

CS-UY 4563: Lecture 2

Simple Linear Regression

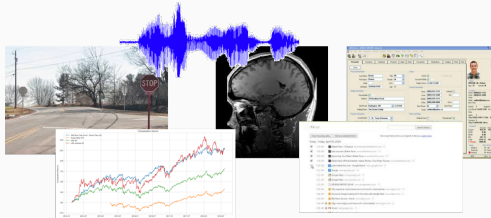
NYU Tandon School of Engineering, Prof. Christopher Musco

- Please enroll for Piazza. Only about 60% of class has.
- First lab assignment: `lab_housing_partial.ipynb`
 - Due **next Tuesday, 2/4 at 11:59pm.**
 - Go through the simple regression demonstration `demo_auto_mpg.ipynb`.
 - There will be a corresponding written homework released shortly.

BASIC GOAL

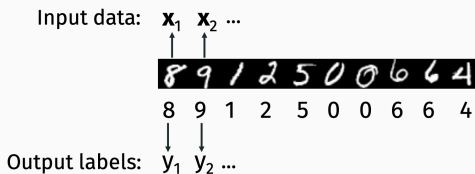
Goal: Develop algorithms to make decisions or predictions based on data.

- **Input:** A single piece of data (an image, audio file, patient healthcare record, MRI scan).



- **Output:** A prediction or decision (this image is a stop sign, this stock will go up 10% next quarter, turn the car right).

Step 1: Collect and label many input/output pairs (\mathbf{x}_i, y_i) . For our digit images, we have each $\mathbf{x}_i \in \mathbb{R}^{28 \times 28}$ and $y_i \in \{0, 1, \dots, 9\}$.



This is called the **training dataset**.

Step 2: Learn from the examples we have.

- Have the computer automatically find some function $f(\mathbf{x})$ such that $f(\mathbf{x}_i) = y_i$ for most (\mathbf{x}_i, y_i) in our training data set (by searching over many possible functions).

In **supervised learning** every input x_i in our training dataset comes with a desired output y_i (typically generated by a human, or some other process).

Types of supervised learning:

- **Classification** – predict a discrete class label.
- **Regression** – predict a continuous value.
 - Dependent variable, response variable, target variable, lots of different names for y_i .

Another example of supervised classification: **Face Detection**.



Each input data example x_i is an image. Each output y_i is 1 if the image contains a face, 0 otherwise.

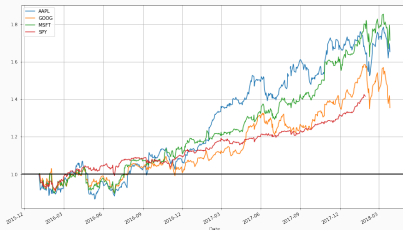
- Harder than digit recognition, but we now have very reliable methods (used in nearly all digital cameras, phones, etc.)

Other examples of supervised classification:

- Object detection (Input: image, Output: dog or cat)
- Spam detection (Input: email text, Output: spam or not)
- Medical diagnosis (Input: patient data, Output: disease condition or not)
- Credit decision making (Input: financial data, Output: offer loan or not)

SUPERVISED LEARNING

Example of supervised regression: **Stock Price Prediction.**



Each input x is a vector of metrics about a company (sales volume, PE ratio, earning reports, historical price data).

Each output y_i is the **price of the stock** 3 months in the future.

Other examples of supervised regression:

- Home price prediction (Inputs: square footage, zip code, number of bathrooms, Output: Price)
- Car price prediction (Inputs: make, model, year, miles driven, Output: Price)
- Weather prediction (Inputs: weather data at nearby stations, Output: tomorrows temperature)
- Robotics/Control (Inputs: information about environment and current position at time t , Output: estimate of position at time $t + 1$)

Later in the class we will talk about other models:

- **Unsupervised learning** (no labels or response variable)
 - Clustering
 - Representation Learning
- **Reinforcement learning**
 - Game playing

You might also hear about semi-supervised learning or active learning – these categories aren't always cut and dry.

In **supervised learnings** every input \mathbf{x}_i in our training dataset comes with a desired output y_i (typically generated by a human, or some other process).

