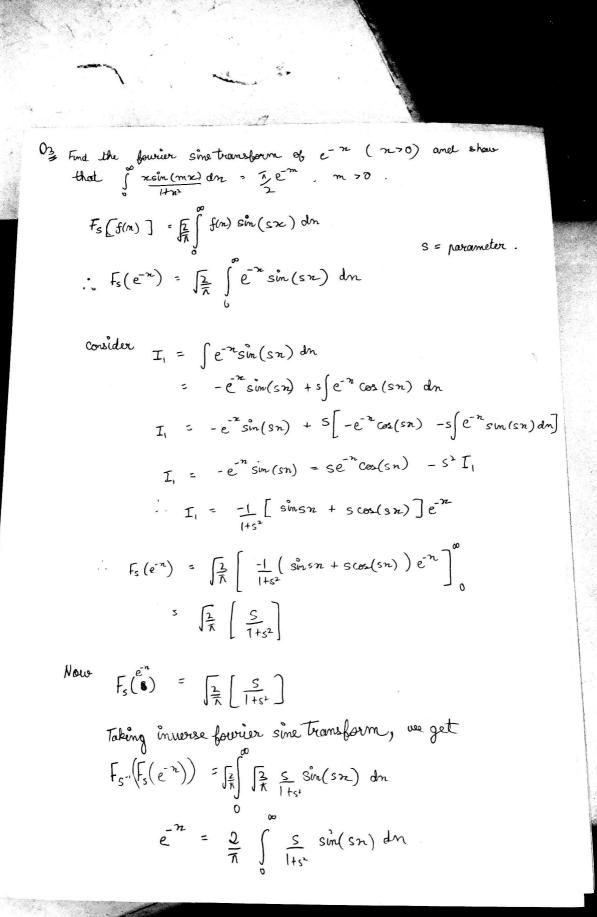
Of Do fourier sino and contransforms of en exist? Explain.

Solution for existence of forwirer sine and cosine transforms, f(n) should be absolutely integratable in  $(-\infty, \infty)$  i.e.  $\int_{-\infty}^{\infty} |f(n)| dn < \infty$  [or should be finite].

for  $f(n) = e^{nc}$ , the integral blows up at  $+\infty$ , hence its sine and cosine fourier transforms does not exist.

 $O_{Z}$  Find the former cosine Transform of f(n) = 1  $1+n^{2}$ 



substitute 
$$x = m$$
 and  $s = n$ 

$$e^{-m} = \frac{2}{h} \int_{0}^{\infty} \frac{n}{1+n^{2}} \sin(mn) dn$$

$$\frac{\pi}{a} e^{-m} = \int_{0}^{\infty} \frac{n}{1+n^{2}} \sin(nn) dn$$

Hence, poroud .

Oy Find the bowier transform of the bunction  $f(n) = e^{-an^2}$  a>0, and hence find the bowier transfor of  $e^{-n\frac{\pi}{2}}$ 

$$F(f(n)) = \int_{2\pi}^{\pi} \int_{-\infty}^{\infty} f(n) e^{i\omega n}$$

$$F(e^{-nx^{2}}) = \int_{2\pi}^{\pi} \int_{-\infty}^{\infty} e^{-an^{2}} e^{isn} dn$$

$$= \int_{2\pi}^{\pi} \int_{-\infty}^{\infty} e^{-(an^{2} - isn)} e^{-(an^{2} - isn + i^{2}s^{2} - i^{2}s^{2})}$$

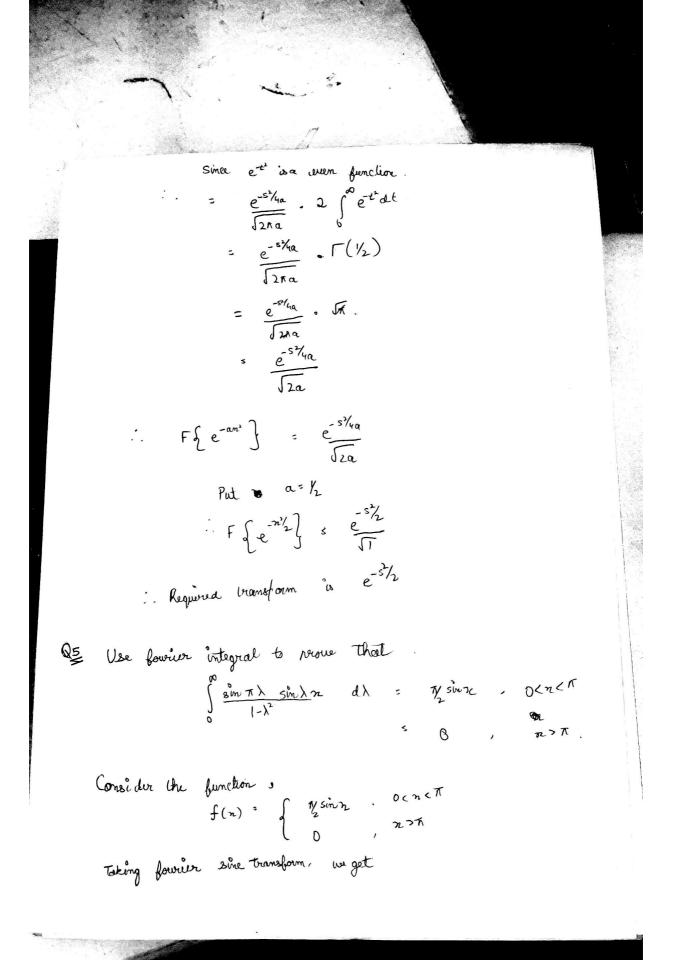
$$= \int_{2\pi}^{\pi} \int_{-\infty}^{\infty} e^{-(an^{2} - isn + i^{2}s^{2} - i^{2}s^{2})} dn$$

