MIE1622 Assignment 2

Risk-Based and Robust Portfolio Selection Strategies

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Summary

This assignment builds upon the previous one. Apart from the prior four mean-variance portfolio selection strategies (Buy and Hold, Equally weighted, Minimum variance, and Maximum Sharpe ratio), we will investigate three more risk-based and robust portfolio selection strategies: Equal risk contributions, leveraged equal risk contributions and robust mean-variance optimization portfolio selection strategies. The primary objective is to minimize the risk of the portfolio and also achieve an acceptable level of return.

To evaluate the performance of the portfolio selection approaches, this study investigates seven different strategies, over three trading periods: from 2020 to 2021, from 2008 to 2009, and year 2022. This study also provides insights into the daily portfolio values after re-balancing the portfolio and the dynamic changes in portfolio allocation of the 20 assets. By showing the daily portfolio values after re-balancing, this study provides a clear understanding of the impact of re-balancing on portfolio performance and helps to highlight the importance of regular portfolio monitoring and management.

Furthermore, this report also examines the portfolio risk by plotting the maximum drawdown which is the largest drop in portfolio value from its highest point to its lowest point. It can provide investors with a better understanding of the risk-return tradeoff of the portfolio and help them wisely select investment strategies.

Question 1. Implement Investment Strategies in Python

As we have already covered the first four portfolio selection strategies in the Assignment 1 report, I will now shift my focus to the remaining three strategies. This study is based on an initial portfolio consisting of 20 stocks, with a total portfolio value of $1000012.93.

The fifth strategy implemented is 'Equal risk contributions' (ERC), which aims to distribute the risk of the portfolio among all of its assets equally. With that being said, portfolios with higher volatility will be given a lower weight in the portfolio than those with lower volatility. Therefore, the corresponding non-linear optimization objective function and the constraints are stated as follows:

$$\begin{aligned} \min_{w} \sum_{i=1}^{n}\sum_{j=1}^{n}(w_i(Qw)_i - w_j(Qw)_j)^2 \\ \text{s.t.} \sum_{i=1}^{n} W_i = 1 \\ W \geq 0 \end{aligned}$$

The sixth strategy implemented is 'Leveraged Equal Risk Contributions', which is an aggressive variation of the Equal Risk Contributions portfolio strategy. The investor would short a risk-free asset through the trading period while borrowing money with a long 200% position in the ERC portfolio. This approach can magnify prospective returns while still maintaining the same level of risk for all assets in the portfolio.

The seventh strategy implemented is 'Robust mean-variance optimization' (RMVO), which is a useful technique that aims to compute optimal portfolio weights that can maximize the expected return while minimizing the risk. It considers the uncertainty both in risk and return, so we set up the constraints for the optimization problem as follows:

$$\begin{aligned} \max \quad & \mu^T w \\ \text{s.t. } & w^T Q w \leq \epsilon_1 \\ & \|\Theta^{\frac{1}{2}} w\| \leq \epsilon_2 \\ \text{s.t.} & \sum_{i=1}^{n} W_i = 1 \\ & W \geq 0 \end{aligned}$$

Question 2. Analysis of Results for years 2020 and 2021

The bi-monthly portfolio values of re-balancing during the 12 trading periods from 2020 to 2021 for four portfolio selection strategies can be found in the appendix.

Figure 1 presents the daily portfolio values for the first trading period over the years 2020 and 2021 for the seven distinct investment strategies. Upon visual inspection, Upon visual inspection, it is apparent that the 'Maximum Sharpe Ratio' strategy outperformed the other six strategies, with the highest portfolio values throughout the trading period in both years. The performance of this strategy is exceptional, with a significant difference from the remaining six strategies. This observation indicates that the 'Maximum Sharpe Ratio' strategy is the most effective approach for investing assets in terms of return.'Equally weighted', 'Leveraged equal risk contribution', 'Equal risk contribution' and 'Robust mean variance optimization' portfolio selection approaches all have relatively similar performances, and their values are steadily increasing with the market trend. These approaches present an excellent alternative to the 'Maximum Sharpe Ratio' strategy, as they provide stable returns with limited risk.

