Problem 8.7

Consider binary classification with a flat prior, mirror-image CCSDs (i.e., $p(s \mid C = 1) = p(-s \mid C = -1)$), and a measurement distribution $p(x \mid s)$ that is symmetric around s (though not necessarily Gaussian). Show that the MAP observer has the decision rule

$$\text{report } C = 1 \quad \text{if } x > 0.$$

Solution. With a flat prior over classes the MAP rule compares

$$\log \frac{p(C = 1 \mid x)}{p(C = -1 \mid x)} \;=\; \log \frac{p(x \mid C = 1)}{p(x \mid C = -1)} \;\gtrless_{C=-1}^{C=1}\; 0.$$

By marginalization

$$p(x \mid C) = \int p(x \mid s)\, p(s \mid C)\, ds.$$

Mirror-image symmetry and measurement symmetry ($p(x \mid s) = p(-x \mid -s)$) give

$$p(x \mid C = -1) = \int p(x \mid s)\, p(s \mid C = -1)\, ds = \int p(x \mid s)\, p(-s \mid C = 1)\, ds = \int p(-x \mid s)\, p(s \mid C = 1)\, ds = p(-x \mid C$$

Hence

$$\log \frac{p(C = 1 \mid x)}{p(C = -1 \mid x)} = \log p(x \mid C = 1) \;-\; \log p(-x \mid C = 1),$$

which is positive exactly when $x > 0$. Therefore the MAP decision rule is

$$\text{report } C = 1 \quad \Longleftrightarrow \quad x > 0.$$