# Curve Tracing

23 Aug 2K19

$f(x) = 4x^3 - 12x^2 = 0 \qquad x \in (-\infty, \infty)$

$x = 0, 3$

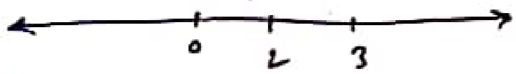
| Interval | $x<0$ | $0<x<3$ | $x>3$ |
|---|---|---|---|
| sign of $f'(x)$ | $-ve$ | $-ve$ | $+ve$ |
| Nature | ↓ | ↓ | ↑ |

$f'(x) = 12x^2 - 24x$

$= 12x(x-2)$

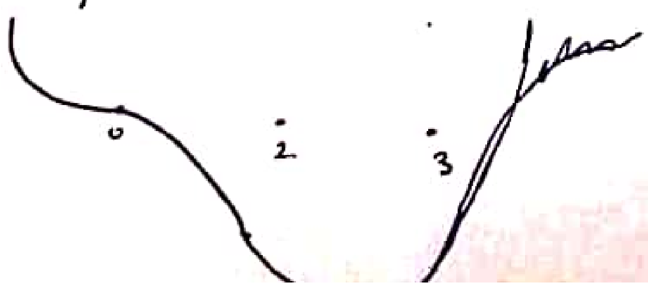
| Interval | $x<0$ | $0<x<2$ | $x>2$ |
|---|---|---|---|
| Sign of $f''(x)$ | $+ve$ | $-ve$ | $+ve$ |
| Nature ∪ or ∩ | ∪ | ∩ | ∪ |

- Things to know to draw graph
  - Domain, Range
  - Odd, even
  - continuity
  - Concave up or concave down
  - Asymptotes
  - Pt. of inflection
  - Imaginery Region




| Interval | $x<0$ | $0<x<2$ | $2<x<3$ | $x>3$ |
|---|---|---|---|---|
| | ↓ | ↓ | ↓ | ↑ |

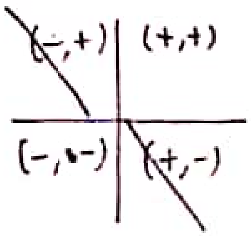
combining



Q: $x^{5/3} = f(x)$ $\qquad x \in (-\infty, \infty)$

$y^3 = x^5$ $\qquad$ there is imaginery region $f^n$ does not exist in II, IV quad.

- when $x$ is $-ve$ $y$ cant be $+ve$ and vica verca.

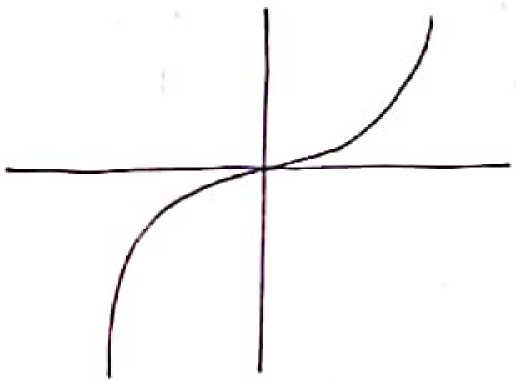



$y' = \frac{5}{3}x^{2/3}$

critical pt $= 0$


$f''(x) = \frac{5}{3}\cdot\frac{2}{3}x^{-\frac{1}{3}}$

| Interval | $x<0$ | $x>0$ |
|---|---|---|
| ~~Sign of~~ Sign $f'(x)$ | $+ve$ | $+ve$ |
| Sign $f''(x)$ | $-ve$ | $+ve$ |
| | $\cap$ | $\cup$ |

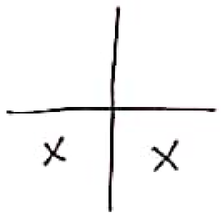


Q: $f(x) = \frac{(x+1)^2}{1+x^2}$ (always $+ve$)

$$f'(x) = \frac{(1+x^2)\cdot 2(x+1) - (x+1)^2(2x)}{(1+x^2)^2}$$

$$= \frac{\cancel{2x}+2+\cancel{2x^3}+2x^2-[\cancel{2x^3}+4x^2+\cancel{2x}]}{(1+x^2)^2}$$

$$= \frac{2-2x^2}{1+x^2} = \frac{2(1-x^2)}{1+x^2}$$




critical pts

$f'(x) = 0$. $2(1-x^2) = 0$

$x = \pm 1$

| ~~Interval~~ Nature | $-\infty < x < -1$ | $-1 < x < 1$ | $x > 1$ |
|---|---|---|---|
| sign | $-ve$ | $+ve$ | $-ve$ |
| $\uparrow/\downarrow$ | $\downarrow$ | $\uparrow$ | $\downarrow$ |

$$f''(x) = \frac{4x(x^2-3)}{(1+x^2)^2}$$

| Nature | $x < -\sqrt{3}$ | $-\sqrt{3} < x < 0$ | $0 < x < \sqrt{3}$ | $x > \sqrt{3}$ |
|---|---|---|---|---|
| | $-ve$ | $+ve$ | $-ve$ | $+ve$ |
| | $\cap$ | $\cup$ | $\cap$ | $\cup$ |

