Problem 10.4

Problem 10.4 Refer back to section 10.2.2 on same–different judgments of binary stimuli. Unless $p_{\text{same}} = 0.5$, the decision rule $d > 0$ does not simplify analytically. However, we can still visualize the decision rule in (x_1, x_2) space: in which regions of this space does the Bayesian observer respond "same," and in which regions "different"? Our goal is to get an intuition for how σ and p_{same} interact to affect optimal behavior.

(a) Choose $\mu = 1$. Consider three values of σ (0.5, 1, and 2), and three values of p_{same} (0.4, 0.5, and 0.6). For each combination of σ and p_{same}, create a plot that shows the boundary between the "respond same" and "respond different" regions in (x_1, x_2) space, where either measurement can take values between -2 and 2. Display the nine plots in a 3×3 grid for easy comparison.

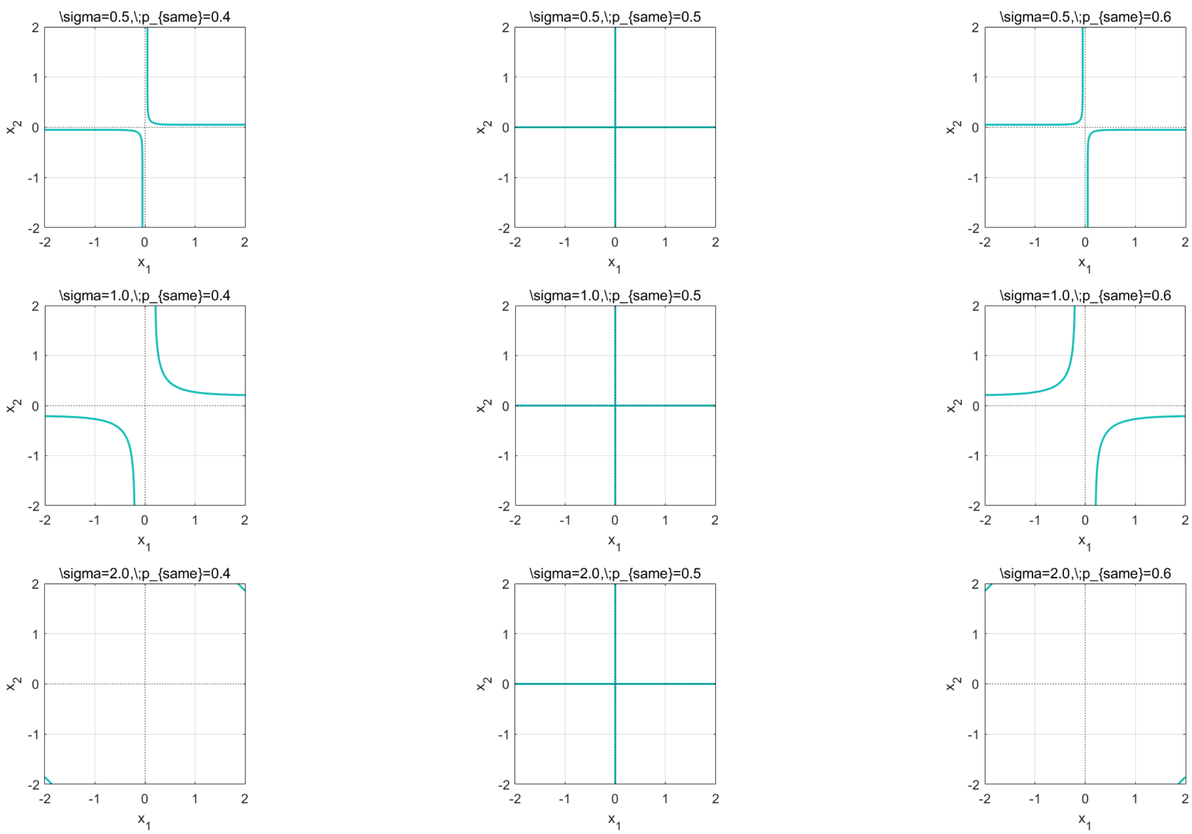

Figure 1: Decision boundary $d = 0$ for combinations of measurement noise σ and prior p_{same}. Rows (top to bottom): $\sigma = 0.5, 1, 2$. Columns (left to right): $p_{\text{same}} = 0.4, 0.5, 0.6$. Inside each boundary curve the observer reports "same."

(b) Describe and interpret the effects of σ and p_{same} on the decision boundary.

Answer.

- When $p_{\text{same}} = 0.5$ (middle column of Fig. 1), the boundary is exactly the two axes ($x_1 = 0$ and $x_2 = 0$), matching $(x_1) = (x_2)$.
- For $p_{\text{same}} < 0.5$ (left column), the prior favors "different," so the "same" region shrinks: only large, same-signed measurements fall inside the boundary.
- For $p_{\text{same}} > 0.5$ (right column), the prior favors "same," expanding the "same" region so that even modest evidence leads to "same."
- Smaller noise σ (top row) yields very steep likelihood ratios, producing sharply curved boundaries near the axes.
- Larger noise σ (bottom row) flattens the likelihood, so the boundary becomes more diagonal and the prior p_{same} dominates the decision.