effective masses of the electron and hole are not exactly equal, then the effective density of states functions $g_c(E)$ and $g_v(E)$ will not be exactly symmetrical about the midgap energy. The Fermi level for the intrinsic semiconductor will then shift slightly from the midgap energy in order to obtain equal electron and hole concentrations.
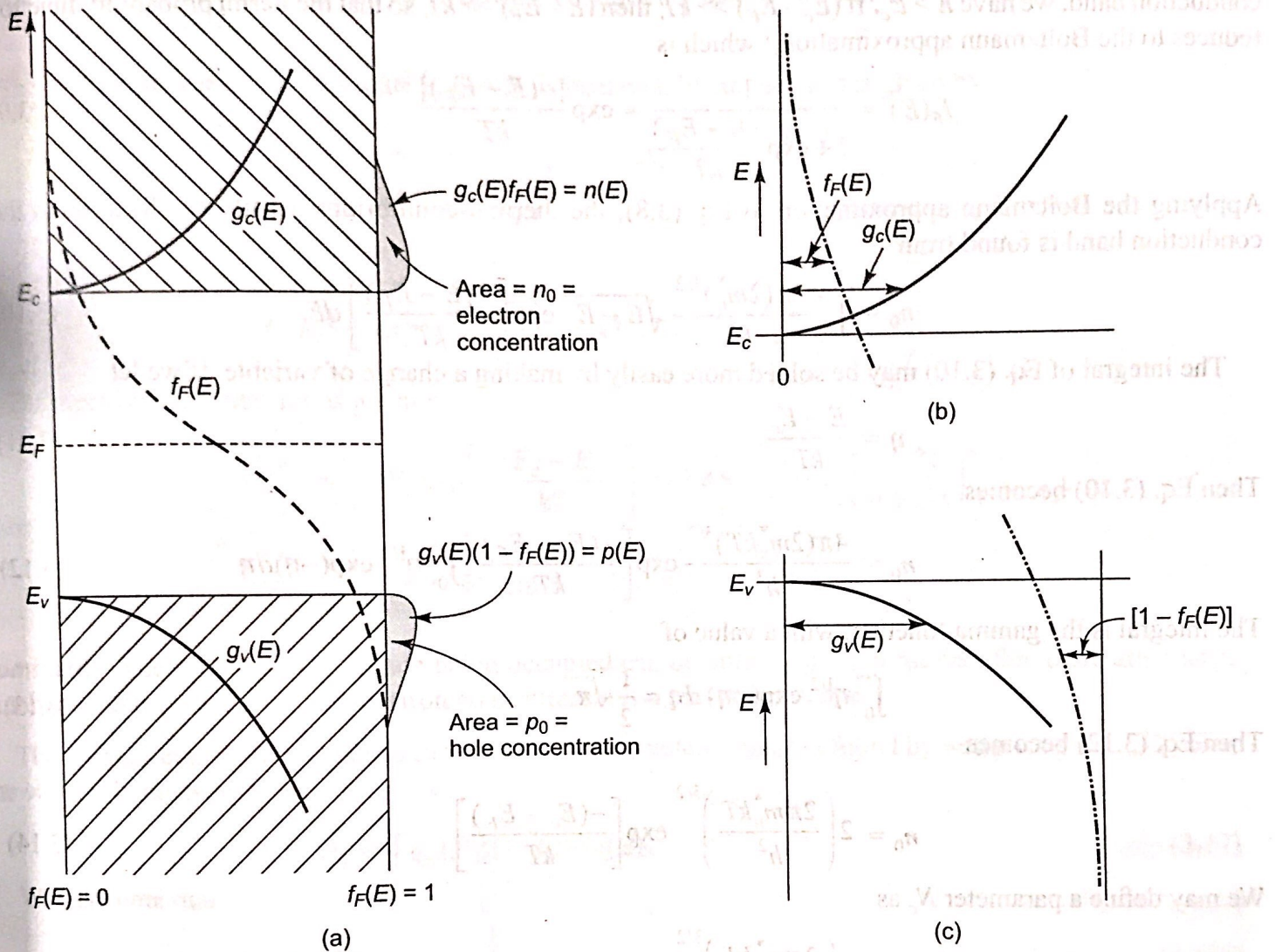

Fig. 3.9 *(a) Density of states functions, Fermi–Dirac probability function, and areas representing electron and hole concentrations for the case when E_F is near the midgap energy; (b) expanded view near the conduction band energy; and (c) expanded view near the valence band energy*

3.2.2 The n_0 and p_0 Equations

We have argued that the Fermi energy for an intrinsic semiconductor is near midgap. In deriving the equations for the thermal-equilibrium concentration of electrons n_0 and the thermal-equilibrium concentration of holes p_0, we will not be quite so restrictive. We will see later that, in particular situations, the Fermi energy can deviate from this midgap energy. We will assume initially, however, that the Fermi level remains within the bandgap energy.

