

In [1]:

```
import tensorflow as tf
#import tensorflow_datasets as tfds
```

## Baseline model

In [2]:

```
# Load Data
# tfds.list_builders()
# (train, test), info = tfds.load("mnist", split=['train', 'test'], with_info=True, as_supervised=True)
(x_train, y_train), (x_test, y_test) = tf.keras.datasets.mnist.load_data()
#(x_train, y_train), (x_test, y_test) = tf.keras.datasets.fashion_mnist.load_data()
```

Downloading data from <https://storage.googleapis.com/tensorflow/tf-keras-datasets/mnist.npz>  
11493376/11490434 [=====] - 0s 0us/step  
11501568/11490434 [=====] - 0s 0us/step

In [3]:

```
# Baseline model definition
model = tf.keras.Sequential([
    tf.keras.layers.Flatten(input_shape=(28, 28)),
    tf.keras.layers.Dense(16, activation='relu'),
    tf.keras.layers.Dense(16, activation='relu'),
    tf.keras.layers.Dense(10, activation='softmax')
])
```

In [4]:

```
# Baseline model compilation
model.compile(optimizer='adam',
              loss='sparse_categorical_crossentropy',
              metrics=['sparse_categorical_accuracy'])
```

In [5]:

```
# Baseline model fitting
history = model.fit(x_train, y_train,
                    batch_size=128,
                    epochs=100,
                    validation_data=(x_test, y_test),
                    verbose=2)
```

```
Epoch 1/100
469/469 - 2s - loss: 3.0030 - sparse_categorical_accuracy: 0.2711 - val_loss: 1.6179 - val_sparse_categorical_accuracy: 0.4191 - 2s/epoch - 5ms/step
Epoch 2/100
469/469 - 2s - loss: 1.4428 - sparse_categorical_accuracy: 0.4830 - val_loss: 1.3313 - val_sparse_categorical_accuracy: 0.4995 - 2s/epoch - 4ms/step
Epoch 3/100
469/469 - 2s - loss: 1.2584 - sparse_categorical_accuracy: 0.5262 - val_loss: 1.1994 - val_sparse_categorical_accuracy: 0.5413 - 2s/epoch - 5ms/step
Epoch 4/100
469/469 - 2s - loss: 1.1566 - sparse_categorical_accuracy: 0.5508 - val_loss: 1.1276 - val_sparse_categorical_accuracy: 0.5585 - 2s/epoch - 5ms/step
Epoch 5/100
469/469 - 2s - loss: 1.0960 - sparse_categorical_accuracy: 0.5685 - val_loss: 1.0801 - val_sparse_categorical_accuracy: 0.5861 - 2s/epoch - 5ms/step
Epoch 6/100
469/469 - 2s - loss: 1.0202 - sparse_categorical_accuracy: 0.6041 - val_loss: 1.0075 - val_sparse_categorical_accuracy: 0.6164 - 2s/epoch - 4ms/step
Epoch 7/100
469/469 - 2s - loss: 0.9718 - sparse_categorical_accuracy: 0.6199 - val_loss: 0.9835 - val_sparse_categorical_accuracy: 0.6279 - 2s/epoch - 3ms/step
Epoch 8/100
469/469 - 1s - loss: 0.9337 - sparse_categorical_accuracy: 0.6355 - val_loss: 0.9531 - val_sparse_categorical_accuracy: 0.6477 - 1s/epoch - 2ms/step
Epoch 9/100
469/469 - 1s - loss: 0.8968 - sparse_categorical_accuracy: 0.6641 - val_loss: 0.9002 - val_sparse_categorical_accuracy: 0.6796 - 1s/epoch - 2ms/step
Epoch 10/100
469/469 - 1s - loss: 0.8245 - sparse_categorical_accuracy: 0.7085 - val_loss: 0.8013 - val_sparse_categorical_accuracy: 0.7446 - 1s/epoch - 2ms/step
Epoch 11/100
469/469 - 1s - loss: 0.7212 - sparse_categorical_accuracy: 0.7652 - val_loss: 0.7072 - val_sparse_categorical_accuracy: 0.7914 - 1s/epoch - 2ms/step
```

Epoch 12/100  
469/469 - 1s - loss: 0.6356 - sparse\_categorical\_accuracy: 0.8150 - val\_loss: 0.6343 - val\_sparse\_categorical\_accuracy: 0.8227 - 1s/epoch - 2ms/step

Epoch 13/100  
469/469 - 1s - loss: 0.5478 - sparse\_categorical\_accuracy: 0.8456 - val\_loss: 0.5593 - val\_sparse\_categorical\_accuracy: 0.8567 - 1s/epoch - 2ms/step

Epoch 14/100  
469/469 - 1s - loss: 0.4759 - sparse\_categorical\_accuracy: 0.8668 - val\_loss: 0.4906 - val\_sparse\_categorical\_accuracy: 0.8776 - 1s/epoch - 2ms/step

Epoch 15/100  
469/469 - 1s - loss: 0.4035 - sparse\_categorical\_accuracy: 0.8884 - val\_loss: 0.4248 - val\_sparse\_categorical\_accuracy: 0.8959 - 1s/epoch - 2ms/step

Epoch 16/100  
469/469 - 1s - loss: 0.3590 - sparse\_categorical\_accuracy: 0.9013 - val\_loss: 0.4028 - val\_sparse\_categorical\_accuracy: 0.9026 - 1s/epoch - 2ms/step

Epoch 17/100  
469/469 - 1s - loss: 0.3270 - sparse\_categorical\_accuracy: 0.9095 - val\_loss: 0.3650 - val\_sparse\_categorical\_accuracy: 0.9107 - 1s/epoch - 2ms/step

Epoch 18/100  
469/469 - 1s - loss: 0.2986 - sparse\_categorical\_accuracy: 0.9184 - val\_loss: 0.3508 - val\_sparse\_categorical\_accuracy: 0.9152 - 1s/epoch - 2ms/step

Epoch 19/100  
469/469 - 1s - loss: 0.2795 - sparse\_categorical\_accuracy: 0.9241 - val\_loss: 0.3176 - val\_sparse\_categorical\_accuracy: 0.9214 - 1s/epoch - 2ms/step

Epoch 20/100  
469/469 - 1s - loss: 0.2627 - sparse\_categorical\_accuracy: 0.9278 - val\_loss: 0.3132 - val\_sparse\_categorical\_accuracy: 0.9251 - 1s/epoch - 2ms/step

Epoch 21/100  
469/469 - 1s - loss: 0.2505 - sparse\_categorical\_accuracy: 0.9317 - val\_loss: 0.2973 - val\_sparse\_categorical\_accuracy: 0.9257 - 1s/epoch - 2ms/step

Epoch 22/100  
469/469 - 1s - loss: 0.2356 - sparse\_categorical\_accuracy: 0.9351 - val\_loss: 0.3092 - val\_sparse\_categorical\_accuracy: 0.9251 - 1s/epoch - 2ms/step

Epoch 23/100  
469/469 - 1s - loss: 0.2246 - sparse\_categorical\_accuracy: 0.9383 - val\_loss: 0.2807 - val\_sparse\_categorical\_accuracy: 0.9332 - 1s/epoch - 2ms/step

Epoch 24/100  
469/469 - 1s - loss: 0.2153 - sparse\_categorical\_accuracy: 0.9402 - val\_loss: 0.2752 - val\_sparse\_categorical\_accuracy: 0.9339 - 1s/epoch - 2ms/step

Epoch 25/100  
469/469 - 1s - loss: 0.2054 - sparse\_categorical\_accuracy: 0.9436 - val\_loss: 0.2642 - val\_sparse\_categorical\_accuracy: 0.9336 - 1s/epoch - 2ms/step

Epoch 26/100  
469/469 - 1s - loss: 0.2000 - sparse\_categorical\_accuracy: 0.9440 - val\_loss: 0.2523 - val\_sparse\_categorical\_accuracy: 0.9347 - 1s/epoch - 2ms/step

Epoch 27/100  
469/469 - 1s - loss: 0.1919 - sparse\_categorical\_accuracy: 0.9475 - val\_loss: 0.2554 - val\_sparse\_categorical\_accuracy: 0.9349 - 1s/epoch - 2ms/step

Epoch 28/100  
469/469 - 1s - loss: 0.1869 - sparse\_categorical\_accuracy: 0.9478 - val\_loss: 0.2470 - val\_sparse\_categorical\_accuracy: 0.9376 - 1s/epoch - 2ms/step

Epoch 29/100  
469/469 - 1s - loss: 0.1832 - sparse\_categorical\_accuracy: 0.9491 - val\_loss: 0.2448 - val\_sparse\_categorical\_accuracy: 0.9403 - 1s/epoch - 2ms/step

Epoch 30/100  
469/469 - 1s - loss: 0.1763 - sparse\_categorical\_accuracy: 0.9504 - val\_loss: 0.2420 - val\_sparse\_categorical\_accuracy: 0.9378 - 1s/epoch - 2ms/step

Epoch 31/100  
469/469 - 1s - loss: 0.1720 - sparse\_categorical\_accuracy: 0.9513 - val\_loss: 0.2346 - val\_sparse\_categorical\_accuracy: 0.9424 - 1s/epoch - 2ms/step

Epoch 32/100  
469/469 - 1s - loss: 0.1697 - sparse\_categorical\_accuracy: 0.9529 - val\_loss: 0.2447 - val\_sparse\_categorical\_accuracy: 0.9398 - 1s/epoch - 2ms/step

Epoch 33/100  
469/469 - 1s - loss: 0.1673 - sparse\_categorical\_accuracy: 0.9534 - val\_loss: 0.2414 - val\_sparse\_categorical\_accuracy: 0.9406 - 1s/epoch - 2ms/step

Epoch 34/100  
469/469 - 1s - loss: 0.1625 - sparse\_categorical\_accuracy: 0.9542 - val\_loss: 0.2318 - val\_sparse\_categorical\_accuracy: 0.9399 - 1s/epoch - 2ms/step

Epoch 35/100  
469/469 - 1s - loss: 0.1620 - sparse\_categorical\_accuracy: 0.9538 - val\_loss: 0.2272 - val\_sparse\_categorical\_accuracy: 0.9443 - 1s/epoch - 2ms/step

Epoch 36/100  
469/469 - 1s - loss: 0.1616 - sparse\_categorical\_accuracy: 0.9538 - val\_loss: 0.2394 - val\_sparse\_categorical\_accuracy: 0.9416 - 1s/epoch - 2ms/step

Epoch 37/100  
469/469 - 1s - loss: 0.1542 - sparse\_categorical\_accuracy: 0.9559 - val\_loss: 0.2286 - val\_sparse\_categorical\_accuracy: 0.9450 - 1s/epoch - 2ms/step

Epoch 38/100  
469/469 - 1s - loss: 0.1522 - sparse\_categorical\_accuracy: 0.9573 - val\_loss: 0.2256 - val\_sparse\_categorical\_accuracy: 0.9456 - 1s/epoch - 2ms/step

Epoch 39/100  
469/469 - 1s - loss: 0.1502 - sparse\_categorical\_accuracy: 0.9571 - val\_loss: 0.2302 - val\_sparse\_categorical\_accuracy: 0.9571 - 1s/epoch - 2ms/step

tegorical\_accuracy: 0.9405 - 1s/epoch - 2ms/step  
Epoch 40/100  
469/469 - 1s - loss: 0.1505 - sparse\_categorical\_accuracy: 0.9570 - val\_loss: 0.2255 - val\_sparse\_ca  
tegorical\_accuracy: 0.9414 - 1s/epoch - 2ms/step  
Epoch 41/100  
469/469 - 1s - loss: 0.1456 - sparse\_categorical\_accuracy: 0.9590 - val\_loss: 0.2253 - val\_sparse\_ca  
tegorical\_accuracy: 0.9454 - 1s/epoch - 2ms/step  
Epoch 42/100  
469/469 - 1s - loss: 0.1462 - sparse\_categorical\_accuracy: 0.9580 - val\_loss: 0.2241 - val\_sparse\_ca  
tegorical\_accuracy: 0.9433 - 1s/epoch - 2ms/step  
Epoch 43/100  
469/469 - 1s - loss: 0.1443 - sparse\_categorical\_accuracy: 0.9585 - val\_loss: 0.2223 - val\_sparse\_ca  
tegorical\_accuracy: 0.9445 - 1s/epoch - 2ms/step  
Epoch 44/100  
469/469 - 1s - loss: 0.1428 - sparse\_categorical\_accuracy: 0.9590 - val\_loss: 0.2248 - val\_sparse\_ca  
tegorical\_accuracy: 0.9430 - 1s/epoch - 2ms/step  
Epoch 45/100  
469/469 - 1s - loss: 0.1432 - sparse\_categorical\_accuracy: 0.9589 - val\_loss: 0.2244 - val\_sparse\_ca  
tegorical\_accuracy: 0.9459 - 1s/epoch - 2ms/step  
Epoch 46/100  
469/469 - 1s - loss: 0.1378 - sparse\_categorical\_accuracy: 0.9603 - val\_loss: 0.2344 - val\_sparse\_ca  
tegorical\_accuracy: 0.9426 - 1s/epoch - 2ms/step  
Epoch 47/100  
469/469 - 1s - loss: 0.1382 - sparse\_categorical\_accuracy: 0.9606 - val\_loss: 0.2294 - val\_sparse\_ca  
tegorical\_accuracy: 0.9411 - 1s/epoch - 2ms/step  
Epoch 48/100  
469/469 - 1s - loss: 0.1351 - sparse\_categorical\_accuracy: 0.9619 - val\_loss: 0.2331 - val\_sparse\_ca  
tegorical\_accuracy: 0.9450 - 1s/epoch - 2ms/step  
Epoch 49/100  
469/469 - 1s - loss: 0.1382 - sparse\_categorical\_accuracy: 0.9603 - val\_loss: 0.2260 - val\_sparse\_ca  
tegorical\_accuracy: 0.9433 - 1s/epoch - 2ms/step  
Epoch 50/100  
469/469 - 1s - loss: 0.1355 - sparse\_categorical\_accuracy: 0.9608 - val\_loss: 0.2403 - val\_sparse\_ca  
tegorical\_accuracy: 0.9423 - 1s/epoch - 2ms/step  
Epoch 51/100  
469/469 - 1s - loss: 0.1320 - sparse\_categorical\_accuracy: 0.9624 - val\_loss: 0.2339 - val\_sparse\_ca  
tegorical\_accuracy: 0.9420 - 1s/epoch - 2ms/step  
Epoch 52/100  
469/469 - 1s - loss: 0.1348 - sparse\_categorical\_accuracy: 0.9615 - val\_loss: 0.2305 - val\_sparse\_ca  
tegorical\_accuracy: 0.9452 - 1s/epoch - 2ms/step  
Epoch 53/100  
469/469 - 1s - loss: 0.1298 - sparse\_categorical\_accuracy: 0.9622 - val\_loss: 0.2496 - val\_sparse\_ca  
tegorical\_accuracy: 0.9415 - 1s/epoch - 2ms/step  
Epoch 54/100  
469/469 - 1s - loss: 0.1310 - sparse\_categorical\_accuracy: 0.9624 - val\_loss: 0.2336 - val\_sparse\_ca  
tegorical\_accuracy: 0.9453 - 1s/epoch - 2ms/step  
Epoch 55/100  
469/469 - 1s - loss: 0.1280 - sparse\_categorical\_accuracy: 0.9632 - val\_loss: 0.2402 - val\_sparse\_ca  
tegorical\_accuracy: 0.9443 - 1s/epoch - 2ms/step  
Epoch 56/100  
469/469 - 1s - loss: 0.1289 - sparse\_categorical\_accuracy: 0.9632 - val\_loss: 0.2415 - val\_sparse\_ca  
tegorical\_accuracy: 0.9427 - 1s/epoch - 2ms/step  
Epoch 57/100  
469/469 - 1s - loss: 0.1276 - sparse\_categorical\_accuracy: 0.9632 - val\_loss: 0.2377 - val\_sparse\_ca  
tegorical\_accuracy: 0.9444 - 1s/epoch - 2ms/step  
Epoch 58/100  
469/469 - 1s - loss: 0.1283 - sparse\_categorical\_accuracy: 0.9625 - val\_loss: 0.2410 - val\_sparse\_ca  
tegorical\_accuracy: 0.9453 - 1s/epoch - 2ms/step  
Epoch 59/100  
469/469 - 1s - loss: 0.1242 - sparse\_categorical\_accuracy: 0.9642 - val\_loss: 0.2393 - val\_sparse\_ca  
tegorical\_accuracy: 0.9433 - 1s/epoch - 2ms/step  
Epoch 60/100  
469/469 - 1s - loss: 0.1238 - sparse\_categorical\_accuracy: 0.9641 - val\_loss: 0.2477 - val\_sparse\_ca  
tegorical\_accuracy: 0.9430 - 1s/epoch - 2ms/step  
Epoch 61/100  
469/469 - 1s - loss: 0.1243 - sparse\_categorical\_accuracy: 0.9646 - val\_loss: 0.2605 - val\_sparse\_ca  
tegorical\_accuracy: 0.9410 - 1s/epoch - 2ms/step  
Epoch 62/100  
469/469 - 1s - loss: 0.1239 - sparse\_categorical\_accuracy: 0.9636 - val\_loss: 0.2542 - val\_sparse\_ca  
tegorical\_accuracy: 0.9432 - 1s/epoch - 2ms/step  
Epoch 63/100  
469/469 - 1s - loss: 0.1245 - sparse\_categorical\_accuracy: 0.9643 - val\_loss: 0.2476 - val\_sparse\_ca  
tegorical\_accuracy: 0.9432 - 1s/epoch - 2ms/step  
Epoch 64/100  
469/469 - 1s - loss: 0.1216 - sparse\_categorical\_accuracy: 0.9646 - val\_loss: 0.2563 - val\_sparse\_ca  
tegorical\_accuracy: 0.9441 - 959ms/epoch - 2ms/step  
Epoch 65/100  
469/469 - 1s - loss: 0.1196 - sparse\_categorical\_accuracy: 0.9654 - val\_loss: 0.2561 - val\_sparse\_ca  
tegorical\_accuracy: 0.9421 - 964ms/epoch - 2ms/step  
Epoch 66/100  
469/469 - 1s - loss: 0.1181 - sparse\_categorical\_accuracy: 0.9658 - val\_loss: 0.2533 - val\_sparse\_ca  
tegorical\_accuracy: 0.9439 - 990ms/epoch - 2ms/step  
Epoch 67/100

