Advanced methods: Hybridization and computationally expensive problems

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Overview

Overview

Learning outcomes

Learning outcomes

After this lecture, the student will:

- Know what is meant by hybridization in the context of interactive multiobjective optimization.
- Know when and how hybridization can and should be applied.
- Know different ways to tackle computationally expensive multiobjective optimization problems.
- Know what is group decision making especially in the context of interactive multiobjective optimization.

Hybridization: Motivation

- So far, we have learned about scalarization-based and evolutionary multiobjective optimization methods (MCDM and EMO methods).
- Scalarization-based methods can find, under certain conditions, solutions that are guaranteed to be (weakly) Pareto optimal, or even properly Pareto optimal.
- Evolutionary methods can evolve a population of solutions at the same time resulting in a non-dominated set of solutions that can be used to represent the Pareto optimal front of a problem.

Hybridization: Motivation

- MCDM methods can compute accurate Pareto optimal solutions, but only one at a time.

- EMO methods can compute multiple solutions simultaneously, but their Pareto optimality cannot be guaranteed.

Hybridization: Motivation

- Would it perhaps make sense to combine MCDM and EMO methods in some ways?
- For instance, even if the solutions found by EMO methods are not guaranteed to be Pareto optimal, we can guide the process to at least produce a wide range of different, non-dominated solutions.
  - This is surely something a DM interested in solving the underlying multiobjective optimization problem is interested in!
- Once the DM has identified interesting solutions, we could use MCDM method to try and improve the solutions further, and possibly even guaranteeing some kind of Pareto optimality!
- Would not this be nice?

Hybridization

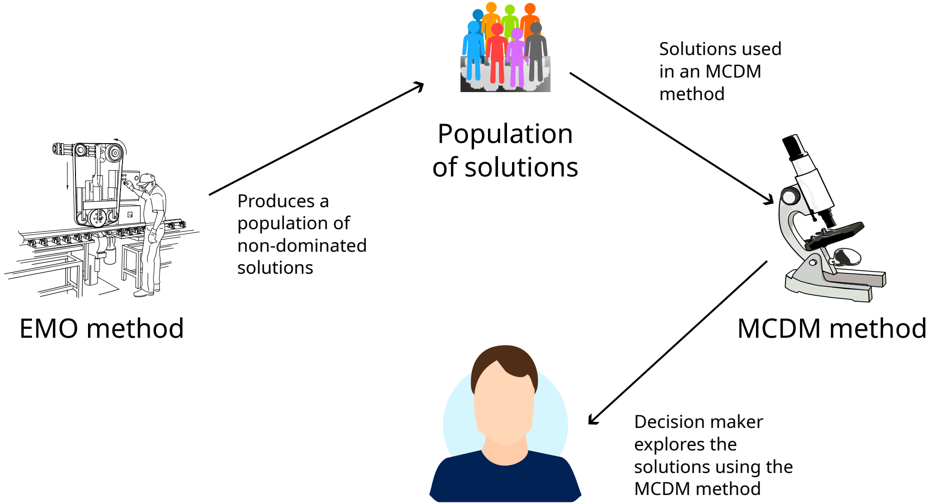
- The word hybrid means the mixture of two different things.
- Consequently, the verb to hybridize means making a mixture of two different things.
- In the field of multiobjective optimization, the exact meaning of, e.g., a hybrid method or hybridization of methods, is not well defined. This is because not much research has been conducted in combining methods of different kinds.

Hybridization

- In this lecture, we will talk about two distinct ways of hybridization:
  1. Combining two or more different methods.
  2. Combining features or parts, or both, of various methods into a new method.
- Way I to hybridize does not involve creating a new method per se, while way II to hybridize is a way to create wholly new methods by combining features from existing methods.
- Obviously, nothing is stopping anyone from employing both ways of hybridization at the same time...

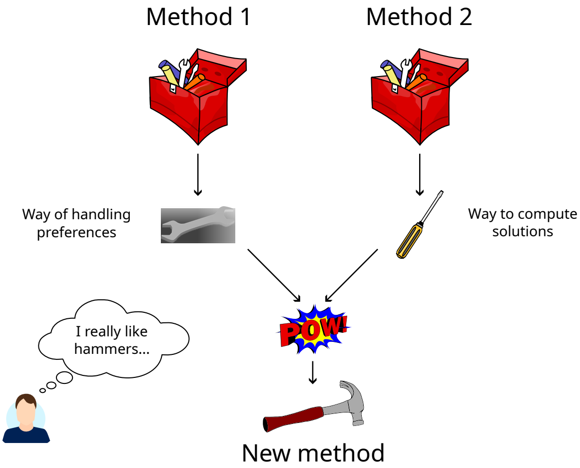
Hybridization

Example 1: Hybridization of type I.



Hybridization

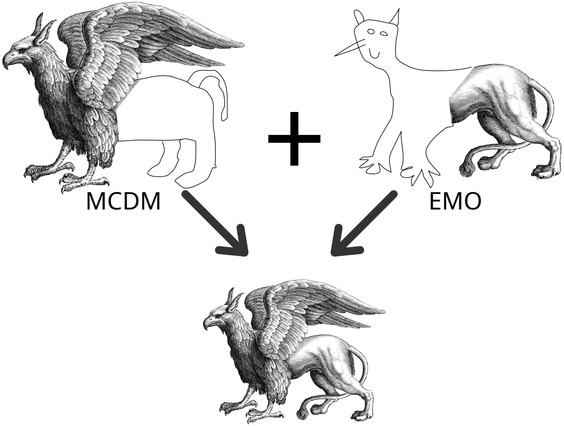
Example 2: Hybridization of type II.



