Problem 10.1

Problem 10.1 Think of a real-world problem, not covered in this chapter, that would require same-different judgments. Formulate it in Bayesian terms.

Answer. A common forensic task is to decide whether two latent fingerprints, measured at a crime scene and on a suspect's file, originate from the same finger ("same") or from different fingers ("different"). We can cast this as a same-different judgment in Bayesian form exactly as in Section 10.2:

1. World state Let

   $$C = \begin{cases} 1, & \text{fingerprints from the } \textit{same} \text{ finger,} \\ 2, & \text{fingerprints from } \textit{different} \text{ fingers.} \end{cases}$$

   We assume a prior

   $$p(C = 1) = p_{\text{same}}, \quad p(C = 2) = 1 - p_{\text{same}}.$$

2. Latent stimuli Denote by s_1 and s_2 the true minutiae-feature vectors of the two prints. Under $C = 1$ they are identical draws from a population prior $p(s)$:

   $$s_1 = s_2 \sim p(s).$$

   Under $C = 2$, they are independent:

   $$s_1 \sim p(s), \quad s_2 \sim p(s).$$

3. Measurements The forensic examiner obtains noisy measurements x_1 and x_2 of each fingerprint feature vector. We model conditional independence and Gaussian noise:

   $$p(x_1, x_2 \mid s_1, s_2) = p(x_1 \mid s_1)\, p(x_2 \mid s_2),$$
   $$p(x_i \mid s_i) = \mathcal{N}\big(x_i; s_i, \sigma^2\big), \quad i = 1, 2.$$

4. Inference The observer computes the posterior odds (log posterior ratio, LPR):

   $$d = \log \frac{p(C = 1 \mid x_1, x_2)}{p(C = 2 \mid x_1, x_2)} = \log \frac{p(x_1, x_2 \mid C = 1)}{p(x_1, x_2 \mid C = 2)} + \log \frac{p_{\text{same}}}{1 - p_{\text{same}}}, \tag{1}$$

   $$p(x_1, x_2 \mid C = 1) = \int p(x_1 \mid s)\, p(x_2 \mid s)\, p(s)\, \mathrm{d}s, \tag{2}$$

   $$p(x_1, x_2 \mid C = 2) = \iint p(x_1 \mid s_1)\, p(s_1)\; p(x_2 \mid s_2)\, p(s_2)\, \mathrm{d}s_1\, \mathrm{d}s_2. \tag{3}$$

   The Bayesian decision rule is

   $$\hat{C} = \begin{cases} 1, & d > 0 \text{ (report "same")}, \\ 2, & d \le 0 \text{ (report "different")}. \end{cases}$$

This formulation parallels exactly the binary same-different model , now applied to fingerprint matching.