469/469 - 1s - loss: 0.1212 - sparse\_categorical\_accuracy: 0.9652 - val\_loss: 0.2525 - val\_sparse\_categorical\_accuracy: 0.9406 - 985ms/epoch - 2ms/step  
Epoch 68/100  
469/469 - 1s - loss: 0.1205 - sparse\_categorical\_accuracy: 0.9651 - val\_loss: 0.2674 - val\_sparse\_categorical\_accuracy: 0.9413 - 971ms/epoch - 2ms/step  
Epoch 69/100  
469/469 - 1s - loss: 0.1195 - sparse\_categorical\_accuracy: 0.9654 - val\_loss: 0.2618 - val\_sparse\_categorical\_accuracy: 0.9408 - 997ms/epoch - 2ms/step  
Epoch 70/100  
469/469 - 1s - loss: 0.1150 - sparse\_categorical\_accuracy: 0.9662 - val\_loss: 0.2888 - val\_sparse\_categorical\_accuracy: 0.9385 - 940ms/epoch - 2ms/step  
Epoch 71/100  
469/469 - 1s - loss: 0.1173 - sparse\_categorical\_accuracy: 0.9653 - val\_loss: 0.2604 - val\_sparse\_categorical\_accuracy: 0.9449 - 965ms/epoch - 2ms/step  
Epoch 72/100  
469/469 - 1s - loss: 0.1165 - sparse\_categorical\_accuracy: 0.9659 - val\_loss: 0.2680 - val\_sparse\_categorical\_accuracy: 0.9420 - 980ms/epoch - 2ms/step  
Epoch 73/100  
469/469 - 1s - loss: 0.1184 - sparse\_categorical\_accuracy: 0.9663 - val\_loss: 0.2780 - val\_sparse\_categorical\_accuracy: 0.9396 - 1s/epoch - 2ms/step  
Epoch 74/100  
469/469 - 1s - loss: 0.1169 - sparse\_categorical\_accuracy: 0.9667 - val\_loss: 0.2584 - val\_sparse\_categorical\_accuracy: 0.9443 - 1s/epoch - 2ms/step  
Epoch 75/100  
469/469 - 1s - loss: 0.1126 - sparse\_categorical\_accuracy: 0.9676 - val\_loss: 0.2807 - val\_sparse\_categorical\_accuracy: 0.9418 - 1s/epoch - 2ms/step  
Epoch 76/100  
469/469 - 1s - loss: 0.1144 - sparse\_categorical\_accuracy: 0.9668 - val\_loss: 0.2665 - val\_sparse\_categorical\_accuracy: 0.9415 - 1s/epoch - 2ms/step  
Epoch 77/100  
469/469 - 1s - loss: 0.1134 - sparse\_categorical\_accuracy: 0.9677 - val\_loss: 0.2557 - val\_sparse\_categorical\_accuracy: 0.9418 - 997ms/epoch - 2ms/step  
Epoch 78/100  
469/469 - 1s - loss: 0.1147 - sparse\_categorical\_accuracy: 0.9662 - val\_loss: 0.2807 - val\_sparse\_categorical\_accuracy: 0.9391 - 992ms/epoch - 2ms/step  
Epoch 79/100  
469/469 - 1s - loss: 0.1111 - sparse\_categorical\_accuracy: 0.9674 - val\_loss: 0.2645 - val\_sparse\_categorical\_accuracy: 0.9431 - 980ms/epoch - 2ms/step  
Epoch 80/100  
469/469 - 1s - loss: 0.1110 - sparse\_categorical\_accuracy: 0.9682 - val\_loss: 0.2781 - val\_sparse\_categorical\_accuracy: 0.9434 - 1s/epoch - 2ms/step  
Epoch 81/100  
469/469 - 1s - loss: 0.1125 - sparse\_categorical\_accuracy: 0.9675 - val\_loss: 0.2814 - val\_sparse\_categorical\_accuracy: 0.9419 - 1s/epoch - 2ms/step  
Epoch 82/100  
469/469 - 1s - loss: 0.1128 - sparse\_categorical\_accuracy: 0.9678 - val\_loss: 0.2738 - val\_sparse\_categorical\_accuracy: 0.9420 - 1s/epoch - 2ms/step  
Epoch 83/100  
469/469 - 1s - loss: 0.1068 - sparse\_categorical\_accuracy: 0.9697 - val\_loss: 0.2858 - val\_sparse\_categorical\_accuracy: 0.9418 - 1s/epoch - 2ms/step  
Epoch 84/100  
469/469 - 1s - loss: 0.1114 - sparse\_categorical\_accuracy: 0.9678 - val\_loss: 0.2829 - val\_sparse\_categorical\_accuracy: 0.9419 - 1s/epoch - 2ms/step  
Epoch 85/100  
469/469 - 1s - loss: 0.1112 - sparse\_categorical\_accuracy: 0.9675 - val\_loss: 0.2902 - val\_sparse\_categorical\_accuracy: 0.9391 - 1s/epoch - 3ms/step  
Epoch 86/100  
469/469 - 1s - loss: 0.1093 - sparse\_categorical\_accuracy: 0.9683 - val\_loss: 0.2996 - val\_sparse\_categorical\_accuracy: 0.9386 - 1s/epoch - 3ms/step  
Epoch 87/100  
469/469 - 1s - loss: 0.1108 - sparse\_categorical\_accuracy: 0.9678 - val\_loss: 0.2928 - val\_sparse\_categorical\_accuracy: 0.9409 - 985ms/epoch - 2ms/step  
Epoch 88/100  
469/469 - 1s - loss: 0.1047 - sparse\_categorical\_accuracy: 0.9702 - val\_loss: 0.2977 - val\_sparse\_categorical\_accuracy: 0.9389 - 1s/epoch - 2ms/step  
Epoch 89/100  
469/469 - 1s - loss: 0.1066 - sparse\_categorical\_accuracy: 0.9691 - val\_loss: 0.2918 - val\_sparse\_categorical\_accuracy: 0.9415 - 1s/epoch - 2ms/step  
Epoch 90/100  
469/469 - 1s - loss: 0.1091 - sparse\_categorical\_accuracy: 0.9678 - val\_loss: 0.2890 - val\_sparse\_categorical\_accuracy: 0.9422 - 1s/epoch - 2ms/step  
Epoch 91/100  
469/469 - 1s - loss: 0.1079 - sparse\_categorical\_accuracy: 0.9688 - val\_loss: 0.2734 - val\_sparse\_categorical\_accuracy: 0.9435 - 1s/epoch - 2ms/step  
Epoch 92/100  
469/469 - 1s - loss: 0.1065 - sparse\_categorical\_accuracy: 0.9685 - val\_loss: 0.2901 - val\_sparse\_categorical\_accuracy: 0.9397 - 957ms/epoch - 2ms/step  
Epoch 93/100  
469/469 - 1s - loss: 0.1062 - sparse\_categorical\_accuracy: 0.9690 - val\_loss: 0.2884 - val\_sparse\_categorical\_accuracy: 0.9402 - 1s/epoch - 2ms/step  
Epoch 94/100  
469/469 - 1s - loss: 0.1064 - sparse\_categorical\_accuracy: 0.9688 - val\_loss: 0.2858 - val\_sparse\_categorical\_accuracy: 0.9411 - 972ms/epoch - 2ms/step

```
Epoch 95/100
469/469 - 1s - loss: 0.1062 - sparse_categorical_accuracy: 0.9692 - val_loss: 0.2869 - val_sparse_categorical_accuracy: 0.9406 - 959ms/epoch - 2ms/step
Epoch 96/100
469/469 - 1s - loss: 0.1068 - sparse_categorical_accuracy: 0.9684 - val_loss: 0.3067 - val_sparse_categorical_accuracy: 0.9418 - 961ms/epoch - 2ms/step
Epoch 97/100
469/469 - 1s - loss: 0.1058 - sparse_categorical_accuracy: 0.9688 - val_loss: 0.2916 - val_sparse_categorical_accuracy: 0.9390 - 974ms/epoch - 2ms/step
Epoch 98/100
469/469 - 1s - loss: 0.1047 - sparse_categorical_accuracy: 0.9695 - val_loss: 0.2817 - val_sparse_categorical_accuracy: 0.9441 - 976ms/epoch - 2ms/step
Epoch 99/100
469/469 - 1s - loss: 0.1067 - sparse_categorical_accuracy: 0.9694 - val_loss: 0.2854 - val_sparse_categorical_accuracy: 0.9412 - 1s/epoch - 2ms/step
Epoch 100/100
469/469 - 1s - loss: 0.1055 - sparse_categorical_accuracy: 0.9692 - val_loss: 0.2929 - val_sparse_categorical_accuracy: 0.9427 - 978ms/epoch - 2ms/step
```

In [6]:

```
# Baseline model evaluation
model.evaluate(x_test, y_test, verbose=2)
```

```
313/313 - 0s - loss: 0.2929 - sparse_categorical_accuracy: 0.9427 - 336ms/epoch - 1ms/step
```

Out[6]:

```
[0.29285579919815063, 0.9427000284194946]
```

The baseline model with `kernel_initializer='glorot_uniform'`, `bias_initializer='zeros'`

## Try different models

**Model1: adam optimizer with learning rate= $e^{-3}$ , random\_uniform initializer, dropout regularization with rate=0.1.**

In [7]:

```
model1 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.1),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_uniform')
])
```

In [8]:

```
model1.compile(optimizer=tf.keras.optimizers.Adam(0.001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])
```

In [9]:

```
history1 = model1.fit(x_train, y_train,
                      batch_size=128,
                      epochs=100,
                      validation_data=(x_test, y_test),
                      verbose=2)
```

```
Epoch 1/100
469/469 - 20s - loss: 0.7836 - sparse_categorical_accuracy: 0.8702 - val_loss: 0.1379 - val_sparse_categorical_accuracy: 0.9723 - 20s/epoch - 43ms/step
Epoch 2/100
469/469 - 19s - loss: 0.1312 - sparse_categorical_accuracy: 0.9690 - val_loss: 0.0790 - val_sparse_categorical_accuracy: 0.9771 - 19s/epoch - 41ms/step
Epoch 3/100
469/469 - 19s - loss: 0.0880 - sparse_categorical_accuracy: 0.9751 - val_loss: 0.0567 - val_sparse_categorical_accuracy: 0.9836 - 19s/epoch - 41ms/step
Epoch 4/100
469/469 - 20s - loss: 0.0770 - sparse_categorical_accuracy: 0.9774 - val_loss: 0.0571 - val_sparse_categorical_accuracy: 0.9774 - 20s/epoch - 41ms/step
```

ategorical accuracy: 0.9821 - 20s/epoch - 42ms/step  
Epoch 5/100  
469/469 - 20s - loss: 0.0696 - sparse\_categorical\_accuracy: 0.9796 - val\_loss: 0.0512 - val\_sparse\_c  
ategorical accuracy: 0.9841 - 20s/epoch - 42ms/step  
Epoch 6/100  
469/469 - 20s - loss: 0.0640 - sparse\_categorical\_accuracy: 0.9809 - val\_loss: 0.0633 - val\_sparse\_c  
ategorical accuracy: 0.9793 - 20s/epoch - 42ms/step  
Epoch 7/100  
469/469 - 20s - loss: 0.0640 - sparse\_categorical\_accuracy: 0.9808 - val\_loss: 0.0501 - val\_sparse\_c  
ategorical accuracy: 0.9842 - 20s/epoch - 42ms/step  
Epoch 8/100  
469/469 - 20s - loss: 0.0609 - sparse\_categorical\_accuracy: 0.9817 - val\_loss: 0.0451 - val\_sparse\_c  
ategorical accuracy: 0.9855 - 20s/epoch - 42ms/step  
Epoch 9/100  
469/469 - 20s - loss: 0.0573 - sparse\_categorical\_accuracy: 0.9820 - val\_loss: 0.0491 - val\_sparse\_c  
ategorical accuracy: 0.9829 - 20s/epoch - 42ms/step  
Epoch 10/100  
469/469 - 20s - loss: 0.0555 - sparse\_categorical\_accuracy: 0.9828 - val\_loss: 0.0438 - val\_sparse\_c  
ategorical accuracy: 0.9865 - 20s/epoch - 42ms/step  
Epoch 11/100  
469/469 - 19s - loss: 0.0544 - sparse\_categorical\_accuracy: 0.9829 - val\_loss: 0.0529 - val\_sparse\_c  
ategorical accuracy: 0.9820 - 19s/epoch - 42ms/step  
Epoch 12/100  
469/469 - 19s - loss: 0.0536 - sparse\_categorical\_accuracy: 0.9830 - val\_loss: 0.0525 - val\_sparse\_c  
ategorical accuracy: 0.9825 - 19s/epoch - 42ms/step  
Epoch 13/100  
469/469 - 19s - loss: 0.0512 - sparse\_categorical\_accuracy: 0.9835 - val\_loss: 0.0395 - val\_sparse\_c  
ategorical accuracy: 0.9876 - 19s/epoch - 42ms/step  
Epoch 14/100  
469/469 - 19s - loss: 0.0491 - sparse\_categorical\_accuracy: 0.9845 - val\_loss: 0.0504 - val\_sparse\_c  
ategorical accuracy: 0.9837 - 19s/epoch - 42ms/step  
Epoch 15/100  
469/469 - 20s - loss: 0.0486 - sparse\_categorical\_accuracy: 0.9845 - val\_loss: 0.0432 - val\_sparse\_c  
ategorical accuracy: 0.9864 - 20s/epoch - 42ms/step  
Epoch 16/100  
469/469 - 20s - loss: 0.0471 - sparse\_categorical\_accuracy: 0.9851 - val\_loss: 0.0424 - val\_sparse\_c  
ategorical accuracy: 0.9866 - 20s/epoch - 42ms/step  
Epoch 17/100  
469/469 - 19s - loss: 0.0476 - sparse\_categorical\_accuracy: 0.9847 - val\_loss: 0.0470 - val\_sparse\_c  
ategorical accuracy: 0.9845 - 19s/epoch - 42ms/step  
Epoch 18/100  
469/469 - 19s - loss: 0.0470 - sparse\_categorical\_accuracy: 0.9851 - val\_loss: 0.0429 - val\_sparse\_c  
ategorical accuracy: 0.9867 - 19s/epoch - 41ms/step  
Epoch 19/100  
469/469 - 19s - loss: 0.0452 - sparse\_categorical\_accuracy: 0.9858 - val\_loss: 0.0475 - val\_sparse\_c  
ategorical accuracy: 0.9847 - 19s/epoch - 41ms/step  
Epoch 20/100  
469/469 - 20s - loss: 0.0471 - sparse\_categorical\_accuracy: 0.9851 - val\_loss: 0.0401 - val\_sparse\_c  
ategorical accuracy: 0.9870 - 20s/epoch - 42ms/step  
Epoch 21/100  
469/469 - 20s - loss: 0.0445 - sparse\_categorical\_accuracy: 0.9854 - val\_loss: 0.0481 - val\_sparse\_c  
ategorical accuracy: 0.9859 - 20s/epoch - 42ms/step  
Epoch 22/100  
469/469 - 21s - loss: 0.0442 - sparse\_categorical\_accuracy: 0.9857 - val\_loss: 0.0385 - val\_sparse\_c  
ategorical accuracy: 0.9870 - 21s/epoch - 45ms/step  
Epoch 23/100  
469/469 - 20s - loss: 0.0438 - sparse\_categorical\_accuracy: 0.9858 - val\_loss: 0.0442 - val\_sparse\_c  
ategorical accuracy: 0.9868 - 20s/epoch - 42ms/step  
Epoch 24/100  
469/469 - 20s - loss: 0.0420 - sparse\_categorical\_accuracy: 0.9868 - val\_loss: 0.0451 - val\_sparse\_c  
ategorical accuracy: 0.9853 - 20s/epoch - 42ms/step  
Epoch 25/100  
469/469 - 19s - loss: 0.0410 - sparse\_categorical\_accuracy: 0.9865 - val\_loss: 0.0494 - val\_sparse\_c  
ategorical accuracy: 0.9855 - 19s/epoch - 42ms/step  
Epoch 26/100  
469/469 - 20s - loss: 0.0431 - sparse\_categorical\_accuracy: 0.9861 - val\_loss: 0.0444 - val\_sparse\_c  
ategorical accuracy: 0.9846 - 20s/epoch - 42ms/step  
Epoch 27/100  
469/469 - 20s - loss: 0.0409 - sparse\_categorical\_accuracy: 0.9864 - val\_loss: 0.0439 - val\_sparse\_c  
ategorical accuracy: 0.9871 - 20s/epoch - 42ms/step  
Epoch 28/100  
469/469 - 20s - loss: 0.0416 - sparse\_categorical\_accuracy: 0.9863 - val\_loss: 0.0401 - val\_sparse\_c  
ategorical accuracy: 0.9865 - 20s/epoch - 42ms/step  
Epoch 29/100  
469/469 - 20s - loss: 0.0412 - sparse\_categorical\_accuracy: 0.9867 - val\_loss: 0.0407 - val\_sparse\_c  
ategorical accuracy: 0.9877 - 20s/epoch - 42ms/step  
Epoch 30/100  
469/469 - 20s - loss: 0.0407 - sparse\_categorical\_accuracy: 0.9869 - val\_loss: 0.0431 - val\_sparse\_c  
ategorical accuracy: 0.9866 - 20s/epoch - 42ms/step  
Epoch 31/100  
469/469 - 20s - loss: 0.0406 - sparse\_categorical\_accuracy: 0.9870 - val\_loss: 0.0454 - val\_sparse\_c  
ategorical accuracy: 0.9869 - 20s/epoch - 42ms/step  
Epoch 32/100

