

Point Estimation

• Point estimation is the process of using sample data to estimate the value of an unknown population parameter.

• The estimated value is called a point estimate or simply an estimate.

• The estimate is often denoted by a symbol with a hat over it, such as $\hat{\theta}$.

• For example, if we want to estimate the mean of a population, we would use the sample mean as our point estimate.

• The sample mean is often denoted by \bar{x} .

• Other common point estimates include the sample proportion (\hat{p}) and the sample standard deviation (s).

• Point estimation is a fundamental concept in statistics and is used in many different applications.

• For example, it is used in quality control to estimate the percentage of defective items in a population.

• It is also used in medicine to estimate the effectiveness of a new treatment.

• In general, point estimation is a useful tool for making inferences about populations based on sample data.

• However, it is important to remember that point estimates are just estimates and may not always be accurate or precise.

• For example, if we take a small sample, our estimate may be biased or have a large standard error.

• Therefore, it is important to use appropriate sampling methods and statistical techniques to obtain accurate and reliable point estimates.

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① Point Estimation

Estimator - statistical quantity $T = t(X_1, \dots, X_n)$
 Estimate - observed value $t = t(x_1, \dots, x_n)$

① MME

$$j\text{-moment of } X: M_j^1(\theta) = E[X^j]$$

$$\mu_j^1 = m_j^1$$

invariance property

$M_j^1 = \frac{1}{n} \sum_{i=1}^n (X_i)^j \rightarrow$ functions of the sample r.v. & ind. of model parameter

$$M_j^1 = E[X^j] = M_j(\theta) \rightarrow M_j^1 = \bar{x}_n$$

$$M_j^1 = M_j^1(\theta_1, \dots, \theta_K) \quad j=1, \dots, K \quad \text{w.r.t. } \theta_1, \dots, \theta_K$$

$\hat{\theta}_1, \dots, \hat{\theta}_K$ are MMEs, $\theta_1, \dots, \theta_K$ are functions of r.v. of the sample

estimator of $g(\theta) = g(\text{estimator of } \theta)$

② MLE

$$L(\theta) = f(x_1; \theta) \times \dots \times f(x_n; \theta) = \prod_{i=1}^n f(x_i; \theta)$$

$$L(\hat{\theta}) = f(x_1, \dots, x_n; \hat{\theta}) = \max_{\theta \in \Omega} f(x_1, \dots, x_n; \theta)$$

invariance property: $\hat{\theta}$ is the MLE of θ & $T(\theta)$ is a function of θ , then $T(\hat{\theta})$ is the MLE of $T(\theta)$

③ Criteria for Estimators

① Unbiased estimator: $E[T(\theta)] = T(\theta)$ for all $\theta \in \Omega$

i.e. for $\text{EXP}(\theta)$, $E[\bar{X}] = \theta$, $E[\hat{\theta}] = \theta$

② Among unbiased estimators we prefer the one that has smallest variance.

③ UMVUE: { if $E[T^*(\theta)] = T(\theta)$

| for any other unbiased estimator T of $T(\theta)$ $\text{Var}[T^*(\theta)] \leq \text{Var}[T(\theta)]$ for all $\theta \in \Omega$.

④ CRLB

$$\text{Var}[T(\theta)] \geq \frac{(T'(\theta))^2}{E\left[\left(\frac{\partial}{\partial \theta} \log f(x_1, \dots, x_n; \theta)\right)^2\right]}$$

$$\text{Var}[T(\theta)] \geq \frac{(T''(\theta))^2}{n E\left[\left(\frac{\partial^2}{\partial \theta^2} \log f(x_1, \dots, x_n; \theta)\right)^2\right]}$$

$$\text{F.g. } \frac{\partial}{\partial \theta} \log f(x; \theta) = -\frac{1}{\theta} + \frac{x}{\theta^2} = \frac{x-\theta}{\theta^2} \quad E\left[\left(\frac{x-\theta}{\theta^2}\right)^2\right] = \frac{1}{\theta^4} E[(x - E(x))^2] = \frac{1}{\theta^4} \text{Var}[x] = \frac{1}{\theta^2}$$

① $E\left[\left(\frac{\partial}{\partial \theta} \log f(x; \theta)\right)^2\right] = E\left[\frac{\partial^2}{\partial \theta^2} \log f(x; \theta)\right]$ ② unbiased $T(\theta)$ attains CRLB. linear function of $\sum_{i=1}^n \frac{\partial^2}{\partial \theta^2} \log f(x_i; \theta)$

Thm: If there is an unbiased estimator of $T(\theta)$ with $\text{Var} = \text{CRLB}$, then only for linear function of $T(\theta)$ unbiased estimators will exist with variance = CRLB

$$E[(x - \mu)^2] = \text{Var}(x) + (E(x) - \mu)^2$$

$$\sum_{i=1}^n \frac{\partial^2}{\partial \theta^2} \log f(x_i; \theta) = \text{d}(T - T(\theta)) \text{ as } E(x) = \mu$$

⑤ Efficiency

$$\text{re}(T, T^*) = \frac{\text{Var}[T^*]}{\text{Var}[T]}$$

$$b(T) = E[T(\theta)] - T(\theta)$$

$$\text{MSE}(T) = E(T - T(\theta))^2$$

$$= \text{Var}(T) + (b(T))^2$$

⑥ Large-Sample properties.

① Simple consistency: $\lim_{n \rightarrow \infty} P\{T_n - T(\theta) \leq \epsilon\} = 1$ $T_n \xrightarrow{P} T(\theta)$

② MSE consistency: $\lim_{n \rightarrow \infty} E\{T_n - T(\theta)\}^2 = 0$

③ Asymptotically unbiased $\lim_{n \rightarrow \infty} E[T_n] = T(\theta)$

9.4.1 $f(T_n)$ is MSE consistent if Asym. unbiased + $\lim_{n \rightarrow \infty} \text{Var}(f(T_n)) = 0$

9.4.2 If $f(T_n)$ is MSE consistent, it is also simply consistent

9.4.3 $f(T_n)$ is simply consistent, for $t(w)$ & $g(t_w)$ is continuous $f(g(T_n))$ is simply consistent for $g(t(w))$

④ Asymptotic Efficiency: are $(T_n, T_n^*) = \lim_{n \rightarrow \infty} \frac{\text{Var}(T_n^*)}{\text{Var}(T_n)}$

Let T be an estimator of $T(\theta)$

$$\text{E}T = T(\theta)$$

AND $\Rightarrow T$ is an UMVUE of $T(\theta)$

$$\text{Var}T = \text{CRVB}$$