CHAPTER 11

Nuclear Structure

Nuclear magnetic resonance is the basis of a high-resolution method of imaging body tissues. The screen shows a computer-constructed cross section of the head of the person lying inside the powerful magnet at the rear.

Thus far we have been able to regard the nucleus of an atom merely as a tiny, positively charged object whose only roles are to provide the atom with most of its mass and to hold its electrons in thrall. The chief properties (except mass) of atoms, molecules, solids, and liquids can all be traced to the behavior of atomic electrons, not to the behavior of nuclei. Nevertheless, the nucleus turns out to be of paramount importance in the grand scheme of things. To begin with, the very existence of the various elements is due to the ability of nuclei to possess multiple electric charges. Furthermore, the energy involved in almost all natural processes can be traced to nuclear reactions and transformations. And the liberation of nuclear energy in reactors and weapons has affected all our lives in one way or another.

11.1 NUCLEAR COMPOSITION

Atomic nuclei of the same element have the same numbers of protons but can have different numbers of neutrons

The electron structure of the atom was understood before even the composition of its nucleus was known. The reason is that the forces that hold the nucleus together are vastly stronger than the electric forces that hold the electrons to the nucleus, and it is correspondingly harder to break apart a nucleus to find out what is inside. Changes in the electron structure of an atom, such as those that occur when a photon is emitted or absorbed or when a chemical bond is formed or broken, involve energies of only a few electronvolts. Changes in nuclear structure, on the other hand, involve energies in the MeV range, a million times greater.

An ordinary hydrogen atom has as its nucleus a single proton, whose charge is $+e$ and whose mass is 1836 times that of the electron. All other elements have nuclei that contain neutrons as well as protons. As its name suggests, the neutron is uncharged; its mass is slightly greater than that of the proton. Neutrons and protons are jointly called **nucleons.**

The **atomic number** of an element is the number of protons in each of its atomic nuclei, which is the same as the number of electrons in a neutral atom of the element. Thus the atomic number of hydrogen is 1, of helium 2, of lithium 3, and of uranium 92. All nuclei of a given element do not necessarily have equal numbers of neutrons. For instance, although over 99.9 percent of hydrogen nuclei are just single protons, a few also contain a neutron, and a very few two neutrons, along with the proton (Fig. 11.1). The varieties of an element that differ in the numbers of neutrons their nuclei contain are called its **isotopes.**

The hydrogen isotope **deuterium** is stable, but **tritium** is radioactive and eventually changes into an isotope of helium. The flux of cosmic rays from space continually replenishes the earth's tritium by nuclear reactions in the atmosphere. Only about 2 kg of tritium of natural origin is present at any time on the earth, nearly all of it in the oceans. **Heavy water** is water in which deuterium atoms instead of ordinary hydrogen atoms are combined with oxygen atoms.

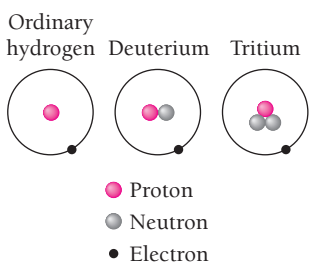


Figure 11.1 The isotopes of hydrogen.

The conventional symbols for nuclear species, or **nuclides,** follow the pattern ${}^{A}_{Z}X$,

where X = chemical symbol of the element
Z = atomic number of the element
= number of protons in the nucleus
A = mass number of the nuclide
= number of nucleons in the nucleus

James Chadwick (1891–1974) was educated at the University of Manchester in England and remained there to work on gamma-ray emission under Rutherford. In Germany to investigate beta decay when World War I broke out, Chadwick was interned as an enemy alien. After the war he joined Rutherford at Cambridge, where he used alpha-particle scattering to show that the atomic number of an element equals its nuclear charge. Rutherford and Chadwick suggested an uncharged particle as a nuclear constituent but could not find a way to detect it experimentally.

Then, in 1930, the German physicists W. Bothe and H. Becker found that an uncharged radiation able to penetrate lead is emitted by beryllium bombarded with alpha particles from polonium (Fig. 11.2). Irene Curie and her husband Frederic Joliot, working in France in 1932, discovered that this mysterious radiation could knock protons with energies up to 5.7 MeV out of a paraffin slab. They assumed the radiation consisted of gamma rays (photons more energetic than x-rays) and, on the basis that the protons were knocked out of the hydrogen-rich paraffin in Compton collisions, calculated that the gamma-ray photon energy had to be at least 55 MeV. But this was far too much energy to be produced by the alpha particles interacting with beryllium nuclei.

Chadwick proposed instead that neutral particles with about the same mass as the proton are responsible, in which case their energy need be only 5.7 MeV since a particle colliding head on with another particle of the same mass can transfer all of its KE to the latter. Other experiments confirmed his hypothesis, and he received the Nobel Prize in 1935 for his part in the discovery of the neutron. (Chadwick did not immediately regard the neutron as an elementary particle but instead as "a small dipole, or perhaps better as a proton embedded in an electron." The idea that the neutron is actually an elementary particle was first put forward by the Russian physicist Dmitri Iwanenko.) During World War II Chadwick headed the British group that participated in developing the atomic bomb.

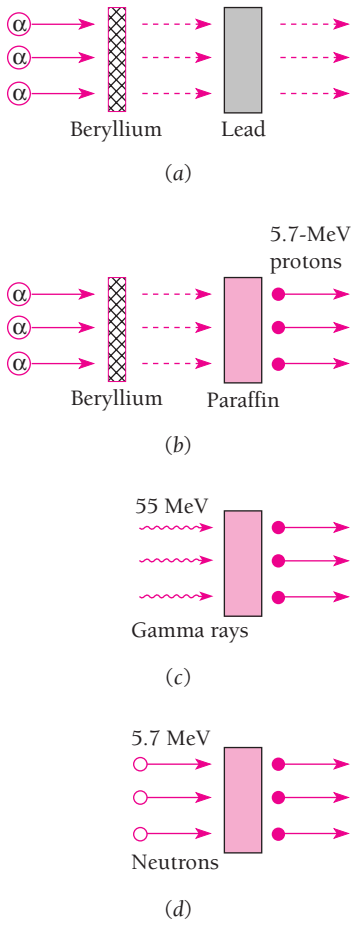
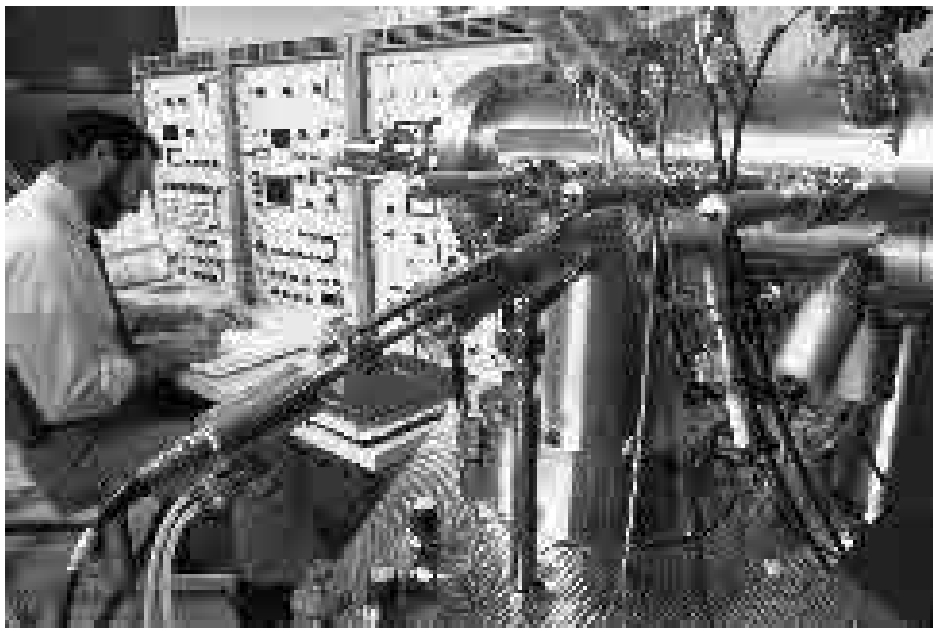


Figure 11.2 (*a*) Alpha particles incident on a beryllium foil cause the emission of a very penetrating radiation. (*b*) Protons of up to 5.7 MeV are ejected when the radiation strikes a paraffin slab. (*c*) If the radiation consists of gamma rays, their energies must be at least 55 MeV. (*d*) If the radiation consists of neutral particles of approximately proton mass, their energies need not exceed 5.7 MeV.

Hence ordinary hydrogen is ${}^{1}_{1}\mathrm{H}$, deuterium is ${}^{2}_{1}\mathrm{H}$, and the two isotopes of chlorine ($Z = 17$), whose nuclei contain 18 and 20 neutrons respectively, are ${}^{35}_{17}\mathrm{Cl}$ and ${}^{37}_{17}\mathrm{Cl}$. Because every element has a characteristic atomic number, Z is often omitted from the symbol for a nuclide: ${}^{35}\mathrm{Cl}$ (read as "chlorine 35") instead of ${}^{35}_{17}\mathrm{Cl}$.

Atomic Masses

Atomic masses refer to the masses of neutral atoms, not of bare nuclei. Thus an atomic mass always includes the masses of its Z electrons. Atomic masses are expressed in



Mass spectrometer being used to study the composition of semiconductor crystals.

mass units (u), which are so defined that the mass of a $^{12}_{6}C$ atom, the most abundant isotope of carbon, is exactly 12 u. The value of a mass unit is

Atomic mass unit $1\ \text{u} = 1.66054 \times 10^{-27}\ \text{kg}$

The energy equivalent of a mass unit is 931.49 MeV. Table 11.1 gives the masses of the proton, neutron, electron, and $^{1}_{1}H$ atom in various units, including the MeV/c^2. The advantage of using this unit is that the energy equivalent of a mass of, say, 10 MeV/c^2 is simply $E = mc^2 = 10$ MeV.

Table 11.2 gives the compositions of the isotopes of hydrogen and chlorine. Chlorine in nature consists of about three-quarters of the ^{35}Cl isotope and one-quarter of the ^{37}Cl isotope, which yields the average atomic mass of 35.46 u that chemists use (see Table 7.2). The chemical properties of an element are determined by the number and arrangement of the electrons in its atoms. Since the isotopes of an element have almost identical electron structures in their atoms, it is not surprising that the two isotopes of chlorine, for instance, have the same yellow color, the same suffocating odor, the same efficiency as poisons and bleaching agents, and the same ability to combine with metals. Because boiling and freezing points depend somewhat on atomic mass, they are slightly different for the two isotopes, as are their densities. Other physical

Table 11.1 Some Masses in Various Units

Particle	Mass (kg)	Mass (u)	Mass (MeV/c^2)
Proton	1.6726×10^{-27}	1.007276	938.28
Neutron	1.6750×10^{-27}	1.008665	939.57
Electron	9.1095×10^{-31}	5.486×10^{-4}	0.511
$^{1}_{1}H$ atom	1.6736×10^{-27}	1.007825	938.79

Table 11.2 The Isotopes of Hydrogen and Chlorine Found in Nature

	Properties of Element		Properties of Isotope				
Element	Atomic Number	Average Atomic Mass, u	Protons in Nucleus	Neutrons in Nucleus	Mass Number	Atomic Mass, u	Relative Abundance, Percent
Hydrogen	1	1.008	1	0	1	1.008	99.985
			1	1	2	2.014	0.015
			1	2	3	3.016	Very small
Chlorine	17	35.46	17	18	35	34.97	75.53
			17	20	37	36.97	24.47

properties of isotopes may very more dramatically with mass number: tritium is radioactive, for instance, whereas ordinary hydrogen and deuterium are not.

Nuclear Electrons

Nuclide masses are always very close to being integral multiples of the mass of the hydrogen atom, as we can see in Table 11.2. Before the discovery of the neutron, it was tempting to regard all nuclei as consisting of protons together with enough electrons to neutralize the positive charge of some of them. This hypothesis is buttressed by the fact that certain radioactive nuclei spontaneously emit electrons, a phenomenon called beta decay. However, there are some strong arguments against the idea of nuclear electrons.

1 Nuclear size. In Example 3.7 we saw that an electron confined to a box of nuclear dimensions must have an energy of more than 20 MeV, whereas electrons emitted during beta decay have energies of only 2 or 3 MeV, an order of magnitude smaller. A similar calculation for protons gives a minimum energy of around 0.2 MeV, which is entirely plausible.
2 Nuclear spin. Protons and electrons are fermions with spins (that is, spin quantum numbers) of $\frac{1}{2}$. Thus nuclei with an even number of protons plus electrons should have 0 or integral spin, those with an odd number of protons plus electrons should have half-integral spins. This prediction is not obeyed. For instance, if a deuterium nucleus, ${}_1^2\mathrm{H}$, consisted of two protons and an electron, its nuclear spin should be $\frac{1}{2}$ or $\frac{3}{2}$, but in fact is observed to be 1.
3 Magnetic moment. The proton has a magnetic moment only about 0.15 percent that of the electron. If electrons are part of a nucleus, its magnetic moment ought to be of the order of magnitude of that of the electron. However, observed nuclear magnetic moments are comparable with that of the proton, not with that of the electron.
4 Electron-nuclear interaction. The forces that hold the constituents of a nucleus together lead to typical binding energies of around 8 MeV per particle. If some electrons can bind this strongly to protons in the nucleus of an atom, how can the other electrons in the atom remain outside the nucleus? Furthermore, when fast electrons are scattered by nuclei, they behave as though acted upon solely by electric forces, whereas the scattering of fast protons shows that a different force also acts on them.

Despite these difficulties, the hypothesis of nuclear electrons was not universally abandoned untill the discovery of the neutron in 1932. When he wrote a book on nuclear physics published the year before, George Gamow felt so uneasy about the accepted proton-electron model of the nucleus that he marked each section dealing with nuclear electrons with a skull and crossbones. When the publisher objected, Gamow replied that "It has never been my intention to scare the poor readers more than the text itself will undoubtedly do," and replaced the skull and crossbones with a less dramatic symbol.

11.2 SOME NUCLEAR PROPERTIES

Small in size, a nucleus may have angular momentum and a magnetic moment

The Rutherford scattering experiment provided the first estimates of nuclear sizes. In that experiment, as we saw in Chap. 4, an incident alpha particle is deflected by a target nucleus in a manner consistent with Coulomb's law provided the distance between them exceeds about 10^{-14} m. For smaller separations Coulomb's law is not obeyed because the nucleus no longer appears as a point charge to the alpha particle.

Since Rutherford's time a variety of experiments have been performed to determine nuclear dimensions, with particle scattering still a favored technique. Fast electrons and neutrons are ideal for this purpose, since an electron interacts with a nucleus only through electric forces while a neutron interacts only through specifically nuclear forces. Thus electron scattering provides information on the distribution of charge in a nucleus and neutron scattering provides information on the distribution of nuclear matter. In both cases the de Broglie wavelength of the particle must be smaller than the radius of the nucleus under study. What is found is that the volume of a nucleus is directly proportional to the number of nucleons it contains, which is its mass number A. This suggests that the density of nucleons is very nearly the same in the interiors of all nuclei.

If a nuclear radius is R, the corresponding volume is $\frac{4}{3}\pi R^3$ and so R^3 is proportional to A. This relationship is usually expressed in inverse form as

Nuclear radii

$$R = R_0 A^{1/3} \tag{11.1}$$

The value of R_0 is

$$R_0 \approx 1.2 \times 10^{-15}\ \text{m} \approx 1.2\ \text{fm}$$

It is necessary to be indefinite in expressing R_0 because, as Fig. 11.3 shows, nuclei do not have sharp boundaries. Despite this, the values of R from Eq. (11.1) are representative of effective nuclear sizes. The value of R_0 is slightly smaller when it is deduced from electron scattering, which implies that nuclear matter and nuclear charge are not identically distributed through a nucleus.

Nuclei are so small that the unit of length appropriate in describing them is the **femtometer** (fm), equal to 10^{-15} m. The femtometer is sometimes called the **fermi** in

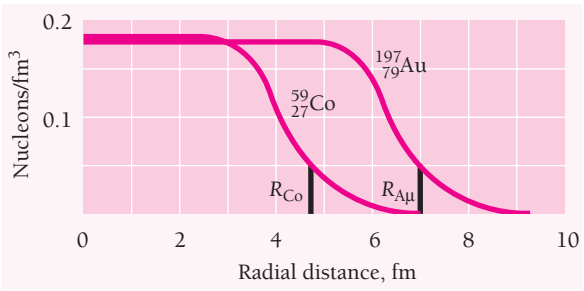


Figure 11.3 The density of nucleons in $^{59}_{27}$Co (cobalt) and $^{197}_{79}$Au (gold) nuclei plotted versus radial distance from the center. The values of the nuclear radius given by $R = 1.2A^{1/3}$ fm are indicated.

honor of Enrico Fermi, a pioneer in nuclear physics. From Eq. (11.1) we find that the radius of the $^{12}_{6}$C nucleus is

$$R \approx (1.2)(12)^{1/3} \text{ fm} \approx 2.7 \text{ fm}$$

Similarly, the radius of the $^{107}_{47}$Ag nucleus is 5.7 fm and that of the $^{238}_{92}$U nucleus is 7.4 fm.

Example 11.1

Find the density of the $^{12}_{6}$C nucleus.

Solution

The atomic mass of $^{12}_{6}$C is 12 u. Neglecting the masses and binding energies of the six electrons, we have for the nuclear density

$$\rho = \frac{m}{\frac{4}{3}\pi R^3} = \frac{(12 \text{ u})(1.66 \times 10^{-27} \text{ Kg/u})}{(\frac{4}{3}\pi)(2.7 \times 10^{-15} \text{ m})^3} = 2.4 \times 10^{17} \text{ kg/m}^3$$

This figure—equivalent to 4 billion tons per cubic inch!—is essentially the same for all nuclei. We learned in Sec. 9.11 of the existence of neutron stars, which consist of atoms that have been so compressed that their protons and electrons have interacted to become neutrons. Neutrons in such an assembly, as in a stable nucleus, do not undergo radioactive decay as do free neutrons. The densities of neutron stars are comparable with that of nuclear matter: a neutron star packs the mass of 1.4 to 3 suns into a sphere only about 10 km in radius.

Example 11.2

Find the repulsive electric force on a proton whose center is 2.4 fm from the center of another proton. Assume the protons are uniformly charged spheres of positive charge. (Protons actually have internal structures, as we shall learn in Chapter 13.)

Solution

Everywhere outside a uniformly charged sphere the sphere is electrically equivalent to a point charge located at the center of the sphere. Hence

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{(2.4 \times 10^{-15} \text{ m})^2} = 40 \text{ N}$$

This is equivalent to 9 lb, a familiar enough amount of force—but it acts on a particle whose mass is less than 2×10^{-27} kg! Evidently the attractive forces that bind protons into nuclei despite such repulsions must be very strong indeed.

Spin and Magnetic Moment

Protons and neutrons, like electrons, are fermions with spin quantum numbers of $s = \frac{1}{2}$. This means they have spin angular momenta **S** of magnitude

$$S = \sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar = \frac{\sqrt{3}}{2}\hbar \qquad (11.2)$$

and spin magnetic quantum numbers of $m_s = \pm\frac{1}{2}$ (see Fig. 7.2).

As in the case of electrons, magnetic moments are associated with the spins of protons and neutrons. In nuclear physics, magnetic moments are expressed in **nuclear magnetons** (μ_N), where

Nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p} = 5.051 \times 10^{-27}\ \text{J/T} = 3.152 \times 10^{-8}\ \text{eV/T} \tag{11.3}$$

Here m_p is the proton mass. The nuclear magneton is smaller than the Bohr magneton of Eq. (6.42) by the ratio of the proton mass to the electron mass, which is 1836. The spin magnetic moments of the proton and neutron have components in any direction of

Proton

$$\mu_{pz} = \pm 2.793\ \mu_N$$

Neutron

$$\mu_{nz} = \mp 1.913\ \mu_N$$

There are two possibilities for the signs of μ_{pz} and $\mu_{nz'}$ depending on whether m_s is $-\frac{1}{2}$ or $+\frac{1}{2}$. The $\pm$ sign is used for μ_{pz} because $\boldsymbol{\mu}_{pz}$ is in the same direction as the spin **S**, whereas $\mp$ is used for μ_{nz} because $\boldsymbol{\mu}_{nz}$ is opposite to **S** (Fig. 11.4).

At first glance it seems odd that the neutron, with no net charge, has a spin magnetic moment. But if we assume that the neutron contains equal amounts of positive and negative charge, a spin magnetic moment could arise even with no net charge. As we shall find in Chap. 13, such a picture has experimental support.

The hydrogen nucleus ${}_1^1\text{H}$ consists of a single proton, and its total angular momentum is given by Eq. (11.2). A nucleon in a more complex nucleus may have orbital angular momentum due to motion inside the nucleus as well as spin angular momentum. The total angular momentum of such a nucleus is the vector sum of the spin and orbital angular momenta of its nucleons, as in the analogous case of the electrons of an atom. This subject will be considered further in Sec. 11.6.

When a nucleus whose magnetic moment has the z component μ_z is in a constant magnetic field **B,** the magnetic potential energy of the nucleus is

Magnetic energy

$$U_m = -\mu_z B \tag{11.4}$$

This energy is negative when $\boldsymbol{\mu}_z$ is in the same direction as **B** and positive when $\boldsymbol{\mu}_z$ is opposite to **B**. In a magnetic field, each angular momentum state of the nucleus is therefore split into components, just as in the Zeeman effect in atomic electron states. Figure 11.5 shows the splitting when the angular momentum of the nucleus is due to the spin of a single proton. The energy difference between the sublevels is

$$\Delta E = 2\mu_{pz} B \tag{11.5}$$

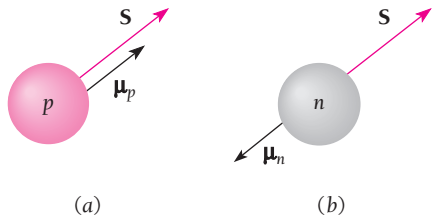


Figure 11.4 (*a*) The spin magnetic moment $\boldsymbol{\mu}_p$ of the proton is in the same direction as its spin angular momentum **S**. (*b*) In the case of the neutron, $\boldsymbol{\mu}_n$ is opposite to **S**.

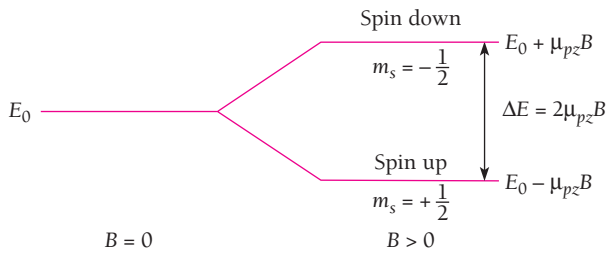


Figure 11.5 The energy levels of a proton in a magnetic field are split into spin-up ($\mathbf{S}_z$ parallel to $\mathbf{B}$) and spin-down ($\mathbf{S}_z$ antiparallel to $\mathbf{B}$) sublevels.

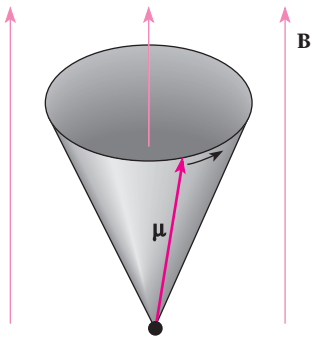


Figure 11.6 A nuclear magnetic moment $\boldsymbol{\mu}$ precesses around an external magnetic field $\mathbf{B}$ with a frequency called the Larmor frequency that is proportional to B.

A photon with this energy will be emitted when a proton in the upper state flips its spin to fall to the lower state. A proton in the lower state can be raised to the upper one by absorbing a photon of this energy. The photon frequency ν_L that corresponds to ΔE is

Larmor frequency for protons

$$\nu_L = \frac{\Delta E}{h} = \frac{2\mu_{pz}B}{h} \tag{11.6}$$

This is equal to the frequency with which a magnetic dipole precesses around a magnetic field (Fig. 11.6). It is named for Joseph Larmor, who derived ν_L from classical physics for an orbiting electron in a magnetic field; his result can be generalized to any magnetic dipole.

Example 11.3

(*a*) Find the energy difference between the spin-up and spin-down states of a proton in a magnetic field of $B = 1.000$ T (which is quite strong). (*b*) What is the Larmor frequency of a proton in this field?

Solution

(*a*) The energy difference is

$$\Delta E = 2\mu_{pz}B = (2)(2.793)(3.153 \times 10^{-8}\ \text{eV/T})(1.000\ \text{T}) = 1.761 \times 10^{-7}\ \text{eV}$$

If an electron rather than a proton were involved, ΔE would be considerably greater.
(*b*) The Larmor frequency of the proton in this field is

$$\nu_L = \frac{\Delta E}{h} = \frac{1.761 \times 10^{-7}\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 4.258 \times 10^{7}\ \text{Hz} = 42.58\ \text{MHz}$$

From Fig. 2.2 we see that em radiation of this frequency is in the lower end of the microwave part of the spectrum.

Nuclear Magnetic Resonance

Suppose we put a sample of some substance that contains nuclei with spins of $\frac{1}{2}$ in a magnetic field $\mathbf{B}$. The spins of most of these nuclei will become aligned parallel to $\mathbf{B}$

Applications of NMR

NMR turns out to be far more useful than just as a way to find nuclear magnetic moments. The electrons around a nucleus partly shield it from an external magnetic field to an extent that depends on the chemical environment of the nucleus. The **relaxation time** needed for the nuclei to drop to the lower state after having been excited also depends on this environment. These properties of NMR enable chemists to use NMR spectroscopy to help unravel details of chemical structures and reactions. For instance, the hydrogen nuclei in the CH_3, CH_2 and OH groups have slightly different resonant frequencies in the same magnetic field. All of these frequencies appear in the NMR spectrum of ethanol with a 3:2:1 ratio of intensities. Ethanol molecules are known to contain two C atoms, six H atoms, and one O atom, so they must consist of the three above groups linked together. The formula CH_3CH_2OH thus better represents methanol than C_2H_6O, which merely lists the atoms in its molecules. The intensity ratio 3:2:1 corroborates this picture since the CH_3 group has three H atoms, CH_2 has two, and OH has one. The NMR spectra of other spin-$\frac{1}{2}$ nuclei, such as ^{13}C and ^{32}P, are also of great help to chemists.

In medicine, NMR is the basis of an imaging method with higher resolution than x-ray tomography. In addition, NMR imaging is safer because rf radiation, unlike x radiation, has too little quantum energy to disrupt chemical bonds and so cannot harm living tissue. What is done is to use a nonuniform magnetic field, which means that the resonance frequency for a particular nucleus depends on the position of the nucleus in the field. Because our bodies are largely water, H_2O, proton NMR is usually employed. By changing the direction of the field gradient, an image that shows the proton density in a thin (3–4 mm) slice of the body can then be constructed by a computer. Relaxation times can also be mapped, which is useful because they are different in diseased tissue. In medicine, NMR imaging is called just magnetic resonance imaging, or MRI, to avoid frightening patients with the word "nuclear."

(spin-up) because this is the lowest energy state; see Fig. 11.5. If we now supply em radiation at the Larmor frequency ν_L to the sample, the nuclei will receive the right amount of energy to flip their spins to the higher state (spin-down). This phenomenon is called **nuclear magnetic resonance** (NMR) and it gives a way to determine nuclear magnetic moments experimentally. In one method, radio frequency (rf) radiation is supplied at a fixed frequency by a coil around the sample, and B is varied until the energy absorbed is a maximum. The resonance frequency is then the Larmor frequency for that value of B, from which μ can be calculated. Another method is to apply a broad-spectrum rf pulse and then measure the frequency (which will be ν_L) of the radiation the sample gives off as its excited nuclei return to the lower energy state.

11.3 STABLE NUCLEI

Why some combinations of neutrons and protons are more stable than others

Not all combinations of neutrons and protons form stable nuclei. In general, light nuclei ($A < 20$) contain approximately equal numbers of neutrons and protons, while in heavier nuclei the proportion of neutrons becomes progressively greater. This is evident from Fig. 11.7, which is a plot of N versus Z for stable nuclides.

The tendency for N to equal Z follows from the existence of nuclear energy levels. Nucleons, which have spins of $\frac{1}{2}$, obey the exclusion principle. As a result, each nuclear energy level can contain two neutrons of opposite spins and two protons of opposite

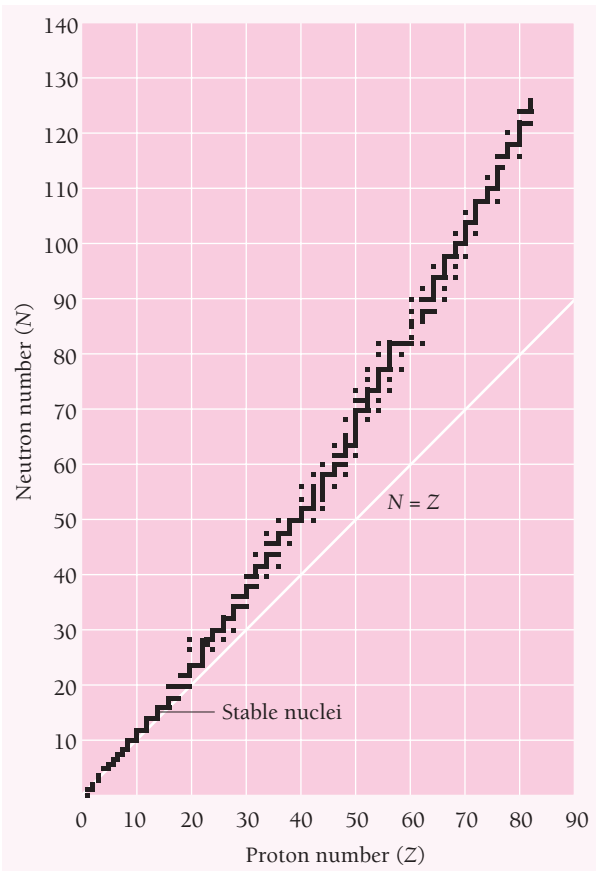


Figure 11.7 Neutron-proton diagram for stable nuclides. There are no stable nuclides with $Z = 43$ or 61, with $N = 19, 35, 39, 45, 61, 89, 115, 126$, or with $A = Z + N = 5$ or 8. All nuclides with $Z > 83$, $N > 126$, and $A > 209$ are unstable.

spins. Energy levels in nuclei are filled in sequence, just as energy levels in atoms are, to achieve configurations of minimum energy and therefore maximum stability. Thus the boron isotope $^{12}_{5}\mathrm{B}$ has more energy than the carbon isotope $^{12}_{6}\mathrm{C}$ because one of its neutrons is in a higher energy level, and $^{12}_{5}\mathrm{B}$ is accordingly unstable (Fig. 11.8). If

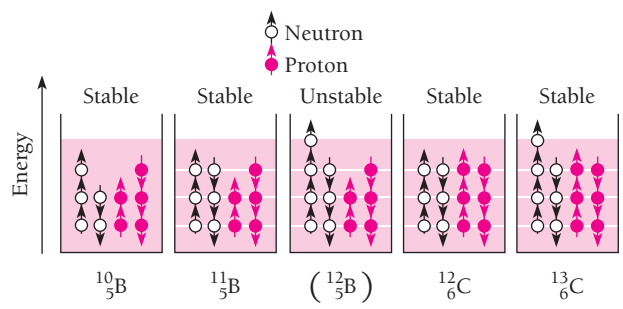


Figure 11.8 Simplified energy-level diagrams of some boron and carbon isotopes. The exclusion principle limits the occupancy of each level to two neutrons of opposite spin and two protons of opposite spin. Stable nuclei have configurations of minimum energy.

created in a nuclear reaction, a ${}^{12}_{5}\mathrm{B}$ nucleus changes by beta decay into a stable ${}^{12}_{6}\mathrm{C}$ nucleus in a fraction of a second.

