



Adaptive vibration reshaping based milling chatter suppression

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ABSTRACT

As a major obstacle to high performance milling, chatter will inevitably decrease tool life, material removal efficiency and workpiece surface quality. Active control based on actuators for chatter suppression has been developed for a long time. However, in traditional active control, the feedback signals are usually time domain vibration signals, which will mitigate both chatter frequencies and normal frequencies, including rotation frequency and its frequency multiplications. In fact, rotation frequency and its frequency multiplications belong to normal cutting phenomenon, which don't need to be suppressed. Therefore, in traditional control strategies, more energy is wasted due to normal frequencies, which cannot obtain the optimal performance and causes the saturation effect of actuators. In order to solve this problem, this paper realized milling chatter suppression based on the adaptive vibration reshaping, which can precisely modify and control the milling vibration frequencies in frequency domain. Hence, the required control forces provided by actuators will decrease a lot, which can optimize the actuator performance and alleviate the saturation effect. Simulation results show that the adaptive vibration reshaping can suppress chatter frequencies effectively without changing rotation frequency and its frequency multiplications, which satisfies the initial control requirement for optimization of actuator performance. Besides, the convergence analysis, the noise resistance performance analysis, the control delay analysis and the chatter frequencies identification errors analysis are also presented for better facilitation to milling chatter suppression. Finally, contrastive milling tests are implemented on a three-axis milling machine. Experimental results show the designed control algorithm can decrease chatter frequencies, while having a little influence on normal frequencies.

1. Introduction

In milling process, the frequent occurrence of chatter will decrease productivity, tool life and surface quality of workpiece [1]. More and more scholars are attracted in chatter avoidance or suppression. To address this problem, three categories of methods have been put forward: parameters selection based on stability analysis, passive control and active control [2].

Parameters selection is based on stability lobe diagram (SLD), which can be obtained by semi-discretization method [3], full-discretization method [4] and so on. A large number of researchers are devoted to how to obtain the SLD with higher accuracy [5–7] and efficiency [3,4,8–10]. For example, Totis et al. [11] revealed the “symmetry breaking mechanism” of milling dynamics based on runout, variable pitch, forced vibration and regenerative effect. However, the accurate SLD can just be applied to parameters selection, but cannot enlarge the stability region and improve material removal efficiency.

The passive control strategies focus on machine tool structure

design or additional devices for chatter mitigation, such as variable pitch [12,13] or variable helix [14,15], passive vibration absorbers [16,17] and mechanical dampers [18]. However, due to the time varying property and uncertainty of cutting process, the passive control cannot show good performance without extra energy input.

Active control is an effective method for chatter suppression, which can satisfy the real-time requirements of the online control system and realize high performance and high efficiency cutting. Different actuators can be integrated into spindle, such as electrostrictive actuator [19], active magnetic bearing [20], piezoelectric stack [21–24]. With extra energy input, the active control system is able to change the dynamics property of machine tool and afford additional forces for chatter suppression, so that the stable cutting region can be expanded. However, in traditional active control, time domain signals are usually used as the feedback signals, which will mitigate both chatter frequencies and normal frequencies as seen in Fig. 9. (b) in Ref. [25], Fig. 15 in Ref. [26], Fig. 10 in Ref. [27] and Fig. 38. (b) in Ref. [28]. In fact, rotation frequency and its frequency multiplications belong to normal cutting

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phenomenon, which don't need to be suppressed. Therefore, actuators in traditional control strategies have to waste much energy for rotation frequency and its frequency multiplications, which cannot obtain the optimal performance and cause the saturation effect of actuators.