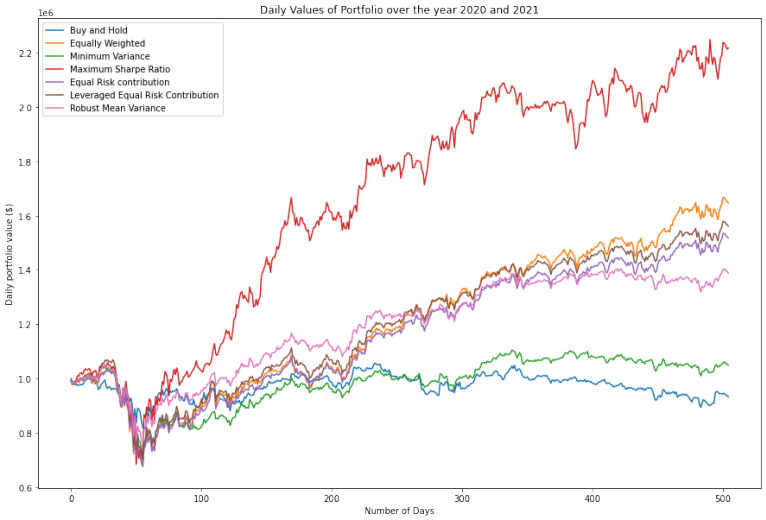

Figure 1

Figure 2 presents the maximum drawdown for these seven strategies, which measures the largest loss of investment and shows the relative risk. All seven portfolio selection strategies encounter the maximum drawdown at period 2 due to market fluctuation which can also be reflected in Figure 1. However, it is noteworthy that this superior performance comes at the cost of increased risk. The 'Maximum Sharpe Ratio' strategy also experiences the highest drawdown, while it quickly adapts to the market demonstrating its hedging capacity to some degree.

The investing strategies of 'Minimum variance' and 'Buy and hold' are now not advised for investors due to their unsatisfied performances in the current market circumstances. Despite the ongoing bull market, these strategies have failed to yield profits, indicating that they may not be the most suitable options for investors seeking substantial returns.

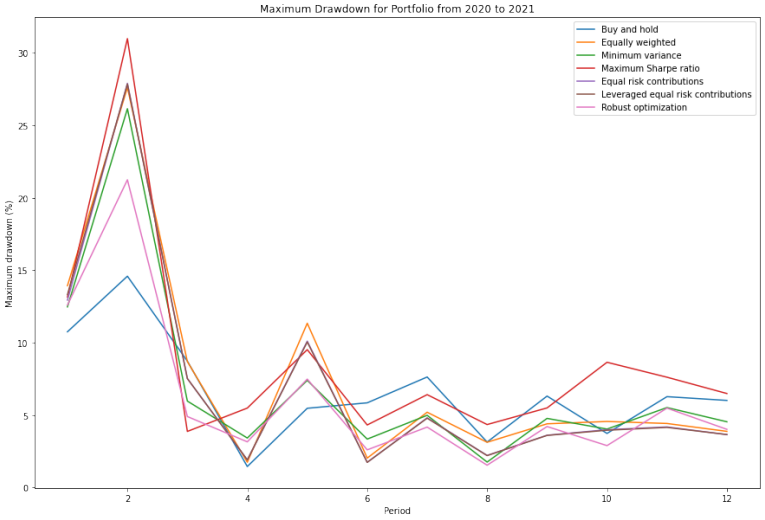

Figure 2

Figures 3, 4 and 5 are representations of the dynamic alterations in the portfolio allocations for the 'Minimum Variance', 'Maximum Sharpe Ratio', and RMVO portfolio re-balancing strategies respectively. The lines depicted in these figures represent the proportionate weight of different stocks within the portfolio. We can observe that both 'Minimum variance' and RMVO tend to hold stock for a long period of time, and the stock 'VZ' dominates the whole trading period. In contrast, 'Maximum Sharpe Ratio' strategy tends to buy and sell stocks in a short time in order to gain profit and not a single stock is held for a long time. Overall, RMVO strategy reduces trading compared to the rest two strategies and is capable of mitigating risk.

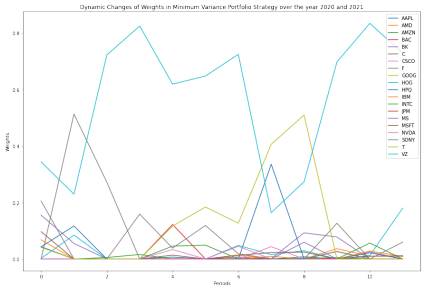

Figure 3

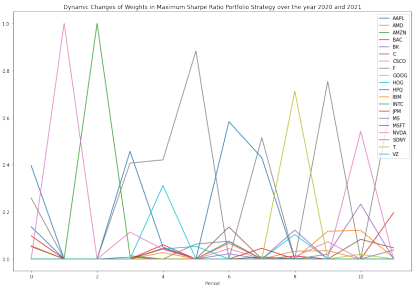

Figure 4

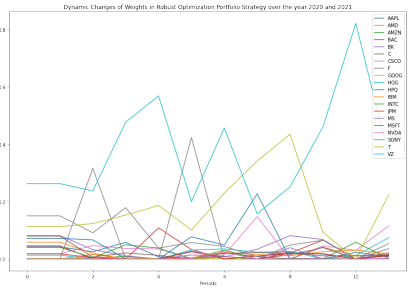

Figure 5

Having taken into account various factors, including the current market situation and available investment strategies, I have come to the conclusion that the 'Maximum Sharpe Ratio' strategy is the most suitable option among the seven of them. This decision is based on a thorough analysis of the strategy's historical performance, which has consistently delivered higher returns and growth values compared to the other investment approaches. Despite this strategy also comes with high risk, the economy and market just recovered from the recession caused by the global pandemic, Covid-19. The 'Maximum Sharpe Ratio' strategy is capable of effectively managing risk while also generating the highest profits in the current bull market.