The preceding argument is only part of the story. Protons are positively charged and repel one another electrically. This repulsion becomes so great in nuclei with more than 10 protons or so that an excess of neutrons, which produce only attractive forces, is required for stability. Thus the curve of Fig. 11.7 departs more and more from the $N = Z$ line as Z increases. Even in light nuclei N may exceed Z, but (except in ${}^{1}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$) is never smaller; ${}^{11}_{5}\mathrm{B}$ is stable, for instance, but not ${}^{11}_{6}\mathrm{C}$.

Sixty percent of stable nuclides have both even Z and even N; these are called "even-even" nuclides. Nearly all the others have either even Z and odd N (even-odd nuclides) or odd Z and even N (odd-even nuclides), with the numbers of both kinds being about equal. Only five stable odd-odd nuclides are known: ${}^{2}_{1}\mathrm{H}$, ${}^{6}_{3}\mathrm{Li}$, ${}^{10}_{5}\mathrm{Be}$, ${}^{14}_{7}\mathrm{N}$, and ${}^{180}_{73}\mathrm{Ta}$. Nuclear abundances follow a similar pattern of favoring even numbers for Z and N. Only about one in eight of the atoms of which the earth is composed has a nucleus with an odd number of protons, for instance.

These observations are consistent with the presence of nuclear energy levels that can each contain two particles of opposite spin. Nuclei with filled levels have less tendency to pick up other nucleons than those with partly filled levels and hence were less likely to participate in the nuclear reactions involved in the formation of the elements.

Nuclear Decay

Nuclear forces are limited in range, and as a result nucleons interact strongly only with their nearest neighbors. This effect is referred to as the **saturation** of nuclear forces. Because the coulomb repulsion of the protons is appreciable throughout the entire nucleus, there is a limit to the ability of neutrons to prevent the disruption of a large nucleus. This limit is represented by the bismuth isotope ${}^{209}_{83}\mathrm{Bi}$, which is the heaviest stable nuclide. All nuclei with $Z > 83$ and $A > 209$ spontaneously transform themselves into lighter ones through the emission of one or more alpha particles, which are ${}^{4}_{2}\mathrm{He}$ nuclei:

Alpha decay

$$ {}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\mathrm{He} $$

Parent nucleus → Daughter nucleus + Alpha particle

Since an alpha particle consists of two protons and two neutrons, an alpha decay reduces the Z and the N of the original nucleus by two each. If the resulting daughter nucleus has either too small or too large a neutron/proton ratio for stability, it may beta-decay to a more appropriate configuration. In negative beta decay, a neutron is transformed into a proton and an electron is emitted:

Beta decay

$$ n^0 \rightarrow p^+ + e^- $$

In positive beta decay, a proton becomes a neutron and a positron is emitted:

Positron emission

$$ p^+ \rightarrow n^0 + e^+ $$

Thus negative beta decay decreases the proportion of neutrons and positive beta decay increases it. A process that competes with positron emission is the capture by a

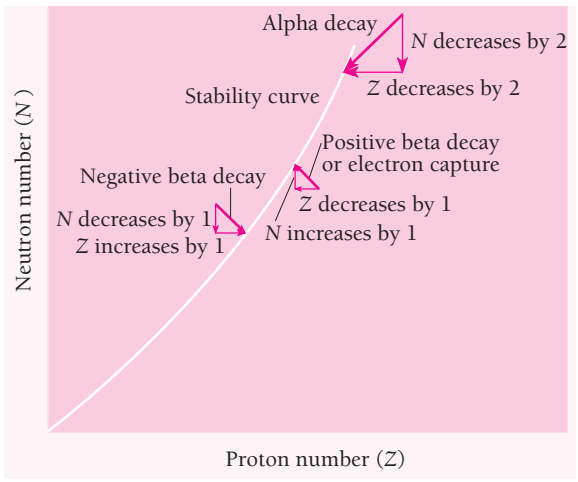


Figure 11.9 Alpha and beta decays permit an unstable nucleus to reach a stable configuration.

nucleus of an electron from its innermost shell. The electron is absorbed by a nuclear proton which is thereby transformed into a neutron:

Electron capture $$p^+ + e^- \rightarrow n^0$$

Figure 11.9 shows how alpha and beta decays enable stability to be achieved. Radioactivity is considered in more detail in Chap. 12, where we will find that another particle, the neutrino, is also involved in beta decay and electron capture.

11.4 BINDING ENERGY

The missing energy that keeps a nucleus together

The hydrogen isotope deuterium, ${}^2_1\text{H}$, has a neutron as well as a proton in its nucleus. Thus we would expect the mass of the deuterium atom to be equal to that of an ordinary ${}^1_1\text{H}$ atom plus the mass of a neutron:

Mass of ${}^1_1\text{H}$ atom	1.007825 u
+ mass of neutron	+1.008665 u
Expected mass of ${}^2_1\text{H}$ atom	2.016490 u

However, the measured mass of the ${}^2_1\text{H}$ atom is only 2.014102 u, which is 0.002388 u *less* than the combined masses of a ${}^1_1\text{H}$ atom and a neutron (Fig. 11.10).

What comes to mind is that the "missing" mass might correspond to energy given off when a ${}^2_1\text{H}$ nucleus is formed from a free proton and neutron. The energy equivalent of the missing mass is

$$\Delta E = (0.002388 \text{ u})(931.49 \text{ MeV/u}) = 2.224 \text{ MeV}$$

To test this interpretation of the missing mass, we can perform experiments to see how much energy is needed to break apart a deuterium nucleus into a separate neutron and

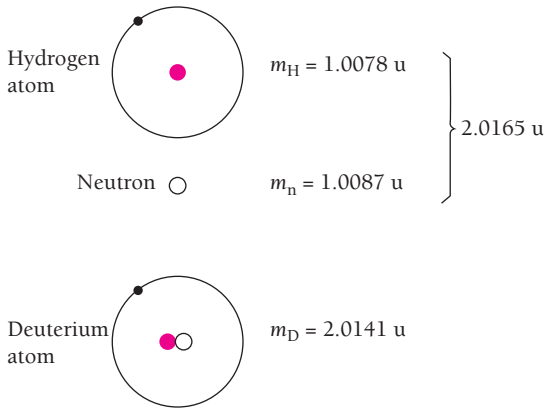


Figure 11.10 The mass of a deuterium atom ($^2_1$H) is less than the sum of the masses of a hydrogen atom ($^1_1$H) and a neutron. The energy equivalent of the missing mass is called the binding energy of the nucleus.

proton. The required energy indeed turns out to be 2.224 MeV (Fig. 11.11). When less energy than 2.224 MeV is given to a $^2_1$H nucleus, the nucleus stays together. When the added energy is more than 2.224 MeV, the extra energy goes into kinetic energy of the neutron and proton as they fly apart.

Deuterium atoms are not the only ones that have less mass than the combined masses of the particles they are composed of—*all* atoms are like that. The energy equivalent of the missing mass of a nucleus is called the **binding energy** of the nucleus. The greater its binding energy, the more the energy that must be supplied to break up the nucleus.

The binding energy E_b in MeV of the nucleus $^A_Z X$, which has $N = A - Z$ neutrons, is given by

$$E_b = [Zm(^1_1\text{H}) + Nm(n) - m(^A_Z X)](931.49 \text{ MeV/u}) \qquad (11.7)$$

where $m(^1_1\text{H})$ is the atomic mass of $^1_1$H, $m(n)$ is the neutron mass, and $m(^A_Z X)$ is the atomic mass of $^A_Z X$, all in mass units. As mentioned before, atomic masses, not nuclear masses, are used in such calculations; the electron masses subtract out.

Nuclear binding energies are strikingly high. The range for stable nuclei is from 2.224 MeV for $^2_1$H (deuterium) to 1640 MeV for $^{209}_{83}$Bi (an isotope of the metal bismuth). To appreciate how high binding energies are, we can compare them with more familiar energies in terms of kilojoules of energy per kilogram of mass. In these units, a typical binding energy is 8×10^{11} kJ/kg—800 billion kJ/kg. By contrast, to boil water

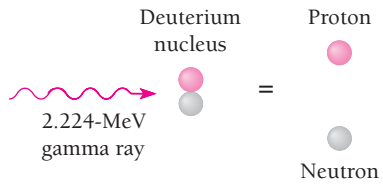


Figure 11.11 The binding energy of the deuterium nucleus is 2.224 MeV. A gamma ray whose energy is 2.224 MeV or more can split a deuterium nucleus into a proton and neutron. A gamma ray whose energy is less than 2.224 MeV cannot do this.

involves a heat of vaporization of a mere 2260 kJ/kg, and even the heat given off by burning gasoline is only 4.7×10^4 kJ/kg, 17 million times smaller.

Example 11.4

The binding energy of the neon isotope $^{20}_{10}$Ne is 160.647 MeV. Find its atomic mass.

Solution

Here $Z = 10$ and $N = 10$. From Eq. (11.7),

$$m(^A_Z X) = [Zm(^1_1\text{H}) + Nm(n)] - \frac{E_b}{931.49 \text{ MeV/u}}$$

$$m(^{20}_{10}\text{Ne}) = [10(1.007825 \text{ u}) + 10(1.008665)] - \frac{160.647 \text{ MeV}}{931.49 \text{ MeV/u}} = 19.992 \text{ u}$$

Binding Energy per Nucleon

The **binding energy per nucleon** for a given nucleus is an average found by dividing its total binding energy by the number of nucleons it contains. Thus the binding energy per nucleon for $^2_1$H is (2.2 MeV)/2 = 1.1 MeV/nucleon, and for $^{209}_{83}$Bi it is (1640 MeV)/209 = 7.8 MeV/nucleon.

Figure 11.12 shows the binding energy per nucleon plotted against the number of nucleons in various atomic nuclei. The greater the binding energy per nucleon, the

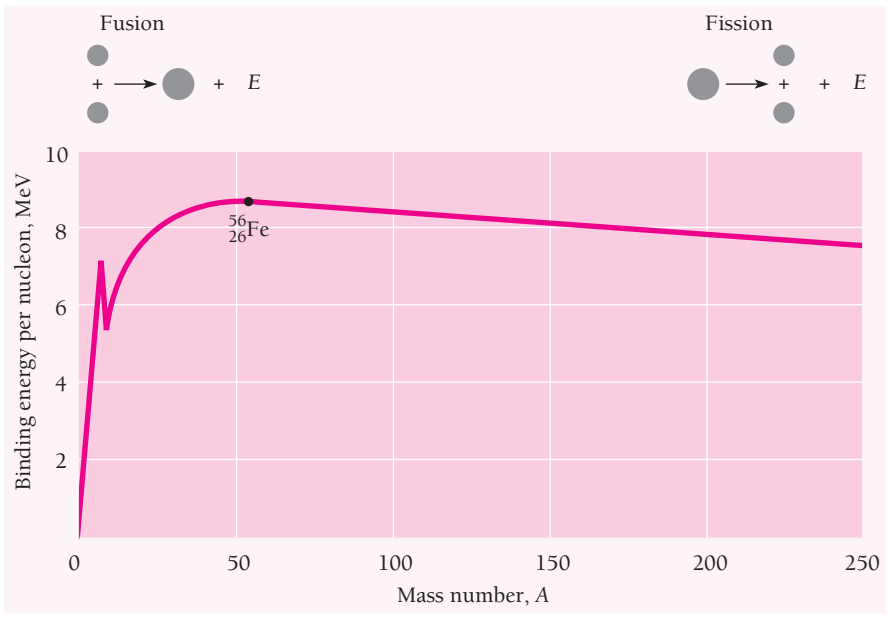


Figure 11.12 Binding energy per nucleon as a function of mass number. The peak at $A = 4$ corresponds to the exceptionally stable $^4_2$He nucleus, which is the alpha particle. The binding energy per nucleon is a maximum for nuclei of mass number $A = 56$. Such nuclei are the most stable. When two light nuclei join to form a heavier one, a process called fusion, the greater binding energy of the product nucleus causes energy to be given off. When a heavy nucleus is split into two lighter ones, a process called fission, the greater binding energy of the product nuclei also causes energy to be given off.

The Strong Interaction

The short-range attractive forces between nucleons arise from the **strong interaction.** (There is another fundamental interaction affecting nucleons called the **weak interaction** that will be discussed in Chaps. 12 and 13.) The strong interaction is what holds nucleons together to form nuclei, and it is powerful enough to overcome the electric repulsion of the positively charged protons in nuclei provided neutrons are also present to help. If the strong interaction were a little stronger—perhaps only 1 percent would be enough—two protons could stick together without any neutrons needed. In this case, when the universe came into being in the Big Bang (Sec. 13.8), all its protons would have joined into diprotons almost as soon as they appeared. Then there would be no individual protons to undergo the fusion reactions that power the stars and have created the chemical elements. The universe would be a very different place from what it is today, and we would not exist.

more stable the nucleus is. The graph has its maximum of 8.8 MeV/nucleon when the total number of nucleons is 56. The nucleus that has 56 protons and neutrons is $^{56}_{26}\mathrm{Fe}$, an iron isotope. This is the most stable nucleus of them all, since the most energy is needed to pull a nucleon away from it.

Two remarkable conclusions can be drawn from the curve of Fig. 11.12. The first is that if we can somehow split a heavy nucleus into two medium-sized ones, each of the new nuclei will have *more* binding energy per nucleon than the original nucleus did. The extra energy will be given off, and it can be a lot. For instance, if the uranium nucleus $^{235}_{92}\mathrm{U}$ is broken into two smaller nuclei, the binding energy difference per nucleon is about 0.8 MeV. The total energy given off is therefore

$$\left(0.8\ \frac{\text{MeV}}{\text{nucleon}}\right)(235\ \text{nucleons}) = 188\ \text{MeV}$$

This is a truly enormous amount of energy to be produced in a single atomic event. As we know, ordinary chemical reactions involve rearrangements of the electrons in atoms and liberate only a few electronvolts per reacting atom. Splitting a heavy nucleus, which is called **nuclear fission,** thus involves 100 million times more energy per atom than, say, the burning of coal or oil.

The other notable conclusion is that joining two light nuclei together to give a single nucleus of medium size also means more binding energy per nucleon in the new nucleus. For instance, if two $^{2}_{1}\mathrm{H}$ deuterium nuclei combine to form a $^{4}_{2}\mathrm{He}$ helium nucleus, over 23 MeV is released. Such a process, called **nuclear fusion,** is also a very effective way to obtain energy. In fact, nuclear fusion is the main energy source of the sun and other stars.

The graph of Fig. 11.12 has a good claim to being the most significant in all of science. The fact that binding energy exists at all means that nuclei more complex than the single proton of hydrogen can be stable. Such stability in turn accounts for the existence of the elements and so for the existence of the many and diverse forms of matter we see around us (and for us, too). Because the curve peaks in the middle, we have the explanation for the energy that powers, directly or indirectly, the evolution of the universe: it comes from the fusion of light nuclei to form heavier ones.

Example 11.5

(*a*) Find the energy needed to remove a neutron from the nucleus of the calcium isotope $^{42}_{20}\mathrm{Ca}$. (*b*) Find the energy needed to remove a proton from this nucleus. (*c*) Why are these energies different?

Solution

(*a*) Removing a neutron from $^{42}_{20}$Ca leaves $^{41}_{20}$Ca. From the table of atomic masses in the Appendix the mass of $^{41}_{20}$Ca plus the mass of a free neutron is

$$40.962278 \text{ u} + 1.008665 \text{ u} = 41.970943 \text{ u}$$

The difference between this mass and the mass of $^{42}_{20}$Ca is 0.012321 u, so the binding energy of the missing neutron is`

$$(0.012321 \text{ u})(931.49 \text{ MeV/u}) = 11.48 \text{ MeV}$$

(*b*) Removing a proton from $^{42}_{20}$Ca leaves the potassium isotope $^{41}_{19}$K. A similar calculation gives a binding energy of 10.27 MeV for the missing proton.
(*c*) The neutron was acted upon only by attractive nuclear forces whereas the proton was also acted upon by repulsive electric forces that decrease its binding energy.

11.5 LIQUID-DROP MODEL

A simple explanation for the binding-energy curve

The short-range force that binds nucleons so securely into nuclei is by far the strongest type of force known. Unfortunately the nuclear force is not as well understood as the electromagnetic force, and the theory of nuclear structure is less complete than the theory of atomic structure. However, even without a full understanding of the nuclear force, much progress has been made in devising nuclear models able to account for prominent aspects of nuclear properties and behavior. We shall examine some of the concepts embodied in these models in this section and the next.

While the attractive forces that nucleons exert upon one another are very strong, their range is short. Up to a separation of about 3 fm, the nuclear attraction between two protons is about 100 times stronger than the electric repulsion between them. The nuclear interactions between protons and protons, between protons and neutrons, and between neutrons and neutrons appear to be identical.

As a first approximation, we can think of each nucleon in a nucleus as interacting solely with its nearest neighbors. This situation is the same as that of atoms in a solid, which ideally vibrate about fixed positions in a crystal lattice, or that of molecules in a liquid, which ideally are free to move about while maintaining a fixed intermolecular distance. The analogy with a solid cannot be pursued because a calculation shows that the vibrations of the nucleons about their average positions would be too great for the nucleus to be stable. The analogy with a liquid, on the other hand, turns out to be extremely useful in understanding certain aspects of nuclear behavior. This analogy was proposed by George Gamow in 1929 and developed in detail by C. F. von Weizsäcker in 1935.

Let us see how the picture of a nucleus as a drop of liquid accounts for the observed variation of binding energy per nucleon with mass number. We start by assuming that the energy associated with each nucleon-nucleon bond has some value U. This energy is actually negative since attractive forces are involved, but is usually written as positive because binding energy is considered a positive quantity for convenience.

Because each bond energy U is shared by two nucleons, each has a binding energy of $\frac{1}{2}U$. When an assembly of spheres of the same size is packed together into the smallest volume, as we suppose is the case of nucleons within a nucleus, each interior sphere

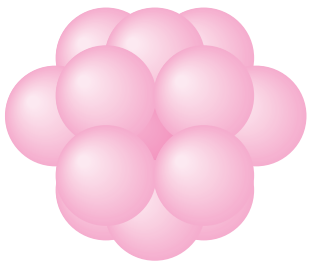


Figure 11.13 In a tightly packed assembly of identical spheres, each interior sphere is in contact with 12 others.

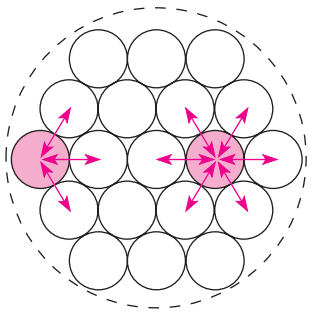


Figure 11.14 A nucleon at the surface of a nucleus interacts with fewer other nucleons than one in the interior of the nucleus and hence its binding energy is less. The larger the nucleus, the smaller the proportion of nucleons at the surface.

has 12 other spheres in contact with it (Fig. 11.13). Hence each interior nucleon in a nucleus has a binding energy of $(12)(\frac{1}{2}U)$ or 6 U. If all A nucleons in a nucleus were in its interior, the total binding energy of the nucleus would be

$$E_v = 6\,AU \tag{11.8}$$

Equation (11.8) is often written simply as

Volume energy

$$E_v = a_1 A \tag{11.9}$$

The energy E_v is called the **volume energy** of a nucleus and is directly proportional to A.

Actually, of course, some nucleons are on the surface of every nucleus and therefore have fewer than 12 neighbors (Fig. 11.14). The number of such nucleons depends on the surface area of the nucleus in question. A nucleus of radius R has an area of $4\pi R^2 = 4\pi R_0^2 A^{2/3}$. Hence the number of nucleons with fewer than the maximum number of bonds is proportional to $A^{2/3}$, reducing the total binding energy by

Surface energy

$$E_s = -a_2 A^{2/3} \tag{11.10}$$

The negative energy E_s is called the **surface energy** of a nucleus. It is most significant for the lighter nuclei since a greater fraction of their nucleons are on the surface. Because natural systems always tend to evolve toward configurations of minimum potential energy, nuclei tend toward configurations of maximum binding energy. Hence a nucleus should exhibit the same surface-tension effects as a liquid drop, and in the absence of other effects it should be spherical, since a sphere has the least surface area for a given volume.

The electric repulsion between each pair of protons in a nucleus also contributes toward decreasing its binding energy. The **coulomb energy** E_c of a nucleus is the work that must be done to bring together Z protons from infinity into a spherical aggregate the size of the nucleus. The potential energy of a pair of protons r apart is equal to

$$V = -\frac{e^2}{4\pi\epsilon_0 r}$$

Since there are $Z(Z-1)/2$ pairs of protons,

$$E_c = \frac{Z(Z-1)}{2}\,V = -\frac{Z(Z-1)e^2}{8\pi\epsilon_0}\left(\frac{1}{r}\right)_{\text{av}} \tag{11.11}$$

where $(1/r)_{\text{av}}$ is the value of $1/r$ averaged over all proton pairs. If the protons are uniformly distributed throughout a nucleus of radius R, $(1/r)_{\text{av}}$ is proportional to $1/R$ and hence to $1/A^{1/3}$, so that

Coulomb energy

$$E_c = -a_3\frac{Z(Z-1)}{A^{1/3}} \tag{11.12}$$

The coulomb energy is negative because it arises from an effect that opposes nuclear stability.

This is as far as the liquid-drop model itself can go. Let us now see how the result compares with reality.

The total binding energy E_b of a nucleus ought to be the sum of its volume, surface, and coulomb energies:

$$E_b = E_v + E_s + E_c = a_1A - a_2A^{2/3} - a_3\frac{Z(Z-1)}{A^{1/3}} \tag{11.13}$$

The binding energy per nucleon is therefore

$$\frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3\frac{Z(Z-1)}{A^{4/3}} \tag{11.14}$$

Each of the terms of Eq. (11.14) is plotted in Fig. 11.15 versus A, together with their sum E_b/A. The coefficients were chosen to make the E_b/A curve resemble as closely as possible the empirical binding energy per nucleon curve of Fig. 11.12. The fact that the theoretical curve can be made to agree so well with the empirical one means that the analogy between a nucleus and a liquid drop has at least some validity.

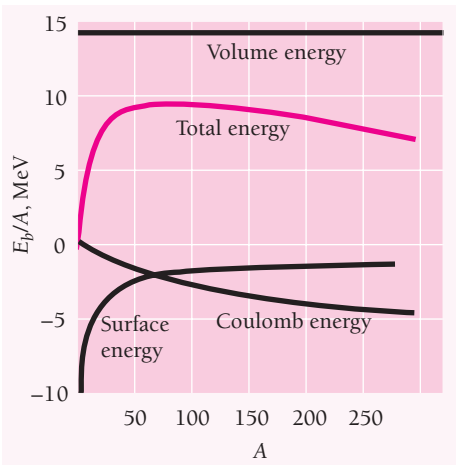


Figure 11.15 The binding energy per nucleon is the sum of the volume, surface, and coulomb energies.

Corrections to the Formula

The binding-energy formula of Eq. (11.13) can be improved by taking into account two effects that do not fit into the simple liquid-drop model but which make sense in terms of a model that provides for nuclear energy levels. (We will see in the next section how these apparently very different approaches can be reconciled.) One of these effects occurs when the neutrons in a nucleus outnumber the protons, which means that higher energy levels have to be occupied than would be the case if N and Z were equal.

Let us suppose that the uppermost neutron and proton energy levels, which the exclusion principle limits to two particles each, have the same spacing ϵ, as in Fig. 11.16. In order to produce a neutron excess of, say, $N - Z = 8$ without changing A, $\frac{1}{2}(N - Z) = 4$ neutrons would have to replace protons in an original nucleus in which $N = Z$. The new neutrons would occupy levels higher in energy by $2\epsilon = 4\epsilon/2$ than those of the protons they replace. In the general case of $\frac{1}{2}(N - Z)$ new neutrons, each must be raised in energy by $\frac{1}{2}(N - Z)\epsilon/2$. The total work needed is

$$\Delta E = (\text{number of new neutrons}) \left(\frac{\text{energy increase}}{\text{new neutron}} \right)$$

$$= \left[\frac{1}{2}(N - Z) \right] \left[\frac{1}{2}(N - Z) \frac{\epsilon}{2} \right] = \frac{\epsilon}{8}(N - Z)^2$$

Because $N = A - Z$, $(N - Z)^2 = (A - 2Z)^2$, and

$$\Delta E = \frac{\epsilon}{8}(A - 2Z)^2 \tag{11.15}$$

As it happens, the greater the number of nucleons in a nucleus, the smaller is the energy level spacing ϵ, with ϵ proportional to $1/A$. This means that the **asymmetry energy** E_a due to the difference between N and Z can be expressed as

Asymmetry energy

$$E_a = -\Delta E = -a_4 \frac{(A - 2Z)^2}{A} \tag{11.16}$$

The asymmetry energy is negative because it reduces the binding energy of the nucleus.

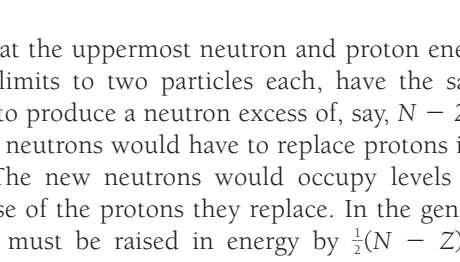


Figure 11.16 In order to replace 4 protons in a nucleus with $N = Z$ by 4 neutrons, the work $(4)(4\epsilon/2)$ must be done. The resulting nucleus has 8 more neutrons than protons.

The last correction term arises from the tendency of proton pairs and neutron pairs to occur (Sec. 11.3). Even-even nuclei are the most stable and hence have higher binding energies than would otherwise be expected. Thus such nuclei as ${}^{4}_{2}\mathrm{He}$, ${}^{12}_{6}\mathrm{C}$, and ${}^{16}_{8}\mathrm{O}$ appear as peaks on the empirical curve of binding energy per nucleon. At the other extreme, odd-odd nuclei have both unpaired protons and neutrons and have relatively low binding energies. The **pairing energy** E_p is positive for even-even nuclei, 0 for odd-even and even-odd nuclei, and negative for odd-odd nuclei, and seems to vary with A as $A^{-3/4}$. Hence

Pairing energy

$$E_p = (\pm, 0)\frac{a_5}{A^{3/4}} \tag{11.17}$$

The final expression for the binding energy of a nucleus of atomic number Z and mass number A, which was first obtained by C. F. von Weizsäcker in 1935, is

Semiempirical binding-energy formula

$$E_b = a_1 A - a_2 A^{2/3} - a_3\frac{Z(Z-1)}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A}(\pm, 0)\frac{a_5}{A^{3/4}} \tag{11.18}$$

A set of coefficients that gives a good fit with the data is as follows:

$$a_1 = 14.1\ \text{MeV} \qquad a_2 = 13.0\ \text{MeV} \qquad a_3 = 0.595\ \text{MeV}$$
$$a_4 = 19.0\ \text{MeV} \qquad a_5 = 33.5\ \text{MeV}$$

Other sets of coefficients have also been proposed. Equation (11.18) agrees better with observed binding energies than does Eq. (11.13), which suggests that the liquid-drop model, though a good approximation, is not the last word on the subject.

Example 11.6

The atomic mass of the zinc isotope ${}^{64}_{30}\mathrm{Zn}$ is 63.929 u. Compare its binding energy with the prediction of Eq. (11.18).

Solution

The binding energy of ${}^{64}_{30}\mathrm{Zn}$ is, from Eq. (11.7),

$$E_b = [(30)(1.007825\ \text{u}) + (34)(1.008665\ \text{u}) - 63.929\ \text{u}](931.49\ \text{MeV/u}) = 559.1\ \text{MeV}$$

The semiempirical binding energy formula, using the coefficients in the text, gives

$$E_b = (14.1\ \text{MeV})(64) - (13.0\ \text{MeV})(64)^{2/3} - \frac{(0.595\ \text{MeV})(30)(29)}{(64)^{1/3}}$$
$$- \frac{(19.0\ \text{MeV})(16)}{64} + \frac{33.5\ \text{MeV}}{(64)^{3/4}} = 561.7\ \text{MeV}$$

The plus sign is used for the last term because ${}^{64}_{30}\mathrm{Zn}$ is an even-even nucleus. The difference between the observed and calculated binding energies is less than 0.5 percent.