[illegible]

```
Epoch 60/100
469/469 - 18s - loss: 0.0345 - sparse_categorical_accuracy: 0.9885 - val_loss: 0.0417 - val_sparse_c
ategorical_accuracy: 0.9868 - 18s/epoch - 39ms/step
Epoch 61/100
469/469 - 19s - loss: 0.0334 - sparse_categorical_accuracy: 0.9887 - val_loss: 0.0467 - val_sparse_c
ategorical_accuracy: 0.9865 - 19s/epoch - 40ms/step
Epoch 62/100
469/469 - 19s - loss: 0.0335 - sparse_categorical_accuracy: 0.9892 - val_loss: 0.0401 - val_sparse_c
ategorical_accuracy: 0.9888 - 19s/epoch - 40ms/step
Epoch 63/100
469/469 - 19s - loss: 0.0334 - sparse_categorical_accuracy: 0.9888 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9896 - 19s/epoch - 40ms/step
Epoch 64/100
469/469 - 19s - loss: 0.0345 - sparse_categorical_accuracy: 0.9891 - val_loss: 0.0404 - val_sparse_c
ategorical_accuracy: 0.9886 - 19s/epoch - 41ms/step
Epoch 65/100
469/469 - 19s - loss: 0.0333 - sparse_categorical_accuracy: 0.9886 - val_loss: 0.0401 - val_sparse_c
ategorical_accuracy: 0.9883 - 19s/epoch - 41ms/step
Epoch 66/100
469/469 - 18s - loss: 0.0330 - sparse_categorical_accuracy: 0.9891 - val_loss: 0.0397 - val_sparse_c
ategorical_accuracy: 0.9881 - 18s/epoch - 39ms/step
Epoch 67/100
469/469 - 19s - loss: 0.0320 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0434 - val_sparse_c
ategorical_accuracy: 0.9880 - 19s/epoch - 40ms/step
Epoch 68/100
469/469 - 19s - loss: 0.0326 - sparse_categorical_accuracy: 0.9890 - val_loss: 0.0386 - val_sparse_c
ategorical_accuracy: 0.9887 - 19s/epoch - 40ms/step
Epoch 69/100
469/469 - 18s - loss: 0.0346 - sparse_categorical_accuracy: 0.9886 - val_loss: 0.0396 - val_sparse_c
ategorical_accuracy: 0.9870 - 18s/epoch - 39ms/step
Epoch 70/100
469/469 - 19s - loss: 0.0312 - sparse_categorical_accuracy: 0.9897 - val_loss: 0.0404 - val_sparse_c
ategorical_accuracy: 0.9872 - 19s/epoch - 40ms/step
Epoch 71/100
469/469 - 19s - loss: 0.0316 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0372 - val_sparse_c
ategorical_accuracy: 0.9896 - 19s/epoch - 41ms/step
Epoch 72/100
469/469 - 19s - loss: 0.0325 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0390 - val_sparse_c
ategorical_accuracy: 0.9875 - 19s/epoch - 40ms/step
Epoch 73/100
469/469 - 19s - loss: 0.0323 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0392 - val_sparse_c
ategorical_accuracy: 0.9881 - 19s/epoch - 41ms/step
Epoch 74/100
469/469 - 20s - loss: 0.0316 - sparse_categorical_accuracy: 0.9891 - val_loss: 0.0451 - val_sparse_c
ategorical_accuracy: 0.9871 - 20s/epoch - 42ms/step
Epoch 75/100
469/469 - 20s - loss: 0.0320 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0435 - val_sparse_c
ategorical_accuracy: 0.9875 - 20s/epoch - 42ms/step
Epoch 76/100
469/469 - 20s - loss: 0.0310 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0402 - val_sparse_c
ategorical_accuracy: 0.9880 - 20s/epoch - 42ms/step
Epoch 77/100
469/469 - 20s - loss: 0.0310 - sparse_categorical_accuracy: 0.9898 - val_loss: 0.0416 - val_sparse_c
ategorical_accuracy: 0.9881 - 20s/epoch - 42ms/step
Epoch 78/100
469/469 - 19s - loss: 0.0319 - sparse_categorical_accuracy: 0.9891 - val_loss: 0.0489 - val_sparse_c
ategorical_accuracy: 0.9862 - 19s/epoch - 41ms/step
Epoch 79/100
469/469 - 19s - loss: 0.0309 - sparse_categorical_accuracy: 0.9900 - val_loss: 0.0438 - val_sparse_c
ategorical_accuracy: 0.9867 - 19s/epoch - 41ms/step
Epoch 80/100
469/469 - 19s - loss: 0.0315 - sparse_categorical_accuracy: 0.9896 - val_loss: 0.0392 - val_sparse_c
ategorical_accuracy: 0.9871 - 19s/epoch - 41ms/step
Epoch 81/100
469/469 - 19s - loss: 0.0312 - sparse_categorical_accuracy: 0.9899 - val_loss: 0.0380 - val_sparse_c
ategorical_accuracy: 0.9881 - 19s/epoch - 41ms/step
Epoch 82/100
469/469 - 19s - loss: 0.0310 - sparse_categorical_accuracy: 0.9896 - val_loss: 0.0385 - val_sparse_c
ategorical_accuracy: 0.9886 - 19s/epoch - 40ms/step
Epoch 83/100
469/469 - 19s - loss: 0.0314 - sparse_categorical_accuracy: 0.9897 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9868 - 19s/epoch - 41ms/step
Epoch 84/100
469/469 - 19s - loss: 0.0312 - sparse_categorical_accuracy: 0.9893 - val_loss: 0.0381 - val_sparse_c
ategorical_accuracy: 0.9882 - 19s/epoch - 41ms/step
Epoch 85/100
469/469 - 19s - loss: 0.0310 - sparse_categorical_accuracy: 0.9897 - val_loss: 0.0363 - val_sparse_c
ategorical_accuracy: 0.9892 - 19s/epoch - 40ms/step
Epoch 86/100
469/469 - 19s - loss: 0.0310 - sparse_categorical_accuracy: 0.9899 - val_loss: 0.0408 - val_sparse_c
ategorical_accuracy: 0.9871 - 19s/epoch - 40ms/step
Epoch 87/100
469/469 - 19s - loss: 0.0300 - sparse categorical accuracy: 0.9896 - val loss: 0.0400 - val sparse c
```



```

ategorical_accuracy: 0.9874 - 19s/epoch - 40ms/step
Epoch 88/100
469/469 - 18s - loss: 0.0292 - sparse_categorical_accuracy: 0.9906 - val_loss: 0.0408 - val_sparse_c
ategorical_accuracy: 0.9868 - 18s/epoch - 39ms/step
Epoch 89/100
469/469 - 19s - loss: 0.0297 - sparse_categorical_accuracy: 0.9902 - val_loss: 0.0368 - val_sparse_c
ategorical_accuracy: 0.9885 - 19s/epoch - 40ms/step
Epoch 90/100
469/469 - 19s - loss: 0.0309 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0394 - val_sparse_c
ategorical_accuracy: 0.9881 - 19s/epoch - 40ms/step
Epoch 91/100
469/469 - 18s - loss: 0.0307 - sparse_categorical_accuracy: 0.9902 - val_loss: 0.0411 - val_sparse_c
ategorical_accuracy: 0.9870 - 18s/epoch - 39ms/step
Epoch 92/100
469/469 - 18s - loss: 0.0293 - sparse_categorical_accuracy: 0.9904 - val_loss: 0.0406 - val_sparse_c
ategorical_accuracy: 0.9870 - 18s/epoch - 39ms/step
Epoch 93/100
469/469 - 18s - loss: 0.0321 - sparse_categorical_accuracy: 0.9892 - val_loss: 0.0418 - val_sparse_c
ategorical_accuracy: 0.9870 - 18s/epoch - 39ms/step
Epoch 94/100
469/469 - 18s - loss: 0.0308 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0393 - val_sparse_c
ategorical_accuracy: 0.9878 - 18s/epoch - 39ms/step
Epoch 95/100
469/469 - 19s - loss: 0.0309 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0432 - val_sparse_c
ategorical_accuracy: 0.9861 - 19s/epoch - 40ms/step
Epoch 96/100
469/469 - 19s - loss: 0.0287 - sparse_categorical_accuracy: 0.9904 - val_loss: 0.0413 - val_sparse_c
ategorical_accuracy: 0.9876 - 19s/epoch - 40ms/step
Epoch 97/100
469/469 - 18s - loss: 0.0309 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0422 - val_sparse_c
ategorical_accuracy: 0.9879 - 18s/epoch - 39ms/step
Epoch 98/100
469/469 - 18s - loss: 0.0292 - sparse_categorical_accuracy: 0.9906 - val_loss: 0.0430 - val_sparse_c
ategorical_accuracy: 0.9871 - 18s/epoch - 39ms/step
Epoch 99/100
469/469 - 19s - loss: 0.0295 - sparse_categorical_accuracy: 0.9900 - val_loss: 0.0402 - val_sparse_c
ategorical_accuracy: 0.9872 - 19s/epoch - 40ms/step
Epoch 100/100
469/469 - 19s - loss: 0.0298 - sparse_categorical_accuracy: 0.9901 - val_loss: 0.0388 - val_sparse_c
ategorical_accuracy: 0.9886 - 19s/epoch - 39ms/step

```

In [10]:

```
model1.evaluate(x_test, y_test, verbose=2)
```

```
313/313 - 1s - loss: 0.0388 - sparse_categorical_accuracy: 0.9886 - 1s/epoch - 4ms/step
```

Out[10]:

```
[0.038794007152318954, 0.9886000156402588]
```

**Model2: adam optimizer with learning rate= $e^{-4}$ , random\_uniform initializer, dropout regularization with rate=0.1.**

In [11]:

```

model2 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.1),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_uniform')
])

```

In [12]:

```

model2.compile(optimizer=tf.keras.optimizers.Adam(0.0001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])

```

In [13]:

```
history2 = model2.fit(x_train, y_train,  
                      batch_size=128,  
                      epochs=100,  
                      validation_data=(x_test, y_test),  
                      verbose=2  
                      )
```

```
Epoch 1/100  
469/469 - 20s - loss: 2.0015 - sparse_categorical_accuracy: 0.5278 - val_loss: 1.6410 - val_sparse_c  
ategorical_accuracy: 0.7318 - 20s/epoch - 42ms/step  
Epoch 2/100  
469/469 - 19s - loss: 1.4567 - sparse_categorical_accuracy: 0.7738 - val_loss: 1.1932 - val_sparse_c  
ategorical_accuracy: 0.8433 - 19s/epoch - 41ms/step  
Epoch 3/100  
469/469 - 19s - loss: 1.0406 - sparse_categorical_accuracy: 0.8818 - val_loss: 0.8561 - val_sparse_c  
ategorical_accuracy: 0.9373 - 19s/epoch - 40ms/step  
Epoch 4/100  
469/469 - 19s - loss: 0.7266 - sparse_categorical_accuracy: 0.9367 - val_loss: 0.5614 - val_sparse_c  
ategorical_accuracy: 0.9594 - 19s/epoch - 40ms/step  
Epoch 5/100  
469/469 - 18s - loss: 0.4995 - sparse_categorical_accuracy: 0.9529 - val_loss: 0.3910 - val_sparse_c  
ategorical_accuracy: 0.9697 - 18s/epoch - 39ms/step  
Epoch 6/100  
469/469 - 19s - loss: 0.3532 - sparse_categorical_accuracy: 0.9598 - val_loss: 0.2602 - val_sparse_c  
ategorical_accuracy: 0.9748 - 19s/epoch - 40ms/step  
Epoch 7/100  
469/469 - 19s - loss: 0.2644 - sparse_categorical_accuracy: 0.9646 - val_loss: 0.2017 - val_sparse_c  
ategorical_accuracy: 0.9766 - 19s/epoch - 40ms/step  
Epoch 8/100  
469/469 - 19s - loss: 0.2083 - sparse_categorical_accuracy: 0.9675 - val_loss: 0.1561 - val_sparse_c  
ategorical_accuracy: 0.9779 - 19s/epoch - 40ms/step  
Epoch 9/100  
469/469 - 19s - loss: 0.1704 - sparse_categorical_accuracy: 0.9696 - val_loss: 0.1235 - val_sparse_c  
ategorical_accuracy: 0.9788 - 19s/epoch - 41ms/step  
Epoch 10/100  
469/469 - 19s - loss: 0.1438 - sparse_categorical_accuracy: 0.9722 - val_loss: 0.1037 - val_sparse_c  
ategorical_accuracy: 0.9793 - 19s/epoch - 41ms/step  
Epoch 11/100  
469/469 - 19s - loss: 0.1252 - sparse_categorical_accuracy: 0.9732 - val_loss: 0.0928 - val_sparse_c  
ategorical_accuracy: 0.9799 - 19s/epoch - 40ms/step  
Epoch 12/100  
469/469 - 19s - loss: 0.1102 - sparse_categorical_accuracy: 0.9749 - val_loss: 0.0792 - val_sparse_c  
ategorical_accuracy: 0.9823 - 19s/epoch - 41ms/step  
Epoch 13/100  
469/469 - 19s - loss: 0.1014 - sparse_categorical_accuracy: 0.9754 - val_loss: 0.0752 - val_sparse_c  
ategorical_accuracy: 0.9819 - 19s/epoch - 40ms/step  
Epoch 14/100  
469/469 - 19s - loss: 0.0925 - sparse_categorical_accuracy: 0.9763 - val_loss: 0.0666 - val_sparse_c  
ategorical_accuracy: 0.9818 - 19s/epoch - 40ms/step  
Epoch 15/100  
469/469 - 19s - loss: 0.0848 - sparse_categorical_accuracy: 0.9778 - val_loss: 0.0615 - val_sparse_c  
ategorical_accuracy: 0.9839 - 19s/epoch - 41ms/step  
Epoch 16/100  
469/469 - 19s - loss: 0.0804 - sparse_categorical_accuracy: 0.9787 - val_loss: 0.0597 - val_sparse_c  
ategorical_accuracy: 0.9838 - 19s/epoch - 41ms/step  
Epoch 17/100  
469/469 - 19s - loss: 0.0766 - sparse_categorical_accuracy: 0.9786 - val_loss: 0.0571 - val_sparse_c  
ategorical_accuracy: 0.9837 - 19s/epoch - 40ms/step  
Epoch 18/100  
469/469 - 19s - loss: 0.0728 - sparse_categorical_accuracy: 0.9796 - val_loss: 0.0538 - val_sparse_c  
ategorical_accuracy: 0.9842 - 19s/epoch - 41ms/step  
Epoch 19/100  
469/469 - 19s - loss: 0.0707 - sparse_categorical_accuracy: 0.9800 - val_loss: 0.0531 - val_sparse_c  
ategorical_accuracy: 0.9837 - 19s/epoch - 40ms/step  
Epoch 20/100  
469/469 - 19s - loss: 0.0672 - sparse_categorical_accuracy: 0.9809 - val_loss: 0.0507 - val_sparse_c  
ategorical_accuracy: 0.9843 - 19s/epoch - 40ms/step  
Epoch 21/100  
469/469 - 19s - loss: 0.0648 - sparse_categorical_accuracy: 0.9816 - val_loss: 0.0502 - val_sparse_c  
ategorical_accuracy: 0.9845 - 19s/epoch - 40ms/step  
Epoch 22/100  
469/469 - 19s - loss: 0.0630 - sparse_categorical_accuracy: 0.9817 - val_loss: 0.0498 - val_sparse_c  
ategorical_accuracy: 0.9853 - 19s/epoch - 41ms/step  
Epoch 23/100  
469/469 - 19s - loss: 0.0619 - sparse_categorical_accuracy: 0.9822 - val_loss: 0.0467 - val_sparse_c  
ategorical_accuracy: 0.9841 - 19s/epoch - 41ms/step  
Epoch 24/100  
469/469 - 19s - loss: 0.0585 - sparse_categorical_accuracy: 0.9827 - val_loss: 0.0452 - val_sparse_c  
ategorical_accuracy: 0.9854 - 19s/epoch - 41ms/step  
Epoch 25/100  
469/469 - 19s - loss: 0.0586 - sparse_categorical_accuracy: 0.9818 - val_loss: 0.0483 - val_sparse_c
```

[illegible]

[illegible]

```

Epoch 81/100
469/469 - 19s - loss: 0.0336 - sparse_categorical_accuracy: 0.9892 - val_loss: 0.0380 - val_sparse_c
ategorical_accuracy: 0.9885 - 19s/epoch - 41ms/step
Epoch 82/100
469/469 - 19s - loss: 0.0353 - sparse_categorical_accuracy: 0.9886 - val_loss: 0.0371 - val_sparse_c
ategorical_accuracy: 0.9885 - 19s/epoch - 41ms/step
Epoch 83/100
469/469 - 19s - loss: 0.0343 - sparse_categorical_accuracy: 0.9888 - val_loss: 0.0377 - val_sparse_c
ategorical_accuracy: 0.9879 - 19s/epoch - 41ms/step
Epoch 84/100
469/469 - 19s - loss: 0.0338 - sparse_categorical_accuracy: 0.9891 - val_loss: 0.0406 - val_sparse_c
ategorical_accuracy: 0.9874 - 19s/epoch - 41ms/step
Epoch 85/100
469/469 - 19s - loss: 0.0331 - sparse_categorical_accuracy: 0.9896 - val_loss: 0.0371 - val_sparse_c
ategorical_accuracy: 0.9883 - 19s/epoch - 41ms/step
Epoch 86/100
469/469 - 19s - loss: 0.0327 - sparse_categorical_accuracy: 0.9892 - val_loss: 0.0408 - val_sparse_c
ategorical_accuracy: 0.9879 - 19s/epoch - 41ms/step
Epoch 87/100
469/469 - 19s - loss: 0.0331 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9892 - 19s/epoch - 41ms/step
Epoch 88/100
469/469 - 19s - loss: 0.0336 - sparse_categorical_accuracy: 0.9886 - val_loss: 0.0376 - val_sparse_c
ategorical_accuracy: 0.9887 - 19s/epoch - 41ms/step
Epoch 89/100
469/469 - 19s - loss: 0.0332 - sparse_categorical_accuracy: 0.9893 - val_loss: 0.0407 - val_sparse_c
ategorical_accuracy: 0.9885 - 19s/epoch - 41ms/step
Epoch 90/100
469/469 - 19s - loss: 0.0334 - sparse_categorical_accuracy: 0.9888 - val_loss: 0.0408 - val_sparse_c
ategorical_accuracy: 0.9879 - 19s/epoch - 41ms/step
Epoch 91/100
469/469 - 19s - loss: 0.0337 - sparse_categorical_accuracy: 0.9890 - val_loss: 0.0367 - val_sparse_c
ategorical_accuracy: 0.9894 - 19s/epoch - 41ms/step
Epoch 92/100
469/469 - 19s - loss: 0.0325 - sparse_categorical_accuracy: 0.9894 - val_loss: 0.0376 - val_sparse_c
ategorical_accuracy: 0.9877 - 19s/epoch - 41ms/step
Epoch 93/100
469/469 - 19s - loss: 0.0320 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0393 - val_sparse_c
ategorical_accuracy: 0.9881 - 19s/epoch - 41ms/step
Epoch 94/100
469/469 - 19s - loss: 0.0326 - sparse_categorical_accuracy: 0.9892 - val_loss: 0.0372 - val_sparse_c
ategorical_accuracy: 0.9895 - 19s/epoch - 41ms/step
Epoch 95/100
469/469 - 19s - loss: 0.0314 - sparse_categorical_accuracy: 0.9897 - val_loss: 0.0395 - val_sparse_c
ategorical_accuracy: 0.9886 - 19s/epoch - 41ms/step
Epoch 96/100
469/469 - 19s - loss: 0.0318 - sparse_categorical_accuracy: 0.9898 - val_loss: 0.0372 - val_sparse_c
ategorical_accuracy: 0.9889 - 19s/epoch - 41ms/step
Epoch 97/100
469/469 - 19s - loss: 0.0322 - sparse_categorical_accuracy: 0.9896 - val_loss: 0.0369 - val_sparse_c
ategorical_accuracy: 0.9889 - 19s/epoch - 41ms/step
Epoch 98/100
469/469 - 19s - loss: 0.0317 - sparse_categorical_accuracy: 0.9895 - val_loss: 0.0397 - val_sparse_c
ategorical_accuracy: 0.9887 - 19s/epoch - 41ms/step
Epoch 99/100
469/469 - 19s - loss: 0.0315 - sparse_categorical_accuracy: 0.9898 - val_loss: 0.0399 - val_sparse_c
ategorical_accuracy: 0.9889 - 19s/epoch - 41ms/step
Epoch 100/100
469/469 - 19s - loss: 0.0311 - sparse_categorical_accuracy: 0.9899 - val_loss: 0.0377 - val_sparse_c
ategorical_accuracy: 0.9892 - 19s/epoch - 41ms/step

```

In [14]:

```
model2.evaluate(x_test, y_test, verbose=2)
```

```
313/313 - 1s - loss: 0.0377 - sparse_categorical_accuracy: 0.9892 - 1s/epoch - 5ms/step
```

Out[14]:

```
[0.037735216319561005, 0.9891999959945679]
```

**Model3: adam optimizer with learning rate= $e^{-3}$ , random\_normal initializer, dropout regularization with rate=0.1.**

In [15]:

```
model3 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.1),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_normal')
])
```

In [16]:

```
model3.compile(optimizer=tf.keras.optimizers.Adam(0.001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])
```

