vec{\boldsymbol{x}})\boldsymbol{\theta}^* + \boldsymbol{w}$$

得

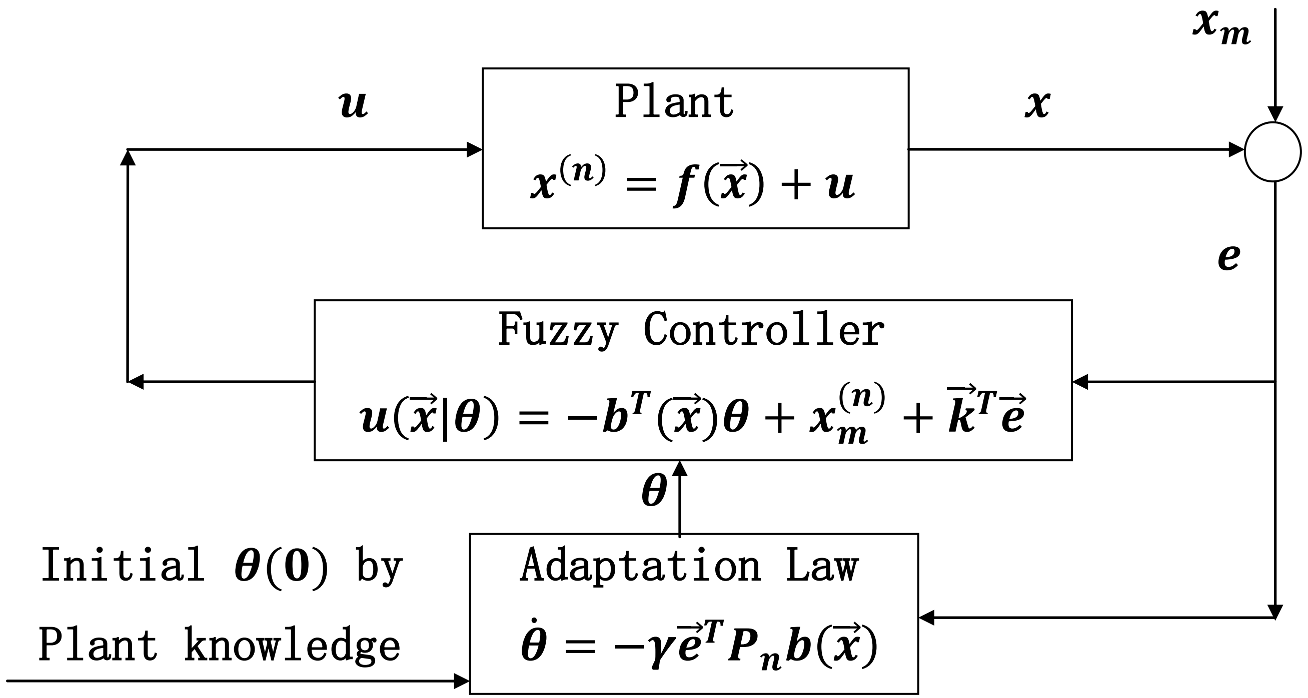
$$\boldsymbol{x}^{(\boldsymbol{n})} = \boldsymbol{b}^{\boldsymbol{T}}(\vec{\boldsymbol{x}})(\boldsymbol{\theta}^* - \boldsymbol{\theta}) + \boldsymbol{x}_{\boldsymbol{m}}^{(\boldsymbol{n})} + \vec{\boldsymbol{k}}^{\boldsymbol{T}}\vec{\boldsymbol{e}} + \boldsymbol{w}$$

设 Lyapunov 函数

$$\boldsymbol{V} = \frac{\mathbf{1}}{\mathbf{2}}\vec{\boldsymbol{e}}^{\boldsymbol{T}}\boldsymbol{P}\vec{\boldsymbol{e}} + \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\gamma}}|\boldsymbol{\theta}^* - \boldsymbol{\theta}|^{\mathbf{2}}$$

对 $\boldsymbol{V}$ 求导数并使其尽可能小，得到参数 $\boldsymbol{\theta}$ 的自适应律

$$\dot{\boldsymbol{\theta}} = -\boldsymbol{\gamma}\vec{\boldsymbol{e}}^{\boldsymbol{T}}\boldsymbol{P}_{\boldsymbol{n}}\boldsymbol{b}(\vec{\boldsymbol{x}})$$



Theorem: If the state x, the parameters θ, and the minimum approximation error w are bounded, then:

(a) The tracking error satisfies

$$\int_0^t |e(\tau)|^2 \, d\tau \leq a + b \int_0^t |w(\tau)|^2 \, d\tau$$

(b) If w is squared integrable, then $\lim_{t\to\infty} |e(t)| = 0$.

二、直接模糊自适应控制

考虑非线性系统

$$\boldsymbol{x^{(n)} = f(\vec{x}) + u}$$

其中 $\vec{x} = \left(x, \dot{x}, \dots, x^{(n-1)}\right)^T$ 为状态向量，u 为控制，而非线性函数 $f(\vec{x})$ 未知。取控制器为模糊系统：

$$u = \hat{f}(\vec{x}|\theta) = b^T(\vec{x})\theta$$

我们的任务：确定模糊系统参数 θ 的动态变化律使得 $e = x - x_m \to 0$。

如果 $f(\vec{x})$ 已知，则控制器

$$u^* = -f(\vec{x}) + x_m^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e$$

使得

$$e^{(n)} + k_1 e^{(n-1)} + \cdots + k_n e = 0$$

我们可以设计 $\vec{k} = \left(k_1, \dots, k_n\right)^{\mathrm{T}}$ 使得 $e \to 0$。

由于 $f(\vec{x})$ 未知，所以最优控制器 u^* 也未知，我们用模糊控制器 $u = b^T(\vec{x})\theta$ 逼近最优控制器 u^*。

设其最优参数为

$$\boldsymbol{\theta}^* = \arg\min_{\theta}[\boldsymbol{sup}_{\vec{x}\in \boldsymbol{R}^n}(\boldsymbol{u}^* - \boldsymbol{b}^T(\vec{\boldsymbol{x}})\boldsymbol{\theta})]$$

则

$$\boldsymbol{u}^* = \boldsymbol{b}^T(\vec{\boldsymbol{x}})\boldsymbol{\theta}^* + \boldsymbol{w}$$

得

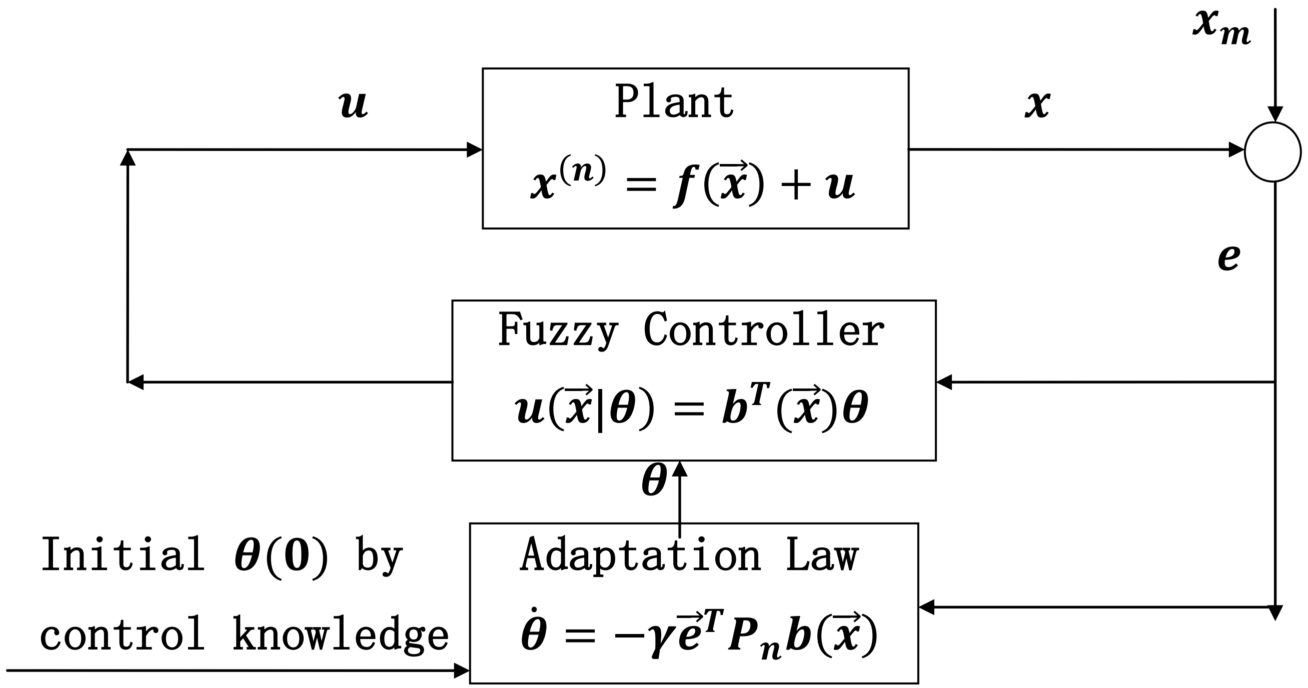
$$\boldsymbol{x}^{(n)} = \boldsymbol{b}^T(\vec{\boldsymbol{x}})(\boldsymbol{\theta}^* - \boldsymbol{\theta}) + \boldsymbol{x}_m^{(n)} + \vec{\boldsymbol{k}}^T\vec{\boldsymbol{e}} + \boldsymbol{w}$$