## Asymptotes

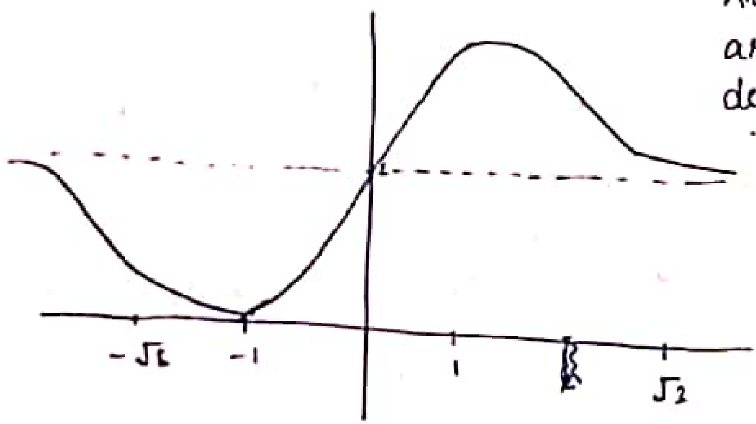
$$y = \frac{x^2+2x+1}{1+x^2}$$

divide N and D by $x^2$ $\lim_{x \to \pm\infty}$

$$f(x) = \frac{1+\frac{2}{x}+\frac{1}{x^2}}{1+\frac{1}{x^2}}$$

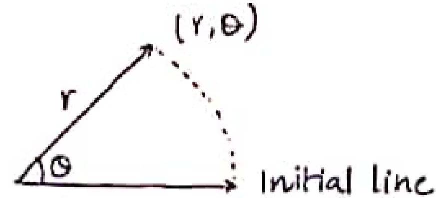
$\boxed{y=1}$ — this is asymptotes.

| $-\infty$ to $\sqrt{3}$ | $-\sqrt{3}$ to $-1$ | $-1$ to $1$ | $1$ to ~~$2\sqrt{3}$~~ | ~~$2$ to $\sqrt{2}$~~ | $\sqrt{3}$ to $\infty$ |
|---|---|---|---|---|---|
| ↓U | ↓U | ↑U | ↑∩ | ~~↓∩~~ | ↓U |

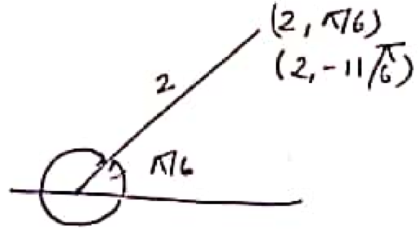



- Asymptotes are always denoted by ........

## Polar Coordinates




$\underbrace{r = f(\theta)}$ polar coordinates




Find all the polar coordinates of $(2, \pi/6)$

$\pi/6,\ \pi/6 \pm 2\pi,\ \pi/6 \pm 3\pi,\ \pi/6 \pm 4\pi \ldots$

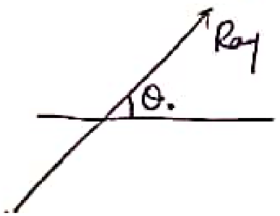
for $r = -2$ $\frac{\pi}{6} \pm n\pi$, $n \in (0, \infty)$

for $r = +2$ $\frac{-5\pi}{6},\ \frac{-5\pi}{6} \pm 2\pi,\ \frac{-5\pi}{6} \pm 4\pi,\ \frac{5\pi}{6} \pm 6\pi \ldots$

### Polar equation and graph

$r = a$ is a circle with radius $a$.

$\theta = \theta_0 \longrightarrow$ ray

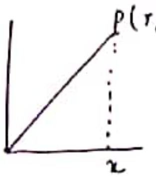


1) $r=1$ and $r=1$

unit circle.