Example 11.7

Isobars are nuclides that have the same mass number A. Derive a formula for the atomic number of the most stable isobar of a given A and use it to find the most stable isobar of $A = 25$.

Solution

To find the value of Z for which the binding energy E_b is a maximum, which corresponds to maximum stability, we must solve $dE_b/dZ = 0$ for Z. From Eq. (11.18) we have

$$\frac{dE_b}{dZ} = -\frac{a_3}{A^{1/3}}(2Z - 1) + \frac{4a_4}{A}(A - 2Z) = 0$$

$$Z = \frac{a_3A^{-1/3} + 4a_4}{2a_3A^{-1/3} + 8a_4A^{-1}} = \frac{0.595A^{-1/3} + 76}{1.19A^{-1/3} + 152A^{-1}}$$

For $A = 25$ this formula gives $Z = 11.7$, from which we conclude that $Z = 12$ should be the atomic number of the most stable isobar of $A = 25$. This nuclide is ${}^{25}_{12}\mathrm{Mg}$, which is in fact the only stable $A = 25$ isobar. The other isobars, ${}^{25}_{11}\mathrm{Na}$ and ${}^{25}_{13}\mathrm{Al}$, are both radioactive.

11.6 SHELL MODEL

Magic numbers in the nucleus

The basic assumption of the liquid-drop model is that each nucleon in a nucleus interacts only with its nearest neighbors, like a molecule in a liquid. At the other extreme, the hypothesis that each nucleon interacts chiefly with a general force field produced by all the other nucleons also has a lot of support. The latter situation is like that of electrons in an atom, where only certain quantum states are permitted and no more than two electrons, which are fermions, can occupy each state. Nucleons are also fermions, and several nuclear properties vary periodically with Z and N in a manner reminiscent of the periodic variation of atomic properties with Z.

The electrons in an atom may be thought of as occupying positions in "shells" designated by the various principal quantum numbers. The degree of occupancy of the outermost shell is what determines certain important aspects of an atom's behavior. For instance, atoms with 2, 10, 18, 36, 54, and 86 electrons have all their electron shells completely filled. Such electron structures have high binding energies and are exceptionally stable, which accounts for the chemical inertness of the rare gases.

The same kind of effect is observed with respect to nuclei. Nuclei that have 2, 8, 20, 28, 50, 82, and 126 neutrons or protons are more abundant than other nuclei of similar mass numbers, suggesting that their structures are more stable. Since complex nuclei arose from reactions among lighter ones, the evolution of heavier and heavier nuclei became retarded when each relatively inert nucleus was formed, which accounts for their abundance.

Other evidence also points up the significance in nuclear structure of the numbers 2, 8, 20, 28, 50, 82, and 126, which have become known as **magic numbers.** An example is the observed pattern of nuclear electric quadrupole moments, which are measures of how much nuclear charge distributions depart from sphericity. A spherical nucleus has no quadrupole moment, while one shaped like a football has a positive

Maria Goeppert-Mayer (1906–1972) was the daughter of the pediatrician of Max Born's children, and she studied at Göttingen under Born. As Born recalled, "She went through all my courses with great industry and conscientiousness, yet remained at the same time a gay and witty member of Göttingen society, fond of parties, of laughter, dancing, and jokes. . . . After she got her doctor's degree with a very good thesis on a problem of quantum mechanics, she married a young American, Joseph Mayer, who worked with me on problems of crystal theory. Both had brilliant careers in the U.S.A., always remaining together." At the University of Chicago in 1948 Goeppert-Mayer reopened the question of periodicities in nuclear stability, which had remained a mystery since their discovery in the early 1930s, and devised a shell model that agreed with the data. J. H. D. Jensen in Germany published a similar theory independently at the same time, and both received the Nobel Prize in 1963 for their work.

moment and one shaped like a pumpkin has a negative moment. Nuclei of magic N and Z are found to have zero quadrupole moments and hence are spherical, while other nuclei are distorted in shape.

The **shell model** of the nucleus is an attempt to account for the existence of magic numbers and certain other nuclear properties in terms of nucleon behavior in a common force field.

Because the precise form of the potential-energy function for a nucleus is not known, unlike the case of an atom, a suitable function $U(r)$ has to be assumed. A reasonable guess on the basis of the nuclear density curves of Fig. 11.3 is a square well with rounded corners. Schrödinger's equation for a particle in a potential well of this kind is then solved, and it is found that stationary states of the system occur that are characterized by quantum numbers n, l, and m_l whose significance is the same as in the analogous case of stationary states of atomic electrons. Neutrons and protons occupy separate sets of states in a nucleus because the latter interact electrically as well as through the specifically nuclear charge. However, the energy levels that come from such a calculation do not agree with the observed sequence of magic numbers. Using other potential-energy functions, for instance that of the harmonic oscillator, gives no better results. Something essential is missing from the picture.

How Magic Numbers Arise

The problem was finally solved independently by Maria Goeppert-Mayer and J. H. D. Jensen in 1949. They realized that it is necessary to incorporate a spin-orbit interaction whose magnitude is such that the consequent splitting of energy levels into sublevels is many times larger than the analogous splitting of atomic energy levels. The exact form of the potential-energy function then turns out not to be critical, provided that it more or less resembles a square well.

The shell theory assumes that *LS* coupling holds only for the very lightest nuclei, in which the l values are necessarily small in their normal configurations. In this scheme, as we saw in Chap. 7, the intrinsic spin angular momenta $\mathbf{S}_i$ of the particles concerned (the neutrons form one group and the protons another) are coupled together into a total spin momentum $\mathbf{S}$. The orbital angular momenta $\mathbf{L}_i$ are separately coupled together into a total orbital momentum $\mathbf{L}$. Then $\mathbf{S}$ and $\mathbf{L}$ are coupled to form a total angular momentum $\mathbf{J}$ of magnitude $\sqrt{J(J+1)}\,\hbar$.

After a transition region in which an intermediate coupling scheme holds, the heavier nuclei exhibit ***jj* coupling.** In this case the $\mathbf{S}_i$ and $\mathbf{L}_i$ of each particle are first coupled to

form a $\mathbf{J}_i$ for that particle of magnitude $\sqrt{j(j+1)}\hbar$. The various $\mathbf{J}_i$ then couple together to form the total angular momentum $\mathbf{J}$. The *jj* coupling scheme holds for the great majority of nuclei.

When an appropriate strength is assumed for the spin-orbit interaction, the energy levels of either class of nucleon fall into the sequence shown in Fig. 11.17. The levels

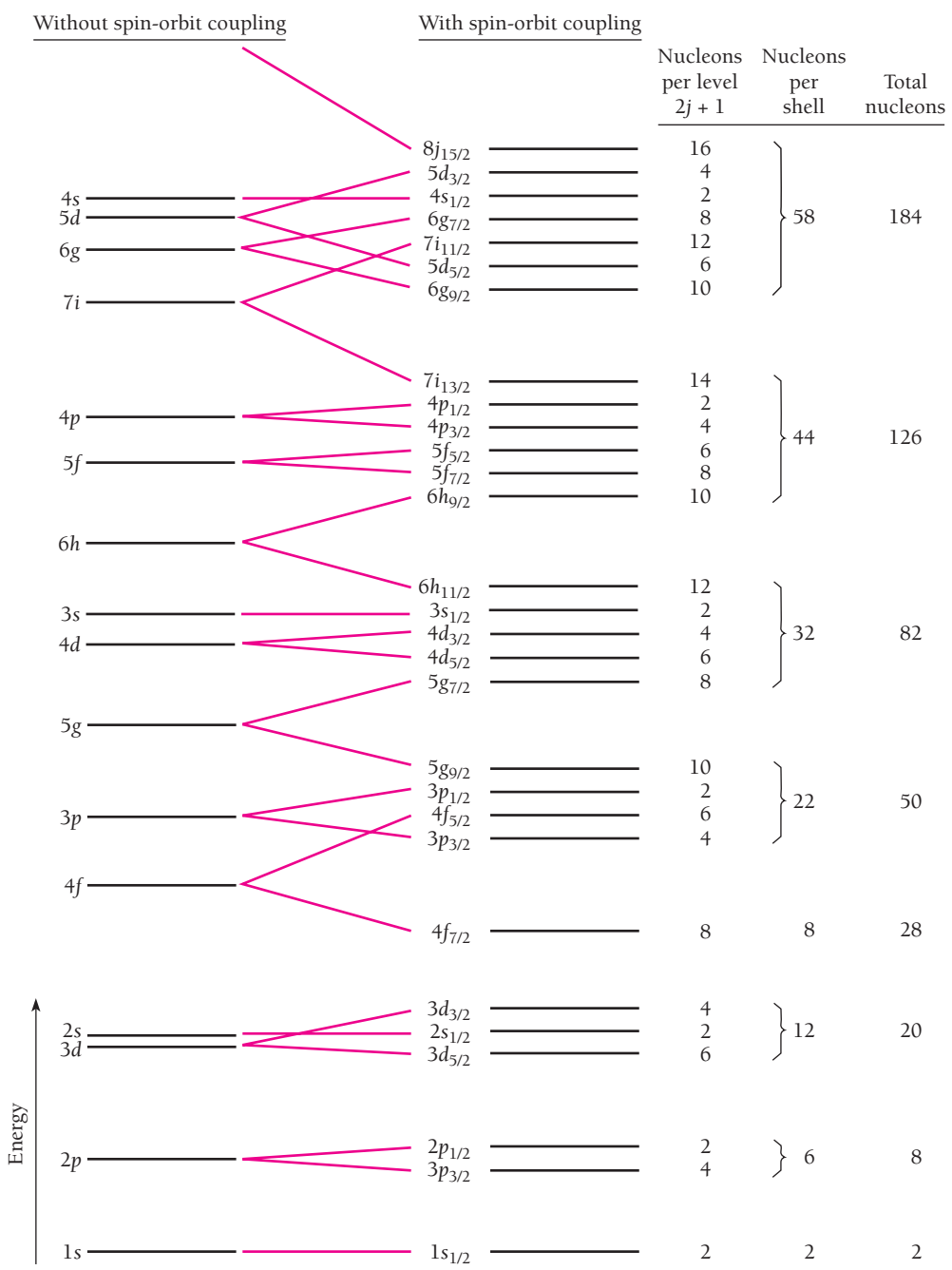


Figure 11.17 Sequence of neutron and proton energy levels according to the shell model (not to scale). The numbers in the right-hand column correspond to the magic numbers expected on the basis of this sequence.

are designated by a prefix equal to the total quantum number n, a letter that indicates l for each particle in that level according to the usual pattern (s, p, d, f, g, . . . corresponding, respectively, to $l = 0, 1, 2, 3, 4, \ldots$), and a subscript equal to j. The spin-orbit interaction splits each state of given j into $2j + 1$ substates, since there are $2j + 1$ allowed orientations of $\mathbf{J}_i$. Large energy gaps appear in the spacing of the levels at intervals that are consistent with the notion of separate shells. The number of available nuclear states in each nuclear shell is, in ascending order of energy, 2, 6, 12, 8, 22, 32, and 44. Hence shells are filled when there are 2, 8, 20, 28, 50, 82, and 126 neutrons or protons in a nucleus.

The shell model accounts for several nuclear phenomena in addition to magic numbers. To begin with, the very existence of energy sublevels that can each be occupied by two particles of opposite spin explains the tendency of nuclear abundances to favor even Z and even N as discussed in Sec. 11.3.

The shell model can also predict nuclear angular momenta. In even-even nuclei, all the protons and neutrons should pair off to cancel out one another's spin and orbital angular momenta. Thus even-even nuclei ought to have zero nuclear angular momenta, as observed. In even-odd and odd-even nuclei, the half-integral spin of the single "extra" nucleon should be combined with the integral angular momentum of the rest of the nucleus for a half-integral total angular momentum. Odd-odd nuclei each have an extra neutron and an extra proton whose half-integral spins should yield integral total angular momenta. Both these predictions are experimentally confirmed.

Reconciling the Models

If the nucleons in a nucleus are so close together and interact so strongly that the nucleus can be considered as analogous to a liquid drop, how can these same nucleons be regarded as moving independently of each other in a common force field as required by the shell model? It would seem that the points of view are mutually exclusive, since a nucleon moving about in a liquid-drop nucleus must surely undergo frequent collisions with other nucleons.

A closer look shows that there is no contradiction. In the ground state of a nucleus, the neutrons and protons fill the energy levels available to them in order of increasing energy in such a way as to obey the exclusion principle (see Fig. 11.8). In a collision, energy is transferred from one nucleon to another, leaving the former in a state of reduced energy and the latter in one of increased energy. But all the available levels of lower energy are already filled, so such an energy transfer can take place only if the exclusion principle is violated. Of course, it is possible for two indistinguishable nucleons of the same kind to merely exchange their respective energies, but such a collision is hardly significant since the system remains in exactly the same state it was in initially. In essence, then, the exclusion principle prevents nucleon-nucleon collisions even in a tightly packed nucleus and thereby justifies the independent-particle approach to nuclear structure.

Both the liquid-drop and shell models of the nucleus are, in their very different ways, able to account for much that is known of nuclear behavior. The **collective model** of Aage Bohr (Niels Bohr's son) and Ben Mottelson combines features of both models in a consistent scheme that has proved quite successful. The collective model takes into account such factors as the nonspherical shape of all but even-even nuclei and the centrifugal distortion experienced by a rotating nucleus. The detailed theory is able to account for the spacing of excited nuclear levels inferred from the gamma-ray spectra of nuclei and in other ways.

Island of Stability

As mentioned in Sec. 11.3, the short range of the strong interaction means that the largest stable nucleus is that of the bismuth isotope $^{209}_{83}$Bi. All nuclei with $Z > 83$ and $A > 209$ undergo radioactive decays until they reach a stable configuration. We can think of the stable nuclei in Fig. 11.7 as representing a peninsula of stability in a sea of instability.

In general, the farther from the peninsula of stability a nucleus is, the faster it decays. For nuclei heavier than $^{209}_{83}$Bi, lifetimes become shorter and shorter with increasing size until they are only milliseconds for $Z = 107$, 108, and 109. (Such superheavy nuclei are created in the laboratory by bombarding targets of heavy atoms with beams of lighter ones.) Since a nucleus with magic numbers of protons or neutrons is exceptionally stable, the question arises whether there might be an island of relative stability among the superheavy nuclei.

In the case of neutrons, Fig. 11.17 shows that the next magic number after $N = 126$ is $N = 184$. For protons the situation is complicated by their electric potential energy, which becomes significant relative to the purely nuclear potential energy (which is independent of charge) when Z is large. The electric potential has a greater effect on proton levels of low l because it is stronger near the nuclear center where the probability densities of such levels are concentrated (see Fig. 6.8). In consequence, the order of proton levels changes from that shown in Fig. 11.17 to make $Z = 114$ a proton magic number instead of $Z = 126$.

A nucleus with $Z = 114$ and $N = 184$ would therefore be doubly magic. This nucleus and nuclei near it in Z and N ought to form an island of stability in the sea of instability that is (so to speak) northeast of the tip of the peninsula of stability in Fig. 11.7.

In 1998 Russian physicists directed a beam of the calcium isotope $^{48}_{20}$Ca at a target of the plutonium isotope $^{244}_{94}$Pu to create a nucleus of $Z = 114$ and $N = 175$. Magic in proton number and not far from the middle of the island of stability, this nucleus has a half-life (the time needed for half a sample to decay; see Sec. 12.2) of 30.4 s. As expected, this half-life is much longer than those of nuclei near but outside the island of stability.

When the idea of an island of stability first came up in 1966, it was thought that perhaps the nucleus of $Z = 114$, $N = 184$ might have a half-life in the billions of years. Later calculations gave more modest estimates that range from less than a hundred years to millions of years. When this doubly magic nucleus is eventually produced, we will know. In the meantime, physicists at the Lawrence Berkeley National Laboratory in California have managed to sail past the island of stability to create nuclei of $Z = 116$.

11.7 MESON THEORY OF NUCLEAR FORCES

Particle exchange can produce either attraction or repulsion

In Chap. 8 we saw how a molecule is held together by the exchange of electrons between adjacent atoms. Is it possible that a similar mechanism operates inside a nucleus, with its component nucleons being held together by the exchange of particles of some kind among them?

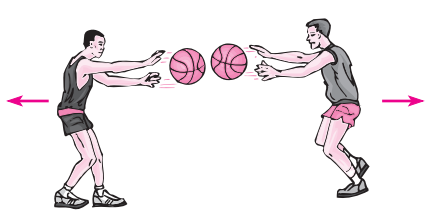
The first approach to this question was made in 1932 by Heisenberg, who suggested that electrons and positrons shift back and forth between nucleons. A neutron, for instance, might emit an electron and become a proton, while a proton absorbing the electron would become a neutron. However, calculations based on beta-decay data showed that the forces resulting from electron and positron exchange by nucleons would be too small by the huge factor of 10^{14} to be significant in nuclear structure.

Hideki Yukawa (1907–1981) grew up in Kyoto, Japan, and attended the university there. After receiving his doctorate at Osaka, he returned to Kyoto where he spent the rest of his career. In the early 1930s Yukawa tackled the problem of what keeps an atomic nucleus together despite the repulsive forces its protons exert on one another. The interaction must be extremely strong but limited in range, and Yukawa found it could be explained on the basis of the exchange between nucleons of particles whose mass is in the neighborhood of 200 electron masses: "Could the neutrons and protons be playing catch?" In 1936, the year after Yukawa published his proposal, a particle of such intermediate mass was found in cosmic rays by C. D. Anderson, who had earlier discovered the positron, and others. But, this particle, today called the muon, did not interact strongly with nuclei, as it should have. The mystery was not cleared up until 1947 when British physicist C. F. Powell discovered the pion, which has the properties Yukawa predicted but decays rapidly into the longer-lived (and hence easier-to-detect) muon. (The pion and muon were originally called the π and μ mesons by Powell because, according to legend, these were the only Greek letters on his typewriter.) Yukawa received the Nobel Prize in 1949, the first Japanese to do so.

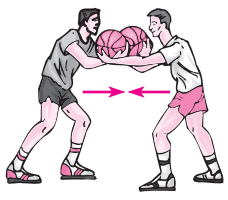
The Japanese physicist Hideki Yukawa was more successful with his 1935 proposal that particles intermediate in mass between electrons and nucleons are responsible for nuclear forces. Today these particles are called **pions.** Pions may be charged (π^+, π^-) or neutral (π^0), and are members of a class of elementary particles collectively called **mesons.** The word pion is a contraction of the original name π meson.

According to Yukawa's theory, every nucleon continually emits and reabsorbs pions. If another nucleon is nearby, an emitted pion may shift across to it instead of returning to its parent nucleon. The associated transfer of momentum is equivalent to the action of a force. Nuclear forces are repulsive at very short range as well as being attractive at greater nucleon-nucleon distances; otherwise the nucleons in a nucleus would mesh together. One of the strengths of the meson theory of such forces is that it can account for both these properties. Although there is no simple way to explain how this comes about, a rough analogy may make it less mysterious.

Let us imagine two boys exchanging basketballs (Fig. 11.18). If they throw the balls at each other, the boys move backward, and when they catch the balls thrown at them,



Repulsive force due to particle exchange



Attractive force due to particle exchange

Figure 11.18 Attractive and repulsive forces can both arise from particle exchange.

their backward momentum increases. Thus this method of exchanging basketballs has the same effect as a repulsive force between the boys. If the boys snatch the basketballs from each other's hands, however, the result will be equivalent to an attractive force acting between them.

A fundamental problem presents itself at this point. If nucleons constantly emit and absorb pions, why are neutrons and protons never found with other than their usual masses? The answer is based upon the uncertainty principle. The laws of physics refer to measurable quantities only, and the uncertainty principle limits the accuracy with which certain combinations of measurements can be made. The emission of a pion by a nucleon which does not change in mass—a clear violation of the law of conservation of energy—can take place provided that the nucleon reabsorbs it or absorbs another pion emitted by a neighboring nucleon so soon afterward that *even in principle* it is impossible to determine whether or not any mass change has actually been involved.

From the uncertainty principle in the form

$$\Delta E\,\Delta t \geq \frac{\hbar}{2} \tag{3.26}$$

an event in which an amount of energy ΔE is not conserved is not prohibited so long as the duration of the event does not exceed $\hbar/2\Delta E$. This condition lets us estimate the pion mass.

Let us assume that a pion travels between nucleons at a speed of $v \sim c$ (actually $v < c$, of course); that the emission of a pion of mass m_π represents a temporary energy discrepancy of $\Delta E \sim m_\pi c^2$ (this neglects the pion's kinetic energy); and that $\Delta E\,\Delta t \sim \hbar$. Nuclear forces have a maximum range r of about 1.7 fm, and the time Δt needed for the pion to travel this far (Fig. 11.19) is

$$\Delta t = \frac{r}{v} \sim \frac{r}{c}$$

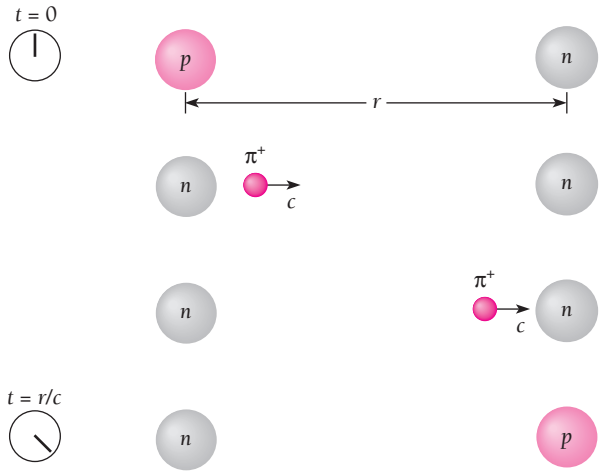


Figure 11.19 The uncertainty principle permits the creation, transfer, and disappearance of a pion to occur without violating conservation of energy provided that the sequence takes place fast enough. Here a positive pion emitted by a proton is absorbed by a neutron; as a result, the proton becomes a neutron and the neutron becomes a proton.

We therefore have

$$\Delta E\,\Delta t \sim \hbar$$

$$(m_\pi c^2)\left(\frac{r}{c}\right) \sim \hbar$$

$$m_\pi \sim \frac{\hbar}{rc} \tag{11.19}$$

which gives a value for m_π of

$$m_\pi \sim \frac{1.05 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1.7 \times 10^{-15}\ \text{m})(3 \times 10^{8}\ \text{m/s})} \sim 2 \times 10^{-28}\ \text{kg}$$

This rough figure is about 220 times the rest mass m_e of the electron.

Discovery of the Pion

A dozen years after Yukawa's proposal, particles with the properties he had predicted were actually discovered. The rest mass of charged pions is 273 m_e and that of neutral pions is 264 m_e, not far from the above estimate.

Two factors contributed to the belated discovery of the free pion. First, enough energy must be supplied to a nucleon so that its emission of a pion conserves energy. Thus at least $m_\pi c^2$ of energy, about 140 MeV, is required. To furnish a stationary nucleon with this much energy in a collision, the incident particle must have considerably more kinetic energy than $m_\pi c^2$ in order that momentum as well as energy be conserved. Particles with kinetic energies of several hundred MeV are therefore required to produce free pions, and such particles are found in nature only in the diffuse stream of cosmic radiation that bombards the earth. Hence the discovery of the pion had to await the development of sufficiently sensitive and precise methods of investigating cosmic-ray

Virtual Photons

Some years before Yukawa's work, particle exchange had been suggested as the mechanism of electromagnetic forces. In this case the particles are photons which, being massless, are not limited in range by Eq.(11.19). However, the greater the distance between two charges, the smaller must be the energies of the photons that pass between them (and hence the less the momenta of the photons and the weaker the resulting force) in order that the uncertainty principle not be violated. For this reason electric forces decrease with distance. Because the photons exchanged in the interactions of electric charges cannot be detected, they are called **virtual photons.** As in the case of pions, they can become actual photons if enough energy is somehow supplied to liberate them from the energy-conservation constraint.

The idea of photons as carriers of electromagnetic forces is attractive on many counts, an obvious one being that it explains why such forces are transmitted with the speed of light and not, say, instantaneously. As subsequently developed, the full theory is called quantum electrodynamics (see Sec. 6.9). Its conclusions have turned out to be in extraordinanly precise agreement with the data on such phenomena as the photoelectric and Compton effects, pair production and annihilation, bremsstrahlung, and photon emission by excited atoms. Unfortunately the details of the theory are too mathematically complex to consider here.

interactions. Later high-energy accelerators were placed in operation which gave the necessary particle energies, and the profusion of pions that were created with their help could be studied readily.

The second reason for the lag between the prediction and experimental discovery of the pion is its instability; the mean lifetime of the charged pion is only 2.6×10^{-8} s and that of the neutral pion is 8.4×10^{-17} s. The lifetime of the π^0 is so short, in fact, that its existence was not established until 1950. The modes of decay of the π^+, π^-, and π^0 are described in Chap. 13. Heavier mesons than the pion have also been discovered, some over a thousand times the electron mass. The contribution of these mesons to nuclear forces is, by Eq. (11.19), limited to shorter distances than those characteristic of pions.

EXERCISES

I hear, and I forget. I see, and I remember. I do, and I understand. —Anon.

11.1 Nuclear Composition

1. State the number of neutrons and protons in each of the following: ${}^{6}_{3}\mathrm{Li}$; ${}^{22}_{10}\mathrm{Ne}$; ${}^{94}_{40}\mathrm{Zr}$; ${}^{180}_{72}\mathrm{Hf}$.

2. Ordinary boron is a mixture of the ${}^{10}_{5}\mathrm{B}$ and ${}^{11}_{5}\mathrm{B}$ isotopes and has a composite atomic mass of 10.82 u. What percentage of each isotope is present in ordinary boron?

11.2 Some Nuclear Properties

3. Electrons of what energy have wavelengths comparable with the radius of a ${}^{197}_{79}\mathrm{Au}$ nucleus? (*Note*: A relativistic calculation is needed.)

4. The greater the atomic number of an atom, the larger its nucleus and the closer its inner electrons are to the nucleus. Compare the radius of the ${}^{238}_{92}\mathrm{U}$ nucleus with the radius of its innermost Bohr orbit.

5. It is believed possible on the basis of the shell model that the nuclide of $Z = 110$ and $A = 294$ may be exceptionally long-lived. Estimate its nuclear radius.

6. Show that the nuclear density of ${}^{1}_{1}\mathrm{H}$ is over 10^{14} times greater than its atomic density. (Assume the atom to have the radius of the first Bohr orbit.)

7. Compare the magnetic potential energies (in eV) of an electron and of a proton in a magnetic field of 0.10 T.

8. One type of magnetometer is based on proton precession. What is the Larmor frequency of a proton in the earth's magnetic field where its magnitude is 3.00×10^{-5} T? In what part of the em spectrum is radiation of this frequency?

9. A system of a million distinguishable protons is in thermal equilibrium at 20°C in a 1.00-T magnetic field. More of the protons are in the lower-energy spin-up state than in the higher-energy spin-down state. (*a*) On the average, how many more? (*b*) Repeat the calculation for a temperature of 20 K. (*c*) What do these results suggest about how strongly such a system will absorb em radiation at the Larmor frequency? (*d*) Could such a system in principle be used as the basis of a laser? If not, why not?

11.3 Stable Nuclei

10. The Appendix at the back of the book lists all known stable nuclides. Are there any for which $Z > N$? Why are such nuclides so rare (or absent)?

11. What limits the size of a stable nucleus?

12. What happens to the atomic number and mass number of a nucleus when it (*a*) emits an alpha particle, (*b*) emits an electron, (*c*) emits a position, (*d*) captures an electron?

13. Which nucleus would you expect to be more stable, ${}^{7}_{3}\mathrm{Li}$ or ${}^{8}_{3}\mathrm{Li}$; ${}^{13}_{6}\mathrm{C}$ or ${}^{15}_{6}\mathrm{C}$?

14. Both ${}^{14}_{8}\mathrm{O}$ and ${}^{19}_{8}\mathrm{O}$ undergo beta decay. Which would you expect to emit a positron and which an electron? Why?

11.4 Binding Energy

15. Find the binding energy per nucleon in ${}^{20}_{10}\mathrm{Ne}$ and in ${}^{56}_{26}\mathrm{Fe}$.

16. Find the binding energy per nucleon in ${}^{79}_{35}\mathrm{Br}$ and in ${}^{197}_{79}\mathrm{Au}$.

17. Find the energies needed to remove a neutron from ${}^{4}_{2}\mathrm{He}$, then to remove a proton, and finally to separate the remaining neutron and proton. Compare the total with the binding energy of ${}^{4}_{2}\mathrm{He}$.

18. The binding energy of ${}^{24}_{12}\mathrm{Mg}$ is 198.25 MeV. Find its atomic mass.

19. Show that the potential energy of two protons 1.7 fm (the maximum range of nuclear forces) apart is of the correct order of magnitude to account for the difference in binding energy between ${}^{3}_{1}\mathrm{H}$ and ${}^{3}_{2}\mathrm{He}$. How does this result bear upon the question of the dependence of nuclear forces on electric charge?

20. The neutron decays in free space into a proton and an electron. What must be the minimum binding energy contributed by a neutron to a nucleus in order that the neutron not decay inside the nucleus? How does this figure compare with the observed binding energies per nucleon in stable nuclei?

11.5 Liquid-Drop Model

21. Use the semiempirical binding-energy formula to calculate the binding energy of $^{40}_{20}$Ca. What is the percentage discrepancy between this figure and the actual binding energy?

22. Two nuclei with the same mass number for which $Z_1 = N_2$ and $Z_2 = N_1$, so that their atomic numbers differ by 1, are called **mirror isobars;** for example, $^{15}_{7}$N and $^{15}_{8}$O. The constant a_3 in the coulomb energy term of Eq. (11.18) can be evaluated from the mass difference between two mirror isobars, one of which is odd-even and the other even-odd (so that their pairing energies are zero). (*a*) Derive a formula for a_3 in terms of the mass difference between two such nuclei, their mass number A, the smaller atomic number Z of the pair, and the masses of the hydrogen atom and the neutron. (*Hint:* First show that $(A - 2Z)^2 = 1$ for both nuclei.) (*b*) Evaluate a_3 for the case of the mirror isobars $^{15}_{7}$N and $^{15}_{8}$O.