Types of supervised learning:

- **Classification** – predict a discrete class label.
- **Regression** – predict a continuous value.
 - Dependent variable, response variable, target variable, lots of different names for y_i .

Motivating example: Predict the highway miles per gallon (MPG) of a car given quantitative information about its engine.
Demo in `demo_auto_mpg.ipynb`.

What factors might matter?

PREDICTING MPG

Data set available from the UCI Machine Learning Repository:
<https://archive.ics.uci.edu/>.



UCI
Machine Learning Repository
Center for Machine Learning and Intelligent Systems

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<p>Latest News:</p> <p>09-24-2018: Welcome to the new Repository admins Dheeru Dua and Elr Karna Taniseldoul 04-08-2013: Welcome to the new Repository admins Kevin Bache and Mohre Lohmeyer 03-01-2010: Note from donor regarding Netflix data 10-16-2009: Two new data sets have been added. 06-16-2009: Several data sets have been added. 03-24-2008: New data sets have been added! 06-25-2007: Two new data sets have been added: UJI Pen Characters, MAGIC Gamma Telescope</p> <p>Featured Data Set: Ozone Level Defection</p>  <p>Task: Classification Data Type: Multivariate, Sequential, Time-Series # Attributes: 73 # Instances: 2539</p> <p>Two ground ozone level data sets are included in this collection. One is the eight hour peak set (eighty data), the other is the one hour peak set (one-hr data). Those data were collected from 1988 to 2004 at the Houston, Galveston and Breaux area.</p>	<p>Newest Data Sets:</p> <p>10-06-2019:  WISDM Smartphone and Smartwatch Activity and Biometrics Dataset</p> <p>09-30-2019:  Hepatitis C Virus (HCV) for Egyptian patients</p> <p>09-23-2019:  QSAR fish toxicity</p> <p>09-23-2019:  QSAR aquatic toxicity</p> <p>09-21-2019:  Online Retail II</p> <p>09-20-2019:  Human Activity Recognition from Continuous Ambient Sensor Data</p> <p>09-20-2019:  Beijing Multi-Site Air-Quality Data</p> <p>09-20-2019:  MEX</p> <p>07-30-2019:  PPG-DaUJA</p> <p>07-24-2019:  Ovarian Predictors data set</p> <p>07-22-2019:  Alcohol QCM Sensor Dataset</p> <p>07-16-2019:  Incident management process enriched event log</p>	<p>Most Popular Data Sets (hits since 2007):</p> <p>3099401:  Iris</p> <p>1711996:  Adult</p> <p>1328924:  Wine</p> <p>1126437:  Heart Diseases</p> <p>1120386:  Wine Quality</p> <p>1116403:  Car Evaluation</p> <p>1110558:  Breast Cancer Wisconsin (Diagnostic)</p> <p>1101179:  Bank Marketing</p> <p>935256:  Human Activity Recognition Using Smartphones</p> <p>885144:  Abalone</p> <p>839187:  Forest Fires</p> <p>586581:  Poker Hand</p>
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This place is a great resource for projects!

PREDICTING MPG

Datasets from UCI (and many other places) comes as tab, space, or comma delimited files.

