Problem B.8

This problem is about the famous Monty Hall problem [155]. You are on a game show. The host shows you three doors. Behind one of them, a prize is hidden. You choose one door. The host, who knows behind which door the prize lies, opens a remaining door that does not contain the prize. The host then gives you the opportunity to switch your choice to the remaining unopened door or to stay with your original choice. Your door of choice gets opened and you receive the prize if it is there.

(a) To maximize the probability of receiving the prize, what should you do?

Answer: I should always switch. The probability of winning by switching is $\frac{2}{3}$, while the probability of winning by staying is only $\frac{1}{3}$.

(b) If there are N doors and the host opens m of them (where $m < n - 1$), what is the probability of receiving the prize under the best strategy?

Answer:

$$p(\text{win if stay}) = \frac{1}{N}, \qquad p(\text{win if switch}) = \frac{N-1}{N} \cdot \frac{1}{N-m-1} = \frac{N-1}{N(N-m-1)}.$$

(c) Would the answer to (a) change if the host did not know which of the two remaining doors contained a prize, but the one he opens just happens not to contain the prize? Explain.

Answer: Yes. In that case, the host's choice is not informative. The two remaining doors are symmetric, so:

$$p(\text{your door} \mid H) = p(\text{other door} \mid H) = \frac{1}{2}.$$

Switching or staying would have equal probability.

(d) Speculate on why most people believe it does not matter whether you stay or switch.

Answer: Most people intuitively assume equal probability between the two unopened doors (50–50), forgetting that the host's action is conditional on knowing where the prize is. This conditioning creates a bias that increases the probability for the remaining unopened door.