SIG787 - MATHEMATICS FOR ARTIFICIAL INTELLIGIENCE

ASSESSMENT-1

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(2). Consider the following function:

$$f(x) = \frac{x}{1+x|x|}$$

(i) Find the domain of $f(x)$.

When $f(x)$ is fraction or rational, the denominator should be always positive, which is not equal to zero.

$\Rightarrow 1+x|x| \neq 0.$

The function has a absolute value $|x|$

$f(x)$ is positive, $x > 0$

$f(x)$ is 0, $x = 0$

$f(x)$ is negative, $x < 0$

$\Rightarrow f(x)$ is negative, $-1 < x < 0$

$f(x)$ is not defined, $x = -1$

$\therefore$ So the function will take the value from $-\infty$ to ∞ except $\boxed{x = -1}$

Domain of $f(x)$ is $(-\infty, -1) \cup (-1, \infty)$

ii, Find all x - intercept , y - intercept

① x - intercept , which is $y=0$

$$f(x)=0$$

$$\frac{x}{1+x|x|}=0$$

$$\Rightarrow x=0$$

② y - intercept , which is $x=0$

$$y=\frac{0}{1+0|0|}$$

$$y=0$$

$\therefore$ The x - intercept is at $x=0$ and y - intercept is at $y=0$.

iii, Rewrite the function as piecewise function.

$$f(x)=\frac{x}{1+x|x|}$$

$$\Rightarrow |x|=\pm x$$

$$+x \Rightarrow \frac{x}{1+x(x)}=\frac{x}{1+x^2}$$

$$-x \Rightarrow \frac{x}{1+x(-x)}=\frac{x}{1-x^2}$$

$$f(x)=\begin{cases} \frac{x}{1+x^2}, & x \geq 0 \\ \frac{x}{1-x^2} & 0 < x < -1 \\ & \text{not defined if } x=-1 \\ \frac{x}{1-x^2} & x < -1 \end{cases}$$

iv) Find all the stationary points and classify them

To find the stationary points with the first derivative of $f(x)$

$f(x) = \frac{x}{1+x^2}$

$\left[\because \frac{u(x)}{v(x)}\right]' = \frac{u'(x)\,v(x) - u(x)\cdot v'(x)}{v(x)^2}$

$f'(x) = \frac{(1+x^2)\cdot 1 - x\cdot 2x}{(1+x^2)^2}$

$= \frac{1+x^2-2x^2}{(1+x^2)^2}$

$\boxed{f'(x) = \frac{1-x^2}{(1+x^2)^2}}$

i) set $f'(x) = 0$

$\frac{1-x^2}{(1+x^2)^2} = 0$

$1 - x^2 = 0$

$+x^2 = +1$

$x = \pm 1$

since $x \geq 0$

$x = 1$, $x = -1$

$x = 1$ is a critical point

ii) $x = 1$

$f(1) = \frac{1}{1+1^2}$

$= \frac{1}{2}$

$x = 1$, $y = \frac{1}{2}$

stationary points are $(1, \frac{1}{2})$

$f(x) = \frac{x}{1-x}$

$f'(x) = \frac{(1-x^2)1 - x\cdot(-2x)}{(1-x^2)^2}$

$= \frac{1-x^2+2x^2}{(1-x^2)^2}$

$\boxed{f'(x) = \frac{1+x^2}{(1-x^2)^2}}$

i) set $f'(x) = 0$

$\frac{1+x^2}{(1-x^2)^2} = 0$

$1 + x^2 = 0$

$x^2 = -1$

$x = i$

It has a imaginary value ~~so no stationary points~~

since $f(x) \neq 0$

there are no stationary points

v) Determine the intervals for which the function is increasing, and the intervals for which the function is decreasing. 1

we check the first derivative of the function

1 For $f'(x) = \frac{1-x^2}{(1+x^2)^2}$

when $0 < x < 1$

$f(0) = \frac{1-0}{1+0} = 1 = +ve.$

$f'(x)$ is Positive.

when $x > 1$

$f(2) = \frac{1-4}{(1+4)^2} = \frac{-3}{25} = -ve$

Function is increasing for the intervals $0 < x < 1$ and decreasing for the intervals $x > 1$

2 For $f'(x) = \frac{1+x^2}{(1-x^2)^2}$

when $x < -1$

$1+x^2$ is +ve

$1-x^2$ is negative

$f'(x)$ is negative

when $-1 < x < 0$

$1+x^2$ is +ve

$1-x^2$ is +ve

$f'(x)$ is +ve.

