Basic Electrical Engg – EE102
(Lecture Notes-Magnetic Circuits & Transformers)

Topics Covered (Part-3)

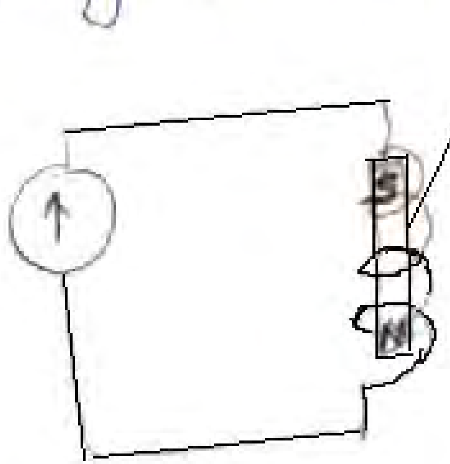
- ✓ Composite Magnetic Circuits
- ✓ Magnetic Leakage and Fringing
- ✓ Kirchhoff's Laws for Magnetic Circuits
- ✓ Solution of Magnetic Circuits
- ✓ Significance of Airgap in Magnetic Circuits
- ✓ Magnetization Characteristics
- ✓ Experimental Determination of B-H Curve
- ✓ Hysteresis Loss, Eddy Current Loss
- Electromagnetic Induction, Fleming's Right Hand Rule
- Self Inductance, Mutual Inductance, Coefficient of Coupling
- Dot Convention, Coupled Coils in Series
- Numerical Examples

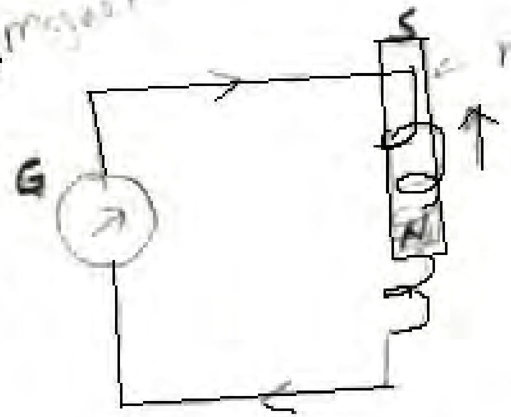
References & Further Reading

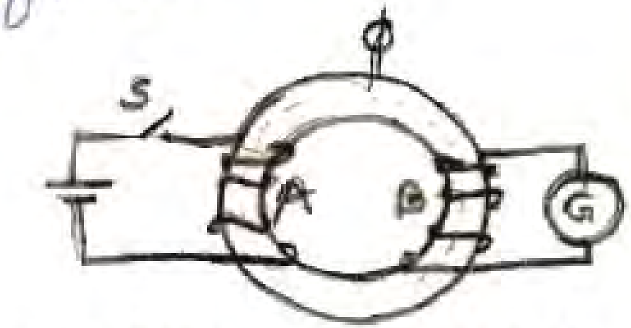
- Vincent Del Toro, Electrical Engineering Fundamentals, Prentice-Hall of India Private Limited.
- Edward Huges, Electrical and Electronic Technology, Pearson Education Limited.
- Rajendra Prasad, Fundamentals of Electrical Engineering, PHI Learning Private Limited.
- Basic Electrical Engineering (Available online : https://nptel.ac.in/ content/ storage2/ courses/108105053/ pdf/L-21(TB)(ET)%20((EE)NPTEL).pdf
- Magnetic Circuits (Available online : https://ocw.mit.edu/ zcourses/electrical-engineering-and-computer-science/6-007-electromagnetic-energy-from-motors-to-lasers-spring-2011/lecture-notes/MIT6_007S11_lec11.pdf

ElectroMagnetic Induction

Faraday (1831) → Electrical machines, Transformer.







S → Close → G → Deflect

S → Open → G → Deflect → Opposite

emf induced – rate of change of flux linkage (λ)

$$e = \frac{d\lambda}{dt} = N\frac{d\phi}{dt}.$$

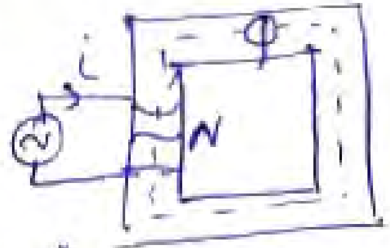
Direction of emf. → Fleming's Right Hand Rule

Hold the thumb, first finger, second finger of right hand in mutually $\perp^r$ dirn with thumb pointing in the dirn of motion and first finger in the dirn of field, then the second finger will points in the dirn of emf.

