Basic Electrical Engg – EE102
(Lecture Notes – 3 Phase AC Circuits)

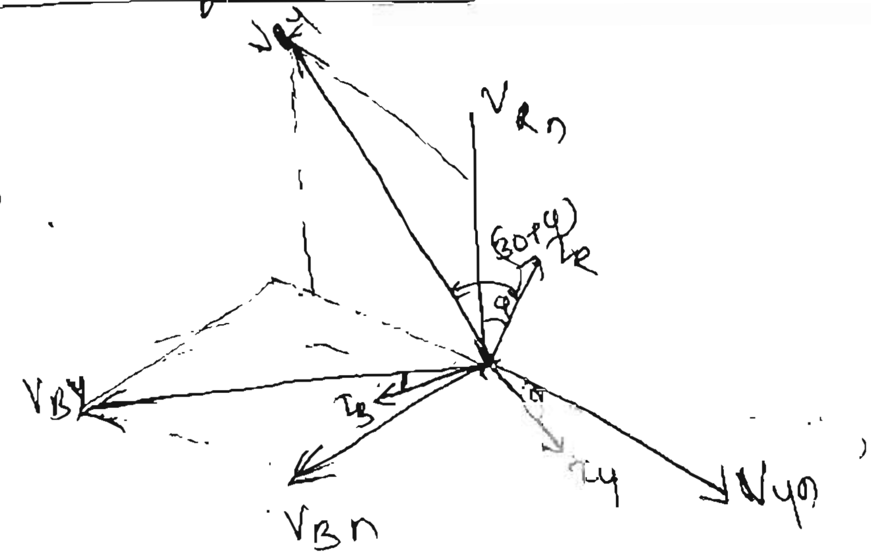
<u>Topics Covered</u>

- 3 Phase EMF Generation
- Delta and Star Connection
- Line and Phase Quantities
- Solution of 3 Phase Circuits
- Balanced Supply and Balanced Load
- Phasor Diagram
- 3 Phase Power Measurement by 2-Wattmeter Method

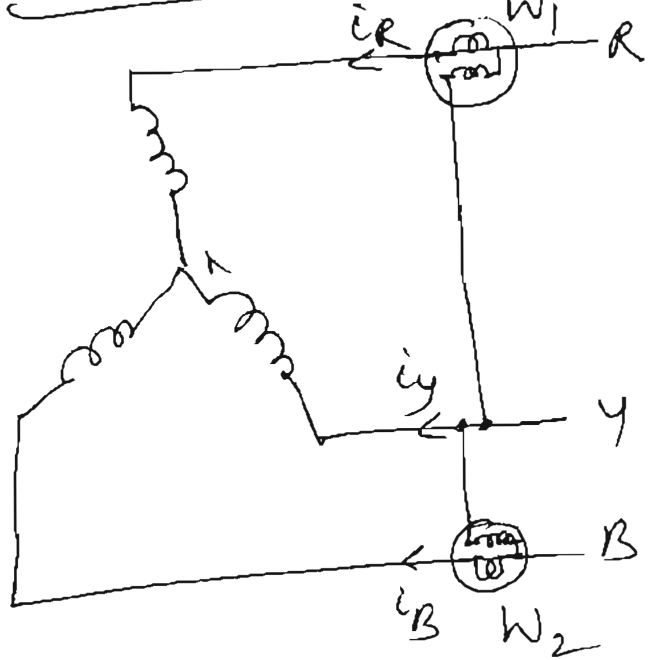
Dr Mini Sreejeth, Lecture Notes – 3 Phase AC Circuits