2) $\theta = \pi/6$ , $\theta = 7\pi/6$

3) $r=1$, $r=2$ $\quad 0 \le \theta \le \pi/2$




$y = r\cos\theta$

$y = r\sin\theta$

$r^2 = x^2+y^2$

$\tan\theta = y/x$

$r\cos\theta = 2$

$x = 2$

$r^2\cos\theta\sin\theta = 4$

$(r\cos\theta)(r\sin\theta) = 4$

$xy = 4$

$r = 1 + 2r\cos\theta$

$r^2 = 1 + 4r^2\cos^2\theta + 4r\cos\theta$

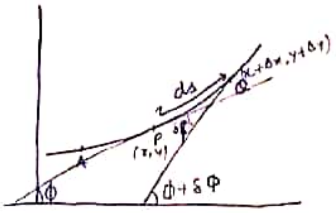
$x^2+y^2 = 1 + 4x^2 + 4x$

$x^2 + 4x^2 + 4x + y^2 - 1 = 0$

## Curvature

$\delta\phi$ is the angle through which the tangent at P turns as the point P moves to the point Q along the curve.

Def. : Rate of change of angle wrt to the arc $\delta s$ as $Q \xrightarrow[\text{to}]{\text{approaches}} P$ along the curve is called curvature.




$\widehat{AP} = s$

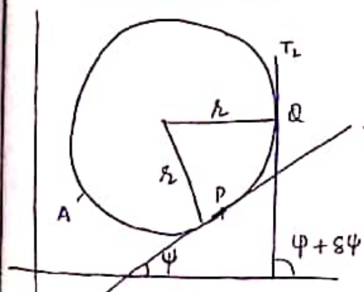
$\widehat{AQ} = s + \delta s$

$\widehat{PQ} = \delta s$

$$\text{Curvature} = \lim_{Q\to P} \frac{\delta\phi}{\delta s}$$

$$= \lim_{\delta s \to 0} \frac{\delta\phi}{ds}$$

## Curvature a circle




$\widehat{PQ} = \delta s$

By def. of curvature at P,

$$= \lim_{Q\to P} \frac{\delta\psi}{\delta s}$$

$$= \lim_{\delta s\to 0} \frac{\delta\psi}{\delta s}$$

$$= \lim_{\delta s\to 0} \frac{\cancel{\delta\psi}}{r\,\cancel{\delta\psi}}$$

$$\boxed{k = \frac{1}{r}}$$

$$\text{curvature} = \frac{1}{\text{Radius}}$$

r ~~also known as~~ radius of curvature. $= \frac{1}{k} = \rho$

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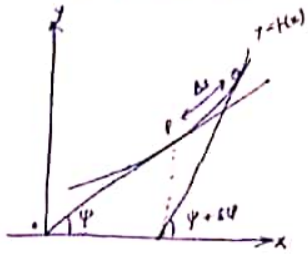
* Radius of curvature (cartesian coordinates)

Let $y = f(x)$ be the curve




$\tan\psi = \frac{dy}{dx}$

diff both sides wrt s

$\sec^2\psi \frac{d\psi}{ds} = \frac{d}{dx}\left(\frac{dy}{dx}\right) \cdot \frac{dx}{ds}$

$\sec^2\psi \frac{d\psi}{ds} = \frac{d^2y}{dx^2} \cdot \frac{dx}{ds}$

$= \dfrac{\frac{d^2y}{dx^2}}{\frac{ds}{dx}}$

$\frac{d\psi}{ds} = \dfrac{\frac{d^2y}{dx^2}}{\frac{ds}{dx} \cdot \sec^2\psi}$

$= \dfrac{\frac{d^2y}{dx^2}}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}$

Q: Find curvature of the parabola $y^2 = 2x$ at $(2,2)$

curvature $= \left.\dfrac{y''}{(1+(y')^2)^{3/2}}\right|_{2,2}$

$y^2 = 2x$

$2y\frac{dy}{dx} = 2$

$\frac{dy}{dx} = \frac{1}{y} = \frac{1}{\sqrt{2x}}$

$\frac{d^2y}{dx^2} = \frac{-1}{2\sqrt{2}} x^{-3/2}$

$k = \dfrac{-\frac{x^{-3/2}}{2\sqrt{2}}}{\left(1+\frac{1}{\sqrt{2x}}\right)^{3/2}}$

$\dfrac{\cancel{-\frac{1}{2\sqrt{2}}} \times \frac{1}{2\sqrt{2}}}{\cancel{1+}\left(3/2\right)^{3/2}} = \dfrac{-1/8}{(1+1/4)^{3/2}}$

$= \left|\frac{-1}{5\sqrt{5}}\right|$

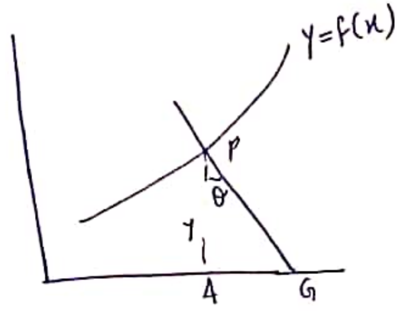
For a function -ve curvature has no value.