Thumb → motion.
First finger → field
Second finger → Direction of emf/current

Self and Mutual Inductances

Self inductance (L)



$\phi \rightarrow$ induced by i ~~and~~ in the coil and links with this coil

$$e = -\frac{d\lambda}{dt} = \frac{d\lambda}{di} \cdot \frac{di}{dt}$$

$$= \frac{N d\phi}{di} \cdot \frac{di}{dt} = L\frac{di}{dt}$$

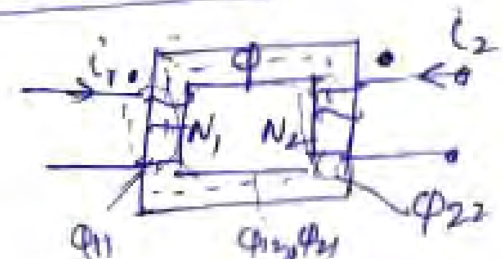
Where $L = \frac{N d\phi}{di} \rightarrow$ self inductance.

flux linkage per Ampere

unit henry $\rightarrow$ $\boxed{L = \frac{N\phi}{I}}$

Mutual Inductance (M)

The flux produced by one coil links with the other coil too



Mutual inductance b/w the coils is defined as

$$M_{12} = \frac{\lambda_{12}}{i_1} = N_2 \frac{\phi_{12}}{i_1} \quad (1)$$

$N_2\phi_{12} \rightarrow$ flux linkage of coil 2 due to current in coil 1.

Illy $M_{21} = \frac{\lambda_{21}}{i_2} = \frac{N_1 \phi_{21}}{i_2} \quad (2)$

Coefficient of coupling

$\phi_1, \phi_2 \rightarrow$ flux established by i_1 & i_2 resply

Then $\phi_1 = \phi_{11} + \phi_{12}$

$\phi_2 = \phi_{22} + \phi_{21}$

Where $\phi_{11}, \phi_{22} \rightarrow$ flux linking with coil 1 & 2 resply.

$\phi_{12}, \phi_{21} \rightarrow$ flux linking with both coils.

$$k = \sqrt{\frac{\phi_{12}}{\phi_1} \cdot \frac{\phi_{21}}{\phi_2}} \quad (3)$$

Induced emf
- → Statically induced emf (Transformer emf)
  - → Self induced → flux → same coil
  - → mutually induced emf.
    ↓ flux of a neighbouring coil changes.
- → Dynamically induced emf (motional emf - electrical machines)

Statically induced emf → ~~B is const~~ coil is fixed → no motion
flux changes with time.

Dynamically induced emf
- → B is constant with t and stationary in space - but coils moves relation to B. (DC generator)
- → B is constant with t ~~on~~ but moves in space and coil is fixed. (AC generator)

Dynamically induced emf (e) = $Blv \sin\theta$

when $\theta = 90^0$ $\qquad \underline{\underline{e = Blv}}$

B - flux density, Wb/m^2
l - conductor length.
v → speed at which conductor cuts the flux.
θ → Angle b/w field & plane of conductor

But from eqn ① & ②.

$$\varphi_{12} = \frac{M_{12} i_1}{N_2} \quad \& \quad \varphi_{21} = \frac{M_{21} i_2}{N_1}$$

sub for φ_{12} & φ_{21} in eqn ③

$$k = \sqrt{\frac{M_{12} M_{21}}{L_1 L_2}}$$

$$\frac{\varphi_1 N_1}{i_1} = L_1$$

$$\frac{\varphi_2 N_2}{i_2} = L_2$$

$$\boxed{k = \frac{M}{\sqrt{L_1 L_2}}} \quad \text{if } M_{12} = M_{21}$$

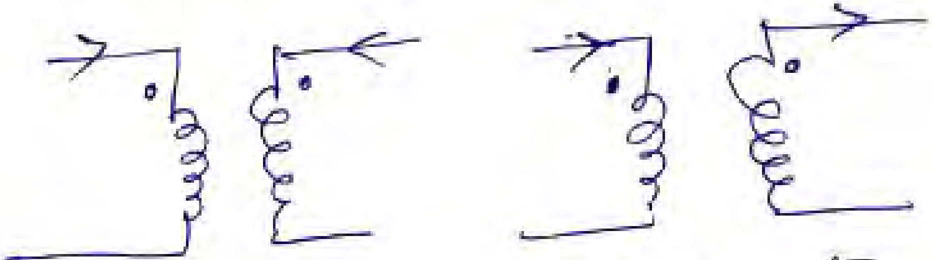
k is close to 1 for closely coupled coils

$k < 1$ for loosely coupled coils

DOT convention

→ is used to indicate the direction of mmf in mutually coupled coils

→ mutually induced emf may aid or oppose the self induced emf depending on whether their mmfs are addition or subtraction.



