Solution

1 Generative assumptions

(a) **Tuning curve of neuron i:**

$$f_i(s) = a_i \exp\!\Big[-\tfrac{(s-s_i)^2}{2\sigma_{tc,i}^2}\Big].$$

(b) **Response variability (Poisson):**

$$p(r_i \mid s) = \frac{f_i(s)^{\,r_i}}{r_i!}\, e^{-f_i(s)}, \qquad r_i \in \mathbb{N}.$$

(c) **Independence across neurons:**

$$p(\mathbf{r} \mid s) = \prod_i p(r_i \mid s).$$

2 Full log-likelihood

$$\log \mathcal{L}_{\text{full}}(s) = \sum_i r_i \log f_i(s) \;-\; \sum_i f_i(s). \tag{1}$$

3 Constant-sum approximation

For a dense, approximately homogeneous population,

$$\sum_i f_i(s) \approx C \quad \text{for } s \text{ in the region of interest.} \tag{2}$$

Because C is independent of s, the second term in (1) is an additive constant and does not affect the maximiser or the curvature. Retaining only the data-dependent term,

$$\log \mathcal{L}_{\text{red}}(s) \doteq -\tfrac{1}{2} \sum_i \frac{r_i}{\sigma_{tc,i}^2}\,(s-s_i)^2. \tag{3}$$

4 Mode of the likelihood

Differentiate (3) and set to zero:

$$\frac{\mathrm{d}}{\mathrm{d}s} \log \mathcal{L}_{\text{red}}(s) = -\sum_i \frac{r_i}{\sigma_{tc,i}^2}(s-s_i) = 0 \implies \hat{s} = \frac{\sum_i \frac{r_i s_i}{\sigma_{tc,i}^2}}{\sum_i \frac{r_i}{\sigma_{tc,i}^2}}.$$

5 Width of the likelihood

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2} \log \mathcal{L}_{\text{red}}(s) = -\sum_i \frac{r_i}{\sigma_{tc,i}^2}, \qquad \sigma_{\text{like}}^2 = \Big(\sum_i \frac{r_i}{\sigma_{tc,i}^2}\Big)^{-1}.$$

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