1 Chapter 14 The Neural Likelihood Function

Problem 14.2 *We assume a population of nine independent Poisson neurons with Gaussian tuning curves and preferred orientations from* -40 *to* 40 *in steps of* 10. *The tuning curve parameters have values* $g = 10$, $b = 0$, *and* $\sigma_{\text{tc}} = 20$. *A stimulus* $s = 0$ *is presented to this population. What is the probability that all neurons stay silent?*

Answer

So each neuron follows a Poisson distribution, and the probability that a neuron is not activated given its firing rate λ_i is:

$$P(k_i = 0) = e^{-\lambda_i}.$$

Then the probability of all neurons being silent would be:

$$P(\text{all dead}) = \prod_{i=1}^{9} e^{-\lambda_i} = e^{-\sum_{i=1}^{9} \lambda_i}.$$

The tuning curve for each neuron is Gaussian:

$$\lambda_i = g \cdot \exp\left(-\frac{(s-\theta_i)^2}{2\sigma_{\text{tc}}^2}\right) + b,$$

where,

- $g = 10$ is the gain,
- $b = 0$ is the baseline firing rate,
- $\sigma_{\text{tc}} = 20$ is the tuning curve width,
- $\theta_i \in \{-40, -30, -20, -10, 0, 10, 20, 30, 40\}$ are the preferred stimuli of each neuron,
- $s = 0$ is the presented stimulus.

Then, we can compute the total expected spike count:

$$\sum_{i=1}^{9} \lambda_i = \sum_{i=1}^{9} 10 \cdot \exp\left(-\frac{\theta_i^2}{2 \cdot 20^2}\right).$$

$$\sum_{i=1}^{9} \lambda_i = 10 \cdot \sum_{\theta_i=-40}^{40,\text{step}=10} \exp\left(-\frac{\theta_i^2}{800}\right)$$

= (...simplification part is too consuming, so fed to ChatGPT...)

$$= 10 \cdot \left[e^{-2} + e^{-1.125} + e^{-0.5} + e^{-0.125} + 1 + e^{-0.125} + e^{-0.5} + e^{-1.125} + e^{-2}\right].$$

$$\approx 10 \cdot [0.1353 + 0.3247 + 0.6065 + 0.8825 + 1 + 0.8825 + 0.6065 + 0.3247 + 0.1353]$$

$$= 10 \cdot 4.898 = 48.98.$$

Therefore, the probability that all neurons stay silent is:

$$P(\text{all silent}) = e^{-48.98} \approx 5.4 \times 10^{-22}.$$

We can see that this is an extremely small probability.
This means that it is very unlikely for all neurons to be inactivated when the stimulus is aligned with the center of the tuning distribution.

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