$\longrightarrow E$

Fig. 13.12 (*b*)

Example 2. *What is the Fermi energy for the free electron gas in silver? What is the speed of an electron with this enrgy? [N/V = 5.8 × 10^{23} /m^3]*

Solution: With $N/V = 5.8 \times 10^{28}$ /m^3, Equation (7) becomes

$$E_F = (3\pi^2)^{2/3} \frac{(1.05 \times 10^{-34}\,\text{Js})^2}{2 \times 9.1 \times 10^{-31}\,\text{kg}} (5.8 \times 10^{28}/\text{m}^3)^{2/3} \quad ...(1)$$

The electron with this energy have the speed

$$v_F = \sqrt{\frac{2E_F}{m_e}} = 1.4 \times 10^6 \text{ m/s} \quad \textbf{Ans.} \quad ...(2)$$

Thus the typical speeds of the electrons in the Fermi gas are quite large. For comparison, note that if we wanted to give the molecules of a classical gas typical speeds of this order of magnitude, we would have to heat the gas to a temperature of 6×10^4 K and this is a striking illustration of the difference between the classical gas and the Fermi gas.

Example 3. *Consider silver in the metallic state, with one free electron/atom. Calculate the fermi energy. Given density of silver = 10.5 gm/cm^3 and its atomic weight = 108.*

Solution: We have $\dfrac{N}{V} = \dfrac{\text{Atoms}}{\text{Volume}} = \dfrac{(\text{Atoms/mole}) \times (\text{mass/volume})}{\text{mass/mole}} = \dfrac{N_0 \times \rho}{\omega}$

Where N_0 = Avogadro number = 6.02×10^{23} Atoms/mole.

ρ = density of Silver = 10.5 gm/cm^3.

ω = Atomic mass of Silver = 108 gm/mole.

$$\therefore \quad \frac{N}{V} = \frac{6.02 \times 10^{23} \text{ atom/mole} \times 10.5 \text{ gm/cm}^3}{108 \text{ gm/mole}}$$

$$= 5.9 \times 10^{22} \text{ free electrons/cm}^3 = 5.9 \times 10^{28} \text{ free electrons/m}^3$$

Now Fermi energy is given by

$$E_F = \frac{h^2}{8m_e}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$$= \frac{6.6 \times 10^{-34}\ Js}{8 \times 9.1 \times 10^{-31}\ kg} \times \left(\frac{3 \times 5.9 \times 10^{28}/m^3}{\pi}\right)^{2/3}$$

$$= 8.8 \times 10^{-19}\ \text{Joule} = \mathbf{5.4\ eV\ Ans.}$$

Example 4. *If the Fermi energy of a metal is 10 eV, what is the corresponding classical temperature?*

Solution: We have the relation

$$E = \frac{3}{2}kT = \frac{3}{5}E_F$$

$$T = \frac{2E_F}{5k} = \frac{2 \times (10 \times 1.602 \times 10^{-19} C)}{5 \times (1.381 \times 10^{-23})} = \mathbf{4.64 \times 10^4\ K\ Ans.}$$

Example 5. *There are about 2.5×10^{28} free electrons/ m^3 in sodium. Calculate its Fermi energy, Fermi velocity and Fermi temperature ($h = 6.62 \times 10^{-34}$ J-s).*

Solution: $\frac{N}{V}$ = No. of free electrons/unit volume of metal = $2.5 \times 10^{28}/m^3$

We have the relation

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\cdot\frac{N}{V}\right)^{2/3} = \frac{(6.62 \times 10^{-34})^2}{2 \times (9.1 \times 10^{-31})}\left(\frac{3}{8\pi} \times 2.5 \times 10^{28}\right)^{2/3}$$

$$= 5 \times 10^{-19}\ J = \frac{5 \times 10^{-19}}{1.6 \times 10^{-19}} = \mathbf{3.1\ eV}$$

This is the max. K.E. of free electrons at absolute zero. Let v_F be the Fermi velocity, so we have

$$\frac{1}{2}mv_F^2 = E_F$$

$$\therefore \quad v_F = \left(\frac{2E_F}{m}\right)^{1/2} = \left[\frac{2 \times 5 \times 10^{-19}}{9.1 \times 10^{-30}}\right]^{1/2} = 1.047\ ms^{-1}$$

