Q1> Prove $1+2+3 \cdots + n = n(n+1)/2$ using induction method

Soln:- let for $n=1$.

$\therefore n = n(n+1)/2 \Rightarrow 1 = \frac{1(1+1)}{2} \Rightarrow 1 = 1$ LHS = RHS true

let for $n=k$.

$1+2+3+4 \cdots k = \frac{k(k+1)}{2}$ - ①

let for $n=k+1$

$1+2+3+4 \cdots (k+1) = \frac{(k+1)(k+2)}{2}$ - ②

take eqn ~~add~~ add $(k+1)$ both side

$1+2+3+4 \cdots k+(k+1) = \frac{k(k+1)}{2} + (k+1)$

$1+2+3+4 \cdots k+(k+1) = \frac{k(k+1)+2(k+1)}{2} = \frac{(k+2)(k+1)}{2}$

which is equal to the RHS of eqn ② hence proved.

$1+2+3 \cdots + n = n(n+1)/2$

Q2) i) Check whether the given sets are equal set: $A=\{1,2,3,4\}$ and $B=\{2,4,1,3\}$

Ans-) Acc. to set theory order doesn't matter and the element in B are present in B and all are unique so $A=B$

ii) write the subsets for the set $A=\{1,3,5,7\}$

$\Rightarrow n=4$, total subsets $= 2^4 = 16$

Subsets are $= \{\phi, \{1\}, \{3\}, \{5\}, \{7\}, \{1,3\}, \{1,5\}, \{1,7\}, \{3,5\}, \{3,7\}, \{5,7\}, \{1,3,5\}, \{1,3,7\}, \{1,5,7\}, \{3,5,7\}, \{1,3,5,7\}\}$

iii) write the set $A=\{1,2,3,4,5 \cdots\}$ in set builder form

A> $A = \{x \mid x \in N)$

iv) If $A=\{1,3,5,7,9,11\}$ and $B=\{1,2,3,12\}$ then find A-B and B-A.

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Q4) A binary string w is "framed" if $|w| \geq 4$ and it begins with '01' and ends with '10'. Create a simple 5-state NFA that recognizes all framed binary strings, and then convert that to an equivalent DFA. Make sure every state in your final DFA has exactly one transition out for every symbol in the alphabet.

Sol. $L = \{0110, 011110, 010010 \ldots\}$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$q_0 = q_0$, $q_f = q_4$

NFA

$\delta(q_0, 0) = q_1$

$\delta(q_1, 1) = q_2$

$\delta(q_2, 1) = q_3$

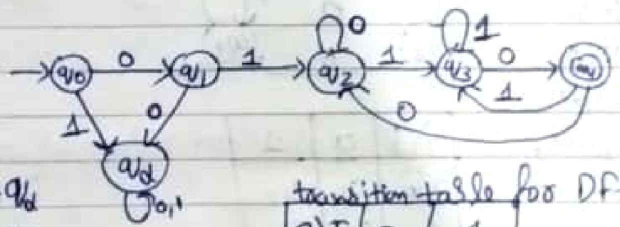
$\delta(q_3, 0) = q_4$

transition table for NFA

S\I	0	1
$\rightarrow q_0$	q_1	—
q_1	—	q_2
q_2	—	q_3
q_3	q_4	—
q_4	—	—

Now, change this to DFA (where we have to define all the input)

DFA



$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$q_0 = q_0$, $F = q_4$

$\delta(q_0, 0) = q_1$, $\delta(q_0, 1) = q_d$

$\delta(q_1, 0) = q_d$, $\delta(q_1, 1) = q_2$

$\delta(q_2, 0) = q_2$, $\delta(q_2, 1) = q_3$

$\delta(q_3, 0) = q_4$, $\delta(q_3, 1) = q_3$

$\delta(q_4, 0) = q_2$, $\delta(q_4, 1) = q_3$

transition table for DFA

S\I	0	1
$\rightarrow q_0$	q_1	q_d
q_1	q_d	q_2
q_2	q_2	q_3
q_3	q_4	q_3
q_4	q_2	q_3

Q5) Construct a DFA for the language over $\{0,1\}^*$ such that it contains '000' as a substring.