$$= e^{-s^{2}/4a} \int_{-\infty}^{\infty} e^{-[hn - is/2a]^{2}} dn$$

$$det \int_{2\pi}^{\pi} \int_{-\infty}^{\infty} e^{-\frac{is}{2a}} dn = \frac{dt}{\sqrt{2a}}$$

$$= e^{-s^{2}/4a} \int_{-\infty}^{\infty} e^{-\frac{is}{2a}} dt$$

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$$F_{S}(fn)) = \int_{\pi}^{\infty} \int_{0}^{\infty} f(n) \sin(\lambda n) dn$$

$$= \int_{\pi}^{2} \int_{0}^{\pi} f \sin x \sin(\lambda n) dn + \int_{\pi}^{0} \sin(\lambda n) dn$$

$$= \int_{\pi}^{\pi} \int_{0}^{\pi} \cos x \sin(\lambda n) dn$$

$$= \int_{2}^{\pi} \int_{2}^{\pi} \frac{\cos(\lambda - 1)n - \sin(\lambda + 1)n}{\lambda - 1} dn$$

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$$= \int_{2}^{\pi} \int_{2}^{\infty} \frac{\sin(\lambda - 1)n}{\lambda - 1} - \frac{\sin(\lambda + 1)n}{\lambda + 1} dn$$

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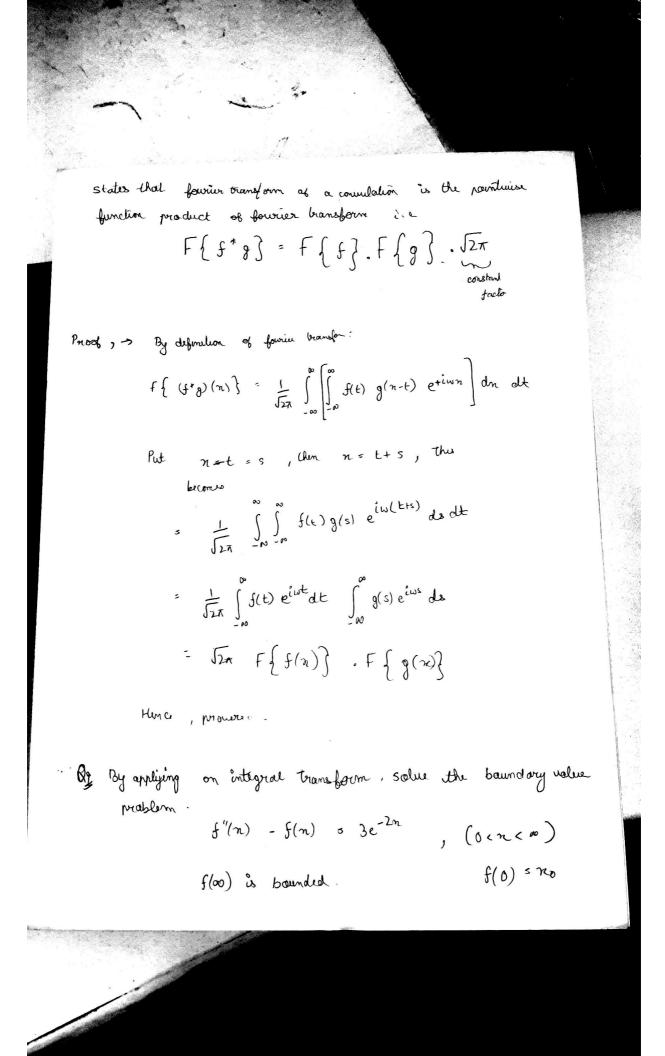
Hence proved