23. The coulomb energy of Z protons uniformly distributed throughout a spherical nucleus of radius R is given by

$$E_C = \frac{3}{5}\frac{Z(Z-1)e^2}{4\pi\epsilon_0 R}$$

(*a*) On the assumption that the mass difference ΔM between a pair of mirror isobars is entirely due to the difference Δm between the $^1_1$H and neutron masses and to the difference between their coulomb energies, derive a formula for R in terms of ΔM, Δm, and Z, where Z is the atomic number of the nucleus with the smaller number of protons. (*b*) Use this formula to find the radii of the mirror isobars $^{15}_{7}$N and $^{15}_{8}$O.

24. Use the formula for E_c of Exercise 23 to calculate a_3 in Eq. (11.12). If this figure is not the same as the value of 0.60 MeV quoted in the text, can you think of any reasons for the difference?

25. (*a*) Find the energy needed to remove a neutron from ^{81}Kr, from ^{82}Kr, and from ^{83}Kr. (*b*) Why is the figure for ^{82}Kr so different from the others?

26. Which isobar of $A = 75$ does the liquid-drop model suggest is the most stable?

27. Use the liquid-drop model to establish which of the mirror isobars $^{127}_{52}$Te and $^{127}_{53}$I decays into the other. What kind of decay occurs?

11.6 Shell Model

28. According to the **Fermi gas model** of the nucleus, its protons and neutrons exist in a box of nuclear dimensions and fill the lowest available quantum states to the extent permitted by the exclusion principle. Since both protons and neutrons have spins of $\frac{1}{2}$ they are fermions and obey Fermi-Dirac statistics. (*a*) Find an equation for the Fermi energy in a nucleus under the assumption that $A = 2Z$. Note that the protons and neutrons must be considered separately. (*b*) What is the Fermi energy in such a nucleus for $R_0 = 1.2$ fm? (*c*) In heavier nuclei, $A > 2Z$. What effect will this have on the Fermi energies for each type of particle?

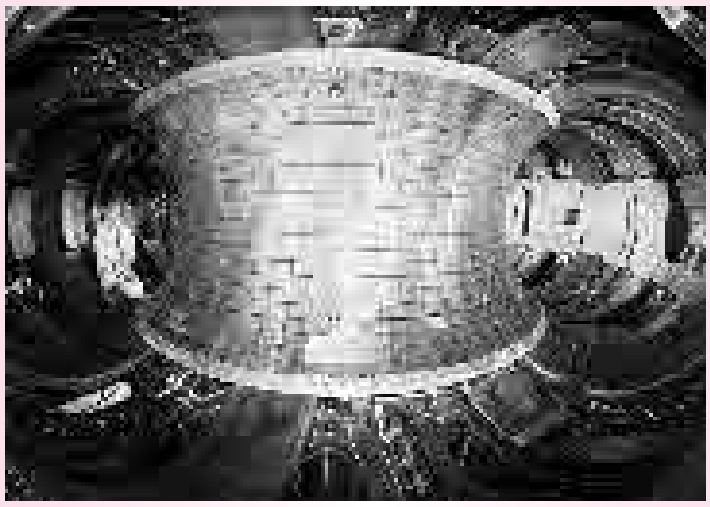
29. A simplified model of the deuteron consists of a neutron and a proton in a square potential well 2 fm in radius and 35 MeV deep. Is this model consistent with the uncertainty principle?

11.7 Meson Theory of Nuclear Forces

30. Van der Waals forces are limited to very short ranges and do not have an inverse-square dependence on distance, yet nobody suggests that the exchange of a special mesonlike particle is responsible for such forces. Why not?

CHAPTER 12

Nuclear Transformations



Interior of the Tokamak Fusion Test Reactor at the Princeton Plasma Physics Laboratory. In December 1993 this reactor produced 6.2 MW of fusion power for 4 s from a deuterium-tritium plasma confined by strong magnetic fields.

Despite the strength of the forces that hold nucleons together to form an atomic nucleus, many nuclides are unstable and spontaneously change into other nuclides by radioactive decay. And all nuclei can be transformed by reactions with nucleons or other nuclei that collide with them. In fact, all complex nuclei came into being in the first place through successive nuclear reactions, some in the first few minutes after the Big Bang and the rest in stellar interiors. The principal aspects of radioactivity and nuclear reactions are considered in this chapter.

12.1 RADIOACTIVE DECAY

Five kinds

No single phenomenon has played so significant a role in the development of nuclear physics as radioactivity, which was discovered in 1896 by Antoine Becquerel. Three features of radioactivity are extraordinary from the perspective of classical physics:

1 When a nucleus undergoes alpha or beta decay, its atomic number Z changes and it becomes the nucleus of a different element. Thus the elements are not immutable, although the mechanism of their transformation would hardly be recognized by an alchemist.
2 The energy liberated during radioactive decay comes from *within* individual nuclei without external excitation, unlike the case of atomic radiation. How can this happen? Not until Einstein proposed the equivalence of mass and energy could this puzzle be understood.
3 Radioactive decay is a statistical process that obeys the laws of chance. No cause-effect relationship is involved in the decay of a particular nucleus, only a certain probability per unit time. Classical physics cannot account for such behavior, although it fits naturally into the framework of quantum physics.

The radioactivity of an element arises from the radioactivity of one or more of its isotopes. Most elements in nature have no radioactive isotopes, although such isotopes can be prepared artificially and are useful in biological and medical research as "tracers." (The procedure is to incorporate a radionuclide in a chemical compound and follow what happens to the compound in a living organism by monitoring the radiation from the nuclide.) Other elements, such as potassium, have some stable isotopes and some radioactive ones; a few, such as uranium, have only radioactive isotopes.

The early experimenters, among them Rutherford and his coworkers, distinguished three components in the radiations from radionuclides (Figs. 12.1 and 12.2). These components

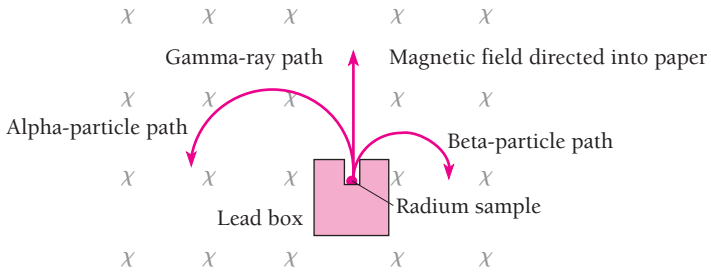
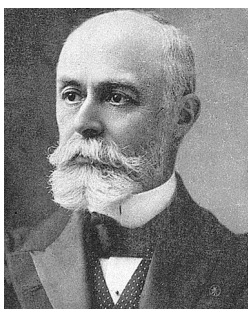


Figure 12.1 The radiations from a radium sample may be analyzed with the help of a magnetic field. Alpha particles are deflected to the left, hence they are positively charged; beta particles are deflected to the right, hence they are negatively charged; and gamma rays are not affected, hence they are unchanged.



Antoine-Henri Becquerel (1852–1908) was born and educated in Paris. His grandfather, father, and son were also physicists, all of them in turn professors at the Paris Museum of Natural History. Like his grandfather and father, Becquerel specialized in fluorescence and phosphorescence, phenomena in which a substance absorbs light at one frequency and reemits it at another, lower frequency.

In 1895 Roentgen had detected x-rays by the fluorescence they cause in an appropriate material. When he learned of this early in 1896, Becquerel wondered whether the reverse process might not occur, with intense light stimulating a fluorescent material to give off x-rays. He placed a fluorescent uranium salt on a photographic plate covered with black paper, exposed the arrangement to the sun, and indeed found the plate fogged when he had developed it. Becquerel then tried to repeat the experiment, but clouds obscured the sun for several days. He developed the plates anyway, expecting them to be clear, but to his surprise they were just as fogged as before. In a short time he had identified the source of the penetrating radiation as the uranium in the fluorescent salt. He was also able to show that the radiation ionized gases and that part of it consisted of fast charged particles.

Although Becquerel's discovery was accidental, he realized its importance at once and explored various aspects of the radioactivity of uranium for the rest of his life. He received the Nobel Prize in physics in 1903.

were called alpha, beta, and gamma, which were eventually identified as ${}^{4}_{2}\mathrm{He}$ nuclei, electrons, and high-energy photons respectively. Later, positron emission and electron capture were added to the list of decay modes. Figure 12.3 shows the five ways in which an unstable nucleus can decay, together with the reason for the instability. (The neutrinos given off when nuclei emit or absorb electrons are discussed in Sec. 12.5.) Examples of the nuclear transformations that accompany the various decays are given in Table 12.1.

Table 12.1 Radioactive Decay†

Decay	Transformation	Example
Alpha decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\mathrm{He}$	${}^{238}_{29}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$
Beta decay	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^{-}$	${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e^{-}$
Positron emission	${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + e^{+}$	${}^{64}_{29}\mathrm{Cu} \rightarrow {}^{64}_{28}\mathrm{Ni} + e^{+}$
Electron capture	${}^{A}_{Z}X + e^{-} \rightarrow {}^{A}_{Z-1}Y$	${}^{64}_{29}\mathrm{Cu} + e^{-} \rightarrow {}^{64}_{28}\mathrm{Ni}$
Gamma decay	${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma$	${}^{87}_{38}\mathrm{Sr}^{*} \rightarrow {}^{87}_{38}\mathrm{Sr} + \gamma$

†The * denotes an excited nuclear state and γ denotes a gamma-ray photon.

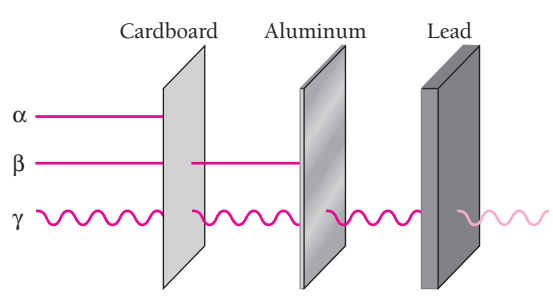


Figure 12.2 Alpha particles from radioactive materials are stopped by a piece of cardboard. Beta particles penetrate the cardboard but are stopped by a sheet of aluminum. Even a thick slab of lead may not stop all the gamma rays.

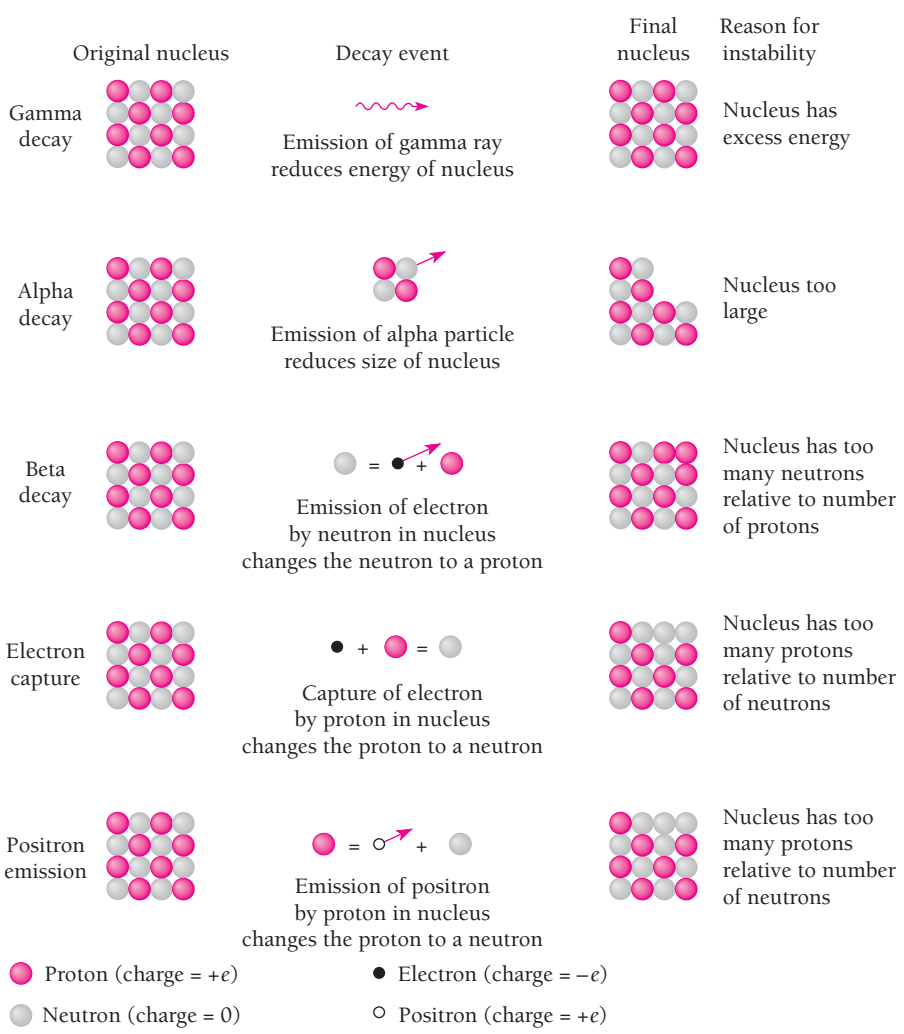


Figure 12.3 Five kinds of radioactive decay.

Example 12.1

The helium isotope $^{6}_{2}\mathrm{He}$ is unstable. What kind of decay would you expect it to undergo?

Solution

The most stable helium nucleus is $^{4}_{2}\mathrm{He}$, all of whose neutrons and protons are in the lowest possible energy levels (see Sec. 11.3). Since $^{6}_{2}\mathrm{He}$ has four neutrons whereas $^{4}_{2}\mathrm{He}$ has only two, the instability of $^{6}_{2}\mathrm{He}$ must be due to an excess of neutrons. This suggests that $^{6}_{2}\mathrm{He}$ undergoes negative beta decay to become the lithium isotope $^{6}_{3}\mathrm{Li}$ whose neutron/proton ratio is more consistent with stability:

$$^{6}_{2}\mathrm{He} \rightarrow {}^{6}_{3}\mathrm{Li} + e^{-}$$

This is, in fact, the manner in which $^{6}_{2}\mathrm{He}$ decays.

Radioactivity and the Earth

Most of the energy responsible for the geological history of the earth can be traced to the decay of the radioactive uranium, thorium, and potassium isotopes it contains. The earth is believed to have come into being perhaps 4.5 billion years ago as a cold aggregate of smaller bodies that consisted largely of metallic iron and silicate minerals that had been circling the sun. Heat of radioactive origin accumulated in the interior of the infant earth and in time led to partial melting. The influence of gravity then caused the iron to migrate inward to form the molten core of today's planet; the geomagnetic field comes from electric currents in this core. The lighter silicates rose to form the rocky mantle around the core that makes up about 80 percent of the earth's volume. Most of the earth's radioactivity is now concentrated in the upper mantle and the crust (the relatively thin outer shell), where the heat it produces escapes and cannot collect to remelt the earth. The steady stream of heat is more than enough to power the motions of the giant plates into which the earth's surface is divided and the mountain building, earthquakes, and volcanoes associated with these motions.

Activity

The **activity** of a sample of any radioactive nuclide is the rate at which the nuclei of its constituent atoms decay. If N is the number of nuclei present in the sample at a certain time, its activity R is given by

Activity

$$R = -\frac{dN}{dt} \tag{12.1}$$

The minus sign is used to make R a positive quantity since dN/dt is, of course, intrinsically negative. The SI unit of activity is named after Becquerel:

$$1 \text{ becquerel} = 1 \text{ Bq} = 1 \text{ decay/s}$$

The activities encountered in practice are usually so high that the megabecquerel ($1 \text{ MBq} = 10^6 \text{ Bq}$) and gigabecquerel ($1 \text{ GBq} = 10^9 \text{ Bq}$) are more often appropriate.

The traditional unit of activity is the **curie** (Ci), which was originally defined as the activity of 1 g of radium, $^{226}_{88}\text{Ra}$. Because the precise value of the curie changed as methods of measurement improved, it is now defined arbitrarily as

$$1 \text{ curie} = 1 \text{ Ci} = 3.70 \times 10^{10} \text{ decays/s} = 37 \text{ GBq}$$

The activity of 1 g of radium is a few percent smaller. Ordinary potassium has an activity of about 0.7 microcurie ($1\ \mu\text{Ci} = 10^{-6}$ Ci) per kilogram because it contains a small proportion of the radioisotope $^{40}_{19}\text{K}$.

Radiation Hazards

The various radiations from radionuclides ionize matter through which they pass. X-ray ionize matter, too. All ionizing radiation is harmful to living tissue, although if the damage is slight, the tissue can often repair itself with no permanent effect. Radiation hazards are easy to underestimate because there is usually a delay, sometimes of many years, between an exposure and some of its possible consequences. These consequences include cancer, leukemia, and changes in the DNA of reproductive cells that lead to children with physical deformities and mental handicaps.

Radiation dosage is measured in **sieverts** (Sv), where 1 Sv is the amount of any radiation that has the same biological effect as those produced when 1 kg of body tissue absorbs 1 joule of x-rays or gamma rays. Although radiobiologists disagree about the exact relationship between radiation exposure and the likelihood of developing cancer, there is no question that such a link exists. The International Commission on Radiation Protection estimates an average risk factor of 0.05 Sv^{-1}. This means that the chances of dying from cancer as a result of radiation are 1 in 20 for a dose of 1 Sv, 1 in 20,000 for a dose of 1 mSv (1 mSv = 0.001 Sv), and so on.

Figure 12.4 shows the chief sources of radiation dosage on a worldwide basis. The most important single source is the radioactive gas radon, a decay product of radium whose own origin traces back to the decay of uranium. Uranium is found in many common rocks, notably granite. Hense radon, colorless and odorless, is present nearly everywhere, though usually in amounts too small to endanger health. Problems arise when houses are built in uranium-rich regions, since it is impossible to prevent radon from entering such houses from the ground under them. Surveys show that millions of American homes have radon concentrations high enough to pose a nonneglible cancer risk. As a cause of lung cancer, radon is second only to cigarette smoking. The most effective method of reducing radon levels in an existing house in a hazardous region seems to be to extract air with fans from underneath the ground floor and disperse it into the atmosphere before it can enter the house.

Other natural sources of radiation dosage include cosmic rays from space and radionuclides present in rocks, soil, and building materials. Food, water, and the human body itself contain small amounts of radionuclides of such elements as potassium and carbon.

Many useful processes involve ionizing radiation. Some employ such radiation directly, as in the x-rays and gamma rays used in medicine and industry. In other cases the radiation is an unwanted but inescapable byproduct, notably in the operation of nuclear reactors and in the disposal of their wastes. In many countries the dose limit for workers (about 9 million wordwide) whose jobs involve ionizing radiation is 20 mSv per year. For the general public, which has no choice in the matter, the dose limit for nonbackground radiation is 1 mSv per year.

An appropriate balance between risk and benefit is not always easy to find where radiation is concerned. This seems particularly true for medical x-ray exposures, many of which are made for no strong reason and do more harm than good. The once "routine" x-raying of symptomless young women to search for breast cancer is now generally believed to have increased, not decreased, the overall death rate due to cancer. Particularly dangerous is the x-raying of pregnant women, until not long ago another "routine" procedure, which dramatically increases the chance of cancer in their children. Of course, x-rays have many valuable applications in medicine. The

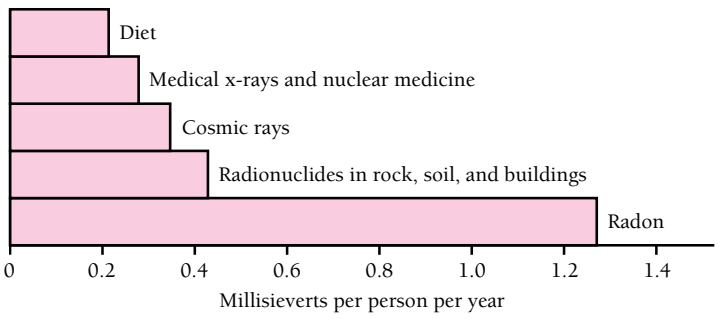


Figure 12.4 The chief sources of radiation dosage averaged around the world. The total is 2.7 mSv, but actual dosages vary widely. For instance, radon concentrations are not the same everywhere, some people receive much more medical radiation than others, cosmic rays are more intense at high altitudes (frequent fliers may get double the sea-level dose, residents of high-altitude cities up to five times as much), and so on. Nuclear power stations are responsible for less than 0.1 percent of the total, though accidents can raise the amount in affected areas to dangerous levels.

point is that every exposure should have a definite justification that outweighs the risk involved. An ordinary chest x-ray using modern equipment involves a radiation dose of about 0.017 mSv, much less than in the past. However, a CT chest scan (Sec. 2.5) involves the considerable dose of 8 mSv. CT scans of children pose especially serious risks and need equally serious justification.

12.2 HALF-LIFE

Less and less, but always some left

Measurements of the activities of radioactive samples show that, in every case, they fall off exponentially with time. Figure 12.5 is a graph of R versus t for a typical radionuclide. We note that in every 5.00-h period, regardless of when the period starts, the activity drops to half of what it was at the start of the period. Accordingly the **half-life** $T_{1/2}$ of the nuclide is 5.00 h.

Every radionuclide has a characteristic half-life. Some half-lives are only a millionth of a second, others are billions of years. One of the major problems faced by nuclear power plants is the safe disposal of radioactive wastes since some of the nuclides present have long half-lives.

The behavior illustrated in Fig. 12.5 means that the time variation of activity follows the formula

Activity law
$$R = R_0 e^{-\lambda t} \tag{12.2}$$

where λ, called the **decay constant,** has a different value for each radionuclide. The connection between decay constant λ and half-life $T_{1/2}$ is easy to find. After a half-life has elapsed, that is, when $t = T_{1/2}$, the activity R drops to $\frac{1}{2}R_0$ by definition. Hence

$$\tfrac{1}{2}R_0 = R_0 e^{-\lambda T_{1/2}}$$
$$e^{\lambda T_{1/2}} = 2$$

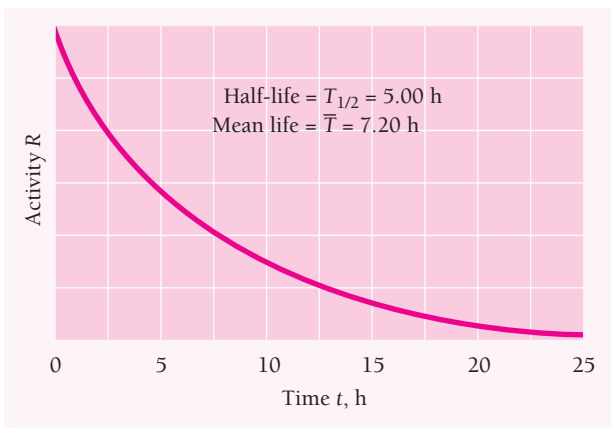


Figure 12.5 The activity of a radionuclide decreases exponentially with time. The half-life is the time needed for an initial activity to drop by half. The mean life of a radionuclide is 1.44 times its half-life [Eq. (12.7)].

Taking natural logarithms of both sides of this equation,

$$\lambda T_{1/2} = \ln 2$$

Half-life

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \tag{12.3}$$

The decay constant of the radionuclide whose half-life is 5.00 h is therefore

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5.00\text{ h})(3600\text{ s/h})} = 3.85 \times 10^{-5}\text{ s}^{-1}$$

The larger the decay constant, the greater the chance a given nucleus will decay in a certain period of time.

The activity law of Eq. (12.2) follows if we assume a constant probability λ per unit time for the decay of each nucleus of a given nuclide. With λ as the probability per unit time, $\lambda\ dt$ is the probability that any nucleus will undergo decay in a time interval dt. If a sample contains N undecayed nuclei, the number dN that decay in a time dt is the product of the number of nuclei N and the probability $\lambda\ dt$ that each will decay in dt. That is,

$$dN = -N\lambda\ dt \tag{12.4}$$

where the minus sign is needed because N decreases with increasing t.

Equation (12.4) can be rewritten

$$\frac{dN}{N} = -\lambda\ dt$$

and each side can now be integrated:

$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt$$

$$\ln N - \ln N_0 = -\lambda t$$

Radioactive decay

$$N = N_0 e^{-\lambda t} \tag{12.5}$$

This formula gives the number N of undecayed nuclei at the time t in terms of the decay probability per unit time λ of the nuclide involved and the number N_0 of undecayed nuclei at $t = 0$.

Figure 12.6 illustrates the alpha decay of the gas radon, $^{222}_{86}$Rn, whose half-life is 3.82 days, to the polonium isotope $^{218}_{84}$Po. If we start with 1.00 mg of radon in a closed container, 0.50 mg will remain after 3.82 days, 0.25 mg will remain after 7.64 days, and so on.

Example 12.2

How long does it take for 60.0 percent of a sample of radon to decay?

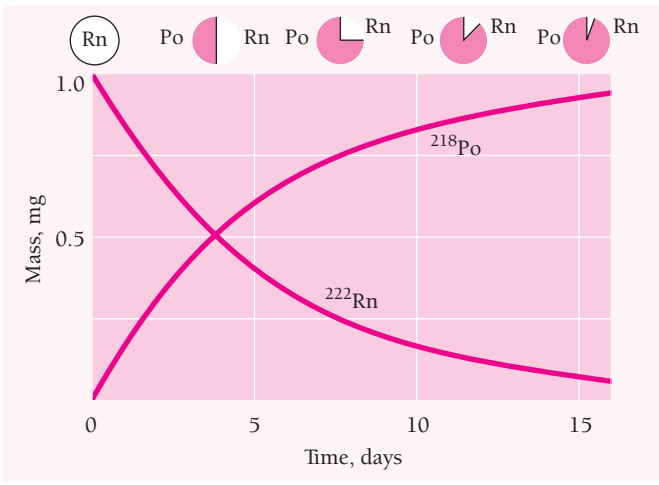


Figure 12.6 The alpha decay of ^{222}Rn to ^{218}Po has a half-life of 3.8 d. The sample of radon whose decay is graphed here had an initial mass of 1.0 mg.

Solution

From Eq. (12.5)

$$\frac{N}{N_0} = e^{-\lambda t} \qquad -\lambda t = \ln\frac{N}{N_0} \qquad \lambda t = \ln\frac{N_0}{N}$$

$$t = \frac{1}{\lambda}\ln\frac{N_0}{N}$$

Here $\lambda = 0.693/T_{1/2} = 0.693/3.82$ d and $N = (1 - 0.600)\,N_0 = 0.400N_0$, so that

$$t = \frac{3.82\text{ d}}{0.693}\ln\frac{1}{0.400} = 5.05\text{ d}$$

The fact that radioactive decay follows the exponential law of Eq. (12.2) implies that this phenomenon is statistical in nature. Every nucleus in a sample of a radionuclide has a certain probability of decaying, but there is no way to know in advance *which* nuclei will actually decay in a particular time span. If the sample is large enough—that is, if many nuclei are present—the actual fraction of it that decays in a certain time span will be very close to the probability for any individual nucleus to decay.

To say that a certain radioisotope has a half-life of 5 h, then, signifies that every nucleus of this isotope has a 50 percent change of decaying in every 5-h period. This does *not* mean a 100 percent probability of decaying in 10 h. A nucleus does not have a memory, and its decay probability per unit time is constant until it actually does decay. A half-life of 5 h implies a 75 percent probability of decay in 10 h, which increases to 87.5 percent in 15 h, to 93.75 percent in 20 h, and so on, because in every 5-h interval the probability is 50 percent.

It is worth keeping in mind that the half-life of a radionuclide is not the same as its **mean lifetime** $\overline{T}$. The mean lifetime of a nuclide is the reciprocal of its decay probability per unit time:

$$\overline{T} = \frac{1}{\lambda} \tag{12.6}$$

Hence

Mean lifetime

$$\overline{T} = \frac{1}{\lambda} = \frac{T_{1/2}}{0.693} = 1.44T_{1/2} \tag{12.7}$$

$\overline{T}$ is nearly half again more than $T_{1/2}$. The mean lifetime of a radionuclide whose half-life is 5.00 h is

$$\overline{T} = 1.44T_{1/2} = (1.44)(5.00\text{ h}) = 7.20\text{ h}$$

Since the activity of a radioactive sample is defined as

$$R = -\frac{dN}{dt}$$

we see that, from Eq. (12.5),

$$R = \lambda N_0 e^{-\lambda t}$$

This agrees with the activity law of Eq. (12.2) if $R_0 = \lambda N_0$, or, in general, if

Activity

$$R = \lambda N \tag{12.8}$$

Example 12.3

Find the activity of 1.00 mg of radon, ^{222}Rn, whose atomic mass is 222 u.

Solution

The decay constant of radon is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(3.8\text{ d})(86{,}400\text{ s/d})} = 2.11 \times 10^{-6}\text{ s}^{-1}$$

The number N of atoms in 1.00 mg of ^{222}Rn is

$$N = \frac{1.00 \times 10^{-6}\text{ kg}}{(222\text{ u})(1.66 \times 10^{-27}\text{ kg/u})} = 2.71 \times 10^{18}\text{ atoms}$$

Hence

$$R = \lambda N = (2.11 \times 10^{-6}\text{ s}^{-1})(2.71 \times 10^{18}\text{ nuclei})$$
$$= 5.72 \times 10^{12}\text{ decays/s} = 5.72\text{ TBq} = 155\text{ Ci}$$

Example 12.4

What will the activity of the above radon sample be exactly one week later?

Solution

The activity of the sample decays according to Eq. (12.2). Since $R_0 = 155$ Ci here and

$$\lambda t = (2.11 \times 10^{-6}\text{ s}^{-1})(7.00\text{ d})(86{,}400\text{ s/d}) = 1.28$$

we find that

$$R = R_0 e^{-\lambda t} = (155\text{ Ci})e^{-1.28} = 43\text{ Ci}$$

Radiometric Dating

Radioactivity makes it possible to establish the ages of many geological and biological specimens. Because the decay of any particular radionuclide is independent of its environment, the ratio between the amounts of that nuclide and its stable daughter in a specimen depends on the latter's age. The greater the proportion of the daughter nuclide, the older the specimen. Let us see how this procedure is used to date objects of biological origin using **radiocarbon,** the beta-active carbon isotope $^{14}_{6}C$.