We can define Fermi temperature T_F from the relation

$$E_F = kT_F$$

$$\therefore \quad T_F = \frac{E_F}{k} = \frac{5 \times 10^{-19}}{1.38 \times 10^{-23}} = 3.623 \times 10^4\ K \quad \textbf{Ans.}$$

Example 6. *Calculate the Fermi energy of sodium assuming that metal has one free electron/atom. (Given density of sodium = 970 kg/m³).*

Solution: In this case

$$\frac{N}{V} = \frac{N}{M/\rho} = \frac{6.02 \times 10^{26}}{23/970} = 2.54 \times 10^{28}$$

Fermi energy is given by the relation

$$E_F = \frac{h^2}{2m}\left(\frac{3}{8\pi}\cdot\frac{N}{V}\right)^{2/3} = \frac{(6.625 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}}\left(\frac{3}{8\pi} \times 2.54 \times 10^{28}\right)^{2/3}$$

$$= 5.11 \times 10^{-19}\ J = \frac{5.11 \times 10^{19}}{1.6 \times 10^{-19}}\ eV = \mathbf{3.19\ eV} \quad \textbf{Ans.}$$

Example 7. *Calculate the extent of the energy range between F (E) = 0.9 and F (E) = 0.1 at a temperature of 200 K and express it as a function of E_F which is 3 eV.*

Solution: We have the relation

$$F(E) = \frac{1}{1 + e^{(E-E_F)/k_BT}} = \frac{1}{1 + e^{-(E_F-E)/k_BT}}$$

(a) In this case $F(E) = 0.9$

and let $(E_F - E)/k_\beta T = x$

$$\therefore \quad 0.9 = \frac{1}{1+e^{-x}}$$

or $$0.9 + 0.9\,e^{-x} = 1$$

or $$0.9\,e^{-x} = 1 - 0.9 = 0.1$$

$$\therefore \quad e^{-x} = \frac{1}{9} \quad \text{or} \quad e^{x} = 9$$

Taking log on both sides

$$x = \log_e 9 = 2.3026 \times \log_{10} 9 = 2.198$$

$$\frac{E_F - E}{k_\beta \cdot T} = 2.198$$

or $$E_F - E = 2.198 \times k_\beta \cdot T$$

or $$k_\beta T = \frac{E_F - E}{2.198}$$

Now $$k_\beta T = \frac{1.38 \times 10^{-23} \times 200}{1.6 \times 10^{-19}}\,\text{eV} = 0.017\ \text{eV}.$$

$$\therefore \quad E_F - E = 2.198 \times 0.017 = 0.037\,\text{eV}$$

So $$E = E_F - 0.037 = 3 - 0.037 = \mathbf{2.963\ eV}$$

(b) In this case $F(E) = 0.1$

$$\therefore \quad 0.1 = \frac{1}{1+e^{(E_1 - E_F)/k_\beta \cdot T}}$$

$$= \frac{1}{1+e^{x}} \qquad \left(\text{put } x = \frac{E_1 - E_F}{k_\beta \cdot T}\right)$$

or $$1 = 0.1 + 0.1\,e^{x}$$

$$\therefore \quad e^{x} = 9$$

Taking log both sides $$x = 2.3026 \log_{10} 9$$

or $$\frac{E_1 - E_F}{0.017} = 2.3026 \times 0.954$$

$$\therefore \quad E_1 - E_F = 0.017 \times 2.3026 \times 0.954 = 0.037\ \text{eV}$$

$$\therefore \quad E_1 = E_F + 0.037 = 3 + 0.037 = 3.037\ \text{eV}$$

$$\therefore \quad \Delta E = E_1 - E = 0.037 - 2.963 = 0.074\ \text{eV}$$

or $$\frac{\Delta E}{E_F} = \frac{0.074}{3} = 0.025 = \mathbf{2.5\%} \quad \textbf{Ans.}$$

Example 8. *Calculate the temperature at which there is one per cent probability that a state, with an energy 0.5 eV, above Fermi energy will be occupied by an electron.*

