

## Linear Mixed Models Study Guide

### Fixed vs. Random Effects

- Fixed effects are things we want to estimate.
- Random effects can be variables we are not necessarily interested in, but are measured and their variation needs to be accounted.
- They need to be included when an experimental design includes non-independence to avoid pseudo replication.

### Linear Mixed Model Expression

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{b} + \boldsymbol{\epsilon}$$

$$\mathbf{b} \sim N(0, \psi_{\theta}), \quad \boldsymbol{\epsilon} \sim N(0, \Lambda_{\theta})$$

- $\mathbf{y}$  is vector of outcome/response
- $\mathbf{X}$  is model matrices for fixed effects
- $\mathbf{Z}$  is model matrices for random effects
- $\boldsymbol{\beta}$  is the effect of covariates on mean of outcome
- $\mathbf{b}$  is the random effects
- $\boldsymbol{\epsilon}$  is the residual errors

### Linear Mixed Model Assumptions

1. There is a continuous response variable.
2. The structure of dependency of this model is correct (i.e. choices about random and fixed variables).
3. Observations within each unit may not be independent, but units are independent.
4. The random effects follows normal distributions and have constant variance.
5. The within-unit residual errors follows normal distributions and have constant variance.

\*Linear mixed models are robust to violations of some of the assumptions.

### Correlated Errors

- Random intercepts
  - A subject is above average in one observation, then it is likely to be above average in other observations.
  - R: (using lme4) `lmer(y~(1|a))` random intercept of a
- Random slopes
  - The difference between two subjects would get bigger as an other variable increases.
- Random slopes and intercepts
  - R: (using lme4) `lmer(y~(1 + a|b))` random slope of a and random intercept of b