Perception Assignment 5

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12/12/2023

1 Question 1

1.1 a

If both cues are independent Gaussian random variables and they are unbiased. Then it implies that the value of these two cues are equal to the actual value of the depth.

1.2 b

Naming the two cues $\text{cue}_1$and cue_2, so the weights will be weight_1 and weight_2, where :
$\text{weight}_1 = \frac{{\sigma_2}^2}{{\sigma_1}^2 + {\sigma_2}^2}$ and $\text{weight}_2 = \frac{{\sigma_1}^2}{{\sigma_1}^2 + {\sigma_2}^2}$, and $\text{weight}_1 + \text{weight}_2 = 1$.
Therefore, $\text{weight}_1 = \frac{{\sigma_2}^2}{{\sigma_1}^2 + {\sigma_2}^2} = \frac{\text{Var}(\text{cue}_2)}{\text{Var}(\text{cue}_1) + \text{Var}(\text{cue}_2)} = \frac{1}{4+1} = 0.2$. Thus, $\text{weight}_2 = 0.8$.

1.3 c

Because $\text{weight}_1 = \text{weight}_2 = 0.5$, so $\text{Var}(\text{cue}_2) = 0.5^2$ x $\text{Var}(\text{cue}_1) + 0.5^2$ x $\text{Var}(\text{cue}_2)$.
Therefore, the variance of cue_2 is 1.6. And the variance of cue_1 is 1.18. The difference is approximately 1.25.
The optimal variance is $\frac{1.25}{0.8} = 1.56$.

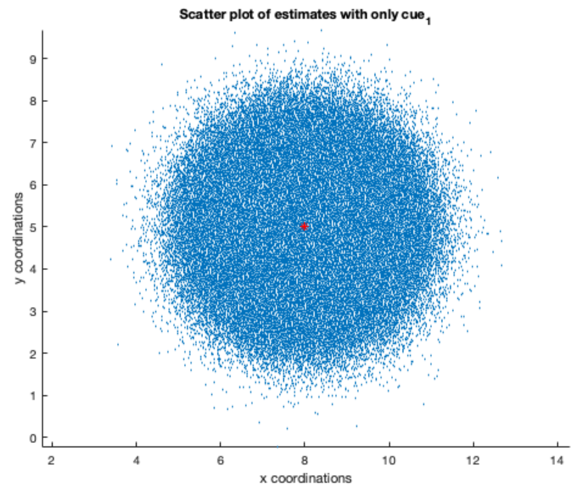
1.4 d

For the averaged-weighted cues in 1c, the estimated variance is 1.25. In original, the lowest variance is 1. Therefore, for averages-weights, both variances are higher than the lowest variance. Thus, it's better for David to just use the lower variance and discard the other.

2 Question 2

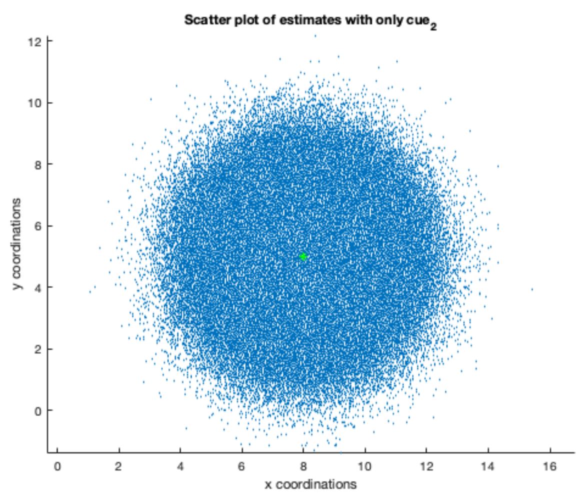
2.1 a

Figure 1: Presentation of Michael's performance with only cue_1



Michael's possibility of hitting the target with only cue 1 is 0.3931.

Figure 2: Presentation of Michael's performance with only cue_1



Michael's possibility of hitting the target with only cue 2 is 0.2212.

