MIE1622 Assignment 4

Asset Pricing

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Summary

This assignment compares the European call and put options computed from Black-Scholes and Monte Carlo simulations. The Black-Scholes model is based on several assumptions about the market, including that the underlying asset's price follows a geometric Brownian motion and that there are no transaction costs, dividends, or arbitrage opportunities available. The underlying present stock price is 100, the strike is at 105, the years to expiry is 1, the risk-free rate is 0.05, the drift is 0.05 and the volatility is 0.2. In addition, the assignment also includes the pricing procedure for the barrier knock-in options where the barrier is set at 110. It is important to assess whether each asset path has passed the barrier in order to calculate the payment for each asset path in barrier options. The type of barrier option and whether the barrier level has been reached during the life of the option then determine the payoff.

Analyze your results

A total of five pricing functions for each strategy is computed in the last session, and the corresponding full results are attached in the Appendix.

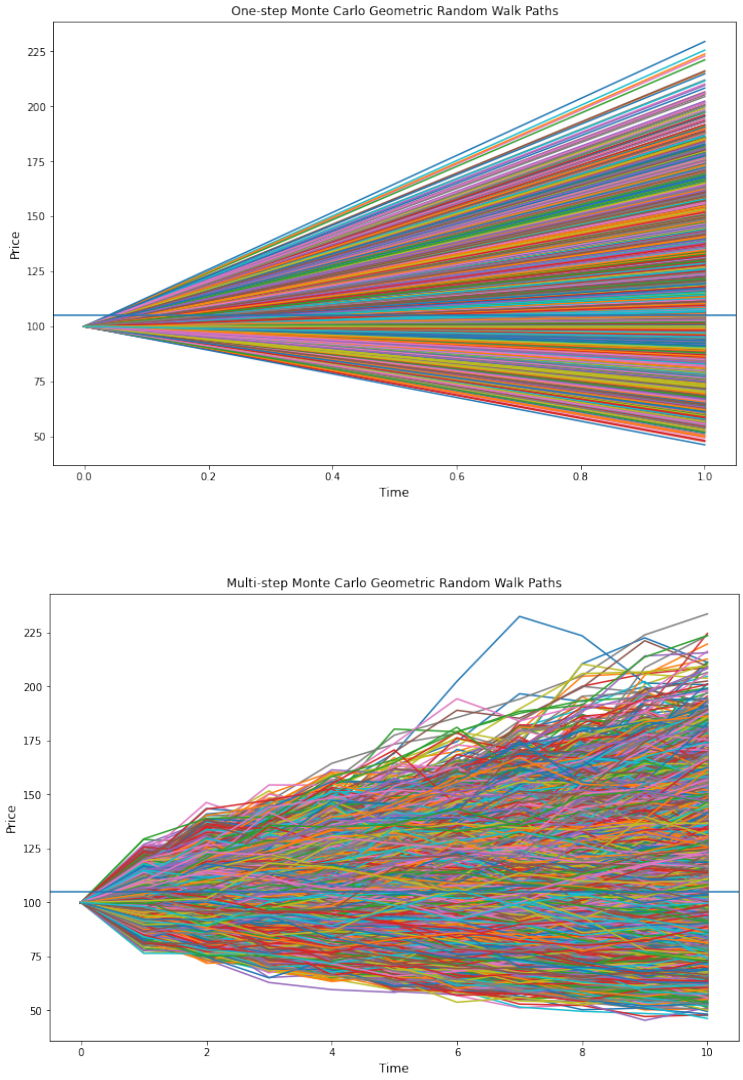

Figure 1

Figure 1 is the Monte Carlo geometric random walk paths for one-step and multi-step European options separately. It can be observed that the distribution of stock prices appears to become more dispersed as time progresses, and multi-step simulations result in greater price fluctuations in comparison to single-step simulations. The horizontal blue line at 105 is the strike price that refers to the predetermined price at which an option can be traded.

