Department of Applied Mathematics

Delhi Technological University, Delhi 110042

ASSIGNMENT-II

2019-2020

Course: Mathematics-II **Code: MA-102**

1. Find the general solution of the following homogeneous differential equations

   a. $y'' - 8y' + 16y = 0$,

   b. $y'''' - 4y''' + 8y'' - 8y' + 4y = 0$,

   c. $4y'''' - 4y''' - 23y'' + 12y' + 36y = 0$.

2. Find the general solution of the following non-homogeneous differential equations

   a. $(D^2 + a^2)y = \cot ax$,

   b.$(D^3 - D^2 - 6D)y = x^2 + 1 + 3^x$,

   c. $(D^4 + D^2 + 1)y = e^{-x/2}\cos\left(\frac{\sqrt{3}}{2}\right)$,

   d. $(D^4 + 2D^2 + 1)y = x^2 \cos x$.

3. Consider the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

   a. Show that e^x and xe^x are two linearly independent solutions for all $x \in \mathbb{R}$.

   b. Write the general solution of the given equation.

   c. Find a particular solution which satisfies $y(0) = 1$ $y'(0) = 4$.

4. Consider $(D - m_1)(D - m_2)y = 0$ such that $m_1 \neq m_2$ and m_1, $m_2 \in \mathbb{R}$. Then show that the general solution of the equation reads $(C_1 x + C_2)e^{m_1 x}$.

5. Find the solution of following differential equations using method of variation of parameters

   a. $y'' - 3y + 2y = \frac{e^x}{1 + e^x}$,

   b. $y'' - y = e^{-x}\sin(e^{-x}) + \cos(e^{-x})$,

   c. $y'' + y = \frac{1}{1 + \sin x}$.

6. Find the complete solution of the following Euler-Cauchy equation

   a. $x^3\frac{d^3y}{dx^3} - 4x^2\frac{d^2y}{dx^2} + 8x\frac{dy}{dx} - 8y = 4\ln x$,

   b. $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \ln x \sin(\ln x)$.

7. Solve the following system of simultaneous differential equations

   a. $\dfrac{dx}{dt} + 4x + 3y = t$ and $\dfrac{dy}{dt} + 2x + 5y = e^t$,

   b. $\dfrac{d^2x}{dt^2} + 2x - y = 0$ and $\dfrac{d^2y}{dt^2} - x + 2y = 0$.

8. Solve the initial value problems

$$\frac{d^2y}{dx^2} + y = \sin{(x + a)}, \quad y(0) = y'(0) = 0.$$

9. The positions of a particle executing simple harmonic motion at the end of 1st, 2nd and 3rd second of its motion are x_1, x_2 and x_3 respectively. Show that the time period is $2\pi/\cos^{-1}\left(\dfrac{x_1 + x_3}{2x_2}\right)$.

10. Consider the equation

$$y'' + y = 0 \tag{1}$$

   a. Verify that the boundary value problem for equation (1) with boundary conditions $y(0) = 1$, $y(\pi/2) = 1$ has a unique solution.

   b. Verify that the boundary value problem for equation (1) with boundary conditions $y(0) = 1$, $y(\pi) = 1$ has no solution.

   c. Verify that the boundary value problem for equation (1) with boundary conditions $y(0) = 1$, $y(2\pi) = 1$ has infinite many solutions.

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