ASSIGNMENT – 3

Theory of Computation

1) CFG:

$S \to AB$

$A \to aa / aaA$

$B \to bB / \varepsilon$

(a) $V = \{S, A, B\}$

$\varepsilon = \{a, b\}$

$S = \{S\}$

(b) Left most derivation tree

$S \to AB$

$S \to aaAB$ $(A \to aaA)$

$S \to aaaaBb$ $(A \to aa)$

$S \to aaaabB$ $(B \to bB)$

$S \to aaaabbB$ $(B \to bB)$

$S \to aaaabb$ $(B \to \varepsilon)$

Right most derivation tree

$S \to AB$

$S \to AbB$ $(B \to bB)$

$S \to AbbB$ $(B \to bB)$

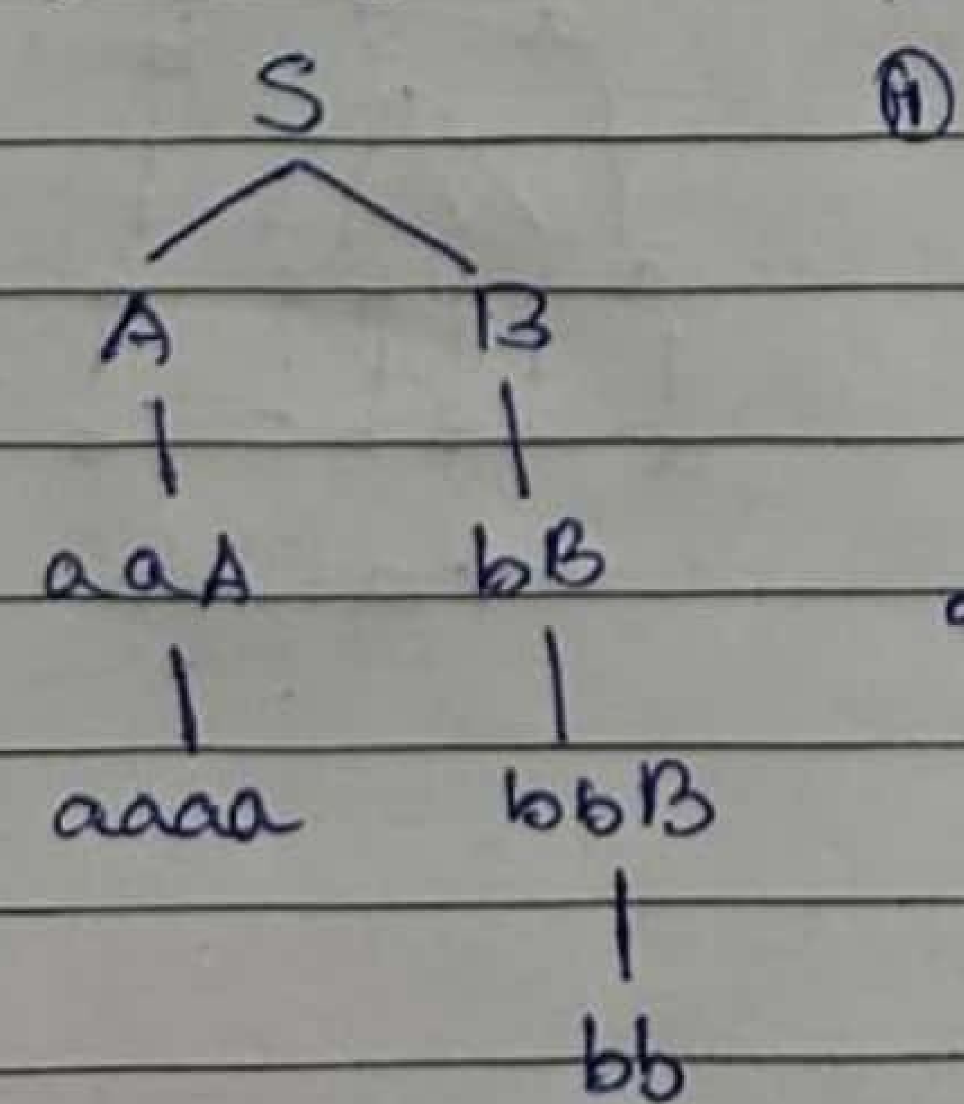
$S \to Abb$ $(B \to \varepsilon)$

$S \to aaAbb$ $(A \to aaA)$

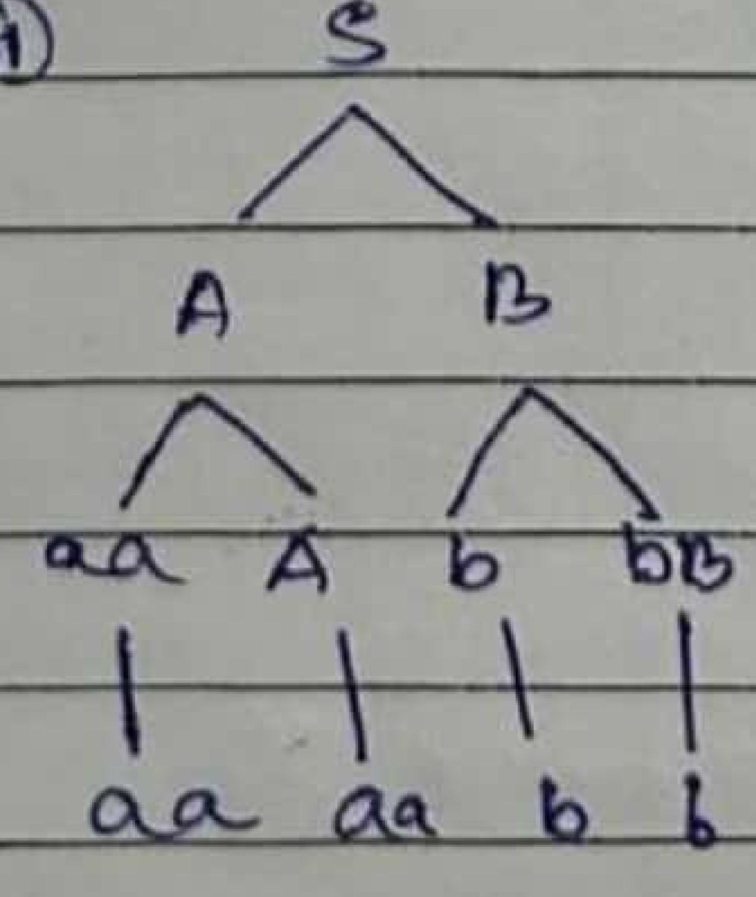
$S \to aaaabb$ $(A \to aa)$

(c) Parse tree

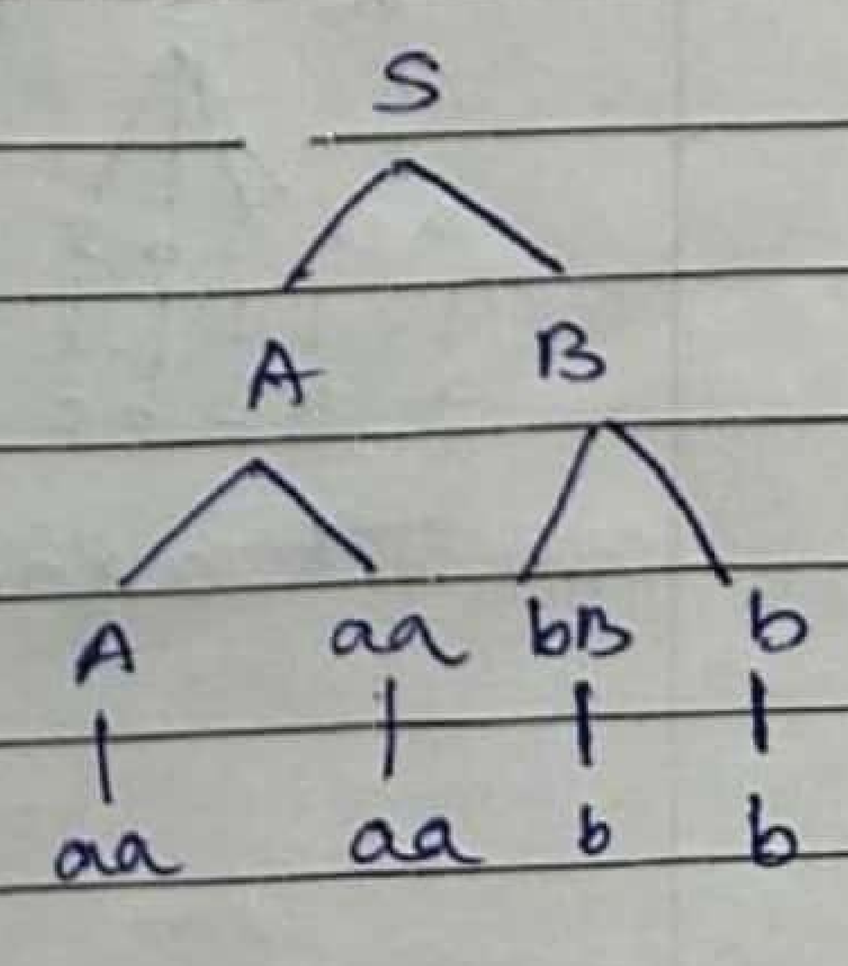
LMD (aaaabb)



(1)



(2)



to check Regularity

$\{a^n b^m / n \geq 1, m > 0, n \text{ is even}\}$

strings:

aa, aab, aaaab

