1 Chapter 2: Using Bayes' Rule

Problem 2.7 ***Prove, using Bayes' rule, that when the likelihood function is perfectly flat (has the same value for all hypotheses), the posterior distribution is identical to the prior distribution.***

If the likelihood function is flat, then it means for all possible hypotheses, they have the same likelihood. Thus,

$$P(Obs \mid Hypo) = constant, \quad \forall Hypo$$

So that Bayes' rule would now be:

$$P(Hypo \mid Obs) = \frac{c \cdot P(Hypo)}{P(Obs)}$$

Now, to compute P(Obs), which is the **normalization term**, it would be:

$$P(Obs) = \sum_{Hypo} P(Obs \mid Hypo)P(Hypo) = \sum_{Hypo} c \cdot P(Hypo) = c \sum_{Hypo} P(Hypo)$$

(note: Thanks Zoe for reminding!)

Since $P(Hypo)$ is a probability distribution, it sums to 1:

$$P(Obs) = c \cdot 1 = c$$

Substituting $P(Obs) = c$ back into the posterior:

$$P(Hypo \mid Obs) = \frac{c \cdot P(Hypo)}{c} = P(Hypo)$$

Therefore, when the likelihood function is flat (constant for all hypotheses), the posterior is identical to the prior.