Department of Applied Mathematics
Delhi Technological University

Assignment-III
B.Tech I Semester Academic Year 2019-20

Course: Mathematics-I **Code: MA-101**

1. If $u = \log(x^2 + y^2) + \tan^{-1} \frac{y}{x}$, show that
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

2. If $z = f(x, y)$ and $x = e^u + e^{-v}$, $y = e^{-u} + e^v$, prove that
$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$$

3. If $x^2 + y^2 + z^2 - 2xyz = 1$, show that $\frac{dx}{\sqrt{(1-x^2)}} + \frac{dy}{\sqrt{(1-y^2)}} + \frac{dz}{\sqrt{(1-z^2)}} = 0$

4. If $u = f(x, y)$ and $x = r\cos\theta$, $y = r\sin\theta$, prove that
$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$$

5. Verify Euler's Theorem, When
   (a) $f(x, y) = ax^2 + 2hxy + by^2$
   (b) $f(x, y) = \frac{x^3+y^3}{x+y}$

6. If $u = \tan^{-1} \frac{x^3+y^3}{x+y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ and
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$$

7. If $u = \tan^{-1} \frac{y^2}{x}$, prove that
$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin^2 u \sin 2u$$

8. Expand the function $f(x, y) = e^x \log(1 + y)$ in powers of x and y upto terms of third degree.

9. Expand the function $f(x, y) = x^y$ in powers of $(x - 1)$ and $(y - 1)$ upto terms of third degree.

10. Examine the function $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$ for extreme values. (Ans. min for $(\sqrt{2}, -\sqrt{2}), (-\sqrt{2}, \sqrt{2})$, Saddle point $(0,0)$)

11. If the sides of a plane triangle ABC vary in such a way that its circumradius remains constant then prove that

$$\frac{da}{\cos A} + \frac{db}{\cos B} + \frac{dc}{\cos C} = 0$$

12. A balloon is in the form of right circular cylinder of radius 1.5 m and length 4 m and is surmounted by hemispherical ends. If the radius is increased by 0.01 m and the length by 0.05 m, then find the approximate percentage change in the volume of the balloon. (Ans. 2.39%)

13. Given $x+y+z = a$, find the maximum value of $x^m y^n z^p$. (Ans. $\frac{a^{(m+n+p)} m^m n^n p^p}{(m+n+p)^{(m+n+p)}}$)

14. Show that the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $\frac{8abc}{3\sqrt{3}}$.

15. Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

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