Let us take the string aaaab $|w| \geq n, m$

case 1

$\underbrace{aa}_{x}\underbrace{aa}_{y}\underbrace{b}_{z}$

for xy^iz ; $i \geq 0$

→ aaaaaab

Satisfied

case 2

$\underbrace{a}_{x}\underbrace{aaa}_{y}\underbrace{b}_{z}$

xy^iz

aaaaaaab

NOt

case 3

$\underbrace{aa}_{x}\underbrace{a}_{y}\underbrace{ab}_{z}$

xy^iz

⇒ aaaaab NO

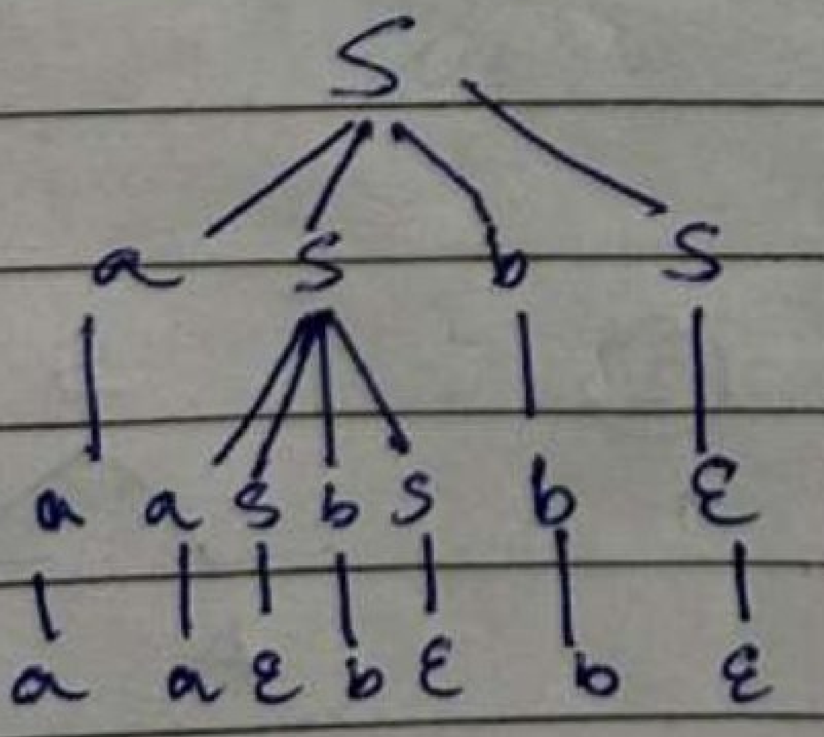
so it is not regular

2) ambiguity.

