1 Chapter 3 Bayesian Inference under Measurement Noise

Problem 3.10 In section 3.4.1, we remarked that for Gaussian priors and likelihoods, posterior uncertainty — defined as standard deviation — is always smaller than both prior uncertainty and likelihood uncertainty. Construct an example involving non-Gaussian priors or likelihoods in which this property no longer holds.

In general, if either the prior or likelihood is not Gaussian, the posterior does not necessarily have lower variance. One example could be that, the prior and likelihood are not Gaussian distribution, but rather bimodal distribution. For example, in speech perception study of distinguishing whether the sound is "pa" or "ba" (McGurk Effect), when the voice onset time is ambiguous, participants tend to have two categories of perception. In this case, if the prior has two peaks and the likelihood has two peaks at different locations, the posterior could remain bimodal, meaning there is no single well-defined peak. So in this case, the posterior can actually have higher variance than either prior or the likelihood because of the increased uncertainty.