housing.data

auto-mpg.data

Users > christophermusco > Desktop > auto-mpg.data

1	18.0	8	307.0	130.0	3504.	12.0	70	1	"chevrolet chevelle malibu"
2	15.0	8	350.0	165.0	3693.	11.5	70	1	"buick skylark 320"
3	18.0	8	318.0	150.0	3436.	11.0	70	1	"plymouth satellite"
4	16.0	8	304.0	150.0	3433.	12.0	70	1	"amc rebel sst"
5	17.0	8	302.0	140.0	3449.	10.5	70	1	"ford torino"
6	15.0	8	429.0	198.0	4341.	10.0	70	1	"ford galaxie 500"
7	14.0	8	454.0	220.0	4354.	9.0	70	1	"chevrolet impala"
8	14.0	8	440.0	215.0	4312.	8.5	70	1	"plymouth fury iii"
9	14.0	8	455.0	225.0	4425.	10.0	70	1	"pontiac catalina"
10	15.0	8	390.0	190.0	3850.	8.5	70	1	"amc ambassador dpl"
11	15.0	8	383.0	170.0	3563.	10.0	70	1	"dodge challenger se"
12	14.0	8	340.0	160.0	3609.	8.0	70	1	"plymouth 'cuda 340"
13	15.0	8	400.0	150.0	3761.	9.5	70	1	"chevrolet monte carlo"
14	14.0	8	455.0	225.0	3086.	10.0	70	1	"buick estate wagon (sw)"
15	24.0	4	113.0	95.00	2372.	15.0	70	3	"toyota corona mark ii"
16	22.0	6	198.0	95.00	2833.	15.5	70	1	"plymouth duster"
17	18.0	6	199.0	97.00	2774.	15.5	70	1	"amc hornet"
18	21.0	6	200.0	85.00	2587.	16.0	70	1	"ford maverick"
19	27.0	4	97.00	88.00	2130.	14.5	70	3	"datsun pl510"
20	26.0	4	97.00	46.00	1835.	20.5	70	2	"volkswagen 1131 deluxe sedan"
21	25.0	4	110.0	87.00	2672.	17.5	70	2	"peugeot 504"
22	24.0	4	107.0	90.00	2430.	14.5	70	2	"audi 100 ls"
23	25.0	4	104.0	95.00	2375.	17.5	70	2	"saab 99e"
24	26.0	4	121.0	113.0	2234.	12.5	70	2	"bmw 2002"
25	21.0	6	199.0	90.00	2648.	15.0	70	1	"amc gremlin"
26	10.0	8	360.0	215.0	4615.	14.0	70	1	"ford f250"

PREDICTING MPG

Check dataset description to know what each column means.

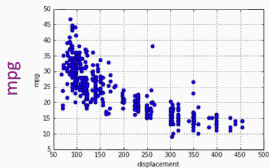
Users > christophermusco > Desktop > auto-mpg.data

y_1	1	18.0	8	307.0	130.0	3504.	12.0	70	1	"chevrolet chevelle malibu"	x_1
y_2	2	15.0	8	350.0	165.0	3693.	11.5	70	1	"buick skylark 320"	x_2
y_3	3	18.0	8	318.0	150.0	3436.	11.0	70	1	"plymouth satellite"	x_3
	4	16.0	8	304.0	150.0	3433.	12.0	70	1	"amc rebel sst"	
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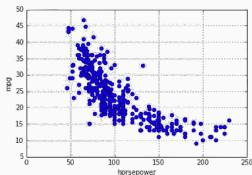
'mpg', 'cylinders', 'displacement', 'horsepower', 'weight',
'acceleration', 'model year', 'origin', 'car name'

LIBRARIES FOR INITIAL DATA READING

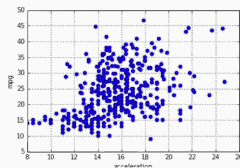
- Use **pandas** for reading data from delimited files. Stores data in a type of table called a “data frame” but this is just a wrapper around a **numpy** array.
- Use **matplotlib** for initial exploration.



Displacement



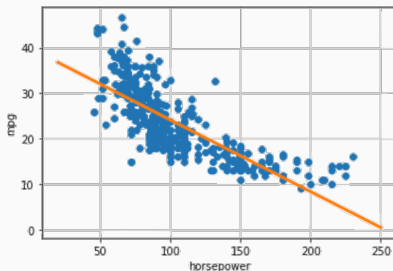
Horsepower



Acceleration

SIMPLE LINEAR REGRESSION

Linear regression from a Machine Learning (not a Statistics) perspective. Our first supervised machine learning model.

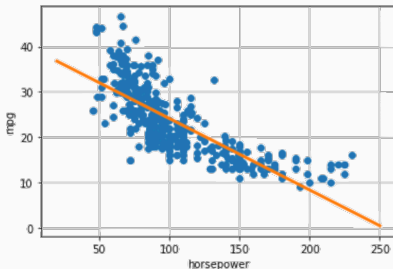


Only focus on one predictive variable at a time (e.g. horsepower). This is why it's called simple linear regression.

