Classical & Quantum stats

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Ques-1

Total no. of proton per unit volume,

$$\frac{N}{u} = \int_0^\infty n(\nu)\, d\nu$$

$$n(\nu)\, d\nu = \frac{u(\nu)\, d\nu}{h\nu}$$

$$N = V \int_0^\infty \frac{u(\nu)\, d\nu}{h\nu} = \frac{8\pi V}{c^3} \int_0^\infty \frac{\nu^2 d\nu}{e^{h\nu/kt} - 1}$$

Let $\frac{h\nu}{Kt} = x$ $\qquad \nu = \frac{ktx}{h}$; $d\nu = \left(\frac{kT}{h}\right) dx$

$$N = 8\pi V \left(\frac{kT}{hc}\right)^3 \underbrace{\int_0^\infty \frac{x^2 dx}{e^x - 1}}_{2.404}$$

$$\boxed{N = 2.03 \times 10^{10} \text{ protons}}$$

average energy

$$E = \int_0^\infty \frac{u(\nu)\, d\nu}{n(\nu)\, d\nu} = \frac{aT^4}{N/V}$$

since $a = \frac{4\sigma}{c}$ and $N = 2.405 \left[8\pi V \left(\frac{kT}{hc}\right)^3\right]$

$$E = \frac{\sigma c^2 h^3 T}{(2.405)(2\pi k^3)}$$

$$= 3.73 \times 10^{-20} J$$

$$\boxed{E = 0.233 eV}$$

Ques-2

$$P(V)\,dv = 4\pi\left[\frac{m}{2\pi K_B T}\right]^{3/2} \exp\left[\frac{-mv^2}{2K_B T}\right] v^2\,dv$$

we have $m = \frac{32}{6.02 \times 10^{23}}\ gm = 5.32 \times 10^{-26}\ kg$

$K_B = 1.38 \times 10^{-23}\ J/K$

$T = 200K$

$v = 100\ m/s$

$dv = 101 - 100 = 1\ m/s$

Putting these values -

$$\boxed{P(V)\,dv = 6.13 \times 10^{-4}}$$

Ques-3

$$V_{av} = 1.59\sqrt{\frac{K_B T}{m}}$$

$$\frac{V_O}{V_H} = \sqrt{\frac{T_O}{T_H}\,\frac{m_H}{m_O}}$$

$$\frac{1}{2} = \sqrt{\frac{300 \times 1}{T_H \times 16}}$$

$$\boxed{T_H = 75K}$$

Ques-4

$$V_{rms} = 1.73\sqrt{\frac{K_B T}{m}}$$

$$V_{mp} = 1.41\sqrt{\frac{K_B T}{m}} \quad ; \quad \frac{V_{rms}}{V_{mp}} = \sqrt{\frac{3}{2}}$$

Ques-5

$$W = n! \prod_{i=1}^{k} \frac{(g_i)^{n_i}}{n_i!}$$

$g_1 = g_2 = g_3 = \frac{1}{3}$, $n = 5$, $k = 3$

$$W = 5! \prod_{i=1}^{3} \frac{(g_i)^{n_i}}{n_i!}$$

$$= 5! \frac{(g_1)^{n_1} (g_2)^{n_2} (g_3)^{n_3}}{n_1!\, n_2!\, n_3!}$$

$$= \frac{5!}{n_1!\, n_2!\, n_3!} \left(\frac{1}{3}\right)^{n_1+n_2+n_3}$$

$$= \frac{5!}{3^5\, n_1!\, n_2!\, n_3!}$$

this should be maximum

n_1	n_2	n_3	$\frac{5!}{n_1!\, n_2!\, n_3!}$
5	0	0	1
4	1	0	5
3	2	0	10
3	1	1	20
2	2	1	30

Most probable macrostates are - (2,2,1) , (2,1,2) , (1,2,2)

Ques-6

$$N(\nu)d\nu = \frac{8\pi \nu^2 d\nu}{c^3}$$

No of harmonic oscillators per unit volume b/w ν and $\nu + d\nu$

By Planck's radiation law -

$$\langle \varepsilon \rangle = \frac{h\nu}{e^{h\nu/KT} - 1}$$

$$E\nu d\nu = \frac{8\pi h\nu^3}{c^3} \frac{d\nu}{\left(e^{h\nu/KT} - 1\right)}$$

$$\nu = c/\lambda \quad , \quad d\nu = \frac{-c\,d\lambda}{\lambda^2}$$

and freq. is inc $\Rightarrow$ $U(\nu)d\nu = -U(\lambda)d\lambda$

$$-U(\lambda)\,d\lambda = \frac{8\pi h \cancel{c^3}}{\lambda^3 \cancel{c^3}} \cdot \frac{c\,d\lambda}{\lambda^2 \left(e^{hc/\lambda KT} - 1\right)}$$

$$U(\lambda)d\lambda = \frac{8\pi h c\, d\lambda}{\lambda^5 \left(e^{hc/\lambda KT} - 1\right)}$$