设 Lyapunov 函数

$$\boldsymbol{V} = \frac{1}{2}\vec{\boldsymbol{e}}^T\boldsymbol{P}\vec{\boldsymbol{e}} + \frac{1}{2\boldsymbol{\gamma}}|\boldsymbol{\theta}^* - \boldsymbol{\theta}|^2$$

对 $\boldsymbol{V}$ 求导数并使其尽可能小，得到参数 $\boldsymbol{\theta}$ 的自适应律

$$\dot{\boldsymbol{\theta}} = -\boldsymbol{\gamma}\vec{\boldsymbol{e}}^T\boldsymbol{P}_n\boldsymbol{b}(\vec{\boldsymbol{x}})$$



<u>Theorem</u>: If the state x, the parameters $\boldsymbol{\theta}$, and the minimum approximation error w are bounded, then:

(c) The tracking error satisfies

$$\int_0^t |e(\tau)|^2 \, d\tau \le a + b \int_0^t |w(\tau)|^2 \, d\tau$$

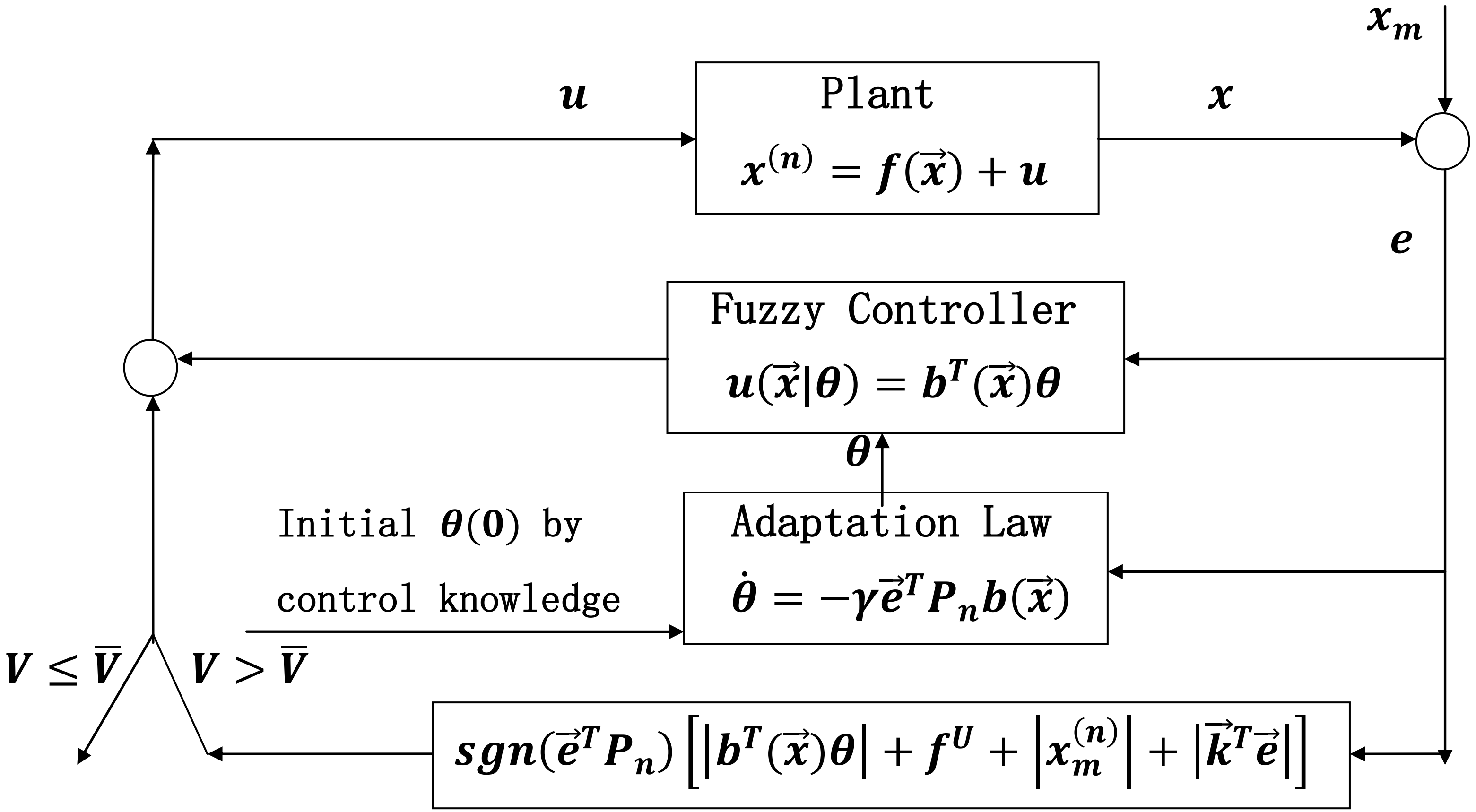
(d) If w is squared integrable, then $\lim_{t\to\infty} |e(t)| = 0$.

三、鲁棒模糊自适应控制

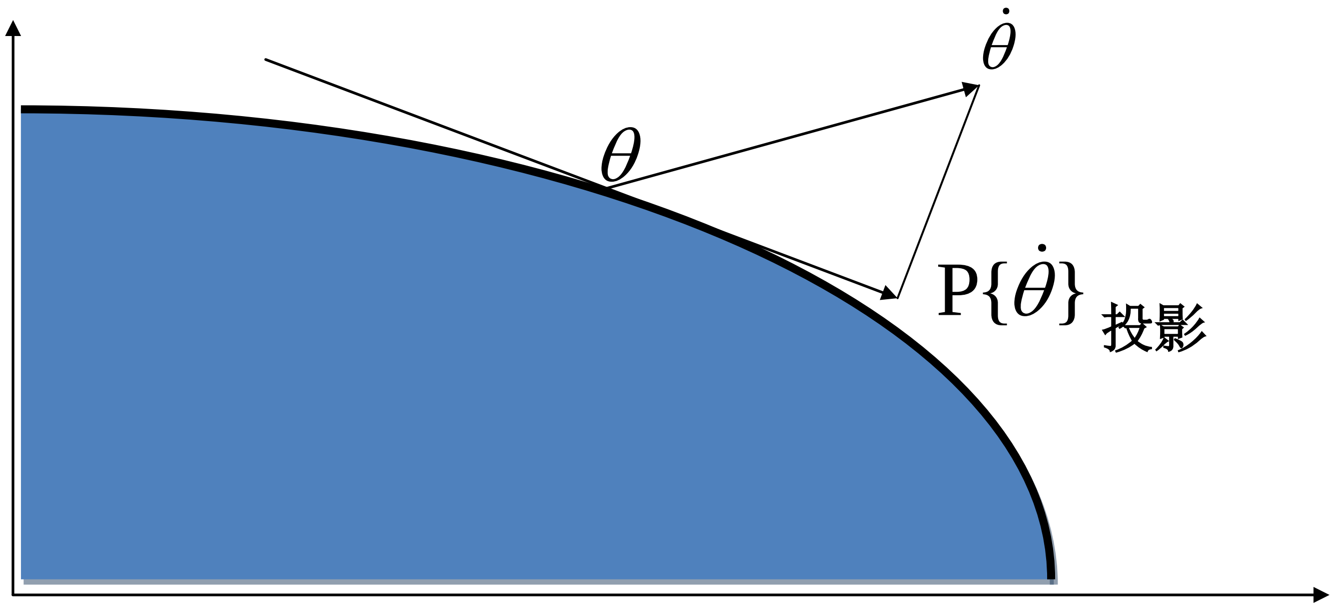
1. 用监视控制器保证状态变量 $\vec{x}$ 有界

$$u = b^T(\vec{x})\theta + u_s$$

$$u_s = I^* sgn(\vec{e}^T P_n)\left[\left|b^T(\vec{x})\theta\right| + f^U + \left|x_m^{(n)}\right| + \left|\vec{k}^T\vec{e}\right|\right]$$



2. 用投影算法保证控制器参数 $\boldsymbol{\theta}$ 有界



$$\dot{\boldsymbol{\theta}} = \begin{cases} -\gamma \vec{e}^T P_n b(\vec{x}) & if\ |\boldsymbol{\theta}| < M\ or\ |\boldsymbol{\theta}| = M\ and\ \vec{e}^T P_n \boldsymbol{\theta}^T b(\vec{x}) \geq 0 \\ P\{-\gamma \vec{e}^T P_n b(\vec{x})\} & if\ \ |\boldsymbol{\theta}| = M\ and\ \vec{e}^T P_n \boldsymbol{\theta}^T b(\vec{x}) < 0 \end{cases}$$

$$P\{-\gamma \vec{e}^T P_n b(\vec{x})\} = -\gamma \vec{e}^T P_n b(\vec{x}) + \gamma \vec{e}^T P_n \frac{\boldsymbol{\theta}\boldsymbol{\theta}^T b(\vec{x})}{|\boldsymbol{\theta}|^2}$$

五、模糊與情网络及其在金融预测中的应用

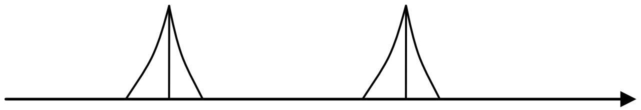
- **Modeling Opinions**

Existing models (DeGroot, Hegselmann-Krause, etc.)