Hybridization

Can you think of (new) ways of hybridizing methods? Let us ponder for 5 minutes. You can think aloud with your peers if you like.

Hybridization



Best of both worlds!

Hybridization

- To give a more concrete example of hybridization, let us look at the IOPIS method [1].
- The central idea of the method is the preference incorporated space, or PIS.
- The original multiobjective optimization is formulated in the PIS as:

PIS problem

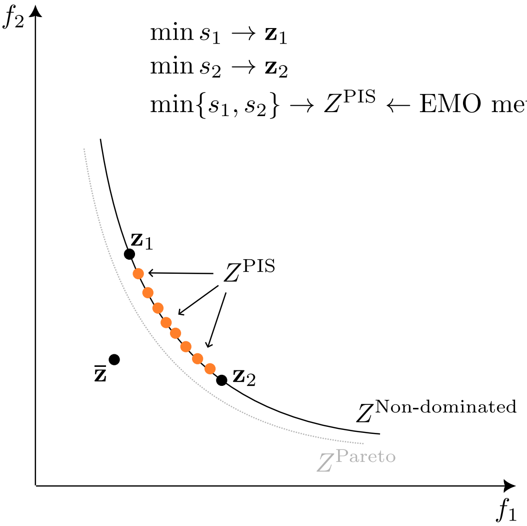
$$\min_{\mathbf{x} \in S} \mathbf{s}(F(\mathbf{x}), \bar{\mathbf{z}}) = \{s_1(F(\mathbf{x}), \bar{\mathbf{z}}), s_2(F(\mathbf{x}), \bar{\mathbf{z}}), \ldots, s_q(F(\mathbf{x}), \bar{\mathbf{z}})\}. \tag{1}$$

Hybridization

- In other words, instead of minimizing one scalarized problem, we instead minimize q scalarized problems with different scalarizing functions and treat their values as objectives.
- The DM must provide a reference point $\bar{\mathbf{z}}$.
- The PIS problem (1) is then solved using some EMO method producing a set of non-dominated solutions.
- New preference points may be supplied to get different solution sets.

Hybridization

$$\min s_1 \rightarrow \mathbf{z}_1$$
$$\min s_2 \rightarrow \mathbf{z}_2$$
$$\min\{s_1, s_2\} \rightarrow Z^{\text{PIS}} \leftarrow \text{EMO method}$$



Hybridization

- IOPIS can be combined with any existing EMO method.
- IOPIS is a nice way to reduce the dimension of a multiobjective optimization problem. This also leads to a computationally less demanding problem.
- IOPIS produces many solutions that are all close to the reference point. By changing the scalarization functions used, one can affect what sort of solutions are computed.
- Experiments have shown that solving the PIS problem produces better results than solving the original problem without information about the reference point.
- IOPIS requires a reference point and information about the nadir point as well.

Hybridization

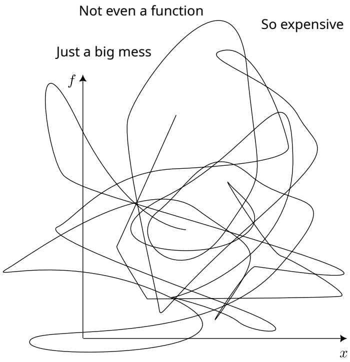
- Hybridization is still a novel concept in the field of multiobjective optimization.
- One of the main challenges in developing and implementing new hybrid methods is the lack of a common framework for multiobjective optimization that has various implementations of different methods, both of the MCDM kind and EMO kind.
- One of the main motivations behind developing DESDEO [2] was to have different methods, of both kinds, implemented in the same framework, and in a modular fashion so that both types of hybridization could be explored.
- There is a lot to do in terms of researching new hybrid methods; there are a lot of low hanging fruit!

Surrogate-based problems

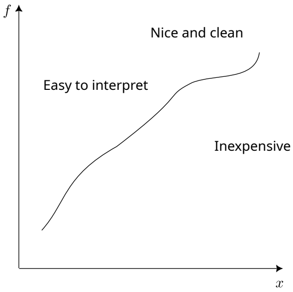
Surrogate-based problems

- When it comes to real-life multiobjective optimization problems, things are seldom simple.
- Problems are often data-based (or data-driven, meaning the same thing) or simulation-based, or based on both, which in practice means that there are no analytical objective functions to be optimized.
- Real-life problems are also often computationally very expensive!
- One way to address these issues is to use *surrogates models*, which are used to approximate the objective functions of the problem in a computationally less expensive way.

Surrogate-based problems



Typical* real-life problem



Surrogate model

*Interpretation of "typical" is subjective to the authors own bias and experience.

Surrogate-based problems

- But what is a surrogate?
- Typically, a surrogate is a simpler model of the original, messy problem. The surrogate is fitted so that it best approximates the original problem.
- Typical surrogates include, but are not limited to:
    - linear regression and other regression models,
    - Gaussian processes (Kriging models),
    - support vector machines,
    - (deep) neural networks, and
    - other machine learning models.
- With surrogates, it is important to remember that the surrogate is an approximation of the original problem. Good surrogate models should therefore also convey some information about their (local) accuracy.

Surrogate-based problems

- Often the original formulation of the multiobjective optimization problem may change during an optimization process. For instance, new data could become available.
- This new data can then be used to update the surrogate model on the fly.
- Solving a multiobjective optimization and updating it during the solution process as new information becomes available is known as *online multiobjective optimization*.
- When the information available does not change during the optimization process, it is known as *offline multiobjective optimization*.