Q Normal at a pt $P$ on the curve

$$2abx = bx^2 + ay^2$$

meet the $x$ axis at pt $g$. Show that at pt $P$

$$\rho = \frac{PG^3}{b^2}$$




$$\frac{y}{PG} = \cos\theta$$

$$PG = y\sec\theta$$

$$= y\sqrt{1+\tan^2\theta}$$

$$= y\sqrt{1+(y')^2}$$

By calculation

$$\rho = \frac{1}{a^3b^3}\left[a^2y^2+b^2(a-x)^2\right]^{3/2}$$

$PG \longrightarrow$ Length of normal to the curve at $P$

$$PG = y\sqrt{1+(y')^2}$$

$$y = \sqrt{1+\left(\frac{b(a-x)}{ay}\right)^2}$$

$$= \frac{1}{a}\left[a^2y^2+b^2a^2+b^2x^2-2abx\right]$$

$$\frac{PG}{b^2} = \frac{\left(a^2y^2+b^2(a-x)^2\right)^{3/2}}{a^3b^2} = \rho$$

- Radius of curvature at a parametric curve

$x = x(t)$

$y = y(t)$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \quad \text{—} (1)$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'}{x'} \quad \text{—} (2)$$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}\left(\frac{y'}{x'}\right)$$

$$= \left[\frac{x'y''-y'x''}{(x')^2}\right]\frac{1}{x'}$$

$$= \frac{x'y''-y'x''}{(x')^3} \quad \text{—} (3)$$

Finally,

$$\rho = \frac{(x'^2+y'^2)^{3/2}}{x'y''-y'x''}$$

Radius of curvature

Q: Find Radius of curvature at any pt $t$ of the

$$x = a(t+\sin t)$$
$$y = a(1-\cos t)\Big|_{t=\pi/2}.$$

A. $$\rho = \frac{(x'^2+y'^2)^{3/2}}{x'y''-y'x''}$$

$\frac{dx}{dt} = a(1+\cos t)$ $\qquad x'' = -a\sin t$

$\frac{dy}{dt} = a(\sin t)$ $\qquad y'' = a\cos t$

$$\rho = \frac{[(a(1+\cos t))^2 + [a\sin t]^2]^{3/2}}{a^2(1+\cos t)\cos t + a^2\sin^2 t}$$

$$= \frac{[[a(1+\cos t)]^2 + a^2\sin^2 t]^{3/2}}{a^2\cos t + a^2}$$

- solving,

$= a\,2^{3/2}(1+\cos t)^{1/2}$

$= a\,2^{3/2}(2\cos^2 t/2)^{1/2}$

$= a \cdot 2^2 \cdot \cos t/2$

$= 4a\cos t/2$

## Radius of curvature at (0,0) / Newtonian Method

To convert a $f^n$ in infinite series it should be diff and cont.

Formula for taylor series

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \ldots$$

Now,

RoC at (0,0)

$$y = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots$$

$$\rho = \frac{(1+y'^2)^{3/2}}{y''} = \frac{(1+p^2)^{3/2}}{q} \longrightarrow y = px + q\frac{x^2}{2!} \ldots$$

If a ~~cur~~ curve passing through origin and x axis is tangent at (0,0) [or || to x axis]

$x = 0$

$y(0) = 0$

$y'(0) = 0 \longrightarrow p = 0$

$y'(0) = p$

$y''(0) = q$

$$\rho = \frac{1}{q}$$

$$y = p(x) + \frac{qx^2}{2!} \ldots$$

$$y = \frac{qx^2}{2} \ldots$$

divide by $x^2$ and $x \to 0$

$\lim_{x\to 0} \frac{2y}{x^2} = q$

$$\boxed{\rho = \lim_{x\to 0} \frac{x^2}{2y}}$$

Similarly
RoC at the (0,0) when tangent is ||al to y axis

$$\boxed{\rho = \lim_{y\to 0} \frac{y^2}{2x}}$$

Q. Find RoC $|_{(0,0)}$

1) $x^3+y^3-2x^2+6y=0$

2) $2x^4+4x^3y+xy^4+6y^3-3x^2-2xy+y^2-4x=0$

* Eqⁿ of tangent at (0,0) can be found by equating to 0 the lowest degree term

Ans 1) for eqⁿ of tangent

$6y=0$

$\boxed{y=0}$

hence x axis is tangent

Now divide eqⁿ by 2y and take $x \to 0$

$\frac{x^3}{2y} + \frac{y^3}{2y} - \frac{2x^2}{2y} + \frac{\cancel{6y}^{3}}{\cancel{2y}} = 0$