In order to solve this problem, this paper realized milling chatter suppression based on the adaptive vibration reshaping, which can precisely modify and control the milling vibration frequencies in frequency domain. Hence, the required control forces provided by actuators will decrease a lot, which can optimize the actuator performance and help to avoid the saturation effect. Kuo et al. [29] proposed the active noise equalizer (ANE) for the first time, which can be applied to narrowband feedforward adaptive control. Later, Liu et al. [30] adopted this algorithm to multi-harmonic amplitude and phase for active noise and vibration reshaping. Considering reference frequency mismatch, Liu et al. [31] proposed a novel narrowband active noise and vibration control algorithm without orthogonal pair-wise reference frequency regulator to compensate the reference frequency mismatch problem. However, these researches didn't consider the influence of delay item and focused on the ordinary differential equation rather than the delay differential equation that can describe milling process. Based on the ANE, this paper will study the line spectrum milling chatter suppression considering delay effect.

The remainder of this paper is organized as follows. In Section 2, the milling dynamic model and time domain simulation are described, which afford the foundation models and simulation technique. Then, milling chatter suppression based on adaptive vibration reshaping is introduced in Section 3. In Section 4, the simulation results are presented. In Section 5, the developed algorithm is verified by contrastive milling tests with and without control. In Section 6, conclusions with a brief discussion on future works are presented.

2. Milling dynamic model and time domain simulation

The milling process can be simplified as the spring-mass-damping vibration model with two degrees of freedom considering flexible tool and rigid workpiece. Without considering the coupling effect in two orthogonal directions, the governing equation can be given by Ref. [32].

$$\begin{bmatrix} m_x & 0 \\ 0 & m_y \end{bmatrix} \begin{bmatrix} \ddot{x}(t) \\ \ddot{y}(t) \end{bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} F_x(t) \\ F_y(t) \end{bmatrix} \quad (1)$$

where m_x , m_y , c_x , c_y , k_x and k_y are the modal mass, modal damping and modal stiffness in the flexible directions, respectively; $x(t)$ and $y(t)$ are the displacements in the x and y directions; $F_x(t)$ and $F_y(t)$ are the milling forces in the two directions.

Time domain simulation is usually utilized to investigate the dynamic behavior under a specific cutting condition. The milling dynamic equation Eq. (1) can be solved numerically with Euler approximation [33].

$$\begin{aligned} \ddot{x}(t_i) &= \frac{F_x(t_i) - c_x \dot{x}(t_i) - k_x x(t_i)}{m_x}, & \ddot{y}(t_i) &= \frac{F_y(t_i) - c_y \dot{y}(t_i) - k_y y(t_i)}{m_y} \end{aligned} \quad (2)$$

$$\dot{x}(t_i) = \dot{x}(t_{i-1}) + \ddot{x}(t_i)dt, \quad \dot{y}(t_i) = \dot{y}(t_{i-1}) + \ddot{y}(t_i)dt \quad (3)$$

$$x(t_i) = x(t_{i-1}) + \dot{x}(t_i)dt, \quad y(t_i) = y(t_{i-1}) + \dot{y}(t_i)dt \quad (4)$$

where i is an integer number starting from one and dt is the time step.

For the milling dynamic equation Eq. (1), the semi-discretization method is adopted for stability analysis. The simulation parameters are as listed in Table 1. The stability lobe diagram (SLD) under this condition is presented in Fig. 1. From the SLD, 17 groups of cutting parameters are selected for time domain simulation in this section. The red circle indicates chatter while the red asterisk represents stable milling.

Table 1

The time domain simulation parameters.

Simulation parameters	value
Tool teeth number	4
Tangential cutting force coefficient (N/m ²)	5.36e8
Normal cutting force coefficient (N/m ²)	1.87e8
Feed per tooth (m/tooth)	0.001
Tool diameter (m)	0.01275
Helix angle (°)	30
Modal mass in the x direction (kg)	0.192
Modal mass in the y direction (kg)	0.192
Modal damping in the x direction	25.17
Modal damping in the y direction	25.17
Modal stiffness in the x direction (N/m)	1.34e6
Modal stiffness in the y direction (N/m)	1.34e6
Milling type	up milling
Radial milling depth (m)	0.006375

