# 1 Chapter 8 Binary Classification

**Problem 8.6** *In this problem, we discuss unmodeled errors. Suppose that we perform an experiment with binary responses ($r = 0$ or $r = 1$) and that $p(r \mid s)$ expresses the predicted probability of the observer's response—under an arbitrary model, Bayesian or non-Bayesian—when the stimulus is $s$.*

(a) Suppose that the observer accidentally presses the wrong key on a proportion $\lambda$ of all trials. How does this change the predicted probability of the observer's response? Remark, much of the human psychophysics literature makes such an assumption and calls the relevant effect lapse.

(b) Suppose that the observer makes a random guess on a proportion $g$ of all trials (e.g., because he sometimes did not pay attention and didn't see the stimulus). How does this change the predicted probability of the observer's response?

## Answer

(a) *Lapse errors.* On each trial, with probability $\lambda$ the response is flipped. Therefore the new response distribution is

$$p_{\text{lapse}}(r \mid s) \;=\; (1-\lambda)\, p(r \mid s) \;+\; \lambda\, p(1-r \mid s).$$

In words: with probability $1-\lambda$ the model's original $p(r \mid s)$ holds, and with probability $\lambda$, the observer lapses and flips their intended response (so $r \to 1-r$).

(b) *Random guesses.* On a proportion $g$ of trials the observer guesses uniformly between $\{0, 1\}$. Thus

$$p_{\text{guess}}(r \mid s) \;=\; (1-g)\, p(r \mid s) \;+\; g\, \tfrac{1}{2},$$

since a pure guess gives each response probability 1/2. Equivalently,

$$p_{\text{guess}}(r \mid s) = (1-g)\, p(r \mid s) + \tfrac{g}{2}.$$