1 Chapter 14 The Neural Likelihood Function

Problem 14.5 *Assume a population of independent Poisson neurons responding to a nonnegative stimulus s. Each neuron has a linear tuning curve, that is,*

$$f_i(s) = a_i s. \tag{14.26}$$

Show that for any pattern of activity in this population, the normalized neural likelihood function is a gamma distribution, and find expressions for its parameters.

Answer

In equation

$$f_i(s) = a_i s$$

, where $s \geq 0$ is the stimulus and $a_i > 0$ is a neuron-specific constant.

Then the likelihood for a single neuron is given by the Poisson distribution:

$$p(r_i \mid s) = \frac{(a_i s)^{r_i} e^{-a_i s}}{r_i!}$$

Since the neurons are independent, the joint likelihood is:

$$p(\mathbf{r} \mid s) = \prod_{i=1}^{N} \frac{(a_i s)^{r_i} e^{-a_i s}}{r_i!} = \left(\prod_{i=1}^{N} \frac{a_i^{r_i}}{r_i!} \right) s^{\sum r_i} e^{-s \sum a_i}$$

Let:

$$R = \sum_{i=1}^{N} r_i, \quad A = \sum_{i=1}^{N} a_i$$

Then:

$$p(\mathbf{r} \mid s) \propto s^R e^{-As}$$

This is the unnormalized form of a Gamma distribution in s:

$$\text{Gamma}(s \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} s^{\alpha-1} e^{-\beta s}$$

Comparing:

$$\alpha = R + 1, \quad \beta = A$$

So, we could know that the posterior likelihood over s is a Gamma distribution:

$$p(s \mid \mathbf{r}) \sim \text{Gamma}(s \mid R + 1, A)$$

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