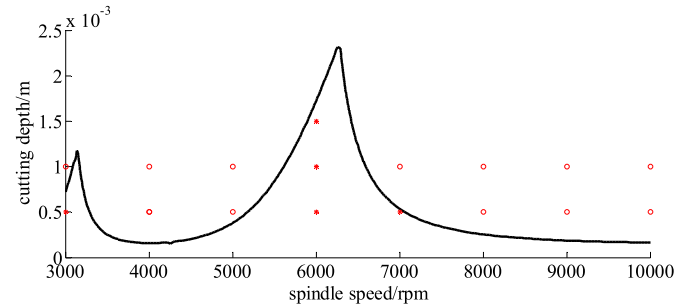


Fig. 1. The SLD in milling process.

The time domain simulation results are in good agreement with the SLD and verify the correctness of time domain simulation technique.

3. Milling chatter suppression based on adaptive vibration reshaping

As one of the most popular algorithms for active vibration reshaping, active noise equalizer (ANE) is adopted in this section for chatter line spectrum control. Similar with FXLMS algorithm, the structure of ANE are presented in Fig. 2. The reference signal is the milling force, which can be transformed into primary noise $d(n)$ through primary path $P(n)$. Then, the sinusoids generator can identify the reference signal frequency ω_p and generate a pair of orthogonal signals as follows

$$x_a(n) = \cos(\omega_p n), \quad x_b(n) = \sin(\omega_p n) \quad (5)$$

The controller output is the linear combination of these two signals

$$y(n) = w_a(n)x_a(n) + w_b(n)x_b(n) \quad (6)$$

where $w_a(n)$ and $w_b(n)$ are the coefficients of controller. The output of controller contains two branches: a canceling branch and a balancing branch, into whom the gains $1 - \beta$ and β are inserted. Therefore, the output of these two branches can be written as

$$y_c(n) = (1 - \beta)y(n), \quad y_b(n) = \beta y(n) \quad (7)$$

The tooltip displacement $e(n)$ is the output of ANE, which can be expressed by

$$e(n) = d(n) - y_c(n) * s(n) \quad (8)$$

where $s(n)$ is the impulse response function of secondary path; $*$ stands for convolution operation. In order to control signals frequency accurately, the pseudo error signal $e_s(n)$ is fed back to the adaptive system, which can be defined as