In [17]:

```
history3 = model3.fit(x_train, y_train,
                      batch_size=128,
                      epochs=100,
                      validation_data=(x_test, y_test),
                      verbose=2)
```

```
Epoch 1/100
469/469 - 20s - loss: 0.8023 - sparse_categorical_accuracy: 0.8349 - val_loss: 0.1346 - val_sparse_c
ategorical_accuracy: 0.9747 - 20s/epoch - 43ms/step
Epoch 2/100
469/469 - 19s - loss: 0.1295 - sparse_categorical_accuracy: 0.9692 - val_loss: 0.0701 - val_sparse_c
ategorical_accuracy: 0.9804 - 19s/epoch - 41ms/step
Epoch 3/100
469/469 - 19s - loss: 0.0902 - sparse_categorical_accuracy: 0.9745 - val_loss: 0.0799 - val_sparse_c
ategorical_accuracy: 0.9751 - 19s/epoch - 42ms/step
Epoch 4/100
469/469 - 20s - loss: 0.0754 - sparse_categorical_accuracy: 0.9780 - val_loss: 0.0484 - val_sparse_c
ategorical_accuracy: 0.9856 - 20s/epoch - 42ms/step
Epoch 5/100
469/469 - 19s - loss: 0.0703 - sparse_categorical_accuracy: 0.9785 - val_loss: 0.0463 - val_sparse_c
ategorical_accuracy: 0.9851 - 19s/epoch - 42ms/step
Epoch 6/100
469/469 - 19s - loss: 0.0635 - sparse_categorical_accuracy: 0.9806 - val_loss: 0.0459 - val_sparse_c
ategorical_accuracy: 0.9844 - 19s/epoch - 41ms/step
Epoch 7/100
469/469 - 19s - loss: 0.0607 - sparse_categorical_accuracy: 0.9813 - val_loss: 0.0570 - val_sparse_c
ategorical_accuracy: 0.9822 - 19s/epoch - 41ms/step
Epoch 8/100
469/469 - 19s - loss: 0.0591 - sparse_categorical_accuracy: 0.9813 - val_loss: 0.0443 - val_sparse_c
ategorical_accuracy: 0.9861 - 19s/epoch - 41ms/step
Epoch 9/100
469/469 - 19s - loss: 0.0558 - sparse_categorical_accuracy: 0.9816 - val_loss: 0.0440 - val_sparse_c
ategorical_accuracy: 0.9860 - 19s/epoch - 41ms/step
Epoch 10/100
469/469 - 19s - loss: 0.0519 - sparse_categorical_accuracy: 0.9837 - val_loss: 0.0464 - val_sparse_c
ategorical_accuracy: 0.9853 - 19s/epoch - 41ms/step
Epoch 11/100
469/469 - 19s - loss: 0.0504 - sparse_categorical_accuracy: 0.9839 - val_loss: 0.0418 - val_sparse_c
ategorical_accuracy: 0.9864 - 19s/epoch - 41ms/step
Epoch 12/100
469/469 - 19s - loss: 0.0516 - sparse_categorical_accuracy: 0.9839 - val_loss: 0.0476 - val_sparse_c
ategorical_accuracy: 0.9849 - 19s/epoch - 41ms/step
Epoch 13/100
469/469 - 19s - loss: 0.0485 - sparse_categorical_accuracy: 0.9851 - val_loss: 0.0414 - val_sparse_c
ategorical_accuracy: 0.9856 - 19s/epoch - 41ms/step
Epoch 14/100
469/469 - 19s - loss: 0.0476 - sparse_categorical_accuracy: 0.9851 - val_loss: 0.0426 - val_sparse_c
ategorical_accuracy: 0.9857 - 19s/epoch - 41ms/step
Epoch 15/100
469/469 - 19s - loss: 0.0491 - sparse_categorical_accuracy: 0.9841 - val_loss: 0.0448 - val_sparse_c
ategorical_accuracy: 0.9861 - 19s/epoch - 40ms/step
Epoch 16/100
469/469 - 19s - loss: 0.0459 - sparse_categorical_accuracy: 0.9855 - val_loss: 0.0406 - val_sparse_c
ategorical_accuracy: 0.9874 - 19s/epoch - 41ms/step
Epoch 17/100
469/469 - 19s - loss: 0.0439 - sparse_categorical_accuracy: 0.9861 - val_loss: 0.0414 - val_sparse_c
```

ategorical accuracy: 0.9855 - 19s/epoch - 41ms/step  
Epoch 18/100  
469/469 - 19s - loss: 0.0451 - sparse\_categorical\_accuracy: 0.9856 - val\_loss: 0.0384 - val\_sparse\_c  
ategorical accuracy: 0.9885 - 19s/epoch - 41ms/step  
Epoch 19/100  
469/469 - 19s - loss: 0.0444 - sparse\_categorical\_accuracy: 0.9855 - val\_loss: 0.0417 - val\_sparse\_c  
ategorical accuracy: 0.9874 - 19s/epoch - 41ms/step  
Epoch 20/100  
469/469 - 19s - loss: 0.0425 - sparse\_categorical\_accuracy: 0.9867 - val\_loss: 0.0495 - val\_sparse\_c  
ategorical accuracy: 0.9848 - 19s/epoch - 41ms/step  
Epoch 21/100  
469/469 - 19s - loss: 0.0418 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0434 - val\_sparse\_c  
ategorical accuracy: 0.9860 - 19s/epoch - 41ms/step  
Epoch 22/100  
469/469 - 20s - loss: 0.0416 - sparse\_categorical\_accuracy: 0.9863 - val\_loss: 0.0445 - val\_sparse\_c  
ategorical accuracy: 0.9861 - 20s/epoch - 42ms/step  
Epoch 23/100  
469/469 - 20s - loss: 0.0405 - sparse\_categorical\_accuracy: 0.9867 - val\_loss: 0.0448 - val\_sparse\_c  
ategorical accuracy: 0.9867 - 20s/epoch - 42ms/step  
Epoch 24/100  
469/469 - 20s - loss: 0.0411 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0485 - val\_sparse\_c  
ategorical accuracy: 0.9845 - 20s/epoch - 42ms/step  
Epoch 25/100  
469/469 - 20s - loss: 0.0410 - sparse\_categorical\_accuracy: 0.9868 - val\_loss: 0.0395 - val\_sparse\_c  
ategorical accuracy: 0.9876 - 20s/epoch - 42ms/step  
Epoch 26/100  
469/469 - 19s - loss: 0.0412 - sparse\_categorical\_accuracy: 0.9865 - val\_loss: 0.0457 - val\_sparse\_c  
ategorical accuracy: 0.9866 - 19s/epoch - 42ms/step  
Epoch 27/100  
469/469 - 19s - loss: 0.0397 - sparse\_categorical\_accuracy: 0.9873 - val\_loss: 0.0382 - val\_sparse\_c  
ategorical accuracy: 0.9876 - 19s/epoch - 42ms/step  
Epoch 28/100  
469/469 - 19s - loss: 0.0385 - sparse\_categorical\_accuracy: 0.9873 - val\_loss: 0.0407 - val\_sparse\_c  
ategorical accuracy: 0.9865 - 19s/epoch - 41ms/step  
Epoch 29/100  
469/469 - 19s - loss: 0.0383 - sparse\_categorical\_accuracy: 0.9875 - val\_loss: 0.0378 - val\_sparse\_c  
ategorical accuracy: 0.9877 - 19s/epoch - 41ms/step  
Epoch 30/100  
469/469 - 19s - loss: 0.0407 - sparse\_categorical\_accuracy: 0.9870 - val\_loss: 0.0391 - val\_sparse\_c  
ategorical accuracy: 0.9877 - 19s/epoch - 41ms/step  
Epoch 31/100  
469/469 - 19s - loss: 0.0369 - sparse\_categorical\_accuracy: 0.9878 - val\_loss: 0.0467 - val\_sparse\_c  
ategorical accuracy: 0.9847 - 19s/epoch - 41ms/step  
Epoch 32/100  
469/469 - 19s - loss: 0.0392 - sparse\_categorical\_accuracy: 0.9873 - val\_loss: 0.0349 - val\_sparse\_c  
ategorical accuracy: 0.9880 - 19s/epoch - 41ms/step  
Epoch 33/100  
469/469 - 19s - loss: 0.0370 - sparse\_categorical\_accuracy: 0.9880 - val\_loss: 0.0401 - val\_sparse\_c  
ategorical accuracy: 0.9876 - 19s/epoch - 41ms/step  
Epoch 34/100  
469/469 - 19s - loss: 0.0385 - sparse\_categorical\_accuracy: 0.9875 - val\_loss: 0.0437 - val\_sparse\_c  
ategorical accuracy: 0.9868 - 19s/epoch - 41ms/step  
Epoch 35/100  
469/469 - 19s - loss: 0.0355 - sparse\_categorical\_accuracy: 0.9884 - val\_loss: 0.0390 - val\_sparse\_c  
ategorical accuracy: 0.9876 - 19s/epoch - 41ms/step  
Epoch 36/100  
469/469 - 19s - loss: 0.0371 - sparse\_categorical\_accuracy: 0.9876 - val\_loss: 0.0348 - val\_sparse\_c  
ategorical accuracy: 0.9890 - 19s/epoch - 41ms/step  
Epoch 37/100  
469/469 - 20s - loss: 0.0358 - sparse\_categorical\_accuracy: 0.9884 - val\_loss: 0.0407 - val\_sparse\_c  
ategorical accuracy: 0.9868 - 20s/epoch - 42ms/step  
Epoch 38/100  
469/469 - 20s - loss: 0.0355 - sparse\_categorical\_accuracy: 0.9881 - val\_loss: 0.0372 - val\_sparse\_c  
ategorical accuracy: 0.9884 - 20s/epoch - 42ms/step  
Epoch 39/100  
469/469 - 20s - loss: 0.0353 - sparse\_categorical\_accuracy: 0.9883 - val\_loss: 0.0414 - val\_sparse\_c  
ategorical accuracy: 0.9861 - 20s/epoch - 42ms/step  
Epoch 40/100  
469/469 - 19s - loss: 0.0357 - sparse\_categorical\_accuracy: 0.9884 - val\_loss: 0.0423 - val\_sparse\_c  
ategorical accuracy: 0.9867 - 19s/epoch - 41ms/step  
Epoch 41/100  
469/469 - 19s - loss: 0.0357 - sparse\_categorical\_accuracy: 0.9884 - val\_loss: 0.0435 - val\_sparse\_c  
ategorical accuracy: 0.9876 - 19s/epoch - 41ms/step  
Epoch 42/100  
469/469 - 19s - loss: 0.0344 - sparse\_categorical\_accuracy: 0.9883 - val\_loss: 0.0414 - val\_sparse\_c  
ategorical accuracy: 0.9872 - 19s/epoch - 41ms/step  
Epoch 43/100  
469/469 - 19s - loss: 0.0330 - sparse\_categorical\_accuracy: 0.9890 - val\_loss: 0.0399 - val\_sparse\_c  
ategorical accuracy: 0.9880 - 19s/epoch - 41ms/step  
Epoch 44/100  
469/469 - 20s - loss: 0.0348 - sparse\_categorical\_accuracy: 0.9886 - val\_loss: 0.0388 - val\_sparse\_c  
ategorical accuracy: 0.9874 - 20s/epoch - 42ms/step  
Epoch 45/100



[illegible]



Epoch 73/100  
469/469 - 20s - loss: 0.0290 - sparse\_categorical\_accuracy: 0.9903 - val\_loss: 0.0390 - val\_sparse\_categorical\_accuracy: 0.9884 - 20s/epoch - 42ms/step

Epoch 74/100  
469/469 - 20s - loss: 0.0295 - sparse\_categorical\_accuracy: 0.9901 - val\_loss: 0.0524 - val\_sparse\_categorical\_accuracy: 0.9836 - 20s/epoch - 42ms/step

Epoch 75/100  
469/469 - 19s - loss: 0.0293 - sparse\_categorical\_accuracy: 0.9901 - val\_loss: 0.0427 - val\_sparse\_categorical\_accuracy: 0.9876 - 19s/epoch - 41ms/step

Epoch 76/100  
469/469 - 20s - loss: 0.0296 - sparse\_categorical\_accuracy: 0.9897 - val\_loss: 0.0377 - val\_sparse\_categorical\_accuracy: 0.9883 - 20s/epoch - 42ms/step

Epoch 77/100  
469/469 - 19s - loss: 0.0285 - sparse\_categorical\_accuracy: 0.9905 - val\_loss: 0.0424 - val\_sparse\_categorical\_accuracy: 0.9868 - 19s/epoch - 41ms/step

Epoch 78/100  
469/469 - 20s - loss: 0.0274 - sparse\_categorical\_accuracy: 0.9908 - val\_loss: 0.0447 - val\_sparse\_categorical\_accuracy: 0.9867 - 20s/epoch - 42ms/step

Epoch 79/100  
469/469 - 20s - loss: 0.0275 - sparse\_categorical\_accuracy: 0.9909 - val\_loss: 0.0396 - val\_sparse\_categorical\_accuracy: 0.9878 - 20s/epoch - 43ms/step

Epoch 80/100  
469/469 - 20s - loss: 0.0283 - sparse\_categorical\_accuracy: 0.9903 - val\_loss: 0.0387 - val\_sparse\_categorical\_accuracy: 0.9886 - 20s/epoch - 42ms/step

Epoch 81/100  
469/469 - 20s - loss: 0.0276 - sparse\_categorical\_accuracy: 0.9907 - val\_loss: 0.0430 - val\_sparse\_categorical\_accuracy: 0.9883 - 20s/epoch - 43ms/step

Epoch 82/100  
469/469 - 20s - loss: 0.0284 - sparse\_categorical\_accuracy: 0.9898 - val\_loss: 0.0436 - val\_sparse\_categorical\_accuracy: 0.9871 - 20s/epoch - 43ms/step

Epoch 83/100  
469/469 - 21s - loss: 0.0275 - sparse\_categorical\_accuracy: 0.9906 - val\_loss: 0.0436 - val\_sparse\_categorical\_accuracy: 0.9874 - 21s/epoch - 44ms/step

Epoch 84/100  
469/469 - 20s - loss: 0.0274 - sparse\_categorical\_accuracy: 0.9907 - val\_loss: 0.0429 - val\_sparse\_categorical\_accuracy: 0.9877 - 20s/epoch - 43ms/step

Epoch 85/100  
469/469 - 20s - loss: 0.0288 - sparse\_categorical\_accuracy: 0.9902 - val\_loss: 0.0404 - val\_sparse\_categorical\_accuracy: 0.9878 - 20s/epoch - 42ms/step

Epoch 86/100  
469/469 - 20s - loss: 0.0268 - sparse\_categorical\_accuracy: 0.9909 - val\_loss: 0.0511 - val\_sparse\_categorical\_accuracy: 0.9849 - 20s/epoch - 42ms/step

Epoch 87/100  
469/469 - 20s - loss: 0.0284 - sparse\_categorical\_accuracy: 0.9905 - val\_loss: 0.0430 - val\_sparse\_categorical\_accuracy: 0.9881 - 20s/epoch - 43ms/step

Epoch 88/100  
469/469 - 20s - loss: 0.0277 - sparse\_categorical\_accuracy: 0.9906 - val\_loss: 0.0382 - val\_sparse\_categorical\_accuracy: 0.9881 - 20s/epoch - 43ms/step

Epoch 89/100  
469/469 - 21s - loss: 0.0260 - sparse\_categorical\_accuracy: 0.9912 - val\_loss: 0.0399 - val\_sparse\_categorical\_accuracy: 0.9888 - 21s/epoch - 44ms/step

Epoch 90/100  
469/469 - 21s - loss: 0.0288 - sparse\_categorical\_accuracy: 0.9898 - val\_loss: 0.0419 - val\_sparse\_categorical\_accuracy: 0.9886 - 21s/epoch - 44ms/step

Epoch 91/100  
469/469 - 21s - loss: 0.0276 - sparse\_categorical\_accuracy: 0.9908 - val\_loss: 0.0398 - val\_sparse\_categorical\_accuracy: 0.9887 - 21s/epoch - 45ms/step

Epoch 92/100  
469/469 - 21s - loss: 0.0271 - sparse\_categorical\_accuracy: 0.9911 - val\_loss: 0.0386 - val\_sparse\_categorical\_accuracy: 0.9896 - 21s/epoch - 44ms/step

Epoch 93/100  
469/469 - 20s - loss: 0.0253 - sparse\_categorical\_accuracy: 0.9915 - val\_loss: 0.0418 - val\_sparse\_categorical\_accuracy: 0.9884 - 20s/epoch - 42ms/step

Epoch 94/100  
469/469 - 20s - loss: 0.0267 - sparse\_categorical\_accuracy: 0.9909 - val\_loss: 0.0432 - val\_sparse\_categorical\_accuracy: 0.9879 - 20s/epoch - 43ms/step

Epoch 95/100  
469/469 - 20s - loss: 0.0263 - sparse\_categorical\_accuracy: 0.9909 - val\_loss: 0.0390 - val\_sparse\_categorical\_accuracy: 0.9882 - 20s/epoch - 42ms/step

Epoch 96/100  
469/469 - 19s - loss: 0.0269 - sparse\_categorical\_accuracy: 0.9910 - val\_loss: 0.0397 - val\_sparse\_categorical\_accuracy: 0.9878 - 19s/epoch - 41ms/step

Epoch 97/100  
469/469 - 19s - loss: 0.0287 - sparse\_categorical\_accuracy: 0.9905 - val\_loss: 0.0381 - val\_sparse\_categorical\_accuracy: 0.9884 - 19s/epoch - 41ms/step

Epoch 98/100  
469/469 - 19s - loss: 0.0260 - sparse\_categorical\_accuracy: 0.9908 - val\_loss: 0.0411 - val\_sparse\_categorical\_accuracy: 0.9894 - 19s/epoch - 41ms/step

Epoch 99/100  
469/469 - 20s - loss: 0.0248 - sparse\_categorical\_accuracy: 0.9918 - val\_loss: 0.0419 - val\_sparse\_categorical\_accuracy: 0.9886 - 20s/epoch - 42ms/step

Epoch 100/100  
469/469 - 20s - loss: 0.0261 - sparse\_categorical\_accuracy: 0.9915 - val\_loss: 0.0384 - val\_sparse\_categorical\_accuracy: 0.9886 - 20s/epoch - 42ms/step

ategorical\_accuracy: 0.9879 - 20s/epoch - 42ms/step

In [18]:

```
model3.evaluate(x_test, y_test, verbose=2)
```

313/313 - 1s - loss: 0.0384 - sparse\_categorical\_accuracy: 0.9879 - 1s/epoch - 4ms/step

Out[18]:

```
[0.038362883031368256, 0.9879000186920166]
```

**Model4: adam optimizer with learning rate= $e^{-4}$ , random\_normal initializer, dropout regularization with rate=0.1.**

In [19]:

```
model4 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.1),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_normal')
])
```

In [20]:

```
model4.compile(optimizer=tf.keras.optimizers.Adam(0.0001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])
```

In [21]:

```
history4 = model4.fit(x_train, y_train,
                      batch_size=128,
                      epochs=100,
                      validation_data=(x_test, y_test),
                      verbose=2)
```

Epoch 1/100

469/469 - 20s - loss: 2.0166 - sparse\_categorical\_accuracy: 0.4317 - val\_loss: 1.6639 - val\_sparse\_categorical\_accuracy: 0.6980 - 20s/epoch - 43ms/step