Measurement of Power in a Three phase system

3 W/m method
2 W/m method
Single W/m → Balanced load.



2 Wattmeter method



W/m reading of $W_1 = i_R V_{RY} \cos(30+\varphi)$

$|||^{ly}$ W/m reading of $W_2 = i_B V_{BY} \cos(30-\varphi)$

$W_1 + W_2 = V_L I_L (2\cos 30 \cos\varphi)$

$$W_1 + W_2 = 2 \times \frac{\sqrt{3}}{2} V_L I_L \cos\varphi$$
$$= \boxed{\sqrt{3} V_L I_L \cos\varphi} \quad \text{--- (1)}$$
$$W_1 - W_2 = V_L I_L (-2 \sin 30 \sin\varphi)$$
$$= -V_L I_L \sin\varphi \quad \text{--- (2)}$$

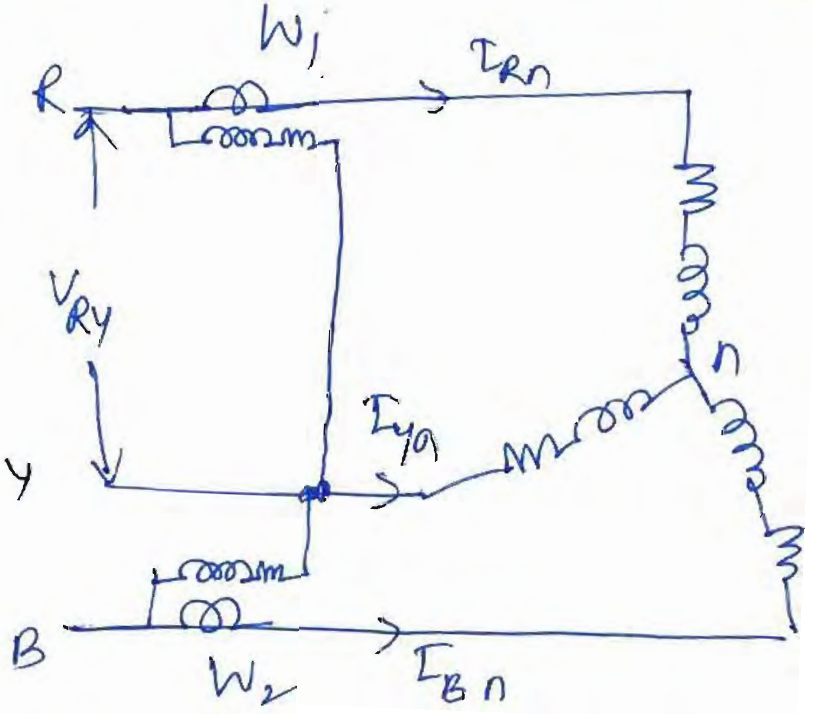
from (1) & (2).

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{-V_L I_L \sin\varphi}{\sqrt{3} V_L I_L \cos\varphi} = \frac{-\tan\varphi}{\sqrt{3}}$$

$$\therefore \boxed{\varphi = \tan^{-1}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)} \qquad \boxed{Pf = \cos\varphi}$$

11 (b)

Two Wattmeter method of Power measurement in a 3 phase circuit



Total inst. $P = W$.

$W = i_{Rn}V_{Rn} + i_{yn}V_{yn} + i_{Bn}V_{Bn}$ — (1)

$W_1 = i_{Rn} \cdot V_{RY} = i_{Rn}(V_{Rn} - V_{yn})$

$W_2 = i_{Bn}V_{BY} = i_{Bn}(V_{Bn} - V_{yn})$

$W_1 + W_2 = i_{Rn}(V_{Rn} - V_{yn}) + i_{Bn}(V_{Bn} - V_{yn})$

$= i_{Rn}V_{Rn} + i_{Bn}V_{Bn} - V_{yn}(i_{Rn} + i_{Bn})$

But $i_{Rn} + i_{Bn} + i_{yn} = 0$

or $i_{yn} = -(i_{Rn} + i_{Bn})$

$\therefore W_1 + W_2 = i_{Rn}V_{Rn} + i_{Bn}V_{Bn} + V_{yn}i_{yn}$ — (2)

From (1) & (2) $\Rightarrow$ $\boxed{\text{Total } P = W_1 + W_2}$

Total Power, $P = W_1 + W_2$.

