BCD3103 Algebra Lineal y Ecuaciones Diferenciales Tarea I

Rachit Vargas lopez

1 Sea u = (2,−5,4,6,−3) y v = (5,−2,1,−7,−4)

a) 4u − 3v

$$\begin{aligned} 4\text{u} &= (8,\ -20,\ 16,\ 24,\ -12) \\ 3\text{v} &= (15,\ -6,\ 3,\ -21,\ -12) \\ &= (-7,\ -14,\ 13,\ 45,\ 0) \end{aligned}$$

b) 5u + 3v

$$\begin{aligned} 5\text{u} &= (10,\ -25,\ 20,\ 30,\ -15) \\ 3\text{v} &= (15,\ -6,\ 3,\ -21,\ -12) \\ &= (25,\ -31,\ 23,\ 9,\ -27) \end{aligned}$$

c) u · v

$$\begin{aligned} \text{u} \cdot \text{v} &= [(2 \cdot 5) + (-5 \cdot -2) + (4 \cdot 1) + (6 \cdot -7) + (-3 \cdot -4)] \\ &= (10 + 10 + 4 - 42 + 12) \\ &= -6 \end{aligned}$$

d) ‖u‖ y ‖v‖

$$\begin{aligned} \|\text{u}\| &= \sqrt{2^2 + (-5)^2 + 4^2 + 6^2 + (-3)^2} \\ \|\text{u}\| &= \sqrt{4 + 25 + 16 + 36 + 9} \\ \|\text{u}\| &= \sqrt{90} \\ \|\text{u}\| &\approx 9.486 \end{aligned}$$

$$\begin{aligned} \|\text{v}\| &= \sqrt{5^2 + (-2)^2 + 1^2 + (-7)^2 + (-4)^2} \\ \|\text{v}\| &= \sqrt{25 + 4 + 1 + 49 + 16} \\ \|\text{v}\| &= \sqrt{95} \\ \|\text{v}\| &\approx 9.746 \end{aligned}$$

e) proj(u,v)

$$\begin{aligned} \tfrac{u \cdot v}{\|u\|^2} \cdot v &= \tfrac{-6}{95} \cdot (5, -2, 1, -7, -4) \\ &= (\tfrac{-6}{19}, \tfrac{12}{95}, \tfrac{-6}{95}, \tfrac{42}{95}, \tfrac{24}{95}) \end{aligned}$$

f) d(u, v)

$$\begin{aligned} \text{d(u, v)} &= \sqrt{(u_1 - v_1)^2 .... + (u_n - v_n)^2} \\ &= \sqrt{(2-5)^2 + (-5 - -2)^2 + (4-1)^2 + (6 - -7)^2 + (-3 - -4)^2} \\ &= \sqrt{197} \\ &\approx 14.035 \end{aligned}$$

2 Determinar el vector v identificado con el segmento de linea dirigido $\overrightarrow{PQ}$ para los puntos P(1, −8, −4,6) y Q(3, −5, 2, −4)

1. v = (Q_1 − P_1, ... , Q_n − P_n)
2. Q − P = ((3 − 1), (−5 − (−8)), (2 − (−4)), (−4 − 6))
3. v = (2, 3, 6, −10)

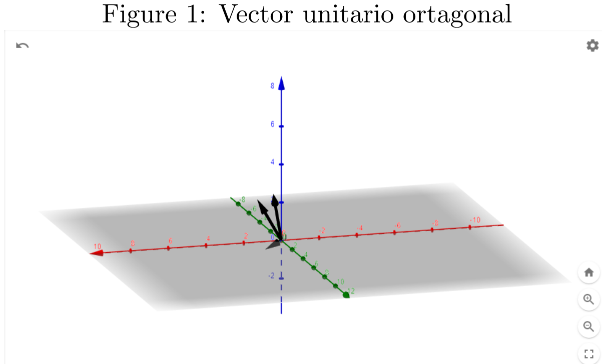
3 Determinar un vector unitario w ortogonal a u $= [1, 2, 3]$ y v $= [1, -1, 2]$

1. $|uxv|$

$$\begin{vmatrix} \text{i} & \text{j} & \text{k} \\ 1 & 2 & 3 \\ 1 & -1 & 2 \end{vmatrix}$$

2. a $= (7, -1, -3)$
3. w $= \frac{a}{\|a\|}$
4. $\|a\| = \sqrt{(7)^2 + (-1)^2 + (-3)^2} = \sqrt{59}$
5. w $= [\frac{(7,-1,-3)}{\sqrt{60}}]$
6. w $= (0.91132238, 0.13018891, -0.39056673)$