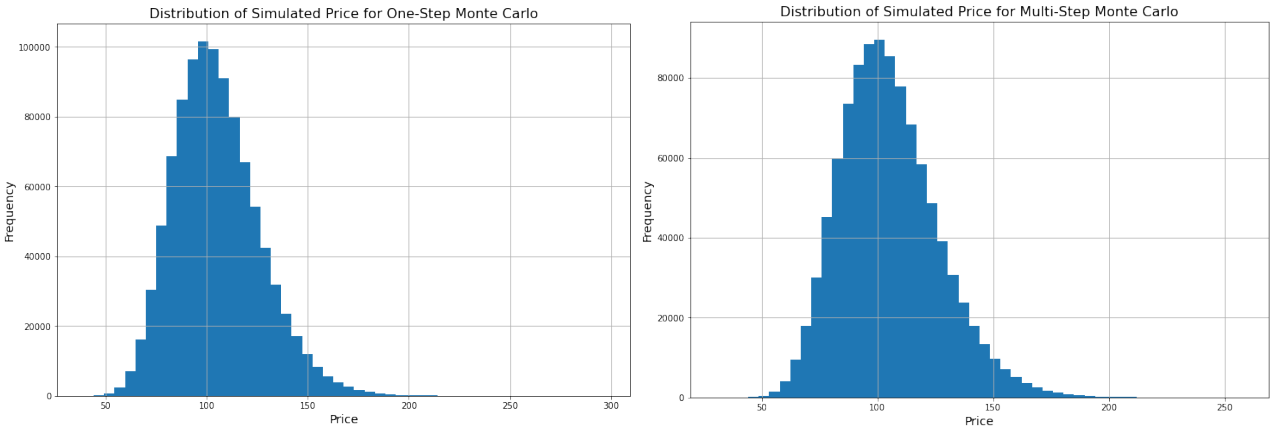

Figure 2

Figure 2 provides a visual inspection of the distribution for simulated prices in terms of the histogram. We can find similar information that both histograms are right-skewed.

Table 1: Prices for Different Strategies of European Options

	Black-Scholes	One-step MC	Multi-step MC
Call Options	8.02135	8.00949	8.00956
Put Options	7.90044	7.89118	7.89961

Table 1 shows the comparison of the three pricing strategies. Among the three of them, Black-Scholes method produces the highest prices for both call and put options, with a call option value of 8.02135, and a put option value of 7.90044. Muli-step and one-step Monte Carlo simulations have relatively similar results, while multi-step Monte Carlo returns slightly higher prices.

Table 2: Prices for Different Strategies of Barrier Options

	One-step MC	Multi-step MC
Call Options	7.79794	7.94183
Put Options	0.0	1.19110

The prices of Barrier call and put options exhibit significant disparity in contrast to the prices of European call and put options. The call option prices for barriers under one-step and multi-step simulation are slighter lower than the European options, with values of 7.79794 and 7.794183. While the pricing of Barrier put options is remarkably low, the one for one-step simulation is 0.0 and the one for multi-step simulation is 1.19110, in comparison to the average European put option price of 7.9.

In this case, the option has a barrier price of 110, a current market price of 100, and a strike price of 105. If the underlying asset's price remains below the barrier level, the option will remain inactive, and the holder will not gain any profit. This is why the price of the barrier put option is low since it is less likely to become active.

However, if the underlying asset's price does rise above the barrier level of 110, the option becomes active and the holder can exercise it, earning a profit based on the difference between the strike price and the market price. On the other hand, if the asset's price remains between the barrier and strike prices, the option will not be exercised, resulting in no profit for the holder. Due to the added condition of the barrier level, both barrier call and put options are priced lower than their European counterparts

Table 3: Barrier Options Price with Volatility Increased or Decreased by 10%

	Call Options	Put Options
One-step MC (Original)	7.79794	0.0
One-step MC (Increased)	8.60798	0.0
One-step MC (Decreased)	6.98216	0.0
Multi-step MC (Original)	7.94183	1.19110
Multi-step MC (Increased)	8.74389	1.49735
Multi-step MC (Decreased)	7.13571	0.90855

Based on Table 3, it can be inferred that as volatility increases, the prices of both barrier call and put options are higher than the original results. Conversely, as volatility decreases, the prices of both options decrease. It suggests that for higher volatility, the stock prices are more likely to exceed the barrier and reach higher levels, thus resulting in higher prices for Barrier call and put options.

Discuss possible strategies to obtain the same prices from two procedures

In this session, I have developed a new function that utilizes a for loop to test various combinations of time steps and scenarios. The aim is to find the optimal parameter values that produce the most accurate call and put option prices using the Monte Carlo simulation compared to the Black-Scholes method.

After running the for loop function, I obtained the following results:
The optimal step times and scenarios for call options are 128 and 1000000, with a difference of 0.0014028148377889238 in option prices. The optimal step times and scenarios for put options are 128 and 100000, with a difference of 0.0016723057266121444 in option prices.

Appendix

Call Options and Put Options Prices:

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One-step MC price of an Barrier call option with volatility increased by 10% is 8.607979730529676
One-step MC price of an Barrier put option with volatility increased by 10% is 0.0
Multi-step MC price of an Barrier call option with volatility increased by 10% is 8.743894592387987
Multi-step MC price of an Barrier put option with volatility increased by 10% is 1.497348729805694
One-step MC price of an Barrier call option with volatility decreased by 10% is 6.9821650969789495
One-step MC price of an Barrier put option with volatility decreased by 10% is 0.0
Multi-step MC price of an Barrier call option with volatility decreased by 10% is 7.135707335889736
Multi-step MC price of an Barrier put option with volatility decreased by 10% is 0.908559596804964
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