Ans $A-B = \{5,7,9\}$
$B-A = \{2,13\}$

iii) Find $A\cup(B\cup C)$, if $A=\{1,3,5\}$, $B=\{2,4,6\}$ and $C=\{1,5,7\}$

Ans: $(A\cup(B\cup C)) = \{1,3,5\}\cup(\{2,4,6\}\cup\{1,5,7\})$
$= \{1,3,5\}\cup\{1,2,4,5,6,7\}$
$= \{1,2,3,4,5,6,7\}$

Q3) Consider the regular language $L = \{w \mid w \in \{0,1\}^*$ and the 4th symbol from the begining of w is 0.

1) Design a DFA that recognizes L, and draw the state diagram clearly marking start state and final state. Your DFA should have 6 states, with a single final/accepting state.

2) ReDraw your state diagram from part a, turning the start state into a final state, turning the final state into the start state, and reversing the direction of each edge.

3) Is this a valid finite automaton, and if so it deterministic or nondeterministic. Briefly explain why.

4) Are any states in this new state diagram unreachable (and so unnecessary)? If so remove them and draw the new diagram.

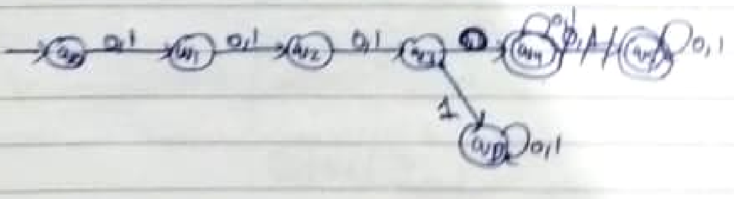
Ans:

A) $L = \{w \mid w \in \{0,1\}^+\}$

$Q = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$q_0 = q_0$

$q_F = q_4$



for NFA =

S\I	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	q_F	—
q_2	—	q_F
q_F	q_F	q_F

$\delta:(q_0,1) = \{q_0,$, $\delta:(q_0,0) = \{q_0,$
$\delta:(q_0,1) = q_2$, $\delta:(q_0,0) = q_1$
$\delta:(q_1,0) = q_F$, $\delta:(q_2,1) = q_F$
$\delta:(q_F,0) = q_F$, $\delta:(q_F,1) = q_F$

now for DFA

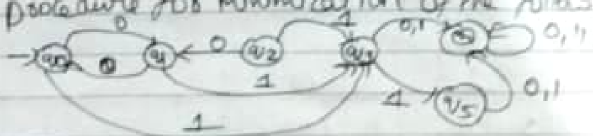
let $A = q_0$
$B = \{q_0, q_1\}$
$C = \{q_0, q_2\}$
$d = \{q_0, q_1, q_F\}$
$E = \{q_0, q_2, q_F\}$

S\I	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_2, q_F\}$
$\{q_0, q_1, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$
$\{q_0, q_2, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$

S\I	0	1
$\rightarrow A$	B	C
B	D	C
C	B	E
D	D	E
E	D	E

$\delta:(A,0) = B, \delta:(A,1) = C$
$\delta:(B,0) = D, \delta:(B,1) = C$
$\delta:(C,0) = B, \delta:(C,1) = E$
$\delta:(D,0) = D ; \delta:(D,1) = E$
$\delta:(E,0) = E , \delta:(E,1) = E$

Q8 > Explain procedure for minimization of the following finite Automata



S\I	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_3
q_3	q_4	q_4
q_4	q_4	q_4
q_5	q_4	q_4

Step 1: Remove the unreachable states here q_2

S\I	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_3
q_3	q_4	q_4
q_4	q_4	q_4
q_5	q_4	q_4