Solution: We have the relation

$$F(E) = \frac{1}{1+e^{(E-E_F)/k_\beta \cdot T}}$$

In the case $$E - E_F = 0.5\ \text{eV}$$

and $$F(E) = 1\% = \frac{1}{100}$$

$$\therefore \quad \frac{1}{100} = \frac{1}{1+e^{x}} \quad \text{where } x = \frac{0.5}{k_\beta \cdot T}$$

$$\therefore \quad 0.01 = \frac{1}{1+e^{x}}$$

$$0.01 + 0.01\, e^x = 1$$

$$0.01\, e^x = 0.99$$

$$\therefore \quad e^x = \frac{0.99}{0.01} = 99$$

Taking log on both sides

$$x = 2.3026 \log_{10} 99$$

$$\frac{0.5}{k_\beta \cdot T} = 2.3026 \log_{10} 99$$

$$\therefore \quad k_\beta . T = \frac{0.5}{2.3026 \log_{10} 99} = 0.109\,\text{eV}$$

$$T = \frac{0.109 \times 1.6 \times 10^{-99}}{1.38 \times 10^{-23}} = 1264 \text{ K}$$

Hence $\quad T = 1264$ **kelvin Ans.**

Example 9. *Find the fermi energy in copper on the assumption that each copper atom contributes one free electron to the electron gas. The density of copper is 8.94×10^3 kg/m^3 and its atomic mass = 63.5 u. Given u = 1.66×10^{-27} kg.*

Solution: In this case $\rho = 8.94 \times 10^3$ kg/m^3, mass of atom = 63.5 u

Number of free electrons/unit volume, $n = \dfrac{N}{V}$

So $$n = \frac{N}{V} = \frac{\text{atoms}}{m^3} = \frac{\text{mass/m}^2}{\text{mass/atoms}}$$

$$n = \frac{8.94 \times 10^3 \text{ kg/m}^3}{(63.5u)(1.66 \times 10^{-27} \text{kg}/u)} = 8.48 \times 10^{28} \text{ atoms/m}^3$$

$$= 8.48 \times 10^{28} \text{ electrons/m}^3$$

Now Fermi Energy is given by

$$E_F = \frac{h^2}{8m_e}\left(\frac{3N}{\pi V}\right)^{2/3} = \frac{h^2}{8m_e} \cdot \left(\frac{3n}{\pi}\right)^{2/3}$$

$$E_F = \frac{(6.63 \times 10^{-34})^2}{8 \times 9.11 \times 10^{-31}}\left(\frac{3 \times 8.48 \times 10^{28}}{3.14}\right)^{2/3}$$

$$= 0.603 \times 10^{-37} \times (8.10 \times 10^{28})^{2/3} = 1.13 \times 10^{-18} J$$

$$= \frac{1.13 \times 10^{-18}}{1.6 \times 10^{-19}} eV = 7.04\, eV \textbf{ Ans.}$$

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Example 11. *A metallic wire has a resistivity of 1.42 × 10–8 Ωm. For an electric field of 0.14 V/m, find (a) average drift velocity of electrons and (b) mean collision time assuming that there are 6 × 10^{28} electrons/m^3.*

Solution: In this case $\rho = 1.42 \times 10^{-8}$ Ωm, $E = 0.14$ V/m, $n = 6 \times 10^{28}$ electrons/m^3.

(*a*) Resistivity of a metal is given by the relation

$$\rho = \frac{m_e}{ne^2\tau}$$

∴ Mean collision time,

$$\tau = \frac{m_e}{ne^2\rho}$$

$$= \frac{9.11\times10^{-31}}{(6\times10^{28})(1.6\times10^{-19})(1.42\times10^{-8})}$$

∴ τ = **4.236 × 10^{-14} *s* Ans.**

Example 12. *(a) Determine the number density of carriers in a copper wire assuming thee is one carrier (electron) per copper atom. (b) The maximum recommended current in a 14 gauge copper wire (radius = 081 mm, A = 2.1 × 10^{-6} m^2) used in household circuits is 15 A. Determine the drift speed of the electrons in such a case. (Give ρ = 8.95 × 103 kg/m^3 and M = 63.5 g/mol).*

Solution: Given I = 15 *A*, *A* = 2.1 × 10^{-6} m^2, ρ = 8.95 × 10^3 kg/m^3 and *M* = 63.5 g/mol.