The equation for the thermal-equilibrium concentration of electrons may be found by integrating Eq. (3.6) over the conduction band energy, or

$$n_0 = \int g_c(E) f_F(E) \, dE \tag{3.8}$$

The lower limit of integration is E_c and the upper limit of integration should be the top of the allowed conduction band energy. However, since the Fermi probability function rapidly approaches zero with increasing energy as indicated in Fig. 3.9(a), we can take the upper limit of integration to be infinity.

We are assuming that the Fermi energy is within the forbidden-energy bandgap. For electrons in the conduction band, we have $E > E_c$. If $(E_c - E_F) \gg kT$, then $(E - E_F) \gg kT$, so that the Fermi probability function reduces to the Boltzmann approximation,[1] which is

$$f_F(E) = \frac{1}{1 + \exp\dfrac{(E - E_F)}{kT}} \approx \exp\frac{[-(E - E_F)]}{kT} \tag{3.9}$$

Applying the Boltzmann approximation to Eq. (3.8), the thermal-equilibrium density of electrons in the conduction band is found from

$$n_0 = \int_{E_c}^{\infty} \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \exp\left[\frac{-(E - E_F)}{kT}\right] dE \tag{3.10}$$

The integral of Eq. (3.10) may be solved more easily by making a change of variable. If we let

$$\eta = \frac{E - E_c}{kT} \tag{3.11}$$

Then Eq. (3.10) becomes

$$n_0 = \frac{4\pi(2m_n^* kT)^{3/2}}{h^3} \exp\left[\frac{-(E_c - E_F)}{kT}\right] \int_0^{\infty} \eta^{1/2} \exp(-\eta) d\eta \tag{3.12}$$

The integral is the gamma function, with a value of

$$\int_0^{\infty} \eta^{1/2} \exp(-\eta) \, d\eta = \frac{1}{2}\sqrt{\pi} \tag{3.13}$$

Then Eq. (3.12) becomes

$$n_0 = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_c - E_F)}{kT}\right] \tag{3.14}$$

We may define a parameter N_c as

$$N_c = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2} \tag{3.15}$$

so that the thermal-equilibrium electron concentration in the conduction band can be written as

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] \tag{3.16}$$

The parameter N_c is called the *effective density of states function in the conduction band*. If we were to assume that $m_n^* = m_0$, then the value of the effective density of states function at $T = 300$ K is $N_c = 2.5 \times 10^{19}$ cm^{-3}, which is the order of magnitude of N_c for most semiconductors. If the effective mass of the electron is larger or smaller than m_0, then the value of the effective density of states function changes accordingly, but is still of the same order of magnitude.

---

[1] The Maxwell–Boltzmann and Fermi–Dirac distribution functions are within 5 percent of each other when $E - E_F \approx 3kT$ (see Fig. 3.8). The $\gg$ notation is then somewhat misleading to indicate when the Boltzmann approximation is valid, although it is commonly used.

Example 3.6

Objective *To calculate the probability that a state in the conduction band is occupied by an electron and calculate the thermal equilibrium electron concentration in silicon at T = 300 K.*

Assume the Fermi energy is 0.25 eV below the conduction band. The value of N_c for silicon at $T = 300$ K is $N_c = 2.8 \times 10^{19}$ cm^{-3}.

The probability that an energy state at $E = E_c$ is occupied by an electron is given by

$$f_F(E_c) = \frac{1}{1 + \exp\left(\dfrac{E_c - E_F}{kT}\right)} \approx \exp\left[\frac{-(E_c - E_F)}{kT}\right]$$

or

$$f_F(E_c) = \exp\left(\frac{-0.25}{0.0259}\right) = 6.43 \times 10^{-5}$$

The electron concentration is given by

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = (2.8 \times 10^{19}) \exp\left(\frac{-0.25}{0.0259}\right)$$

or

$$n_0 = 1.8 \times 10^{15} \text{ cm}^{-3}$$

Comment The probability of a state being occupied can be quite small, but the fact that there are a large number of states means that the electron concentration is a reasonable value.

The thermal-equilibrium concentration of holes in the valence band is found by integrating Eq. (3.7) over the valence band energy, or

$$P_0 = \int g_v(E)[1 - f_F(E)] \, dE \tag{3.17}$$

We may note that

$$1 - f_F(E) = \frac{1}{1 + \exp\left(\dfrac{E_F - E}{kT}\right)} \tag{3.18a}$$

For energy states in the valence band, $E < E_v$. If $(E_F - E_v) \gg kT$ (the Fermi function is still assumed to be within the bandgap), then we have a slightly different form of the Boltzmann approximation. Equation (3.18a) may be written as

$$1 - f_F(E) = \frac{1}{1 + \exp\left(\dfrac{E_F - E}{kT}\right)} \approx \exp\left[\frac{-(E_F - E)}{kT}\right] \tag{3.18b}$$

Applying the Boltzmann approximation of Eq. (3.18b) to Eq. (3.17), we find the thermal-equilibrium concentration of holes in the valence band is

$$P_0 = \int_{-\infty}^{E_v} \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \, \exp\left[\frac{-(E_F - E)}{kT}\right] dE \tag{3.19}$$

where the lower limit of integration is taken as minus infinity instead of the bottom of the valence band. The exponential term decays fast enough so that this approximation is valid.

