Department of Applied Mathematics

Delhi Technological University

Assignment-II

Course: Mathematics-I **Code: MA-101**

1. Find the Maclaurin series for (i) $\tan^{-1} x$ (ii) $\ln(1-x)$ (iii) $e^{\cos x}$.

   Ans.

   (i) $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9} - \dots \qquad (-1 < x < 1)$

   (ii) $-x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots.$

   (iii) $e\left(1 - \frac{1}{2}x^2 + \frac{1}{6}x^4 + \dots\right).$

2. Find the Taylor series at the stated point.

   (i) $\sin x$; $x = \frac{\pi}{4}$ (ii) $\frac{1}{2x+3}$; $x = 1$.

   Ans.

   (i) $\frac{\sqrt{2}}{2}\left[1 + \frac{x - \pi/4}{1!} + \frac{(x-\pi/4)^2}{2!} + \frac{(x-\pi/4)^3}{3!} + \dots\right].$

   (ii) $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{5^{n+1}}(x-1)^n$, where $|x-1| < \frac{5}{2}$.

3. Trace the curves

   (i) $x^3 + y^3 = 3\,axy$ (ii) $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{x}{a}\right)^{2/3} = 1$

4. Trace the curves

   (i) $x = a(t + \sin t),\ y = a(1 - \cos t)$, where $-\pi \le t \le \pi$

   (ii) $r = a(1 + \cos\theta)$

5. Find the curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at its vertices.

   Ans. Curvature at $(a, 0)$ is $\frac{a}{b^2}$, at $(0, b)$ is $\frac{b}{a^2}$.

6. Find the volume of the solid of revolution obtained by rotating about the x-axis the region in the first quadrant bounded by the curve $x^{2/3} + y^{2/3} = \alpha^{2/3}$ and the coordinate axes.

   Ans. $V = \frac{16\,\pi\alpha^3}{105}$.

7. Find the arc length of $y = \frac{1}{3}\sqrt{x}(3 - x)$ for $0 \le x \le 3$.

   Ans. $2\sqrt{3}$

8. Let R be the region bounded by the curves $y = x^2 - 4x + 6$ and $y = x + 2$. Find the volume of the solid generated when R is rotated about the x-axis.
   Ans. $\dfrac{162\pi}{5}$

9. Compute the surface area of the solid generated by the revolution of the astroid $x = a\cos^3 t,\quad y = a\sin^3 t$, about the y-axis.
   Ans. $\dfrac{12\pi a^2}{5}$.

10. For the curve $y = \dfrac{\alpha x}{\alpha + x}$, prove that
$$\left(\frac{2\rho}{\alpha}\right)^{2/3} = \left(\frac{y}{x}\right)^2 + \left(\frac{x}{y}\right)^2,$$
   where ρ is the radius of curvature at (x, y).

11. Find the area common to the cardioids $r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta)$.
   Ans. $\left(\dfrac{3\pi}{2} - 4\right)a^2$

12. At what point(s) does $(2x)^2 + (3y)^2 = 36$ have minimum radius of curvature?
   Ans. $(-3, 0),\ (3, 0)$.

13. Find the arc length of the curve $y = x^{3/2}$ from $(1, 1)$ to $(2, 2\sqrt{2})$.
   Ans. $\dfrac{22\sqrt{22} - 13\sqrt{13}}{27} \approx 2.09$.

14. Find the volume of the solid generated when the region enclosed by $y = \sqrt{x},\quad y = 2$ and $x = 0$ is revolved about the y-axis.
   Ans. $\dfrac{32\pi}{5}$

15. Find the area of the surface that is generated by revolving the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.
   Ans. $\dfrac{\pi}{27}(10^{3/2} - 1) \approx 3.56$.

16. A nose cone for a space reentry vehicle is designed so that a cross section, taken x feet from the tip and perpendicular to the axis of symmetry, is a circle of radius $\dfrac{1}{4}x^2$ feet. Find the volume of the nose cone given that its length is 20 feet.
   Ans. $40,000\pi$ cubic feet.

End