Delhi Technological University
Department of Applied Mathematics
Mathematics-II(MA-102)
Assignment-IV

1. Find the Laplace transforms of the following functions:
   (i) $e^{-3t}(2cos(5t) - 3sin(5t))$.$[Ans : \frac{2s-9}{s^2+6s+34}]$
   (ii) $\sqrt{t}e^{3t}$.$[Ans : \frac{\sqrt{\pi}}{2}\frac{1}{(s-3)^{\frac{3}{2}}}]$
   (iii) $tsin(at)$.$[Ans : \frac{2as}{(s^2+a^2)^2}]$
   (iv) $\frac{cos(\sqrt{t})}{\sqrt{t}}$.$[Ans : \sqrt{\frac{\pi}{s}}e^{\frac{-1}{4s}}]$
   (v) $|t-1| + |t+1|$, $t \geq 0$.$[Ans : \frac{2}{s}(1 + \frac{e^{-s}}{s})]$
2. Find the Laplace transform of the following periodic functions

$$(i) \quad f(t) = sin(\omega t), \quad 0 < t < \frac{\pi}{\omega}$$
$$= 0, \qquad \frac{\pi}{\omega} < t < \frac{2\pi}{\omega}.[Ans : \frac{1}{1-e^{\frac{-\pi s}{\omega}}}\frac{\omega}{s^2+\omega^2}]$$

$$(ii) \quad f(t) = t, \quad 0 < t < \pi$$
$$= \pi - t, \ \pi < t < 2\pi.[Ans : \frac{1}{1-e^{-2\pi s}}\{\frac{\pi}{s}(e^{-2\pi s} - e^{-\pi s}) + \frac{1}{s^2}(1 + e^{-2\pi s} - 2e^{-\pi s})\}]$$

3. Using multiplicative property, find the Laplace transform of
   (a) t^3e^{-3t}.$[Ans : \frac{6}{(s+3)^4}]$
   (b) $te^{-t}sin(3t)$.$[Ans : \frac{6(s+1)}{(s^2+2s+10)^2}]$
4. Using division property, find the Laplace transform of
   (a) $\frac{1-e^t}{t}$.$[Ans : log(\frac{s-1}{s+1})]$
   (b) $\frac{cos(at)-cos(bt)}{t}$.$[Ans : \frac{1}{2}log\frac{s^2+a^2}{s^2+b^2}]$

5. Evaluate
   (a) $L\{e^{-t}\int_0^t \frac{sin(t)}{t}dt\}.[Ans: \frac{1}{s+1}cot^{-1}(s+1)]$
   (b) $L\{t\int_0^t \frac{e^{-t}sin(t)}{t}dt\}.[Ans: \frac{s+(s^2+2s+2)cot^{-1}(s+1)}{s^2(s^2+2s+2)}]$
   (c) $\int_0^\infty \frac{sin(at)}{t}dt,\ a>0.[Ans: \frac{\pi}{2}]$
   (d) $\int_0^\infty e^{-t}\frac{cos(at)-cos(bt)}{t}dt.[Ans: \frac{1}{2}log\frac{s^2+a^2}{s^2+b^2}]$
6. Find the inverse Laplace transform of the following
   (a) $\frac{s^3-3s+4}{s^3}.[Ans: 1-3t+2t^2]$
   (b) $\frac{s+2}{s^2-4s+13}.[Ans: e^{2t}cos(3t)+\frac{4}{3}e^{2t}sin(3t)]$
   (c) $\frac{4s+5}{(s-1)^2(s+2)}.[Ans: \frac{1}{3}e^t+3te^t-\frac{1}{3}e^{-2t}]$
   (d) $\frac{(s+2)^2}{(s^2+4s+8)^2}.[Ans: e^{-2t}(\frac{sin(2t)}{4}+\frac{tcos(2t)}{2}]$
   (e) $\frac{s}{(s^2+a^2)^2}.[Ans: \frac{1}{2a}tsin(at)]$
   (f) $log\frac{s+1}{s-1}.[Ans: 2\frac{sinh(t)}{t}]$
7. Use Convolution theorem to evaluate
   (a) $L^{-1}\{\frac{s}{(s^2+a^2)^2}\}.[Ans: \frac{1}{2a}tsin(at)]$
   (b) $L^{-1}\{\frac{1}{(s^2+1)(s^2+9)}\}.[Ans: \frac{1}{8}(sin(t)-\frac{sin(3t)}{3})]$
8. Use Laplace transform method to solve the following problems
   (a) $\frac{d^2x}{dt^2}-2\frac{dx}{dt}+x=e^t, given\ \ x(0)=2, \frac{dx}{dt}=-1\ at\ t=0.[Ans: x=2e^t-3te^t+\frac{1}{2}t^2e^t]$
   (b) $\frac{d^3y}{dx^3}+2\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=0, given\ \ y(x)=0, \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=6\ at\ x=0.[Ans: y=e^x-3e^{-x}+2e^{-2x}]$
   (c) $t\frac{d^2y}{dt^2}+2\frac{dy}{dt}+ty=cos(t), given\ \ y(0)=1.[Ans: y=\frac{1}{2}(1+\frac{2}{t})sin(t)]$