Cosmic rays are high-energy atomic nuclei, chiefly protons, that circulate through the Milky Way galaxy of which the sun is a member. About 10^{18} of them reach the earth each second. When they enter the atmosphere, they collide with the nuclei of atoms in their paths to produce showers of secondary particles. Among these secondaries are neutrons that can react with nitrogen nuclei in the atmosphere to form radiocarbon with the emission of a proton:

Formation of radiocarbon

$$^{14}_{7}N + {}^{1}_{0}n \rightarrow {}^{14}_{6}C + {}^{1}_{1}H$$

The proton picks up an electron and becomes a hydrogen atom. Radiocarbon has too many neutrons for stability and beta decays into $^{14}_{7}N$ with a half-life of about 5760 years. Although the radiocarbon decays steadily, the cosmic-ray bombardment constantly replenishes the supply. A total of perhaps 90 tons of radiocarbon is distributed around the world at the present time.

Shortly after their formation, radiocarbon atoms combine with oxygen molecules to form carbon dioxide molecules. Green plants take in carbon dioxide and water which they convert into carbohydrates in the process of photosynthesis, so that every plant contains some radiocarbon. Animals eat plants and thereby become radioactive themselves. Because the mixing of radiocarbon is efficient, living plants and animals all have the same ratio of radiocarbon to ordinary carbon (^{12}C).

When plants and animals die, however, they no longer take in radiocarbon atoms, but the radiocarbon they contain keeps decaying away to ^{14}N. After 5760 years, then, they have only one-half as much radiocarbon left—relative to their total carbon content—as they had as living matter, after 11,520 years only one-fourth as much, and so on. By determining the proportion of radiocarbon to ordinary carbon it is therefore possible to evaluate the ages of ancient objects and remains of organic origin. This elegant method permits the dating of mummies, wooden implements, cloth, leather, charcoal from campfires, and similar artifacts from ancient civilizations as much as 50,000 years old, about nine half-lives of ^{14}C.

Example 12.5

A piece of wood from the ruins of an ancient dwelling was found to have a ^{14}C activity of 13 disintegrations per minute per gram of its carbon content. The ^{14}C activity of living wood is 16 disintegrations per minute per gram. How long ago did the tree die from which the wood sample came?

Solution

If the activity of a certain mass of carbon from a plant or animal that was recently alive is R_0 and the activity of the same mass of carbon from the sample to be dated is R, then from Eq. (12.2)

$$R = R_0 e^{-\lambda t}$$

To solve for the age t we proceed as follows:

$$e^{\lambda t} = \frac{R_0}{R} \qquad \lambda t = \ln \frac{R_0}{R} \qquad t = \frac{1}{\lambda} \ln \frac{R_0}{R}$$

From Eq. (12.3) the decay constant λ of radiocarbon is $\lambda = 0.693/T_{1/2} = 0.693/5760$ y. Here $R_0/R = 16/13$ and so

$$t = \frac{1}{\lambda} \ln \frac{R_0}{R} = \frac{5760 \text{ y}}{0.693} \ln \frac{16}{13} = 1.7 \times 10^3 \text{ y}$$

Radiocarbon dating is limited to about 50,000 years whereas the earth's history goes back 4.5 or so billion years Geologists accordingly use radionuclides of much longer half-lives to date rocks (Table 12.2). In each case it is assumed that all the stable daughter nuclides found in a particular rock sample came from the decay of the parent nuclide. Although the thorium and uranium isotopes in the table do not decay in a single step as do ^{40}K and ^{87}Rb, the half-lives of the intermediate products are so short compared with those of the parents that only the latter need be considered.

If the number of atoms of a parent nuclide in a sample is N and the number of atoms of both parent and daughter is N_0, then from Eq. (12.5)

Geological dating

$$t = \frac{1}{\lambda} \ln \frac{N_0}{N}$$

The precise significance of the time t depends on the nature of the rock involved. It may refer to the time at which the minerals of the rock crystallized, for instance, or it may refer to the most recent time at which the rock cooled below a certain temperature.

The most ancient rocks whose ages have been determined are found in Greenland and are believed to be 3.8 billion years old. Lunar rocks and meteorites as well as terrestrial rocks have been dated by the methods of Table 12.2. Some lunar samples apparently solidified 4.6 billion years ago, which is very soon after the solar system came into being. Because the youngest rocks found on the moon are 3 billion years old, the inference is that although the lunar surface was once molten and there were widespread volcanic eruptions for some time afterwards, all such activity must have ceased 3 billion years ago. To be sure, the lunar surface has been disturbed in a variety of small-scale ways since it cooled, but apparently meteoroid bombardment was responsible for most of them.

Table 12.2 Geological Dating Methods

Method	Parent Radionuclide	Stable Daughter Nuclide	Half-Life, Billion Years
Potassium-argon	^{40}K	^{40}Ar	1.3
Rubidium-strontium	^{87}Rb	^{87}Sr	47
Thorium-lead	^{232}Th	^{208}Pb	13.9
Uranium-lead	^{235}U	^{207}Pb	0.7
Uranium-lead	^{238}Pb	^{206}Pb	4.5

Astronaut Charles M. Duke, Jr., collecting rocks from the surface of the moon during the Apollo 16 expedition in 1972. The rocks were dated radiometrically. The youngest was found to be 3 billion years old, so igneous activity such as volcanic eruptions must have stopped at that time.

12.3 RADIOACTIVE SERIES

Four decay sequences that each end in a stable daughter

Most of the radionuclides found in nature are members of four **radioactive series**, with each series consisting of a succession of daughter products all ultimately derived from a single parent nuclide.

The reason that there are exactly four series follows from the fact that alpha decay reduces the mass number of a nucleus by 4. Thus the nuclides whose mass numbers are all given by $A = 4n$, where n is an integer, can decay into one another in descending order of mass number. The other three series have mass numbers specified by $A = 4n + 1$, $4n + 2$, and $4n + 3$. The members of these series, too, can decay into one another.

Table 12.3 lists the four radioactive series. The half-life of neptunium is so short compared with the age of the solar system that members of this series are not found on the earth today. They have, however, been produced in the laboratory by bombarding other heavy nuclei with neutrons, as described later. The sequence of alpha and beta decays that lead from parent to stable end product is shown in Fig. 12.7 for the uranium series. The decay chain branches at ^{214}Bi, which may decay either by alpha or beta emission. The alpha decay is followed by a beta decay and the beta decay is followed by an alpha decay, so both branches lead to ^{210}Pb.

Table 12.3 Four Radioactive Series

Mass Numbers	Series	Parent	Half-Life, Years	Stable End Product
$4n$	Thorium	$^{232}_{90}$Th	1.39×10^{10}	$^{208}_{82}$Pb
$4n + 1$	Neptunium	$^{237}_{93}$Np	2.25×10^{6}	$^{209}_{83}$Bi
$4n + 2$	Uranium	$^{238}_{92}$U	4.47×10^{9}	$^{206}_{82}$Pb
$4n + 3$	Actinium	$^{235}_{92}$U	7.07×10^{8}	$^{207}_{82}$Pb

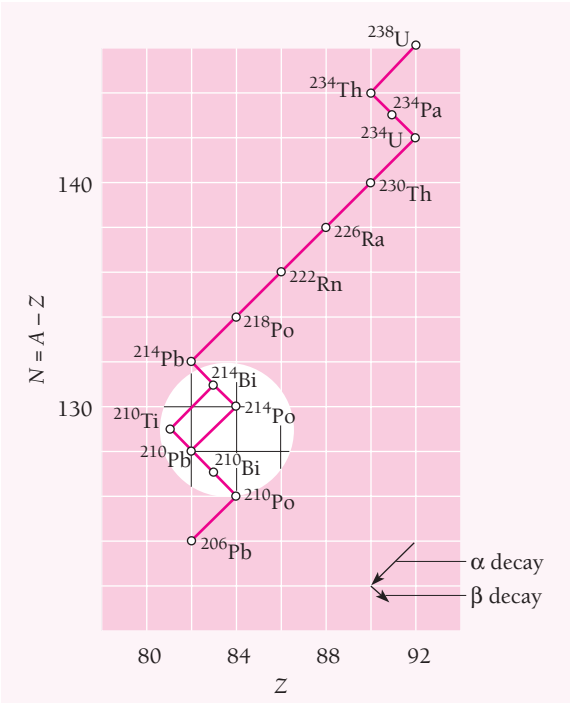


Figure 12.7 The uranium decay series ($A = 4n + 2$). The decay of $^{214}_{83}$Bi may proceed either by alpha emission and then beta emission or in the reverse order.

Marie Sklodowska Curie (1867–1934) was born in Poland, at that time under Russia's oppressive domination. Following high school, she worked as a governess until she was twenty-four so that she could study science in Paris, where she had barely enough money to survive. In 1894 Marie married Pierre Curie, eight years older and already a noted physicist. In 1897, just after the birth of her daughter Irene (who was to win a Nobel Prize in physics herself in 1935), Marie began to investigate the newly discovered phenomenon of radioactivity—her word—for her doctoral thesis.

The year before, Becquerel had found that uranium emitted a mysterious radiation. Marie, after a search of all the known elements, learned that thorium did so as well. She then examined various minerals for radioactivity. Her studies showed that the uranium ore pitchblende was far more radioactive than its uranium content would suggest. Marie and Pierre together went on to identify first polonium, named for her native Poland, and then radium as the sources of the additional activity. With the primitive facilities that were all they could afford (they had to use their own money), they had succeeded by 1902 in purifying a tenth of a gram of radium from several tons of ore, a task that involved immense physical as well as intellectual labor.

Together with Becquerel, the Curies shared the 1903 Nobel Prize in physics. Pierre ended his acceptance speech with these words: "One may also imagine that in criminal hands radium might become very dangerous, and here one may ask if humanity has anything to gain by learning the secrets of nature, if it is ready to profit from them, or if this knowledge is not harmful. . . . I am among those who think . . . that humanity will obtain more good than evil from the new discoveries."

In 1906 Pierre was struck and killed by a horse-drawn carriage in a Paris street. Marie continued work on radioactivity, still in an inadequate laboratory, and won the Nobel Prize in chemistry in 1911. Not until her scientific career was near an end did she have proper research facilities. Even before Pierre's death, both Curies had suffered from ill health because of their exposure to radiation, and much of Marie's later life was marred by radiation-induced ailments, including the leukemia from which she died.

Several alpha-radioactive nuclides whose atomic numbers are less than 82 are found in nature, though they are not very abundant.

The intermediate members of each decay series have much shorter half-lives than their parent nuclide. As a result, if we start with a sample of N_A nuclei of a parent nuclide A, after a period of time an equilibrium situation will come about in which each successive daughter B, C, . . . decays at the same rate as it is formed. Thus the activities R_A, R_B, R_C, . . . are all equal at equilibrium, and since $R = \lambda N$ we have

Radioactive equilibrium

$$N_A\lambda_A = N_B\lambda_B = N_C\lambda_C = \cdots \tag{12.9}$$

Each number of atoms N_A, N_B, N_C, . . . decreases exponentially with the decay constant λ_A of the parent nuclide, but Eq. (12.9) remains valid at any time. Equation (12.9) can be used to establish the decay constant (or half-life) of any member of the series if the decay constant of another member and their relative proportions in a sample are known.

Example 12.6

The atomic ratio between the uranium isotopes ^{238}U and ^{234}U in a mineral sample is found to be 1.8×10^4. The half-life of ^{234}U is $T_{1/2}(234) = 2.5 \times 10^5$ y. Find the half-life of ^{238}U.

Solution

Since $T_{1/2} = 0.693/\lambda$, from Eq. (12.9) we have

$$T_{1/2}(238) = \frac{N(238)}{N(234)}T_{1/2}(234)$$
$$= (1.8 \times 10^4)(2.5 \times 10^5 \text{ y}) = 4.5 \times 10^9 \text{ y}$$

This method is convenient for finding the half-lives of very long-lived and very short-lived radionuclides that are in equilibrium with other radionuclides whose half-lives are easier to measure.

12.4 ALPHA DECAY

Impossible in classical physics, it nevertheless occurs

Because the attractive forces between nucleons are of short range, the total binding energy in a nucleus is approximately proportional to its mass number A, the number of nucleons it contains. The repulsive electric forces between protons, however, are of unlimited range, and the total disruptive energy in a nucleus is approximately proportional to Z^2 [Eq. (11.12)]. Nuclei which contain 210 or more nucleons are so large that the short-range nuclear forces that hold them together are barely able to counterbalance the mutual repulsion of their protons. Alpha decay occurs in such nuclei as a means of increasing their stability by reducing their size.

Why are alpha particles emitted rather than, say, individual protons or $^3_2$He nuclei? The answer follows from the high binding energy of the alpha particle. To escape from a nucleus, a particle must have kinetic energy, and only the alpha-particle mass is sufficiently smaller than that of its constituent nucleons for such energy to be available.

To illustrate this point, we can compute, from the known masses of each particle and the parent and daughter nuclei, the energy Q released when various particles are emitted by a heavy nucleus. This is given by

Disintegration energy

$$Q = (m_i - m_f - m_x)c^2 \tag{12.10}$$

where m_i = mass of initial nucleus
m_f = mass of final nucleus
m_x = particle mass

We find that the emission of an alpha particle in some cases is energetically possible, but other decay modes would need energy supplied from outside the nucleus. Thus alpha decay in ${}^{232}_{92}\text{U}$ is accompanied by the release of 5.4 MeV, while 6.1 MeV would be needed for a proton to be emitted and 9.6 MeV for a ${}^{3}_{2}\text{He}$ nucleus to be emitted. The observed disintegration energies in alpha decay agree with the predicted values based upon the nuclear masses involved.

The kinetic energy KE_α of the emitted alpha particle is never quite equal to the disintegration energy Q because, since momentum must be conserved, the nucleus recoils with a small amount of kinetic energy when the alpha particle emerges. It is easy to show (see Exercise 23) from momentum and energy conservation that KE_α is related to Q and the mass number A of the original nucleus by

Alpha-particle energy

$$\text{KE}_\alpha \approx \frac{A-4}{A} Q \tag{12.11}$$

The mass numbers of nearly all alpha emitters exceed 210, and so most of the disintegration energy appears as the kinetic energy of the alpha particle.

Example 12.7

The polonium isotope ${}^{210}_{84}\text{Po}$ is unstable and emits a 5.30-MeV alpha particle. The atomic mass of ${}^{210}_{84}\text{Po}$ is 209.9829 u and that of ${}^{4}_{2}\text{He}$ is 4.0026 u. Identify the daughter nuclide and find its atomic mass.

Solution

(*a*) The daughter nuclide has an atomic number of $Z = 84 - 2 = 82$ and a mass number of $A = 210 - 4 = 206$. Since $Z = 82$ is the atomic number of lead, the symbol of the daughter nuclide is ${}^{206}_{82}\text{Pb}$.
(*b*) The disintegration energy that follows from an alpha-particle energy of 5.30 MeV is

$$Q = \frac{A}{A-4}\text{KE}_\alpha = \left(\frac{210}{210-4}\right)(5.30\ \text{MeV}) = 5.40\ \text{MeV}$$

The mass equivalent of this Q value is

$$m_Q = \frac{5.40\ \text{MeV}}{931\ \text{MeV/u}} = 0.0058\ \text{u}$$

Hence

$$m_f = m_i - m_\alpha - m_Q = 209.9829\ \text{u} - 4.0026\ \text{u} - 0.0058\ \text{u} = 205.9745\ \text{u}$$

Tunnel Theory of Alpha Decay

While a heavy nucleus can, in principle, spontaneously reduce its bulk by alpha decay, there remains the problem of *how* an alpha particle can actually escape the nucleus. Figure 12.8 is a plot of the potential energy U of an alpha particle as a function of its distance r from the center of a certain heavy nucleus. The height of the potential barrier is about 25 MeV, which is equal to the work that must be done against the repulsive electric force to bring an alpha particle from infinity to a position adjacent to the nucleus but just outside the range of its attractive forces. We may therefore regard an alpha particle in such a nucleus as being inside a box whose walls require an energy of 25 MeV to be surmounted. However, decay alpha particles have energies that range from 4 to 9 MeV, depending on the particular nuclide involved—16 to 21 MeV short of the energy needed for escape.

Although alpha decay is inexplicable classically, quantum mechanics provides a straightforward explanation. In fact, the theory of alpha decay, developed independently in 1928 by Gamow and by Gurney and Condon, was greeted as an especially striking confirmation of quantum mechanics.

In the Appendix to this chapter we shall find that even a simplified treatment of the problem of the escape of an alpha particle from a nucleus gives results in agreement with experiment. Gurney and Condon made these observations in their paper: "It has hitherto been necessary to postulate some special arbitrary 'instability' of the nucleus; but in the following note it is pointed out that disintegration is a natural consequence of the laws of quantum mechanics without any special hypothesis. . . . Much has been written about the explosive violence with which the α-particle is hurled from its place in the nucleus. But from the process pictured above, one would rather say that the particle slips away almost unnoticed."

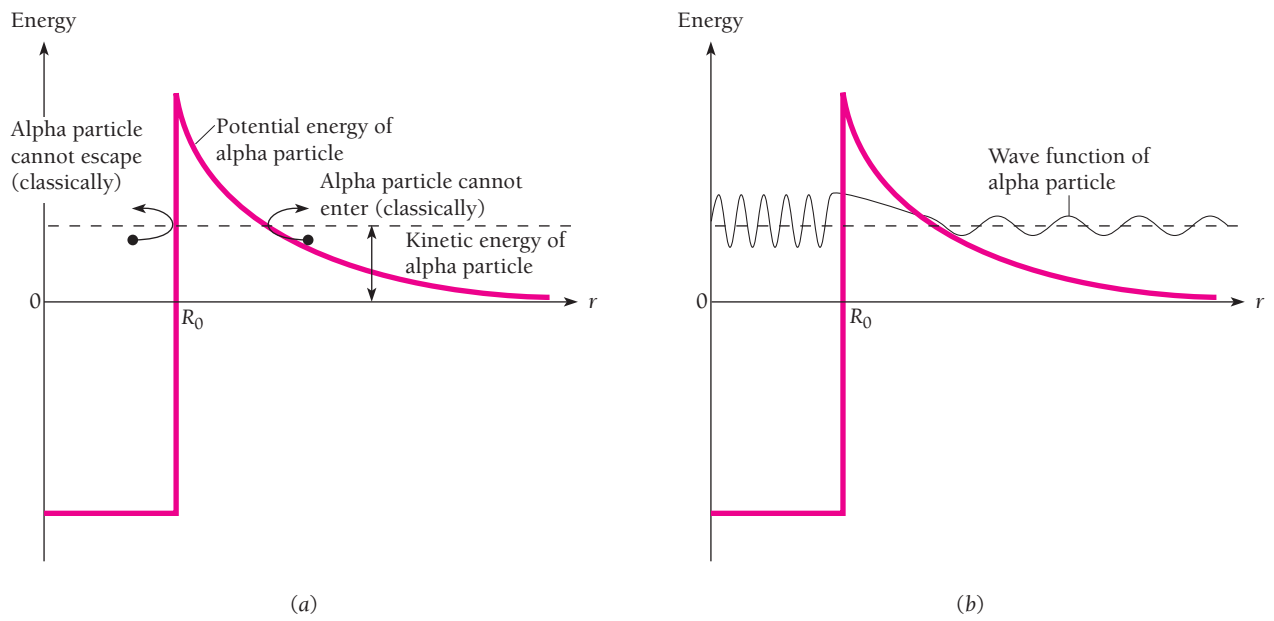


Figure 12.8 (*a*) In classical physics, an alpha particle whose kinetic energy is less than the height of the potential barrier around a nucleus cannot enter or leave the nucleus, whose radius is R_0. (*b*) In quantum physics, such an alpha particle can tunnel through the potential barrier with a probability that decreases with the height and thickness of the barrier.

George Gamow (1904–1968), born and educated in Russia, did his first important work at Göttingen in 1928 when he developed the theory of alpha decay, the first application of quantum mechanics to nuclear physics. (Edward U. Condon and Ronald W. Gurney, working together, arrived at the same theory independently of Gamow at about the same time). In 1929 he proposed the liquid-drop model of the nucleus. After periods in Copenhagen, Cambridge, and Leningrad, Gamow went to the United States in 1934 where he was first at George Washington University and later at the University of Colorado. In 1936 Gamow collaborated with Edward Teller on an extension of Fermi's theory of beta decay. Much of his later research was concerned with astrophysics, notably on the evolution of stars, where he showed that as a star uses up its supply of hydrogen in thermonuclear reactions, it becomes hotter, not cooler. Gamow also did important work on the origin of the universe (he and his students predicted the 2.7-K remnant radiation from the Big Bang) and on the formation of the elements. His books for the general public introduced many people to the concepts of modern physics.

The basic notions of this theory are:

1 An alpha particle may exist as an entity within a heavy nucleus.
2 Such a particle is in constant motion and is held in the nucleus by a potential barrier.
3 There is a small—but definite—likelihood that the particle may tunnel through the barrier (despite its height) each time a collision with it occurs.

According to the last assumption, the decay probability per unit time λ can be expressed as

Decay constant

$$\lambda = \nu T \tag{12.12}$$

Here ν is the number of times per second an alpha particle within a nucleus strikes the potential barrier around it and T is the probability that the particle will be transmitted through the barrier.

If we suppose that at any moment only one alpha particle exists as such in a nucleus and that it moves back and forth along a nuclear diameter,

Collision frequency

$$\nu = \frac{v}{2R_0} \tag{12.13}$$

where v is the alpha-particle velocity when it eventually leaves the nucleus and R_0 is the nuclear radius. Typical values of v and R_0 might be 2×10^7 m/s and 10^{-14} m respectively, so that

$$\nu \approx 10^{21}\ \text{s}^{-1}$$

The alpha particle knocks at its confining wall 10^{21} times per second and yet may have to wait an average of as much as 10^{10} y to escape from some nuclei!

As developed in the Appendix to this chapter, the tunnel theory for the decay constant λ gives the formula

Alpha decay constant

$$\log_{10} \lambda = \log_{10} \left(\frac{v}{2R_0}\right) + 1.29Z^{1/2}R_0^{1/2} - 1.72ZE^{-1/2} \tag{12.14}$$

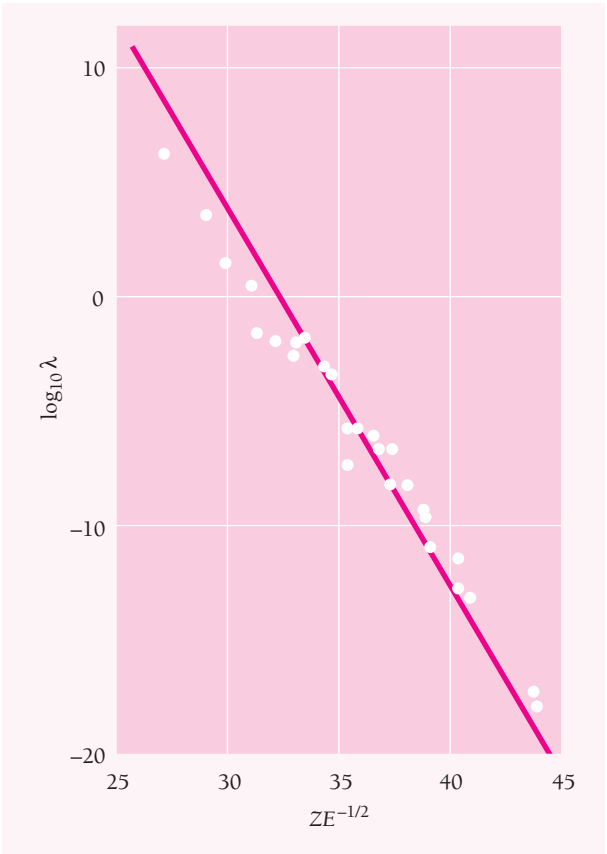


Figure 12.9 Experimental verification of the theory of alpha decay.

Here v is the alpha-particle velocity in m/s and E its energy in MeV, R_0 is the nuclear radius in fermis, and Z is the atomic number of the daughter nucleus. Figure 12.9 is a plot of $\log_{10} \lambda$ versus $ZE^{-1/2}$ for a number of alpha-radioactive nuclides. The straight line fitted to the experimental data has the -1.72 slope predicted throughout the entire range of decay constants. We can use the position of the line to determine R_0, the nuclear radius. The result is just about what is obtained from nuclear scattering experiments. This approach thus constitutes an independent means of determining nuclear sizes.

Equation (12.14) predicts that the decay constant λ, and hence the half-life, should vary strongly with the alpha-particle energy E. This is indeed the case. The slowest decay is that of $^{232}_{90}$Th, whose half-life is 1.3×10^{10} y, and the fastest decay is that of $^{212}_{84}$Po, whose half-life is 3.0×10^{-7} s. Whereas its half-life is 10^{24} greater, the alpha-particle energy of $^{232}_{90}$Th (4.05 MeV) is only about half that of $^{212}_{84}$Po (8.95 MeV).

12.5 BETA DECAY

Why the neutrino should exist and how it was discovered

Like alpha decay, beta decay is a means whereby a nucleus can alter its composition to become more stable. Also like alpha decay, beta decay has its puzzling aspects: the

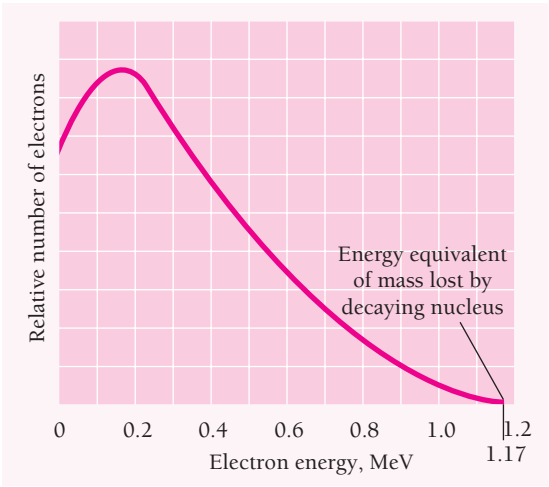


Figure 12.10 Energy spectrum of electrons from the beta decay of $^{210}_{83}$Bi.

conservation principles of energy, linear momentum, and angular momentum are all apparently violated in beta decay.

1 The electron energies observed in the beta decay of a particular nuclide are found to vary *continuously* from 0 to a maximum value KE_{max} characteristic of the nuclide. Figure 12.10 shows the energy spectrum of the electrons emitted in the beta decay of $^{210}_{83}$Bi; here $KE_{max} = 1.17$ MeV. The maximum energy

$$E_{max} = mc^2 + KE_{max}$$

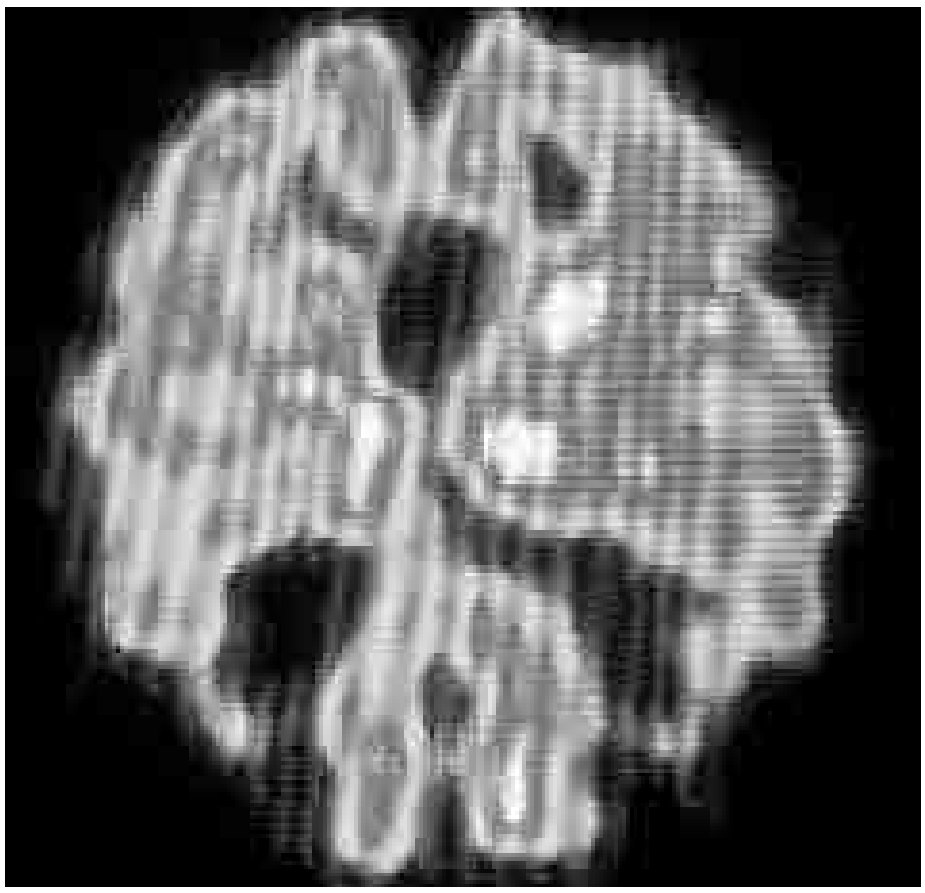
carried off by the decay electron is equal to the energy equivalent of the mass difference between the parent and daughter nuclei. Only seldom, however, is an emitted electron found with an energy of KE_{max}.
2 When the directions of the emitted electrons and of the recoiling nuclei are observed, they are almost never exactly opposite as required for linear momentum to be conserved.
3 The spins of the neutron, proton, and electron are all $\frac{1}{2}$. If beta decay involves just a neutron becoming a proton and an electron, spin (and hence angular momentum) is not conserved.

