

MathTools HW1

%% Question 4
% 2024-9-13

a) If the retinal neuron is only reaction at certain location, then in general the expression of that neuron can be $[0, \dots, v_1, \dots, v_2, \dots, v_3, \dots, v_4, \dots, v_5]$. In short, it would be as stated in question, $v = [v_1, v_2, v_3, v_4, v_5]$.

Given that the vector is $v = [v_1, v_2, v_3, v_4, v_5]$ and the weighted output is $r = v_1 + 2.3v_2 + 1.5v_3 + v_4 + 6v_5$, we can obtain that the system, let's say w , would be $w = [1, 2.3, 1.5, 1, 6]$.

Therefore, the system is linear.

b) Since it is asked to get the maximum output with unit-length input, then $\|v\| = 1$. The question is asking $r = WV$ to be maximum, thus r to be maximum.

Understanding that $WV = \|w\| \cdot \|v\| \cdot \cos(\theta)$. Then, $\cos(\theta)$ need to be largest, where when θ is 0, $\cos(\theta)$ would be 1. In geometry representation, the vector v and w would need to be in same direction. Following calculation please see attached hand written section.

In conclusion, thinking geometrically and in 2D, the dot product will reach its maximum when the vectors lie in the same line and in positive values.

c) To reach the minimal output, the vector needed would be any unit vector that has 0 input anywhere else, but 1 at one of the smallest weighted term. For example, in w , 1st and 4th term is weighted 1, so the input can be:

$v(\min) = [1, 0, 0, 0, 0]$; or, $v(\min) = [0, 0, 0, 1, 0]$.

For a vector to be physically realizable, it means that it can actually occur in reality, thus the vector can not be all-zeros.