Given, $S \to aSbS | \varepsilon$

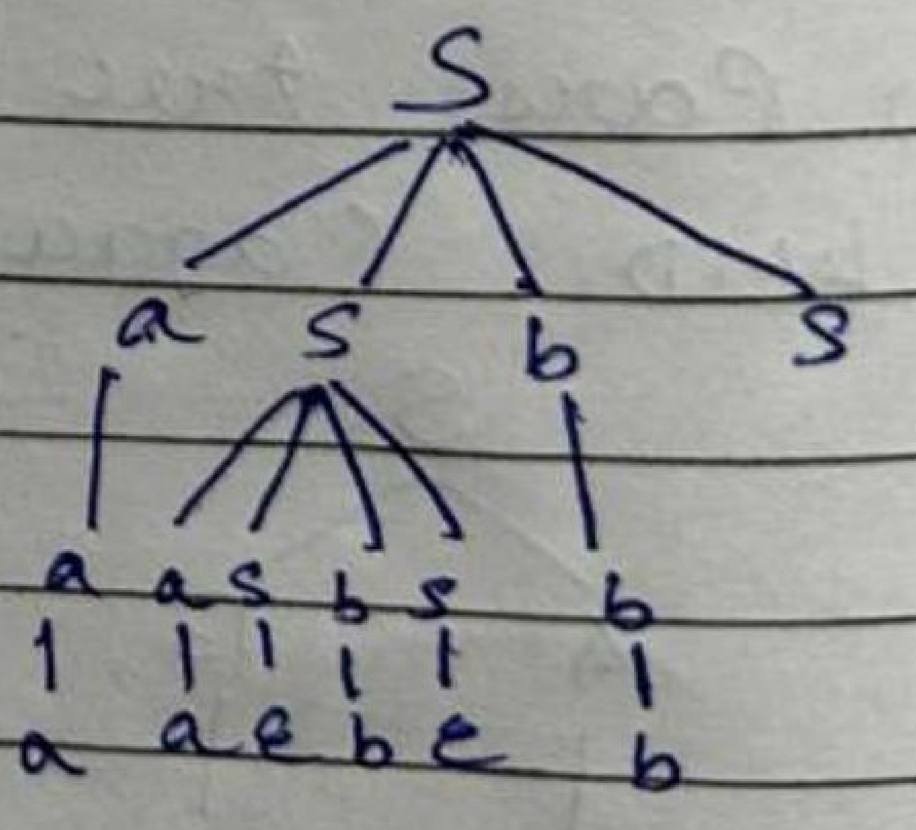
(a) two parse tree for the string

LMD. (aabb)



→ aabb

RMD



→ aabb

(b) G is ambiguous if there exists a string $w \in L(G)$ have two different parse tree (equivalently two different leftmost derivations or two different rightmost derivations).
we have same parse tree for the string aabb that means it is unambiguous.

(c) A non-ambiguous grammar generating the same language.
the given grammar is non-ambiguous

$S \to AB / \epsilon$

$A \to aAb / ab$

4) CFG:

$S \to aA / bB$

$A \to aA / \epsilon$

$B \to bB / \epsilon$

(a) Removing ϵ-production

$S \to aA / bB / a / b$

$A \to aA / a$

$B \to bB / b$

for unit $\to$ there is no unit production.