M - positive

M - negative

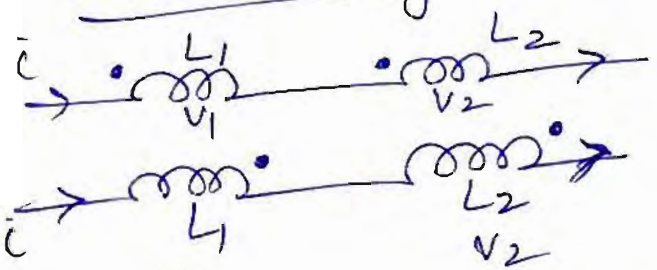
→ when i is enter both coils (or leave both coils) at dot marked terminals mmfs are addition, M is +ve.

→

→ If current enters the dot marked terminal in one coil but leaves the dot marked terminal at the other coil, the mmfs oppose each other, M is −ve.

Coupled coils in series

Series aiding connection



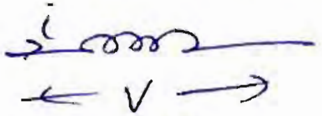
$$v_1 = L_1\frac{di}{dt} + M\frac{di}{dt} = (L_1+M)\frac{di}{dt}$$

$$V_2 = L_2\frac{di}{dt} + M\frac{di}{dt} = (L_2+M)\frac{di}{dt}$$

both case (a) & (b)

$$\therefore V = V_1+V_2 = (L_1+L_2+2M)\frac{di}{dt} \quad \text{—(1)}$$

If L_{eq} →



$$V = L_{eq}\frac{di}{dt} \quad \text{—(2)}$$

from (1) & (2).

$$\boxed{L_{eq} = L_1+L_2+2M}$$ Series aiding.

Series opposing connection.

or

current leaving dot

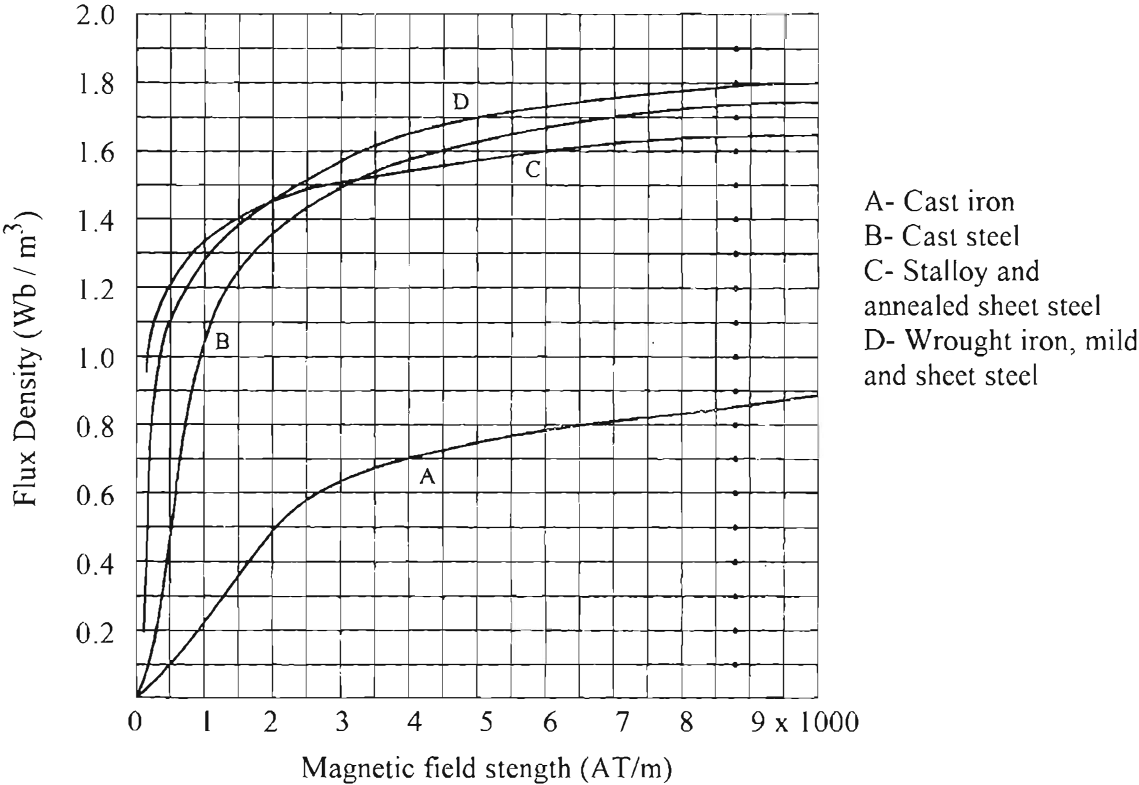
current entering the dot.