(*a*) Number of free electrons/unit volume, $n = \frac{N_A\rho}{M}$

$$n = \frac{(6.02\times10^{23}\,\text{mol}^{-1})(8.95\times10^6\,\text{g/m}^3)}{(63.5\,\text{g/mol})}$$

= **8.48 × 10^{28} electrons/m^3**

(*b*) Drift velocity of the electrons may be written as

$$v_d = \frac{I}{nAe}$$

$$= \frac{15}{8.48\times10^{28}\times2.1\times10^{-6}\times1.6\times10^{-19}}$$

= **5.3 × 10^{-4} m/s Ans.**

Example 13. *Find the drift velocity of the free electrons in a copper wire whose cross sectional area is 1.0 mm^2 when the wire carries a current of 1.0 A. Assume that each copper atom contributes one electron to the electron gas. (Given n = 8.5 × 10^{28} electrons/m^3).*

Solution: In this case *A* = 10 mm^2 = 1.0 × 10^{-6} m^2, *I* = 10 *A*

Drift velocity of the free electrons is given by the relation

$$v_d = \frac{I}{nAe}$$

$$= \frac{1.0}{(8.5\times10^{28})(1.0\times10^{-6})(1.6\times10^{-19})}$$

= **7.4 × 10^{-4} m/s Ans.**

Example 14. *A uniform silver has a resistivity of 1.54 × 10^{-8} Ωm at room temperature. For an electric field along the wire of 1 V/cm, calculate (a) the drift velocity (b) the mobility and (c) the relaxation time of electrons assuming that there are 5.8 × 10^{28} electrons/m^3 of the material.*

Solution: We have ρ = 1.54 × 10^{-8} Ωm, *E* = 100 V/m, *n* = 5.8 × 10^{28} electrons/m^3, τ = ?, v_d = ? and μ = ?

(*a*) Mean collision time may be written as $\tau = \frac{m_e}{ne^2\rho}$

$$= \frac{9.11\times10^{-31}}{(5.8\times10^{28})(1.6\times10^{-19})^2(1.54\times10^{-8})}$$

$$= \mathbf{3.98 \times 10^{-14}\ s\ Ans.}$$

(b) Drift velocity, is given by $v_d = \left(\frac{eE}{m_e}\right)\tau$

$$= \frac{(1.6\times10^{-19})(100)(3.98\times10^{-14})}{9.11\times10^{-31}} = \mathbf{0.7\,m/s\ Ans.}$$

(c) Relation for mobility is $\mu = \frac{v_d}{E}$

$$= \frac{0.7}{10^2} = \mathbf{7\times10-3\,m^2/Vs\ Ans.}$$

Example 15. *Find the relaxation time of conduction electrons in a metal of resistivity 1.54 × 10^{-8} Ωm, if the metal has 5.8 × 10^{28} electrons/m³.*

Solution: In this case $\rho = 1.54 \times 10^{-8}\ \Omega m$, $n = 5.8 \times 10^{28}$ electrons/m³ and $\tau = ?$

Resistivity of the metal, $\rho = \frac{m_e}{ne^2\tau}$, so we have

Relaxation time, $\tau = \frac{m_e}{ne^2\rho}$

$$= \frac{9.11\times10^{31}}{(5.8\times10^{28})(1.6\times10^{-19})(1.54\times10^{-8})}$$

$$= \mathbf{3.97 \times 10^{-14}\ s\ Ans.}$$

Example 16. *Calculate the mobility of electrons in copper assuming that each atom contributes one free electron for conduction. Resistivity of copper = 1.7 × 10^{-8} Ωm. Atomic weight = 63.54. Density = 8.96 × 10^3 kg/m³ and Avogadro's number 6.025 × 10^{23}/mole.*

Solution: Given that $\rho = 1.7 \times 10^{-8}\ \Omega m$, $M = 63.54$, $D = 8.96 \times 10^3$ kg/m³, $N_A = 6.025 \times 10^{23}$/ mole, Number of free electrons/atom = 1 and $\mu = ?$

Free electrons/unit volume, $n = \frac{N}{V} = N_A \cdot \frac{\rho}{M}$

$$= \frac{(6.025\times10^{26})(8.96\times10^3)}{63.54}$$

$$= \mathbf{8.50 \times 10^{28}/m^3\ Ans.}$$

Example 17. *Calculate the drift velocity and thermal velocity of free electrons in copper at room temperature, (300 K), when a copper wire of length 3 m and resistance 0022 Ω carries a current of 15 A. Given μ_{cu} = 4.3 × 10^{-3} m²/Vs.*