Surrogate-based problems

Example 2: Surrogate assisted interactive multiobjective optimization in energy system design of buildings [3].

- Four objectives in optimizing the configuration of an energy systems of a large business building complex:
  1. Minimize initial investment cost,
  2. Minimize annual operation cost,
  3. Minimize annual carbon dioxide emissions, and
  4. Maximize resilience.
- The objective functions (apart from the first one) are simulation-based and expensive to evaluate.
- There are ten decision variables with only box-constraints.

Surrogate-based problems

- To solve the problem, a new interactive method is developed: interactive K-RVEA.
- Interactive K-RVEA is a Kriging-assisted interactive version of the vector guided evolutionary algorithm (RVEA).
- Interactive K-RVEA can incorporate a DM's preference (reference point) in the evolutionary optimization process.
- The surrogate model used is updated in each iteration based on the preference of the DM and the accuracy of the solutions produced by optimizing the surrogate models.
- The original problem is used during the interactive process to compute new solutions that best match the DM's expressed preferences.
- The evaluations of the original problem are minimal.

Surrogate-based problems

Example 3: Optimistic NAUTILUS Navigator: O-NAUTILUS [4].

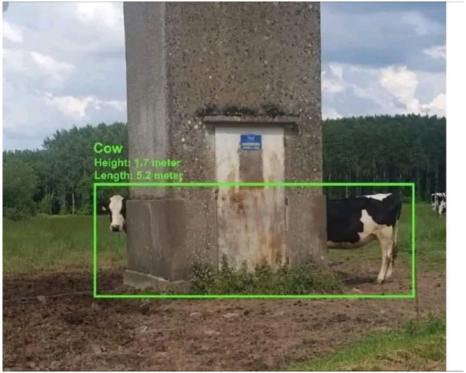
- An extension of NAUTILUS Navigator for online data-driven interactive multiobjective optimization.
- In addition to showing reachable ranges, like in NAUTILUS Navigator, O-NAUTILUS can show the optimistic ranges of solutions. These solutions are predicted (based on information from the surrogate) to have good objective values. These solutions form the *optimistic front*.
- As a surrogate, O-NAUTILUS utilizes Kriging models.

Surrogate-based problems

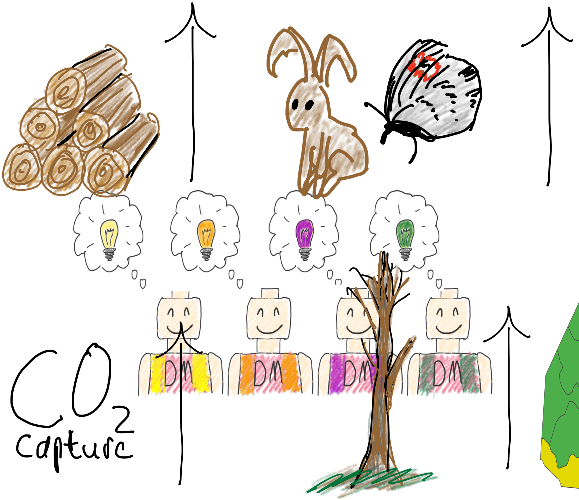
- O-NAUTILUS allows the DM to conduct function evaluations that hopefully lead to interesting solutions.
- No resources are wasted to compute solutions that are not interesting according to the preferences of the DM.
- Computed solutions can then be used to further increase the accuracy of the surrogate model used, which can help the DM to find more interesting solutions near their previous preferences.

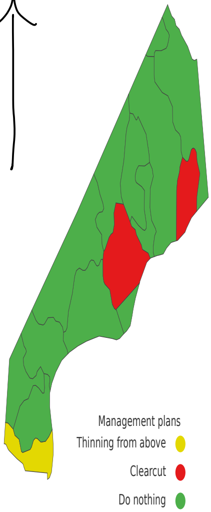
Surrogate-based problems

- Remember that surrogates are often approximations of the original problem. Results may differ from reality.



Group decision making in multiobjective optimization





Background

When is a decision collective?

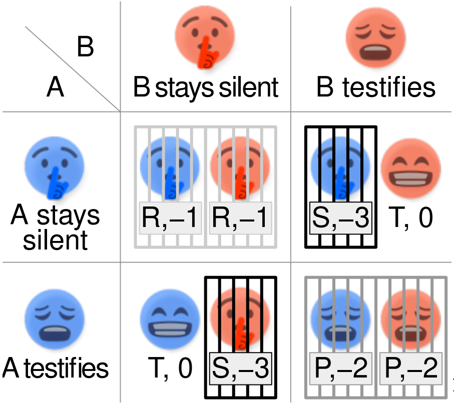
Background II

When there are multiple participants
that need to make a decision as a collective
aiming to make the best choice.

Background III

- **Game Theory**
- **Negotiations**
- **Group Decisions**

- Several players acting super-rationally (e.g. according to utility functions).
- The actions are determined by individual strategy of each player or by a binding agreement among all players.
- Well-known examples such as prisoner's dilemma.



[1]

[1]"Prisoners dilemma.svg" by cmglee, is licensed under CC BY-SA 4.0. .

Negotiation is

- .. a decision-making process by which two or more parties (people, organizations, nations, etc.) communicate and exchange ideas, arguments and offers with the intention of satisfying their needs and achieving objectives by educating and informing their counterparts and changing relationships through conferring for an agreement (Kersten, 2004).
- . . . a process in which two or more independent, concerned parties may make a collective choice, or no choice at all (Kilgour & Eden, 2010).
- In other words, the parties cannot be forced to negotiate, and the negotiations may end without making a choice at all. Any recent examples on how tough negotiations can be come to mind?
- Classic integrative negotiation situation example: The Orange Example.