Step 2: Split the table in 2 parts 1) Finalist 2) non finalist

finalist

S\I	0	1
q_4	q_4	q_4

non-finalist

S\I	0	1
$\to q_0$	q_1	q_3
q_1	q_0	q_3
q_3	q_4	q_4
q_5	q_4	q_4

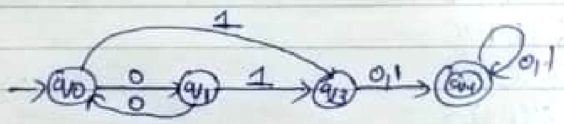
Step 3: in both table merge the two states with same transition function. here in non finalist $\delta(q_3,0) = \delta(q_5,0) = q_4$; $\delta(q_3,1) = \delta(q_5,1) = q_4$

Change q_5 with q_3 then,

S\I	0	1
$\to q_0$	q_1	q_3
q_1	q_0	q_3
q_3	q_4	q_4

Step 4: now merge the both tables and our draw the minimized DFA

S\I	0	1
$\to q_0$	q_1	q_3
q_1	q_0	q_3
q_3	q_4	q_4
q_4	q_4	q_4



Q9) Construct a DFA equivalet to NFA, $M = \{\{q, p, r\}, \{0,1\}, \delta, p, \{q, s\}\}$ where δ is defined in the following table

δ	0	1
~~p~~	~~{p,s}~~	~~{p}~~
~~q~~	~~{R}~~	~~R~~
~~r~~	~~S~~	~~–~~
~~s~~	~~S~~	~~S~~

δ	0	1
p	{q,s}	{q}
q	{r}	{q,r}
r	{s}	{p}
s	–	{p}

from here ⇒ $Q = \{p, q, r, s\}$

$\Sigma = \{0, 1\}$

$F = \{q, s\}$

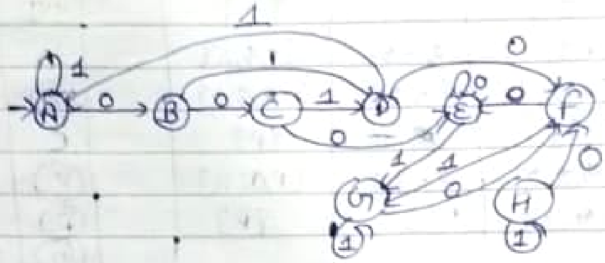
$\delta(p,0) = q$; $\delta(p,0) = s$, $\delta(p,1) = q$

$\delta(q,0) = r$, $\delta(q,1) = q$, $\delta(q,1) = r$

$\delta(r,0) = s$, $\delta(r,1) = p$, $\delta(s,1) = p$

Let → $A = \{P\}$, $B = \{P,Q\}$, $C = \{P,Q,R\}$, $D = \{P,R\}$, $E = \{P,Q,R,S\}$
$F = \{P,Q,S\}$, $G = \{P,R,S\}$, $H = \{P,S\}$

S\I	0	1
→A	B	A
B	C	D
C	E	D
D	F	A
E	E	G
F	E	G
G	F	G
H	F	H



Q13> Construct a NFA accepting all strings in $\{a,b\}^*$ with either two consecutive a's or two consecutive b's.

→ $L = \{aa, bb, \ldots\}$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{a, b\}$

$q_0 = q_0$

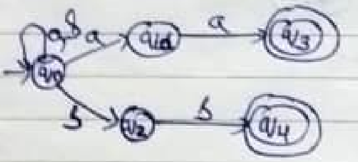
$F = q_3, q_4$

$\delta : (q_0, a) = q_0, q_1$

$\delta : (q_0, b) = q_0, q_2$

$\delta : (q_1, a) = q_3$

$\delta : (q_2, b) = q_4$



S\I	a	b
→q_0	q_0, q_1	q_0, q_2
q_1	q_3	—
q_2	—	q_4
q_3	—	—
q_4	—	—

Q14> Give the DFA accepting the following language: set of all strings beginning with a 1 that when interpreted as a binary integer is a multiple of 5.