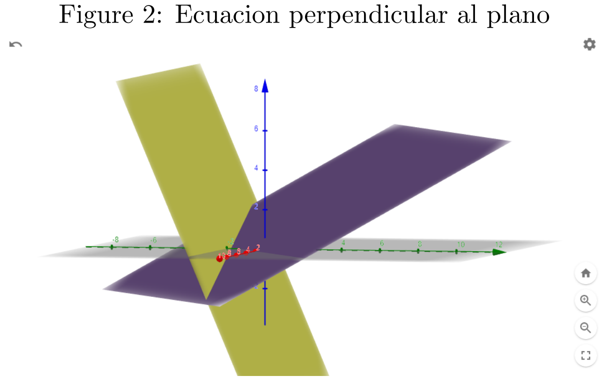
Figure 1: Vector unitario ortagonal



4 Determinar la ecuacion de la linea L perpendicular al plano $2x - 3y + 7z = 4$ y que contiene el punto P$(1, -5, 7)$

1. P$(1, -5, 7)$ 2. v $(2, -3, 7)$
3. $= (x, y, z) = (1, -5, 7) + \lambda\ (2, -3, 7)$
4. $= \frac{x-1}{2} = \frac{y+5}{-3} = \frac{z-7}{7}$
5. $e_1 = -3x + 3 = 2y + 10$
6. $e_1 = 3x + 2y + 7 = 0$
7. $e_2 = 7y + 35 = -3z + 21$
8. $e_2 = 7y + 3z + 14 = 0$

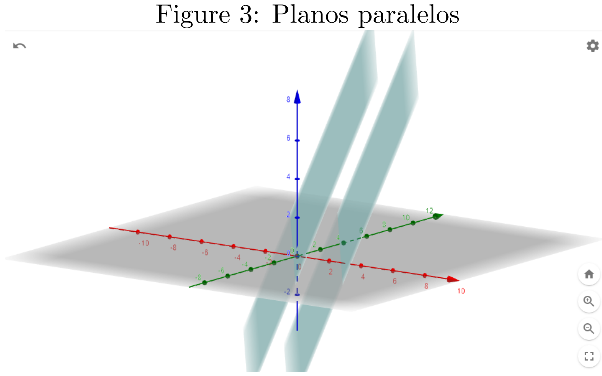
Figure 2: Ecuacion perpendicular al plano



5 Determinar la ecuacion del plano H paralelo a $4x+3y-2z = 11$ y conteniendo el punto $Q = (2, -1, 3)$

1. $Q = (2, -1, 3)$
2. $v = (4, 3, -2)$
3. $(x-2, y+1, z-3) \cdot (4, 3, -2)$
4. $[4(x-2) + 3(y+1) - 2(z-3)] = 0$
5. $4x - 8 + 3y + 3 - 2z + 6 = 0$
6. $4x + 3y - 2z + 1 = 0$

Figure 3: Planos paralelos



6 Calcular (u x v) dados $u = 3i - 4j + 2k$ y $v = 21i + 5j - 3k$

1. $|uxv|$

$$\begin{vmatrix} \text{i} & \text{j} & \text{k} \\ 3 & -4 & 2 \\ 21 & 5 & -3 \end{vmatrix}$$

2. $|uxv| = ((12 - 10), (-9 - 42), (15 - (-84)))$
3. $|uxv| = (2i, -51j, 99k)$

7 Vectores complejos, sea u $= (1 + 7i, 2 - 6i)$ y v $= (5 - 2i, 3 - 4i)$ determine:

a. $u + v$

$$\begin{gathered} \text{u} = (1 + 7i, 2 - 6i) \\ \text{v} = (5 - 2i, 3 - 4i) \\ = (6 + 5i, 5 - 10i) \end{gathered}$$

b. $(3 + i)u$

$$\begin{gathered} (3 + i)u = (3 + i) \cdot (1 + 7i, 2 - 6i) \\ (3 + i)u = [(3 + 21i + i + 7(-1)), (6 - 18i + 2i - 6(-1))] \\ (3 + i)u = (22i - 4, -16i + 12) \end{gathered}$$

c. (u · v)

$$\begin{gathered} (1 + 7i, 2 - 6i) \cdot (5 - 2i, 3 - 4i) \\ = [(5 - 2i) + (35i - 14(-1)) \ , \ (6 - 8i) + (18i + 24(-1))] \\ = (33i + 19, -26i - 18) \end{gathered}$$

d. $\|\mathrm{u}\|$ y $\|\mathrm{v}\|$

$$\|\mathrm{u}\| = (1)^2 + (7)^2 + (2)^2 + (-6)^2$$
$$\|\mathrm{u}\| = 1 + 49 + 4 + 36$$
$$\|\mathrm{u}\| = \sqrt{90}$$
$$\|\mathrm{v}\| = (5)^2 + (-2)^2 + (3)^2 + (-4)^2$$
$$\|\mathrm{v}\| = 25 + 4 + 9 + 16$$
$$\|\mathrm{u}\| = \sqrt{54}$$

8 Ejercicio de matrices

$$A = \begin{pmatrix} 9, & 1, & -3 \\ 6, & -5, & 4 \\ 7, & 2, & 6 \end{pmatrix}$$

$$B = \begin{pmatrix} 12, & -7, & -7 \\ 0, & -2, & 4 \end{pmatrix}$$

8.1 Determine:

(a) $A + B$ (Las dimensiones de la matriz no son iguales. No hay solucion)