Question 3. Test your trading strategies for years 2008 and 2009

In the period between 2008 and 2009, the financial crisis caused significant turbulence in the stock market, the 'Minimum Variance Portfolio' strategy appears as the best approach to investment. It allocates funds to

stocks with the lowest volatility and avoids substantial losses to the highest degree. According to Figure 6, the 'Minimum variance' strategy remains relatively stable reaches to a break-even point at the end of the period, it indicated that there was no net profit or loss for the investor. Unlike the previous trading periods, where the 'Maximum Sharpe Ratio' strategy had performed well, it turned out to be the worst strategy during this period. The portfolio experienced a dramatic loss around the 270th day, and the portfolio values failed to recover thereafter.

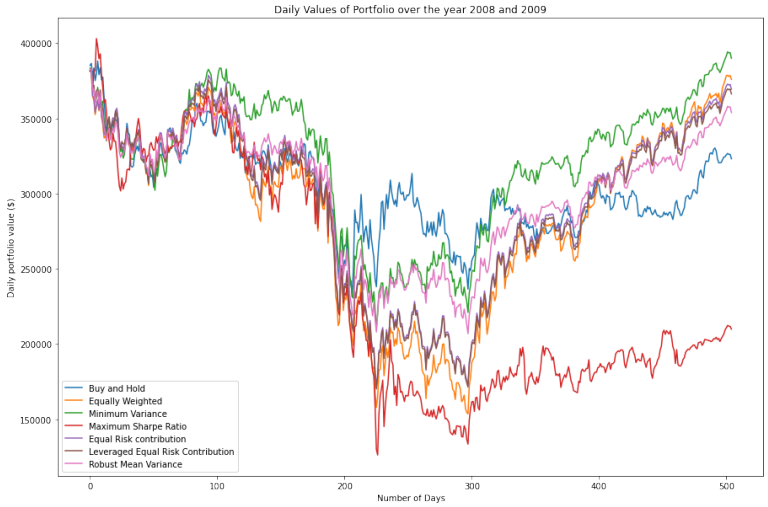

Figure 6

Figure 7 provides valuable insights into the performance of various investment strategies over the trading period. One key finding from the figure is that the 'Maximum Sharpe Ratio' strategy experienced the greatest drawback among all strategies. Conversely, the 'Equally Weighted' strategy suffered the least, while the portfolio value doesn't rise significantly after that.

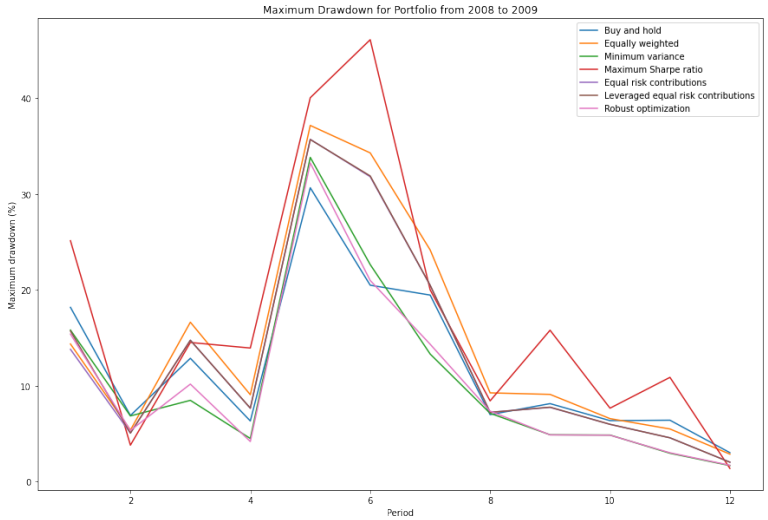

Figure 7

As for the dynamic change of portfolio, the 'Maximum Sharpe Ratio' strategy trades frequently due to the market crisis, and RMVO strategy reduces trading compared to the rest.