$Pf = \cos\left[\tan^{-1}\left(\frac{W_2 - W_1}{W_1 + W_2}\right)\sqrt{3}\right]$

When i) $\phi = 0$ $\quad W_1 = W_2$ $\quad$ Load - purely Resistive, Pf = unity

ii) $\phi = 60^\circ$ $\quad W_1 = 0$ $\quad Pf = 0.5$

iii) $\phi < 60$ $\quad W_1$ & W_2 +ve $\quad Pf$ 0.5 & 1

$P = W_1 + W_2$

iv) $\phi > 60$ $\quad$ Reading of W_1 (-ve)

$P \rightarrow$ Diff b/w W/m readings

$Pf < 0.5$

v) $\phi = 90^\circ$ $\quad Pf = 0$ $\quad |W_1| = |W_2|$ $\quad W_1$ +ve & W_2 (-ve) Total Power zero

Sl No	Power factor Pf, phase angle ϕ	W/m reading W_1	W/m reading W_2	Remark
1.	$Pf = 1$ ($\phi = 0$)	+ve	+ve	$W_1 = W_2$
2.	$0.5 < Pf < 1$ $60^\circ > \phi > 0$	+ve	+ve	$W_1 > W_2$
		+ve	0	$P = W_1$
3.	$Pf = 0.5$			
4	$0 < Pf < 0.5$ $90 > \phi > 60^\circ$	+ve	-ve	$\|W_1\| > \|W_2\|$
5	$Pf = 0$ $\phi = 90^\circ$	+ve	-ve	$\|W_1\| = \|W_2\|$ $P = 0$

3 phase circuits

A star connected 3φ load has a resistance of 9Ω and inductive reactance of 11Ω in each phase. It is fed by 3 phase 400V, 50 Hz supply. (a) write phasor expressions for voltage across each phase, line voltages and line currents (b) find total apparent power, active power and reactive power

a) $V_p = \frac{400}{\sqrt{3}} = 230.95\,V$

$= 230.95\angle 0\,V$

Let $V_{an} = V_p\angle 0 = \leftarrow 115.475 - j200\,V$.

$V_{bn} = V_p\angle -120 = \quad -115.475 + j200\,V$

$V_{cn} = V_p\angle -240 =$

The line voltages are.

$V_{ab} = V_{an} - V_{bn} = 346.425 + j200 = \underline{400\angle 30°\,V}$

$V_{bc} = V_{bn} - V_{cn} = -j400 = \underline{400\angle -90°\,V}$

$V_{ca} = V_{cn} - V_{an} = 346.425 + j200 = 400\angle -210\,V$

$Z = 9 + j11 = \underline{14.21}\angle 50.71°\,\Omega$

$\cos\theta = 0.633$
$\sin\theta = 0.774$

$\therefore I_a = \frac{V_{an}}{Z} = \frac{230\angle 0}{14.21\angle 50.71} = \underline{16.25\angle -50.71\,A}$

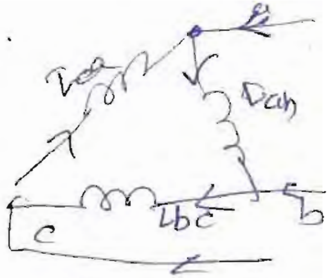
$I_b = \frac{V_{bn}}{Z} = \frac{230.95\angle -120}{14.21\angle 50.71} = \underline{16.25\angle -170.71\,A}$

