

1/31/18

## Lecture 2

$y \in \{0, 1\} = \mathcal{Y}$  output space

true system (not a model... but it's sometimes called the "true model")  
It's not a model because it IS reality

$y = t(z_1, z_2, z_3)$   
 $z_1$  = has sufficient funds  
 $z_2$  = unforeseen emergency  
 $z_3$  = criminal intent

Big Problem:  $\{z_1, z_2, z_3\}$  are unobservable

Impossible: get  $\{z_1, z_2, z_3\}, t$

Next Best: try to define and collect information "related" to the causal inputs.

$\begin{cases} X_1: \text{salary. How } \text{~~much~~ \text{ to measure: avg. over 5 yr.}} \\ X_2: \text{previous loan repayment } \in \{0, 1\} \text{ ever defaulted} \\ X_3: \text{previous crime type (no crime, infraction, misdemeanor, felon)} \end{cases}$   
 (Very difficult to obtain these  $X$ s) ( $X_3$  not #'s)

Business-as-usual: Use what you have or use what is cheaply obtainable

$(\{z_1, z_2, z_3\} \approx \text{some of the same info} \approx \{x_1, x_2, x_3\})$

Bob's information:

$\vec{X} = [x_1, x_2, x_3]$   $\rightarrow$  "covariates", "Predictors"  
 $\rightarrow$  "features", "variables", "characteristics", "attributes", "regressors"  
 $\rightarrow$  "observation", "record", "object", "input", "independent variables"

$\vec{X} = [x_1, x_2, x_3] \in \mathcal{X}$  "covariate space"

$x_1 \in \mathbb{R}$  "continuous variable"

$x_2 \in \{0, 1\}$  "binary ~~or~~ dummy variable"

$x_3$  is a "categorical variable" with 4 "levels" (unique possible values)

(A) Code it numerically,  $x_3 \in \{0, 1, 2, 3\}$  This should only be done if the categorical predictor is "ordinal"



(B) Dummification

$X_3 \rightarrow X_{3a}, X_{3b}, X_{3c}, X_{3d}$

binary                      binary

no crime                      misdemeanor

binary                      binary

infraction                      felony

$x_3 = \text{misdeemeanor}$   $\vec{x} = [ \cdot \cdot 0 0 1 0 ]$

$$p \equiv 3 \rightarrow 6$$

Goal: do the best we can in explaining  $y$  by creating model  $f$ ,  
the approx., i.e. the best functional relationship we can get.

Does  $y = f(x_1, x_2, x_3)$ ? No...  
ever

Instead,  $y \approx f(x_1, x_2, x_3)$  ↙ comes from ignorance  
 $y = f(x_1, x_2, x_3) + \delta$   $\delta = t(\vec{x}) - f(\vec{x})$

How do we get  $f$ ?

First note there is no analytical solution e.g.  $h(x) = x^2$  find  $\min h$ .  
Instead use an "empirical solution" i.e. use data.

⇒ "Learning from data"

"Supervised Learning" uses historical examples of records and their responses.

It requires 3 ingredients:

$$\textcircled{1} \mathbb{D} := \{ \langle x_1, y_1 \rangle, \langle x_2, y_2 \rangle, \dots, \langle x_n, y_n \rangle \}$$

$x_1$  is Bill's characteristics and  $y_i$  is whether or not he paid back loan

$\vec{x}_2$  is Jill's      ||      ||       $\vec{t}_2$       ||      ||

Let  $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathcal{X}^n$      $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \in \mathcal{Y}^n$      $\dim(\mathcal{X}) = n \times p$   
 $\dim(\mathcal{Y}) = n$