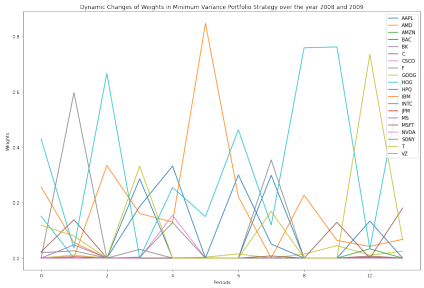

Figure 8

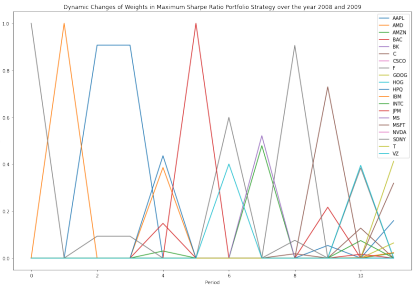

Figure 9

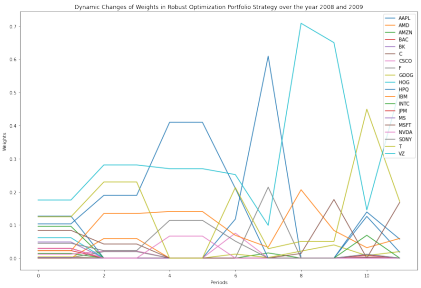

Figure 10

Upon analyzing the data, it can be observed that all investment strategies experienced challenges during the financial crisis, resulting in less than ideal performance. However, the 'Minimum Variance Strategy' was found to be the most effective strategy during the bear market and the 'Equally Weighted' strategy involves provides a balanced and diversified portfolio. In contrast, aggressive strategies like LERC and 'Maximum Sharpe Ratio' experience great loss from the unstable market. For this trading period, I would recommend conservative portfolio selection strategy like 'Minimum variance' to mitigate risk.

Question 4. Test your trading strategies for year 2022

Figure 11 presents the daily portfolio values for the third trading period of 2022, with a consistent downward market trend that is affecting the performance of investment portfolios. Noted that 12 months trading period may not be longer enough to present the market trend. Therefore, portfolio strategies that prioritize risk management, such as the 'Minimum Variance ', 'Buy and hold' and RMVO are still the optimal approaches to manage portfolios according to the graph. 'Maximum Sharpe Ratio' results in a great loss with the portfolio values drops to the lowest around the 190th day.

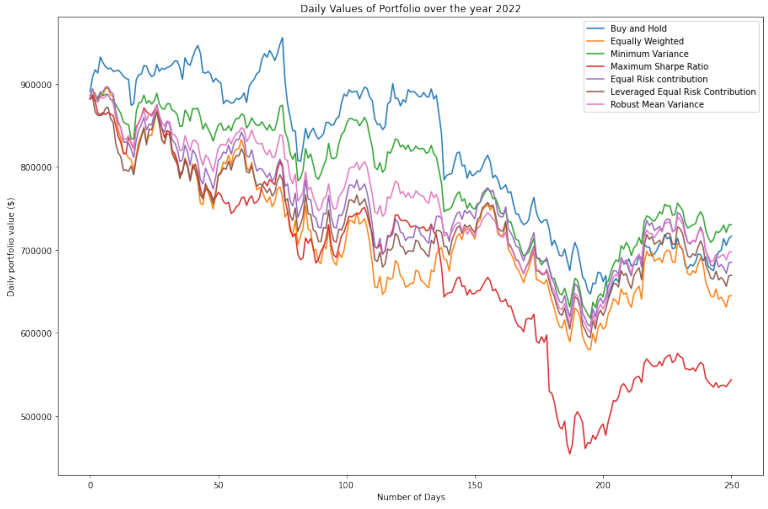

Figure 11

The maximum drawdown of these portfolio selection strategies proves our previous observation that the 'Maximum Sharpe ratio' has the greatest drawdown at the period 5. It shows that this approach is highly affected by market volatility and uncertainty.

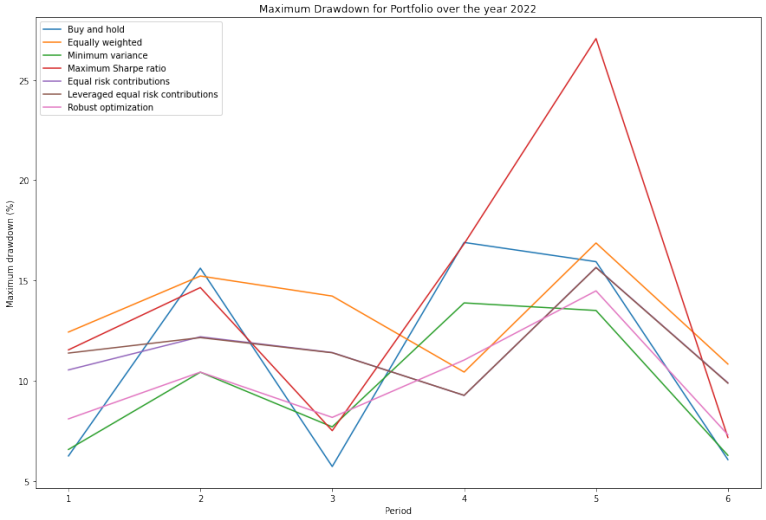

Figure 12

The maximum drawdown for the 2022 period and the 2008-2009 period can provide valuable information about the similarities and differences between these two "recession" periods. 2008-2009 a severe recession caused by global economic crisis, some stocks even experienced maximum drawdown more than 50%. Even though there have been substantial market changes in 2022, especially in the early part of the decade, the drawdown in 2022 is not as severe as the one in 2008-2009. In conclusion, investors should carefully do research on the market and evaluate investing strategies before entering the market, manage risk well while gain return.

