Total No. of Page: 02   Roll No. ....

FIRST SEMESTER   B. Tech.

END SEM EXAMINATION   NOVEMBER-2018

MA – 101 Mathematics I

Time: 3:00 Hours   Max. Marks: 50

Note: Answer any five questions. All questions carry equal marks.

        Assume suitable missing data, if any.

1.

    a) Test the convergence of the following series

$$\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \cdots$$

    b) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{2}\right)$ and determine the value of

    $\sin 91^0$ correct upto four decimal places.

2.

    a) Examine the function $\sin x + \sin y + \sin(x + y)$ for extreme

    points and also find the value of the function at those points.

    b) If $u = cosec^{-1}\left[\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}\right]^{1/2}$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} +$

    $y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + \tan^2 u).$

3.

    a) Change the order of integration in the integral and hence

    $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dx\, dy$ and hence evaluate.

    b) Evaluate $I = \iiint \sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}}\ dx\, dy\, dz$ over the region

    $V = \left\{(x, y, z); x \geq 0, y \geq 0, z \geq 0, \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1\right\}.$

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4.

a) Find the area lying inside the curve $r = a\sin\theta$ and outside the curve $r = a(1 - \cos\theta)$.

b) Find the area of the surface formed by the revolution of $y^2 = 4ax$ about the x-axis, by an arc from the vertex to one end of the latus rectum.

5.

a) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ where Q is the point $(5, 0, 4)$. In what direction it will be maximum? Find also the magnitude of this maximum.

b) Verify Green's theorem in the plane for $\oint_C \{(3x^2 - 8y^2)dx + (4y - 6xy)dy\}$ where C is the boundary of the region defined by $y = x$ and $y = x^2$.

6.

a) Find the total work done in moving a particle in a force field $\vec{f} = 3xy\,\hat{\imath} - 5z\hat{\jmath} + 10x\,\hat{k}$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

b) Prove that $\text{Curl}(\text{Curl } \vec{V}) = grad\; div\; \vec{V} - \nabla^2\vec{V}$

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