1 Chapter 13 Combining Inference with Utility

Problem 13.4 *Show mathematically that if there are only two possible actions, equation (13.34) for the probability of choosing one action reduces to a logistic function of the scaled difference in expected utility between that action and the alternative action.*

Answer

In softmax function, there are:
two actions: a_1 and a_2 with expected utilities U_1 and U_2;
β is the inverse temperature.

$$P(a_1) = \frac{e^{\beta U_1}}{e^{\beta U_1} + e^{\beta U_2}}.$$

We can factor out $e^{\beta U_1}$ from the numerator and denominator:

$$P(a_1) = \frac{1}{1 + e^{\beta(U_2 - U_1)}}.$$

Then we can name $U_1 - U_2$ as ΔU.

$$P(a_1) = \frac{1}{1 + e^{-\beta \Delta U}}.$$

This is exactly the **logistic function**:

$$P(a_1) = \sigma(\beta \Delta U),$$

since $\sigma(z) = \frac{1}{1+e^{-z}}$ is the standard logistic function.

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