$$e_s(n) = e(n) - y_b(n) * s(n) \quad (9)$$

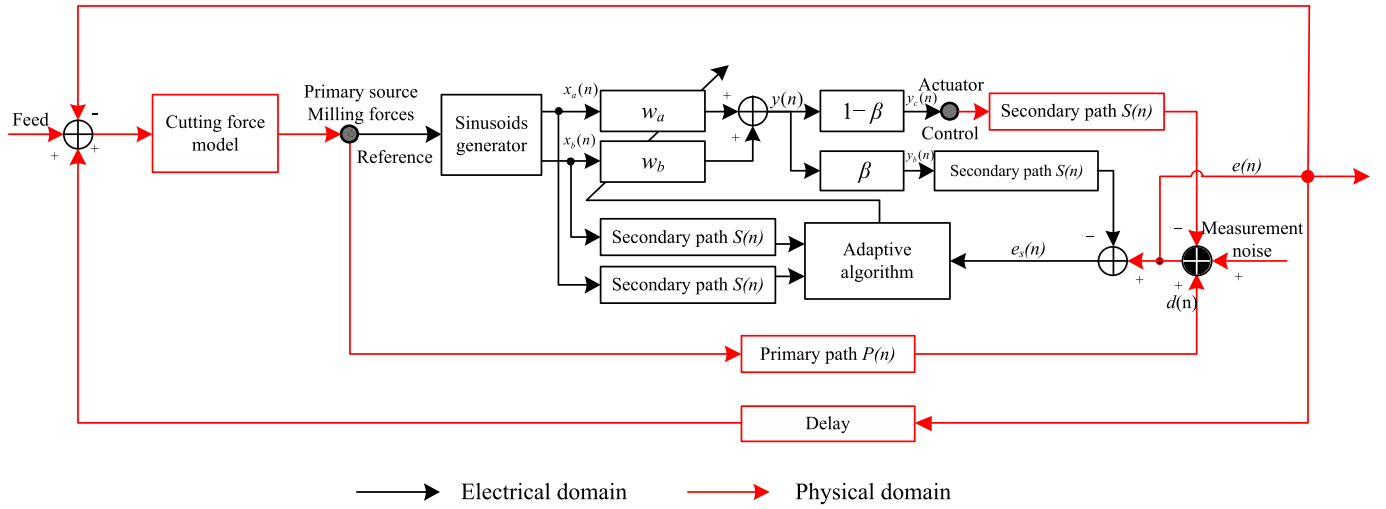


Fig. 2. The block diagram of milling chatter suppression.

According to the stochastic gradient, the renewal equation of controller can be written as

$$w_l(n+1) = w_l(n) + \mu_l e_s(n) x'_l(n), \quad (l = a, b) \quad (10)$$

where $x'_l(n) = x_l(n) * s(n)$, ($l = a, b$) is the filter reference signal; μ_l is the convergence factor. The larger μ_l is, the faster convergence is. However, when μ_l is more than a limited value, the iteration will be divergent. According to Ref. [29], convergence (stability) occurs provided that

$$0 < \mu_l < \frac{2 \cos(\theta - \hat{\theta})}{|S \hat{S}^*|} \quad (11)$$

where θ and $\hat{\theta}$ are the phase of the secondary path and its estimate; S and \hat{S}^* are the discrete Fourier transform component of the impulse response of the secondary path and the complex conjugate of its estimate.

When the adaptive algorithm is convergent, the pseudo error signal $e_s(n)$ will be close to zero. Based on Eqs. (7)–(9), it can be derived that the output of ANE $e(n)$ can be expressed as

$$e(n) = \beta d(n) \quad (12)$$

Based on the above analysis, $e(n)$ and $d(n)$ are the system output with and without ANE control. Therefore, the ANE control can reshape the vibration signals through Eq. (12). With different gains β , the corresponding frequency component can be cancelled ($\beta = 0$), attenuated ($0 < \beta < 1$), holden ($\beta = 1$) and enhanced ($\beta > 1$). For milling chatter suppression, the ultimate aim is to avoid chatter; hence, $\beta = 0$ is required. However, if the selected actuators cannot afford adequate control forces due to saturation effect, $0 < \beta < 1$ is needed for chatter suppression rather avoidance according to the actuator performance.

In milling process, the primary path is the tooltip impulse response function. The secondary path is the impulse response function between actuators and tooltip. The primary source is the milling force. The milling force can produce primary vibration at tooltip through primary path. The control forces from actuators can produce secondary vibration at tooltip through secondary path for chatter control. The total tool vibration is the sum of the above two vibrations.

From the above analysis, it can be known that the adaptive vibration reshaping control can precisely modify and control the vibration frequencies in frequency domain. Hence, the required control forces from actuators will be decreased a lot, which can optimize the actuator performance and help to avoid the saturation effect. It's worth noting that the delay effect is taken into account for line spectrum milling chatter suppression. For better understanding, some explanations about Fig. 2 are presented as follows.