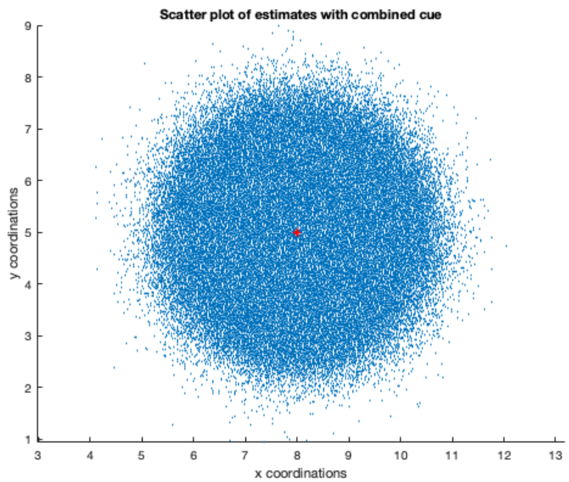
```

clear; close all; clc;

tRadius= 1; % radius of the target
x = randi(10); % The x-coordinate of the center of the target
y = randi(10); % The y-coordinate of the center of the target
trials = 1e6;

var1 = 1; % variance of both Gaussians for cue1
sig1 = sqrt(var1);
var2 = 4; % variance of both Gaussians for cue2
sig2 = sqrt(var2);

% Drawing samples from Gaussians
```

```
X1 = x+ mvnrnd(0, sig1, trials);
X2 = x+ mvnrnd(0, sig2, trials);
Y1 = y+mvnrnd(0, sig1, trials);
Y2 = y +mvnrnd(0, sig2, trials);

figure();
scatter(X1, Y1, 1)
hold on;
plot(x, y, 'r*')
xlabel('x coordinations')
ylabel('y coordinations')
title('Scatter plot of estimates with only cue_1')
axis equal

distance = sqrt((X1 - x).^2 + (Y1 - y).^2);
cCorrect = sum(distance <= tRadius);
pCorrect = cCorrect/trials;

figure();
scatter(X2, Y2, 1)
hold on;
plot(x, y, 'g*')
xlabel('x coordinations')
ylabel('y coordinations')
title('Scatter plot of estimates with only cue_2')
axis equal
distance2 = sqrt((X2 - x).^2 + (Y2 - y).^2);
cCorrect2 = sum(distance2 <= tRadius);
pCorrect2 = cCorrect2/trials;
```

2.2 b

$$X = w^{*}X_1 + (1 - w) * X_2$$
$$\mathrm{Y} = v^{*}Y_1 + (1 - v) * Y_2$$
$$w = \frac{Var(X_2)}{Var(X_1) + Var(X_2)}$$
$$v = \frac{Var(Y_2)}{Var(Y_1) + Var(Y_2)}$$

Figure 3: Scatter plot of estimates with combined cue



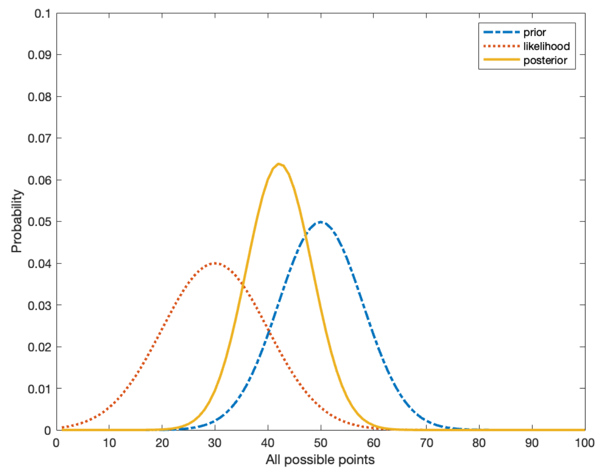
From the scatter plot, we can see that the data points become denser. Meanwhile, Michael's possibility of hitting the target with combined cue is 0.501. These are suggesting that Michael getting higher probability to hit the target correctly.

```
w = var2/(var1 + var2);
v = var2/(var1 + var2);

Xb = w * X1 + (1 - w) * X2;
Yb = v .* Y1 + (1 - v) .* Y2;

figure();
scatter(Xb,Yb, 1)
hold on;
plot(x, y, 'r*')
xlabel('x coordinations')
ylabel('y coordinations')
title('Scatter plot of estimates with combined cue')
axis equal
distance44 = sqrt((Xb - x).^2 + (Yb - y).^2);
cCorrect44 = sum(distance44 <= tRadius);
pCorrect44 = cCorrect44/trials;
```

3 Question 3

3.1 a

Figure 4: Prior, Likelihood and Posterior distributions of Denis



So firstly, the current location of Denis can be framed into a coordinate X, and all possible locations of the shopping mall would be 100.