Epoch 2/100

469/469 - 20s - loss: 1.4940 - sparse\_categorical\_accuracy: 0.7277 - val\_loss: 1.1858 - val\_sparse\_categorical\_accuracy: 0.8278 - 20s/epoch - 42ms/step

Epoch 3/100

469/469 - 20s - loss: 1.0695 - sparse\_categorical\_accuracy: 0.8383 - val\_loss: 0.8614 - val\_sparse\_categorical\_accuracy: 0.9035 - 20s/epoch - 42ms/step

Epoch 4/100

469/469 - 19s - loss: 0.7742 - sparse\_categorical\_accuracy: 0.9013 - val\_loss: 0.5965 - val\_sparse\_categorical\_accuracy: 0.9431 - 19s/epoch - 41ms/step

Epoch 5/100

469/469 - 19s - loss: 0.5527 - sparse\_categorical\_accuracy: 0.9328 - val\_loss: 0.4111 - val\_sparse\_categorical\_accuracy: 0.9583 - 19s/epoch - 41ms/step

Epoch 6/100

469/469 - 19s - loss: 0.4026 - sparse\_categorical\_accuracy: 0.9464 - val\_loss: 0.3003 - val\_sparse\_categorical\_accuracy: 0.9662 - 19s/epoch - 41ms/step

Epoch 7/100

469/469 - 19s - loss: 0.3068 - sparse\_categorical\_accuracy: 0.9553 - val\_loss: 0.2227 - val\_sparse\_categorical\_accuracy: 0.9712 - 19s/epoch - 41ms/step

Epoch 8/100

469/469 - 20s - loss: 0.2411 - sparse\_categorical\_accuracy: 0.9603 - val\_loss: 0.1714 - val\_sparse\_categorical\_accuracy: 0.9726 - 20s/epoch - 42ms/step

Epoch 9/100

469/469 - 20s - loss: 0.1982 - sparse\_categorical\_accuracy: 0.9637 - val\_loss: 0.1399 - val\_sparse\_categorical\_accuracy: 0.9759 - 20s/epoch - 42ms/step

Epoch 10/100

469/469 - 20s - loss: 0.1667 - sparse\_categorical\_accuracy: 0.9660 - val\_loss: 0.1150 - val\_sparse\_categorical\_accuracy: 0.9773 - 20s/epoch - 42ms/step

Epoch 11/100

469/469 - 20s - loss: 0.1431 - sparse\_categorical\_accuracy: 0.9698 - val\_loss: 0.0974 - val\_sparse\_categorical\_accuracy: 0.9786 - 20s/epoch - 42ms/step

Epoch 12/100

469/469 - 19s - loss: 0.1281 - sparse\_categorical\_accuracy: 0.9709 - val\_loss: 0.0850 - val\_sparse\_categorical\_accuracy: 0.9803 - 19s/epoch - 41ms/step

```
Epoch 13/100
469/469 - 19s - loss: 0.1144 - sparse_categorical_accuracy: 0.9729 - val_loss: 0.0769 - val_sparse_c
ategorical_accuracy: 0.9813 - 19s/epoch - 41ms/step
Epoch 14/100
469/469 - 19s - loss: 0.1051 - sparse_categorical_accuracy: 0.9737 - val_loss: 0.0706 - val_sparse_c
ategorical_accuracy: 0.9813 - 19s/epoch - 41ms/step
Epoch 15/100
469/469 - 19s - loss: 0.0979 - sparse_categorical_accuracy: 0.9742 - val_loss: 0.0652 - val_sparse_c
ategorical_accuracy: 0.9820 - 19s/epoch - 41ms/step
Epoch 16/100
469/469 - 20s - loss: 0.0910 - sparse_categorical_accuracy: 0.9757 - val_loss: 0.0614 - val_sparse_c
ategorical_accuracy: 0.9825 - 20s/epoch - 42ms/step
Epoch 17/100
469/469 - 19s - loss: 0.0866 - sparse_categorical_accuracy: 0.9764 - val_loss: 0.0581 - val_sparse_c
ategorical_accuracy: 0.9831 - 19s/epoch - 41ms/step
Epoch 18/100
469/469 - 19s - loss: 0.0807 - sparse_categorical_accuracy: 0.9775 - val_loss: 0.0556 - val_sparse_c
ategorical_accuracy: 0.9831 - 19s/epoch - 41ms/step
Epoch 19/100
469/469 - 19s - loss: 0.0782 - sparse_categorical_accuracy: 0.9779 - val_loss: 0.0530 - val_sparse_c
ategorical_accuracy: 0.9838 - 19s/epoch - 41ms/step
Epoch 20/100
469/469 - 19s - loss: 0.0736 - sparse_categorical_accuracy: 0.9794 - val_loss: 0.0516 - val_sparse_c
ategorical_accuracy: 0.9834 - 19s/epoch - 41ms/step
Epoch 21/100
469/469 - 19s - loss: 0.0722 - sparse_categorical_accuracy: 0.9789 - val_loss: 0.0489 - val_sparse_c
ategorical_accuracy: 0.9834 - 19s/epoch - 41ms/step
Epoch 22/100
469/469 - 19s - loss: 0.0708 - sparse_categorical_accuracy: 0.9796 - val_loss: 0.0494 - val_sparse_c
ategorical_accuracy: 0.9847 - 19s/epoch - 41ms/step
Epoch 23/100
469/469 - 19s - loss: 0.0698 - sparse_categorical_accuracy: 0.9798 - val_loss: 0.0470 - val_sparse_c
ategorical_accuracy: 0.9847 - 19s/epoch - 41ms/step
Epoch 24/100
469/469 - 19s - loss: 0.0668 - sparse_categorical_accuracy: 0.9796 - val_loss: 0.0472 - val_sparse_c
ategorical_accuracy: 0.9842 - 19s/epoch - 41ms/step
Epoch 25/100
469/469 - 19s - loss: 0.0649 - sparse_categorical_accuracy: 0.9806 - val_loss: 0.0455 - val_sparse_c
ategorical_accuracy: 0.9845 - 19s/epoch - 41ms/step
Epoch 26/100
469/469 - 19s - loss: 0.0617 - sparse_categorical_accuracy: 0.9812 - val_loss: 0.0449 - val_sparse_c
ategorical_accuracy: 0.9853 - 19s/epoch - 41ms/step
Epoch 27/100
469/469 - 19s - loss: 0.0612 - sparse_categorical_accuracy: 0.9816 - val_loss: 0.0454 - val_sparse_c
ategorical_accuracy: 0.9847 - 19s/epoch - 40ms/step
Epoch 28/100
469/469 - 19s - loss: 0.0601 - sparse_categorical_accuracy: 0.9819 - val_loss: 0.0433 - val_sparse_c
ategorical_accuracy: 0.9859 - 19s/epoch - 41ms/step
Epoch 29/100
469/469 - 19s - loss: 0.0583 - sparse_categorical_accuracy: 0.9829 - val_loss: 0.0441 - val_sparse_c
ategorical_accuracy: 0.9848 - 19s/epoch - 40ms/step
Epoch 30/100
469/469 - 19s - loss: 0.0582 - sparse_categorical_accuracy: 0.9827 - val_loss: 0.0422 - val_sparse_c
ategorical_accuracy: 0.9856 - 19s/epoch - 40ms/step
Epoch 31/100
469/469 - 19s - loss: 0.0580 - sparse_categorical_accuracy: 0.9823 - val_loss: 0.0422 - val_sparse_c
ategorical_accuracy: 0.9860 - 19s/epoch - 41ms/step
Epoch 32/100
469/469 - 19s - loss: 0.0567 - sparse_categorical_accuracy: 0.9824 - val_loss: 0.0416 - val_sparse_c
ategorical_accuracy: 0.9851 - 19s/epoch - 40ms/step
Epoch 33/100
469/469 - 19s - loss: 0.0566 - sparse_categorical_accuracy: 0.9825 - val_loss: 0.0411 - val_sparse_c
ategorical_accuracy: 0.9861 - 19s/epoch - 41ms/step
Epoch 34/100
469/469 - 19s - loss: 0.0531 - sparse_categorical_accuracy: 0.9837 - val_loss: 0.0413 - val_sparse_c
ategorical_accuracy: 0.9859 - 19s/epoch - 41ms/step
Epoch 35/100
469/469 - 19s - loss: 0.0544 - sparse_categorical_accuracy: 0.9833 - val_loss: 0.0391 - val_sparse_c
ategorical_accuracy: 0.9865 - 19s/epoch - 41ms/step
Epoch 36/100
469/469 - 19s - loss: 0.0530 - sparse_categorical_accuracy: 0.9833 - val_loss: 0.0406 - val_sparse_c
ategorical_accuracy: 0.9857 - 19s/epoch - 40ms/step
Epoch 37/100
469/469 - 19s - loss: 0.0523 - sparse_categorical_accuracy: 0.9838 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9866 - 19s/epoch - 41ms/step
Epoch 38/100
469/469 - 19s - loss: 0.0517 - sparse_categorical_accuracy: 0.9841 - val_loss: 0.0404 - val_sparse_c
ategorical_accuracy: 0.9867 - 19s/epoch - 40ms/step
Epoch 39/100
469/469 - 19s - loss: 0.0517 - sparse_categorical_accuracy: 0.9839 - val_loss: 0.0377 - val_sparse_c
ategorical_accuracy: 0.9875 - 19s/epoch - 40ms/step
Epoch 40/100
469/469 - 19s - loss: 0.0499 - sparse_categorical_accuracy: 0.9841 - val_loss: 0.0392 - val_sparse_c
```

[illegible]

[illegible]

Epoch 96/100  
469/469 - 20s - loss: 0.0343 - sparse\_categorical\_accuracy: 0.9891 - val\_loss: 0.0374 - val\_sparse\_categorical\_accuracy: 0.9883 - 20s/epoch - 42ms/step  
Epoch 97/100  
469/469 - 19s - loss: 0.0346 - sparse\_categorical\_accuracy: 0.9892 - val\_loss: 0.0375 - val\_sparse\_categorical\_accuracy: 0.9880 - 19s/epoch - 41ms/step  
Epoch 98/100  
469/469 - 19s - loss: 0.0347 - sparse\_categorical\_accuracy: 0.9892 - val\_loss: 0.0352 - val\_sparse\_categorical\_accuracy: 0.9883 - 19s/epoch - 40ms/step  
Epoch 99/100  
469/469 - 19s - loss: 0.0347 - sparse\_categorical\_accuracy: 0.9892 - val\_loss: 0.0341 - val\_sparse\_categorical\_accuracy: 0.9885 - 19s/epoch - 41ms/step  
Epoch 100/100  
469/469 - 19s - loss: 0.0340 - sparse\_categorical\_accuracy: 0.9890 - val\_loss: 0.0361 - val\_sparse\_categorical\_accuracy: 0.9884 - 19s/epoch - 40ms/step

In [22]:

```
model4.evaluate(x_test, y_test, verbose=2)
```

313/313 - 1s - loss: 0.0361 - sparse\_categorical\_accuracy: 0.9884 - 1s/epoch - 4ms/step

Out[22]:

```
[0.036136385053396225, 0.9883999824523926]
```

**Model5: adam optimizer with learning rate= $e^{-3}$ , random\_uniform initializer, dropout regularization with rate=0.2.**

In [23]:

```
model5 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.2),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_uniform')
])
```

In [24]:

```
model5.compile(optimizer=tf.keras.optimizers.Adam(0.001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])
```

In [25]:

```
history5 = model5.fit(x_train, y_train,
                      batch_size=128,
                      epochs=100,
                      validation_data=(x_test, y_test),
                      verbose=2)
```

Epoch 1/100  
469/469 - 20s - loss: 0.7364 - sparse\_categorical\_accuracy: 0.8833 - val\_loss: 0.1193 - val\_sparse\_categorical\_accuracy: 0.9731 - 20s/epoch - 42ms/step  
Epoch 2/100  
469/469 - 19s - loss: 0.1364 - sparse\_categorical\_accuracy: 0.9664 - val\_loss: 0.0737 - val\_sparse\_categorical\_accuracy: 0.9803 - 19s/epoch - 41ms/step  
Epoch 3/100  
469/469 - 19s - loss: 0.1003 - sparse\_categorical\_accuracy: 0.9713 - val\_loss: 0.0673 - val\_sparse\_categorical\_accuracy: 0.9790 - 19s/epoch - 41ms/step  
Epoch 4/100  
469/469 - 19s - loss: 0.0868 - sparse\_categorical\_accuracy: 0.9741 - val\_loss: 0.0552 - val\_sparse\_categorical\_accuracy: 0.9828 - 19s/epoch - 41ms/step  
Epoch 5/100  
469/469 - 19s - loss: 0.0773 - sparse\_categorical\_accuracy: 0.9767 - val\_loss: 0.0481 - val\_sparse\_categorical\_accuracy: 0.9838 - 19s/epoch - 41ms/step  
Epoch 6/100  
469/469 - 20s - loss: 0.0723 - sparse\_categorical\_accuracy: 0.9775 - val\_loss: 0.0494 - val\_sparse\_categorical\_accuracy: 0.9842 - 20s/epoch - 42ms/step  
Epoch 7/100  
469/469 - 19s - loss: 0.0706 - sparse\_categorical\_accuracy: 0.9780 - val\_loss: 0.0507 - val\_sparse\_categorical\_accuracy: 0.9831 - 19s/epoch - 41ms/step

Epoch 8/100  
469/469 - 19s - loss: 0.0656 - sparse\_categorical\_accuracy: 0.9797 - val\_loss: 0.0573 - val\_sparse\_categorical\_accuracy: 0.9805 - 19s/epoch - 41ms/step

Epoch 9/100  
469/469 - 19s - loss: 0.0659 - sparse\_categorical\_accuracy: 0.9797 - val\_loss: 0.0458 - val\_sparse\_categorical\_accuracy: 0.9860 - 19s/epoch - 41ms/step

Epoch 10/100  
469/469 - 19s - loss: 0.0624 - sparse\_categorical\_accuracy: 0.9808 - val\_loss: 0.0451 - val\_sparse\_categorical\_accuracy: 0.9855 - 19s/epoch - 41ms/step

Epoch 11/100  
469/469 - 21s - loss: 0.0623 - sparse\_categorical\_accuracy: 0.9812 - val\_loss: 0.0414 - val\_sparse\_categorical\_accuracy: 0.9857 - 21s/epoch - 44ms/step

Epoch 12/100  
469/469 - 24s - loss: 0.0594 - sparse\_categorical\_accuracy: 0.9811 - val\_loss: 0.0402 - val\_sparse\_categorical\_accuracy: 0.9866 - 24s/epoch - 51ms/step

Epoch 13/100  
469/469 - 23s - loss: 0.0589 - sparse\_categorical\_accuracy: 0.9817 - val\_loss: 0.0383 - val\_sparse\_categorical\_accuracy: 0.9871 - 23s/epoch - 49ms/step

Epoch 14/100  
469/469 - 22s - loss: 0.0574 - sparse\_categorical\_accuracy: 0.9815 - val\_loss: 0.0432 - val\_sparse\_categorical\_accuracy: 0.9851 - 22s/epoch - 47ms/step

Epoch 15/100  
469/469 - 22s - loss: 0.0565 - sparse\_categorical\_accuracy: 0.9826 - val\_loss: 0.0466 - val\_sparse\_categorical\_accuracy: 0.9855 - 22s/epoch - 47ms/step

Epoch 16/100  
469/469 - 21s - loss: 0.0565 - sparse\_categorical\_accuracy: 0.9822 - val\_loss: 0.0491 - val\_sparse\_categorical\_accuracy: 0.9842 - 21s/epoch - 45ms/step

Epoch 17/100  
469/469 - 21s - loss: 0.0563 - sparse\_categorical\_accuracy: 0.9815 - val\_loss: 0.0387 - val\_sparse\_categorical\_accuracy: 0.9864 - 21s/epoch - 45ms/step

Epoch 18/100  
469/469 - 21s - loss: 0.0546 - sparse\_categorical\_accuracy: 0.9817 - val\_loss: 0.0467 - val\_sparse\_categorical\_accuracy: 0.9840 - 21s/epoch - 45ms/step

Epoch 19/100  
469/469 - 22s - loss: 0.0555 - sparse\_categorical\_accuracy: 0.9829 - val\_loss: 0.0383 - val\_sparse\_categorical\_accuracy: 0.9870 - 22s/epoch - 46ms/step

Epoch 20/100  
469/469 - 22s - loss: 0.0527 - sparse\_categorical\_accuracy: 0.9833 - val\_loss: 0.0389 - val\_sparse\_categorical\_accuracy: 0.9867 - 22s/epoch - 46ms/step

Epoch 21/100  
469/469 - 21s - loss: 0.0525 - sparse\_categorical\_accuracy: 0.9833 - val\_loss: 0.0372 - val\_sparse\_categorical\_accuracy: 0.9872 - 21s/epoch - 45ms/step

Epoch 22/100  
469/469 - 21s - loss: 0.0520 - sparse\_categorical\_accuracy: 0.9831 - val\_loss: 0.0390 - val\_sparse\_categorical\_accuracy: 0.9871 - 21s/epoch - 44ms/step

Epoch 23/100  
469/469 - 21s - loss: 0.0504 - sparse\_categorical\_accuracy: 0.9838 - val\_loss: 0.0393 - val\_sparse\_categorical\_accuracy: 0.9865 - 21s/epoch - 45ms/step

Epoch 24/100  
469/469 - 21s - loss: 0.0494 - sparse\_categorical\_accuracy: 0.9839 - val\_loss: 0.0384 - val\_sparse\_categorical\_accuracy: 0.9869 - 21s/epoch - 46ms/step

Epoch 25/100  
469/469 - 21s - loss: 0.0494 - sparse\_categorical\_accuracy: 0.9841 - val\_loss: 0.0362 - val\_sparse\_categorical\_accuracy: 0.9887 - 21s/epoch - 46ms/step

Epoch 26/100  
469/469 - 21s - loss: 0.0510 - sparse\_categorical\_accuracy: 0.9836 - val\_loss: 0.0354 - val\_sparse\_categorical\_accuracy: 0.9882 - 21s/epoch - 45ms/step

Epoch 27/100  
469/469 - 21s - loss: 0.0501 - sparse\_categorical\_accuracy: 0.9842 - val\_loss: 0.0353 - val\_sparse\_categorical\_accuracy: 0.9876 - 21s/epoch - 45ms/step

Epoch 28/100  
469/469 - 21s - loss: 0.0506 - sparse\_categorical\_accuracy: 0.9837 - val\_loss: 0.0342 - val\_sparse\_categorical\_accuracy: 0.9883 - 21s/epoch - 44ms/step

Epoch 29/100  
469/469 - 21s - loss: 0.0478 - sparse\_categorical\_accuracy: 0.9846 - val\_loss: 0.0393 - val\_sparse\_categorical\_accuracy: 0.9858 - 21s/epoch - 44ms/step

Epoch 30/100  
469/469 - 21s - loss: 0.0485 - sparse\_categorical\_accuracy: 0.9838 - val\_loss: 0.0342 - val\_sparse\_categorical\_accuracy: 0.9889 - 21s/epoch - 44ms/step

Epoch 31/100  
469/469 - 21s - loss: 0.0472 - sparse\_categorical\_accuracy: 0.9854 - val\_loss: 0.0327 - val\_sparse\_categorical\_accuracy: 0.9887 - 21s/epoch - 44ms/step