Solution: In this case l = 3 m, $R = 0.022\ \Omega$, $I = 15\ A$, $T = 300$ K, $\mu_{cu} = 4.3 \times 10^{-3}$ m²/Vs, $v_d = ?$ and $v_{th} = ?$

Voltage drop across the copper wire is given by

$$V = IR = 15 \times 0.022 = \mathbf{0.33\ V}$$

∴ Electric field, $E = \frac{V}{l} = \frac{0.33}{3} = \mathbf{0.11\ V/m}$

Drift velocity, $v_d = E \times u = 0.11 \times 4.3 \times 10^{-3} = \mathbf{0.473 \times 10^{-3}\ m/s\ Ans.}$

Thermal velocity is written as,

$$v_{th} = \sqrt{\frac{3kT}{m_e}}$$

$$= \sqrt{\frac{3 \times 1.387 \times 10^{-23} \times 300}{9.11 \times 10^{-31}}}$$

$$= \mathbf{1.17 \times 105\ m/s\ Ans.}$$

Example 18. *Calculate the Fermi energy in eV for silver at 0 K, given that the density of silver = 10500 kg/m³, atomic weight = 107.9 and it has one conduction per atom.*

Solution: Given $T = 0\ K$, $D = 10500\ kg/m^3$, $M = 107.9$ and $E_F = ?$

Number of free electrons/unit volume, $n = \frac{N_A}{V} = N_A \frac{\rho}{M}$

$$\therefore \quad n = \frac{(6.025 \times 10^{26})(10500)}{107.9} = \mathbf{5.863 \times 10^{28}/m^3\ Ans.}$$

Fermi energy,

$$E_F = \left(\frac{h^2}{8m_e}\right)\left(\frac{3n}{\pi}\right)^{2/3}$$

$$= \frac{(6.63 \times 10^{-34})}{8 \times 9.11 \times 10^{-31}} \left(\frac{3 \times 5.863 \times 10^{28}}{3.14}\right)$$

$$= (0.603 \times 10^{-37})(5.60 \times 10^{28})^{2/3} = 8.83 \times 10^{-19}\ J$$

$$E_F = \frac{8.83 \times 10^{-19}}{1.60 \times 10^{-19}} = \mathbf{5.518\ eV\ Ans.}$$

Fermi energy for silver is 5.518 eV which corresponds to E_F (0).

$$\therefore \quad E_F(0) = \mathbf{5.518\ eV}$$

Example 19. *Energy over which probability f (E) falls from 0.9 to 0.1%. Over what range of energy, expressed in terms of kT, does the Fermi-Dirac distribution function change from 0.90 to 0.10?*

Solution: F–D function is given by, $f(E) = \frac{1}{e^{(E_b - E_F)/kT} + 1}$

Let E_b represent the energy at which $f(E_b) = 0.1$, so we have,

$$\therefore \quad 0.1 = \frac{1}{e^{(E_b - E_F)/kT} + 1}$$

$$e^{(Eb - EF)/kt} + 1 = \frac{1}{0.1} = 10$$

$$\therefore \quad e^{(Eb - EF)/kT} = 9$$

or $$\frac{E_a - E_F}{kT} = \log_e 9 = 2.303 \log_{10} 9 \qquad ...(1)$$

Let E_a be such that $f(E_a) = 0.90$ in that case

$$0.9 = \frac{1}{e^{(E_a - E_F)/kT} + 1}$$

$$e^{(E_b - E_F)/kT} + 1 = \frac{1}{0.9} = 1.11$$

$$\therefore \quad e^{(E_a - E_F)/kT} = 1.11$$

or $$\frac{E_a - E_F}{kT} = \log_e 0.11 = 2.303 \log_{10} 0.11 \qquad ...(2)$$

Subtracting Equation (2) from Equation (1), we have

$$\frac{E_b - E_F}{kT} - \frac{E_a - E_F}{kT} = 2.303\ (\log_{10} 9 - \log_{10} 0.11) = 2.303 \log_{10}\left(\frac{9}{0.11}\right)$$

$$= 2.303 \times \log_{10}(81.818) = 2.303 \times 1.91 = \mathbf{4.4\ Ans.}$$

So the probability that a state occupied from 90% to 10% over an energy range $E_F - E_n = 4.4\,kT$, so this is the range over which F-D function changes in terms of kT.