For the initial belief of Denis' movement, this belief would be the **prior probability** in Bayesian estimation system. Given that Denis prefers to be near the center of the mall (location 50), this preference is modeled as a Gaussian distribution with a mean (central tendency) of 50 and a variance of 64. The prior probability distribution reflects Marisa's initial belief about Denis' location based on her knowledge of his preferences.

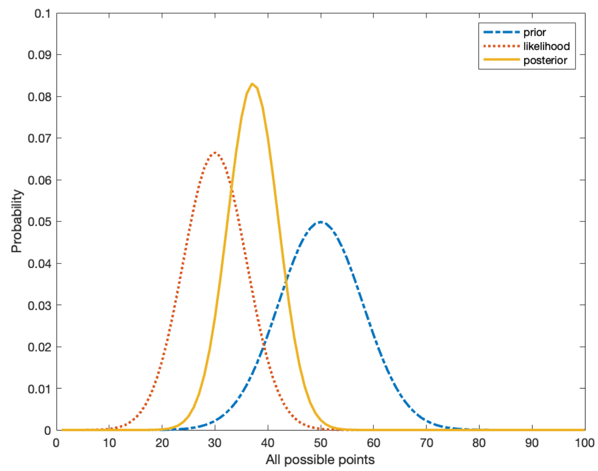
For the coffee cup brand, that is the only brand that Denis drink, so it represents the **likelihood**, in other words, the probability of observing the future evidence.

For Denis' posterior distribution, what we aim to compute, it's the **posterior distribution** in Bayesian estimation system. The posterior probability combines the prior probability and the likelihood, providing a new probability distribution that reflects our updated belief about Denis' location after finding the coffee cup.

```
x_all= 1:100;
Denis_prior_mean = 50;
Denis_prior_sd = sqrt(64);

prior_unnormalized = normpdf(x_all, Denis_prior_mean, Denis_prior_sd);
%prior_normalized_1 = prior_unnormalized ./ max(prior_unnormalized);
prior_normalized_2 = prior_unnormalized ./ sum(prior_unnormalized);

likelihood_mean = 30;
likelihood_sd = sqrt(100);
likelihood_unnormalized = normpdf(x_all, likelihood_mean, likelihood_sd);
%likelihood_normalized_1 = likelihood_unnormalized ./ max(likelihood_unnormalized);
likelihood_normalized_2 = likelihood_unnormalized ./ sum(likelihood_unnormalized);

posterior_unnormalized = prior_normalized_2 .* likelihood_normalized_2;
%posterior_unnormalized = prior_normalized_1 .* likelihood_normalized_1;
posterior_normalized_2 = posterior_unnormalized ./ sum(posterior_unnormalized);

figure();
plot(x_all, prior_normalized_2, '-.','DisplayName', 'prior', 'LineWidth', 2)
hold on;
plot(x_all, likelihood_normalized_2, ':', 'DisplayName', 'likelihood', 'LineWidth', 2)
plot(x_all, posterior_normalized_2, 'DisplayName', 'posterior', 'LineWidth', 2)
xlabel('All possible points')
ylabel('Probability')
ylim([0, 0.1])
legend()
```

3.2 b

Figure 5: Prior, Likelihood and Posterior distributions of Denis



In the second scenario, The mean remains at X=30, but the variance is now smaller, at 36 instead of 100.

The likelihood function, which represents the probability of finding the coffee cup given Denis' location, is still centered at 30 but is now more "confident" (i.e., has less variance). A smaller variance (36 instead of 100) in the likelihood means the evidence (the coffee cup's location) suggests a stronger belief that Denis was near the location where the cup was found.

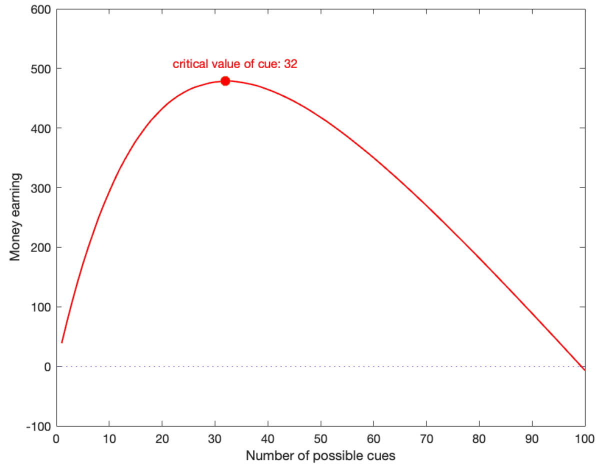
For the posterior distribution, a smaller variance in the likelihood means the likelihood mean (30) will have more influence on the posterior mean. Therefore, the posterior mean will shift closer to 30 compared to the previous scenario. The posterior variance is inversely related to the sum of the reciprocals of the prior and likelihood variances. A smaller variance in the likelihood means the denominator in this calculation increases, resulting in a smaller posterior variance. Hence, the posterior distribution will be narrower, indicating increased confidence in the estimation of Denis' location.