- 1) The sinusoids generator can be realized in two ways. One is to predict the chatter frequencies [34]. However, it requires identification of modal parameters and cutting forces coefficients. Besides, there are many identified chatter frequencies, which will increase the calculation burden and decrease the controller performance. Therefore, this method isn't suitable to actual milling chatter suppression. The another way to identify chatter frequencies is the on-line Fourier transform in milling process. The detailed implementation steps are as follows. (1) The measured signals in time domain can be transformed into frequency domain using Fourier transform. (2) The main vibration frequencies can be found with extremum-search method in frequency domain. (3) Normal frequencies are the integral multiple of rotation frequency while chatter frequencies are not, hence divide the identified frequencies by rotation frequency. (4) If the results are not integers, the identified frequencies are chatter frequencies. Otherwise, they are normal frequencies. (5) Delete normal frequencies from the main vibration frequencies, then chatter frequencies can be obtained.
- 2) The transfer function $P(n)$ is the tooltip impulse response function.
- 3) The transfer function $S(n)$ is the impulse response function between actuators and tooltip.
- 4) The red lines and blocks are in physical domain, which are realized by hardware in control process.
- 5) The black lines and blocks are in electrical domain, which are realized by programs.
- 6) \oplus is the adder in physical domain; \oplus is the logical adder in electrical domain.
- 7) In simulation, cutting forces can be calculated from the feed effect and the regenerative effect. The obtained cutting forces play two roles: (1) Calculating the tooltip vibration by the primary path $P(n)$. (2) Analyzing the cutting force signals to obtain chatter frequencies as the reference frequencies for the subsequent ANE control.
- 8) In experiment, cutting forces are just needed to identify chatter frequencies. However, due to the expensive price and inconvenient installation of the dynamometer, the displacement signals are adopted for chatter frequencies identification.

4. Simulation analysis

The milling parameters are presented in Table 1 with spindle speed 3000 rpm and axial cutting depth 0.01 m. The controller parameters in control process are as follows. The convergence factors in the x and y directions are 0.0015 and 0.0010, respectively. The gains of transfer function $S(n)$ are set up to be a constant for convenience.

In order to show that the developed algorithm requires less actuator

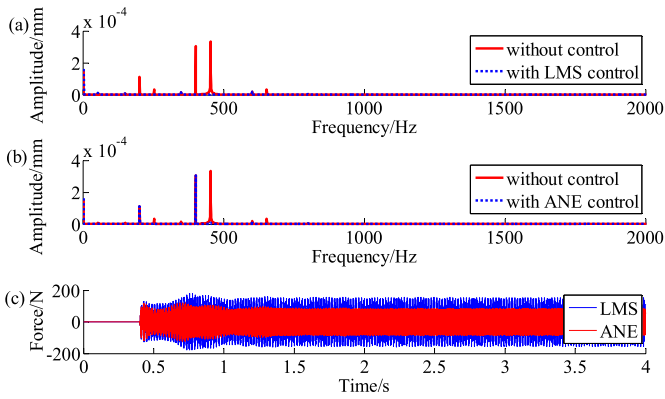


Fig. 3. (a) Tooltip displacement signals in the y direction with LMS control (b) Tooltip displacement signals in the y direction with ANE control (c) The control forces in the y direction with LMS and ANE control.

forces, the time-domain least mean square (LMS) adaptive algorithm [26] is adopted for comparison, which can suppress both chatter frequencies and normal frequencies. The LMS algorithm and the ANE are adopted for chatter suppression, respectively. The simulation results are presented in Fig. 3.

Fig. 3(b) shows that for unstable milling process, normal frequencies keep unchanged but chatter frequencies are suppressed successfully, which indicates the ANE is able to mitigate chatter frequencies without influence on rotation frequency and its harmonics. The simulation results meet the initial control requirement and validate the effectiveness of the adopted algorithm. Fig. 3(a) indicates that the LMS algorithm mitigates both chatter frequencies and normal frequencies. The root mean square values of required control forces in the y direction are 68.5144 N for LMS and 53.4765 N for ANE, respectively. Fig. 3(c) shows that the ANE requires less control forces compared with the LMS algorithm in time domain.

In addition, the convergence analysis, the noise resistance performance analysis, the control delay analysis and the chatter frequencies identification errors analysis are also implemented. The analysis results are presented as follows. (1) The larger μ is, the faster convergence is, the smaller the oscillation is and the better the control effect is. However, when μ is more than a limited value, the ANE algorithm doesn't work. (2) With the signal-to-noise ratio increasing, the control effect gets better and the convergence rate becomes faster. Hence, in actual control process, the measured feedback signals can be considered to go through noise elimination processing for better controller performance. (3) With the control delay increasing, the controller performance decreases. Thus, the control delay is supposed to be as small as possible. (4) With the chatter frequencies identification errors increasing, the controller performance decreases until fail. Therefore, chatter frequencies need to be identified accurately and the narrow-band [35] active noise control can be considered to deal with this problem.