In 1930 Pauli proposed a "desperate remedy": if an uncharged particle of small or zero rest mass and spin $\frac{1}{2}$ is emitted in beta decay together with the electron, the above discrepancies would not occur. This particle, later called the **neutrino** ("little neutral one") by Fermi, would carry off an energy equal to the difference between KE_{max} and the actual KE of the electron (the recoiling nucleus carries away negligible KE). The neutrino's linear momentum also exactly balances those of the electron and the recoiling daughter nucleus.

Subsequently it was found that *two* kinds of neutrinos are involved in beta decay, the neutrino itself (symbol ν) and the **antineutrino** (symbol $\bar{\nu}$). The distinction between them is discussed in Chap. 13. In ordinary beta decay it is an antineutrino that is emitted:

Beta decay
$$n \rightarrow p + e^- + \bar{\nu} \qquad (12.15)$$

The neutrino hypothesis has turned out to be completely successful. The neutrino mass was not expected to be more than a small fraction of the electron mass because KE_{max} is observed to be equal (within experimental error) to the value calculated from the parent-daughter mass difference. The neutrino mass is now believed to be the mass equivalent of at most a few electronvolts. The interaction of neutrinos with matter is extremely feeble. Lacking charge and mass, and not electromagnetic in nature as is the photon, the neutrino can pass unimpeded through vast amounts of matter. A neutrino would have to pass through over 100 *light-years* of solid iron on the average before interacting! The only interaction with matter a neutrino can experience is through a process called inverse beta decay, which we shall consider shortly. Neutrinos are believed to outnumber protons in the universe by about a billion to one.



A positron emission tomography (PET) scan of the brain of a patient with Alzheimer's disease. The lighter the area, the higher the rate of metabolic activity. In PET, a suitable positron-emitting radionuclide (here the oxygen isotope ^{15}O) is injected and allowed to circulate in a patient's body. When a positron encounters an electron, which it does almost at once after being emitted, both are annihilated. From the directions of the resulting pair of gamma rays the location of the annihilation, and hence of the emitting nucleus, can be found. In this way, a map that is accurate to several millimeters of the concetration of the radionuclide can be built up. In a normal brain, metabolic activity produces a similar PET pattern in each hemisphere; here, the irregular appearance of the scan indicates degeneration of brain tissue.

Positions were discovered in 1932 and two years later were found to be spontaneously emitted by certain nuclei. The properties of the positron are identical with those of the electron except that it carries a charge of $+e$ instead of $-e$. Positron emission corresponds to the conversion of a nuclear proton into a neutron, a positron, and a neutrino:

Positron emission

$$p \rightarrow n + e^+ + \nu \tag{12.16}$$

Whereas a neutron outside a nucleus undergoes negative beta decay into a proton (half-life = 10 min 16 s) because its mass is greater than that of the proton, the lighter proton cannot be transformed into a neutron except within a nucleus. Positron emission leads to a daughter nucleus of lower atomic number Z while leaving the mass number A unchanged.

Closely connected with positron emission is electron capture. In electron capture a nucleus absorbs one of its inner atomic electrons, with the result that a nuclear proton becomes a neutron and a neutrino is emitted:

Electron capture

$$p + e^- \rightarrow n + \nu \tag{12.17}$$

Usually the absorbed electron comes from the K shell, and an x-ray photon is emitted when one of the atom's outer electrons falls into the resulting vacant state. The wavelength of the photon will be one of those characteristic of the daughter element, not of the original one, and the process can be recognized on this basis.

Electron capture is competitive with positron emission since both processes lead to the same nuclear transformation. Electron capture occurs more often than positron emission in heavy nuclides because the electrons in such nuclides are relatively close to the nucleus, which promotes their interaction with it. Since nearly all the unstable nuclei found in nature are of high Z, positron emission was not discovered until several decades after electron emission had been established.

Inverse Beta Decay

By comparing Eqs. (12.16) and (12.17) we see that electron capture by a nuclear proton is equivalent to a proton's emission of a positron. Similarly the absorption of an

The Weak Interaction

The nuclear interaction that holds nucleons together to form nuclei cannot account for beta decay. Another short-range fundamental interaction turns out to be responsible: the **weak interaction.** Insofar as the structure of matter is concerned, the role of the weak interaction seems to be confined to causing beta decays in nuclei whose neutron/proton ratios are not appropriate for stability. This interaction also affects elementary particles that are not part of a nucleus and can lead to their transformation into other particles. The name "weak interaction" arose because the other short-range force affecting nucleons is extremely strong, as the high binding energies of nuclei attest. The gravitational interaction is weaker than the weak interaction at distances where the latter is a factor.

Thus four fundamental interactions are apparently sufficient to govern the structure and behavior of the entire physical universe, from atoms to galaxies of stars. In order of increasing strength they are gravitational, weak nuclear, electromagnetic, and strong nuclear. These interactions and how they are related to one another and to the origin and evolution of the universe will be discussed in Chap. 13.

antineutrino is equivalent to the emission of a neutrino, and vice versa. The latter reactions are called **inverse beta decays:**

Inverse beta decay

$$p + \overline{\nu} \rightarrow n + e^+ \qquad (12.18a)$$

$$n + \nu \rightarrow p + e^- \qquad (12.18b)$$

Inverse beta decays have extremely low probabilities, which is why neutrinos and antineutrinos are able to pass through such vast amounts of matter, but these probabilities are not zero. Starting in 1953, a series of experiments was carried out by F. Reines, C. L. Cowan, and others to detect the considerable flux of neutrinos (actually antineutrinos) from the beta decays that occur in a nuclear reactor. A tank of water containing a cadmium compound in solution supplied the protons which were to interact with the incident neutrinos. Surrounding the tank were gamma-ray detectors. Immediately after a proton absorbed a neutrino to yield a positron and a neutron, as in Eq. (12.18*a*), the positron encountered an electron and both were annihilated. The gamma-ray detectors responded to the resulting pair of 0.51-MeV photons. Meanwhile the newly formed neutron migrated through the solution until, after a few microseconds, it was captured by a cadmium nucleus. The new, heavier cadmium nucleus then released about 8 MeV of excitation energy divided among three or four photons, which were picked up by the detectors several microseconds after those from the positron-electron annihilation. In principle, then, the arrival of this sequence of photons at the detector is a sure sign that the reaction of Eq. (12.18*a*) has occurred. To avoid any uncertainty, the experiment was performed with the reactor alternately on and off, and the expected variation in the frequency of neutrino-capture events was observed. In this way the neutrino hypothesis was confirmed.

12.6 GAMMA DECAY

Like an excited atom, an excited nucleus can emit a photon

A nucleus can exist in states whose energies are higher than that of its ground state, just as an atom can. An excited nucleus is denoted by an asterisk after its usual symbol, for instance ${}^{87}_{38}\mathrm{Sr}^*$. Excited nuclei return to their ground states by emitting photons whose energies correspond to the energy differences between the various initial and final states in the transitions involved. The photons emitted by nuclei range in energy up to several MeV, and are traditionally called **gamma rays.**

A simple example of the relationship between energy levels and decay schemes is shown in Fig. 12.11, which pictures the beta decay of ${}^{27}_{12}\mathrm{Mg}$ to ${}^{27}_{13}\mathrm{Al}$. The half-life of the decay is 9.5 min, and it may take place to either of the two excited states of ${}^{27}_{13}\mathrm{Al}$. The resulting ${}^{27}_{13}\mathrm{Al}^*$ nucleus then undergoes one or two gamma decays to reach the ground state.

As an alternative to gamma decay, an excited nucleus in some cases may return to its ground state by giving up its excitation energy to one of the atomic electrons around it. While we can think of this process, which is known as **internal conversion,** as a kind of photoelectric effect in which a nuclear photon is absorbed by an atomic electron, it is in better accord with experiment to regard internal conversion as representing a direct transfer of excitation energy from a nucleus to an electron. The emitted electron has a kinetic energy equal to the lost nuclear excitation energy minus the binding energy of the electron in the atom.

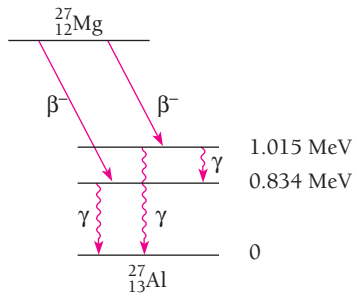


Figure 12.11 Successive beta and gamma emissions in the decay of $^{27}_{12}$Mg to $^{27}_{13}$Al via $^{27}_{13}$Al*.

Most excited nuclei have very short half-lives against gamma decay, but a few remain excited for as long as several hours. The analogy with metastable atomic states is a close one. A long-lived excited nucleus is called an **isomer** of the same nucleus in its ground state. The excited nucleus $^{87}_{38}$Sr* has a half-life of 2.8 h and is accordingly an isomer of $^{87}_{38}$Sr.

12.7 CROSS SECTION

A measure of the likelihood of a particular interaction

Most of what is known about atomic nuclei has come from experiments in which energetic bombarding particles collide with stationary target nuclei. A very convenient way to express the probability that a bombarding particle will interact in a certain way with a target particle employs the idea of **cross section** that was introduced in the Appendix to Chap. 4 in connection with the Rutherford scattering experiment.

What we do is imagine each target particle as presenting a certain area, called its cross section, to the incident particles, as in Fig. 12.12. Any incident particle that is directed at this area interacts with the target particle. Hence the greater the cross section, the greater the likelihood of an interaction. The interaction cross section of a target particle varies with the nature of the process involved and with the energy of the incident particle; it may be greater or less than the geometrical cross section of the particle.

Suppose we have a slab of some material whose area is A and whose thickness is dx (Fig. 12.13). If the material contains n atoms per unit volume, a total of $nA\,dx$ nuclei is in the slab, since its volume is $A\,dx$. Each nucleus has a cross section of σ for some particular interaction, so that the aggregate cross section of all the nuclei in the slab is $nA\sigma\,dx$. If there are N incident particles in a bombarding beam, the number dN that interact with nuclei in the slab is therefore specified by

$$\frac{\text{Interacting particles}}{\text{Incident particles}} = \frac{\text{aggregate cross section}}{\text{target area}}$$

$$\frac{dN}{N} = \frac{nA\sigma\,dx}{A}$$

Cross section

$$= n\sigma\,dx \tag{12.19}$$

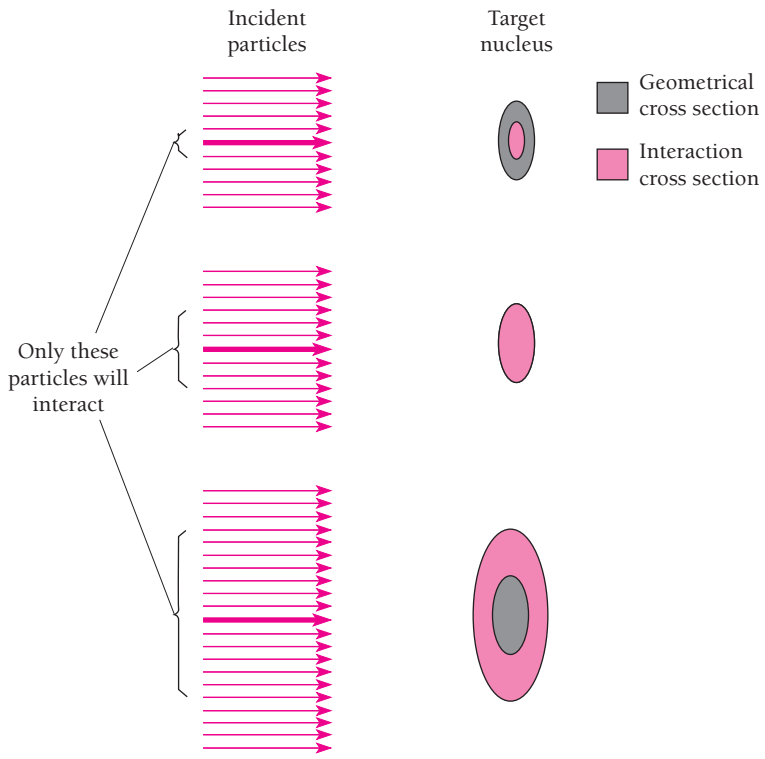


Figure 12.12 A geometrical interpretation of the concept of cross section. The interaction cross section may be smaller than, equal to, or larger than the geometrical cross section. The cross section of a nucleus for a particular interaction is a mathematical way to express the probability that the interaction will occur when a certain particle is incident on the nucleus; the diagram here is nothing more than a helpful visualization.

Now we consider the same beam of particles incident on a slab of finite thickness x. If each particle can interact only once, dN particles may be thought of as being removed from the beam in passing through the first dx of the slab. Hence we need a minus sign in Eq. (12.19), which becomes

$$-\frac{dN}{N} = n\sigma\, dx$$

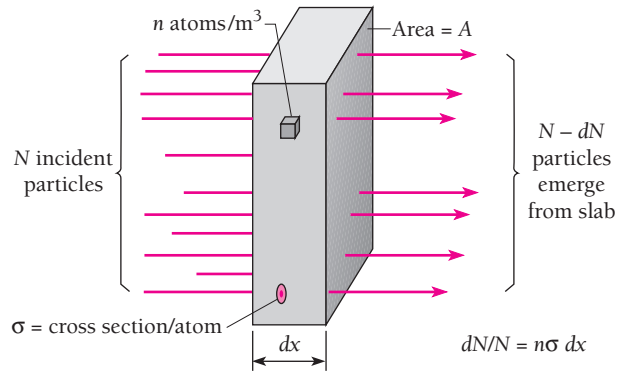


Figure 12.13 The relationship between cross section and beam intensity.

Denoting the initial number of incident particles by N_0, we have

$$\int_{N_0}^{N} \frac{dN}{N} = -n\sigma \int_0^x dx$$

$$\ln N - \ln N_0 = -n\sigma x \tag{12.20}$$

Surviving particles

$$N = N_0 e^{-n\sigma x}$$

The number of surviving particles N decreases exponentially with increasing slab thickness x.

The customary unit for nuclear cross sections is the **barn,** where

$$1 \text{ barn} = 1 \text{ b} = 10^{-28} \text{ m}^2 = 100 \text{ fm}^2$$

Although not an SI unit, the barn is handy because it is of the same order of magnitude as the geometrical cross section of a nucleus. The name comes from a more familiar target cross-sectional area, the side of a barn.

The cross sections for most nuclear reactions depend on the energy of the incident particle. Figure 12.14 shows how the neutron-capture cross section of $^{113}_{48}$Cd varies with neutron energy. This reaction, in which the absorption of a neutron is followed by the emission of a gamma ray, is usually expressed in shorthand form as

$$^{113}\text{Cd}(n, \gamma)^{114}\text{Cd}$$

The narrow peak at 0.176 eV is a resonance effect associated with an excited state in the ^{114}Cd nucleus. Although the ^{113}Cd isotope constitutes only 12 percent of natural cadmium, its capture cross sections for slow neutrons are so great that cadmium is widely used in control rods for nuclear reactors.

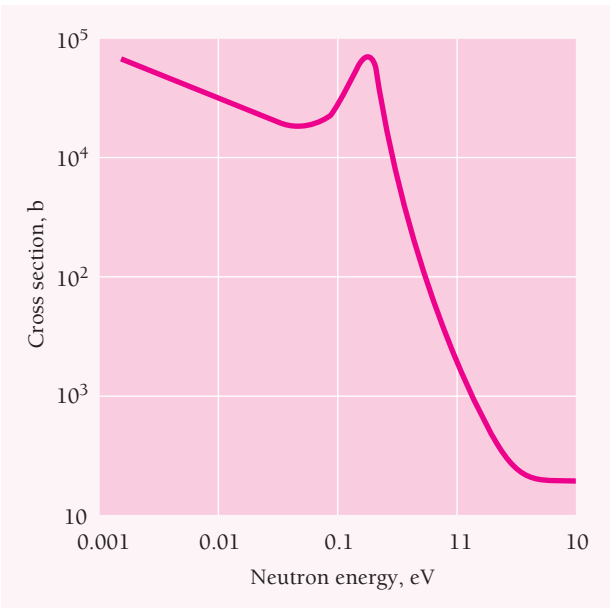


Figure 12.14 The cross section for the reaction $^{113}\text{Cd}(n, \gamma)^{114}\text{Cd}$ varies strongly with neutron energy. In this reaction a neutron is absorbed and a gamma ray is emitted.

Example 12.8

A neutron passing through a body of matter and not absorbed in a nuclear reaction undergoes frequent elastic collisions in which some of its kinetic energy is given up to nuclei in its path. Very soon the neutron reaches thermal equilibrium, which means that it is equally likely to gain or to lose energy in further collisions. At room temperature such a **thermal neutron** has an average energy of $\frac{3}{2}kT = 0.04$ eV and a most probable energy of $kT = 0.025$ eV; the latter figure is usually quoted as the energy of such neutrons.

The cross section of ^{113}Cd for capturing thermal neutrons is 2×10^4 b, the mean atomic mass of natural cadmium is 112 u, and its density is 8.64 g/cm^3 = 8.64×10^3 kg/m^3. (*a*) What fraction of an incident beam of thermal neutrons is absorbed by a cadmium sheet 0.1 mm thick? (*b*) What thickness of cadmium is needed to absorb 99 percent of an incident beam of thermal neutrons?

Solution

(*a*) Since ^{113}Cd constitutes 12 percent of natural cadmium, the number of ^{113}Cd atoms per cubic meter is

$$n = (0.12)\left[\frac{8.64 \times 10^3 \text{ kg/m}^3}{(112 \text{ u/atom})(1.66 \times 10^{-27} \text{ kg/u})}\right]$$
$$= 5.58 \times 10^{27} \text{ atoms/m}^3$$

The capture cross section is $\sigma = 2 \times 10^4$ b $= 2 \times 10^{-24}$ m^2, so

$$n\sigma = (5.58 \times 10^{27} \text{ m}^{-3})(2 \times 10^{-24} \text{ m}^2) = 1.12 \times 10^4 \text{ m}^{-1}$$

From Eq. (12.20), $N = N_0 e^{-n\sigma x}$, so the fraction of incident neutrons that is absorbed is

$$\frac{N_0 - N}{N_0} = \frac{N_0 - N_0 e^{-n\sigma x}}{N_0} = 1 - e^{-n\sigma x}$$

Since $x = 0.1$ mm $= 10^{-4}$ m here,

$$\frac{N_0 - N}{N_0} = 1 - e^{(-1.12 \times 10^4 \text{ m}^{-1})(10^{-4} \text{ m})} = 0.67$$

Two-thirds of the incident neutrons are absorbed.

(*b*) Since we are given that 1 percent of the incident neutrons pass through the cadmium sheet, $N = 0.01N_0$ and

$$\frac{N}{N_0} = 0.01 = e^{-n\sigma x}$$
$$\ln 0.01 = -n\sigma x$$
$$x = \frac{-\ln 0.01}{n\sigma} = \frac{-\ln 0.01}{1.12 \times 10^4 \text{ m}^{-1}} = 4.1 \times 10^{-4} \text{ m} = 0.41 \text{ mm}$$

Cadmium is evidently a very efficient absorber of thermal neutrons.

The mean free path λ of a particle in a material is the average distance it can travel in the material before interacting there. Since $e^{-n\sigma x}\,dx$ is the probability that a particle interact in the interval dx at the distance x, we have, by the same reasoning as that

used in Sec. 5.4,

Mean free path

$$\lambda = \frac{\int_0^\infty x e^{-n\sigma x}\,dx}{\int_0^\infty e^{-n\sigma x}\,dx} = \frac{1}{n\sigma} \tag{12.21}$$

Example 12.9

Find the mean free path of thermal neutrons in ^{113}Cd.

Solution

Since $n\sigma = 1.12 \times 10^4\ \text{m}^{-1}$ here, the mean free path is

$$\lambda = \frac{1}{n\sigma} = \frac{1}{1.12 \times 10^4 \text{m}^{-1}} = 8.93 \times 10^{-5}\text{m} = 0.0893\ \text{mm}$$

Slow Neutron Cross Sections

Although neutrons interact with nuclei only through short-range nuclear forces, reaction cross sections for slow neutrons can be much greater than the geometrical cross sections of the nuclei involved.The geometrical cross section of ^{113}Cd is 1.06 b, for example but its cross section for the capture of thermal neutrons is 20,000 b.

When we recall the wave nature of a moving neutron, though, such discrepancies become less bizarre. The slower a neutron, the greater its de Broglie wavelength λ and the larger the region of space through which we must regard it as being spread out. A fast neutron with a wavelength smaller than the radius R of a target nucleus behaves more or less like a particle when it interacts with the nucleus. The cross section is then approximately geometrical, in the neighborhood of πR^2. Less energetic neutrons behave more like wave packets and interact over larger areas. Although cross sections in the latter case of $\pi\lambda^2$ (which is over 10^7 b for a thermal neutron) are rare, cross sections for nuclear reactions with slow neutrons greatly exceed πR^2, as we have seen.

Reaction Rate

When we know the cross section for a nuclear reaction caused by a beam of incident particles, we can find the rate $\Delta N/\Delta t$ at which the reaction occurs in a given sample of the target material. Let us consider a sample in the form of a slab of area A and thickness x that contains n atoms/m^3, with the particle beam incident normal to one face of the slab. From Eq. (12.20)

$$\frac{\Delta N}{\Delta t} = \frac{N_0 - N}{\Delta t} = \frac{N_0}{\Delta t}(1 - e^{-n\sigma x})$$

If the slab is thin enough so that none of the nuclear cross sections overlaps any others, $n\sigma x \ll 1$. Since $e^{-y} = 1 - y$ for $y \ll 1$, in this case

$$\frac{\Delta N}{\Delta t} = \left(\frac{N_0}{\Delta t}\right) n\sigma x$$

The flux Φ of the beam is the number of incident particles per unit area per unit time, so $\Phi A = N_0/\Delta t$ is their number per unit time. Because Ax is the volume of the sample, the total number of atoms it contains is $n' = nAx$. The reaction rate is therefore just

Reaction rate

$$\frac{\Delta N}{\Delta t} = (\Phi A)(n\sigma x) = \Phi n'\sigma \tag{12.22}$$

Example 12.10

Natural gold consists entirely of the isotope $^{197}_{79}$Au whose cross section for thermal neutron capture is 99 b. When $^{197}_{79}$Au absorbs a neutron, the product is $^{198}_{79}$Au which is beta-radioactive with a half-life of 2.69 d. How long should a 10.0-mg gold foil be exposed to a flux of 2.00×10^{16} neutrons/m$^2 \cdot$ s in order for the sample to have an activity of 200 μCi? Assume that the irradiation period is much shorter than the half-life of $^{198}_{79}$Au so the decays that occur during the irradiation can be neglected.

Solution

The decay constant of $^{198}_{79}$Au is

$$\lambda = \frac{0.693}{(2.69 \text{ d})(86{,}400 \text{ s/d})} = 2.98 \times 10^6 \text{ s}^{-1}$$

The required activity of $R = \Delta N\lambda = 200\mu\text{Ci} = 2.00 \times 10^{-4}$ Ci means that the number of $^{198}_{79}$Au atoms must be

$$\Delta N = \frac{R}{\lambda} = \frac{(2.00 \times 10^{-4} \text{ Ci})(3.70 \times 10^{10} \text{ s}^{-1}/\text{Ci})}{2.98 \times 10^6 \text{ s}^{-1}} = 2.48 \times 10^{12} \text{ atoms}$$

The number of atoms in 10.0 mg $= 1.00 \times 10^{-5}$ kg of $^{197}_{79}$Au is

$$n' = \frac{1.00 \times 10^{-5} \text{ kg}}{(197 \text{ u/atom})(1.66 \times 10^{-27} \text{ kg/u})} = 3.06 \times 10^{19} \text{ atoms}$$

From Eq. (12.22) we find that

$$\Delta t = \frac{N}{\Phi n'\sigma} = \frac{2.48 \times 10^{12} \text{ atoms}}{(2.00 \times 10^{16} \text{ neutrons/m}^2 \cdot \text{s})(3.06 \times 10^{24} \text{ atoms})(99 \times 10^{-28} \text{ m}^2)}$$
$$= 409 \text{ s} = 6 \text{ min } 49 \text{ s}$$

As we assumed, $\Delta t \ll T_{1/2}$.

12.8 NUCLEAR REACTIONS

In many cases, a compound nucleus is formed first

When two nuclei come close together, a **nuclear reaction** can occur that results in new nuclei being formed. Nuclei are positively charged and the repulsion between them keeps them beyond the range where they can interact unless they are moving very fast to begin with. In the sun and other stars, whose internal temperatures range up to

millions of kelvins, many nuclei present have high enough speeds for reactions to be frequent. Indeed, the reactions provide the energy that maintains these temperatures.

In the laboratory, it is easy to produce nuclear reactions on a small scale, either with alpha particles from radionuclides or with protons or heavier nuclei accelerated in various ways. But only one type of nuclear reaction has as yet proved to be a practical source of energy on the earth, namely the fission of certain nuclei when struck by neutrons.

Many nuclear reactions actually involve two separate stages. In the first, an incident particle strikes a target nucleus and the two combine to form a new nucleus, called a **compound nucleus,** whose atomic and mass numbers are respectively the sum of the atomic numbers of the original particles and the sum of their mass numbers. This idea was proposed by Bohr in 1936.

A compound nucleus has no memory of how it was formed, since its nucleons are mixed together regardless of origin and the energy brought into it by the incident particle is shared among all of them. A given compound nucleus may therefore be formed in a variety of ways. To illustrate this, Fig. 12.15 shows six reactions whose product is the compound nucleus $^{14}_{7}\mathrm{N}^*$. (The asterisk signifies an excited state. Compound nuclei are always excited by amounts equal to at least the binding energies of the incident particles in them.) Compound nuclei have lifetimes on the order of 10^{-16} s or so. Although too short to permit actually observing such nuclei directly, such lifetimes are long relative to the 10^{-21} s or so a nuclear particle with an energy of several MeV would need to pass through a nucleus.

A given compound nucleus may decay in one or more ways, depending on its excitation energy. Thus $^{14}_{7}\mathrm{N}^*$ with an excitation energy of, say, 12 MeV can decay in any of the four ways shown in Fig. 12.15. $^{14}_{7}\mathrm{N}^*$ can also simply emit one or more gamma rays whose energies total 12 MeV. However, it *cannot* decay by the emission of a triton ($^{3}_{1}\mathrm{H}$) or a helium-3 ($^{3}_{2}\mathrm{He}$) particle since it does not have enough energy to liberate them. Usually a particular decay mode is favored by a compound nucleus in a specific excited state.

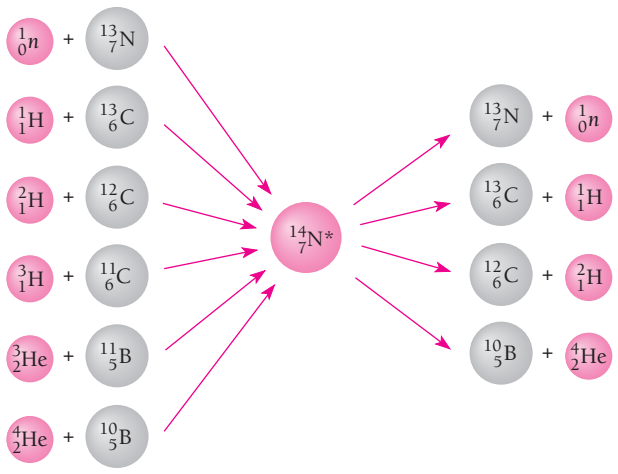


Figure 12.15 Six nuclear reactions whose product is the compound nucleus $^{14}_{7}\mathrm{N}^*$ and four ways in which $^{14}_{7}\mathrm{N}^*$ can decay if its excitation energy is 12 meV. Other decay modes are possible if the excitation energy is greater, fewer are possible if this energy is less. In addition, $^{14}_{7}\mathrm{N}^*$ can simply lose its excitation energy by emitting one or more gamma rays.

The formation and decay of a compound nucleus has an interesting interpretation on the basis of the liquid-drop nuclear model described in Sec. 11.5. In terms of this model, an excited nucleus is analogous to a drop of hot liquid, with the binding energy of the emitted particles corresponding to the heat of vaporization of the liquid molecules. Such a drop of liquid will eventually evaporate one or more molecules, thereby cooling down. The evaporation occurs when random fluctuations in the energy distribution within the drop cause a particular molecule to have enough energy to escape. Similarly, a compound nucleus persists in its excited state until a particular nucleon or group of nucleons happens to gain enough of the excitation energy to leave the nucleus. The time interval between the formation and decay of a compound nucleus fits in nicely with this picture.

Resonance

Information about the excited states of nuclei can be gained from nuclear reactions as well as from radioactive decay. The presence of an excited state may be detected by a peak in the cross section versus energy curve of a particular reaction, as in the neutron-capture reaction of Fig. 12.14. Such a peak is called a **resonance** by analogy with ordinary acoustic or ac circuit resonances. A compound nucleus is more likely to be formed when the excitation energy provided exactly matches one of its energy levels than if the excitation energy has some other value.

The reaction of Fig. 12.14 has a resonance at 0.176 eV whose width (at half-maximum) is $\Gamma = 0.115$ eV. This resonance corresponds to an excited state in ^{114}Cd that decays by the emission of a gamma ray. The mean lifetime τ of an excited state is related to its level width Γ by the formula

Mean lifetime of excited state

$$\tau = \frac{\hbar}{\Gamma} \tag{12.23}$$

This result is in accord with the uncertainty principle in the form $\Delta E\, \Delta t \geq \hbar/2$ if we associate Γ with the uncertainty ΔE in the excitation energy of the state and τ with the uncertainty Δt in the time the state will decay. In the case of the above reaction, the level width of 0.115 eV implies a mean lifetime for the compound nucleus of

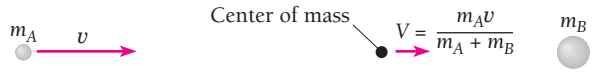
$$\tau = \frac{1.054 \times 10^{-34}\ \text{J} \cdot \text{s}}{(0.115\ \text{eV})(1.60 \times 10^{-19}\ \text{J/eV})} = 5.73 \times 1^{-15}\ \text{s}$$

Center-of-Mass Coordinate System

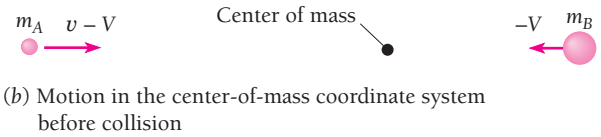
Most nuclear reaction in the laboratory occur when a moving nucleon or nucleus strikes a stationary one. Analyzing such a reaction is simplified when we use a coordinate system that moves with the center of mass of the colliding particles.