$$\therefore V_1 = L_1\frac{di}{dt} - M\frac{di}{dt} = (L_1-M)\frac{di}{dt}$$

$$V_2 = L_2\frac{di}{dt} - M\frac{di}{dt} = (L_2-M)\frac{di}{dt}$$

$$\therefore V_1+V_2 = V = (L_1+L_2-2M)\frac{di}{dt}.$$

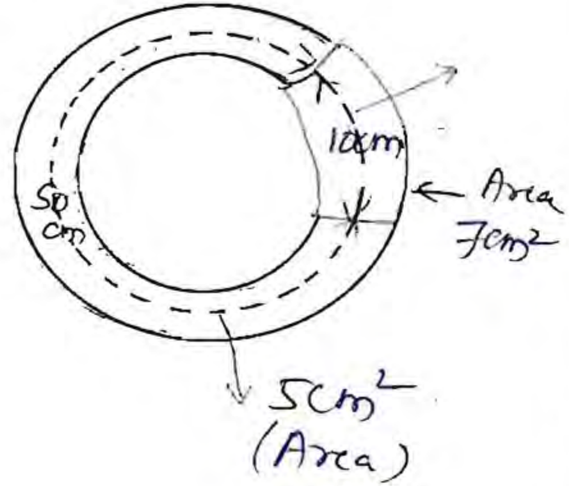
$$\therefore \boxed{L_{eq} = L_1+L_2-2M}$$



Normal magnetization curves for common magnetic materials

Numericals

Ex A ring shaped core shown in fig is made of a ferromagnetic material having a relative permeability of 600. It is required to set up a flux density of 1.6T in the thin section of the core. Find mmf and exciting current of the coil has 300 turns



For the thin section, $B = 1.6\,T$

$$H = \frac{B}{\mu_0 \mu_r} = \frac{1.6}{4\pi \times 10^{-7} \times 600} = \underline{2122.06\ A/m}$$

$$\text{mmf},\ F = H \times l = 2122.06 \times \frac{50}{100}$$
$$= \underline{1061.03\,A}$$

For the thick section

$$\phi = B \times A = 1.6 \times 5 \times 10^{-4} = 8 \times 10^{-4} \quad \text{(same as that of thin section)}$$

$$B = \frac{\phi}{A} = \frac{8 \times 10^{-4}}{7 \times 10^{-4}} = \underline{1.143} \quad ; \quad H = \frac{B}{\mu_0 \mu_r} = \frac{1.143}{4\pi \times 10^{-7} \times 600}$$

$$\therefore H = 1515.76\ A/m \quad ; \quad \text{mmf} = H \times l = 1515.76 \times \frac{10}{100} = \underline{151.58\,A}$$

$$\therefore \text{Total mmf} = 1061.03 + 151.58 = \boxed{1212.61\,AT} \qquad I = \frac{\text{mmf}}{N} = \frac{1212.61}{300} = \boxed{4.042\,A}$$

Num 4 A toroidal core made of steel has a mean diameter of 16cm and a cross-sectional area of $3 cm^2$. Calculate (a) the mmf to produce a flux of 4×10^{-4} Wb and (b) the corresponding values of the reluctance of the core and the relative permeability.