Epoch 32/100  
469/469 - 21s - loss: 0.0476 - sparse\_categorical\_accuracy: 0.9846 - val\_loss: 0.0361 - val\_sparse\_categorical\_accuracy: 0.9883 - 21s/epoch - 44ms/step

Epoch 33/100  
469/469 - 21s - loss: 0.0468 - sparse\_categorical\_accuracy: 0.9847 - val\_loss: 0.0366 - val\_sparse\_categorical\_accuracy: 0.9882 - 21s/epoch - 45ms/step

Epoch 34/100  
469/469 - 21s - loss: 0.0459 - sparse\_categorical\_accuracy: 0.9851 - val\_loss: 0.0352 - val\_sparse\_categorical\_accuracy: 0.9883 - 21s/epoch - 45ms/step

Epoch 35/100  
469/469 - 21s - loss: 0.0478 - sparse\_categorical\_accuracy: 0.9844 - val\_loss: 0.0416 - val\_sparse\_categorical\_accuracy: 0.9844 - 21s/epoch - 45ms/step

ategorical\_accuracy: 0.9867 - 21s/epoch - 45ms/step  
Epoch 36/100  
469/469 - 21s - loss: 0.0471 - sparse\_categorical\_accuracy: 0.9846 - val\_loss: 0.0363 - val\_sparse\_c  
ategorical\_accuracy: 0.9872 - 21s/epoch - 44ms/step  
Epoch 37/100  
469/469 - 21s - loss: 0.0448 - sparse\_categorical\_accuracy: 0.9854 - val\_loss: 0.0371 - val\_sparse\_c  
ategorical\_accuracy: 0.9873 - 21s/epoch - 45ms/step  
Epoch 38/100  
469/469 - 21s - loss: 0.0485 - sparse\_categorical\_accuracy: 0.9844 - val\_loss: 0.0357 - val\_sparse\_c  
ategorical\_accuracy: 0.9881 - 21s/epoch - 45ms/step  
Epoch 39/100  
469/469 - 21s - loss: 0.0456 - sparse\_categorical\_accuracy: 0.9856 - val\_loss: 0.0385 - val\_sparse\_c  
ategorical\_accuracy: 0.9865 - 21s/epoch - 45ms/step  
Epoch 40/100  
469/469 - 21s - loss: 0.0451 - sparse\_categorical\_accuracy: 0.9852 - val\_loss: 0.0362 - val\_sparse\_c  
ategorical\_accuracy: 0.9870 - 21s/epoch - 45ms/step  
Epoch 41/100  
469/469 - 21s - loss: 0.0457 - sparse\_categorical\_accuracy: 0.9844 - val\_loss: 0.0374 - val\_sparse\_c  
ategorical\_accuracy: 0.9879 - 21s/epoch - 45ms/step  
Epoch 42/100  
469/469 - 22s - loss: 0.0445 - sparse\_categorical\_accuracy: 0.9855 - val\_loss: 0.0379 - val\_sparse\_c  
ategorical\_accuracy: 0.9873 - 22s/epoch - 48ms/step  
Epoch 43/100  
469/469 - 21s - loss: 0.0453 - sparse\_categorical\_accuracy: 0.9854 - val\_loss: 0.0378 - val\_sparse\_c  
ategorical\_accuracy: 0.9878 - 21s/epoch - 46ms/step  
Epoch 44/100  
469/469 - 21s - loss: 0.0451 - sparse\_categorical\_accuracy: 0.9850 - val\_loss: 0.0332 - val\_sparse\_c  
ategorical\_accuracy: 0.9886 - 21s/epoch - 45ms/step  
Epoch 45/100  
469/469 - 22s - loss: 0.0441 - sparse\_categorical\_accuracy: 0.9854 - val\_loss: 0.0390 - val\_sparse\_c  
ategorical\_accuracy: 0.9878 - 22s/epoch - 46ms/step  
Epoch 46/100  
469/469 - 21s - loss: 0.0449 - sparse\_categorical\_accuracy: 0.9859 - val\_loss: 0.0377 - val\_sparse\_c  
ategorical\_accuracy: 0.9885 - 21s/epoch - 45ms/step  
Epoch 47/100  
469/469 - 21s - loss: 0.0433 - sparse\_categorical\_accuracy: 0.9858 - val\_loss: 0.0383 - val\_sparse\_c  
ategorical\_accuracy: 0.9874 - 21s/epoch - 45ms/step  
Epoch 48/100  
469/469 - 21s - loss: 0.0431 - sparse\_categorical\_accuracy: 0.9862 - val\_loss: 0.0416 - val\_sparse\_c  
ategorical\_accuracy: 0.9868 - 21s/epoch - 45ms/step  
Epoch 49/100  
469/469 - 21s - loss: 0.0427 - sparse\_categorical\_accuracy: 0.9862 - val\_loss: 0.0455 - val\_sparse\_c  
ategorical\_accuracy: 0.9858 - 21s/epoch - 45ms/step  
Epoch 50/100  
469/469 - 21s - loss: 0.0442 - sparse\_categorical\_accuracy: 0.9859 - val\_loss: 0.0328 - val\_sparse\_c  
ategorical\_accuracy: 0.9887 - 21s/epoch - 45ms/step  
Epoch 51/100  
469/469 - 21s - loss: 0.0429 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0351 - val\_sparse\_c  
ategorical\_accuracy: 0.9876 - 21s/epoch - 44ms/step  
Epoch 52/100  
469/469 - 21s - loss: 0.0417 - sparse\_categorical\_accuracy: 0.9863 - val\_loss: 0.0399 - val\_sparse\_c  
ategorical\_accuracy: 0.9870 - 21s/epoch - 45ms/step  
Epoch 53/100  
469/469 - 21s - loss: 0.0427 - sparse\_categorical\_accuracy: 0.9862 - val\_loss: 0.0338 - val\_sparse\_c  
ategorical\_accuracy: 0.9883 - 21s/epoch - 45ms/step  
Epoch 54/100  
469/469 - 21s - loss: 0.0427 - sparse\_categorical\_accuracy: 0.9860 - val\_loss: 0.0324 - val\_sparse\_c  
ategorical\_accuracy: 0.9889 - 21s/epoch - 45ms/step  
Epoch 55/100  
469/469 - 21s - loss: 0.0447 - sparse\_categorical\_accuracy: 0.9857 - val\_loss: 0.0376 - val\_sparse\_c  
ategorical\_accuracy: 0.9881 - 21s/epoch - 45ms/step  
Epoch 56/100  
469/469 - 21s - loss: 0.0420 - sparse\_categorical\_accuracy: 0.9867 - val\_loss: 0.0371 - val\_sparse\_c  
ategorical\_accuracy: 0.9877 - 21s/epoch - 45ms/step  
Epoch 57/100  
469/469 - 21s - loss: 0.0411 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0413 - val\_sparse\_c  
ategorical\_accuracy: 0.9867 - 21s/epoch - 45ms/step  
Epoch 58/100  
469/469 - 21s - loss: 0.0428 - sparse\_categorical\_accuracy: 0.9859 - val\_loss: 0.0333 - val\_sparse\_c  
ategorical\_accuracy: 0.9890 - 21s/epoch - 45ms/step  
Epoch 59/100  
469/469 - 21s - loss: 0.0412 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0360 - val\_sparse\_c  
ategorical\_accuracy: 0.9877 - 21s/epoch - 46ms/step  
Epoch 60/100  
469/469 - 21s - loss: 0.0434 - sparse\_categorical\_accuracy: 0.9859 - val\_loss: 0.0336 - val\_sparse\_c  
ategorical\_accuracy: 0.9887 - 21s/epoch - 46ms/step  
Epoch 61/100  
469/469 - 21s - loss: 0.0400 - sparse\_categorical\_accuracy: 0.9871 - val\_loss: 0.0359 - val\_sparse\_c  
ategorical\_accuracy: 0.9884 - 21s/epoch - 45ms/step  
Epoch 62/100  
469/469 - 21s - loss: 0.0410 - sparse\_categorical\_accuracy: 0.9869 - val\_loss: 0.0348 - val\_sparse\_c  
ategorical\_accuracy: 0.9886 - 21s/epoch - 45ms/step  
Epoch 63/100



469/469 - 21s - loss: 0.0427 - sparse\_categorical\_accuracy: 0.9861 - val\_loss: 0.0348 - val\_sparse\_categorical\_accuracy: 0.9881 - 21s/epoch - 45ms/step  
Epoch 64/100  
469/469 - 21s - loss: 0.0410 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0352 - val\_sparse\_categorical\_accuracy: 0.9891 - 21s/epoch - 45ms/step  
Epoch 65/100  
469/469 - 21s - loss: 0.0398 - sparse\_categorical\_accuracy: 0.9869 - val\_loss: 0.0355 - val\_sparse\_categorical\_accuracy: 0.9883 - 21s/epoch - 46ms/step  
Epoch 66/100  
469/469 - 22s - loss: 0.0405 - sparse\_categorical\_accuracy: 0.9868 - val\_loss: 0.0331 - val\_sparse\_categorical\_accuracy: 0.9890 - 22s/epoch - 46ms/step  
Epoch 67/100  
469/469 - 22s - loss: 0.0419 - sparse\_categorical\_accuracy: 0.9863 - val\_loss: 0.0325 - val\_sparse\_categorical\_accuracy: 0.9890 - 22s/epoch - 46ms/step  
Epoch 68/100  
469/469 - 22s - loss: 0.0401 - sparse\_categorical\_accuracy: 0.9872 - val\_loss: 0.0341 - val\_sparse\_categorical\_accuracy: 0.9893 - 22s/epoch - 47ms/step  
Epoch 69/100  
469/469 - 22s - loss: 0.0413 - sparse\_categorical\_accuracy: 0.9864 - val\_loss: 0.0409 - val\_sparse\_categorical\_accuracy: 0.9865 - 22s/epoch - 48ms/step  
Epoch 70/100  
469/469 - 21s - loss: 0.0421 - sparse\_categorical\_accuracy: 0.9864 - val\_loss: 0.0356 - val\_sparse\_categorical\_accuracy: 0.9890 - 21s/epoch - 46ms/step  
Epoch 71/100  
469/469 - 21s - loss: 0.0398 - sparse\_categorical\_accuracy: 0.9870 - val\_loss: 0.0368 - val\_sparse\_categorical\_accuracy: 0.9884 - 21s/epoch - 46ms/step  
Epoch 72/100  
469/469 - 21s - loss: 0.0402 - sparse\_categorical\_accuracy: 0.9865 - val\_loss: 0.0326 - val\_sparse\_categorical\_accuracy: 0.9891 - 21s/epoch - 45ms/step  
Epoch 73/100  
469/469 - 21s - loss: 0.0419 - sparse\_categorical\_accuracy: 0.9863 - val\_loss: 0.0332 - val\_sparse\_categorical\_accuracy: 0.9890 - 21s/epoch - 44ms/step  
Epoch 74/100  
469/469 - 21s - loss: 0.0403 - sparse\_categorical\_accuracy: 0.9865 - val\_loss: 0.0360 - val\_sparse\_categorical\_accuracy: 0.9885 - 21s/epoch - 45ms/step  
Epoch 75/100  
469/469 - 21s - loss: 0.0398 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0382 - val\_sparse\_categorical\_accuracy: 0.9874 - 21s/epoch - 45ms/step  
Epoch 76/100  
469/469 - 21s - loss: 0.0389 - sparse\_categorical\_accuracy: 0.9874 - val\_loss: 0.0334 - val\_sparse\_categorical\_accuracy: 0.9883 - 21s/epoch - 44ms/step  
Epoch 77/100  
469/469 - 21s - loss: 0.0401 - sparse\_categorical\_accuracy: 0.9866 - val\_loss: 0.0357 - val\_sparse\_categorical\_accuracy: 0.9886 - 21s/epoch - 45ms/step  
Epoch 78/100  
469/469 - 22s - loss: 0.0392 - sparse\_categorical\_accuracy: 0.9869 - val\_loss: 0.0315 - val\_sparse\_categorical\_accuracy: 0.9888 - 22s/epoch - 46ms/step  
Epoch 79/100  
469/469 - 21s - loss: 0.0384 - sparse\_categorical\_accuracy: 0.9874 - val\_loss: 0.0378 - val\_sparse\_categorical\_accuracy: 0.9878 - 21s/epoch - 46ms/step  
Epoch 80/100  
469/469 - 21s - loss: 0.0412 - sparse\_categorical\_accuracy: 0.9865 - val\_loss: 0.0327 - val\_sparse\_categorical\_accuracy: 0.9896 - 21s/epoch - 45ms/step  
Epoch 81/100  
469/469 - 21s - loss: 0.0381 - sparse\_categorical\_accuracy: 0.9875 - val\_loss: 0.0321 - val\_sparse\_categorical\_accuracy: 0.9891 - 21s/epoch - 45ms/step  
Epoch 82/100  
469/469 - 21s - loss: 0.0401 - sparse\_categorical\_accuracy: 0.9871 - val\_loss: 0.0345 - val\_sparse\_categorical\_accuracy: 0.9897 - 21s/epoch - 45ms/step  
Epoch 83/100  
469/469 - 21s - loss: 0.0409 - sparse\_categorical\_accuracy: 0.9869 - val\_loss: 0.0382 - val\_sparse\_categorical\_accuracy: 0.9880 - 21s/epoch - 45ms/step  
Epoch 84/100  
469/469 - 21s - loss: 0.0391 - sparse\_categorical\_accuracy: 0.9870 - val\_loss: 0.0378 - val\_sparse\_categorical\_accuracy: 0.9874 - 21s/epoch - 45ms/step  
Epoch 85/100  
469/469 - 21s - loss: 0.0386 - sparse\_categorical\_accuracy: 0.9872 - val\_loss: 0.0314 - val\_sparse\_categorical\_accuracy: 0.9904 - 21s/epoch - 45ms/step  
Epoch 86/100  
469/469 - 21s - loss: 0.0377 - sparse\_categorical\_accuracy: 0.9876 - val\_loss: 0.0325 - val\_sparse\_categorical\_accuracy: 0.9900 - 21s/epoch - 45ms/step  
Epoch 87/100  
469/469 - 21s - loss: 0.0382 - sparse\_categorical\_accuracy: 0.9880 - val\_loss: 0.0351 - val\_sparse\_categorical\_accuracy: 0.9889 - 21s/epoch - 45ms/step  
Epoch 88/100  
469/469 - 21s - loss: 0.0401 - sparse\_categorical\_accuracy: 0.9865 - val\_loss: 0.0344 - val\_sparse\_categorical\_accuracy: 0.9892 - 21s/epoch - 45ms/step  
Epoch 89/100  
469/469 - 22s - loss: 0.0397 - sparse\_categorical\_accuracy: 0.9874 - val\_loss: 0.0335 - val\_sparse\_categorical\_accuracy: 0.9894 - 22s/epoch - 46ms/step  
Epoch 90/100  
469/469 - 21s - loss: 0.0387 - sparse\_categorical\_accuracy: 0.9875 - val\_loss: 0.0347 - val\_sparse\_categorical\_accuracy: 0.9890 - 21s/epoch - 46ms/step

Epoch 91/100  
 469/469 - 21s - loss: 0.0378 - sparse\_categorical\_accuracy: 0.9872 - val\_loss: 0.0358 - val\_sparse\_categorical\_accuracy: 0.9889 - 21s/epoch - 46ms/step  
 Epoch 92/100  
 469/469 - 21s - loss: 0.0385 - sparse\_categorical\_accuracy: 0.9869 - val\_loss: 0.0368 - val\_sparse\_categorical\_accuracy: 0.9882 - 21s/epoch - 45ms/step  
 Epoch 93/100  
 469/469 - 21s - loss: 0.0381 - sparse\_categorical\_accuracy: 0.9880 - val\_loss: 0.0346 - val\_sparse\_categorical\_accuracy: 0.9888 - 21s/epoch - 45ms/step  
 Epoch 94/100  
 469/469 - 21s - loss: 0.0394 - sparse\_categorical\_accuracy: 0.9873 - val\_loss: 0.0379 - val\_sparse\_categorical\_accuracy: 0.9872 - 21s/epoch - 45ms/step  
 Epoch 95/100  
 469/469 - 21s - loss: 0.0397 - sparse\_categorical\_accuracy: 0.9864 - val\_loss: 0.0348 - val\_sparse\_categorical\_accuracy: 0.9883 - 21s/epoch - 45ms/step  
 Epoch 96/100  
 469/469 - 21s - loss: 0.0392 - sparse\_categorical\_accuracy: 0.9874 - val\_loss: 0.0387 - val\_sparse\_categorical\_accuracy: 0.9873 - 21s/epoch - 45ms/step  
 Epoch 97/100  
 469/469 - 21s - loss: 0.0382 - sparse\_categorical\_accuracy: 0.9874 - val\_loss: 0.0338 - val\_sparse\_categorical\_accuracy: 0.9891 - 21s/epoch - 45ms/step  
 Epoch 98/100  
 469/469 - 21s - loss: 0.0392 - sparse\_categorical\_accuracy: 0.9868 - val\_loss: 0.0356 - val\_sparse\_categorical\_accuracy: 0.9891 - 21s/epoch - 45ms/step  
 Epoch 99/100  
 469/469 - 21s - loss: 0.0388 - sparse\_categorical\_accuracy: 0.9875 - val\_loss: 0.0322 - val\_sparse\_categorical\_accuracy: 0.9891 - 21s/epoch - 44ms/step  
 Epoch 100/100  
 469/469 - 21s - loss: 0.0381 - sparse\_categorical\_accuracy: 0.9873 - val\_loss: 0.0324 - val\_sparse\_categorical\_accuracy: 0.9889 - 21s/epoch - 45ms/step

In [26]:

```
model5.evaluate(x_test, y_test, verbose=2)
```

313/313 - 2s - loss: 0.0324 - sparse\_categorical\_accuracy: 0.9889 - 2s/epoch - 5ms/step

Out[26]:

```
[0.03238482028245926, 0.9889000058174133]
```

**Model6: adam optimizer with learning rate= $e^{-4}$ , random\_uniform initializer, dropout regularization with rate=0.2.**

In [27]:

```
model6 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.2),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_uniform'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_uniform')
])
```

In [28]:

```
model6.compile(optimizer=tf.keras.optimizers.Adam(0.0001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])
```

In [29]:

```
history6 = model6.fit(x_train, y_train,
                     batch_size=128,
                     epochs=100,
                     validation_data=(x_test, y_test),
                     verbose=2)
```

Epoch 1/100  
 469/469 - 22s - loss: 1.9870 - sparse\_categorical\_accuracy: 0.4950 - val\_loss: 1.5852 - val\_sparse\_categorical\_accuracy: 0.8566 - 22s/epoch - 47ms/step  
 Epoch 2/100  
 469/469 - 21s - loss: 1.3726 - sparse\_categorical\_accuracy: 0.8446 - val\_loss: 1.0536 - val\_sparse\_categorical\_accuracy: 0.9260 - 21s/epoch - 44ms/step

Epoch 3/100  
469/469 - 21s - loss: 0.9182 - sparse\_categorical\_accuracy: 0.9093 - val\_loss: 0.6949 - val\_sparse\_categorical\_accuracy: 0.9536 - 21s/epoch - 45ms/step