5. Experimental verification

5.1. Introduction of experimental equipment

In order to validate the effectiveness of the proposed method, the active control system for milling chatter suppression is established, including two capacitive displacement sensors (CPL290 with sensitivity 0.4 V/ μ m), a data acquisition system (LMS SCADASIII), a NI controller (NI PXI-4481 with two input channels and two output channels), a power amplifier (Pst E01 B4), two piezoelectric stack actuators (Pst 80VS15 with length 82 mm, resonant frequency 12 Hz, maximum displacement 95 μ m and maximum output force 2300 N) and the three-axis milling machine (VMC-V5). The designed ANE algorithm is programmed by Labview and downloaded to the NI controller. During the control process, the vibration displacement signals are collected by the displacement sensors and be transmitted to the controller. The programmed algorithm can output the control voltage based on the displacement signals to the piezoelectric stack actuators through the power amplifier. With the control voltage, the piezoelectric stack actuators can exert the control forces on the milling system for chatter control. In practical control process, the control forces act on the designed toolholder with a rolling bearing directly. The detailed description about the designed toolholder can refer to Ref. [26]. The set-up for milling tests is presented in Fig. 4. The workpiece material Aluminum alloy 6061 with density 2960 kg/m³, elastic modulus 68.9 GPa and Poisson's ratio 0.33. The adopted milling cutter has the following parameters: teeth number 1, helix angle 0°, diameter 12 mm and overhang 75 mm.

5.2. Swept sine experiment for transfer function $S(n)$ identification

According to Fig. 2, the transfer function $S(n)$ needs to be identified. $S(n)$ is the impulse response function between actuator voltage and tooltip displacement, which cannot be obtained by the impact test. Hence, the swept sine experiment is adopted. The signal generator (Tektronix AFG3022C) is used to generate the sinusoidal sweep signals with the following parameters: the start frequency 0 Hz, the stop frequency 2000 Hz, the sweep time 200 s, the hold time 0 s, the return time 200 s. The 'spectral tests' module of LMS data acquisition software (with the sample frequency 4096 Hz) is used to record the voltage and displacement signals, and to calculate the transfer function in frequency domain through 'Hv' estimation. Then, the impulse response function in time domain can be obtained with the inverse Fourier transform.

5.3. Milling tests for validation of the proposed method

In order to validate the effectiveness of the proposed control algorithm, milling tests are implemented on the three-axis milling machine with control on and control off. The milling parameters are as follows. The spindle speed is 5000 rpm; the feed per tooth is 0.1 mm/tooth; up-milling. The convergence factor is set up to be 0.001 and the gain is set



Fig. 4. The set-up for milling tests.