Appendix 1

For trading period 2020-2021, 2008-2009 and 2022:

```
Initial portfolio value = $ 1000013.0

Period 1: start date 01/02/2020, end date 02/28/2020
  Strategy "Buy and Hold", value begin = $ 1000013.00, value end = $ 893956.82
  Strategy "Equally Weighted Portfolio", value begin = $ 990898.24, value end = $ 893208.59
  Strategy "Mininum Variance Portfolio", value begin = $ 992758.41, value end = $ 916240.12
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 990064.37, value end = $ 922095.98
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 991537.08, value end = $ 898332.69
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 990197.08, value end = $ 927867.87
  Strategy "Robust Optimization Portfolio", value begin = $ 992214.57, value end = $ 917833.69

Period 2: start date 03/02/2020, end date 04/30/2020
  Strategy "Buy and Hold", value begin = $ 945076.08, value end = $ 949228.39
  Strategy "Equally Weighted Portfolio", value begin = $ 931395.29, value end = $ 862353.83
  Strategy "Mininum Variance Portfolio", value begin = $ 955988.34, value end = $ 851552.27
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 962082.00, value end = $ 1017240.71
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 937389.72, value end = $ 852606.02
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 967205.78, value end = $ 879435.74
  Strategy "Robust Optimization Portfolio", value begin = $ 963167.53, value end = $ 947878.18

Period 3: start date 05/01/2020, end date 06/30/2020
  Strategy "Buy and Hold", value begin = $ 937916.75, value end = $ 913415.30
  Strategy "Equally Weighted Portfolio", value begin = $ 831101.45, value end = $ 934159.54
  Strategy "Mininum Variance Portfolio", value begin = $ 827264.88, value end = $ 854237.62
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 974390.72, value end = $ 1175795.01
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 822368.09, value end = $ 917455.74
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 848395.89, value end = $ 946519.00
  Strategy "Robust Optimization Portfolio", value begin = $ 919674.19, value end = $ 997796.15

Period 4: start date 07/01/2020, end date 08/31/2020
  Strategy "Buy and Hold", value begin = $ 905419.70, value end = $ 994693.42
  Strategy "Equally Weighted Portfolio", value begin = $ 927755.25, value end = $ 1060727.96
  Strategy "Mininum Variance Portfolio", value begin = $ 856556.48, value end = $ 981500.25
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1219685.93, value end = $ 1606785.15
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 913744.06, value end = $ 1053491.40
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 942685.23, value end = $ 1086864.50
  Strategy "Robust Optimization Portfolio", value begin = $ 1002083.83, value end = $ 1140274.32

Period 5: start date 09/01/2020, end date 10/30/2020
  Strategy "Buy and Hold", value begin = $ 993194.54, value end = $ 971914.18
  Strategy "Equally Weighted Portfolio", value begin = $ 1068338.47, value end = $ 999243.10
  Strategy "Mininum Variance Portfolio", value begin = $ 983247.54, value end = $ 942756.98
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1640764.45, value end = $ 1553142.55
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1061388.82, value end = $ 996158.25
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1094995.64, value end = $ 1027836.96
  Strategy "Robust Optimization Portfolio", value begin = $ 1145514.50, value end = $ 1097205.53

Period 6: start date 11/02/2020, end date 12/31/2020
  Strategy "Buy and Hold", value begin = $ 983801.02, value end = $ 1004435.74
  Strategy "Equally Weighted Portfolio", value begin = $ 1008081.57, value end = $ 1194326.17
  Strategy "Mininum Variance Portfolio", value begin = $ 951192.00, value end = $ 1005965.27
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1551740.49, value end = $ 1789134.34
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1004358.78, value end = $ 1180000.11
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1036297.07, value end = $ 1217277.81
  Strategy "Robust Optimization Portfolio", value begin = $ 1104505.48, value end = $ 1239405.79
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Period 7: start date 01/04/2021, end date 02/26/2021
  Strategy "Buy and Hold", value begin = $ 1005601.39, value end = $ 956244.08
  Strategy "Equally Weighted Portfolio", value begin = $ 1180783.14, value end = $ 1267218.28
  Strategy "Mininum Variance Portfolio", value begin = $ 1003981.40, value end = $ 975148.49
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1737569.08, value end = $ 1851392.00
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1166995.14, value end = $ 1220610.62
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1203901.12, value end = $ 1258225.65
  Strategy "Robust Optimization Portfolio", value begin = $ 1226078.78, value end = $ 1223422.86

Period 8: start date 03/01/2021, end date 04/30/2021
  Strategy "Buy and Hold", value begin = $ 957791.35, value end = $ 1019731.32
  Strategy "Equally Weighted Portfolio", value begin = $ 1297587.56, value end = $ 1398874.11
  Strategy "Mininum Variance Portfolio", value begin = $ 975468.82, value end = $ 1088054.39