To an observer located at the center of mass, the particles have equal and opposite momenta (Fig. 12.16). Hence if a particle of mass m_A and speed v approaches a stationary particle of mass m_B as viewed by an observer in the laboratory, the speed V of the center of mass is defined by the condition

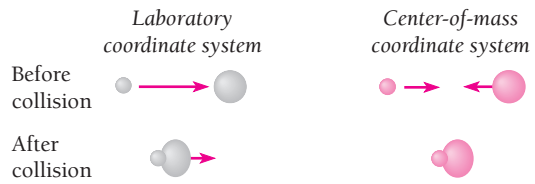
$$m_A(v - V) = m_B V$$



(*a*) Motion in the laboratory coordinate system before collision



(*b*) Motion in the center-of-mass coordinate system before collision



(*c*) A completely inelastic collision as seen in laboratory and center-of-mass coordinate systems

Figure 12.16 Laboratory and center-of-mass coordinate systems.

so that

Speed of center of mass

$$V = \left(\frac{m_A}{m_A + m_B}\right) v \tag{12.24}$$

In most nuclear reactions, $v \ll c$ and a nonrelativistic treatment is sufficient.

In the laboratory system, the total kinetic energy is that of the incident particle only:

Kinetic energy in lab system

$$\mathrm{KE}_{\mathrm{lab}} = \tfrac{1}{2} m_A v^2 \tag{12.25}$$

In the center-of-mass system, both particles are moving and contribute to the total kinetic energy:

$$\begin{aligned} \mathrm{KE}_{\mathrm{cm}} &= \tfrac{1}{2} m_A (v - V)^2 + \tfrac{1}{2} m_B V^2 \\ &= \tfrac{1}{2} m_A v^2 - \tfrac{1}{2} (m_A + m_B) V^2 \\ &= \mathrm{KE}_{\mathrm{lab}} - \tfrac{1}{2} (m_A + m_B) V^2 \end{aligned}$$

Kinetic energy in CM system

$$\mathrm{KE}_{\mathrm{cm}} = \left(\frac{m_B}{m_A + m_B}\right) \mathrm{KE}_{\mathrm{lab}} \tag{12.26}$$

The total kinetic energy of the particles relative to the center of mass is their total kinetic energy in the laboratory system minus the kinetic energy $\frac{1}{2}(m_A + m_B)V^2$ of the moving center of mass. Thus we can regard $\mathrm{KE}_{\mathrm{cm}}$ as the kinetic energy of the relative motion of the particles. When the particles collide, the maximum amount of kinetic energy that can be converted to excitation energy of the resulting compound nucleus while still conserving momentum is $\mathrm{KE}_{\mathrm{cm}}$, which is always less than $\mathrm{KE}_{\mathrm{lab}}$.

The **Q value** of the nuclear reaction

$$A + B \rightarrow C + D$$

is defined as the difference between the rest energies of A and B and the rest energies of C and D:

Q value of nuclear reaction

$$Q = (m_A + m_B - m_C - m_D)c^2 \tag{12.27}$$

If Q is a positive quantity, energy is given off by the reaction. If Q is a negative quantity, enough kinetic energy KE_{cm} in the center-of-mass system must be provided by the reacting particles so that $KE_{cm} + Q \geq 0$.

Example 12.11

Find the minimum kinetic energy in the laboratory system needed by an alpha particle to cause the reaction $^{14}N(\alpha, p)^{17}O$. The masses of ^{14}N, ^{4}He, ^{1}H, and ^{17}O are respectively 14.00307 u, 4.00260 u, 1.00783 u, and 16.99913 u.

Solution

Since the mases are given in atomic mass units, it is easiest to proceed by finding the mass difference between reactants and products in the same units and then multiplying by 931.5 MeV/u. Thus we have

$$Q = (14.00307 \text{ u} + 4.00260 \text{ u} - 1.00783 \text{ u} - 16.99913 \text{ u})\ (931.5 \text{ MeV/u}) = -1.20 \text{ MeV}$$

The minimum kinetic energy KE_{cm} in the center-of-mass system must therefore be 1.20 MeV in order for the reaction to occur. From Eq. (12.26) with the alpha particle as A,

$$KE_{lab} = \left(\frac{m_A + m_B}{m_B}\right) KE_{cm} = \left(\frac{4.00260 + 14.00307}{14.00307}\right)(1.20 \text{ MeV}) = 1.54 \text{ MeV}$$

The cross section for this reaction is another matter. Because both alpha particles and ^{14}N nuclei are positively charged and repel electrically, the greater KE_{cm} is above the threshold of 1.20 MeV, then the greater the cross section and the more likely the reaction will occur.

12.9 NUCLEAR FISSION

Divide and conquer

As we saw in Sec. 11.4, a lot of binding energy will be released if we can break a large nucleus into smaller ones. But nuclei are ordinarily not at all easy to split. What we need is a way to disrupt a heavy nucleus without using more energy than we get back from the process.

The answer came in 1938 with the realization by Lise Meitner that a nucleus of the uranium isotope $^{235}_{92}U$ undergoes fission when struck by a neutron. It is not the impact of the neutron that has this effect. Instead, the $^{235}_{92}U$ nucleus absorbs the neutron to become $^{236}_{92}U$, and the new nucleus is so unstable that almost at once it explodes into two fragments (Fig. 12.17). Later several other heavy nuclides were found to be fissionable by neutrons in similar processes.

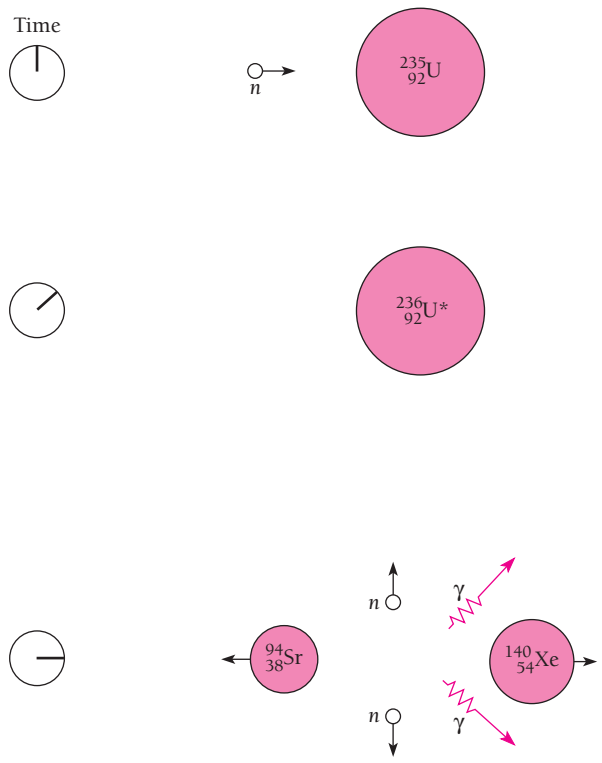


Figure 12.17 In nuclear fission, an absorbed neutron causes a heavy nucleus to split into two parts. Several neutrons and gamma rays are emitted in the process. The smaller nuclei shown here are typical of those produced in the fission of $^{235}_{92}$U and are both radioactive.

Nuclear fission can be understood on the basis of the liquid-drop model of the nucleus (Sec. 11.5). When a liquid drop is suitably excited, it may oscillate in a variety of ways. A simple one is shown in Fig 12.18: the drop in turn becomes a prolate spheroid, a sphere, an oblate spheroid, a sphere, a prolate spheroid again, and so on. The restoring force of its surface tension always returns the drop to spherical shape, but the inertia of the moving liquid molecules causes the drop to overshoot sphericity and go to the opposite extreme of distortion.

Nuclei exhibit surface tension, and so can vibrate like a liquid drop when in an excited state. They also are subject to disruptive forces due to the mutual repulsion of their protons. When a nucleus is distorted from a spherical shape, the short-range restoring force of surface tension must cope with the long-range repulsive force as well as with the inertia of the nuclear matter. If the degree of distortion is small, the surface tension can do this, and the nuclear vibrates back and forth until it eventually loses its excitation energy by gamma decay. If the degree of distortion is too great, however, the surface tension is

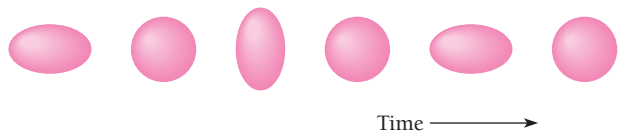


Figure 12.18 The oscillations of a liquid drop.

Lise Meitner (1878–1968), the daughter of a Viennese lawyer, became interested in science when she read about the Curies and radium. She earned her Ph.D. in physics in 1905 at the University of Vienna, only the second woman to obtain a doctorate there. She then went to Berlin where she began research on radioactivity with the chemist Otto Hahn. Their supervisor refused to have a woman in his laboratory, so they started their work in a carpentry shop. Ten years later she was a professor, a department head, and, with Hahn, the discoverer of a new element, protactinium.

In the 1930s the Italian physicist Enrico Fermi found that bombarding heavy elements with neutrons led to the production of other elements. What happened in the case of uranium was especially puzzling, and Meitner and Hahn tried to find out by repeating the experiment. At the time the German persecution of Jews had begun, but Meitner, who was Jewish, was protected by her Austrian citizenship. In 1938 Germany annexed Austria, and Meitner fled to Sweden but kept in touch with Hahn and their younger colleague Fritz Strassmann. Hahn and Strassmann finally concluded that neutrons interact with uranium to produce radium, but Meitner's calculations showed that this was impossible and she urged them to persist in their work. They did, and found to their surprise that the lighter element barium had in fact been created. Meitner surmised that the neutrons had caused the uranium nuclei to split apart and, with her nephew Otto Frisch, developed the theoretical picture of what they called fission.

In January 1939 Hahn and Strassmann published the discovery of fission in a German journal; because Meitner was Jewish, they thought it safer for themselves to ignore her contribution. Meitner and Frisch later published their own paper on fission in an English journal, but it was too late: Hahn disgracefully claimed full credit, and not once in the years that followed acknowledged her role. Hahn alone received the Nobel Prize in physics for discovering fission. Unfortunately Meitner did not live to see a measure of justice: the element of atomic number 109 is called meitnerium in her honor, while the tentative name of hahnium for element 105 was changed in 1997 to dubnium, after the Russian nuclear research center in Dubna.

Niels Bohr carried the news of the discovery of fission to the United States later in 1939, just before the start of World War II, where its military possibilities were immediately recognized. Expecting that German physicists would come to the same conclusion and would start work on an atomic bomb, such a program began in earnest in the United States. By the time it was successful, in 1945, Germany had been defeated, and two atomic bombs exploded over Hiroshima and Nagasaki then ended the war with Japan. It was later learned that the German atomic-bomb effort had amounted to very little. Not long afterward the Soviet Union, Great Britain, and France also developed nuclear weapons, and later China, Israel, South Africa, India, and Pakistan did so as well.

unable to bring back together the now widely separated groups of protons, and the nucleus splits into two parts. This picture of fission is illustrated in Fig. 12.19.

The new nuclei that result from fission are called **fission fragments.** Usually fission fragments are of unequal size (Fig. 12.20). Because heavy nuclei have a greater neutron/proton ratio than lighter ones, the fragments contain an excess of neutrons. To reduce this excess, two or three neutrons are emitted by the fragments as soon as they are formed, and subsequent beta decays bring their neutron/proton ratios to stable values. A typical fission reaction is

$$^{235}_{92}\mathrm{U} + {}^{1}_{0}n \rightarrow {}^{236}_{92}\mathrm{U}^{*} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + {}^{1}_{0}n + {}^{1}_{0}n$$

which was illustrated in Fig. 12.17.

A heavy nucleus undergoes fission when it has enough excitation energy (5 MeV or so) to oscillate violently. A few nuclei, notably ^{235}U, are able to split in two merely by

Figure 12.19 Nuclear fission according to the liquid-drop model.

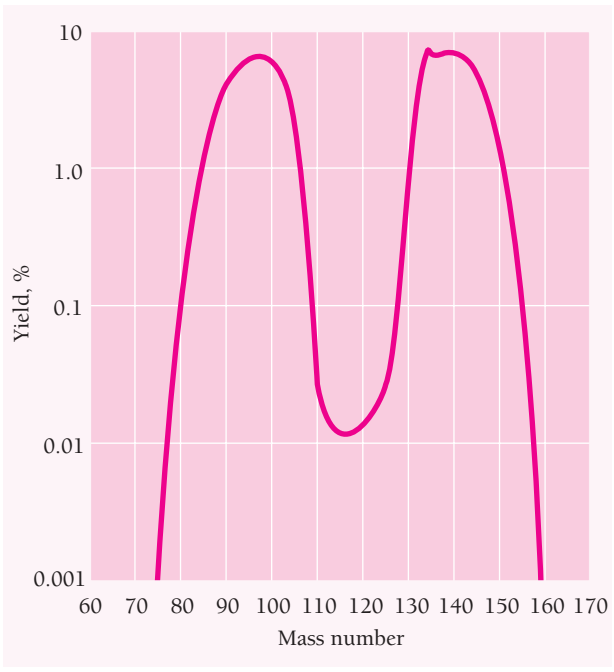


Figure 12.20 The distribution of mass numbers in the fragments from the fission of $^{235}_{92}U$.

absorbing an additional neutron. Other nuclei, notably ^{238}U (which makes up 99.3 percent of natural uranium, with ^{235}U as the remainder) need more excitation energy for fission than the binding energy released when another neutron is absorbed. Such nuclei undergo fission only by reaction with fast neutrons whose kinetic energies exceed about 1 MeV.

Fission can occur after a nucleus is excited by means other than neutron capture, for instance by gamma-ray or proton bombardment. Some nuclides are so unstable as to be capable of spontaneous fission, but they are more likely to undergo alpha decay before this takes place.

A striking aspect of nuclear fission is the magnitude of the energy given off. As we saw earlier, this is in the neighborhood of 200 MeV, a remarkable figure for a single atomic event; chemical reactions liberate only a few electronvolts per event. Most of the energy released in fission goes into the kinetic energy of the fission fragments. In the case of the fission of ^{235}U, about 83 percent of the energy appears as kinetic energy of the fragments, about 2.5 percent as kinetic energy of the neutrons, and about 3.5 percent in the form of instantly emitted gamma rays. The remaining 11 percent is given off in the subsequent beta and gamma decays of the fission fragments.

Shortly after nuclear fission was discovered it was realized that, because fission leads to other neutrons being given off, a self-substaining sequence of fissions should be possible (Fig.12.21). The condition for such a **chain reaction** to occur in an assembly of fissionable material is simple: at least one neutron produced during each fission must, on the average, cause another fission. If too few neutrons cause fissions, the reaction will slow down and stop; if precisely one neutron per fission causes another fission, energy will be released at a constant rate. (which is the case in a **nuclear reactor**); and if the frequency of fissions increases, the energy release will be so rapid that an explosion will occur (which is the case in an **atomic bomb**).

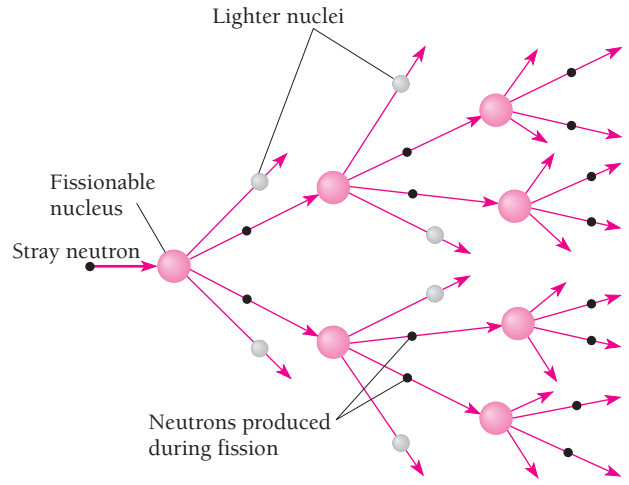


Figure 12.21 Sketch of a chain reaction. The reaction is self-sustaining if at least one neutron from each fission event on the average induces another fission event. If more than one neutron per fission on the average induces another fission, the reaction is explosive.

These situations are respectively called **subcritical, critical,** and **supercritical.** If two neutrons from each fission in an atomic bomb induce further fissions in 10^{-8} s, a chain reaction starting with a single fission will give off 2×10^{13} J of energy in less than 10^{-6} s.

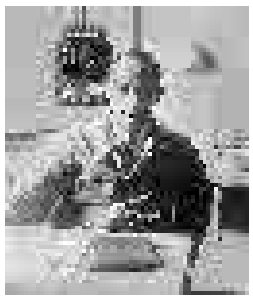
12.10 NUCLEAR REACTORS

$$E_0 = mc^2 + \$\$\$$$

A nuclear reactor is a very efficient source of energy: the fission of 1 g of ^{235}U per day evolves energy at a rate of about 1 MW, whereas 2.6 tons of coal per day must be burned in a conventional power plant to produce 1 MW. The energy given off in a reactor becomes heat, which is removed by a liquid or gas coolant. The hot coolant is then used to boil water, and the resulting steam is fed to a turbine that can power an electric generator, a ship, or a submarine.

Each fission in ^{235}U releases an average of 2.5 neutrons, so no more than 1.5 neutrons per fission can be lost for a self-substaining chain reaction to occur. However, natural uranium contains only 0.7 percent of the fissionable isotope ^{235}U. The more abundant ^{238}U readily captures fast neutrons but usually does not undergo fission as a result. As it happens, ^{238}U has only a small cross section for the capture of *slow* neutrons, whereas the cross section of ^{235}U for slow neutron-induced fission is a whopping 582 barns. Slowing down the fast neutrons that are liberated in fission thus helps prevent their unproductive absorption by ^{238}U and at the same time promotes further fissions in ^{235}U.

To slow down fission neutrons, the uranium in a reactor is mixed with a **moderator,** a substance whose nuclei absorb energy from fast neutrons in collisions without much tendency to capture the neutrons. While the exact amount of energy lost by a moving body that collides elastically with another depends on the details of the interaction, in general the energy transfer is a maximum when the participants are of equal mass



Enrico Fermi (1901–1954) was born in Rome and obtained his doctorate at Pisa. After periods at Göttingen and Leiden working with leading figures in the new quantum mechanics, Fermi returned to Italy. At the University of Rome in 1926 he investigated the statistical mechanics of particles that obey Pauli's exclusion principle, such as electrons; the result is called Fermi-Dirac statistics because Dirac independently arrived at the same conclusions shortly afterward. In 1933 Fermi introduced the concept of the weak interaction and used it together with Pauli's newly postulated neutrino (as Fermi called it) to develop a theory of beta decay able to account for the shape of the electron energy spectrum and the decay half-life.

Later in the 1930s Fermi and a group of collaborators carried out a series of experiments in which radionuclides were produced artificially by bombarding various elements with neutrons; they found slow neutrons especially effective. Some of their results seemed to suggest the formation of transuranic elements. In fact, as Meitner and Hahn were to find later, what they were observing was nuclear fission. In 1938 Fermi received the Nobel Prize for this work, but instead of returning to Mussolini's Fascist Italy, he went to the United States. As part of the atomic-bomb program, Fermi directed the design and construction of the first nuclear reactor at the University of Chicago, which began operating in December 1942, four years after the discovery of fission. After the war Fermi shifted to a different field, high-energy particle physics, where he made important contributions. He died of cancer in 1954, one of the very few physicists of the modern era to combine virtuosity in both theory and experiment. The element of atomic number 100, discovered the year after his death, is called fermium in his honor.

(Fig. 12.22). The greater the difference between the masses, the greater the number of collisions needed to slow a neutron down, and the longer the period in which it is in danger of being captured by a ^{238}U nucleus. The majority of today's commercial reactors use light water both as moderator and as coolant. Each molecule of water contains two hydrogen atoms whose proton nuclei have masses almost identical with that of the neutron, so light water is an efficient moderator.

Unfortunately protons tend to capture neutrons to form deuterons in the reaction $^{1}H(n, \gamma)^{2}H$. Light-water reactors therefore cannot use natural uranium for fuel but need **enriched uranium** whose ^{235}U content has been increased to about 3 percent. Enriched uranium can be produced in several ways. Originally all enriched uranium was produced by gaseous diffusion, with uranium hexafluoride (UF_6) gas being passed through about 2000 successive permeable barriers. Molecules of $^{235}UF_6$ are slightly more likely to diffuse through each barrier than $^{238}UF_6$ because of their smaller mass. A more recent method uses high-speed gas centrifuges for the separation. Still other processes are possible.

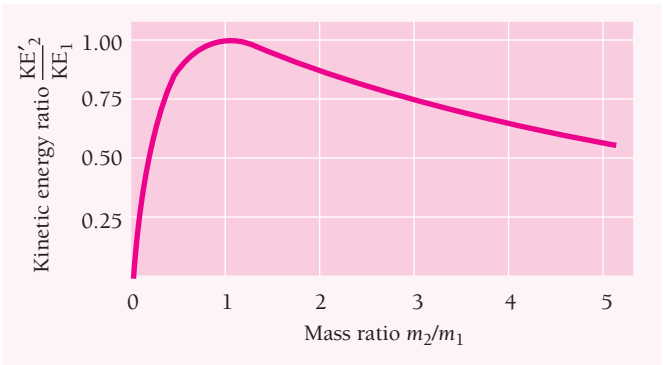
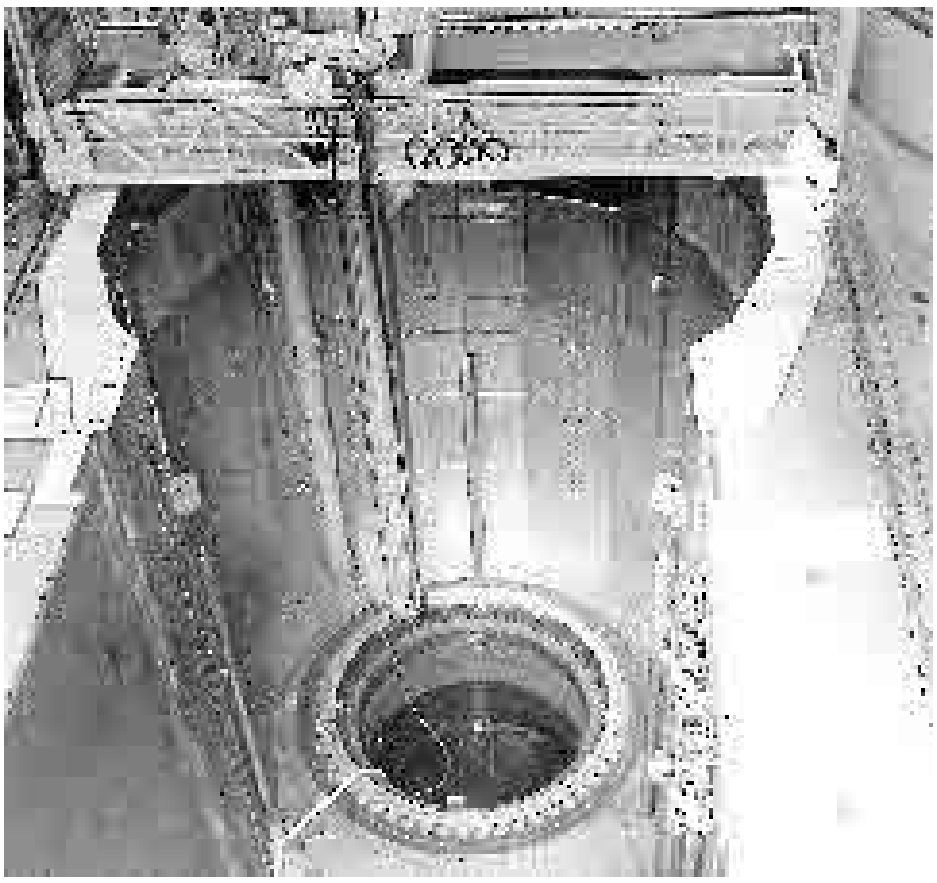


Figure 12.22 Energy transfer in an elastic head-on collision between a moving object of mass m_1 and a stationary object of mass m_2 (see Exercise 59).



Fuel rods being loaded into the core of a 1,129-MW reactor at the William McGuire Nuclear Power Plant in Cornelius, North Carolina.

The fuel for a water-moderated reactor consists of uranium oxide (UO_2) pellets sealed in long, thin tubes. Control rods of cadmium or boron, which are good absorbers of slow neutrons, can be slid in and out of the reactor core to adjust the rate of the chain reaction. In the most common type of reactor, the water that circulates around the fuel in the core is kept at a high pressure, about 155 atmospheres, to prevent boiling. The water, which acts as both moderator and coolant, is passed through a heat exchanger to produce steam that drives a turbine (Fig. 12.23). Such a reactor might contain 90 tons of UO_2 and operate at 3400 MW to yield 1100 MW of electric power. The reactor fuel must be replaced every few years as its ^{235}U content is used up.

Breeder Reactors

Some nonfissionable nuclides can be transmuted into fissionable ones by absorbing neutrons. A notable example is ^{238}U, which becomes ^{239}U when it captures a fast neutron. This uranium isotope beta-decays with a half-life of 24 min into $^{239}_{93}Np$, an isotope of the element neptunium, which is also beta-active. The decay of ^{239}Np has a half-life of 2.3 days and yields $^{239}_{94}Pu$, an isotope of plutonium whose half-life against

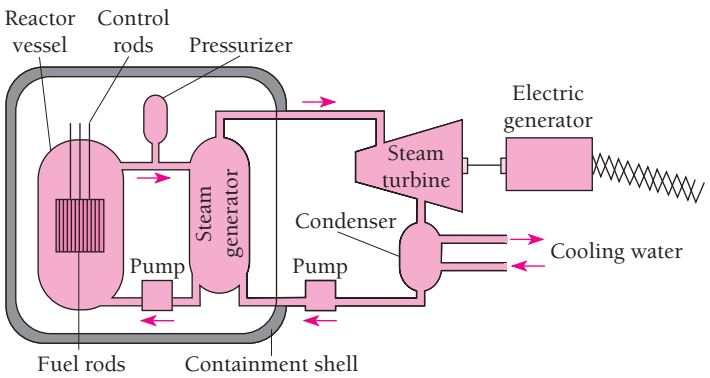


Figure 12.23 Basic design of the most common type of nuclear power plant. Water under pressure is both the moderator and coolant, and transfers heat from the chain reaction in the fuel rods of the core to a steam generator. The resulting steam then passes out of the containment shell, which serves as a barrier to protect the outside world from accidents to the reactor, and is directed to a turbine that drives an electric generator. In a typical plant, the reactor vessel is 13.5 m high and 4.4 m in diameter and weighs 385 tons. It contains about 90 tons of uranium oxide in the form of 50,952 fuel rods each 3.85 m long and 9.5 mm in diameter. Four steam generators are used, instead of the single one shown here, as well as a number of turbine generators.

alpha decay is 24,000 y. The entire sequence is shown in Fig. 12.24. Both neptunium and plutonium are **transuranic elements,** none of which are found on the earth because their half-lives are too short for them to have survived even if they had been present when the earth came into being 4.5 billion years ago.

The plutonium isotope ^{239}Pu is fissionable and can be used as a reactor fuel and for weapons. Plutonium is chemically different from uranium, and its separation from the remaining ^{238}U after neutron irradiation is more easily accomplished than the separation of ^{235}U from the much more abundant ^{238}U in natural uranium.

A **breeder reactor** is one especially designed to produce more plutonium than the ^{235}U it consumes. Because the otherwise useless ^{238}U is 140 times more abundant than the fissionable ^{235}U, the widespread use of breeder reactors would mean that known

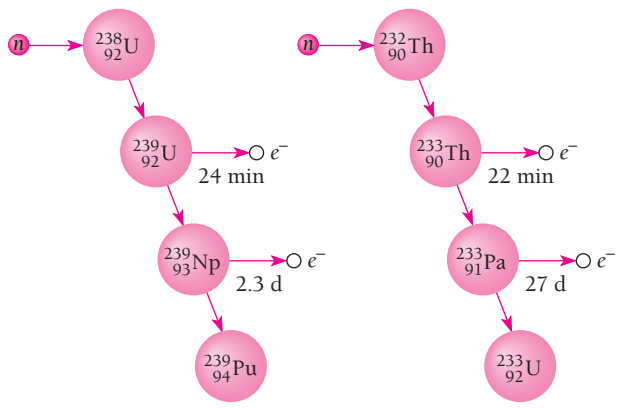


Figure 12.24 ^{238}U and ^{232}Th are "fertile" nuclides. Each becomes a fissionable nuclide after absorbing a neutron and undergoing two beta decays. These transformations are the basis of the breeder reactor, which produces more fuel in the form of ^{239}Pu or ^{233}U than it uses up in the form of ^{235}U.

reserves of uranium could fuel reactors for many centuries to come. Because plutonium can also be used in nuclear weapons (unlike the slightly enriched uranium that fuels ordinary reactors), the widespread use of breeder reactors would also complicate the control of nuclear weapons. Several breeder reactors are operating today, all of them outside the United States. They have proved to be extremely expensive and have had severe operating problems.

Actually, plutonium is already an important nuclear fuel. By the end of the usual three-year fuel cycle in a reactor, after which the fuel rods are replaced, so much plutonium has been produced from the ^{238}U present that more fissions occur in ^{239}Pu than in ^{235}U.

A Nuclear World?