Group decisions

- Several decision makers who have a shared problem in which they must make a decision that is acceptable to all decision makers (Kilgour & Eden 2010).
- The individual decision makers may have conflicting preferences.
- Requires in some way (explicitly or implicitly) defining what *acceptable to all* means.
- *How to decide how to decide?*
- Common concepts to consider: fairness, justice, consensus.

Solving group decision problems

A general approach to solve group decision problems (Keeney, 1993):

- In some way aggregate the DMs' preferences to **a collective preference**.
  - For example, ordinal rankings of the solutions can be aggregated to a collective ranking, a collective preference.
  - Multiple reference points can be aggregated to a collective reference point, a collective preference.
- The collective preference *determines* the most preferred solution that is "best" for the group.

Hotel selection example

Imagine a group of participants choosing a hotel for holiday or for work stay:

	Quality	**Location**	**Cheapness**
A	1	3	5
B	2	5	3
C	5	4	1

What is the best hotel to choose?
How the choice is to be made?
How fair the choice is?

Hotel selection example II

Imagine a group of participants choosing a hotel for holiday or for work stay:

	Quality	**Location**	**Cheapness**
A	1	3	5
B	2	5	3
C	5	4	1

	DM1	**DM2**	**DM3**	**Sum of ranks**
A	1	2	3	6
B	2	1	2	5
C	3	3	1	7

What is the best hotel to choose? How the choice is to be made? How fair the choice is?

Story time

Group decision making: Properties of individual DMs

Different characterizations of groups have been used in the literature, e.g. [5], [6].

- The DMs have their own expertise, attitudes and preferences.
- The DMs may have different roles or degrees of importances in decision making.
- The DMs may or may not be able to communicate in-person.
- The group may prefer to work transparently or anonymously [7].
- The group may prefer making decisions by some specified way [8], based on e.g. aggregating, voting, negotiating or other specific method.

Describing different type of group structures

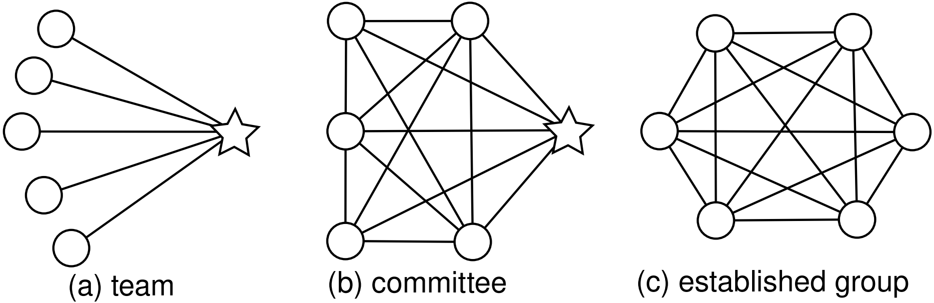
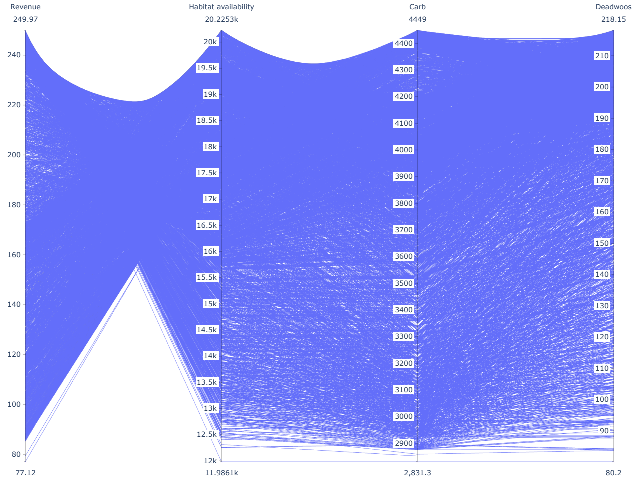


Figure: Group structures according to [7].

Let us get back to MOO world!

'A few' Pareto optimal solutions



Group decision making in multiobjective optimization (GDM-MOO)

GDM-MOO is the combination of group decisions and the peculiarities of multiobjective optimization [9].

- The group of DMs is solving a common multiobjective optimization problem.
- The group aims to find a solution that they can accept as the most preferred solution for the group.
- A GDM-MOO method incorporates multiple preferences from several DMs in some way to the optimization process.
- GDM-MOO methods can be classified to a priori, a posteriori and interactive methods, as MOO methods.

Interactive GDM-MOO: Key characteristics

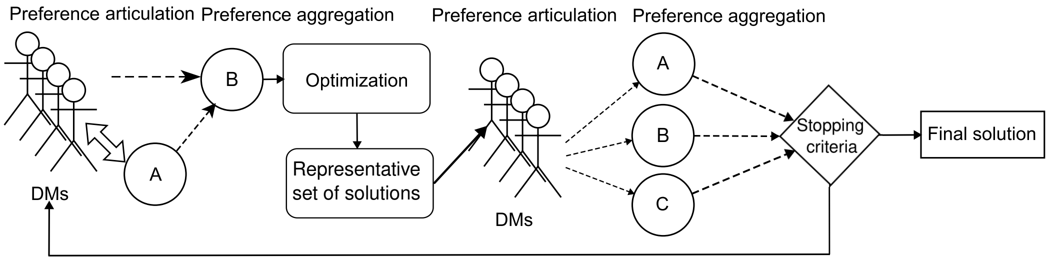
- The decision makers can learn about the problem such as the trade-offs among the objective functions.
- They can learn about the preferences of other DMs.
- $\rightarrow$ Thus, there are necessarily no static preferences.
- However, trade-offs (among the conflicting preferences of DMs) on top of trade-offs (of the conflicting objective functions)!