$\lim_{x\to 0} \left[ \frac{x^3}{2y} + \frac{y^2}{2} - \frac{2x^2}{2y} + 3 \right] = 0$

$\lim_{x\to 0} \left( x \cdot \frac{x^2}{2y} \right) + \lim_{x\to 0} \frac{y^2}{2} - 2 \lim_{x\to 0} \frac{x^2}{2y} + 3 = 0$

$-2\rho + 3 = 0$

it is bcz its at (0,0) so when $x\to 0$ $y \to 0$

$0 + 0 - 2\rho + 3 = 0$

$$\boxed{\rho = 3/2}$$

# Radius of curvature of polar curve

$r = f(\theta)$

$r' = \frac{dr}{d\theta} = r_1$

$r'' = \frac{d^2r}{d\theta^2} = r_2$

$\rho = \frac{(r^2+r_1^2)^{3/2}}{r^2+2r_1^2-rr_2}$

Q: Find RoC at any pt $(r,\theta)$ on the curve

$\sqrt{r}\cos\theta/2 = \sqrt{a}$

$r\cos^2\frac{\theta}{2} = a$

$r = a\sec^2\frac{\theta}{2}$

$r_1 = \frac{2a}{2}\sec^2\frac{\theta}{2}\tan\frac{\theta}{2}$

$r_1 = r\tan\theta/2$

$r_2 = r_1\tan\theta/2 + \frac{r}{2}\sec^2\frac{\theta}{2}$

$\rho = \frac{2r^{3/2}}{a^{1/2}}$

Q: $r = a(1+\cos\theta)$

Prove

$\frac{\rho}{r} = \frac{a}{8}$

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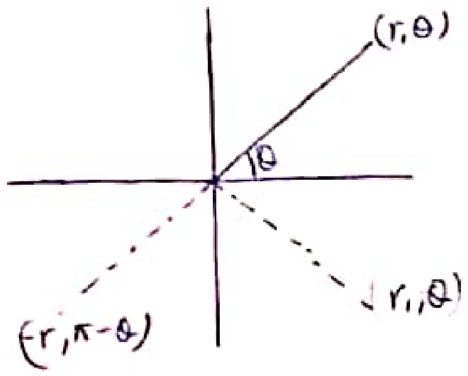
## Graphing polar curves

$r = f(\theta)$

1) **Symmetry abt x axis**

If pt $(r,\theta)$ lie on a graph then point $\{(r,-\theta), (r,\pi-\theta)\}$ will also lie on graph.




2) **Symmetry abt y axis**

If a point $(r,\theta)$ ~~then~~ lie on a graph then pt $\{$ $(r,\pi-\theta)$, $(-r,-\theta)$ also lie on the graph.

3) **Symmetry abt origin**

If a pt $(r,\theta)$ lie on graph then $(-r,\theta)$, $(r,\pi+\theta)$ also lie on graph.

Ex: $r = 1-\cos\theta$

symmetrical abt x axis. You can find that by putting the pts on previous page for each case.

| $\theta$ | $1-\cos\theta = r$ |
|---|---|
| $\pi/4$ | 0 |
| $\pi/3$ | 1/2 |
| $\pi/2$ | 1 |
| $2\pi/3$ | 3/2 |
| $\pi$ | 2 |

