Department of Applied Mathematics
Delhi Technological University
Assignment -IV
MA-101 (Mathematics- I)

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1. Evaluate the following integrals by changing the order of integration:
   a. $\int_0^1 \int_{e^x}^{e} \frac{dy\,dx}{\log y}$ Ans: e-1
   b. $\int_0^a \int_{\sqrt{ax}}^{a} \frac{y^2 dx\,dy}{\sqrt{y^4 - a^2x^2}}$ Ans: $\frac{\pi a^2}{6}$
   c. $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dx\,dy}{\sqrt{y^2+x^2}}$ Ans: $1 - \frac{1}{\sqrt{2}}$
2. Evaluate $\iint \frac{r\,dr\,d\Theta}{\sqrt{a^2+r^2}}$ over one loop of the lemniscate $r^2 = a^2 Cos2\Theta$. Ans: $\frac{4a^2}{3}$
3. Evaluate the given double integral by converting to polar coordinates $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ Ans: $\frac{\pi}{4}$
4. Find, by double integration, the area lying inside the cardioid $r = a(1 + Cos\Theta)$ and outside the circle $r = a$. Ans: $\frac{a^2(\pi+8)}{4}$
5. Find, by double integration, the area enclosed by the curves $y = \frac{3x}{x^2+2}$ and $4y = x^2$.

   Ans: $\frac{3}{2}\log_e 3 - \frac{2}{3}$
6. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dx\,dy\,dz$ Ans: $^1/_{48}$
7. Evaluate the given triple integral by converting into spherical coordinates $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} \frac{dz\,dy\,dx}{\sqrt{x^2+y^2+z^2}}$ Ans: $\frac{\pi(\sqrt{2}-1)}{4}$
8. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. Ans: 16π

9. Using triple integration, find the volume of the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$

Ans: $\frac{4\pi abc}{3}$

10. Find the volume of the portion of the sphere $x^2+y^2+z^2=a^2$ lying inside the cylinder $x^2+y^2=ay.$ Ans: $\frac{2a^3(3\pi-4)}{9}$

11. Prove the following:

a. $\beta\left(m,\frac{1}{2}\right)=2^{2m-1}\,\beta(m,m),$ where β represents the Beta Function.

b. $\Gamma(m).\Gamma\left(m+\frac{1}{2}\right)=\frac{\sqrt{\pi}}{2^{2m-1}}.\Gamma(2m),$ where Γ represents the Gamma Function.

12. Show that $\int_0^1 y^{q-1}(\log\frac{1}{y})^{p-1}\,dy=\frac{\Gamma(p)}{q^p}$, where $p>0$ and $q>0$, and Γ represents the Gamma Function.

13. Show that $\int_a^b (x-a)^{m-1}(b-x)^{n-1}\,dx=(b-a)^{m+n-1}.\ \beta(m,n),$ where β represents the Beta Function.

14. Express the following in terms of Gamma functions:

a. $\int_0^\infty \frac{x^c}{c^x}\,dx$ Ans: $\frac{\Gamma(c+1)}{(\log c)^{c+1}}$

b. $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta$ Ans: $\frac{1}{2}\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)$

15. Prove that $\int_0^1 \frac{x^2\,dx}{\sqrt{1-x^4}}\cdot\int_0^1 \frac{dx}{\sqrt{1+x^4}}=\frac{\pi}{4\sqrt{2}}$