1 Chapter 7 Discrimination and Detection

Problem 7.1 ***In medicine, it is common to encounter the terms sensitivity and specificity for a diagnostic test for a disease.***

These are synonyms for the true-positive rate and the true-negative rate, respectively. In addition, the (objectively correct) prior probability of a disease is called its *prevalence*. The *positive predictive value* (PPV) is the probability that someone has the disease given that they test positive. Use Bayes' rule to show that

$$PPV = \frac{sensitivity \cdot prevalence}{sensitivity \cdot prevalence + (1 - specificity) \cdot (1 - prevalence)}.$$

According to Bayes' Rule,

$$P(disease \mid positive) = \frac{P(positive \mid disease) \cdot P(disease)}{P(positive)}$$

According to the question, we can know that:

- Sensitivity $= P(positive \mid disease)$
- Specificity $= P(negative \mid nodisease)$
- Prevalence $= P(disease)$

According to Bayes' Rule we can know that:

$$P(positive) = P(positive \mid disease) \cdot P(disease) + P(positive \mid nodisease) \cdot P(nodisease)$$

Meanwhile, we can conclude that:

$$P(positive \mid nodisease) = 1 - specificity \quad and \quad P(nodisease) = 1 - prevalence$$

Now, substituting into Bayes' Rule:

$$P(disease \mid positive) = \frac{sensitivity \cdot prevalence}{sensitivity \cdot prevalence + (1 - specificity) \cdot (1 - prevalence)}$$

$$\Rightarrow PPV = \frac{sensitivity \cdot prevalence}{sensitivity \cdot prevalence + (1 - specificity) \cdot (1 - prevalence)}$$