Example 20. *The specific gravity of tungsten is 18.8 and its atomic weight is 184. Assume that there are two electrons per atom. Calculate the numerical values of n and E_F*

Solution: Specific gravity = 18.8, M = 184

Number of electrons/atom = 2, n = ? and E_F = ?

Number of free electrons/unit volume, $n = \dfrac{N}{V} = N_A \cdot \dfrac{\rho}{M}$

$$\therefore\ n = 6.02 \times 10^{23}\ \frac{\text{molecules}}{\text{mole}} \times \frac{1\,\text{mole}}{184\ \text{g}} \times \frac{\text{g}}{cm^3} \times \frac{2\,\text{electrons} \times 1\ \text{atom}}{\text{atom.molecule}}$$

$$n = 1.23 \times 10^{23}\ \frac{\text{electrons}}{\text{cm}^2} = \mathbf{1.23 \times 10^{29}\ electrons/m^3\ Ans.}$$

For tungsten the atomic and the molecular weights are the same. So we have in this case

Fermi energy, $\quad E_F = 3.64 \times 10^{-19} \cdot n^{2/3}$

$$= (3.64 \times 10^{-19})\,(0.123 \times 10^{30})^{2/3} = \mathbf{9\ eV\ Ans.}$$

Example 21. *Calculate the Fermi velocity and the Fermi temperature for the free electrons in gold. Given E_F = 5.53 eV and $\tau = 3.91 \times 10^{-14}$s.*

Solution: We have E_F = 5.53 eV = $5.53 \times 1.6 \times 10^{-19}\,J$, $\tau = 3.91 \times 10^{-14}s$, v_F = ? and λ = ?

So Fermi velocity is given by

$$v_F = \sqrt{\frac{2E_F}{m_e}}$$

$$= \sqrt{\frac{2 \times 5.53 \times 1.6 \times 10^{-19}}{9.11 \times 10^{-31}}} = \mathbf{1.39 \times 10^6\ m/s\ Ans.}$$

and Fermi temperature, $\quad T = \dfrac{E_F}{k} = \dfrac{5.53 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23}} = \mathbf{6.41 \times 10^4\ \mathit{K}\ Ans.}$

Example 22. *In a solid, consider the energy level lying 0.01 eV below Fermi level. What is the probability of this level not being occupied by an electron? (Given kT = 0.026 eV at T = 300 K).*

Solution: Here $E_F - E$ = 0.01 eV.

The probability of an energy level E not being occupied by an electron is given by $[1-f(E)]$, so we have

$$[1 - f(E)] = 1 - \frac{1}{e^{(E-E_F)/kT} + 1} = \frac{1}{e^{(E_F - E)/kT} + 1}$$

$$= \frac{1}{e^{0.01/0.026} + 1} = \frac{1}{e^{0.385} + 1} = \frac{1}{1.47 + 1} = \frac{1}{2.47}$$

$$= \mathbf{0.405\ Ans.}$$

Example 23. *Calculate the mobility and the relaxation time of electrons in copper with the following data: Resistivity of copper = 1.73×10^{-8} Ωm, atomic weight = 63.5, density = 8.92g/cc, Avogadro number = 6.023×10^{23}.*

Solution: Number of free electrons per unit volume may be written as

$$n = \frac{\text{Avogadro number} \times \text{density}}{\text{Atomic weight}}$$

$$= \frac{6.023 \times 10^{23} \times 8.92 \times 10^3}{63.5} = 8.463 \times 10^{25}$$

Mobility of electrons is given by the relation

$$\mu = \frac{1}{\rho n e}$$

$$= \frac{1}{1.73\times10^{-8}\times 8.463\times10^{25}\times1.6\times10^{-19}}$$

= **4.1145 m2/Vs Ans.**

Relaxation time, $\tau = \frac{m}{ne^2\rho}$

$$= \frac{9.11\times10^{-31}}{8.463\times10^{25}\times(1.6\times10^{-19})^2\times1.73\times10^{-8}}$$

= **2.25 × 10^{-11} s Ans.**

Example 24. *Copper is an fcc crystal with lattice constant 3.61 Å and the metal has one free electron per atom. (a) Calculate the Fermi energy in eV for the metal. (b) Calculate its Fermi factor at 300 K for an energy value 0.1 eV higher than E_F.*

Solution: In this case a = 3.61Å = 3.61 × 10^{-10} m, Number of free electrons per atom = 1, T = 300 K, $E = E_F + 0.1$ eV.