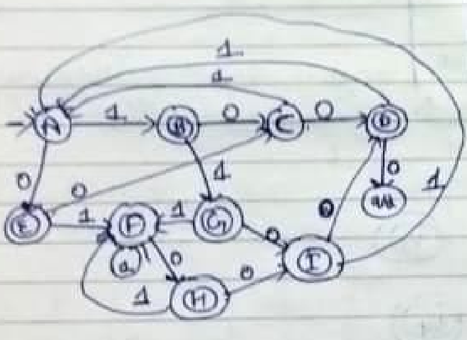
For DFA:

S\I	0	1
{p}	{q,s}	{q}
{q,s}	{r}	{p,q,r}
{q}	{r}	{q,r}
{r}	{s}	{p}
{p,q,r}	{q,r,s}	{p,q,r}
{q,r}	{r,s}	{p,q,r}
{s}	—	{p}
{q,r,s}	{r,s}	{p,q,r}
{q,s}	{s}	{p}

Let {p} = A, {q} = B, {r} = C, {s} = D
{q,s} = E, {p,q,r} = F, {q,r} = G,
{q,r,s} = H, {r,s} = I

new table is

S\I	0	1
→A	E	B
B	C	G
C	D	A
E	C	F
F	H	F
G	I	F
H	I	F
I	D	A
D	~~ad~~	A



Q11) Construct a DFA equivalent to the NFA given below:

S\I	0	1
P	{P,Q}	P
Q	R	R
R	S	–
S	S	S

Soln: For DFA

S\I	0	1
{P}	{P,Q}	{P}
{P,Q}	{P,Q,R}	{P,R}
{P,Q,R}	{P,Q,R,S}	{P,R}
{P,R}	{P,Q,S}	{P}
{P,Q,R,S}	{P,Q,R,S}	{P,R,S}
{P,Q,S}	{P,Q,R,S}	{P,R,S}
{P,R,S}	{P,Q,S}	{P,S}
{P,S}	{P,Q,S}	{P,S}

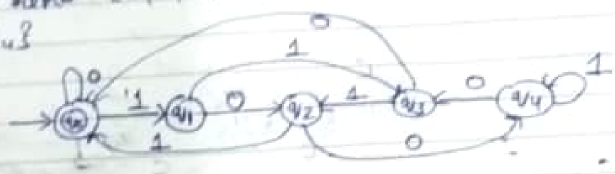
=) for divisible by 5 rem $\Sigma = \{0, 1, 2, 3, 4\}$

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = 0, 1$

$q_0 = q_0$

$F = q_0$



S\I	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

$\delta:(q_0, 0) = q_0$, $\delta(q_0, 1) = q_1$

$\delta:(q_1, 0) = q_2$, $\delta(q_1, 1) = q_3$

$\delta:(q_2, 0) = q_4$, $\delta(q_2, 1) = q_0$

$\delta(q_3, 0) = q_1$, $\delta(q_4, 1) = q_2$

$\delta(q_4, 0) = q_3$, $\delta(q_4, 1) = q_4$

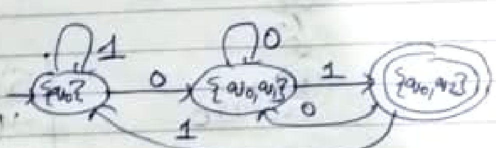
Q15) Construct DFA equivalent to the NFA given below.

S\I	0	1
$\rightarrow q_0$	(q_0, q_1)	q_0
q_1	—	q_2
q_2	—	—

$\delta:(q_0, 0) = q_0, q_1$, $\delta(q_0, 1) = q_0$

$\delta:(q_1, 1) = q_2$

S\I	0	1
$\rightarrow \{q_0\}$	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	q_0



$\delta(\{q_0\}, 0) = \{q_0, q_1\}$, $\delta(\{q_0\}, 1) = \{q_0\}$

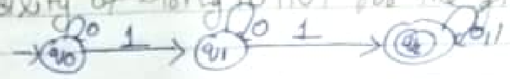
$\delta(\{q_0, q_1\}, 0) = \{q_0, q_1\}$, $\delta(\{q_0, q_1\}, 1) = \{q_0, q_2\}$

$\delta(\{q_0, q_2\}, 0) = \{q_0, q_1\}$, $\delta(\{q_0, q_2\}, 1) = \{q_0\}$

Q18) let $\Sigma = \{a, b, c\}$

1) Draw a DFA that rejects all words for which the last two letters match.