Epoch 4/100  
469/469 - 21s - loss: 0.6200 - sparse\_categorical\_accuracy: 0.9336 - val\_loss: 0.4551 - val\_sparse\_categorical\_accuracy: 0.9615 - 21s/epoch - 45ms/step

Epoch 5/100  
469/469 - 21s - loss: 0.4360 - sparse\_categorical\_accuracy: 0.9464 - val\_loss: 0.3072 - val\_sparse\_categorical\_accuracy: 0.9675 - 21s/epoch - 45ms/step

Epoch 6/100  
469/469 - 21s - loss: 0.3260 - sparse\_categorical\_accuracy: 0.9521 - val\_loss: 0.2255 - val\_sparse\_categorical\_accuracy: 0.9712 - 21s/epoch - 45ms/step

Epoch 7/100  
469/469 - 21s - loss: 0.2557 - sparse\_categorical\_accuracy: 0.9566 - val\_loss: 0.1768 - val\_sparse\_categorical\_accuracy: 0.9733 - 21s/epoch - 45ms/step

Epoch 8/100  
469/469 - 21s - loss: 0.2099 - sparse\_categorical\_accuracy: 0.9594 - val\_loss: 0.1402 - val\_sparse\_categorical\_accuracy: 0.9753 - 21s/epoch - 44ms/step

Epoch 9/100  
469/469 - 21s - loss: 0.1775 - sparse\_categorical\_accuracy: 0.9626 - val\_loss: 0.1177 - val\_sparse\_categorical\_accuracy: 0.9769 - 21s/epoch - 44ms/step

Epoch 10/100  
469/469 - 21s - loss: 0.1553 - sparse\_categorical\_accuracy: 0.9647 - val\_loss: 0.1027 - val\_sparse\_categorical\_accuracy: 0.9772 - 21s/epoch - 44ms/step

Epoch 11/100  
469/469 - 21s - loss: 0.1397 - sparse\_categorical\_accuracy: 0.9660 - val\_loss: 0.0909 - val\_sparse\_categorical\_accuracy: 0.9796 - 21s/epoch - 44ms/step

Epoch 12/100  
469/469 - 21s - loss: 0.1269 - sparse\_categorical\_accuracy: 0.9679 - val\_loss: 0.0836 - val\_sparse\_categorical\_accuracy: 0.9792 - 21s/epoch - 44ms/step

Epoch 13/100  
469/469 - 21s - loss: 0.1164 - sparse\_categorical\_accuracy: 0.9693 - val\_loss: 0.0734 - val\_sparse\_categorical\_accuracy: 0.9813 - 21s/epoch - 44ms/step

Epoch 14/100  
469/469 - 21s - loss: 0.1066 - sparse\_categorical\_accuracy: 0.9713 - val\_loss: 0.0698 - val\_sparse\_categorical\_accuracy: 0.9808 - 21s/epoch - 44ms/step

Epoch 15/100  
469/469 - 20s - loss: 0.1017 - sparse\_categorical\_accuracy: 0.9724 - val\_loss: 0.0647 - val\_sparse\_categorical\_accuracy: 0.9816 - 20s/epoch - 43ms/step

Epoch 16/100  
469/469 - 21s - loss: 0.0972 - sparse\_categorical\_accuracy: 0.9729 - val\_loss: 0.0613 - val\_sparse\_categorical\_accuracy: 0.9825 - 21s/epoch - 44ms/step

Epoch 17/100  
469/469 - 21s - loss: 0.0912 - sparse\_categorical\_accuracy: 0.9740 - val\_loss: 0.0591 - val\_sparse\_categorical\_accuracy: 0.9825 - 21s/epoch - 45ms/step

Epoch 18/100  
469/469 - 21s - loss: 0.0885 - sparse\_categorical\_accuracy: 0.9747 - val\_loss: 0.0581 - val\_sparse\_categorical\_accuracy: 0.9830 - 21s/epoch - 44ms/step

Epoch 19/100  
469/469 - 21s - loss: 0.0859 - sparse\_categorical\_accuracy: 0.9751 - val\_loss: 0.0553 - val\_sparse\_categorical\_accuracy: 0.9833 - 21s/epoch - 44ms/step

Epoch 20/100  
469/469 - 21s - loss: 0.0829 - sparse\_categorical\_accuracy: 0.9756 - val\_loss: 0.0538 - val\_sparse\_categorical\_accuracy: 0.9836 - 21s/epoch - 44ms/step

Epoch 21/100  
469/469 - 21s - loss: 0.0797 - sparse\_categorical\_accuracy: 0.9766 - val\_loss: 0.0532 - val\_sparse\_categorical\_accuracy: 0.9838 - 21s/epoch - 44ms/step

Epoch 22/100  
469/469 - 21s - loss: 0.0787 - sparse\_categorical\_accuracy: 0.9763 - val\_loss: 0.0500 - val\_sparse\_categorical\_accuracy: 0.9838 - 21s/epoch - 44ms/step

Epoch 23/100  
469/469 - 20s - loss: 0.0785 - sparse\_categorical\_accuracy: 0.9767 - val\_loss: 0.0507 - val\_sparse\_categorical\_accuracy: 0.9836 - 20s/epoch - 44ms/step

Epoch 24/100  
469/469 - 20s - loss: 0.0764 - sparse\_categorical\_accuracy: 0.9774 - val\_loss: 0.0486 - val\_sparse\_categorical\_accuracy: 0.9842 - 20s/epoch - 43ms/step

Epoch 25/100  
469/469 - 20s - loss: 0.0739 - sparse\_categorical\_accuracy: 0.9781 - val\_loss: 0.0491 - val\_sparse\_categorical\_accuracy: 0.9851 - 20s/epoch - 43ms/step

Epoch 26/100  
469/469 - 20s - loss: 0.0719 - sparse\_categorical\_accuracy: 0.9782 - val\_loss: 0.0477 - val\_sparse\_categorical\_accuracy: 0.9850 - 20s/epoch - 44ms/step

Epoch 27/100  
469/469 - 20s - loss: 0.0713 - sparse\_categorical\_accuracy: 0.9782 - val\_loss: 0.0470 - val\_sparse\_categorical\_accuracy: 0.9852 - 20s/epoch - 43ms/step

Epoch 28/100  
469/469 - 20s - loss: 0.0705 - sparse\_categorical\_accuracy: 0.9788 - val\_loss: 0.0459 - val\_sparse\_categorical\_accuracy: 0.9861 - 20s/epoch - 44ms/step

Epoch 29/100  
469/469 - 20s - loss: 0.0678 - sparse\_categorical\_accuracy: 0.9788 - val\_loss: 0.0477 - val\_sparse\_categorical\_accuracy: 0.9855 - 20s/epoch - 43ms/step

Epoch 30/100  
469/469 - 20s - loss: 0.0675 - sparse\_categorical\_accuracy: 0.9795 - val\_loss: 0.0452 - val\_sparse\_categorical\_accuracy: 0.9855 - 20s/epoch - 43ms/step

[illegible]

[illegible]

```

Epoch 86/100
469/469 - 21s - loss: 0.0459 - sparse_categorical_accuracy: 0.9852 - val_loss: 0.0393 - val_sparse_c
ategorical_accuracy: 0.9877 - 21s/epoch - 44ms/step
Epoch 87/100
469/469 - 21s - loss: 0.0461 - sparse_categorical_accuracy: 0.9856 - val_loss: 0.0379 - val_sparse_c
ategorical_accuracy: 0.9883 - 21s/epoch - 44ms/step
Epoch 88/100
469/469 - 20s - loss: 0.0470 - sparse_categorical_accuracy: 0.9852 - val_loss: 0.0391 - val_sparse_c
ategorical_accuracy: 0.9877 - 20s/epoch - 43ms/step
Epoch 89/100
469/469 - 20s - loss: 0.0470 - sparse_categorical_accuracy: 0.9848 - val_loss: 0.0401 - val_sparse_c
ategorical_accuracy: 0.9880 - 20s/epoch - 44ms/step
Epoch 90/100
469/469 - 21s - loss: 0.0460 - sparse_categorical_accuracy: 0.9848 - val_loss: 0.0368 - val_sparse_c
ategorical_accuracy: 0.9878 - 21s/epoch - 44ms/step
Epoch 91/100
469/469 - 21s - loss: 0.0463 - sparse_categorical_accuracy: 0.9854 - val_loss: 0.0372 - val_sparse_c
ategorical_accuracy: 0.9875 - 21s/epoch - 44ms/step
Epoch 92/100
469/469 - 20s - loss: 0.0445 - sparse_categorical_accuracy: 0.9858 - val_loss: 0.0380 - val_sparse_c
ategorical_accuracy: 0.9874 - 20s/epoch - 44ms/step
Epoch 93/100
469/469 - 21s - loss: 0.0459 - sparse_categorical_accuracy: 0.9854 - val_loss: 0.0380 - val_sparse_c
ategorical_accuracy: 0.9867 - 21s/epoch - 44ms/step
Epoch 94/100
469/469 - 20s - loss: 0.0449 - sparse_categorical_accuracy: 0.9855 - val_loss: 0.0399 - val_sparse_c
ategorical_accuracy: 0.9876 - 20s/epoch - 44ms/step
Epoch 95/100
469/469 - 20s - loss: 0.0460 - sparse_categorical_accuracy: 0.9850 - val_loss: 0.0372 - val_sparse_c
ategorical_accuracy: 0.9887 - 20s/epoch - 44ms/step
Epoch 96/100
469/469 - 21s - loss: 0.0464 - sparse_categorical_accuracy: 0.9843 - val_loss: 0.0386 - val_sparse_c
ategorical_accuracy: 0.9878 - 21s/epoch - 44ms/step
Epoch 97/100
469/469 - 21s - loss: 0.0449 - sparse_categorical_accuracy: 0.9857 - val_loss: 0.0395 - val_sparse_c
ategorical_accuracy: 0.9866 - 21s/epoch - 44ms/step
Epoch 98/100
469/469 - 21s - loss: 0.0456 - sparse_categorical_accuracy: 0.9854 - val_loss: 0.0366 - val_sparse_c
ategorical_accuracy: 0.9872 - 21s/epoch - 45ms/step
Epoch 99/100
469/469 - 20s - loss: 0.0453 - sparse_categorical_accuracy: 0.9855 - val_loss: 0.0366 - val_sparse_c
ategorical_accuracy: 0.9877 - 20s/epoch - 44ms/step
Epoch 100/100
469/469 - 20s - loss: 0.0452 - sparse_categorical_accuracy: 0.9854 - val_loss: 0.0411 - val_sparse_c
ategorical_accuracy: 0.9862 - 20s/epoch - 43ms/step

```

In [30]:

```
model6.evaluate(x_test, y_test, verbose=2)
```

```
313/313 - 1s - loss: 0.0411 - sparse_categorical_accuracy: 0.9862 - 1s/epoch - 4ms/step
```

Out[30]:

```
[0.041084155440330505, 0.9861999750137329]
```

**Model7: adam optimizer with learning rate= $e^{-3}$ , random\_normal initializer, dropout regularization with rate=0.2.**

In [31]:

```

model7 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.2),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_normal')
])

```

In [32]:

```

model7.compile(optimizer=tf.keras.optimizers.Adam(0.001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])

```

In [33]:

```
history7 = model7.fit(x_train, y_train,  
                      batch_size=128,  
                      epochs=100,  
                      validation_data=(x_test, y_test),  
                      verbose=2  
                      )
```

Epoch 1/100

469/469 - 21s - loss: 0.8860 - sparse\_categorical\_accuracy: 0.8195 - val\_loss: 0.2078 - val\_sparse\_categorical\_accuracy: 0.9562 - 21s/epoch - 45ms/step

Epoch 2/100

469/469 - 20s - loss: 0.1772 - sparse\_categorical\_accuracy: 0.9600 - val\_loss: 0.0927 - val\_sparse\_categorical\_accuracy: 0.9756 - 20s/epoch - 43ms/step

Epoch 3/100

469/469 - 20s - loss: 0.1154 - sparse\_categorical\_accuracy: 0.9681 - val\_loss: 0.0604 - val\_sparse\_categorical\_accuracy: 0.9835 - 20s/epoch - 43ms/step

Epoch 4/100

469/469 - 20s - loss: 0.0967 - sparse\_categorical\_accuracy: 0.9718 - val\_loss: 0.0582 - val\_sparse\_categorical\_accuracy: 0.9820 - 20s/epoch - 44ms/step

Epoch 5/100

469/469 - 20s - loss: 0.0877 - sparse\_categorical\_accuracy: 0.9737 - val\_loss: 0.0518 - val\_sparse\_categorical\_accuracy: 0.9842 - 20s/epoch - 43ms/step

Epoch 6/100

469/469 - 21s - loss: 0.0802 - sparse\_categorical\_accuracy: 0.9758 - val\_loss: 0.0471 - val\_sparse\_categorical\_accuracy: 0.9860 - 21s/epoch - 45ms/step

Epoch 7/100

469/469 - 21s - loss: 0.0769 - sparse\_categorical\_accuracy: 0.9765 - val\_loss: 0.0491 - val\_sparse\_categorical\_accuracy: 0.9833 - 21s/epoch - 45ms/step

Epoch 8/100

469/469 - 21s - loss: 0.0732 - sparse\_categorical\_accuracy: 0.9778 - val\_loss: 0.0452 - val\_sparse\_categorical\_accuracy: 0.9867 - 21s/epoch - 45ms/step

Epoch 9/100

469/469 - 21s - loss: 0.0743 - sparse\_categorical\_accuracy: 0.9770 - val\_loss: 0.0462 - val\_sparse\_categorical\_accuracy: 0.9857 - 21s/epoch - 45ms/step

Epoch 10/100

469/469 - 21s - loss: 0.0687 - sparse\_categorical\_accuracy: 0.9788 - val\_loss: 0.0500 - val\_sparse\_categorical\_accuracy: 0.9837 - 21s/epoch - 44ms/step

Epoch 11/100

469/469 - 20s - loss: 0.0675 - sparse\_categorical\_accuracy: 0.9794 - val\_loss: 0.0443 - val\_sparse\_categorical\_accuracy: 0.9863 - 20s/epoch - 43ms/step

Epoch 12/100

469/469 - 20s - loss: 0.0659 - sparse\_categorical\_accuracy: 0.9796 - val\_loss: 0.0470 - val\_sparse\_categorical\_accuracy: 0.9844 - 20s/epoch - 43ms/step

Epoch 13/100

469/469 - 20s - loss: 0.0660 - sparse\_categorical\_accuracy: 0.9791 - val\_loss: 0.0371 - val\_sparse\_categorical\_accuracy: 0.9871 - 20s/epoch - 43ms/step

Epoch 14/100

469/469 - 20s - loss: 0.0648 - sparse\_categorical\_accuracy: 0.9794 - val\_loss: 0.0430 - val\_sparse\_categorical\_accuracy: 0.9863 - 20s/epoch - 43ms/step

Epoch 15/100

469/469 - 20s - loss: 0.0631 - sparse\_categorical\_accuracy: 0.9794 - val\_loss: 0.0415 - val\_sparse\_categorical\_accuracy: 0.9872 - 20s/epoch - 43ms/step

Epoch 16/100

469/469 - 20s - loss: 0.0632 - sparse\_categorical\_accuracy: 0.9802 - val\_loss: 0.0457 - val\_sparse\_categorical\_accuracy: 0.9854 - 20s/epoch - 44ms/step

Epoch 17/100

469/469 - 20s - loss: 0.0606 - sparse\_categorical\_accuracy: 0.9807 - val\_loss: 0.0410 - val\_sparse\_categorical\_accuracy: 0.9867 - 20s/epoch - 43ms/step

Epoch 18/100

469/469 - 21s - loss: 0.0597 - sparse\_categorical\_accuracy: 0.9813 - val\_loss: 0.0437 - val\_sparse\_categorical\_accuracy: 0.9855 - 21s/epoch - 44ms/step

Epoch 19/100

469/469 - 20s - loss: 0.0605 - sparse\_categorical\_accuracy: 0.9805 - val\_loss: 0.0395 - val\_sparse\_categorical\_accuracy: 0.9882 - 20s/epoch - 44ms/step

Epoch 20/100

469/469 - 20s - loss: 0.0579 - sparse\_categorical\_accuracy: 0.9813 - val\_loss: 0.0444 - val\_sparse\_categorical\_accuracy: 0.9860 - 20s/epoch - 43ms/step

Epoch 21/100

469/469 - 20s - loss: 0.0582 - sparse\_categorical\_accuracy: 0.9813 - val\_loss: 0.0377 - val\_sparse\_categorical\_accuracy: 0.9876 - 20s/epoch - 43ms/step

Epoch 22/100

469/469 - 20s - loss: 0.0575 - sparse\_categorical\_accuracy: 0.9815 - val\_loss: 0.0393 - val\_sparse\_categorical\_accuracy: 0.9863 - 20s/epoch - 43ms/step

Epoch 23/100

469/469 - 20s - loss: 0.0571 - sparse\_categorical\_accuracy: 0.9819 - val\_loss: 0.0359 - val\_sparse\_categorical\_accuracy: 0.9885 - 20s/epoch - 43ms/step

Epoch 24/100

469/469 - 20s - loss: 0.0553 - sparse\_categorical\_accuracy: 0.9819 - val\_loss: 0.0415 - val\_sparse\_categorical\_accuracy: 0.9855 - 20s/epoch - 43ms/step

Epoch 25/100

469/469 - 20s - loss: 0.0568 - sparse\_categorical\_accuracy: 0.9817 - val\_loss: 0.0375 - val\_sparse\_categorical\_accuracy: 0.9855 - 20s/epoch - 43ms/step

[illegible]