Equation (3.19) may be solved more easily by again making a change of variable. If we let

$$\eta' = \frac{E_v - E}{kT}$$

then Eq. (3.19) becomes

$$p_0 = \frac{-4\pi(2m_p^* kT)^{3/2}}{h^3} \exp\left[\frac{-(E_F - E_v)}{kT}\right] \int_{+\infty}^{0} (\eta')^{1/2} \exp(-\eta') d\eta'$$

where the negative sign comes from the differential $dE = -kT d\eta'$. Note that the lower limit of η' becomes $+\infty$ when $E = -\infty$. If we change the order of integration, we introduce another minus sign. From Eq. (3.19), Eq. (3.21) becomes

$$p_0 = \left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2} \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

We may define a parameter N_v as

$$N_v = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

which is called the *effective density of states function in the valence band*. The thermal-equilibrium concentration of holes in the valence band may now be written as

$$\boxed{p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]}$$

The magnitude of N_v is also on the order of 10^{19} cm^{-3} at $T = 300$ K for most semiconductors.

Example 3.7

Objective *Calculate the thermal equilibrium hole concentration in silicon at $T = 400$ K.*

Assume that the Fermi energy is 0.27 eV above the valence band energy. The value of N_v for silicon at $T = 300$ K is $N_v = 1.04 \times 10^{19}$ cm^{-3}.

The parameter values at $T = 400$ K are found as

$$N_v = (1.04 \times 10^{19})\left(\frac{400}{300}\right)^{3/2} = 1.60 \times 10^{19} \text{ cm}^{-3}$$

and

$$kT = (0.0259)\left(\frac{400}{300}\right) = 0.03453 \text{ eV}$$

The hole concentration is then

$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = (1.60 \times 10^{19}) \exp\left(\frac{-0.27}{0.03453}\right)$$

or

$$p_0 = 6.43 \times 10^{15} \text{ cm}^{-3}$$

Comment The parameter values at any temperature can easily be found by using the 300 K values and the temperature dependence.

The effective density of states functions, N_c and N_v, are constant for a given semiconductor material at a fixed temperature. Table 3.1 gives the values of the density of states function and of the effective masses for silicon, gallium arsenide, and germanium. Note that the value of N_c for gallium arsenide is smaller than the typical 10^{19} cm^{-3} value. This difference is due to the small electron effective mass in gallium arsenide.

Table 3.1 *Effective density of states function and effective mass values*

	N_c (cm^{-3})	N_v (cm^{-3})	m_n^*/m_0	m_n^*/m_0
Silicon	2.8×10^{19}	1.04×10^{19}	1.08	0.56
Gallium arsenide	4.7×10^{17}	7.0×10^{18}	0.067	0.48
Germanium	1.04×10^{19}	6.0×10^{18}	0.55	0.37

The thermal equilibrium concentrations of electrons in the conduction band and of holes in the valence band are directly related to the effective density of states constants and to the Fermi energy level.

3.2.3 The Intrinsic Carrier Concentration

For an intrinsic semiconductor, the concentration of electrons in the conduction band is equal to the concentration of holes in the valence band. We may denote n_i and p_i as the electron and hole concentrations, respectively, in the intrinsic semiconductor. These parameters are usually referred to as the intrinsic electron concentration and intrinsic hole concentration. However, $n_i = p_i$ so normally we simply use the parameter n_i as the intrinsic carrier concentration, which refers to either the intrinsic electron or hole concentration.

The Fermi energy level for the intrinsic semiconductor is called the intrinsic Fermi energy, or $E_F = E_{Fi}$. If we apply Eqs (3.16) and (3.24) to the intrinsic semiconductor, then we can write

$$n_0 = n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \tag{3.25}$$

and

$$p_0 = p_i = n_i = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right] \tag{3.26}$$

If we take the product of Eqs (3.25) and (3.26), we obtain

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_{Fi})}{kT}\right] \exp\left[\frac{-(E_{Fi} - E_v)}{kT}\right] \tag{3.27}$$

or

$$n_i^2 = N_c N_v \exp\left[\frac{-(E_c - E_v)}{kT}\right] = N_c N_v \exp\left[\frac{-E_g}{kT}\right] \tag{3.28}$$

where E_g is the bandgap energy. For a given semiconductor material at a constant temperature, the value of n_i is a constant, and independent of the Fermi energy.