$\varphi = 4 \times 10^{-4}$ Wb ; $A = 3 \times 10^{-4} m^2$; $\therefore B = \frac{\varphi}{A} = \frac{4 \times 10^{-4}}{3 \times 10^{-4}} = 1.33 \, Wb/m^2$

From the magnetization Chts, for $B = 1.33 \, Wb/m^2$, $H = 950$ AT/m.

a) $\therefore$ mmf $= Hl = 950 \times \pi \times 16 \times 10^{-2} = \boxed{478 \, AT}$

b) Reluctance, $S = \frac{mmf}{\varphi} = \frac{478}{4 \times 10^{-4}} = \boxed{119.5 \times 10^4 \, AT/Wb}$.

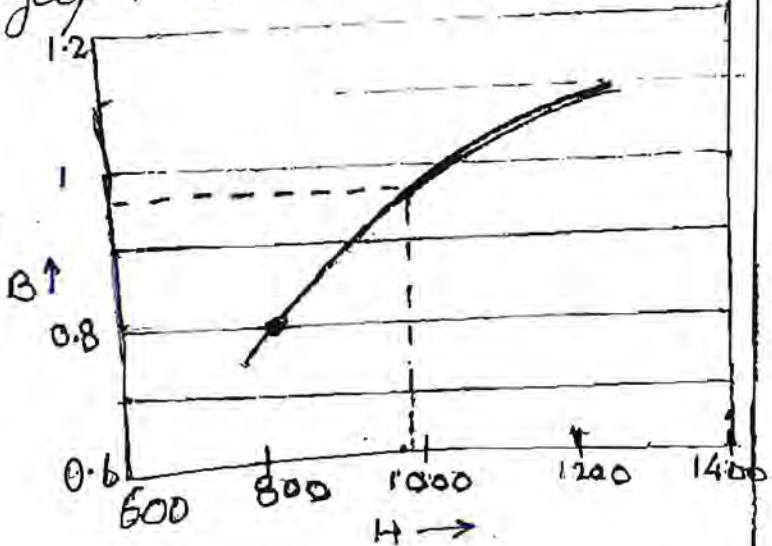
But $S = \frac{l}{\mu A}$; $119.5 \times 10^4 = \frac{\pi \times 16 \times 10^{-2}}{\mu \times 3 \times 10^{-4}}$

$\therefore \mu = 0.1403 \times 10^{-2}$. $\mu = \mu_0 \mu_r$.

$\therefore \mu_r = \frac{0.1403 \times 10^{-2}}{4\pi \times 10^{-7}} = \boxed{1116}$

Num 5 The iron length of an electromagnet with its armature is 40cm and cross sectional area is 5 cm². There is a total air gap of 2mm. Assuming a leakage factor of 1.2, calculate the mmf required to produce a flux of 400μWb in the air gap. B-H curve is as follows:

B (Wb/m²)	0.8	1.0	1.2
H (AT/m)	800	1000	1600



Reluctance for air gap $= \dfrac{2\times10^{-3}}{4\pi\times10^{-3}\times5\times10^{-4}}$

$\left(S = \dfrac{l}{\mu_0 A}\right)$

$= \underline{318.2\times10^{4}}$ AT/Wb

mmf required $= \phi S = 400\times10^{-6}\times318.2\times10^{4} = \boxed{1272.8\text{ AT}}$

Max flux in the iron $\phi_m = \phi \times$ leakage factor $= 400\times10^{-6}\times1.2 = 480\times10^{-6}$ Wb

$\therefore B_{max} = \dfrac{\phi_{max}}{A} = \dfrac{480\times10^{-6}}{5\times10^{-4}} = 0.96$ Wb/m²

From B-H curve corresponding $H = 940$ AT/m

$\therefore$ mmf $= 940\times40\times10^{-2} = \boxed{376\text{AT}}$; Total mmf $= 1272.8+376 = \boxed{1648.8\text{ AT}}$

Num 6 Calculate the inductance of a coil which surrounds a magnetic circuit having reluctance of 6×10^{6} AT/Wb. The number of turns of the coil is 1500. When the rate of change of current in the coil is 200 A/s. Calculate the induced voltage in the coil.

$$L = \frac{N\phi}{I} = \frac{N}{I} \cdot \frac{mmf}{Reluctance} = \frac{N^2}{S} = \frac{1500 \times 1500}{6 \times 10^{6}} = 0.375 H$$

$\therefore$ Induced voltage $v = L\frac{di}{dt} = 0.375 \times 200 = 75V$

Ex 7 A solenoid has 1500 turns of wire wound on a length of 60cm. A search coil of 500 turns enclosing a mean area of 20 sm^2 is placed centrally in the solenoid. Find (a) mutual inductance (b) emf induced in the search coil if current in the solenoid changes at the rate of 250 A/s

