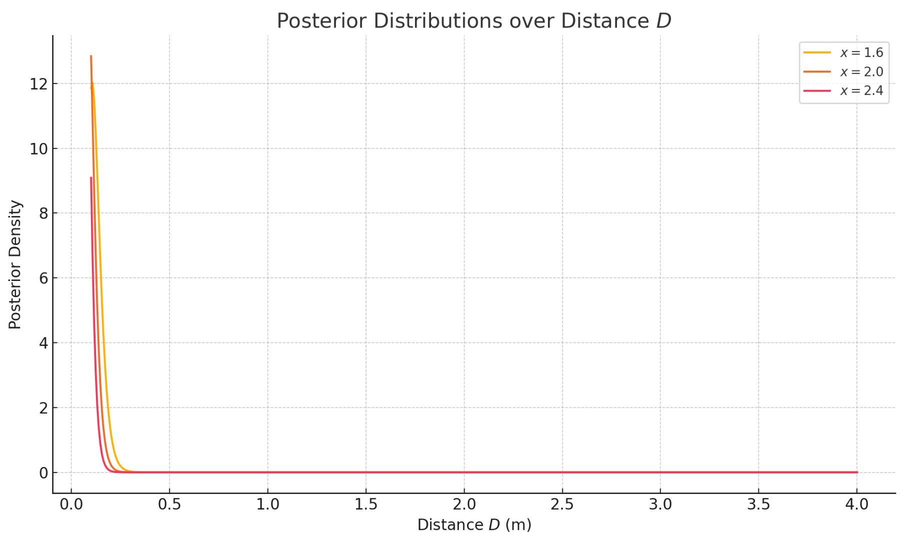
1 Chapter 9 Top-Level Nuisance Variables and Ambiguity

Problem 9.4 *In section 9.2, we used a lognormal distribution over the width w (i.e., $p(w) =$ Lognormal$(w; \mu_w, \sigma_w^2)$). Using 1 meter as the implicit unit in all cases, let us choose $\mu_w = 2$ (cars may be 2 meters wide), $\sigma_w = 0.3$ (most cars are between 1.7 and 2.3 meters wide), and eye diameter $l = 0.025$ (the eye is roughly 2.5 cm in diameter).*

(a) Consider three retinal observations of width, $x = 1.6$, $x = 2$, and $x = 2.4$. For each retinal width, plot the posterior probability density over distance (single plot, three curves, color-coded).

(b) Discuss how these posteriors compare to each other, and why so.

a)



As the retinal width increases, the estimated distance decreases, reflecting the inverse relationship between image size and distance. This aligns with the size–depth relationship: a larger image on the retina suggests the object is closer. The shift in the modes illustrates how sensory input (retinal image size) modulates the inferred distance following a Bayesian framework.

b)
When $x = 2.4$, the retinal image is large, suggesting the object is relatively close. The posterior peaks at a smaller D, and its probability mass is concentrated toward the left (near distances).
When $x = 1.6$, the retinal image is small, indicating that the object is likely farther away. The posterior shifts right, and the peak moves to a larger value of D.
When $x = 2.0$, the posterior falls between the other two, corresponding to a moderate perceived distance.

Under the generative model $x = \frac{lw}{D}$, for a given image size x, a larger distance D implies a larger object width w. Since w is uncertain, the observer infers D by marginalizing over possible w values. This uncertainty over w causes the posterior over D to also be spread out — specifically, lognormal in shape — and its mode to depend on the observed value of x.
Conclusion: As x increases, the observer infers a closer object, but the inference is uncertain due to unknown w.