1 Chapter 3 Bayesian Inference under Measurement Noise

Problem 3.5 ***The figure below shows a likelihood function and a posterior distribution. Both are Gaussian, with $\sigma_{posterior} = 1.2$ and $\sigma_{likelihood} = 1.5$. Assume that the prior is also Gaussian. Which of the following statements is true? Explain. You will need both the plot and the given numbers.***

(a) The prior is centered to the left of the likelihood function and is narrower.

(b) The prior is centered to the left of the likelihood function and is wider than it.

(c) The prior is centered to the right of the likelihood function and is narrower than it.

(d) The prior is centered to the right of the likelihood function and is wider than it.

Since both the likelihood and posterior are Gaussian, we can infer that the prior is also Gaussian. Thus, we can calculate that, the variance of prior would be:

$$\frac{1}{\sigma^2_{posterior}} = \frac{1}{\sigma^2_{likelihood}} + \frac{1}{\sigma^2_{prior}}$$

After plugging the known values, and I'm omitting the steps of plugging in values, we can get that:

$$\frac{1}{\sigma^2_{prior}} = 0.6944 - 0.4444 = 0.25$$

$$\sigma^2_{prior} = 4 \Rightarrow \sigma_{prior} = 2$$

Then, from the plot we can see that the Posterior distribution is to the left of the likelihood distribution. Since Bayesian update is a weighted process of combination of prior and likelihood means, so we can derive that the prior mean would also be to the left of the likelihood mean.
So, from the graph and value calculated, we would know that **(b) The prior is centered to the left of the likelihood function and is wider than it.** is the correct answer.