Problem 10.2

Problem 10.2 Imagine you are a psychophysicist. Come up with an experiment, not covered in this chapter, to study whether people are Bayesian in their same–different judgments.

Proposed Experiment. We will test human judgment of whether two briefly flashed Gabor patches have the *same* orientation or *different* orientations under varying levels of orientation noise. Observers view two sequential Gabors (s_1, s_2) and press a key to report "same" or "different."

1. World state. Let
$$C = \begin{cases} 1, & \text{the two Gabors share the same true orientation,} \\ 2, & \text{they have independently drawn orientations.} \end{cases}$$
We assume a flat prior: $p(C=1) = p(C=2) = 0.5$.

2. Stimuli. True orientations s_1, s_2 are drawn from a uniform prior over $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$.
$$\begin{aligned} &\text{If } C=1, \quad s_1 = s_2 \sim \mathcal{U}\big[-\tfrac{\pi}{4}, \tfrac{\pi}{4}\big], \\ &\text{If } C=2, \quad s_1 \sim \mathcal{U}\big[-\tfrac{\pi}{4}, \tfrac{\pi}{4}\big], \quad s_2 \sim \mathcal{U}\big[-\tfrac{\pi}{4}, \tfrac{\pi}{4}\big] \text{ independently.} \end{aligned}$$

3. Measurements. The observer's internal measurements x_1, x_2 of each orientation are corrupted by Gaussian noise,
$$p(x_i \mid s_i) = \mathcal{N}\big(x_i;\, s_i, \sigma^2\big), \quad i = 1, 2,$$
with known noise standard deviation σ. We assume conditional independence:
$$p(x_1, x_2 \mid s_1, s_2) = p(x_1 \mid s_1)\, p(x_2 \mid s_2).$$

4. Inference. On each trial the observer computes the log posterior ratio (LPR),
$$d = \log \frac{p(C=1 \mid x_1, x_2)}{p(C=2 \mid x_1, x_2)} = \log \frac{p(x_1, x_2 \mid C=1)}{p(x_1, x_2 \mid C=2)} + \log \frac{p(C=1)}{p(C=2)}, \quad (1)$$
$$p(x_1, x_2 \mid C=1) = \int p(x_1 \mid s)\, p(x_2 \mid s)\, p(s)\, \mathrm{d}s, \quad (2)$$
$$p(x_1, x_2 \mid C=2) = \iint p(x_1 \mid s_1)\, p(s_1)\; p(x_2 \mid s_2)\, p(s_2)\, \mathrm{d}s_1\, \mathrm{d}s_2. \quad (3)$$
With $p(C=1) = p(C=2) = 0.5$, the decision rule is
$$\hat{C} = \begin{cases} 1, & d > 0 \quad \text{(report "same")}, \\ 2, & d \le 0 \quad \text{(report "different")}. \end{cases}$$

5. Behavioral test. We vary the true orientation difference $\Delta s = |s_2 - s_1|$ and noise level σ. For each $(\Delta s, \sigma)$ condition, we measure the proportion of "same" responses. A Bayesian observer predicts a psychometric curve $p(\hat{C} = 1 \mid \Delta s, \sigma)$ that falls smoothly from near 1 at $\Delta s = 0$ to 0.5 at large Δs, with steeper decline for smaller σ. By comparing human data to these predictions (via numerical marginalization), we can assess whether people follow Bayesian same–different inference.