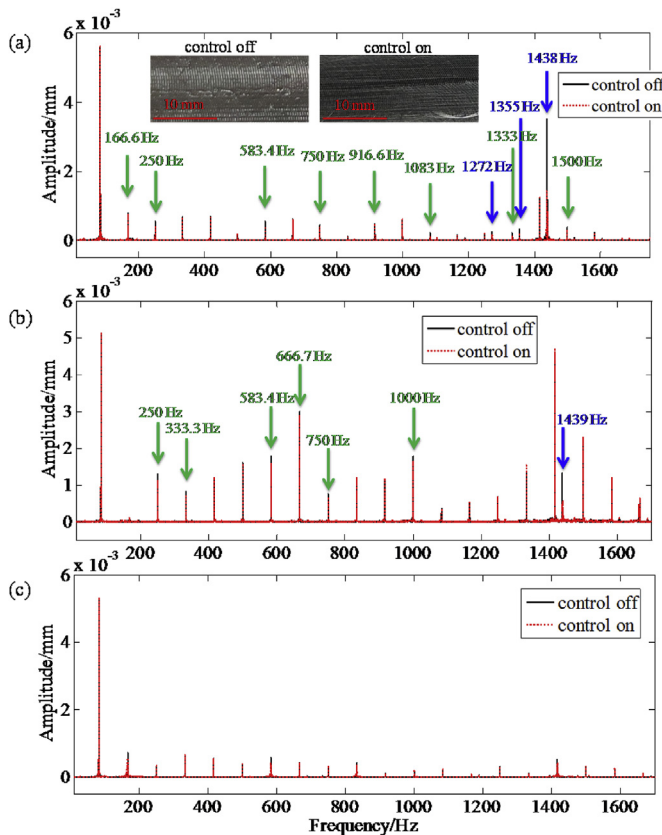


Fig. 5. The measured displacement signals of the x direction in frequency domain (a) axial cutting depth 0.3 mm, radial cutting depth 6 mm (b) axial cutting depth 1 mm, radial cutting depth 1 mm (c) axial cutting depth 0.2 mm, radial cutting depth 4 mm.

up to be 0. Different combinations of axial cutting depth and radial cutting depth (0.3 mm and 6 mm, 1 mm and 1 mm, 0.2 mm and 4 mm) are adopted. The measured displacement signals of the x direction in frequency domain are shown in Fig. 5.

The measured displacement signals under axial cutting depth 0.3 mm and radial cutting depth 6 mm are presented in Fig. 5(a). The blue arrows are the suppressed chatter frequencies, which show the proposed algorithm succeeded in chatter suppression. The gain is 0, however, the amplitude at dominant chatter frequency 1438 Hz decreases to about 40%. The error analysis is presented as follows: (1) The transfer function between actuators and tooltip don't be identified accurately. (2) The uncertainty exists in the milling process. (3) There maybe exist some measurement error. (4) The control delay maybe decrease the controller performance. The green arrows indicate the suppressed normal frequencies (rotation frequency and its harmonics). That may be because the uncertainty exists in the milling process or the normal frequencies are wrongly identified as chatter frequencies. The other frequencies are the unchanged frequencies, such as 83.33 Hz, 333.3 Hz, 500.0 Hz, etc. Due to large amplitudes, they are the main vibration in milling process, which show that the control algorithm has no influence on these normal frequencies. In addition, the photos of finished part are also provided, which shows that the surface with ANE control has smaller roughness and better machining quality.

Fig. 5(b) under axial cutting depth 1 mm and radial cutting depth 1 mm is similar as Fig 5(a), which displays that the chatter frequency (1439 Hz) is mitigated successfully while the change of normal frequencies is very small. Besides, the stable milling displacement signals under axial cutting depth 0.2 mm and radial cutting depth 4 mm are also presented in Fig. 5(c), which shows that there is no chatter frequency and the milling process is stable. Experimental results indicate

that the designed control algorithm doesn't work in stable milling, which won't waste control forces on normal frequencies.

6. Conclusions and future works

In order to optimize the actuator performance and avoid the saturation effect, this paper realized milling chatter suppression based on the adaptive vibration reshaping, which can precisely modify and control the milling vibration frequencies in frequency domain. A large number of numerical simulations are implemented for different cutting parameters. Simulation results show that the adaptive vibration reshaping can suppress chatter frequencies effectively and don't change rotation frequency and its frequency multiplications, which satisfy the initial control requirement for optimization of actuator performance. Besides, the convergence analysis, the noise resistance performance analysis, the control delay analysis and the chatter frequencies identification errors analysis are also implemented for better application to milling chatter suppression.

The adaptive vibration reshaping control can precisely adjust the milling vibration frequencies. Therefore, in the future, the relationship between milling surface roughness and vibration can be studied with this control method. For example, we can erase one frequency or combination of some vibration frequencies to explore the effect of these frequency components on milling surface topography. In addition, the method can also be applied to some other research fields about vibration frequency in milling, even turning, drilling, grinding, etc.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.ijmachtools.2019.04.001>.

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