(a) Reluctance, $S = \frac{l}{\mu_0 A} = \frac{0.6}{4\pi \times 10^{-7} \times 20 \times 10^{-4}} = 2.38 \times 10^{8}$

$$M = \frac{N_1 N_2 k}{S} = \frac{1500 \times 500 \times 1}{2.38 \times 10^{8}} = 3.14 \times 10^{-3} H$$

(b) $\therefore e = M\frac{di}{dt} = 3.14 \times 10^{-3} \times 250 = 0.785 V$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$\phi_{12} = k\phi_1$

$$\therefore M_{12} = \frac{N_2 \phi_{12}}{i_1} = \frac{N_2 k \phi_1}{i_1} = \frac{N_2 k}{i_1} \cdot \frac{N_1 i_1}{S} = \frac{k N_1 N_2}{S}$$

Num7 The combined inductance of two coils connected in series is 0.9H and 0.2H depending on the relative directions of currents in the two coils. The ~~to~~ self inductance of one coil is 0.3H find (a) M (b) L_2 and (c) k.

$L_a = L_1 + L_2 + 2M$.
$L_b = L_1 + L_2 - 2M$.

or $M = \frac{L_a - L_b}{4}$

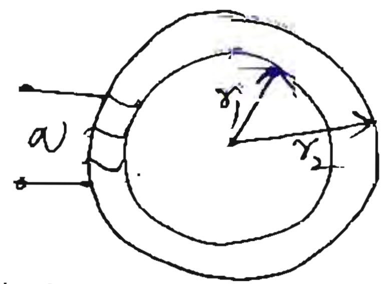
$$M = \frac{0.9 - 0.2}{4} = \boxed{0.175H}$$

$\therefore L_a = L_1 + L_2 + 2M$.

$0.9 = 0.3 + L_2 + 2 \times 0.175$; $\boxed{L_2 = 0.25H}$

$k = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.175}{\sqrt{0.3 \times 0.25}} = 0.639$

Num8 Find the inductance of the toroid shown in fig. Thickness of toroid is b m.



Length of magnetic path $= 2\pi r = 2\pi \times \frac{(r_1 + r_2)}{2}$
$= \pi (r_1 + r_2)$

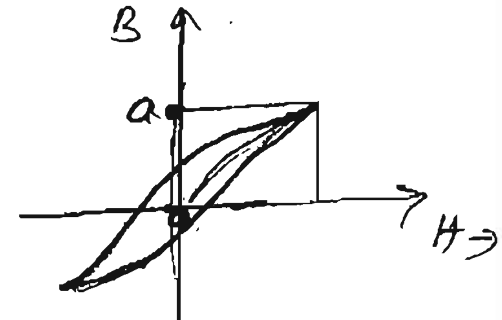
Area of magnetic path $= b(r_2 - r_1)$; $\phi = \frac{mmf}{S} = \frac{Ni \mu_0 \mu_r b(r_2 - r_1)}{\pi (r_1 + r_2)}$

Inductance $L = \frac{N\phi}{i} = \boxed{\frac{N^2 \mu_0 \mu_r b(r_2 - r_1)}{\pi (r_1 + r_2)}}$ Henry

Num 9 A Hysteresis loop is plotted with horizontal axis scale as ~~1cm~~ 1cm = 1000 A/m and vertical axis scale 5cm = 1T. The area of hysteresis loop is 9 cm^2 and overall height is 14cm. Find
(a) Hysteresis loss in Joules/m^3/cycle (b) maximum flux density
(c) Hysteresis loss in watts/kg if density of material is 7800 kg/m^3

a) Area of Hysteresis loop is B H units $= 9 \times 1000 \times \frac{1}{5}$
$= 1800$

Hysteresis loss $= 1800$ J/m^3/cycle



b) on y axis 1cm = 0.2T
14cm $= 14 \times 0.2 = 2.8T$

$\therefore$ Maximum flux density, $B_{max} = \boxed{2.8T}$

c) Hysteresis loss $= 1800 \times 50$ W/m^3
$= 90{,}000$ W/m^3

1 m^3 = 7800 kg.

$\therefore$ Hysteresis loss in W/kg $= \frac{90{,}000}{7800} = 11.54$ W/kg