Function increasing for the intervals $-1 < x < 0$ and decreasing for the intervals $x < -1$

vi, Find the second derivative of the function, and identify all the function is convex or concave.

$$f'(x) = \frac{1-x^2}{(1+x^2)^2}$$

i $$f''(x) = \frac{(1+x^2)^2(-2x) - (1-x^2)2(1+x^2)(2x)}{(1+x^2)^4}$$

$$= \frac{-2x(1+x^2) - 4x(1-x^2)(1+x^2)}{(1+x^2)^4}$$

$$= \frac{-2x - 2x^3 - 4x + 4x^3 . (1+\cancel{x^2})}{(1+x^2)^{\cancel{4}\,3}}$$

$$= \frac{+2x(-1-x^2-2+2x^2)}{(1+x^2)^3}$$

$$\boxed{f''(x) = \frac{2x(x^2-3)}{(1+x^2)^3}}$$

ii $$f'(x) = \frac{1+x^2}{(1-x^2)^2}$$

$$= \frac{(1-x^2)^2(2x) - (x^2+1)2(1-x^2)(-2x)}{(1-x^2)^4}$$

$$= \frac{2x(1-x^2)^2 - (x^2+1)(1-x^2)(-4x)}{(1-x^2)^4}$$

$$= \frac{2x(1-x^2)^2 - (-4x)(x^2+1)(1-x^2)}{(1-x^2)^{\cancel{4}3}}$$

$$= \frac{2x(1-x^2)^2 + 4x^3 + 4x}{(1-x^2)^3}$$

$$= \frac{2x - 2x^3 + 4x^3 + 4x}{(1-x^2)^3}$$

$$= \frac{-2x(1-x^2+2x^2+2)}{(1-x^2)^3}$$

$$\boxed{f''(x) = \frac{-2x(x^2+3}{(1-x^2)3}}$$

sub $x=0$

$$f''(0) = \frac{2(0)(0-3)}{(0+1)^3} = 0$$

For $0 < x < \sqrt{3}$

$2x$ is +ve

x^2-3 is -ve

$(x^2+1)^3$ is +ve

For $x > \sqrt{3}$

$2x$ is +ve

x^2-3 is +ve

$(x^2+1)^3$ is +ve

Function of $0 < x < \sqrt{3}$ is concave

Function of $x > \sqrt{3}$ is convex

~~for~~ $f''(x)$

$$= \frac{-2x(x^2+3)}{(-x^2+1)^3}$$

For $x < -1$

$-2x$ is +ve

x^2+3 is +ve

$(-x^2+1)^3$ is +ve

For $-1 < x < 0$

$-2x$ is +ve

x^2+3 is +ve

$(1-x^2)^3$ is -ve

Function of $x < -1$ is convex

Function of $-1 < x < 0$ is concave

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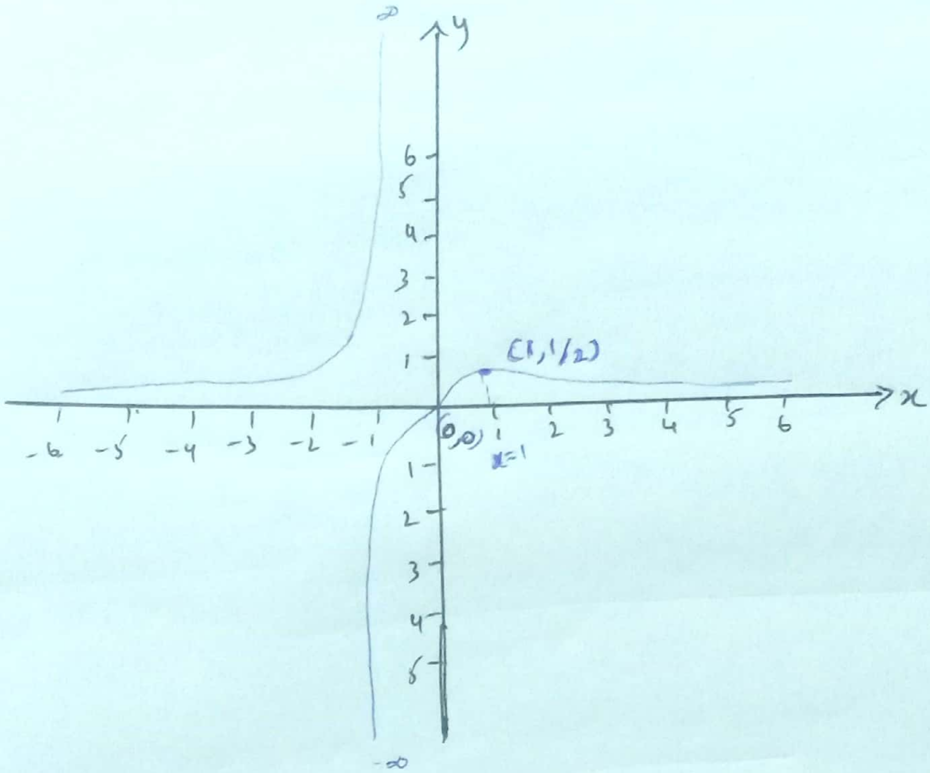
(vii) Sketch the function by hand based on the information you gained through steps (i) to (vi). Label all the important points on the graph of the function.



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