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 1899769.93, value end = $ 2059602.42
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1244693.08, value end = $ 1356128.20
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1282824.38, value end = $ 1397751.19
  Strategy "Robust Optimization Portfolio", value begin = $ 1233597.37, value end = $ 1365634.94

Period 9: start date 05/03/2021, end date 06/30/2021
  Strategy "Buy and Hold", value begin = $ 1022204.61, value end = $ 987842.85
  Strategy "Equally Weighted Portfolio", value begin = $ 1397748.67, value end = $ 1459313.30
  Strategy "Mininum Variance Portfolio", value begin = $ 1087868.73, value end = $ 1076783.13
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2051066.72, value end = $ 2014754.84
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1355086.59, value end = $ 1385979.53
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1396629.81, value end = $ 1427456.69
  Strategy "Robust Optimization Portfolio", value begin = $ 1364543.26, value end = $ 1362202.55

Period 10: start date 07/01/2021, end date 08/31/2021
  Strategy "Buy and Hold", value begin = $ 993283.49, value end = $ 975250.19
  Strategy "Equally Weighted Portfolio", value begin = $ 1466719.25, value end = $ 1517805.45
  Strategy "Mininum Variance Portfolio", value begin = $ 1076813.56, value end = $ 1086661.37
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2013665.31, value end = $ 2121761.08
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1392307.72, value end = $ 1444274.23
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1433887.13, value end = $ 1486918.42
  Strategy "Robust Optimization Portfolio", value begin = $ 1364973.65, value end = $ 1403835.11

Period 11: start date 09/01/2021, end date 10/29/2021
  Strategy "Buy and Hold", value begin = $ 974520.08, value end = $ 949068.41
  Strategy "Equally Weighted Portfolio", value begin = $ 1513571.60, value end = $ 1563474.39
  Strategy "Mininum Variance Portfolio", value begin = $ 1081129.14, value end = $ 1057285.32
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2102424.44, value end = $ 2144868.51
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1438365.38, value end = $ 1454437.54
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1480766.80, value end = $ 1496188.09
  Strategy "Robust Optimization Portfolio", value begin = $ 1395008.35, value end = $ 1364638.49

Period 12: start date 11/01/2021, end date 12/31/2021
  Strategy "Buy and Hold", value begin = $ 951350.41, value end = $ 932471.35
  Strategy "Equally Weighted Portfolio", value begin = $ 1584847.25, value end = $ 1646671.96
  Strategy "Mininum Variance Portfolio", value begin = $ 1054688.50, value end = $ 1048743.13
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 2113787.35, value end = $ 2217141.21
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 1464929.30, value end = $ 1517795.39
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 1506737.42, value end = $ 1560987.14
  Strategy "Robust Optimization Portfolio", value begin = $ 1358187.47, value end = $ 1388305.01
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Initial portfolio value = $ 385097.15

Period 1: start date 01/02/2008, end date 02/29/2008
  Strategy "Buy and Hold", value begin = $ 385097.15, value end = $ 325918.34
  Strategy "Equally Weighted Portfolio", value begin = $ 381649.89, value end = $ 326929.57
  Strategy "Mininum Variance Portfolio", value begin = $ 383262.49, value end = $ 327144.09
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 381265.54, value end = $ 332652.59
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 381848.69, value end = $ 329229.03
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 381237.14, value end = $ 323081.49
  Strategy "Robust Optimization Portfolio", value begin = $ 382119.84, value end = $ 324780.73

Period 2: start date 03/03/2008, end date 04/30/2008
  Strategy "Buy and Hold", value begin = $ 325807.08, value end = $ 349997.20
  Strategy "Equally Weighted Portfolio", value begin = $ 322097.69, value end = $ 354821.22
  Strategy "Mininum Variance Portfolio", value begin = $ 322718.41, value end = $ 365547.71
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 325785.77, value end = $ 344234.61
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 324432.42, value end = $ 361275.86
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 321901.77, value end = $ 358409.38
  Strategy "Robust Optimization Portfolio", value begin = $ 323209.59, value end = $ 347627.06

Period 3: start date 05/01/2008, end date 06/30/2008
  Strategy "Buy and Hold", value begin = $ 357929.49, value end = $ 322881.56
  Strategy "Equally Weighted Portfolio", value begin = $ 366424.15, value end = $ 308970.75
  Strategy "Mininum Variance Portfolio", value begin = $ 372947.72, value end = $ 351399.81
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 349025.01, value end = $ 312518.46
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 372052.61, value end = $ 322765.18
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 369101.36, value end = $ 320250.89