Opinions are numbers: opinion 1 opinion 2

But, opinions cannot be separated with their uncertainties.

opinion 1 opinion 2



agents are very confident

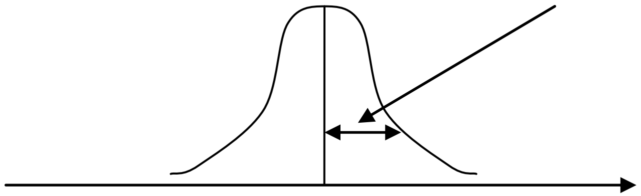
opinion 1 opinion 2

agents are not sure

Our model:

Opinions are Gaussian fuzzy sets

uncertainty



opinion

opinion = center of fuzzy set

opinion uncertainty = standard deviation of fuzzy set

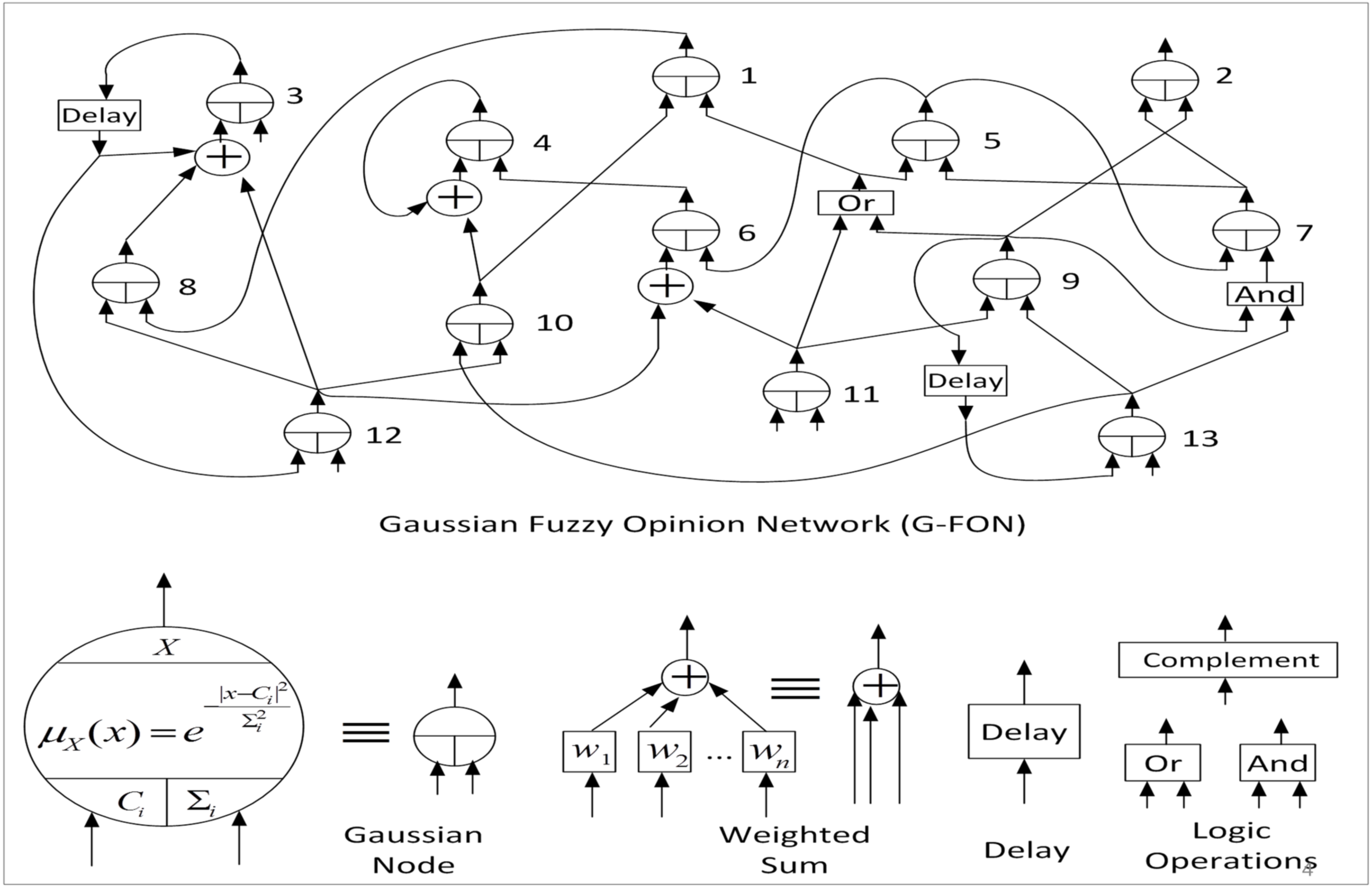
- **<u>Definition of Fuzzy Opinion Networks</u>**

<u>Definition 1</u>: A *<u>Gaussian node i</u>* is a 2-input-1-output node characterized by the Gaussian membership function:

$$\mu_X(x) = e^{-\frac{|x-C_i|^2}{\Sigma_i^2}}$$

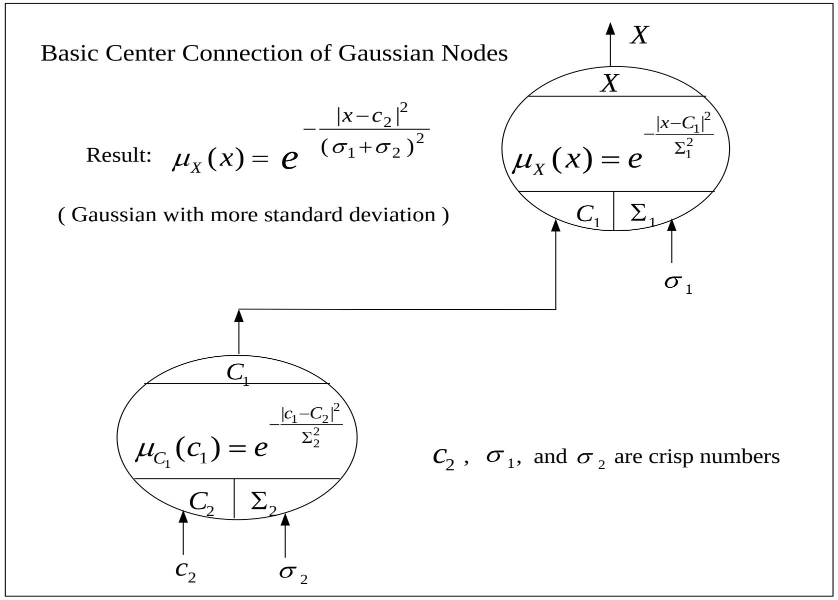
where the center C_i and the standard deviation Σ_i are two input fuzzy sets to the node and the fuzzy set X is the output of the node.

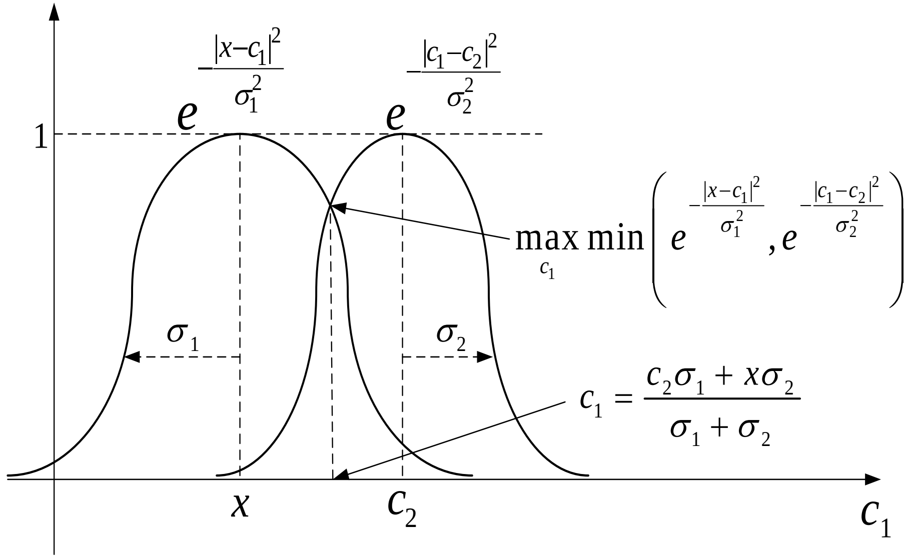
A *<u>Gaussian Fuzzy Opinion Network</u>* (G-FON) is a connection of a number of Gaussian nodes, possibly through some weighted-sum, delay, and/or logic operation elements, as shown in the next figure.