19> Check acceptability of String 101101 for the given automata?



=) 1 0 1 1 0 1

for 1 => $q_0 \to q_1$

for 0 => $q_1 \to q_1$

for 1 = $q_1 \to q_2$

for 1 = $q_2 \to q_2$

for 0 = $q_2 \to q_2$

for 1 = $q_2 \to q_2$

this automata is acceptable for this String.

Q20> for the following NDFA find equivalent DFA

State\Input	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	$\{q_2\}$	ϕ
q_2	ϕ	ϕ
q_3	ϕ	$\{q_4\}$
q_4	ϕ	ϕ.

=) for DFA

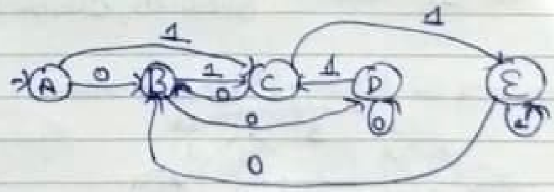
let $A = \{q_0\}$, $B = \{q_0, q_1\}$

$C = \{q_0, q_3\}$, $D = \{q_0, q_1, q_2\}$

$E = \{q_0, q_3, q_4\}$

State\Input	0	1
$\{q_0\}$	$\{q_0, q_1\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3, q_4\}$	$\{q_0, q_1\}$	$\{q_0, q_3, q_4\}$

S\I	0	1
A	B	C
B	D	C
C	B	E
D	D	C
E	B	E

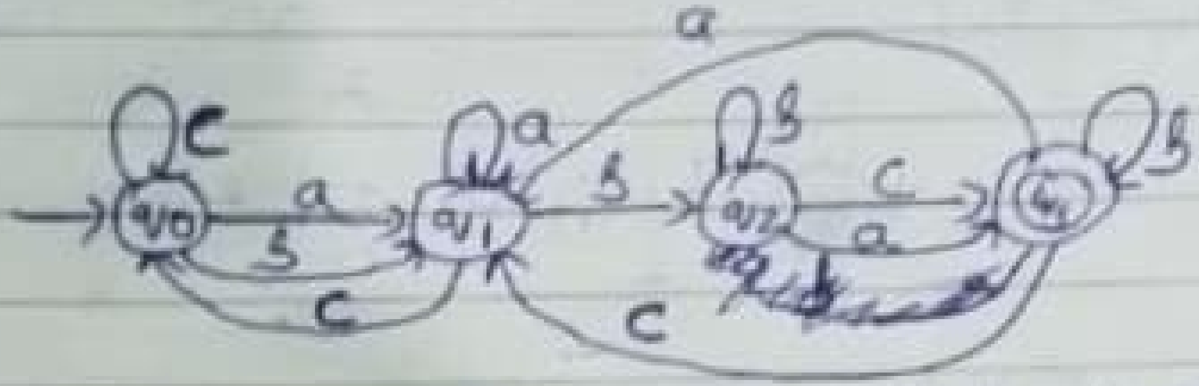


$L = \{ab, c, bbc, aac, bac \ldots\}$

$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{a, b, c\}$

$F = q_3, \; q_0 = q_0$



$\delta \backslash I$	a	b	c
$\rightarrow q_0$	q_1	q_1	q_0
q_1	q_1	q_2	q_0
q_2	q_3	q_2	q_3
q_3	q_1	q_3	q_1

$\delta:(q_0, a) = q_1, \; \delta:(q_0, b) = q_1 ; \; \delta:(q_0, c) = q_0$

$\delta:(q_1, a) = q_1, \; \delta:(q_1, b) = q_2, \; \delta:(q_1, c) = q_0$

$\delta:(q_2, a) = q_3, \; \delta:(q_2, b) = q_2, \; \delta:(q_2, c) = q_3$

$\delta:(q_3, a) = q_1, \; \delta:(q_3, b) = q_3, \; \delta:(q_3, c) = q_1$