  Strategy "Robust Optimization Portfolio", value begin = $ 356160.96, value end = $ 327859.50

Period 4: start date 07/01/2008, end date 08/29/2008
  Strategy "Buy and Hold", value begin = $ 324349.75, value end = $ 326489.53
  Strategy "Equally Weighted Portfolio", value begin = $ 309425.79, value end = $ 315897.27
  Strategy "Mininum Variance Portfolio", value begin = $ 351751.30, value end = $ 356195.63
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 324814.45, value end = $ 314800.05
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 322402.83, value end = $ 326544.86
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 319883.45, value end = $ 323987.74
  Strategy "Robust Optimization Portfolio", value begin = $ 327599.29, value end = $ 331499.60

Period 5: start date 09/02/2008, end date 10/31/2008
  Strategy "Buy and Hold", value begin = $ 333252.73, value end = $ 274022.75
  Strategy "Equally Weighted Portfolio", value begin = $ 316675.00, value end = $ 231420.37
  Strategy "Mininum Variance Portfolio", value begin = $ 348289.18, value end = $ 269010.37
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 306067.46, value end = $ 229119.53
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 326306.02, value end = $ 242218.59
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 323747.05, value end = $ 240386.96
  Strategy "Robust Optimization Portfolio", value begin = $ 327455.78, value end = $ 255365.83

Period 6: start date 11/03/2008, end date 12/31/2008
  Strategy "Buy and Hold", value begin = $ 282342.11, value end = $ 305967.56
  Strategy "Equally Weighted Portfolio", value begin = $ 230011.81, value end = $ 198885.85
  Strategy "Mininum Variance Portfolio", value begin = $ 269394.11, value end = $ 248173.94
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 226378.28, value end = $ 175247.44
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 241380.96, value end = $ 212413.33
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 239554.33, value end = $ 210720.56
  Strategy "Robust Optimization Portfolio", value begin = $ 257377.27, value end = $ 247818.51
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Period 7: start date 01/02/2009, end date 02/27/2009
  Strategy "Buy and Hold", value begin = $ 313366.90, value end = $ 258275.19
  Strategy "Equally Weighted Portfolio", value begin = $ 207366.91, value end = $ 169935.27
  Strategy "Mininum Variance Portfolio", value begin = $ 256222.08, value end = $ 244193.85
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 174617.67, value end = $ 145423.65
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 221286.26, value end = $ 188950.02
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 219548.08, value end = $ 187438.28
  Strategy "Robust Optimization Portfolio", value begin = $ 252858.41, value end = $ 226597.29

Period 8: start date 03/02/2009, end date 04/30/2009
  Strategy "Buy and Hold", value begin = $ 248688.22, value end = $ 286368.72
  Strategy "Equally Weighted Portfolio", value begin = $ 161713.12, value end = $ 260066.85
  Strategy "Mininum Variance Portfolio", value begin = $ 234569.46, value end = $ 319274.90
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 138446.34, value end = $ 180431.92
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 181044.69, value end = $ 271585.64
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 179594.87, value end = $ 269287.37
  Strategy "Robust Optimization Portfolio", value begin = $ 218761.50, value end = $ 288718.53

Period 9: start date 05/01/2009, end date 06/30/2009
  Strategy "Buy and Hold", value begin = $ 287805.37, value end = $ 285824.08
  Strategy "Equally Weighted Portfolio", value begin = $ 259634.59, value end = $ 273277.43
  Strategy "Mininum Variance Portfolio", value begin = $ 316797.53, value end = $ 320211.72
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 179243.52, value end = $ 184965.16
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 271099.77, value end = $ 281611.72
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 268804.39, value end = $ 279254.33
  Strategy "Robust Optimization Portfolio", value begin = $ 287339.22, value end = $ 291357.63

Period 10: start date 07/01/2009, end date 08/31/2009
  Strategy "Buy and Hold", value begin = $ 286766.63, value end = $ 298338.27
  Strategy "Equally Weighted Portfolio", value begin = $ 272967.79, value end = $ 321758.02
  Strategy "Mininum Variance Portfolio", value begin = $ 319907.28, value end = $ 341210.60
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 180487.01, value end = $ 195121.88
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 281619.61, value end = $ 320401.32
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 279253.97, value end = $ 317690.97
  Strategy "Robust Optimization Portfolio", value begin = $ 291263.00, value end = $ 311543.34

Period 11: start date 09/01/2009, end date 10/30/2009
  Strategy "Buy and Hold", value begin = $ 291703.36, value end = $ 290193.57
  Strategy "Equally Weighted Portfolio", value begin = $ 310182.86, value end = $ 328338.22
  Strategy "Mininum Variance Portfolio", value begin = $ 333055.45, value end = $ 349798.81