SIMPLE LINEAR REGRESSION

Dataset:

- $x_1, \dots, x_n \in \mathbb{R}$ (horsepowers of n cars – this is the predictor/independent variable)
- $y_1, \dots, y_n \in \mathbb{R}$ (MPG – this is the response/dependent variable)



SUPERVISED LEARNING DEFINITIONS

- **Model** $f_{\theta}(x)$: Class of equations or programs which map input x to predicted output. We want $f_{\theta}(x_i) \approx y_i$ for training inputs.
- **Model Parameters** θ : Vector of numbers. These are numerical nobs which parameterize our class of models.
- **Loss Function** $L(\theta)$: Measure of how well a model fits our data. Typically some function of $f_{\theta}(x_1) - y_1, \dots, f_{\theta}(x_n) - y_n$

Goal: Choose parameters θ^* which minimize the Loss Function:

$$\theta^* = \arg \min_{\theta} L(\theta)$$

General Supervised Learning

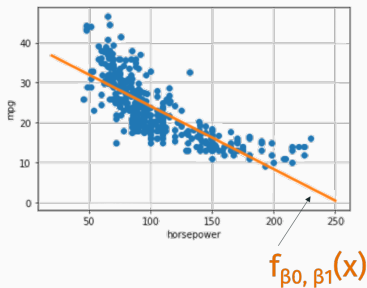
- Model: $f_{\theta}(x)$
- Model Parameters: θ
- Loss Function: $L(\theta)$

Linear Regression

- Model:
- Model Parameters:
- Loss Function:

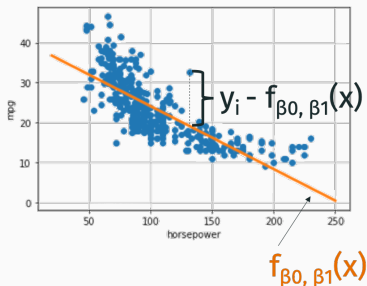
HOW TO MEASURE GOODNESS OF FIT

What is a natural **loss function** for linear regression?



HOW TO MEASURE GOODNESS OF FIT

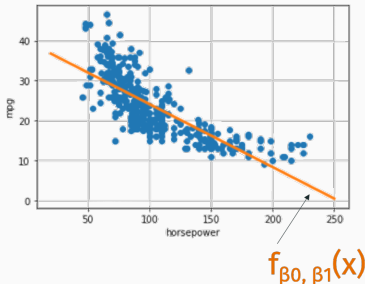
Typical choices are a function of $y_1 - f_{\beta_0, \beta_1}(x_1), \dots, y_n - f_{\beta_0, \beta_1}(x_n)$



- ℓ_2 /Squared Loss: $L(\beta_0, \beta_1) = \sum_{i=1}^n [y_i - f_{\beta_0, \beta_1}(x_i)]^2$.
- ℓ_1 /Least absolute deviations: $L(\beta_0, \beta_1) = \sum_{i=1}^n |y_i - f_{\beta_0, \beta_1}(x_i)|$.
- ℓ_∞ Loss $L(\beta_0, \beta_1) = \max_{i \in 1, \dots, n} |y_i - f_{\beta_0, \beta_1}(x_i)|$.

HOW TO MEASURE GOODNESS OF FIT

We're going to start with the Squared Loss/Sum-of-Squares Loss. Also called "Residual Sum-of-Squares (RSS)"



- Relatively robust to outliers.
- Simple to define, leads to simple algorithms for finding β_0, β_1
- Justifications from classical statistics related to assumptions about Gaussian noise. Will discuss later in the course.

General Supervised Learning

- Model: $f_{\theta}(x)$
- Model Parameters: θ
- Loss Function: $L(\theta)$

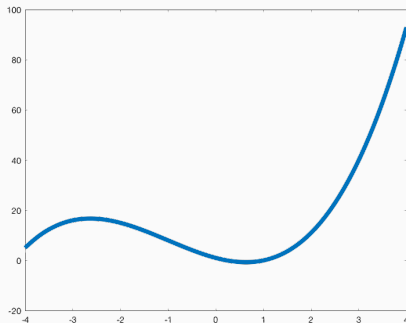
Linear Regression

- Model:
 $f_{\beta_0, \beta_1}(x) = \beta_0 + \beta_1 \cdot x$
- Model Parameters: β_0, β_1
- Loss Function: $L(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - f_{\beta_0, \beta_1}(x_i))^2$

Goal: Choose β_0, β_1 to minimize
 $L(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$.