How interactive GDM-MOO methods are built?

Useful way to think about interactive GDM-MOO methods:

- Problem setting: *What is the problem to be solved, who are the DMs, what is the group structure and so on..*
- Preference aggregation: *How the DMs preferences are aggregated (if they are)?*
- Search: *How the optimization process seeks Pareto optimal solutions?*
- Convergence: *How the group converges on a single Pareto optimal solution?*

General flowchart for interactive GDM-MOO methods



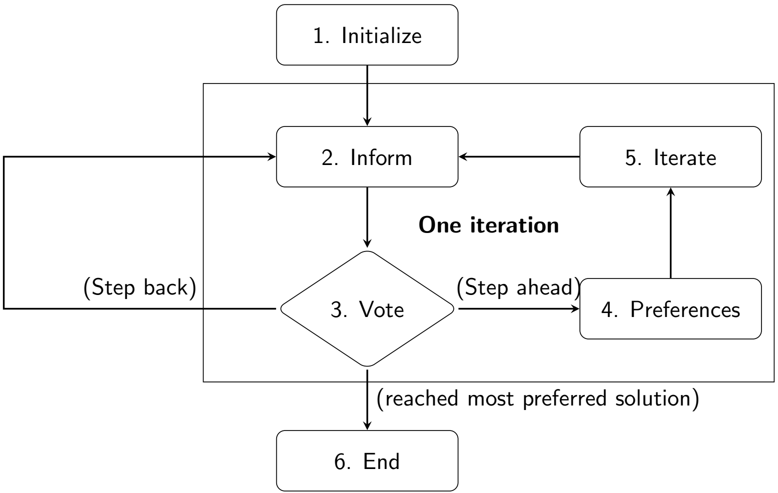
Extending NAUTILUS for GDM-MOO

- NAUTILI extends the NAUTILUS family of trade-off free methods for GDM.
- Paper is under review!
- **Collaborators**: Prof. Kaisa Miettinen, Prof. Francisco Ruiz, Dr. Dmitry Podkopaev, Dr. Babooshka Shavazipour and Dr. Bhupinder Saini and me.

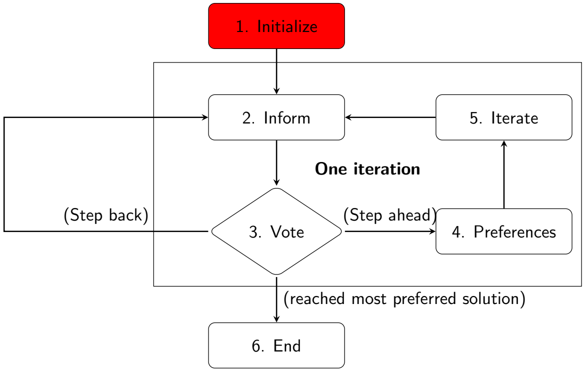
Example NAUTILI

- Problem setting: Collaborative established group solving a shared MOP. Group works online, cannot communicate or share preferences.
- Preference aggregation: Utilizing arithmetic mean as the aggregation operator on the direction of improvement vectors.
- Search: Using achievement scalarizing function with the group improvement direction.
- Convergence: The solution process converges to a single Pareto optimal solution according to the group's preferences.

General NAUTILI scheme

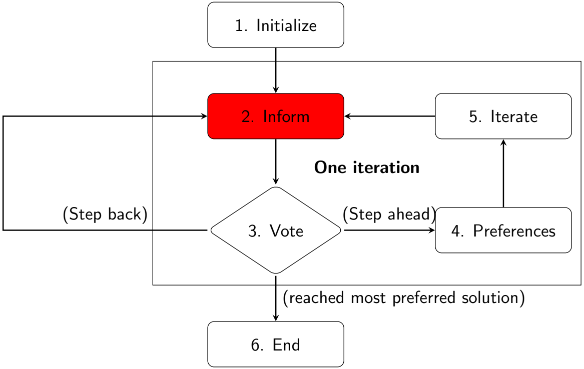


NAUTILI: Initialize



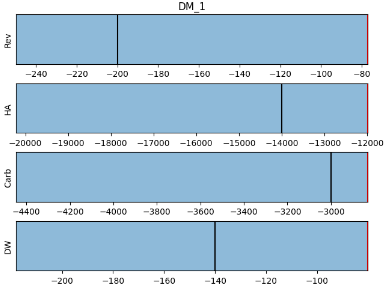
NAUTILI begins with an initialization where:

- The MOP is set.
- The group must decide how many iterations (steps to take) to perform.
- The group must decide the number determining the majority agreement or what does *most* of the group mean.

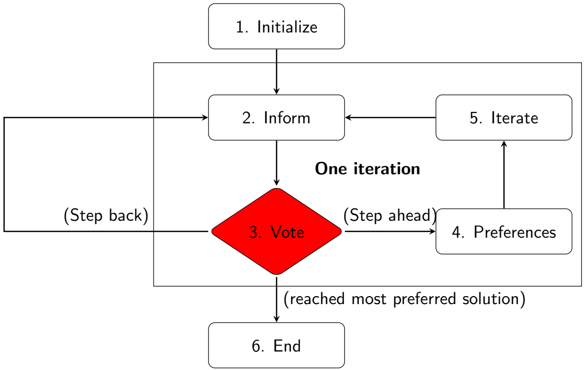


Inform:

- The reachable ranges, the reference point and the current iteration point are visualized to each DM.