In 1951 the first electricity from a nuclear plant was generated in Idaho. Today over 400 reactors in 26 countries produce about 200,000 MW of electric power—the equivalent of nearly 10 million barrels of oil per day. France, Belgium, and Taiwan obtain more than half electricity from reactors, with several other countries close behind (Fig. 12.25). In the United

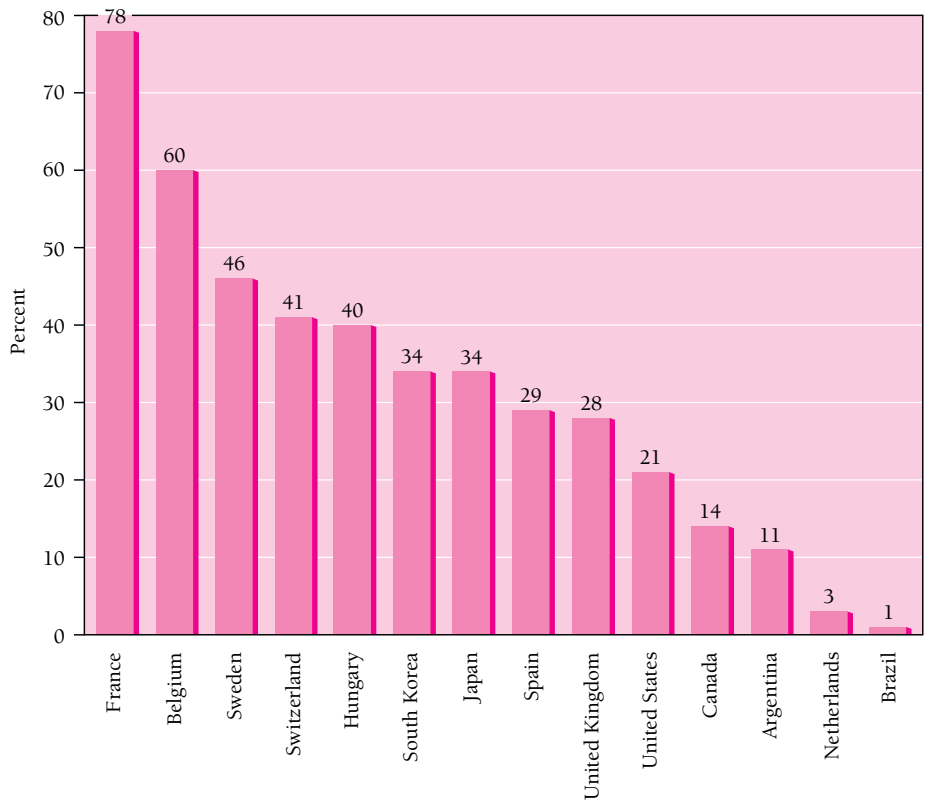


Figure 12.25 Percentage of electric energy in various countries that comes from nuclear power stations. Figures are for 1997.

States, nuclear energy is responsible for 21 percent of generated electricity, slightly more than the world average; there are 103 reactors in 31 states. Yet for all the success of nuclear technology, no new nuclear power stations have been planned in this country since 1979. Why not?

In March 1979, failures in its cooling system disabled one of the reactors at Three Mile Island in Pennsylvania and a certain amount of radioactive material escaped. Although a nuclear reactor cannot explode in the way an atomic bomb does, breakdowns can occur that put large population at risk. Although a true catastrophe was narrowly avoided, the Three Mile Island incident made it clear that the hazards associated with nuclear energy are real.

After 1979 it was inevitable that greater safety would have to be built into new reactors, adding to their already high cost. In addition, demand for electricity in the United States was not increasing as fast as expected, partly because of efforts toward greater efficiency and partly because of a decline in some of industries (such as steel, cars, and chemicals) that are heavy users of electricity. As a result of these factors, new reactors made less economic sense than before, which together with widespread public unease led to a halt in the expansion of nuclear energy in the United States.

Elsewhere the situation was different. Nuclear reactors still seemed the best way to meet the energy needs of many countries without abundant fossil fuel resources. Then in April 1986, a severe accident destroyed a 1000-MW reactor at Chernobyl in what is now Ukraine, then part of the Soviet Union. This was the worst environmental disaster of technological origin in history and contributed to the collapse of the Soviet Union. Over 50 tons of radioactive material escaped and was carried around the world by winds. The radiation released was nearly 200 times the total given off by the Hiroshima and Nagasaki atomic bombs in 1945. Radiation levels in much of Europe rose well above normal for a time and a quarter of a million people were permanently evacuated from the vicinity of Chernobyl. A number of reactor, rescue, and cleanup workers died soon afterward as a result of exposure to radiation, and thousands more became ill. Widespread contamination with radionuclides, particularly of food and water supplies, suggests that cancer will raise the total of people affected manyfold in the years to come. Already about a thousand children, who are especially susceptible, have developed thyroid cancer as a result of ingesting the radioactive iodine isotope ^{131}I; a third of all the children living near Chernobyl who were under 4 years old in 1986 are expected to come down with thyroid cancer eventually.

As in the United States after Three Mile Island, public anxiety over the safety of nuclear programs grew in Europe after Chernobyl. Some countries, for instance Italy, abandoned plans for new reactors. In other countries, for instance France, the logic behind their nuclear programs remained strong enough for them to continue despite Chernobyl.

Quite apart from the safty of reactors themselves is the issue of what to do with the wastes they produce. Even if old fuel rods are processed to separate out the uranium and plutonium they contain, what is left is still highly radioactive. Although a lot of the activity will be gone in a few months and much of the rest in a few hundred years, some of the radionuclides have half-lives in the millions of years. At present perhaps 20,000 tons of spent nuclear fuel is being stored on a temporary basis in the United States (not to mention the vast amount of highly radioactive waste left over from nuclear weapons manufacture that is also awaiting safe storage). Burying nuclear wastes deep underground currently seems to be the best long-term way to dispose of them. The right location is easy to specify but not easy to find: stable geologically with no earthquakes likely, no nearby population centers, a type of rock that does not disintegrate in the presence of heat and radiation but is easy to drill into, and not near groundwater that might become contaminated.

From today's perspective, nuclear energy has important advantages not fully appreciated in the past: it does not produce the air pollution that fossil-fuel burning does, nor the huge quantities of carbon dioxide that are the main contributor to global warming via the greenhouse effect. Together with the rising cost of fossil fuels and increasing demand for electricity, these factors seem likely to lead to the construction of new nuclear reactors in the United States after a delay of over two decades.

12.11 NUCLEAR FUSION IN STARS

How the sun and stars get their energy

Here on the earth, 150 million km from the sun, a surface 1 m^2 in area exposed to the vertical rays of the sun receives energy at a rate of about 1.4 kW. Adding up all the energy radiated by the sun per second gives the enormous total of 4×10^{26} W. And the sun has been emitting energy at this rate for billions of years. Where does it all come from?

The basic energy-producing process in the sun is the fusion of hydrogen nuclei into helium nuclei. This can take place in two different reaction sequences, the most common of which, the **proton-proton cycle,** is shown in Fig. 12.26. The total evolved energy is 24.7 MeV per $^4_2$He nucleus formed.

Since 24.7 MeV is 4×10^{-12} J, the sun's power output of 4×10^{26} W means the sequence of reactions in Fig. 12.26 must occur 10^{38} times per second. The sun consists of 70 percent hydrogen, 28 percent helium, and 2 percent of other elements, so plenty of hydrogen remains for billions of years of further energy production at its current rate. Eventually the hydrogen in the sun's core will be exhausted, and then, as the other reactions described below take over, the sun will swell to become a red giant star and later subside into a white dwarf.

Self-sustaining fusion reactions can occur only under conditions of extreme temperature and density. The high temperature ensures that some nuclei—those in the high-velocity tail of the Maxwell-Boltzmann distribution—have the energy needed to come close enough together to interact, which they do by tunnelling through the electric potential barrier between them. (At the 10^7 K temperature typical of the sun's interior, the average proton kinetic energy is only about 1 keV, whereas the barrier is about 1 MeV, a thousand times higher.) The high density ensures that such collisions are frequent. A further condition for the proton-proton and other multistep cycles is a large reacting mass, such as that of the sun, since much time may elapse between the initial fusion of a particular proton and its eventual incorporation in an alpha particle.

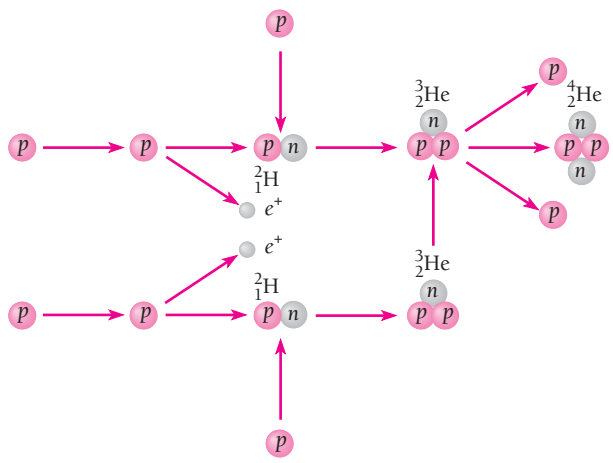


Figure 12.26 The proton-proton cycle. This is the chief nuclear reaction sequence that takes place in stars like the sun and cooler stars. Energy is given off at each step. The net result is the combination of four hydrogen nuclei to form a helium nucleus and two positrons. The neutrinos also produced are not shown.

Hans A. Bethe (1906–) was born in Strasbourg, then part of Germany but today part of France. He studied physics in Frankfurt and Munich and taught at various German universities until 1933, when Hitler came to power. After two years in England he came to the United States where he was professor of physics at Cornell University from 1937 to 1975. He has remained active in research and in public affairs even though formally retired.

Notable among Bethe's many and varied contributions to physics is his 1938 account of the sequences of nuclear reactions that power the sun and stars, for which he received the Nobel Prize in 1967. During World War II he directed the theoretical physics division of the laboratory at Los Alamos, New Mexico, where the atomic bomb was developed. A strong believer in nuclear energy—"it is more necessary now than ever before because of global warming"—Bethe has also been an effective advocate of nuclear disarmament.

The proton-proton cycle dominates in the sun and other stars with less than about 1.5 times the sun's mass. In more massive stars, whose interiors are hotter, the carbon cycle is the main energy source. This cycle proceeds as in Fig. 12.27. The net result again is the formation of an alpha particle and two positrons from four protons, with the evolution of 24.7 MeV. The initial $^{12}_{6}C$ acts as a kind of catalyst for the process, since it reappears at its end. The dependence of the two cycles on temperature is shown in Fig. 12.28.

Formation of Heavier Elements

Fusion reactions that produce helium are not the only ones that occur in the sun and other stars. When all the hydrogen in a star's core has become helium, gravitational

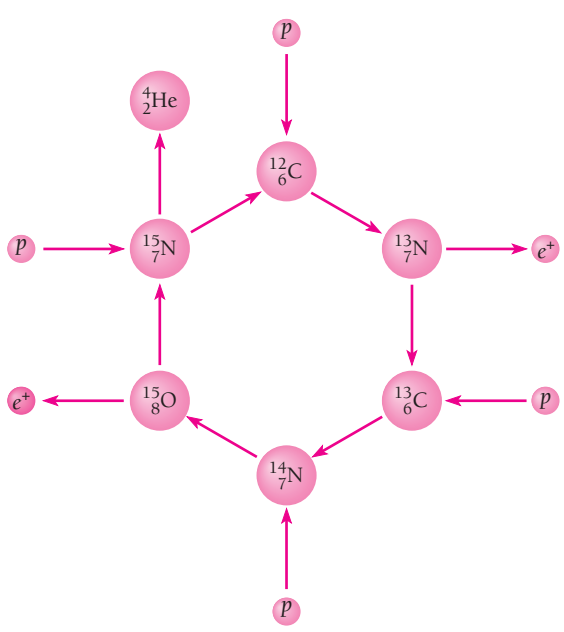


Figure 12.27 The carbon cycle also involves the combination of four hydrogen nuclei to form a helium nucleus with the evolution of energy. The $^{12}_{6}C$ nucleus is unchanged by the series of reactions. This cycle occurs in stars hotter than the sun.

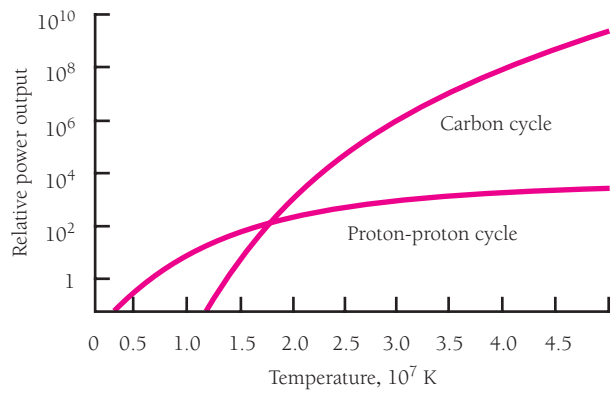


Figure 12.28 How the rates of energy generation for the carbon and proton-proton fusion cycles vary with the temperature of a star's interior. The rates are equal at about 1.8×10^7 K. Note that the power output scale is not linear.

contraction compresses the core and raises its temperature to the 10^8 K needed for helium fusion to begin. This involves the combination of three alpha particles to form a carbon nucleus with the evolution of 7.5 MeV:

$$^4_2\text{He} + {}^4_2\text{He} \rightarrow {}^8_4\text{Be} + \gamma$$

$$^4_2\text{He} + {}^8_4\text{Be} \rightarrow {}^{12}_6\text{C} + \gamma$$

Because the beryllium isotope $^8_4$Be is unstable and breaks apart into two alpha particles with a half-life of only 6.7×10^{-17} s, the second reaction must take place immediately after the first. The sequence is called the triple-alpha reaction.

The smallest stars do not get hot enough (over 10^7 K) to go beyond hydrogen fusion, and helium fusion is as far as a star with the sun's mass gets. But in heavier stars, core temperatures can go even higher, and fusion reactions that involve carbon then become possible. Some examples are

$$^4_2\text{He} + {}^{12}_6\text{C} \rightarrow {}^{16}_8\text{O}$$

$$^{12}_6\text{C} + {}^{12}_6\text{C} \rightarrow {}^{24}_{12}\text{Mg}$$

$$^{12}_6\text{C} + {}^{12}_6\text{C} \rightarrow {}^{20}_{10}\text{Ne} + {}^4_2\text{He}$$

The heavier the star, the higher the eventual temperature of its core, and the larger the nuclei that can be formed. (The high temperatures, of course, are needed to overcome the greater electric repulsion of reacting nuclei with many protons.) In stars more than

The Triple-Alpha Reaction

Because no sufficiently stable nuclides with $A = 5$ or $A = 8$ exist, there is no simple way in which protons, neutrons, and alpha particles can add together in succession to form the nuclei of carbon and elements of still higher atomic number. Eventually it became clear that three alpha particles could react to produce a $^{12}_6$C nucleus in stars whose interiors are sufficiently hot. However, the cross section (Sec. 12.7) for the process seemed much too small for the reaction to be significant. Then, in 1953, the British astronomer Fred Hoyle realized that a resonance associated with the triple-alpha process would greatly enhance its likelihood. Hoyle's calculation indicated that the resonance would correspond to an excited state in $^{12}_6$C of 7.7 MeV. Experiments soon showed that this excited state indeed occurred and increased the cross section by a factor of 10^7, thereby removing the biggest obstacle to understanding the origin of the elements.

about 10 times as massive as the sun, the iron isotope $^{56}_{26}$Fe is reached. This is the nucleus with the greatest binding energy per nucleon (Fig. 11.12). Any reaction between a $^{56}_{26}$Fe nucleus and another nucleus will therefore lead to the breakup of the iron nucleus, not to the formation of a still heavier one.

Then how do nuclides beyond $^{56}_{26}$Fe originate? The answer is through the successive capture of neutrons, with beta decays when needed for appropriate neutron/proton ratios. The neutrons are liberated in such sequences as

$$^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{13}_{7}\mathrm{N} + \gamma$$

$$^{13}_{7}\mathrm{N} \rightarrow {}^{13}_{6}\mathrm{C} + e^{+} + \nu$$

$$^{4}_{2}\mathrm{He} + {}^{13}_{6}\mathrm{C} \rightarrow {}^{16}_{8}\mathrm{O} + {}^{1}_{0}n$$

Neutron-capture reactions in a stellar interior can build up nuclides as far as $^{209}_{83}$Bi, the largest stable nucleus, but no further. The density of neutrons there is not sufficient for them to be captured in rapid enough succession by nuclei of $A > 209$ before such nuclei decay. However, when a very massive star has reached the end of its fuel supply, its core collapses and a violent explosion follows that appears in the sky as a supernova. During the collapse neutrons are produced in abundance, some by the disintegration of neutron-rich nuclei into alpha particles and neutrons in collisions and some by the reaction $e^{-} + p \rightarrow n + \nu$. The huge neutron flux lasts only a matter of seconds, but this is sufficient to produce nuclei with mass numbers up to perhaps 260.

A supernova explosion, which occurs once or twice per century in a galaxy of stars like our own Milky Way, flings into space a large part of the star's mass, which becomes dispersed in interstellar matter. New stars (and their planets, such as our own) that come into being from this matter thus contain the entire spectrum of nuclides, not just the hydrogen and helium of the early universe. We are all made of stardust.

12.12 FUSION REACTORS

The energy source of the future?

Enormous as the energy produced by fission is, the fusion of light nuclei to form heavier ones can give out even more per kilogram of starting materials. It seems possible that nuclear fusion could become the ultimate source of energy on the earth: safe, relatively nonpolluting, and with the oceans themselves supplying limitless fuel.

On the earth, where any reacting mass must be very limited in size, an efficient fusion process cannot involve more than a single step. Two reactions that may eventually power fusion reactors involve the combination of two deuterons to form a triton and a proton,

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + 4.0\ \mathrm{MeV} \qquad (12.28)$$

or their combination to form a $^{3}_{2}$He nucleus and a neutron,

$$^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{1}_{0}n + 3.3\ \mathrm{MeV} \qquad (12.29)$$

Both D-D reactions have about equal probabilities. A major advantage of these reactions is that deuterium is present in seawater and is cheap to extract. Although its

concentration in seawater is only 33 g/m^3, this adds up to a total of about 10^{15} tons of deuterium in the world's oceans. The deuterium in a gallon of seawater can yield as much energy through fusion as 600 gallons of gasoline can through combustion.

The first fusion reactors are more likely to employ a deuterium-tritium mixture because the D-T reaction

$$^3_1\mathrm{H} + {}^2_1\mathrm{H} \rightarrow {}^4_2\mathrm{He} + {}^1_0n + 17.6\ \mathrm{MeV} \tag{12.30}$$

has a higher yield than the others and occurs at lower temperatures. Seawater contains too little tritium to be extracted economically, but it can be produced by the neutron bombardment of the two isotopes of natural lithium:

$$^6_3\mathrm{Li} + {}^1_0n \rightarrow {}^3_1\mathrm{H} + {}^4_2\mathrm{He} \tag{12.31}$$

$$^7_3\mathrm{Li} + {}^1_0n \rightarrow {}^3_1\mathrm{H} + {}^4_2\mathrm{He} + {}^1_0n \tag{12.32}$$

In fact, plans for future fusion reactors include lithium blankets that will make the tritium they need by absorbing neutrons liberated in the fusion reactions.

At the required temperatures, a fusion reactor's fuel will be in the form of a **plasma,** which is a fully ionized gas. **Breakeven** occurs when the energy produced equals the energy input to the reacting plasma. **Ignition,** a more difficult (and perhaps unnecessary) target, occurs when enough energy is produced for the reaction to be self-sustaining.

A successful fusion reactor has three basic conditions to meet:

1 The plasma temperature must be high so that an adequate number of the ions have the speeds needed to come close enough together to react despite their mutual repulsion. Taking into account that many ions have speeds well above the average and that tunneling through the potential barrier reduces the ion energy needed, the minimum temperature for igniting a D-T plasma is about 100 million K, which corresponds to an "ion temperature" of $kT \sim 10$ keV.
2 The plasma density n (in ions/m^3) must be high to ensure that collisions between nuclei are frequent.
3 The plasma of reacting nuclei must remain together for a sufficiently long time τ. How long depends on the product $n\tau$, the confinement quality parameter. In the case of a D-T plasma with $kT \sim 10$ keV, $n\tau$ must be greater than roughly 10^{20} s/m^3 for breakeven, more than that for ignition (Fig. 12.29).

Apart from stellar interiors, the combination of temperature, density, and confinement time needed for fusion thus far has occurred only in the explosion of fission ("atomic") bombs. Incorporating the ingredients for fusion reactions in such a bomb leads to an even more destructive weapon, the "hydrogen" bomb.

Confinement Methods

The approach to the controlled release of fusion energy that has thus far shown the most promise uses a strong magnetic field to confine the reactive plasma. In the Russian-designed **tokamak** scheme, the magnetic field is a modified torus (doughnut) in form (Fig. 12.30). (In Russian, tokamak stands for "toroidal magnetic chamber.") Because the field lines of a purely toroidal field are curved, an ion moving in

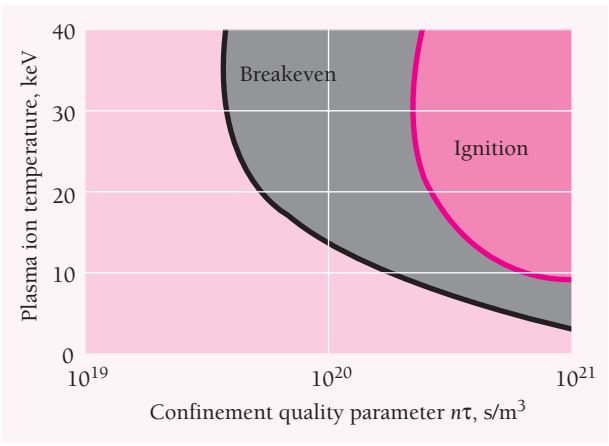
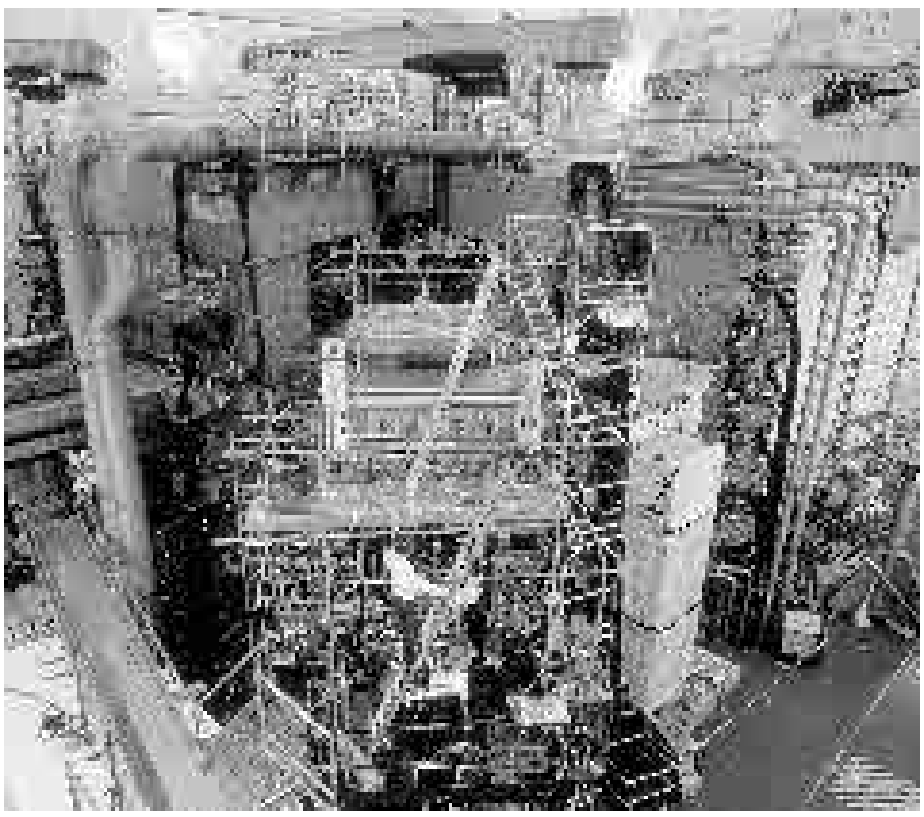


Figure 12.29 Conditions for breakeven (energy output equals energy input) and for ignition (a self-sustaining reaction) in a fusion reactor. Existing reactors have come close to breakeven; the projected International Thermonuclear Experimental Reactor is intended to go beyond it.



The Joint European Torus is an experimental tokamak fusion reactor at Culham, England. The reactor has delivered 16 MW with a power input of 25 MW, encouragingly close to breakeven.

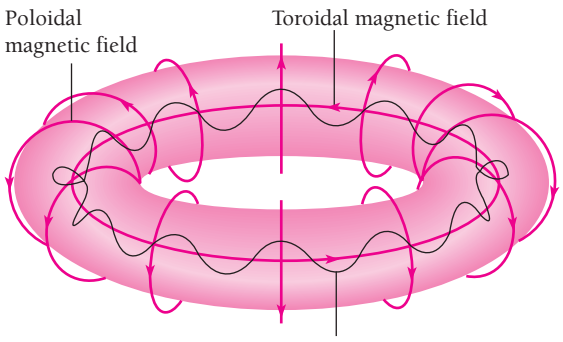


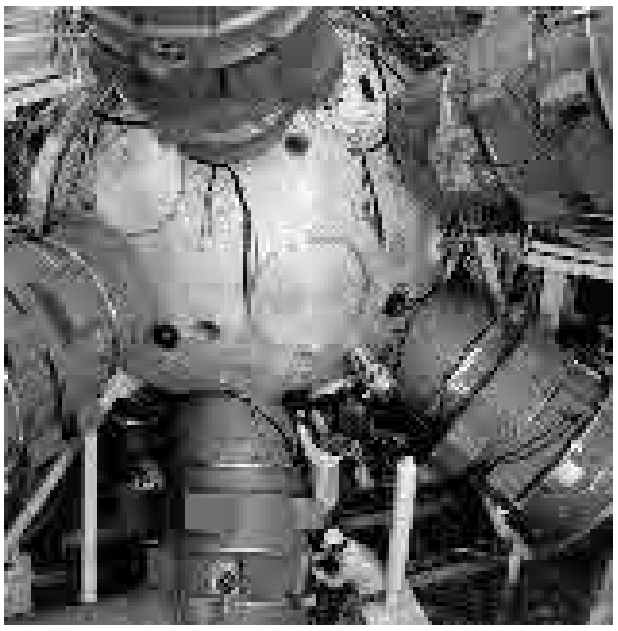
Figure 12.30 In a tokamak, combined toroidal and poloidal magnetic fields confine a plasma.

a helical path around its field lines will drift across the field and escape. To prevent this, a tokamak uses a poloidal field whose field lines are circles around the toroid axis. The poloidal field is produced by a current set up in the plasma itself by the changing field of an electromagnet in the center of the toroid. This current also heats the plasma; once the plasma is sufficiently hot, the current needs little help to continue.

ITER

The planned International Thermonuclear Experimental Reactor (ITER) represents what is hoped to be the final step before practical fusion energy becomes a reality. ITER is currently sponsored by Japan and several European countries; the United States pulled out of the project because of concerns about its original design and cost, and Russia withdrew (except for providing some staff) because it cannot afford to participate. The redesigned ITER is expected to generate 400 MW from deuterium-tritium reactions, to weigh 32,000 tons, to cost $3 billion, and to take 10 to 15 years to build. Superconducting magnets (a large part of the cost) will keep the reacting ions in a doughnut-shaped region whose volume is that of a large house. About 80 percent of the energy released will be carried off by the neutrons that are produced, and these neutrons will be absorbed by lithium pellets in tubes that surround the reaction chamber. Circulating water will carry away the resulting heat; this is the heat that could be used in a commercial reactor to power turbines connected to electric generators.

Even if ITER works as planned, though, not every pessimistic observer of the fusion program is likely to become a convert to the cause. Fusion reactors will certainly be enormously complex and expensive and not wholly safe: lithium is an extremely reactive metal that burns or explodes on contact with water. Also, when lithium absorbs neutrons in the reactions of Eqs. (12.31) and (12.32), radioactive tritium is produced. Hence an accident could be catastrophic. Of course, the optimists could turn out to be correct, and fusion will become the preferred energy source of the future. But even if this happens, many decades lie ahead in which energy problems will remain. Fission reactors employ an established technology and ways exist to make them very safe, but memories of Three Mile Island and Chernobyl, plus continuing questions about the disposal of radioactive wastes, continue to affect their public image. Meanwhile fossil fuels are being used up and burning them produces enough CO_2 to affect weather and climate. Such "green" energy sources as solar cells and wind turbines are unlikely to provide more than a small (though welcome) fraction of energy needs. An energy strategy for the world that is both sensible and widely acceptable is not obvious.



The world's most powerful laser, located at the Lawrence National Laboratory in California, is used in inertial confinement experiments. Its output of 60 kJ per nanosecond (10^{-9} s) pulse is divided into 10 beams that are directed at tiny dueterium-tritium pellets in an effort to induce fusion reactions in them.

The most powerful tokamaks today have attained plasma temperatures of 30 keV and confinement quality $n\tau$ values of 2×10^{19} s/m^3, but not breakeven. Breakeven will probably have to wait for the planned International Thermonuclear Experimental Reactor (ITER).

An entirely different procedure, called **inertial confinement,** uses energetic beams to both heat and compress tiny deuterium-tritium pellets by blasting them from all sides. The result is, in effect, a miniature hydrogen-bomb explosion, and a succession of them could provide a steady stream of energy. If ten 0.1-mg pellets are ignited every second, the average thermal output would be about 1 GW and could yield 300 MW or so of electric power, enough for a city of 175,000 people.

Laser beams have received the most attention for inertial confinement, but electron and proton beams have promise as well. The beam energy is absorbed in the outer layer of the fuel pellet, which blows off outward. Conservation of momentum leads to an inward shock wave that must squeeze the rest of the pellet to about 10^4 times its original density to heat the fuel sufficiently to start fusion reactions. The required beam energy is well beyond the capacity of today's lasers, though perhaps not of future ones. Particle beams are closer to reaching the needed energy but are much harder to focus on the tiny fuel pellets. Research continues, but magnetic confinement seems closer to the goal of a working fusion reactor.