Delay
3
1
2
4
5
Or
6
7
8
9
And
10
11
Delay
12
13
Gaussian Fuzzy Opinion Network (G-FON)
X
$\mu_X(x) = e^{-\frac{|x-C_i|^2}{\Sigma_i^2}}$
C_i
Σ_i
Gaussian Node
w_1
w_2
...
w_n
Weighted Sum
Delay
Delay
Complement
Or
And
Logic Operations
4

Basic Connections of Gaussian Nodes

Connection 1 (Basic Center Connection):





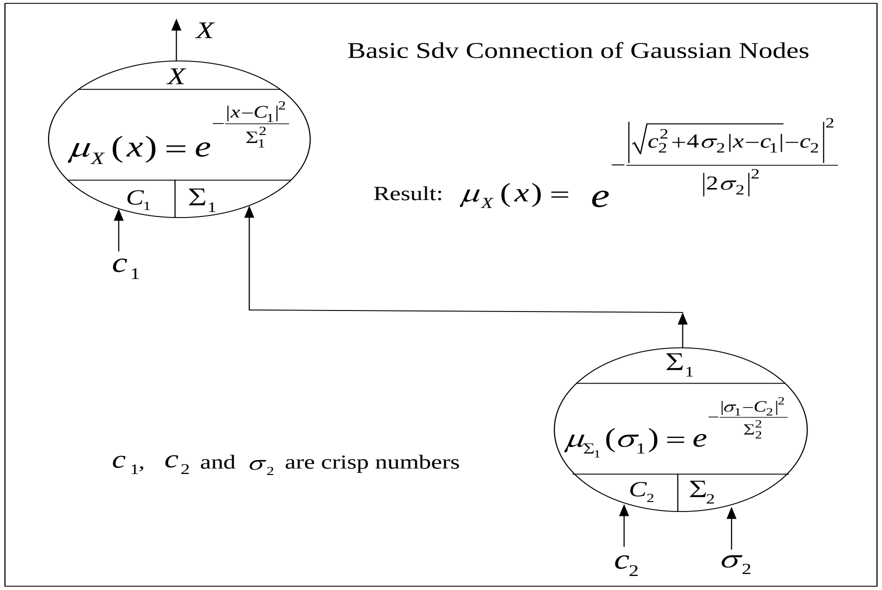
$$\mu_{X|C_1}(x|C_1) = e^{-\frac{|x-C_1|^2}{\sigma_1^2}} \qquad \mu_{C_1}(c_1) = e^{-\frac{|c_1-c_2|^2}{\sigma_2^2}}$$

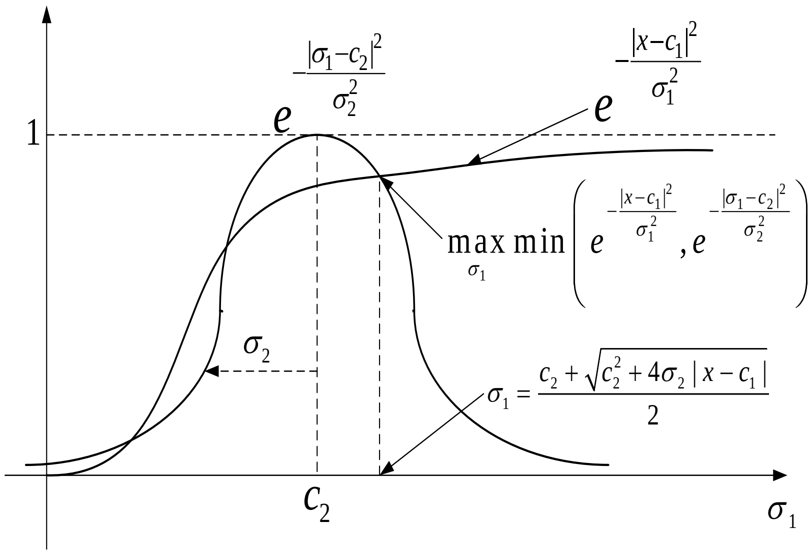
$$\mu_X(x) = \max_{c_1 \in R^n} min\left[e^{-\frac{|x-c_1|^2}{\sigma_1^2}}, e^{-\frac{|c_1-c_2|^2}{\sigma_2^2}}\right]$$

$$\mu_X(x) = e^{-\frac{|x-c_2|^2}{(\sigma_1+\sigma_2)^2}}$$

<u>Main observations</u>: 1) Still Gaussian; 2) Center no change; 3) Sdv’s added up

- **Connection 2 (Basic Standard Deviation Connection):**



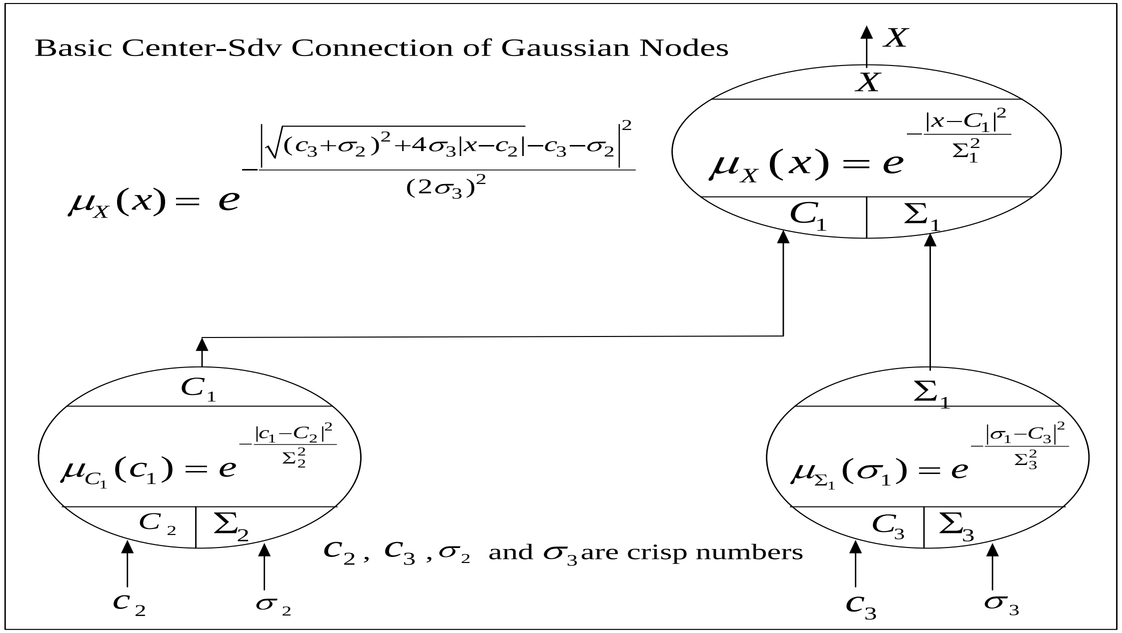


$$\mu_{X|\Sigma_1}(x|\Sigma_1) = e^{-\frac{|x-c_1|^2}{\Sigma_1^2}} \qquad \mu_{\Sigma_1}(\sigma_1) = e^{-\frac{|\sigma_1-c_2|^2}{\sigma_2^2}}$$

$$\mu_X(x) = \max_{\sigma_1 \in R^+} \min\left[e^{-\frac{|x-c_1|^2}{\sigma_1^2}}, e^{-\frac{|\sigma_1-c_2|^2}{\sigma_2^2}}\right] \qquad \mu_X(x) = e^{-\frac{\left|\sqrt{c_2^2+4\sigma_2|x-c_1|}-c_2\right|^2}{(2\sigma_2)^2}}$$

Main observations: 1) Center = c_1 ; 2) Sdv = $\sigma_2 + c_2$; 3) Changing between Gaussian ($c_2 \gg \sigma_2$) and exponential ($c_2 \ll \sigma_2$)

