1 Chapter 3: Bayesian Inference under Measurement Noise

Problem 3.1 ***Let*** s ***be the stimulus of interest,*** x ***the measurement,*** $p_s(s)$ ***the stimulus distribution, and*** $p_{x|s}(x|s)$ ***the measurement distribution.***

(a) Write down the posterior distribution over hypothesized stimulus s, given an observed measurement x_{obs}.

(b) Which of the terms in your expression is called the likelihood function?

(c) What is the difference between the likelihood function and the measurement distribution?

1. (a) Using Bayes' theorem, the posterior distribution is:

$$p_{s|x}(s|x_{obs}) = \frac{p_{x|s}(x_{obs}|s)p_s(s)}{p_x(x_{obs})}$$

$p_{s|x}(s|x_{obs})$ is the Posterior distribution.

$p_{x|s}(x_{obs}|s)$ is the Likelihood function, describing how the measurement x_{obs} is generated given s.

$p_s(s)$ is the Prior distribution over s.

$p_x(x_{obs})$ is measurements distribution, also known as the Normalization term.

2. (b)

$$p_{x|s}(x_{obs}|s)$$

This is the Likelihood function. This function describe the probability of observing x_{obs} given a particular stimulus s. It never sums to 1.

3. (c)

The likelihood function is a function of the hypothesis s, while the measurement distribution is a function of observed data x_{obs}.

Also, likelihood function doesn't integrate to 1 over s, but measurement distribution integrates to 1 overall possible x_{obs}.