1 Chapter 6 Learning as Inference

Problem 6.3 *We start with equation (6.8) for the posterior over r in the island case study.*

(a) Write down an expression for the posterior over r if you observe only rain days.

(b) Plot this posterior for $n_{\text{rain}} = 1, 2, 5, 10$ (four curves in the same plot).

(c) For how many days would it have to rain for the posterior mean to be greater than 0.9?

Answer

(a) Under a uniform prior $p(r) = 1$ and observing n_{rain} rain days (and $n_{\text{dry}} = 0$ dry days), equation (6.8)

$$p(r \mid x) \;\propto\; r^{n_{\text{rain}}}\,(1-r)^{n_{\text{dry}}}$$

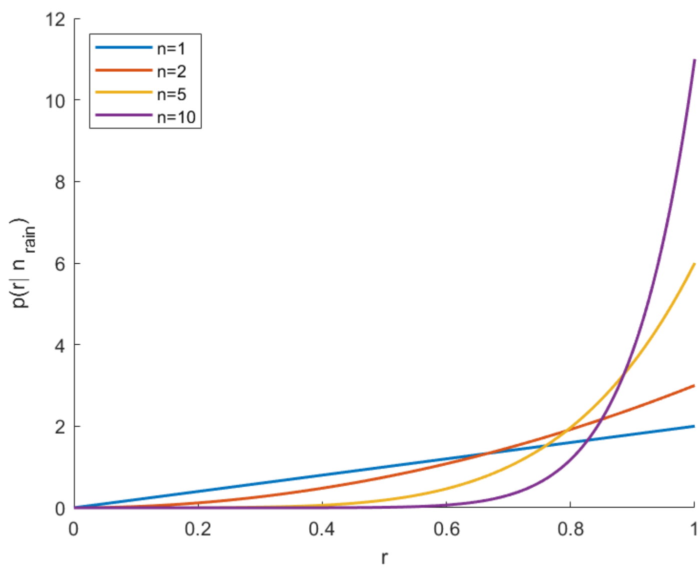
becomes

$$p(r \mid \underbrace{\text{rain}, \ldots, \text{rain}}_{n_{\text{rain}}}) \;\propto\; r^{n_{\text{rain}}}\,.$$

Normalizing over $r \in [0, 1]$ yields a Beta$(n_{\text{rain}} + 1,\, 1)$ density,

$$p(r \mid \text{all rain}) = \;(n_{\text{rain}} + 1)\, r^{n_{\text{rain}}}, \quad 0 \le r \le 1.$$

(b) The four curves $p(r \mid n_{\text{rain}}) = (n_{\text{rain}} + 1) r^{n_{\text{rain}}}$ for $n_{\text{rain}} = 1, 2, 5, 10$ are plotted below.



(c) The posterior mean under Beta$(n_{\text{rain}} + 1, 1)$ is

$$\mathbb{E}[r \mid n_{\text{rain}}] = \frac{(n_{\text{rain}} + 1)}{(n_{\text{rain}} + 1) + 1} = \frac{n_{\text{rain}} + 1}{n_{\text{rain}} + 2}.$$

We require

$$\frac{n_{\text{rain}} + 1}{n_{\text{rain}} + 2} > 0.9 \implies n_{\text{rain}} + 1 > 0.9\,(n_{\text{rain}} + 2) \implies 0.1\,n_{\text{rain}} > 0.8 \implies n_{\text{rain}} > 8.$$

Hence the smallest integer satisfying this is $n_{\text{rain}} = 9$.

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