- **<u>Connection 3 (Basic Center-Sdv Connection):</u>**

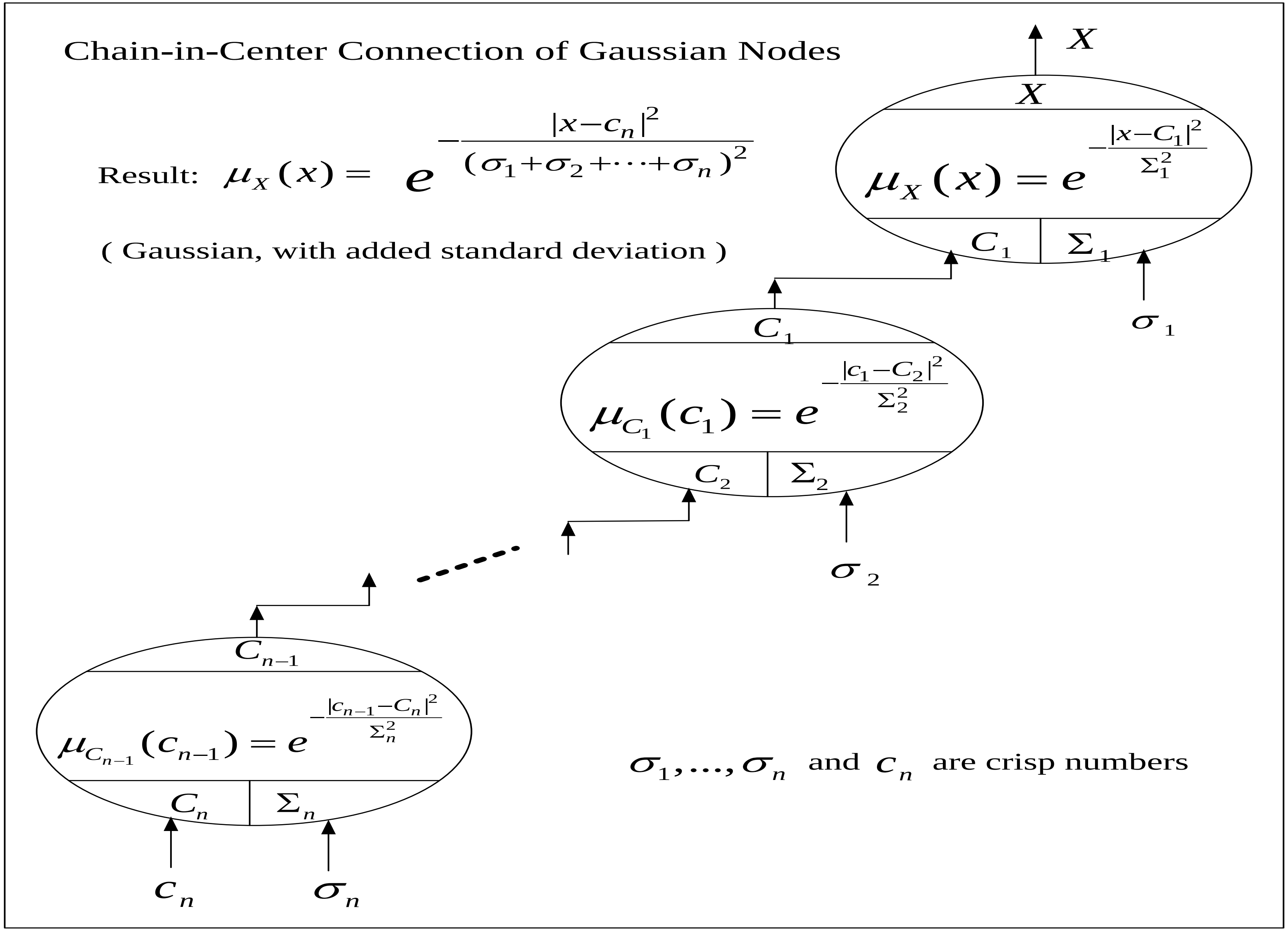


$$\mu_{X|\Sigma_1}(x|\Sigma_1) = e^{-\frac{|x-c_2|^2}{(\Sigma_1+\sigma_2)^2}} \qquad \mu_{\Sigma_1}(\sigma_1) = e^{-\frac{|\sigma_1-c_3|^2}{\sigma_3^2}}$$

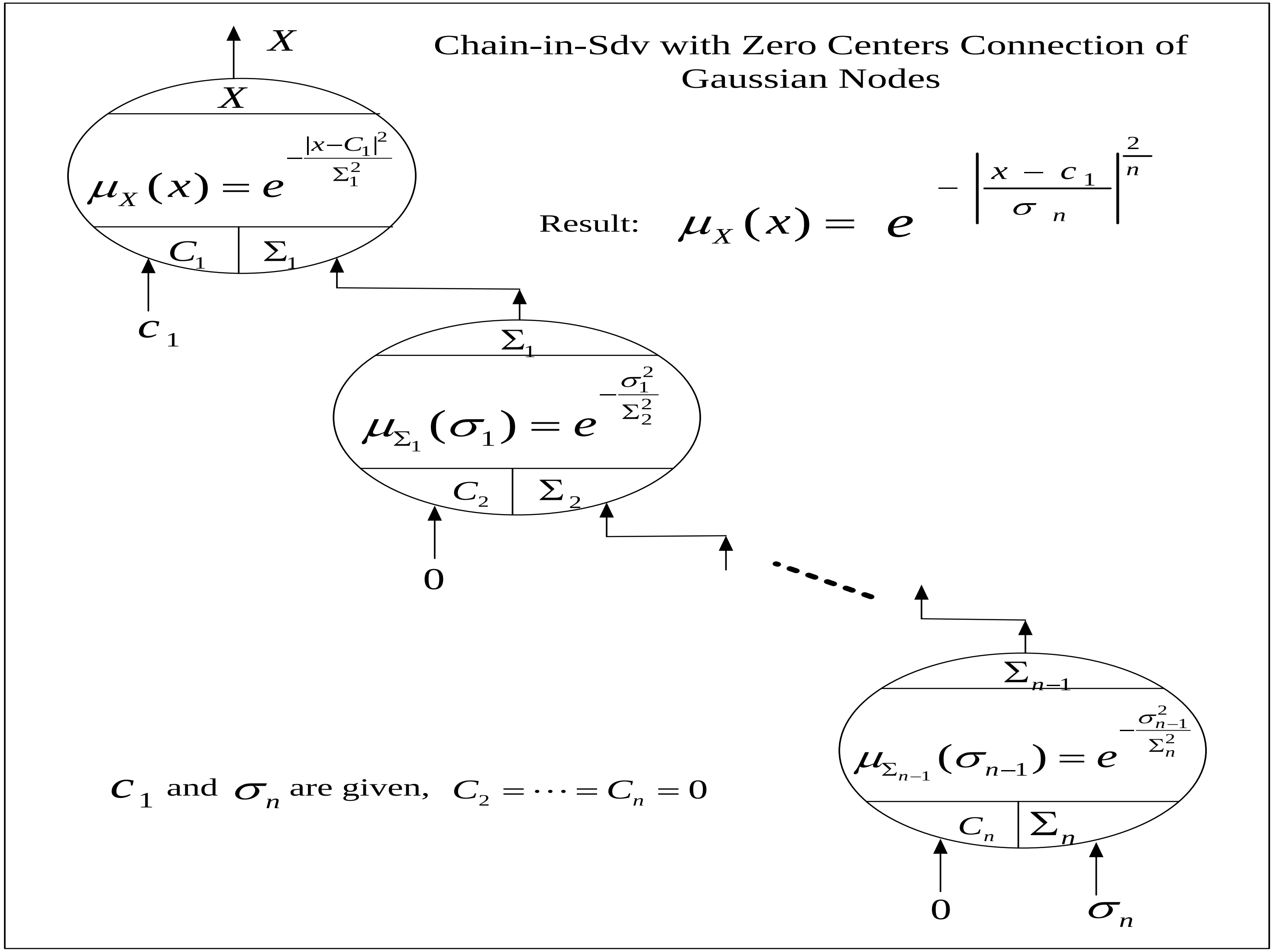
$$\mu_X(x) = \max_{\sigma_1\in R^+} min\left[e^{-\frac{|x-c_2|^2}{(\sigma_1+\sigma_2)^2}}, e^{-\frac{|\sigma_1-c_3|^2}{\sigma_3^2}}\right] \qquad \mu_X(x) = e^{-\frac{\left|\sqrt{(c_3+\sigma_2)^2+4\sigma_3|x-c_2|}-c_3-\sigma_2\right|^2}{(2\sigma_3)^2}}$$

<u>Main observations</u>: 1) Center = c_2 ; 2) Sdv = $c_3+\sigma_2+\sigma_3$; 3) Changing between Gaussian and exponential.