Ques-7

$$W = n! \prod_{n=1}^{k} \frac{(g_i)^{n_i}}{n_i!}$$

$$g_1 = 4 \quad , g_2 = 2 \quad , K = 2 , \quad n = 8$$

$$W = 8! \frac{(4)^{n_i} (2)^{n_2}}{n_1!\, n_2!}$$

$n_1 = 8 \quad , \quad n_2 = 0$

$$W(8,0) = \frac{8!\ 4^8}{8!} = 65536$$

Ques-8

$$e = \frac{8\pi}{3}\left(\frac{2mE_F}{h^2}\right)^{3/2}$$

$E_F = 4.72\,eV = 4.72 \times 1.6 \times 10^{-19}$

$= 7.55 \times 10^{-19}\ J$

$m = 9.1 \times 10^{-31}\ kg \quad , \quad h = 6.63 \times 10^{-34}$

$$e = \frac{8\pi}{3}\left[\frac{2 \times 9.1 \times 10^{-31} \times 7.55 \times 10^{-19}}{(6.63 \times 10^{-34})^2}\right]$$

$$\boxed{e = 4.62 \times 10^{26}\ \text{electrons}/m^3}$$

Ques-9

$$F(E) = \frac{1}{\left[\exp\left(\frac{E-E_f}{K_BT}\right) + 1\right]}$$

$E = E_F + 0.5\,eV \quad ; \quad f(E) = 0.01$

$$0.01 = \frac{1}{\exp\left[\frac{0.5}{K_BT} + 1\right]}$$

$$0.01 \exp\left[\frac{0.5}{K_BT}\right] = 0.99$$

$$T = \frac{0.5}{\ln 99\, K_B} = \frac{0.109}{K_B}$$

$K_B = 1.38 \times 10^{-23}$

$$\boxed{T = 1264K}$$

Ques-10

$$N = \frac{(n+g_i-1)!}{n!\,(g_i-1)!}$$

$n = 8, \quad g_i = 6$

$$N = \frac{13!}{8!\,5!} = 1287$$

Ques-11

$$N = \frac{g_i!}{n!\,(g_i-n)!}$$

$g_i = 15 \quad , \quad n = 10$

$$N = \frac{15!}{10!\,5!} = 3003$$

Ques-12

mass of Na $= 38.18 \times 10^{-24}$ gm

density " " $= 0.999$ gm/cm^3

volume " " $= \dfrac{38.18 \times 10^{-24}}{0.999}$

$= 39.36 \times 10^{-30}\ m^3$

density $= \dfrac{1}{39.36 \times 10^{-30}} = 0.0254 \times 10^{30}\ \dfrac{e^-}{m^3}$

we know,

$$E_F = \frac{h^2}{2m}\left(\frac{3e}{3H}\right)^{2/3}$$

$$E_f = \frac{(6.64\times10^{-34})^2}{2\times9.1\times10^{-31}}\left[\frac{3\times0.025\times10^{30}}{8\times3.14}\right]^{2/3}$$

$$= 0.05021\times10^{-17}\ J$$

$$\boxed{E_f = 3.1375\,eV}$$

Ques-13

Let fermi energy of conduction electron in Be is E_1 and E_2. In cesium,

given –

$e_{Be} = 24.2\times10^{22}/cm^3$

$e_{Ce} = 0.91\times10^{22}/cm^3$

$E_1 = 14.14\,eV$

$E_2 = ?$

$$E_1 = \frac{h^2}{2m_0}\left(\frac{3e_{Be}}{8\pi}\right)^{2/3} \quad \text{— ①}$$

$$E_2 = \frac{h^2}{2m_0}\left(\frac{3e_{Ce}}{8\pi}\right)^{2/3} \quad \text{— ②}$$

$$\frac{E_2}{E_1} = \left(\frac{e_{Ce}}{e_{Be}}\right)^{2/3} = \frac{0.91\times10^{22}}{24.2\times10^{22}}$$

$$\boxed{E_2 = 1.583\,eV}$$

Ques-14

$$n(E)\,dE = \frac{3n}{2} E_f^{-3/2} E^{1/2} dE \quad — (1)$$

we know,

$$E_f = \frac{h^2}{2m}\left(\frac{3n}{8\pi V}\right)^{2/3}$$

$$8\sqrt{2}\frac{\pi}{h^3} Vm^{3/2} = \frac{3}{2} NE^{-3/2}$$

↳ putting in eq (1)

$$n(E)\,dE = 8\sqrt{2}\frac{\pi}{h^3} Vm^{3/2} E^{1/2} dE$$

if n the total no. of electron then

$$n = \int_0^{E_f} n(E)\,dE = 8\sqrt{2}\frac{\pi}{h^3} Vm^{3/2} \int_0^{E_f} E^{1/2} dE$$