Finally, the change in the posterior distribution make sense. The updated likelihood indicates that the evidence (the coffee cup's location) is more strongly indicative of Denis' location than initially thought. As a result, the posterior distribution adjusts to reflect this increased confidence: it shifts towards the location where the cup was found and becomes narrower, indicating a more precise estimate of Denis' location.

```

x_all= 1:100;
Denis_prior_mean = 50;
Denis_prior_sd = sqrt(64);

prior_unnormalized = normpdf(x_all, Denis_prior_mean, Denis_prior_sd);
%prior_normalized_1 = prior_unnormalized ./ max(prior_unnormalized);
prior_normalized_2 = prior_unnormalized ./ sum(prior_unnormalized);

likelihood_mean = 30;
likelihood_sd = sqrt(36);
likelihood_unnormalized = normpdf(x_all, likelihood_mean, likelihood_sd);
%likelihood_normalized_1 = likelihood_unnormalized ./ max(likelihood_unnormalized);
likelihood_normalized_2 = likelihood_unnormalized ./ sum(likelihood_unnormalized);

posterior_unnormalized = prior_normalized_2 .* likelihood_normalized_2;
posterior_normalized_2 = posterior_unnormalized ./ sum(posterior_unnormalized);

figure();
plot(x_all, prior_normalized_2, '-.','DisplayName', 'prior', 'LineWidth', 2)
hold on;
plot(x_all, likelihood_normalized_2, ':', 'DisplayName', 'likelihood', 'LineWidth', 2)
plot(x_all, posterior_normalized_2, 'DisplayName', 'posterior', 'LineWidth', 2)
xlabel('All possible points')
ylabel('Probability')
ylim([0, 0.1])
legend()
```

4 Question 4

Figure 6: This is an image



The maximum number of cues that Roozbeh can use in order to gain most money is 32.
Following procedure in Question 2, the possibility of getting the game correct can be calculated as follow:

```
clear; close all; clc;

trials = 1e6;
x = randi(10).*ones(trials, 1); % x center of all targets
y = randi(10).*ones(trials, 1); % y center of all targets
tRadius= ones(trials, 1); % All radius for ragets
possible_cues = 1:100;
var = 10.*ones(trials, 1); % All variance of targets
%sig = sqrt(10).*ones(trials, 1);
sig = sqrt(10);

money = 1000;
cost = 10;


%empty container
x_choice = nan(trials, length(possible_cues));
y_choice = nan(trials, length(possible_cues));


X_cue = x + sig.*randn(trials, length(possible_cues));
Y_cue = y + sig.*randn(trials, length(possible_cues));


%Generate weighted results
for jj = possible_cues
    w = ones(jj, 1)./jj;
    v = ones(jj, 1)./jj;
    x_choice(:,jj) = X_cue(:, 1:jj)*w;
    y_choice(:,jj) = Y_cue(:, 1:jj)*w;
end

distance_3b = sqrt((x_choice - x).^2 + (y_choice - y).^2);
cCorrect3 = sum(distance_3b <= tRadius);
pCorrect3 = cCorrect3/trials;
```

Now, the possible prize is 1000 dollar. Therefore, all the possible case of winning money can be computed as follow, and the maximum of these possible case would be the highest money to earn:

```
gain = 1000.*pCorrect3;
```

```
lose = cost*possible_cues;

money_in_pocket = gain - lose;
[max_money_gain, max_count] = max(money_in_pocket);

figure();
plot (possible_cues,money_in_pocket, 'LineWidth', 1.2, 'Color', 'r'); hold on;
plot (possible_cues(max_count), max_money_gain, 'o', 'MarkerSize', 8, 'MarkerFaceColor','r')
text(possible_cues(max_count)-10, max_money_gain+30, 'critical value of cue: 32', 'Color', 'r')
xlabel('Number of possible cues')
ylabel('Money earning')
ylim([-100 600]);
yline(0, ':', 'Color', 'b');
```