1) $r = 1-\cos\theta$

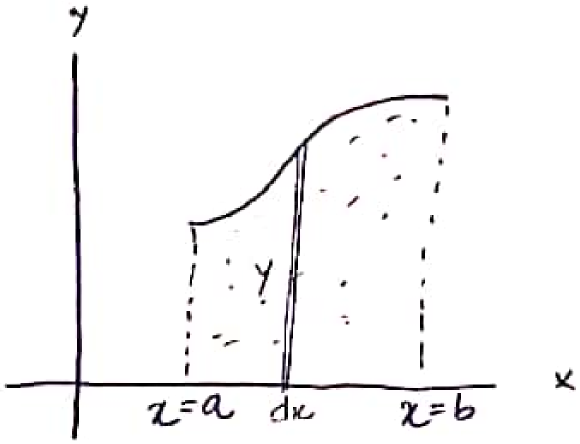
2) $r = 1+\cos\theta$

3) $r^2 = 4\cos\theta$

4) $r^2 = \sin 2\theta$

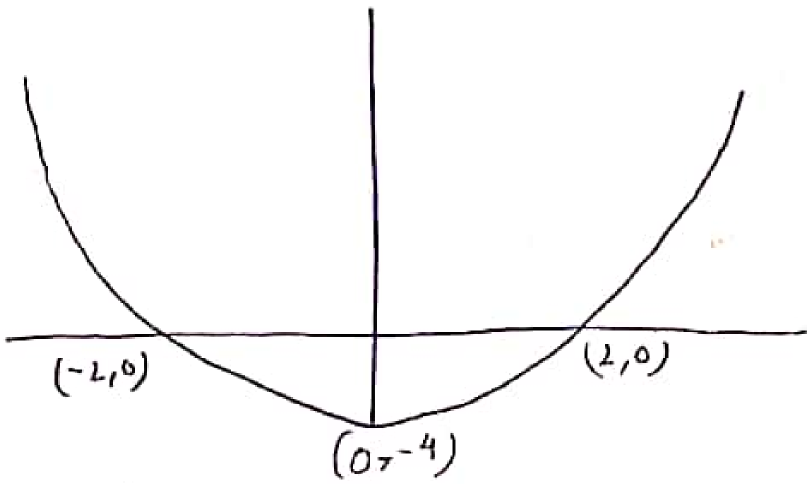
5) $r = \cos 3\theta$

Area of curve: Area under a curve between 2 pts can be found by doing definite integral between two points.
[Only limit/intersection pt of curves are needed]




$$\int_{x=a}^{x=b} y\,dx$$

Q: Find the area of a curve $y = x^2 - 4$ and the x axis.




$$\int_{-2}^{2} x^2 - 4 \, dx$$

$$= \left[\frac{x^3}{3} - 4x\right]_{-2}^{2}$$

$$= \frac{32}{3}$$

Q: Find the common area of parabola $x^2 = ay$ and $y^2 = bx$

$$\left(\frac{x^2}{a}\right)^2 = bx$$

$$\frac{x^{4/3}}{a^2} = b\cancel{x}$$

$$x^3 = a^2 b$$

$$x = \sqrt[3]{a^2 b}$$

$$\int_0^{\sqrt[3]{a^2b}} \sqrt{bx} - \frac{x^2}{a}$$

$$= \cancel{\left[\sqrt{b}\,\frac{x^2}{2} - \frac{x^3}{3a}\right]_0^{\sqrt[3]{a^2b}}}$$

$$= \cancel{\sqrt{b}\,\frac{(a^2b)^{4/3}}{2} - \frac{a^2b}{3\cancel{a}}}$$

$$= \cancel{\frac{b^{7/2}a^{4/3}}{2} - \frac{ab}{3}}$$

$$= \cancel{\frac{3b^{7/2}a^{4/3} - 2ab}{6}}$$

$$= \int_0^{a^{4/3}b^{1/3}} = \frac{ab}{3}$$

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Key facts

$$\Rightarrow \int_0^{\pi/2} \sin^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots \left\langle \begin{array}{l} \frac{1}{2}\,\frac{\pi}{2} \\ 2/3 \end{array}\right.$$

(can be cos too)

$$\Rightarrow \int_0^{\pi/2} \sin^6 x\,dx = \frac{5}{6}\times\frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}$$

$$\int_0^{\pi/2} \sin^5 dx = \frac{4}{3}\times\frac{2}{3} = \frac{8}{15}$$

$$\Rightarrow \int_0^{\pi/2} \cos^q\theta\,\sin^p\theta\,d\theta = \frac{p-1}{p+q}\times\frac{p-3}{p+q-2}\cdots \left\langle \begin{array}{l} \frac{1}{\alpha}\times\int_0^{\pi/2}\cos^2\theta\,dx \\ \frac{2}{\alpha}\times\frac{1}{\alpha-2} \end{array}\right.$$

$$\Rightarrow \int_0^{\pi/2} \sin^p\theta\,\cos\theta\,d\theta \longrightarrow q=1 \longrightarrow \int = \frac{1}{1+p}$$