[illegible]

```

Epoch 81/100
469/469 - 21s - loss: 0.0443 - sparse_categorical_accuracy: 0.9852 - val_loss: 0.0339 - val_sparse_c
ategorical_accuracy: 0.9894 - 21s/epoch - 44ms/step
Epoch 82/100
469/469 - 20s - loss: 0.0429 - sparse_categorical_accuracy: 0.9861 - val_loss: 0.0373 - val_sparse_c
ategorical_accuracy: 0.9882 - 20s/epoch - 44ms/step
Epoch 83/100
469/469 - 21s - loss: 0.0420 - sparse_categorical_accuracy: 0.9863 - val_loss: 0.0361 - val_sparse_c
ategorical_accuracy: 0.9891 - 21s/epoch - 44ms/step
Epoch 84/100
469/469 - 21s - loss: 0.0430 - sparse_categorical_accuracy: 0.9860 - val_loss: 0.0324 - val_sparse_c
ategorical_accuracy: 0.9899 - 21s/epoch - 44ms/step
Epoch 85/100
469/469 - 21s - loss: 0.0422 - sparse_categorical_accuracy: 0.9858 - val_loss: 0.0309 - val_sparse_c
ategorical_accuracy: 0.9910 - 21s/epoch - 44ms/step
Epoch 86/100
469/469 - 21s - loss: 0.0437 - sparse_categorical_accuracy: 0.9859 - val_loss: 0.0341 - val_sparse_c
ategorical_accuracy: 0.9895 - 21s/epoch - 44ms/step
Epoch 87/100
469/469 - 20s - loss: 0.0426 - sparse_categorical_accuracy: 0.9858 - val_loss: 0.0343 - val_sparse_c
ategorical_accuracy: 0.9906 - 20s/epoch - 43ms/step
Epoch 88/100
469/469 - 20s - loss: 0.0433 - sparse_categorical_accuracy: 0.9861 - val_loss: 0.0334 - val_sparse_c
ategorical_accuracy: 0.9898 - 20s/epoch - 44ms/step
Epoch 89/100
469/469 - 20s - loss: 0.0440 - sparse_categorical_accuracy: 0.9853 - val_loss: 0.0326 - val_sparse_c
ategorical_accuracy: 0.9890 - 20s/epoch - 43ms/step
Epoch 90/100
469/469 - 21s - loss: 0.0421 - sparse_categorical_accuracy: 0.9862 - val_loss: 0.0325 - val_sparse_c
ategorical_accuracy: 0.9903 - 21s/epoch - 44ms/step
Epoch 91/100
469/469 - 21s - loss: 0.0414 - sparse_categorical_accuracy: 0.9863 - val_loss: 0.0337 - val_sparse_c
ategorical_accuracy: 0.9895 - 21s/epoch - 44ms/step
Epoch 92/100
469/469 - 21s - loss: 0.0423 - sparse_categorical_accuracy: 0.9861 - val_loss: 0.0323 - val_sparse_c
ategorical_accuracy: 0.9900 - 21s/epoch - 45ms/step
Epoch 93/100
469/469 - 21s - loss: 0.0419 - sparse_categorical_accuracy: 0.9860 - val_loss: 0.0332 - val_sparse_c
ategorical_accuracy: 0.9893 - 21s/epoch - 45ms/step
Epoch 94/100
469/469 - 20s - loss: 0.0436 - sparse_categorical_accuracy: 0.9853 - val_loss: 0.0385 - val_sparse_c
ategorical_accuracy: 0.9879 - 20s/epoch - 43ms/step
Epoch 95/100
469/469 - 21s - loss: 0.0397 - sparse_categorical_accuracy: 0.9864 - val_loss: 0.0324 - val_sparse_c
ategorical_accuracy: 0.9892 - 21s/epoch - 44ms/step
Epoch 96/100
469/469 - 20s - loss: 0.0424 - sparse_categorical_accuracy: 0.9859 - val_loss: 0.0352 - val_sparse_c
ategorical_accuracy: 0.9891 - 20s/epoch - 44ms/step
Epoch 97/100
469/469 - 21s - loss: 0.0418 - sparse_categorical_accuracy: 0.9863 - val_loss: 0.0348 - val_sparse_c
ategorical_accuracy: 0.9894 - 21s/epoch - 44ms/step
Epoch 98/100
469/469 - 21s - loss: 0.0406 - sparse_categorical_accuracy: 0.9861 - val_loss: 0.0336 - val_sparse_c
ategorical_accuracy: 0.9901 - 21s/epoch - 44ms/step
Epoch 99/100
469/469 - 21s - loss: 0.0423 - sparse_categorical_accuracy: 0.9860 - val_loss: 0.0338 - val_sparse_c
ategorical_accuracy: 0.9892 - 21s/epoch - 44ms/step
Epoch 100/100
469/469 - 21s - loss: 0.0415 - sparse_categorical_accuracy: 0.9864 - val_loss: 0.0335 - val_sparse_c
ategorical_accuracy: 0.9891 - 21s/epoch - 44ms/step

```

In [34]:

```
model7.evaluate(x_test, y_test, verbose=2)
```

```
313/313 - 1s - loss: 0.0335 - sparse_categorical_accuracy: 0.9891 - 1s/epoch - 5ms/step
```

Out[34]:

```
[0.03348444402217865, 0.9890999794006348]
```

**Model8: adam optimizer with learning rate= $e^{-4}$ , random\_normal initializer, dropout regularization with rate=0.2.**

In [35]:

```
model8 = tf.keras.Sequential([
    tf.keras.layers.Input(shape=(28, 28, 1)),
    tf.keras.layers.GaussianNoise(0.1),
    tf.keras.layers.Conv2D(20, (5, 5)),
    tf.keras.layers.MaxPooling2D((3, 3)),
    tf.keras.layers.MaxPooling2D((2, 2)),
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dropout(0.2),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.Dense(16, activation='relu', kernel_initializer='random_normal'),
    tf.keras.layers.BatchNormalization(),
    tf.keras.layers.Dense(10, activation='softmax', kernel_initializer='random_normal')
])
```

In [36]:

```
model8.compile(optimizer=tf.keras.optimizers.Adam(0.0001),
               loss='sparse_categorical_crossentropy',
               metrics=['sparse_categorical_accuracy'])
```

In [37]:

```
history8 = model8.fit(x_train, y_train,
                      batch_size=128,
                      epochs=100,
                      validation_data=(x_test, y_test),
                      verbose=2)
```

```
Epoch 1/100
469/469 - 21s - loss: 1.9704 - sparse_categorical_accuracy: 0.4433 - val_loss: 1.6157 - val_sparse_c
ategorical_accuracy: 0.6779 - 21s/epoch - 46ms/step
Epoch 2/100
469/469 - 21s - loss: 1.4665 - sparse_categorical_accuracy: 0.7059 - val_loss: 1.1774 - val_sparse_c
ategorical_accuracy: 0.8522 - 21s/epoch - 44ms/step
Epoch 3/100
469/469 - 21s - loss: 1.0470 - sparse_categorical_accuracy: 0.8489 - val_loss: 0.8101 - val_sparse_c
ategorical_accuracy: 0.9184 - 21s/epoch - 44ms/step
Epoch 4/100
469/469 - 21s - loss: 0.7295 - sparse_categorical_accuracy: 0.9046 - val_loss: 0.5362 - val_sparse_c
ategorical_accuracy: 0.9471 - 21s/epoch - 44ms/step
Epoch 5/100
469/469 - 20s - loss: 0.5152 - sparse_categorical_accuracy: 0.9299 - val_loss: 0.3687 - val_sparse_c
ategorical_accuracy: 0.9582 - 20s/epoch - 44ms/step
Epoch 6/100
469/469 - 21s - loss: 0.3815 - sparse_categorical_accuracy: 0.9420 - val_loss: 0.2610 - val_sparse_c
ategorical_accuracy: 0.9645 - 21s/epoch - 44ms/step
Epoch 7/100
469/469 - 21s - loss: 0.2959 - sparse_categorical_accuracy: 0.9504 - val_loss: 0.2007 - val_sparse_c
ategorical_accuracy: 0.9692 - 21s/epoch - 44ms/step
Epoch 8/100
469/469 - 21s - loss: 0.2400 - sparse_categorical_accuracy: 0.9553 - val_loss: 0.1622 - val_sparse_c
ategorical_accuracy: 0.9712 - 21s/epoch - 44ms/step
Epoch 9/100
469/469 - 20s - loss: 0.2016 - sparse_categorical_accuracy: 0.9583 - val_loss: 0.1337 - val_sparse_c
ategorical_accuracy: 0.9736 - 20s/epoch - 44ms/step
Epoch 10/100
469/469 - 20s - loss: 0.1743 - sparse_categorical_accuracy: 0.9617 - val_loss: 0.1133 - val_sparse_c
ategorical_accuracy: 0.9744 - 20s/epoch - 43ms/step
Epoch 11/100
469/469 - 20s - loss: 0.1546 - sparse_categorical_accuracy: 0.9640 - val_loss: 0.1005 - val_sparse_c
ategorical_accuracy: 0.9748 - 20s/epoch - 43ms/step
Epoch 12/100
469/469 - 20s - loss: 0.1389 - sparse_categorical_accuracy: 0.9662 - val_loss: 0.0902 - val_sparse_c
ategorical_accuracy: 0.9761 - 20s/epoch - 43ms/step
Epoch 13/100
469/469 - 20s - loss: 0.1279 - sparse_categorical_accuracy: 0.9672 - val_loss: 0.0799 - val_sparse_c
ategorical_accuracy: 0.9782 - 20s/epoch - 43ms/step
Epoch 14/100
469/469 - 20s - loss: 0.1199 - sparse_categorical_accuracy: 0.9679 - val_loss: 0.0763 - val_sparse_c
ategorical_accuracy: 0.9794 - 20s/epoch - 43ms/step
Epoch 15/100
469/469 - 20s - loss: 0.1127 - sparse_categorical_accuracy: 0.9685 - val_loss: 0.0710 - val_sparse_c
ategorical_accuracy: 0.9799 - 20s/epoch - 43ms/step
Epoch 16/100
469/469 - 20s - loss: 0.1073 - sparse_categorical_accuracy: 0.9699 - val_loss: 0.0669 - val_sparse_c
ategorical_accuracy: 0.9814 - 20s/epoch - 43ms/step
Epoch 17/100
469/469 - 20s - loss: 0.1027 - sparse_categorical_accuracy: 0.9709 - val_loss: 0.0654 - val_sparse_c
```

[illegible]

[illegible]

```
Epoch 73/100
469/469 - 20s - loss: 0.0549 - sparse_categorical_accuracy: 0.9827 - val_loss: 0.0387 - val_sparse_c
ategorical_accuracy: 0.9868 - 20s/epoch - 43ms/step
Epoch 74/100
469/469 - 20s - loss: 0.0553 - sparse_categorical_accuracy: 0.9831 - val_loss: 0.0382 - val_sparse_c
ategorical_accuracy: 0.9875 - 20s/epoch - 43ms/step
Epoch 75/100
469/469 - 20s - loss: 0.0539 - sparse_categorical_accuracy: 0.9829 - val_loss: 0.0372 - val_sparse_c
ategorical_accuracy: 0.9870 - 20s/epoch - 43ms/step
Epoch 76/100
469/469 - 20s - loss: 0.0548 - sparse_categorical_accuracy: 0.9834 - val_loss: 0.0381 - val_sparse_c
ategorical_accuracy: 0.9873 - 20s/epoch - 43ms/step
Epoch 77/100
469/469 - 20s - loss: 0.0545 - sparse_categorical_accuracy: 0.9825 - val_loss: 0.0383 - val_sparse_c
ategorical_accuracy: 0.9873 - 20s/epoch - 43ms/step
Epoch 78/100
469/469 - 20s - loss: 0.0551 - sparse_categorical_accuracy: 0.9826 - val_loss: 0.0373 - val_sparse_c
ategorical_accuracy: 0.9879 - 20s/epoch - 43ms/step
Epoch 79/100
469/469 - 20s - loss: 0.0537 - sparse_categorical_accuracy: 0.9830 - val_loss: 0.0402 - val_sparse_c
ategorical_accuracy: 0.9862 - 20s/epoch - 43ms/step
Epoch 80/100
469/469 - 20s - loss: 0.0552 - sparse_categorical_accuracy: 0.9827 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9877 - 20s/epoch - 43ms/step
Epoch 81/100
469/469 - 20s - loss: 0.0543 - sparse_categorical_accuracy: 0.9832 - val_loss: 0.0381 - val_sparse_c
ategorical_accuracy: 0.9882 - 20s/epoch - 43ms/step
Epoch 82/100
469/469 - 21s - loss: 0.0540 - sparse_categorical_accuracy: 0.9831 - val_loss: 0.0374 - val_sparse_c
ategorical_accuracy: 0.9877 - 21s/epoch - 44ms/step
Epoch 83/100
469/469 - 21s - loss: 0.0533 - sparse_categorical_accuracy: 0.9830 - val_loss: 0.0386 - val_sparse_c
ategorical_accuracy: 0.9867 - 21s/epoch - 44ms/step
Epoch 84/100
469/469 - 21s - loss: 0.0539 - sparse_categorical_accuracy: 0.9827 - val_loss: 0.0361 - val_sparse_c
ategorical_accuracy: 0.9881 - 21s/epoch - 44ms/step
Epoch 85/100
469/469 - 21s - loss: 0.0526 - sparse_categorical_accuracy: 0.9835 - val_loss: 0.0387 - val_sparse_c
ategorical_accuracy: 0.9880 - 21s/epoch - 44ms/step
Epoch 86/100
469/469 - 20s - loss: 0.0527 - sparse_categorical_accuracy: 0.9833 - val_loss: 0.0379 - val_sparse_c
ategorical_accuracy: 0.9874 - 20s/epoch - 44ms/step
Epoch 87/100
469/469 - 21s - loss: 0.0523 - sparse_categorical_accuracy: 0.9832 - val_loss: 0.0382 - val_sparse_c
ategorical_accuracy: 0.9874 - 21s/epoch - 44ms/step
Epoch 88/100
469/469 - 21s - loss: 0.0536 - sparse_categorical_accuracy: 0.9825 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9875 - 21s/epoch - 44ms/step
Epoch 89/100
469/469 - 20s - loss: 0.0517 - sparse_categorical_accuracy: 0.9838 - val_loss: 0.0385 - val_sparse_c
ategorical_accuracy: 0.9874 - 20s/epoch - 43ms/step
Epoch 90/100
469/469 - 20s - loss: 0.0548 - sparse_categorical_accuracy: 0.9827 - val_loss: 0.0380 - val_sparse_c
ategorical_accuracy: 0.9867 - 20s/epoch - 43ms/step
Epoch 91/100
469/469 - 20s - loss: 0.0526 - sparse_categorical_accuracy: 0.9834 - val_loss: 0.0389 - val_sparse_c
ategorical_accuracy: 0.9876 - 20s/epoch - 43ms/step
Epoch 92/100
469/469 - 20s - loss: 0.0520 - sparse_categorical_accuracy: 0.9841 - val_loss: 0.0384 - val_sparse_c
ategorical_accuracy: 0.9864 - 20s/epoch - 43ms/step
Epoch 93/100
469/469 - 20s - loss: 0.0515 - sparse_categorical_accuracy: 0.9839 - val_loss: 0.0378 - val_sparse_c
ategorical_accuracy: 0.9880 - 20s/epoch - 44ms/step
Epoch 94/100
469/469 - 20s - loss: 0.0509 - sparse_categorical_accuracy: 0.9839 - val_loss: 0.0375 - val_sparse_c
ategorical_accuracy: 0.9875 - 20s/epoch - 43ms/step
Epoch 95/100
469/469 - 20s - loss: 0.0497 - sparse_categorical_accuracy: 0.9842 - val_loss: 0.0385 - val_sparse_c
ategorical_accuracy: 0.9876 - 20s/epoch - 43ms/step
Epoch 96/100
469/469 - 20s - loss: 0.0503 - sparse_categorical_accuracy: 0.9846 - val_loss: 0.0380 - val_sparse_c
ategorical_accuracy: 0.9872 - 20s/epoch - 43ms/step
Epoch 97/100
469/469 - 21s - loss: 0.0518 - sparse_categorical_accuracy: 0.9834 - val_loss: 0.0376 - val_sparse_c
ategorical_accuracy: 0.9884 - 21s/epoch - 44ms/step
Epoch 98/100
469/469 - 20s - loss: 0.0485 - sparse_categorical_accuracy: 0.9849 - val_loss: 0.0364 - val_sparse_c
ategorical_accuracy: 0.9882 - 20s/epoch - 43ms/step
Epoch 99/100
469/469 - 20s - loss: 0.0492 - sparse_categorical_accuracy: 0.9843 - val_loss: 0.0388 - val_sparse_c
ategorical_accuracy: 0.9874 - 20s/epoch - 43ms/step
Epoch 100/100
469/469 - 20s - loss: 0.0492 - sparse_categorical_accuracy: 0.9844 - val_loss: 0.0367 - val_sparse_c
```

ategorical\_accuracy: 0.9885 - 20s/epoch - 43ms/step

In [38]:

```
model8.evaluate(x_test, y_test, verbose=2)
```

313/313 - 1s - loss: 0.0367 - sparse\_categorical\_accuracy: 0.9885 - 1s/epoch - 5ms/step

Out[38]:

```
[0.036694206297397614, 0.9884999990463257]
```

## Plots

In [39]:

```
import matplotlib.pyplot as plt
```

### Plots for different kernel initializers

In [40]:

```
training_accuracy0 = history.history['sparse_categorical_accuracy']
validation_accuracy0 = history.history['val_sparse_categorical_accuracy']

training_accuracy1 = history1.history['sparse_categorical_accuracy']
validation_accuracy1 = history1.history['val_sparse_categorical_accuracy']

training_accuracy2 = history2.history['sparse_categorical_accuracy']
validation_accuracy2 = history2.history['val_sparse_categorical_accuracy']

training_accuracy3 = history3.history['sparse_categorical_accuracy']
validation_accuracy3 = history3.history['val_sparse_categorical_accuracy']

training_accuracy4 = history4.history['sparse_categorical_accuracy']
validation_accuracy4 = history4.history['val_sparse_categorical_accuracy']

training_accuracy5 = history5.history['sparse_categorical_accuracy']
validation_accuracy5 = history5.history['val_sparse_categorical_accuracy']

training_accuracy6 = history6.history['sparse_categorical_accuracy']
validation_accuracy6 = history6.history['val_sparse_categorical_accuracy']

training_accuracy7 = history7.history['sparse_categorical_accuracy']
validation_accuracy7 = history7.history['val_sparse_categorical_accuracy']

training_accuracy8 = history8.history['sparse_categorical_accuracy']
validation_accuracy8 = history8.history['val_sparse_categorical_accuracy']

epochs_range=range(100)

plt.figure(figsize=(8, 8))
plt.subplot(1, 2, 1)
plt.plot(epochs_range, training_accuracy0, label='Train Acc for Baseline')
plt.plot(epochs_range, training_accuracy1, label='Train Acc for Model1')
plt.plot(epochs_range, training_accuracy2, label='Train Acc for Model2')
plt.plot(epochs_range, training_accuracy3, label='Train Acc for Model3')
plt.plot(epochs_range, training_accuracy4, label='Train Acc for Model4')
plt.plot(epochs_range, training_accuracy5, label='Train Acc for Model5')
plt.plot(epochs_range, training_accuracy6, label='Train Acc for Model6')
plt.plot(epochs_range, training_accuracy7, label='Train Acc for Model7')
plt.plot(epochs_range, training_accuracy8, label='Train Acc for Model8')
plt.legend(loc='lower right')
plt.title('Training Accuracy For All Models')

plt.subplot(1, 2, 2)
plt.plot(epochs_range, validation_accuracy0, label='Val Acc for Baseline')
plt.plot(epochs_range, validation_accuracy1, label='Val Acc for Model1')
plt.plot(epochs_range, validation_accuracy2, label='Val Acc for Model2')
plt.plot(epochs_range, validation_accuracy3, label='Val Acc for Model3')
plt.plot(epochs_range, validation_accuracy4, label='Val Acc for Model4')
plt.plot(epochs_range, validation_accuracy5, label='Val Acc for Model5')
plt.plot(epochs_range, validation_accuracy6, label='Val Acc for Model6')
plt.plot(epochs_range, validation_accuracy7, label='Val Acc for Model7')
plt.plot(epochs_range, validation_accuracy8, label='Val Acc for Model8')
plt.legend(loc='lower right')
plt.title('Validation Accuracy For All Models')
plt.show()
```

[illegible]

Figure 4 is a line graph showing the validation accuracy of the proposed model across different training steps (0 to 100). The Y-axis represents Validation Accuracy (0.4 to 1.0). The X-axis represents Training Steps (0 to 100). The Baseline (blue line) starts at approximately 0.42 and reaches about 0.94. Models 1 through 8 (orange, green, red, purple, brown, pink, grey, and yellow lines) start at approximately 0.68 and reach about 0.98.