$\mathbb{R}^3$ and Planes in spaces

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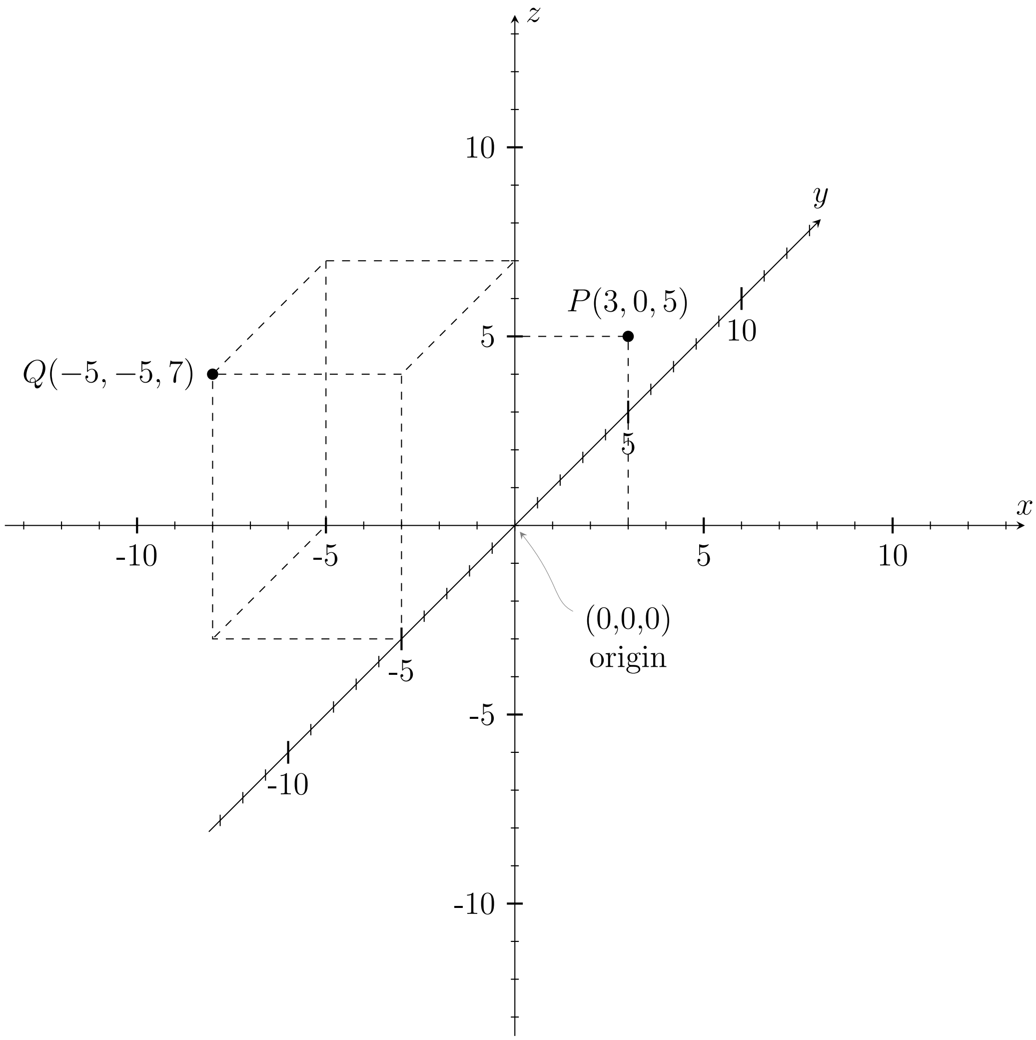
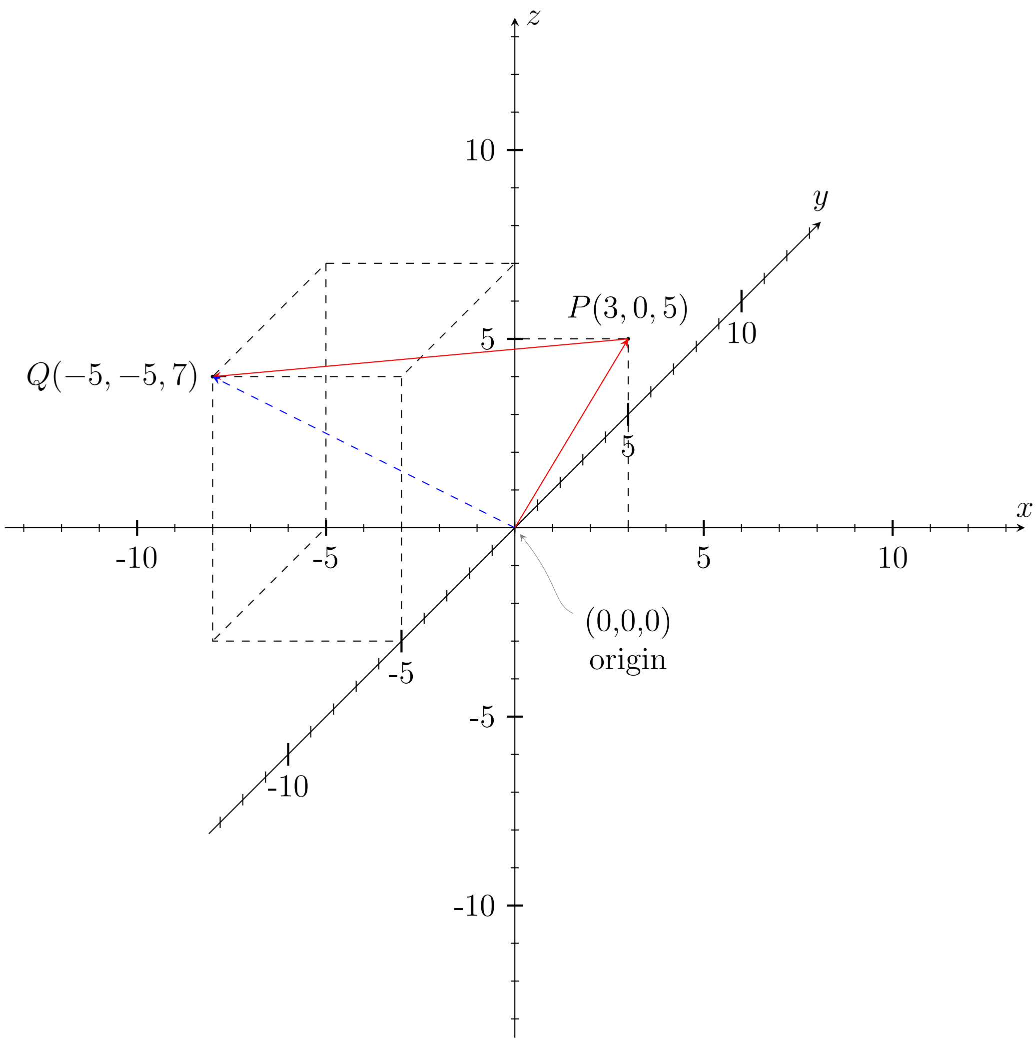
We first recall what $\mathbb{R}^3$ looks like:

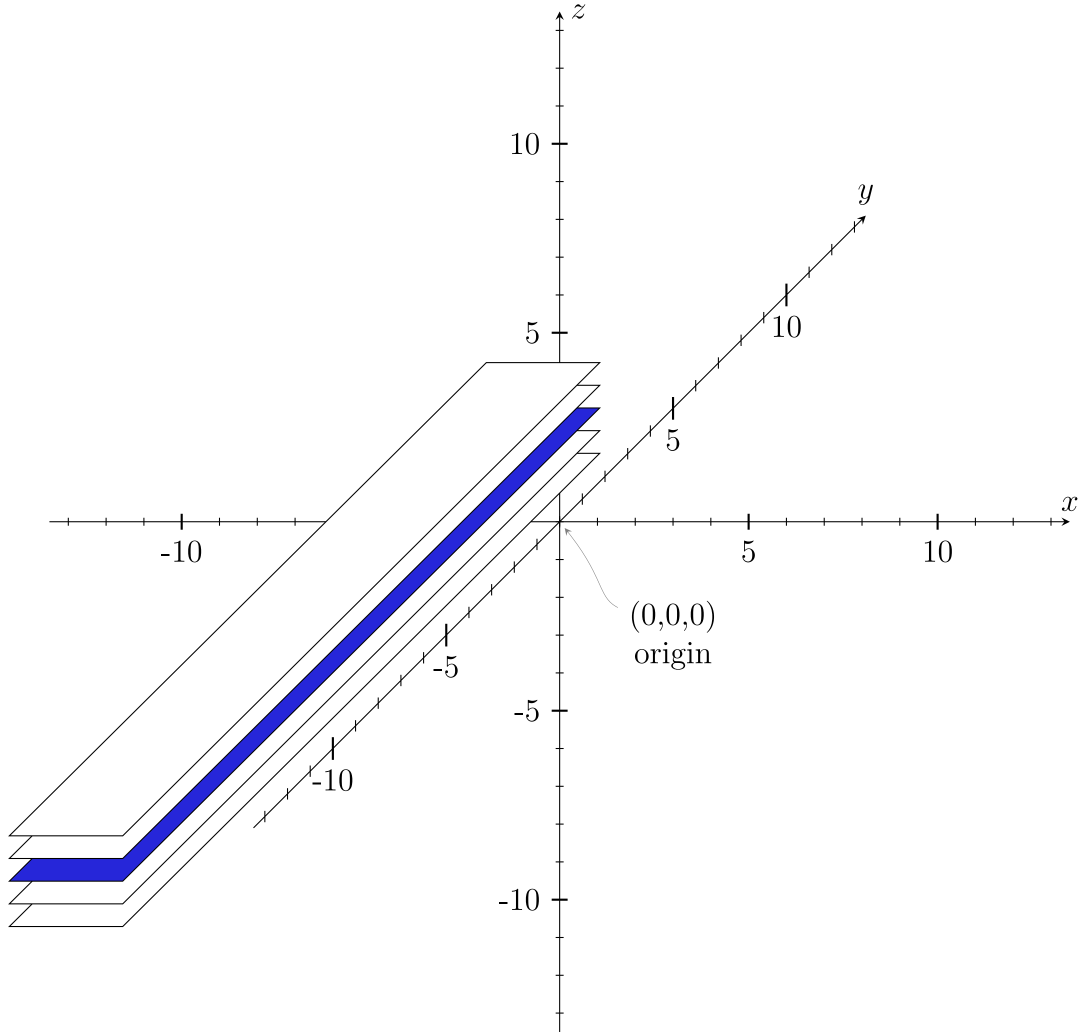


In $\mathbb{R}^3$ we have vectors and position vectors:

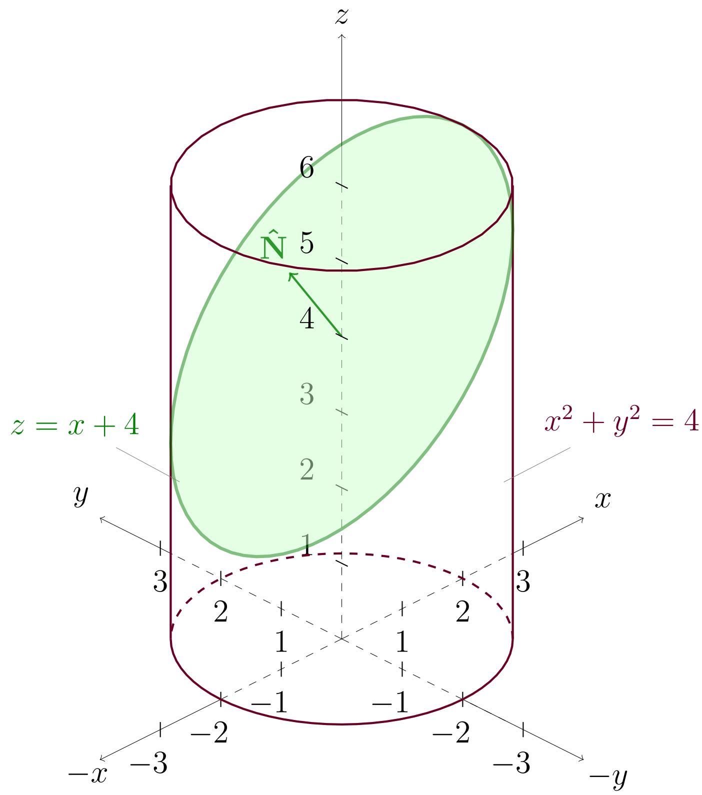


The blue vector above is the vector $\vec{OQ}$ and $\vec{OQ} = \vec{OP} + \vec{PQ}$.

Let's draw a couple of parallel planes in $\mathbb{R}^3$:



One important geometrical intuition we'll use over and over throughout the course is as follows. Whenever you have a surface in $\mathbb{R}^3$, think also of having a plane slice the surface at some given place, and think about the imprint left on the plane after such a slicing. We will find out later how to determine a parametrization of the curve of intersection between objects in $\mathbb{R}^3$. In the example below, a plane with a certain equation intersects a cylinder and the imprint left by this slicing is an ellipse. That ellipse essentially sit on the plane. Cases of importance involve taking planes parallel to the xy-plane at different z-elevation, slicing a repeatedly with such planes and gathering all such imprints and looking at them.



Here's a similar problem just like in class. Suppose an object is being lifted as shown below. The angle α is 45^o. Find the resulting total force. Mg is the gravitational force. f_R is the downward velocity and T is the tension of the string. N is the normal force.