1 Chapter 7 Discrimination and Detection

Problem 7.6 ***Suppose the stimulus*** s ***can take two values:*** $s_+ = 1^\circ$ ***and*** $s_- = -1^\circ$***.***

Suppose that the measurement is normally distributed around s with standard deviation 0.5°*. On a given trial, the observer's measurement is* -0.1°*, and* $s = s_+$ *occurs on 80 percent of trials.*
Would an optimal observer report that the stimulus was s_+ or s_-? Provide all the steps in your reasoning.

So here we want to see that if there is a optimal Bayesian observer, whether they will choose s_+ or s_- given the measurement $x = -0.1^\circ$.

The intuitive logic is that, the observer should choose the stimulus with a higher posterior probability due to the Baye's rule:

$$p(s \mid x) \propto p(x \mid s)p(s)$$

First, we need to compute the likelihood function (assuming it's a Gaussian distribution here), and plug in $\sigma = 0.5$:

$$p(x \mid s) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-s)^2}{2\sigma^2}\right)$$

p(x = -0.1 | $s = s_+$) = $\frac{1}{\sqrt{2\pi(0.5)^2}} \exp\left(-\frac{(-0.1-1)^2}{2(0.5)^2}\right)$
$= \frac{1}{\sqrt{2\pi(0.25)}} \exp\left(-\frac{(-1.1)^2}{0.5}\right)$
$= \frac{1}{\sqrt{2\pi(0.25)}} \exp\left(-\frac{1.21}{0.5}\right)$
$= \frac{1}{\sqrt{0.5\pi}} \exp(-2.42)$

p(x = -0.1 | $s = s_-$) = $\frac{1}{\sqrt{2\pi(0.5)^2}} \exp\left(-\frac{(-0.1+1)^2}{2(0.5)^2}\right)$
$= \frac{1}{\sqrt{0.5\pi}} \exp\left(-\frac{0.81}{0.5}\right)$
$= \frac{1}{\sqrt{0.5\pi}} \exp(-1.62)$

Then,

$$p(s = s_+ \mid x) \propto p(x \mid s_+)p(s_+) = \exp(-2.42) \cdot 0.8$$
$$p(s = s_- \mid x) \propto p(x \mid s_-)p(s_-) = \exp(-1.62) \cdot 0.2$$

Finally, we can copmpare the posteriors:

$$\frac{p(s_+ \mid x)}{p(s_- \mid x)} = \frac{0.8\exp(-2.42)}{0.2\exp(-1.62)} = \frac{0.8}{0.2} \cdot \exp(-0.8) = 4 \cdot e^{-0.8} \approx 4 \cdot 0.4493 = 1.797$$

We can see that the ratio is greater than 1, $p(s_+ \mid x) > p(s_- \mid x)$, so the observer would report the stimulus is s_+.