- **Connection 4 (Chain-in-Center Connection):**

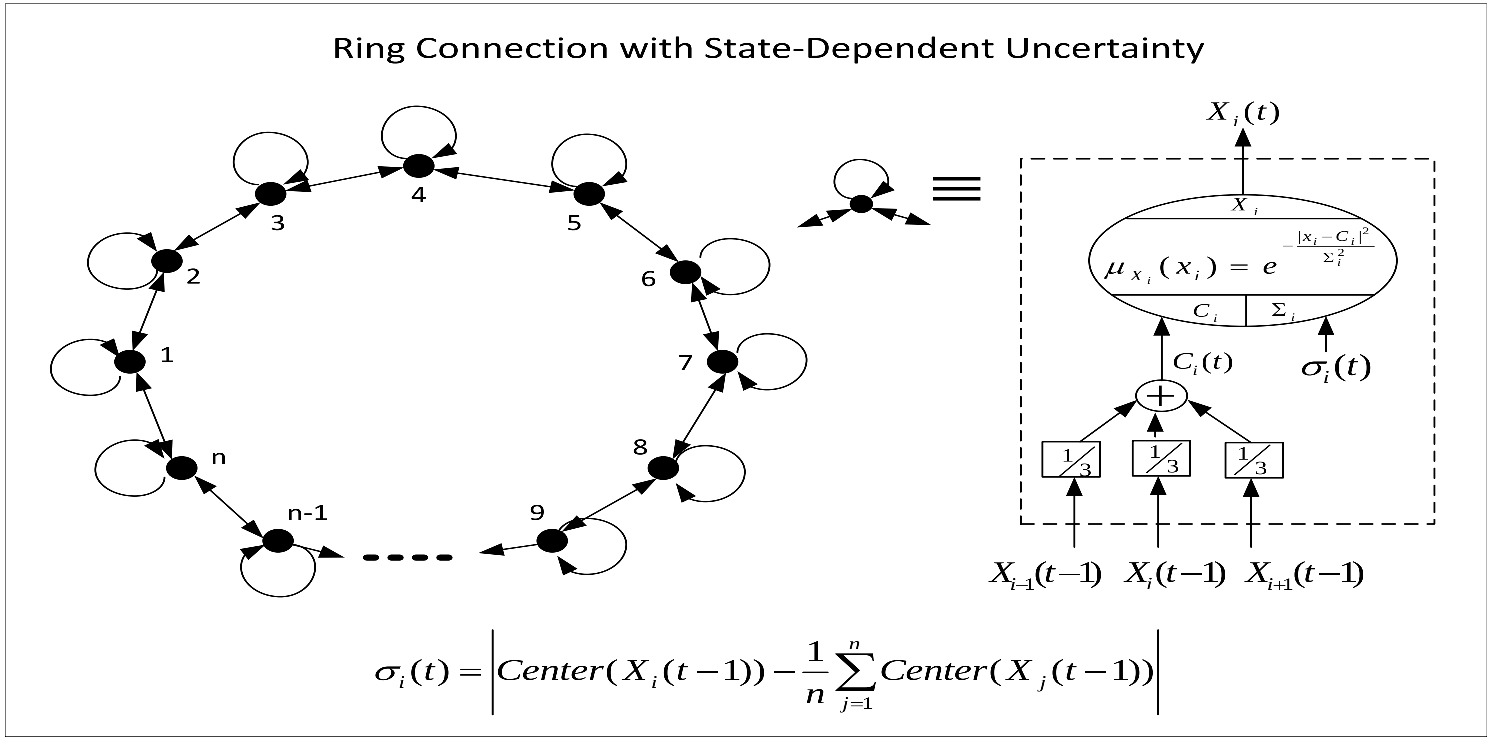


- <u>**Connection 5 (Chain-in-Sdv with Zero Centers Connection):**</u>



Stretched exponential function

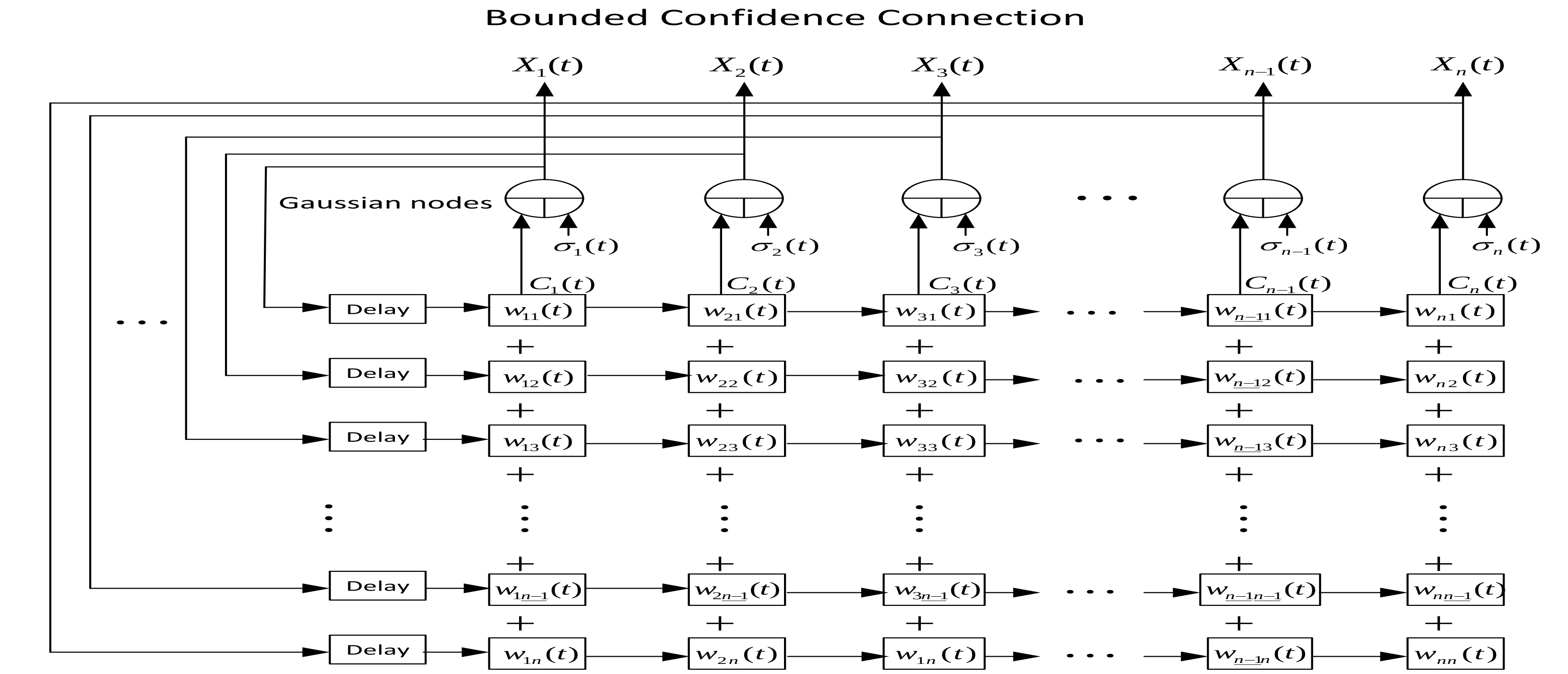
Connection 11 (Ring Connection with State-Dependent Uncertainty):



$$C(t) = WX(t-1) \qquad \sigma_i(t) = \left| \mathrm{Center}\big(X_i(t-1)\big) - \frac{1}{n}\sum_{j=1}^{n} \mathrm{Center}\big(X_j(t-1)\big) \right| \qquad W = \begin{pmatrix} 1/3 & 1/3 & 0 & \cdots & 0 & 1/3 \\ 1/3 & 1/3 & 1/3 & 0 & \cdots & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \cdots & 0 \\ \vdots & & & \vdots & & \\ 0 & \cdots & 0 & 1/3 & 1/3 & 1/3 \\ 1/3 & 0 & \cdots & 0 & 1/3 & 1/3 \end{pmatrix}$$

$$\mathrm{Center}\big(X(t)\big) = W\ \mathrm{Center}\big(X(t-1)\big) \qquad \mathrm{Sdv}\big(X(t)\big) = W\ \mathrm{Sdv}\big(X(t-1)\big) + \sigma(t)$$

- **Connection 13 (Bounded Confidence Connection):**



$$C_i(t) = \sum_{j=1}^{n} w_{ij}(t) X_j(t-1)$$

$$\sigma_i(t) = a\left|Center(X_i(t-1)) - \frac{1}{|N_i(t)|}\sum_{k\in N_i(t)} Center(X_k(t-1))\right|$$ (local reference)

or

$$\sigma_i(t) = a\left|Center(X_i(t-1)) - \frac{1}{n}\sum_{k=1}^{n} Center(X_k(t-1))\right|$$ (global reference)

$$w_{ij}(t) = \begin{cases} \dfrac{\max\limits_x \min\left(\mu_{X_i(t-1)}(x), \mu_{X_j(t-1)}(x)\right)}{\sum\limits_{k\in N_i(t)}\left(\max\limits_x \min\left(\mu_{X_i(t-1)}(x), \mu_{X_k(t-1)}(x)\right)\right)}, & j \in N_i(t) \\ 0, & j \notin N_i(t) \end{cases}$$

$$N_i(t) = \left\{ j \in \{1,\ldots,n\} \mid \max_x \min\left(\mu_{X_i(t-1)}(x), \mu_{X_j(t-1)}(x)\right) > d_i \right\}$$