$I_c = \frac{V_{cn}}{Z} = \frac{230.95\angle -240}{14.21\angle 50.71} = \underline{16.25\angle 69.29°\,A}$

b) Apparent power $= \sqrt{3}\, V_L I_L$

$= \sqrt{3} \times 400 \times 16.25$

$= 11258\ VA$



$P = \sqrt{3}\, V_L I_L \cos\theta = 11258 \times \cos(50.71)$

$= 7126.3 W$

$Q = \sqrt{3}\, V_L I_L \sin\theta = 11258 \times \sin(50.71)$

$= 8713.69\ VAR$

2 Repeat above numerical for delta connected load

let $V_{ab} = 400\angle 0$, $V_{bc} = 400\angle -120°\ V$

$V_{ca} = 400\angle -240.$

$I_{ab} = \frac{V_{ab}}{Z} = \frac{400\angle 0}{14.21\angle 50.71} = 28.15\angle -50.71 = 17.83 - j21.79$

$I_{bc} = \frac{V_{bc}}{Z} = \frac{400\angle -120}{14.21\angle 50.71} = 28.15\angle -170.71°\ A = -27.78 - j4.54\ A$

$I_{ca} = \frac{V_{ca}}{Z} = \frac{400\angle -240}{14.21\angle 50.71} = 28.15\angle 69.29$

$= 9.95 + j26.33\ A$

$I_a = I_{ab} - I_{ca} = (17.83 - j21.79) - (9.95 + j26.33) = 48.76\angle -80.7°$

$= 7.88 - j48.12\ A$

$I_b = I_{bc} - I_{ab} = 48.76\angle 159.3\ A$

$= 48.75\angle 39.3°\ A$

$I_c = I_{ca} - I_{bc}$

Apparent power $= \sqrt{3} V_L I_L$

$= \sqrt{3} \; 400 \times 48.75$

$= \underline{33774 \text{ VA}}$

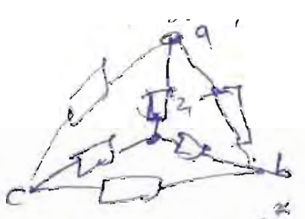
$P = \sqrt{3} V_L I_L \cos\theta = 33774 \times 0.633$

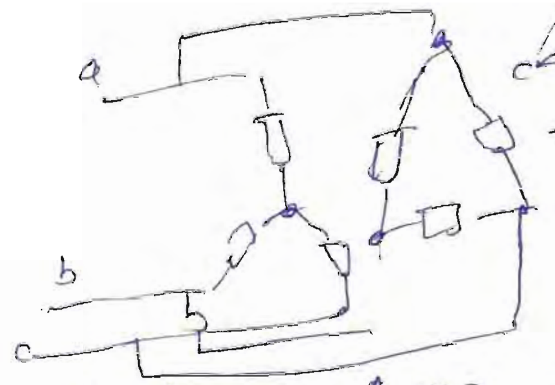
$= \underline{21378.9 \text{ W}}$

$Q = \sqrt{3} V_L I_L \sin\theta = 33774 \times 0.774$

$= \underline{26141.1 \text{ Vars}}$

3 Three equal impedances $12\angle 45^\circ$ are connected in star and another set of three equal impedances $15\angle 60^\circ$ are connected in delta. Both these loads are fed by a 400V 3ϕ system. Find the magnitude of line ct, apparent power, active power, reactive power and overall power factor





$Z_1 = \frac{Z_a Z_b}{Z_a+Z_b+Z_c} = \frac{Z^2}{3Z}$

$= \frac{Z}{3}$

$Z_Y = \frac{Z}{3} = \frac{15\angle 60}{3}$

$= \underline{5\angle 60}$

Each phase has two load impedances $12\angle 45^\circ$ and $5\angle 60^\circ$ in ||el. The equivalent load imp. per phase is

$Z = \frac{12\angle 45 \times 5\angle 60}{12\angle 45 + 5\angle 60} = \frac{60\angle 105}{8.48 + j8.48 + 2.5 + j4.33}$

$= \frac{60\angle 105}{10.98 + j12.81} = \frac{60\angle 105}{16.87\angle 49.4} = \underline{3.557\angle 55.6^\circ}$

