MathTools HW1

```
%% Question 6
%2024-9-25
```

a)

The null space is also called the kernal of a matrix. It refers to the space where all vectors in the inputs space will be mapping to zero in the output space.

The range space refers to the column space of a matrix, the set of all possible linear combinations of its column vectors. No matter what you put in input. The output is always lying in the range space (the possible space that the output vector can lies in).

b)

If the creature has a non-zero null space, that means when it received non-zero pressure inputs, it will result in zero neuronal responses. More specifically, it means that the creature has limited tactile snesory scope, meaning that it's unable to perceive certain pressure stimuli.

```
%c)
S = load('mtxExamples2024.mat');

for idx = 1:5
    fileName = sprintf('mtx%d',idx);
    M = S.(fileName);

    fprintf('--- Processing Matrix %s ---\n', fileName);

    %Goal 1: determine if there's a nullspace
    [U, S_matrix, V] = svd(M); singular_values = diag(S_matrix);

    boundary = 10^-3;

    %if the diagnol of s matrix is zero
    null_columns = find(all(S_matrix < boundary, 1));

    if size(null_columns,2)>0
        fprintf('There is a nullspace of size %d\n', size(null_columns, 2))
        xCord =  V(:, null_columns) * randn(size(null_columns, 2), 1);
        fprintf('The vector in the null space is:')
        disp(xCord);
        yCord = M*xCord;
        fprintf('Sanity check (should be near zero):')
        disp(yCord);
    else
        fprintf('No nullspace exists for this matrix')

    end %if
```

```
    range_rows = find(singular_values > boundary, 2);

    range_indices = find(singular_values >= boundary);
    S_pinv = zeros(size(S_matrix'));

        for i = 1:length(range_indices)
        idk = range_indices(i);
        S_pinv(idk, idk) = 1 / singular_values(idk);
        end %for


    y_range = U(:, range_rows) * randn(size(range_rows, 1), 1);
    %x_range = pinv(M) * y_range; %custom function for pseudoinverse

    M_pinv = V * S_pinv * U';
    x_range = M_pinv * y_range;
    fprintf('The vector in the range space is:')
    y_range_recomputed = M * x_range;
    disp(y_range_recomputed);

    fprintf('Sanity check (should be same as y_range_recomputed):')
    disp(y_range);

end %for
```

```
--- Processing Matrix mtx1 ---
No nullspace exists for this matrix
The vector in the range space is:
    0.2594
   -0.0598
   -0.4061
Sanity check (should be same as y_range_recomputed):
    0.2594
   -0.0598
   -0.4061
--- Processing Matrix mtx2 ---
There is a nullspace of size 2
The vector in the null space is:
   -0.3858
   -0.1679
   -0.1337
Sanity check (should be near zero):
   1.0e-15 *

   -0.0069
   -0.1110
         0
The vector in the range space is:
   -0.1384
   -1.8457
    0.8183
Sanity check (should be same as y_range_recomputed):
   -0.1384
   -1.8457
    0.8183
--- Processing Matrix mtx3 ---
There is a nullspace of size 1
The vector in the null space is:
```

```
   -1.6327
   -1.5201
    0.3517
Sanity check (should be near zero):
   1.0e-14 *

   -0.0278
    0.1082
The vector in the range space is:
    1.1312
   -1.9506
Sanity check (should be same as y_range_recomputed):
    1.1312
   -1.9506
--- Processing Matrix mtx4 ---
No nullspace exists for this matrix
The vector in the range space is:
    0.1405
    1.9364
Sanity check (should be same as y_range_recomputed):
    0.1405
    1.9364
--- Processing Matrix mtx5 ---
There is a nullspace of size 2
The vector in the null space is:
    0.4504
    0.2808
    0.3791
Sanity check (should be near zero):
   1.0e-16 *

   -0.5551
    0.0694
The vector in the range space is:
    0.4181
   -0.0625
Sanity check (should be same as y_range_recomputed):
    0.4181
   -0.0625
```