This is the entire job of any **Supervised Learning Algorithm**.

Univariate function:



$$x^3 + 3 \cdot x^2 - 5 \cdot x + 1$$

- Find all places where derivative $f'(x) = 0$ and check which has the smallest value.

Multivariate function: $L(\beta_0, \beta_1)$

- Find values of β_0, β_1 where all partial derivatives equal 0.
- $\frac{\partial L}{\partial \beta_0} = 0$ and $\frac{\partial L}{\partial \beta_1} = 0$.

MINIMIZING SQUARED LOSS FOR REGRESSION

Multivariate function: $L(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$

- Find values of β_0, β_1 where all partial derivatives equal 0.
- $\frac{\partial L}{\partial \beta_0} = 0$ and $\frac{\partial L}{\partial \beta_1} = 0$.

Some definitions:

- Let $\bar{y} = \frac{1}{n} \sum_{i=1}^n y_i$. \bar{y} is the mean of y .
- Let $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$. \bar{x} is the mean of x .
- Let $\sigma_y^2 = \frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2$. σ_y^2 is the variance of y .
- Let $\sigma_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$. σ_x^2 is the variance of x .
- Let $\sigma_{xy} = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})$. σ_{xy} is the covariance.

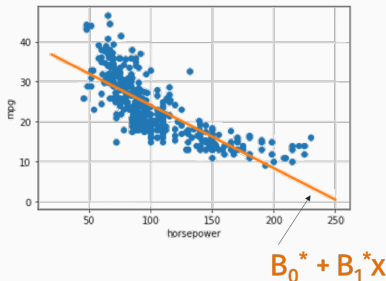
Claim: $L(\beta_0, \beta_1)$ is minimized when:

- $\beta_1 = \sigma_{xy} / \sigma_x^2$
- $\beta_0 = \bar{y} - \beta_1 \bar{x}$

MINIMIZING SQUARED LOSS FOR REGRESSION

Takeaways:

- Minimizing functions is often easy with calculus.
- Tools we will see again: **linearity of derivatives, chain rule.**
- Simple closed form formula for optimal parameters β_0^* and β_1^* for squared-loss!



s Let $L(\beta_0, \beta_1) = \sum_{i=1}^n (y_i - \beta_0 - \beta_1 x_i)^2$.

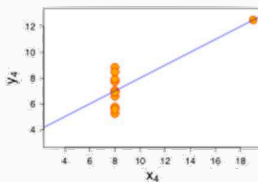
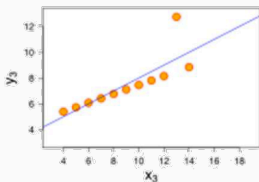
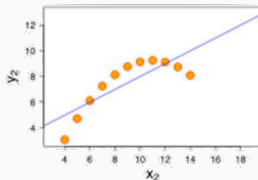
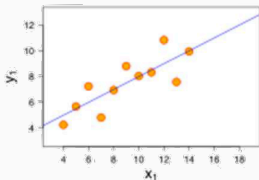
$$R^2 = 1 - \frac{L(\beta_0, \beta_1)}{n\sigma_y^2}$$

is exactly the R^2 value you may remember from statistics.

The smaller the loss, the closer R^2 is to 1, which means we have a better regression fit.

A FEW COMMENTS

Many reasons you might get a poor regression fit:

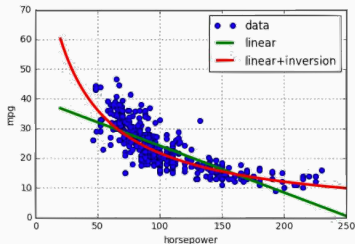
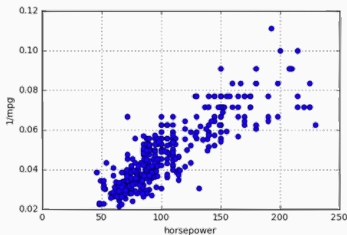


A FEW COMMENTS

Some of these are fixable!

- Remove outliers, use more robust loss function.
- **Non-linear model transformation.**

Fit the model $\frac{1}{\text{mpg}} \approx \beta_0 + \beta_1 \cdot \text{horsepower}$.



Much better fit, same exact learning algorithm!