(b) $AByBA$

$$\text{AB} = \text{No hay solucion, debido a que sus dimensiones no son correctas para multiplicar}$$

$$BA = \begin{pmatrix} 12 & -7 & -7 \\ 0 & -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 9 & 1 & -3 \\ 6 & -5 & 4 \\ 7 & 2 & 8 \end{pmatrix}$$

$$BA = \begin{pmatrix} (12 \cdot 9) + (-7 \cdot 6) + (-7 \cdot 7) & (12 \cdot 1) + (-7 \cdot -5) + (-7 \cdot 2) & (12 \cdot -3) + (-7 \cdot 4) + (-7 \cdot 8) \\ (0 \cdot 9) + (-2 \cdot 6) + (4 \cdot 7) & (0 \cdot 1) + (-2 \cdot -5) + (4 \cdot 2) & (0 \cdot -2) + (-2 \cdot 4) + (4 \cdot 8) \end{pmatrix}$$

$$BA = \begin{pmatrix} 17 & 33 & -120 \\ 16 & 18 & 24 \end{pmatrix}$$

(c) I_3A y $0A$

$$IA = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 9 & 1 & -3 \\ 6 & -5 & 4 \\ 7 & 2 & 8 \end{pmatrix}$$

$$IA = \begin{pmatrix} (1 \cdot 9) + (0 \cdot 6) + (0 \cdot 7) & (1 \cdot 1) + (0 \cdot -5) + (0 \cdot 2) & (1 \cdot -3) + (0 \cdot 4) + (0 \cdot 8) \\ (0 \cdot 9) + (1 \cdot 6) + (0 \cdot 7) & (0 \cdot 1) + (1 \cdot -5) + (0 \cdot 2) & (0 \cdot -3) + (1 \cdot 4) + (0 \cdot 8) \\ (0 \cdot 9) + (0 \cdot 6) + (1 \cdot 7) & (0 \cdot 1) + (0 \cdot -5) + (1 \cdot 2) & (0 \cdot -3) + (0 \cdot 4) + (1 \cdot 8) \end{pmatrix}$$

$$IA = \begin{pmatrix} 9 & 1 & -3 \\ 6 & -5 & 4 \\ 7 & 2 & 8 \end{pmatrix}$$

$$0A = 0 \cdot \begin{pmatrix} 9 & 1 & -3 \\ 6 & -5 & 4 \\ 7 & 2 & 8 \end{pmatrix}$$

$$0A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

(d) A^T y $A + A^T$

$$AT = \begin{pmatrix} 9 & 1 & -3 \\ 6 & -5 & 4 \\ 7 & 2 & 8 \end{pmatrix} = \begin{pmatrix} 9 & 6 & 7 \\ 1 & -5 & 2 \\ -3 & 4 & 8 \end{pmatrix}$$

$$A + AT = \begin{pmatrix} 9 & 1 & -3 \\ 6 & -5 & 4 \\ 7 & 2 & 8 \end{pmatrix} + \begin{pmatrix} 9 & 6 & 7 \\ 1 & -5 & 2 \\ -3 & 4 & 8 \end{pmatrix}$$

$$A + AT = \begin{pmatrix} (9+9) & (1+6) & (-3+7) \\ (6+1) & (-5+-5) & (4+2) \\ (7+-3) & (2+4) & (8+8) \end{pmatrix}$$

$$A + AT = \begin{pmatrix} 18 & 7 & 4 \\ 7 & -10 & 6 \\ 4 & 6 & 16 \end{pmatrix}$$

(e) $C = BB^T$

$$C = BBT = \begin{pmatrix} 12 & -7 & -7 \\ 0 & -2 & 4 \end{pmatrix}$$

$$BT = \begin{pmatrix} 12 & -7 & -7 \\ 0 & -2 & 4 \end{pmatrix}^T = \begin{pmatrix} 12 & 0 \\ -7 & -2 \\ -7 & 4 \end{pmatrix}$$

$$BBT = \begin{pmatrix} 12 & -7 & -7 \\ 0 & -2 & 4 \end{pmatrix} \cdot \begin{pmatrix} 12 & 0 \\ -7 & -2 \\ -7 & 4 \end{pmatrix}$$

$$C = \begin{pmatrix} 242 & -14 \\ -14 & 20 \end{pmatrix} = MatrizSimetrica$$

(f) $detC$

$$|C| = \begin{pmatrix} 242 & -14 \\ -14 & 20 \end{pmatrix}$$

$$(142 \cdot 20) - (-14 \cdot -14))$$
$$|C| = 4,644$$

(g) C^{-1}

Metodo de reduccion de Gauss Jordan

$$C^{-}1 = \begin{pmatrix} 242 & -14 & 1 & 0 \\ -14 & 20 & 0 & 1 \end{pmatrix}$$

$$C^{-}1 = \begin{pmatrix} 1 & 0 & \frac{20}{4644} & \frac{14}{4644} \\ 0 & 1 & \frac{14}{4644} & \frac{242}{4644} \end{pmatrix}$$

$$C^{-}1 = \begin{pmatrix} \frac{20}{4644} & \frac{14}{4644} \\ \frac{14}{4644} & \frac{242}{4644} \end{pmatrix}$$

Figure 4: Desarrollo de metodo de Gauss Jordan