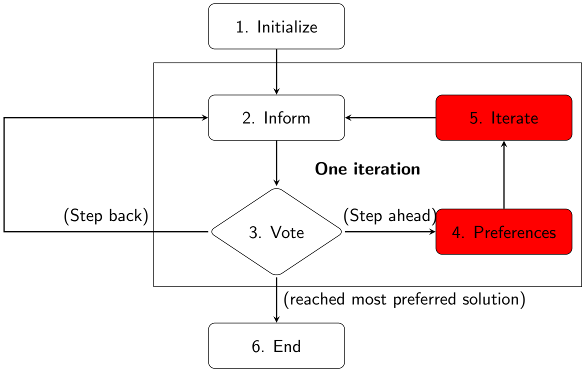
NAUTILI: Vote



Vote:

- If most DMs vote for taking a step back, perform a step back.
- The DMs can vote by pressing a button in the graphical user interface.

NAUTILI: Preferences & Iterate



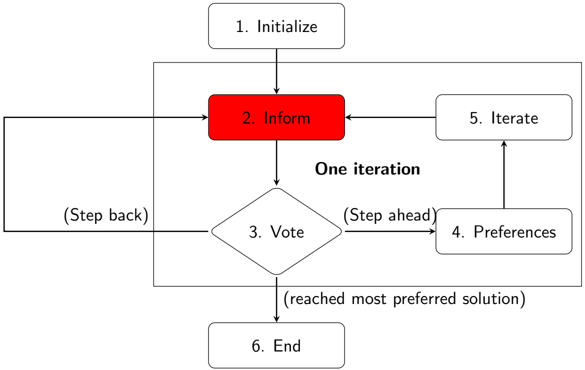
Preferences:

- The DMs give their preference information as reference points.

Iterate:

- Calculate the collective direction of improvement.
- Iterate and go to inform.

NAUTILI: Inform



Inform:

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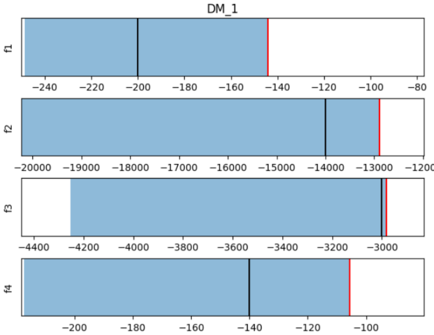
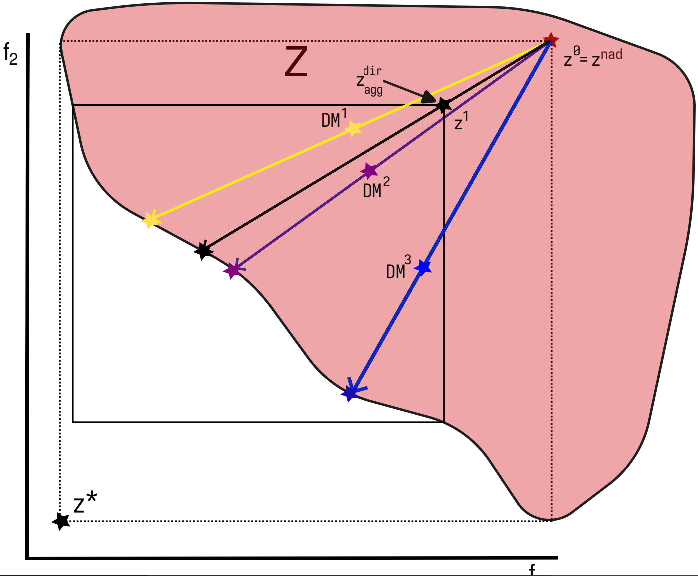
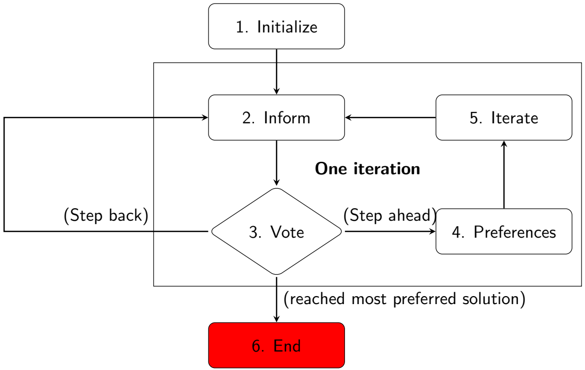


NAUTILI direction of improvement for group of DMs



NAUTILI: Most preferred solution



After last iteration:

- The reachable ranges have shrunk to a single Pareto optimal solution.
- That solution corresponds to the most preferred solution for the group according to the NAUTILI method.

In afternoon demo session from a group and solve a MOP with NAUTILI!