(b) for converting into CNF

$S \to aA$

$S \to XA \; (X \to a)$

$S \to bB$

$S \to YB \; (Y \to b)$

$S \to a$

$S \to b$

$A \to XA \; (X \to a)$

$A \to a$

$B \to YB \; (Y - b)$

$B \to b$

Production Rules :-
{ $S \to XA$, $S \to YB$, $S \to a$, $S \to b$, $A \to XA$, $B \to YB$,
$A \to a$, $B \to b$, $X \to a$, $Y \to b$ }

(c) derivation of string aab
in Original →
$S \to aA$
$S \to aaA$ ($A \to aA$)
$S \to aa$ ($A \to \epsilon$)
Not Possible

in CNF
$S \to XA$
$S \to aA$ ($X \to a$)
$S \to aa$ ($A \to a$)
∴ Not Possible

in both the languages

(d) CNF simplifies parsing (CYK algo), simplifies CFG - PDA constructions and proofs, and gives uniform parse tree that makes dynamic programming recursion straightforward.

5)(a) $L = \{a^n b^n c^n \mid n \geq 0\}$
strings : ϵ, abc, aabbcc
Let us take the string aabbcc $|w| \geq n$
$i \geq 0$, $|y| > 0$
$xy^i z$
for $\underset{x}{aa}\ \underset{y}{bb}\ \underset{z}{cc}$

case 1 → aabbbbcc ✗

case 2 → $\underset{x}{aa}\ \underset{y}{bb}\ \underset{z}{cc}$
→ aabbabbcc ✗

case 3 → $\underset{x}{aa}\ \underset{y}{bb}\ \underset{z}{cc}$ → aabbcbbcc ✗

Not Regular, ∴ Not Context - free

→ Context-free grammar is always Regular.

Pumping lemma S-
if L is an infinite context-free language then there exist an integer $p \geq 1$ such that every string $S \in L$ with $|S| \geq p$, we can write
$S = uvwxy$

$|vwx| \leq p$
$|vx| \geq 1$
$i \geq 0$ for uv^iwx^iy

(d) class of context free language is exactly the class of language accept by pushdown automata
$L = \{a^n b^n c^n\}$ is not CFL, so no PDA

6) $L(G) = \{a^m b^n / m > 0$ and $n \geq 0\}$

$S \to aS / aT$ or $S \to AB$
$T \to bT / \varepsilon$ $A \to aA / a$
$B \to bB / \varepsilon$

7) $S \to aS / aSbS / c$
Let use check its ambiguity by string
→ aacbc [or → $aSbS$, $S \to c$]

LMD:

Different Parse Tree ⇒ ambiguous

Non-ambiguous grammar

$S \to aSB / C$

$C \to aC / c$

8) G =>

Production Rules

$S \to A$

$A \to B$

$B \to a$

By removing unit

$S \to a$

$A \to a$

$B \to a$

9) Production Rules

$S \to A$

$A \to aB$

$B \to c$

by removing useless,

$S \to A$

$A \to aB$

$B \to c$

No change needed.

10) $S \to a/aA/B$

$A \to aBB / \epsilon$

$B \to Aa/b$

S1) Remove ε-production

$S \to a/aA/B$

$A \to aBB$

$B \to Aa/b/a$

Scanned with OKEN Scanner

S_2) Remove unit Rreaduction

$S \to a / aA / b$

$A \to aBB$

$B \to Aa / b / a$

S_3) Converting CNF

$S \to a$

$S \to aA$

$S \to XA \quad (X \to a)$

$S \to b$

$S \to a$

$A \to XBB \quad (x \to a)$

$B \to AX \quad (X \to a)$

$B \to b$

$B \to a$

$\cancel{B} \to XX_1 \quad (X_1 \to BB)$

P: $\{S \to a,\ S \to b,\ S \to XA,\ A \to XX_1,\ B \to AX,\ B \to b,$
$B \to a,\ X \to a,\ X_1 \to BB\}$