1 Chapter 8 Binary Classification

Problem 8.2

In the context of section 8.5, an observer's *75 percent threshold* can be defined as the value of the stimulus for which the observer (who has observation noise of σ) reports "right" 75 percent of trials, minus the value of the stimulus for which the observer reports "right" 50 percent of trials.
How many standard deviations σ does the 75 percent-threshold correspond to?
As a reminder:

$$p(\hat{C} = 1 \mid s) = \Phi_{\text{standard}}\left(\frac{s - k}{\sigma}\right),$$

where Φ_{standard} denotes the cumulative distribution function (CDF) of the standard normal distribution.

1.1 Answer

To simplify the question, it's basically asking to find how many standard deviations σ correspond to the 75% threshold and the 50% threshold.

For stimulus at 75% response rate:

$$\Phi\left(\frac{s_{75} - k}{\sigma}\right) = 0.75.$$

Taking the inverse of the CDF:

$$\frac{s_{75} - k}{\sigma} = \Phi^{-1}(0.75) = 0.674 \Rightarrow s_{75} = k + 0.674\sigma.$$

For stimulus at 50% response rate:

$$\Phi\left(\frac{s_{50} - k}{\sigma}\right) = 0.5.$$

Since $\Phi^{-1}(0.5) = 0$, we get:

$$\frac{s_{50} - k}{\sigma} = 0 \Rightarrow s_{50} = k.$$

Then, we can know that:

$$\text{Threshold} = s_{75} - s_{50} = (k + 0.674\sigma) - k = 0.674\sigma.$$

So, the 75% threshold corresponds to $\boxed{0.674\sigma}$.