$$n = \frac{2\sqrt{2}}{3\pi^2}\frac{1}{h^3} Vm^{3/2}\left(\frac{1}{2}mv_f^2\right)^{3/2}$$

$$n = \frac{Vm^3 v_f^3}{3\pi^2 h^3}$$

as all energy states below fermilevel are filled so -

$$n = \frac{Vm^3 v^3}{3\pi^2 h^3}$$

$$dn = \frac{Vm^3 v^2 dv}{\pi^2 h^3}$$

now avg force e- velocity at absolute zero is -

$$\bar{v} = \frac{1}{n}\int_0^{v} v\,dn$$

$$\bar{v} = \frac{1}{n}\int_0^{v}v\left(\frac{Vm^3}{\pi^2 h^3}v^2\,dv\right)$$

$$\bar{v} = \frac{1}{n}\frac{Vm^3}{\pi^2 h^3}\frac{v_F}{4}\cdot v_F^3$$

$$= \frac{1}{n}\cdot\frac{V}{\pi^2}\cdot\frac{v_F}{4}\left(\frac{3\pi^2 h}{V}\right)$$

$$\boxed{\bar{v} = \frac{3v_F}{4}}$$

Ques-15

	Maxwell Boltzman	Bose-einstein	Fermi-Dirac
Applies to system of	Identical & distinguishable particle	Identical, indistinguishable; doesn't obey exclusion principle	Identical, indistinguishable; obey exclusion principle
Particle category	classical	Bosons	Fermions
Properties of particle	any spin, far enough so that wave func. doesn't overlap	spin 0,1,2--. wave funct. are symm. to interchange of particle level	spin 1/2, 3/2 - - wave funct. are anti-symm. to interchange of particle.
Property of distribution	No limits to no. of particle per states	No limit to no. of particles per states	Never more than 1 particle per states
examples	Molecules of gas	Photons in cavity	Free electrons in metals.

Ques-11

$$n(E)dE = \frac{2\pi n}{(\pi KT)^{3/2}} \sqrt{E}\, e^{-E/KT}\, dE \quad —①$$

(a) avg. molecular energy

$$E = \frac{1}{n}\int_0^\infty E\, n(E)dE$$

$$= \frac{1}{n}\frac{2\pi n}{(\pi KT)^{3/2}}\int_0^\infty E^{3/2} e^{-E/KT}\, dE$$

$$= \frac{2\pi}{(\pi KT)^{3/2}}\int_0^\infty E^{3/2} e^{-E/KT}\, dE$$

using -

$$\int_0^\infty x^{3/2} e^{-ax} dx = \frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$$

$$E = \frac{2\pi}{(\pi KT)^{3/2}}\left[\frac{3}{4}(KT)^2\sqrt{\pi KT}\right]$$

$$\boxed{E = \frac{3}{2}KT}$$

(b) For t_{mp}, we diff ① and equate it to 0

$$n(E)dE = \frac{2\pi n}{(\pi KT)^{3/2}} E^{1/2} e^{-E/KT}\, dE$$

$$\frac{2\pi n}{(\pi KT)^{3/2}}\left[\frac{-1}{KT} e^{-E/KT}\sqrt{E} + \left(\frac{1}{2}\right)^{1/\sqrt{E}} \cdot e^{-E/KT}\right] = 0$$

$$\frac{-1}{KT}\sqrt{E} = \frac{1}{2}E^{-1/2}$$

$$\boxed{t_{mp} = \frac{KT}{2}}$$

Que-17

(a) $n(E)\,dE = 8\sqrt{2}\,\frac{\pi}{h^3}\,Vm^{3/2}\,E^{-1/2}\,dE$ — (1)

If N is no. of electrons in e^- gas, the value of E_f can be found

$$N = \int_0^{E_f} n(E)\,dE$$

$$N = 8\sqrt{2}\,\frac{\pi}{h^3}\,Vm^{3/2}\int_0^{E_f} E^{1/2}\,dE$$

$$N = 8\sqrt{2}\,\frac{\pi}{h^3}\,Vm^{3/2}\cdot\frac{2E^{3/2}}{3}$$

$$\boxed{E_f = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3}}$$

(4) If E_0 is total energy of electrons gas at absolute zero, then -

$$E_0 = \int_0^{E_f} E\,(n(E))\,dE \quad -(2)$$

eq (1) & (2)

$$E_0 = \frac{3}{2}N E_f^{-3/2}\int_0^{E_f} E^{3/2}\,dE$$

$$E_0 = \frac{3}{5}N E_F^{-3/2}\cdot E_F^{5/2}$$

$$\boxed{E_0 = \frac{3}{5}N E_f}$$

The avg free electron energy at absolute zero is -

$$\bar{E} = \frac{E_0}{N} = \frac{3}{5}E_f$$