1 Chapter 6 Learning as Inference

Problem 6.7 *In the island case study of section 6.2, what is the distribution of the number of days elapsed between rain days? Explain.*

Answer

Step 1: Generative model. Each day's weather is a Bernoulli trial:

$$p(x_i = 1) = r, \quad p(x_i = 0) = 1 - r,$$

where $x_i = 1$ denotes "rain on day i."

Step 2: Waiting-time distribution. Define the random variable T as the number of days from one rainy day until the next rainy day (inclusive). Because each day is independent and the process is memoryless, T follows a geometric distribution:

$$p(T = k) \;=\; (1 - r)^{k-1}\, r, \qquad k = 1, 2, 3, \ldots$$

That is, the probability of $k - 1$ dry days followed by a rain day.

Intuition. To wait exactly k days between rains, you must have $(k-1)$ consecutive dry days (each with probability $1 - r$) and then a rainy day (probability r). Hence

$$p(T = k) = (1 - r)^{k-1}\, r.$$