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## Length of a curve

$$= \int_{x=a}^{b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_{\theta=\alpha}^{\beta} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

find length of curve $y = x^{3/2}$ from $x=0 - x=1$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$\int_0^1 \sqrt{1+\left(\frac{3}{2}x^{1/2}\right)^2}\, dx$$

$$\int_0^1 \sqrt{\frac{4+9x}{4}}$$

$$\cdot \frac{1}{2}\int \sqrt{4+9x}\, dx$$

## Taylor's Series — is always convergent

- Proof not in course

$f(x) \rightarrow$ continous in domain $D$

$f(x) \rightarrow$ diffrentiable $n$ times for $n$ is very large integer

then for $a \in D$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \ldots$$

$$\ldots \frac{(x-a)^n}{n!} f^n(a) + R_n$$

$R_n$ is called Remainder term/error term

$$R_n = \frac{(x-a)^{n+1}}{(n+1)!} f^{n+1}(c) \quad , \quad a < c < x$$

For $a=0$

Taylor series is called Maclarin series.

Ex: If is said to expand and approximate upto 3rd degree polynomial,

$$e^x = \underbrace{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}}_{\text{Ans}} \ldots$$

but since we are neglecting an infinitely many terms there will be some error, that error is resorted by $R_n$.

Q: $f(x) = \sin x$

a) expand by taylor series expansion about $x=0$

b) approximate $f(x)$ by taylors polynomial of degree three about $x=0$

and find c such that error satisfies.

$R_3(x) \le 0.001$

tolerable error

Ans. a) about $x=0$

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!}(f''(0)) \ldots$$

where

$$|R_n| = \frac{x^{n+1}}{(n+1)!} f^{n+1}(c)$$

$$= x\cos 0 + 0 \times x^2 + \frac{x^3}{3!}(-\cos 0) \ldots + R_n$$

$$f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots + R_n$$

b) Approx. $f(x)$ as

$f(x)$ as

$$f(x) \approx x - \frac{x^3}{3!}$$

then error $f^n$

$$|R_{n,3}| = \left|\frac{x^4}{4!}\sin c\right| \le \max_c \left|\frac{x^4}{4!}\sin c\right|$$

$$= \max_c \frac{x^4}{4!} \quad \max_c \sin c = 1$$

$$= \max_c \frac{x^4}{4!}$$

$$= \frac{c^4}{4!} \text{ say}$$

$\frac{c^4}{4!} < 0.001$ (given)

$c^4 < 24 \times 0.001$

$c^4 < 0.024$

$c < 0.3936$

Q: Obtain a $4^{th}$ Degree Taylor poly approx of $f(x) = e^{2x}$ about $x=0$

find max error when

$0 < x < 0.5$

Ans) $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f^3(0) \ldots$

$$f(x) = 1 + 2x + 4 \cdot \frac{x^2}{2!} + 8\frac{x^3}{3!} + \frac{16x^4}{4!} \ldots$$

$f(x) = 1+2x+2x^2+\frac{4}{3}x^3+\frac{2}{3}x^4$

error term

$$|R_4(x)| = \frac{x^5}{5!}f^5(c) = \left|\frac{32}{5!}x^5 e^{2x}\right|$$

$$\leq \frac{32}{5!} \max_{0<x<0.5} x^5 \max_{0<x<0.5} e^{2x c}$$

$$= \frac{32}{5!} 5^5 e^{2/2} = \frac{1}{5!}e \leq \frac{2.732}{120}$$

Q: expand the given $f^n$ in the following form.

$$f^n = x^3 \sin(e^{x^2})$$

form

$$= f(\pi/2) + (x-\pi/2) f'(\pi/2) \ldots$$

$a = \pi/2$

$\downarrow$

by comparing to taylor series

Volume of solid of Revolution

$y = f(x)$

Volume of SOR obtained by revolving the curve about $x$ axis the area bounded by the line $x=a$ $x=b$ and $x$ axis

1) $V = \int_a^b \pi y^2 dx$

2) If revolution about $y$ axis

$$V = \int_{y=c}^{y=d} \pi x^2 dy$$

3) If the area bounded by the curve $y=f(x)$ and the line $y=p$ (line ∥ to $x$ axis) then volume of solid of Revolution.

$$V = \pi \int_{x=a}^{x=b} (y-p)^2 dx$$

similarly

4) $V = \pi \int_c^d (x-x_0)^2 dy$

Q: Find volume formed by Revolution of the loop of the curve $y^2(a+x) = x^2(a-x)$ about x axis.

Ans $\frac{2}{3}\pi a^3(3\ln 2 - 2)$.