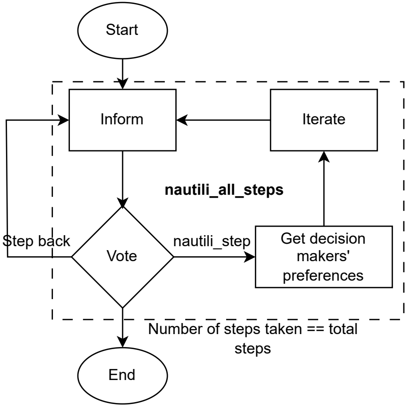
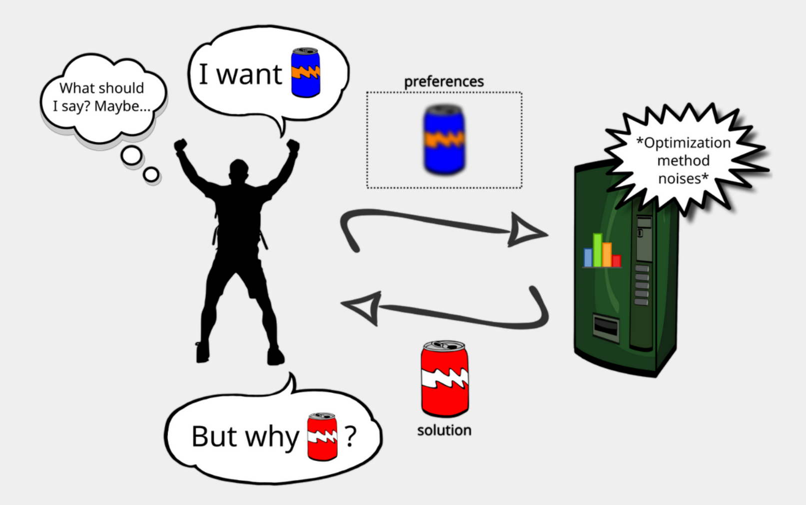


Figure: The overview of the NAUTILI method.

Explainable interactive multiobjective optimization

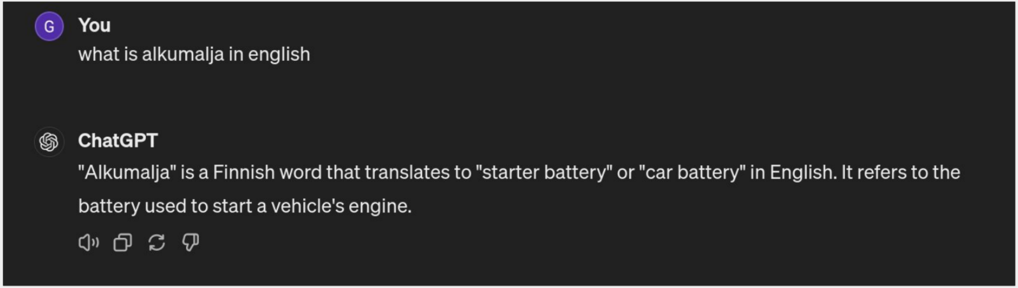
Introduction I



Introduction II

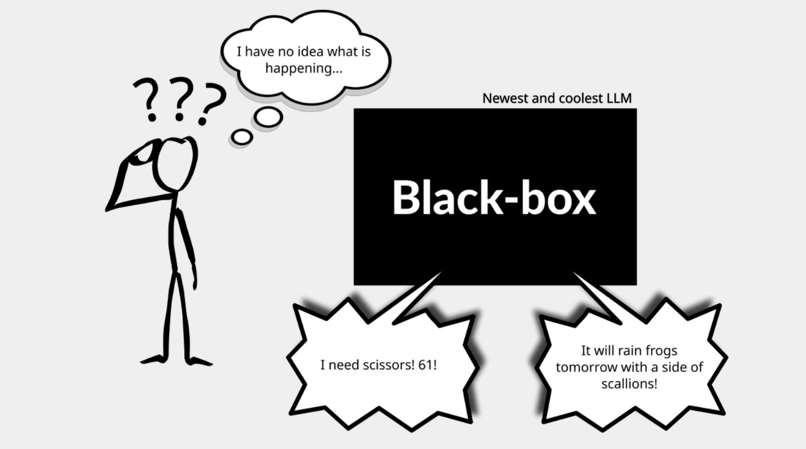
- How does an interactive multiobjective optimization method look like to a decision maker?
- Preferences go in, some solutions come out.
- But why? How? Can we answer these types of questions?
- We are not the only field wondering these sorts of things. . .

Introduction III



"alkumalja" in Finnish means "opening toast"