● 在金融预测中的应用

$\bar{\boldsymbol{p}}_{\boldsymbol{i,t}}$为投资者 $\boldsymbol{i}$ 对股价的预期值，$\boldsymbol{\sigma}_{\boldsymbol{i,t}}$为预期的不确定性，则

$$\ln(p_{t+1}) = \ln(p_t) + \sum_{i=1}^{n} \frac{a_i\left[\ln\left(\bar{p}_{i,t}\right) - \ln(p_t)\right]}{\sigma_{i,t}} + \varepsilon_t$$

$$\bar{p}_{i,t+1} = \frac{1}{|N_i(t)|}\sum_{j\in N_i(t)} \bar{p}_{j,t} \quad , \quad \sigma_{i,t+1} = \frac{1}{|N_i(t)|}\sum_{j\in N_i(t)} \sigma_{j,t} + u_i(t+1)$$

$$N_i(t) = \left\{ j \in \{1, \dots, n\} \mid e^{-\frac{\left|\bar{p}_{i,t}-\bar{p}_{j,t}\right|^2}{\left(\sigma_{i,t}+\sigma_{j,t}\right)^2}} \geq d_i \right\}$$

(a) ***Local Reference*:**

$$u_i(t+1) = b\left|\bar{p}_{i,t} - \frac{1}{|N_i(t)|}\sum_{j\in N_i(t)} \bar{p}_{j,t}\right|$$

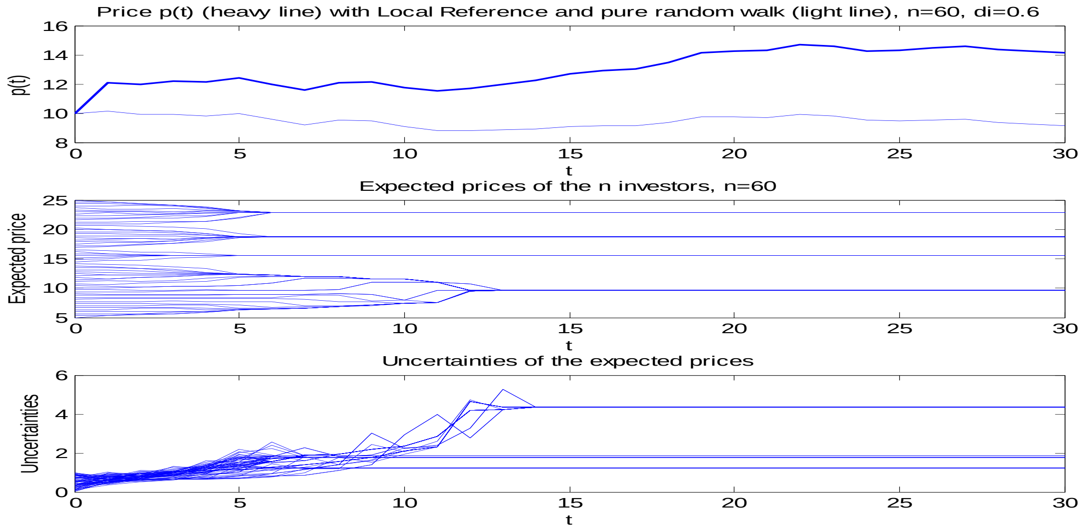
(b) ***Global Reference*:**

$$u_i(t+1) = b\left|\bar{p}_{i,t} - \frac{1}{n}\sum_{j=1}^{n} \bar{p}_{j,t}\right|$$

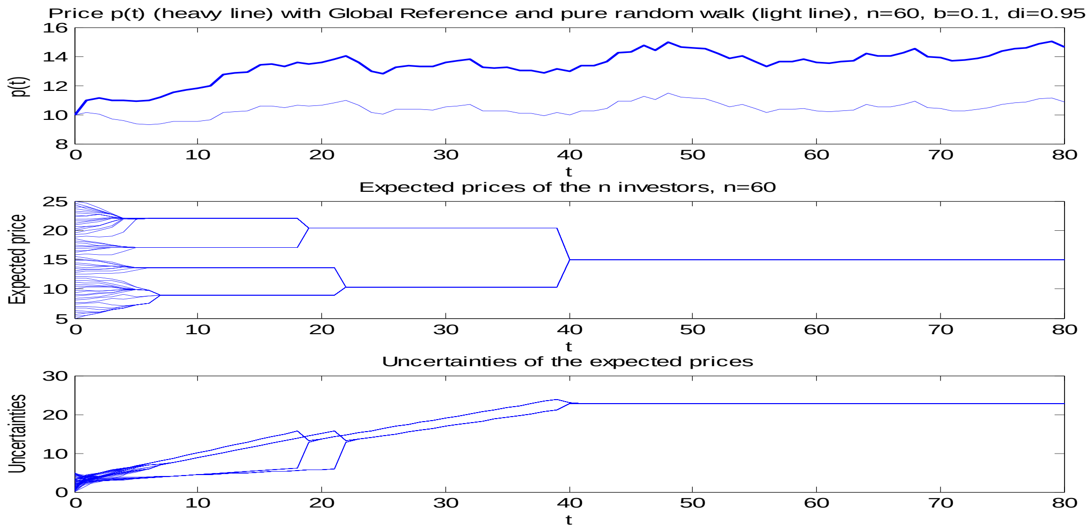
(c) ***Real Price Reference*:**

$$u_i(t+1) = b\left|\bar{p}_{i,t} - p_t\right|$$

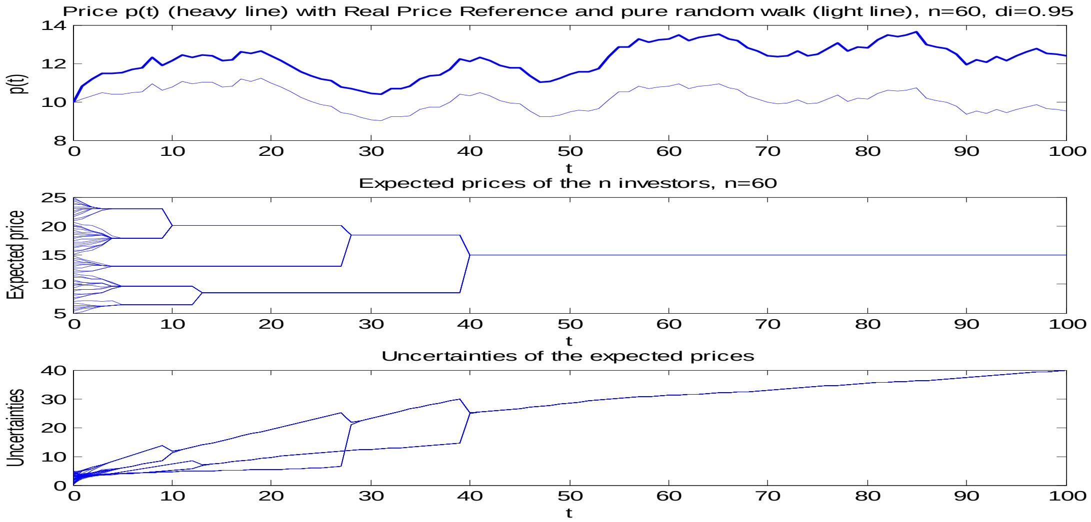
(a) *Local Reference*:



(b) *Global Reference*:



(c) *Real Price Reference*:



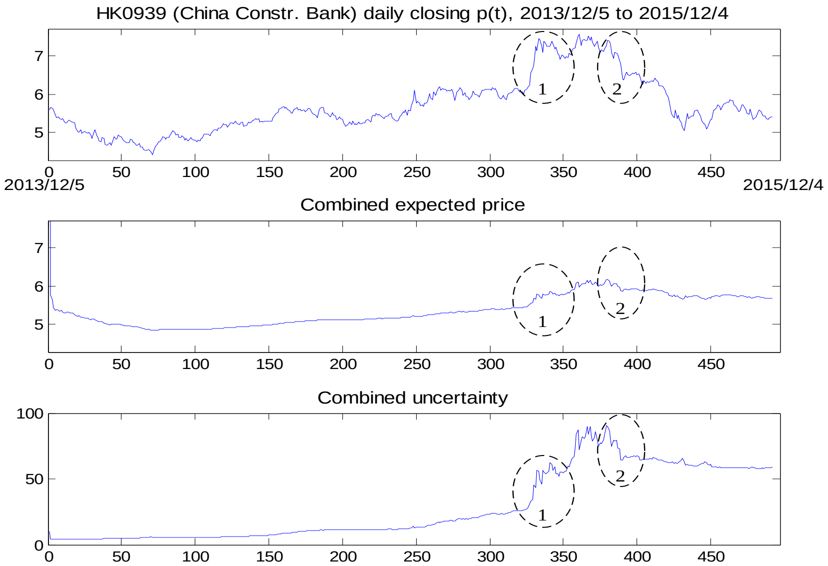
定义投资者综合不确定性为 $\boldsymbol{\sigma_t}$，综合预期价格为 $\boldsymbol{\bar{p}_t}$：