Q. Find volume of solid revolving about region bounded by $y = 3-x^2$ and $y = -1$ about line $y = -1$.

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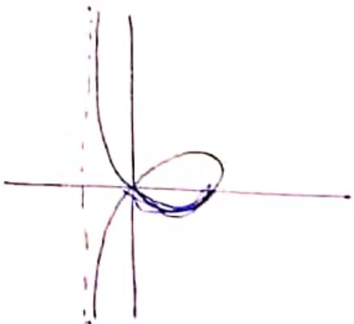
Ans 1) $y^2 = \frac{x^2(a-x)}{(a+x)}$



∘ Volume for upper half, lower half and full will be same.

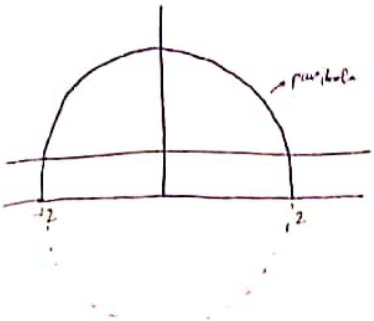
$$V = \int_0^a \pi y^2 dx$$

$$= \pi \int_0^a \frac{x^2(a-x)}{a+x}$$

(Q. Ans 2)

$y \cancel{x^2} = 3 - x^2$

$\sqrt{y-3} = x$

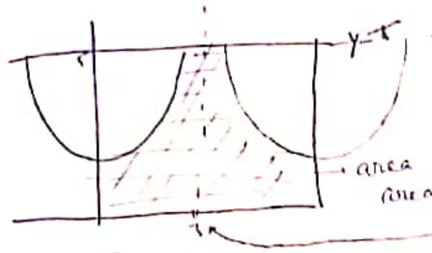



$$= \pi \int_{-2}^{2} (y-1)^2 dx$$

$$= \pi \int_{-2}^{2} (y+1)^2 dx$$

Assgn.

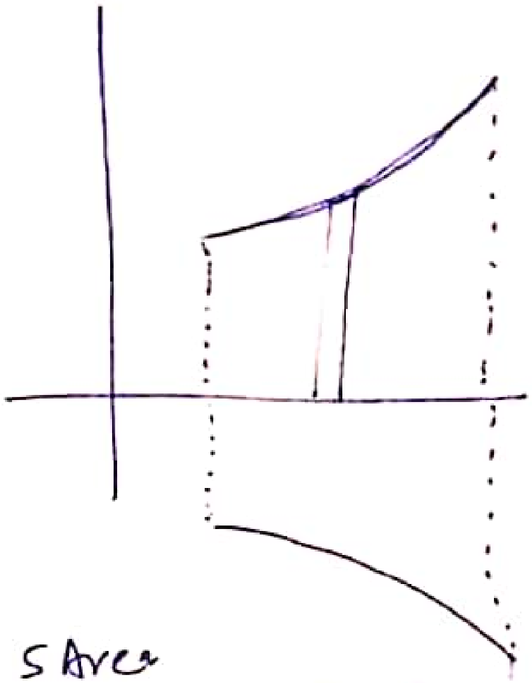
Q: Find volume of solid by revolving a finite region bounded by curve $y = x^2+1$, $y = 5$ about line $x = 3$




area is this bec area line se bound hona change

$$R = \pi \int_{y=1}^{5} (x-3)^2 dy$$

# 5. Area of surface of Revolution



Cylinder

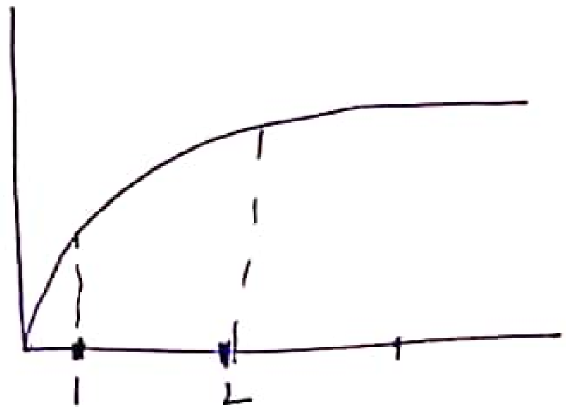
$2\pi r h$

$2\pi y h$ ← length of curve

$= \int_a^b 2\pi y \sqrt{1+y'^2}\, dx$

5 Area

Q. $y = \sqrt{2x}$ about x axis.




$= 2\pi \int_1^2 y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= 2\pi \int_1^2 2\sqrt{x} \sqrt{1+\left(\frac{1}{\sqrt{x}}\right)^2}\, dx$

$= \frac{8\pi}{3}(3\sqrt{3} - 2\sqrt{2})$