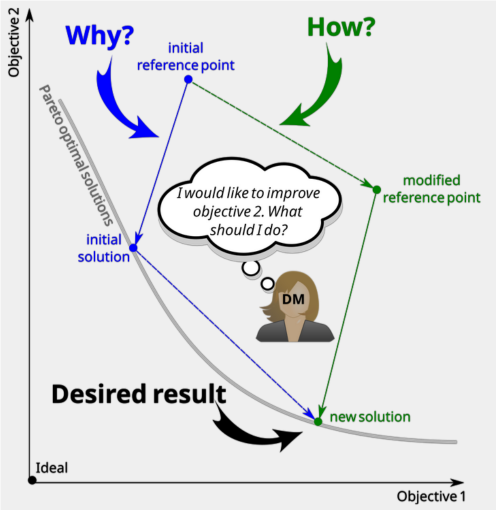
Introduction IV



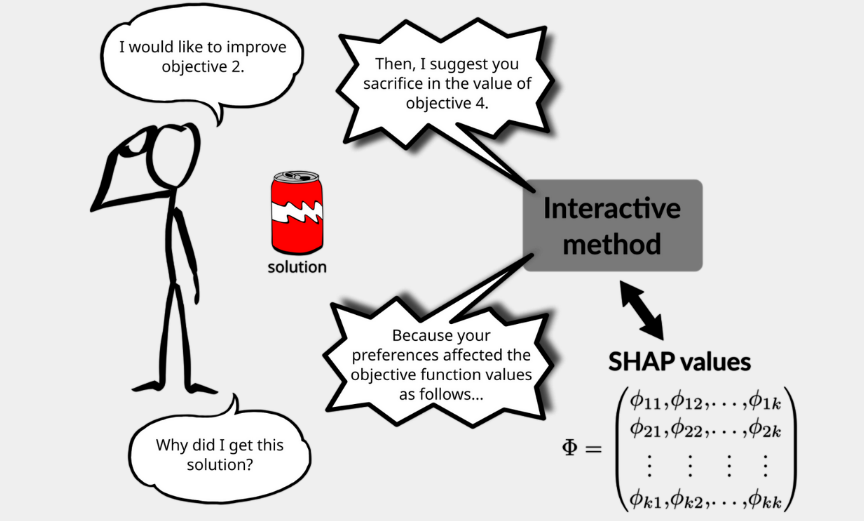
Introduction V

- In the field of **explainable artificial intelligence** [10], they are very much concerned with what happens inside machine learning models.
- The concept of **explainability** has been developed in said field to study and address the lack of transparency introduced by many machine learning models in society.
- What if the same concept and established tools to developed explainable aspects for machine learning models were applied to interactive multiobjective optimization?
- And so, the topic for my PhD thesis [11] was once formulated. . .

The R-XIMO method I



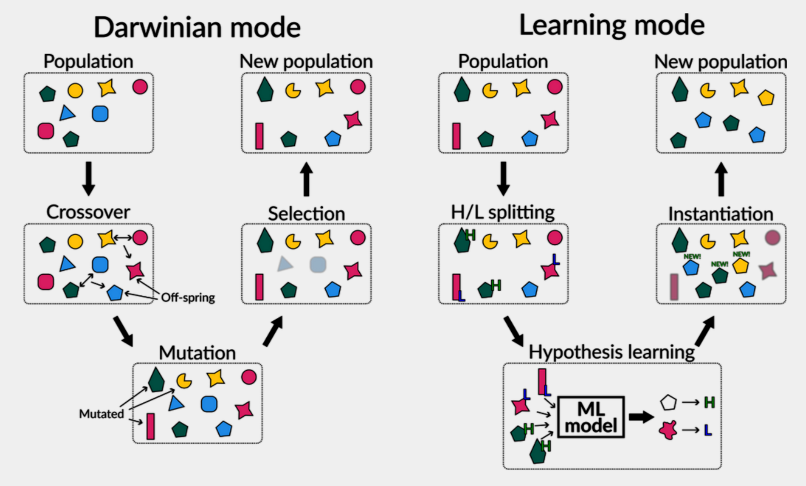
The R-XIMO method II



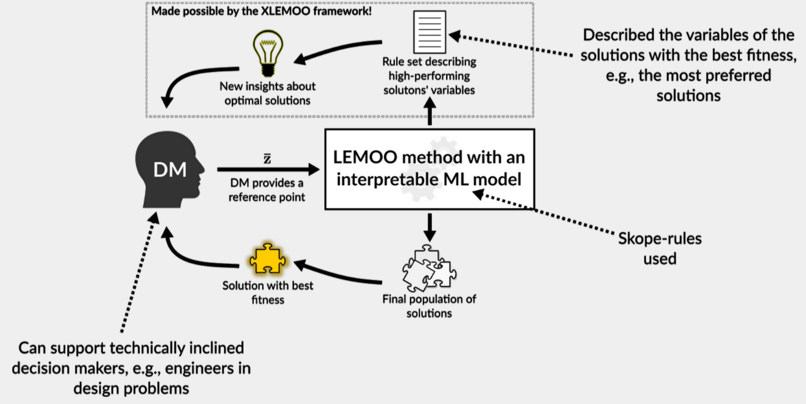
The R-XIMO method III

- The **R-XIMO method** [12] was developed to address the lack of support decision makers face when providing preference information during an interactive multiobjective optimization process.
- The method utilized SHAP-values [13] to figure out how a provided reference point affects the computed solutions by a reference point based interactive method.
- Decision makers get explanation telling them how their preference map to the computed solutions, and based on their desires, suggests how to modify the reference point so that better solutions can be found.
- The central idea utilizing SHAP values can be applied virtually to any method, regardless of preference type.

The XLEMOO method I



The XLEMOO method II



The XLEMOO method III

- The **XLEMOO method** [14] combines evolutionary multiobjective optimization with so-called **learnable evolutionary models** [15].
- In these models, the heuristics of an evolutionary optimization process are enhanced with machine learning to introduce more *intelligent* behavior.
- In the XLEMOO method, this idea has been explored in the context of interactive multiobjective optimization, utilizing an interpretable machine learning model.
- The method can provide additional information about regions in the solution and decision space of the problem to a decision maker. This can be useful in engineering applications, such as design problems.

Other methods I

- Other methods in the direction of explainable interactive multiobjective optimization have also been explored. These are summarized in a recently published book chapter [16].
- There are also recent efforts by out other group members [17].
- This is a very new research direction and it remains to be seen where it will go!

Conclusions

Conclusions

- Whatever we do, we should always keep in mind one important goal: the methods we develop should support decision makers in decision-making when there are multiple conflicting objectives.
- We can make all sorts of assumptions and come up with all kinds of beautiful, mathematically-rooted theories about rationality and whatnot, but sometimes theory can be far from reality—especially if we involve humans!
- And we need to involve humans, because there is the fundamental problem in multiobjective optimization that Pareto optimal solutions cannot be mathematically compared without additional information that is not available in the problem formulation itself.
- This all applies to group decision making also! Additionally, the need to make multiple decision makers confident of the method and the final solution.

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