$V_p = \frac{400}{\sqrt{3}} = \underline{230.95 \text{ V}}$

$I_L = I_p = \frac{230.95}{3.557} = \underline{64.93 \text{ A}}$

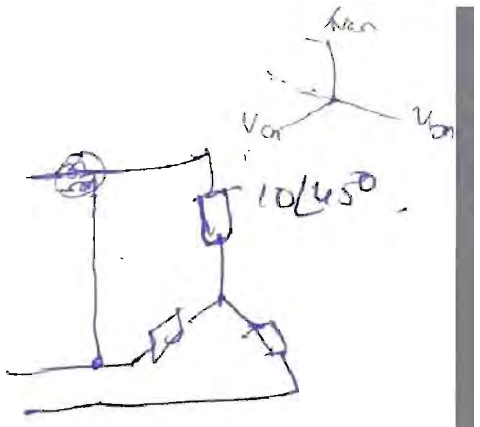
Apparent power $= \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 64.93$
$= \underline{44983.5\,VA}$

$P = \sqrt{3} V_L I_L \cos\theta = 44983.5 \cos(55.6)$
$= \underline{25414.2\,W.}$

$Q = \sqrt{3} V_L I_L \sin\theta = 44983.5 \sin(55.6)$
$= \underline{37116.5\,Vars}$

Power factor $= \cos(55.6) = \underline{0.565}$ lagging.

4. Three similar impedances each $10\angle 45^\circ$ are connected in star across a 220V, 3ϕ ac supply. Find (a) Phase voltages (b) phase cts and line currents (c) readings of two wattmeters connected for measurement of power and (d) using W_1 & W_2 find total power and power factor and verify the results



a) Phase voltage $= \frac{220}{\sqrt{3}} = \underline{127.02V}$

$V_{an} = 127.02\angle 0.$
$V_{bn} = 127.02\angle -120^\circ.$
$V_{cn} = 127.02\angle 120.$

b) Phase ct $I_a = \frac{127.02}{10\angle 45} = \underline{12.7\angle -45} = I_L$

$I_b = \frac{127.02\angle -120}{10\angle -45} = \underline{12.7\angle -165\,A}$

$I_c = \frac{127.02\angle -240}{10\angle -45} = \underline{12.7\angle 75^\circ}$

$W_1 = \underline{I_R}\, \underline{V_{RY}} \cos(30+\phi)$
$= \sqrt{3} V_p I_p \cos(30+\phi)$

c) $W_1 = \sqrt{3} V_p I \cos(30+\phi) = \sqrt{3} \times 127.02 \times 12.7 \cos(30+45)$
$= \underline{723.14\,W}$

$$W_2 = \sqrt{3}\, V_p I_c \cos(30-\varphi)$$
$$= \sqrt{3}(127.02) \times 12.7 \cdot \cos(30-45)$$
$$= \underline{2698.78\,W}$$

d) $$\text{Total } P = W_1 + W_2 = 723.14 + 2698.78$$
$$= \underline{3422.02\,W} \quad \text{— (a)}$$

$$\tan\theta = \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} = \frac{\sqrt{3}[2698.78 - 723.14]}{3422.02} = 1$$

$$\theta = \underline{45^\circ}$$

$$\cos\theta = \underline{0.707}$$

$$\text{Total power} = 3 \times 127.02 \times 12.7 \cos 45^\circ$$
$$= \underline{3422.02\,W} \quad \text{— (b)}$$

It is seen that total power and power factor are same as obtained from the readings of W/ms.

References & Further Reading

- **Vincent Del Toro, Electrical Engineering Fundamentals, Prentice-Hall of India Private Limited.**
- **Edward Huges, Electrical and Electronic Technology, Pearson Education Limited.**
- **Rajendra Prasad, Fundamentals of Electrical Engineering, PHI Learning Private Limited.**
- **Basic Electrical Engineering (Available online : https://nptel.ac.in/courses/108105053/)**