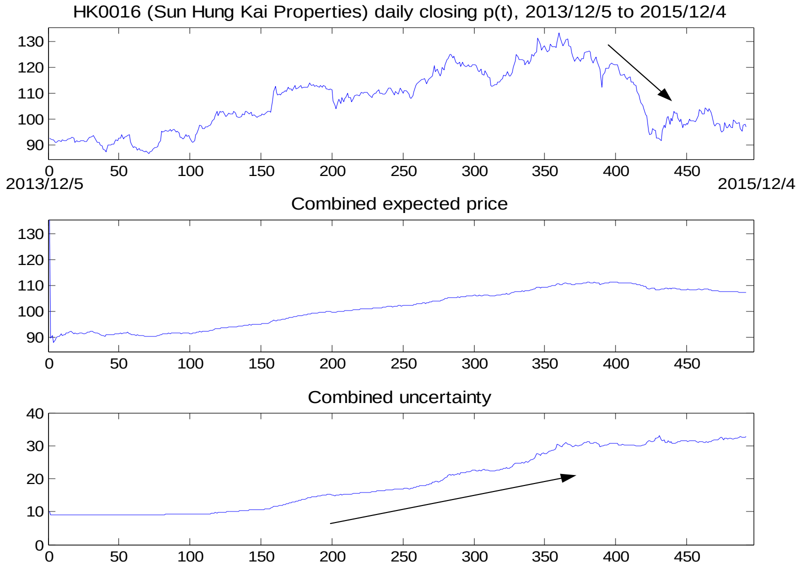
$$\sigma_t = \frac{1}{\sum_{i=1}^{n}\left(\frac{a_i}{\sigma_{i,t}}\right)}, \qquad \bar{p}_t = e^{\sigma_t \sum_{i=1}^{n}\left(\frac{a_i \ln(\bar{p}_{i,t})}{\sigma_{i,t}}\right)}$$

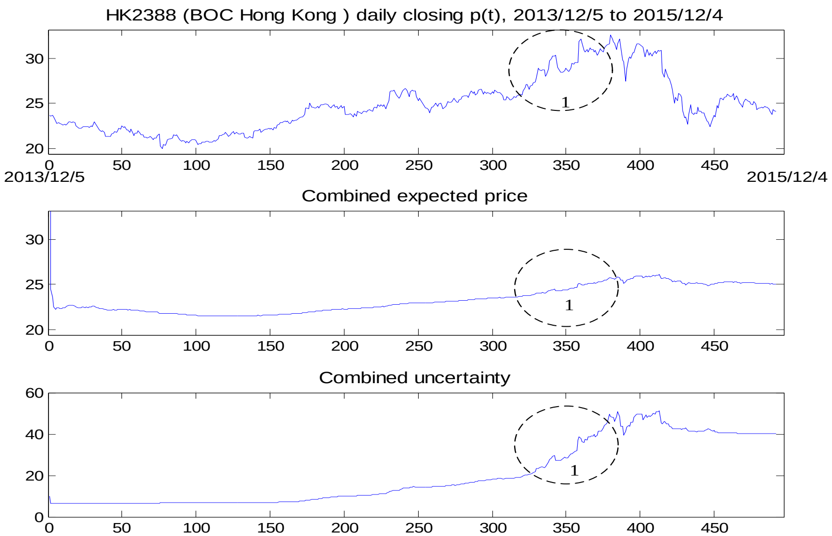
得

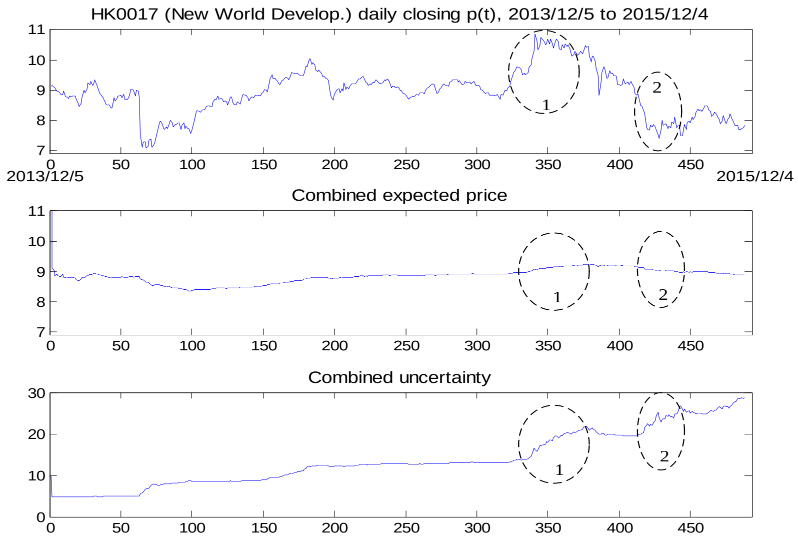
$$\ln(p_{t+1}) = \ln(p_t) + \frac{\ln(\bar{p}_t) - \ln(p_t)}{\sigma_t} + \varepsilon_t$$

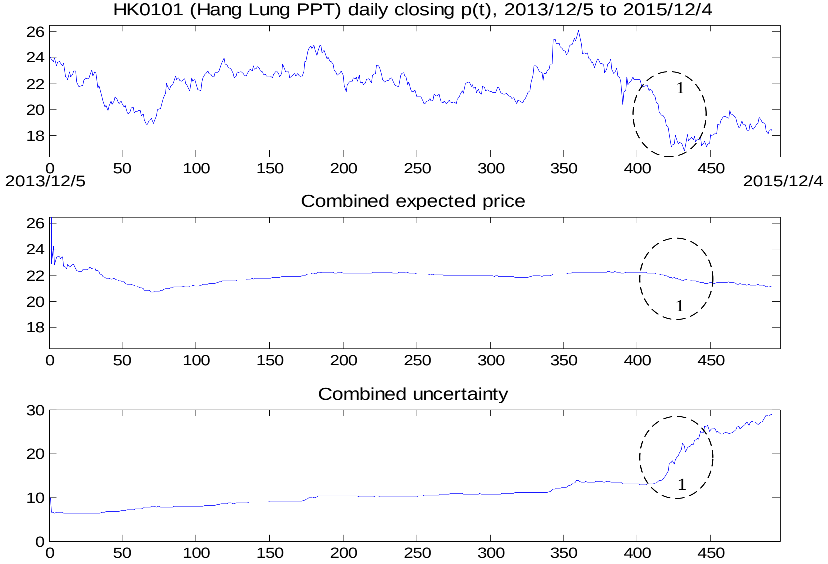
我们可以通过股价数据辨识出 $\boldsymbol{\bar{p}_t}$ 和 $\boldsymbol{\sigma_t}$。

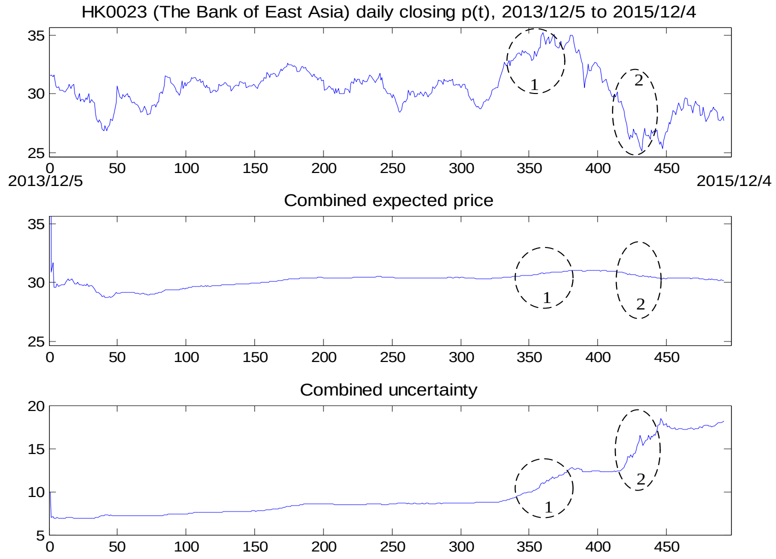

HK0939 (China Constr. Bank) daily closing p(t), 2013/12/5 to 2015/12/4
2013/12/5
2015/12/4
Combined expected price
Combined uncertainty


HK0016 (Sun Hung Kai Properties) daily closing p(t), 2013/12/5 to 2015/12/4
2013/12/5
2015/12/4
Combined expected price
Combined uncertainty


HK2388 (BOC Hong Kong) daily closing p(t), 2013/12/5 to 2015/12/4
2013/12/5
2015/12/4
Combined expected price
Combined uncertainty


HK0017 (New World Develop.) daily closing p(t), 2013/12/5 to 2015/12/4
2013/12/5
2015/12/4
Combined expected price
Combined uncertainty





<u>默顿 PK 牛顿：“从牛顿到默顿”PK“永远的牛顿”</u>