Of Solve the integral equation  $\int_{0}^{\infty} f(n) \cos(sn) dn = 1-s \quad 0 \leq s \leq 1$ and hence show that  $\int_{0}^{\infty} \frac{\sin^2 t}{t^2} dt \cdot \sqrt[n]{2}$ (ansider  $f(s) = \begin{cases} 1-s & 0 \le s \le 1 \\ 0 & s > 1 \end{cases}$ Taking inverse social teranston ; .. we get,  $F_{c}^{-1}\left\{f(\mathbf{s})\right\} \stackrel{s}{=} \int_{\mathbb{R}}^{\infty} \int f(s) \cos s \, ds \, ds$  $s \int_{R} \int_{R} (1-s) \cos(sn) d\lambda + \int_{R}^{\infty} 0 dx$  $= \int_{n}^{2} \left\{ \frac{\sin(sn)}{n} \right|_{b}^{b} + \left\{ \frac{\sin sn}{s} \right\} = \int_{0}^{\infty} \frac{\sin sn}{n} dn$  $\frac{1}{n} \left[ \frac{\cos n}{n^2} \right]_0^1$  $F_{c}^{-1}\left\{f(s)\right\} = \int_{-R}^{L} \frac{1-\cos n}{n^{2}}$ Taking favour cosme bronsfer as get  $f(s) \leq \frac{2}{\pi} \int \frac{1-\cos n}{n^2} \cos/sn/dn$  $1 = \frac{2}{n} \int_{0}^{\infty} \frac{1 - \cos n}{n^2} dn$ Put n = 2n, we get  $\nabla = \left(\frac{2\sin^2 n}{4n^2}\right) \times dn$ 

Of f(s) is the formier transform of f(n), then prove that  $f(f(n))e^{-i\alpha n} = f(s-a)$ By definition of formier transform  $F\{f(n)\} = \int_{-\infty}^{\infty} f(n) e^{isn} dn$   $= \int_{-\infty}^{\infty} f(n) e^{i\alpha n} e^{isn} dn$   $= \int_{-\infty}^{\infty} f(n) e^{i(s-a)n} dn$ 

Of Define considering of two functions f(n) and g(n) and hence prove that forwier transform of consulation of two functions is equal to the product of their forwier transforms. Consulation nepers to a mathematical operations in two function f(n) and g(n), and giving the integral of the raintives multiple cation of the less functions as a function of the amount that one of the original function is translated.

And in case of formier bransforms it is called consultation the for formies bransforms which



Soluleon Applying fourier sine transform on bath sides  $f_s[f''(n)] - f_s[f(n)] = 3f_s[e^{2n}]$  $-s^{2} - f_{s}(\tilde{\omega}) + \frac{s}{\omega} \left[ \frac{1}{2} f(0) - f_{s}(\tilde{\omega}) = 3 f_{s} \left[ e^{2m} \right] \right]$  $F_s(\mathbf{x})$  is form in transform of f(n). Consider ⇒ fs[e<sup>2n</sup>]  $I = \int_{\frac{\pi}{N}}^{\infty} \int_{0}^{\infty} e^{2\pi t} \cdot \sin(sn) dn$  $I = \int_{\frac{\pi}{h}}^{2} \int e^{-2n} \cdot \sin(sn) \, dn$ I's  $\int_{R}^{2} \left[ \frac{e^{2n}}{n} \sin(sn) + \frac{s}{2} \int e^{2n} \cos n \right]$  $I = \int_{\overline{\Lambda}} \left[ \frac{e^{-2n} \operatorname{Sin}(\operatorname{sn})}{-2} + \sum_{n=1}^{\infty} \left[ \frac{e^{-2n} \operatorname{conn}}{-2} - \sum_{n=1}^{\infty} e^{-2n} \operatorname{Sin}(\operatorname{sn}) \right] dn \right]$  $I\left[1+\frac{s^2}{4}\right] = \int_{\frac{1}{\Lambda}}^{\frac{1}{2}} \left[-\frac{1}{2}\right] \left[e^{-2n}\sin n + \frac{s}{2}e^{-2n}\cos(\sin)\right]_0^{\infty}$ = 12 8.  $-s^{2} f_{s}(s) + s \int_{\frac{\pi}{n}}^{2} n_{o} - F_{s}(s) + s \int_{\frac{\pi}{n}}^{2} \frac{s}{s^{2} + 4}$  $F_{s}(s) = \int_{\pi}^{2} \frac{\delta x_{o}}{\kappa^{2+1}} = 3 \int_{\pi}^{2} \frac{s}{(s^{2}+4)(s^{2}+1)}$ 