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 186739.28, value end = $ 186512.54
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 310654.70, value end = $ 329477.87
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 308028.02, value end = $ 326736.52
  Strategy "Robust Optimization Portfolio", value begin = $ 304326.79, value end = $ 318255.94

Period 12: start date 11/02/2009, end date 12/31/2009
  Strategy "Buy and Hold", value begin = $ 288596.05, value end = $ 323101.02
  Strategy "Equally Weighted Portfolio", value begin = $ 329690.48, value end = $ 375806.51
  Strategy "Mininum Variance Portfolio", value begin = $ 347213.39, value end = $ 389876.96

  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 185539.21, value end = $ 209901.43
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 330130.57, value end = $ 369317.40
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 327377.44, value end = $ 366178.56
  Strategy "Robust Optimization Portfolio", value begin = $ 316474.35, value end = $ 353822.36
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Initial portfolio value = $ 890077.15


Period 1: start date 01/03/2022, end date 02/28/2022
  Strategy "Buy and Hold", value begin = $ 890077.15, value end = $ 924072.93
  Strategy "Equally Weighted Portfolio", value begin = $ 881997.57, value end = $ 802549.65
  Strategy "Mininum Variance Portfolio", value begin = $ 885873.66, value end = $ 863328.94
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 881222.74, value end = $ 800309.12
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 882721.54, value end = $ 817347.20
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 881302.55, value end = $ 800648.23
  Strategy "Robust Optimization Portfolio", value begin = $ 883675.91, value end = $ 828053.68

Period 2: start date 03/01/2022, end date 04/29/2022
  Strategy "Buy and Hold", value begin = $ 921940.14, value end = $ 807230.89
  Strategy "Equally Weighted Portfolio", value begin = $ 783067.12, value end = $ 705796.90
  Strategy "Mininum Variance Portfolio", value begin = $ 854828.72, value end = $ 782907.97
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 783852.04, value end = $ 694344.92
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 801448.97, value end = $ 738715.65
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 782765.85, value end = $ 721680.36
  Strategy "Robust Optimization Portfolio", value begin = $ 819303.00, value end = $ 758423.10

Period 3: start date 05/02/2022, end date 06/30/2022
  Strategy "Buy and Hold", value begin = $ 806237.92, value end = $ 877550.83
  Strategy "Equally Weighted Portfolio", value begin = $ 716066.23, value end = $ 654767.51
  Strategy "Mininum Variance Portfolio", value begin = $ 786244.72, value end = $ 820699.01
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 687969.73, value end = $ 735561.85
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 747435.69, value end = $ 710807.74
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 730156.19, value end = $ 694451.31
  Strategy "Robust Optimization Portfolio", value begin = $ 762682.75, value end = $ 763333.31

Period 4: start date 07/01/2022, end date 08/31/2022
  Strategy "Buy and Hold", value begin = $ 892738.72, value end = $ 742946.10
  Strategy "Equally Weighted Portfolio", value begin = $ 656646.38, value end = $ 679030.46
  Strategy "Mininum Variance Portfolio", value begin = $ 825927.32, value end = $ 712502.66
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 732602.04, value end = $ 613030.34
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 714920.77, value end = $ 704408.86
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 698469.83, value end = $ 688169.42
  Strategy "Robust Optimization Portfolio", value begin = $ 768423.48, value end = $ 687380.71

Period 5: start date 09/01/2022, end date 10/31/2022
  Strategy "Buy and Hold", value begin = $ 742641.68, value end = $ 682506.51
  Strategy "Equally Weighted Portfolio", value begin = $ 675547.95, value end = $ 646239.65
  Strategy "Mininum Variance Portfolio", value begin = $ 711049.16, value end = $ 700702.88
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 607830.60, value end = $ 538791.56
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 702763.30, value end = $ 684678.96
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 686553.32, value end = $ 668970.70
  Strategy "Robust Optimization Portfolio", value begin = $ 685726.30, value end = $ 683426.02

Period 6: start date 11/01/2022, end date 12/30/2022
  Strategy "Buy and Hold", value begin = $ 683477.34, value end = $ 716351.42
  Strategy "Equally Weighted Portfolio", value begin = $ 648031.33, value end = $ 644576.31
  Strategy "Mininum Variance Portfolio", value begin = $ 708668.50, value end = $ 730238.06
  Strategy "Maximum Sharpe Ratio Portfolio", value begin = $ 533837.24, value end = $ 543574.01
  Strategy "Equal Risk Contributions Portfolio", value begin = $ 687044.51, value end = $ 684724.08
  Strategy "Leveraged Equal Risk Contributions Portfolio", value begin = $ 671283.81, value end = $ 669037.10
  Strategy "Robust Optimization Portfolio", value begin = $ 687911.03, value end = $ 697257.84
```