For *fcc* structure each until cell consists of 4 atoms and the structure is that of a cube. So that number of free electrons/unit cell, N = 4 × 1 = 4

Volume of the unit cell, $V = a^3 = (3.61 \times 10^{-10})^3 = 47.046 \times 10^{-30}\ m^3$

Number of free electrons/unit volume, $n = \frac{N}{V}$

$$\therefore \quad n = \frac{4}{47.046\times10^{-30}} = 8.5\times10^{28}$$

(*a*) Now Fermi energy, $E_F = \left(\frac{h^2}{8m_e}\right)\left(\frac{3n}{\pi}\right)^{2/3}$

$$= \frac{(6.63\times10^{-34})^2}{8\times9.11\times10^{-31}}\times\left(\frac{3\times8.5\times10^{28}}{314}\right)^{2/3}$$

$$= (0.603 \times 10^{-37})\ (8.12 \times 10^{28})^{2/3}$$

$$= (0.603 \times 10^{-37})\ (148.7 \times 10^{18})$$

$$= 89.67\times10^{-19}\,J = \frac{89.67\times10^{-19}}{1.6\times10^{-19}}\ \text{eV}$$

= **56.04 eV Ans.**

(*b*) Fermi factor may be written as

$$f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$$

In this case $E = E_F + 0.1$ or $E - E_F = 0.1$ and $k = 1.381 \times 10^{-23}$ J/K

$= 1.381 \times 10^{-23} \times 6.24 \times 10^{18}$ eV/K

$= 8.6174 \times 10^{-5}$ eV/K

$$\therefore \quad f(E) = \frac{1}{e^{0.1/86174\times10^{-5}\times300}+1} = \frac{1}{e^{3.868}+1}$$

$$= \frac{1}{47.85+1} = \frac{1}{48.85} = \mathbf{0.0205\ Ans.}$$

Example 25. *The Fermi level for potassium is 2.1 eV. Calculate the velocity of the electron at the Fermi level.*

Solution: Given $E_F = 2.1$ eV, $v_F = ?$

Fermi velocity is given by the relation

$$v_F = \sqrt{\frac{2E_F}{m_e}}$$

$$= \sqrt{\frac{2 \times 2.1 \times 1.6 \times 10^{-19} J}{9.11 \times 10^{-31} \text{ kg}}} = \mathbf{8.6 \times 10^5 \text{ m/s Ans.}}$$

Example 26. *There are 10^{20} conduction electrons per m^3 in a material having resistivity of 0.1 Ωm. Find the charge mobility and the electric field needed to produce a drift velocity of 1 m/s.*

Solution: Given $n = 10^{20}$ electrons/m^3, $\rho = 0.1\,\Omega m$, $v = 1$ m/s

Mobility of electrons is given by the relation

$$\mu = \frac{1}{\rho n e} = \frac{1}{0.1 \times 10^{20} \times 1.6 \times 10^{-19}} = 0.625 \; m^2/Vs$$

∴ Electric field, $E = \frac{v}{\mu} = \frac{1}{0.625} = \mathbf{1.6 \text{ V/m Ans.}}$

5. Knowing ... the knowledge of the Hall voltage developed can be found.

Example 27. *A copper strip 2.0 cm wide and 1.0 mm thick is placed, in a magnetic field with B = 1.5 webers m^{-2}. If a current of 200 A is set up in the strip, calculate the p.d. that appears across the strip if the number of charge carriers is 8.4 × 10^{28} m^{-3}.*

Solution: (*i*) We have

$$E_H = \frac{jB}{ne}$$

Now
$$E_H = \frac{V}{d} \text{ and } j = \frac{I}{A} = \frac{I}{dt}$$

where *t* is the thickness of the strip.

$$\therefore \quad V = \frac{IB}{net} = \frac{200 \text{ Å} \times 1.5 \text{ weber m}^{-2}}{8.4 \times 10^{28} \text{ m}^{-3} \times 1.6 \times 10^{-19} \text{ c} \times 1.0 \times 10^{-3} \text{ m}}$$

$$= 2.2 \times 10^{-5} \text{ volt} = \mathbf{22\ \mu V.}$$

Example 28. *A copper strip 4.0 cm wide and 0.55 mm. thick carries a current of 100 A. If placed in a magnetic field of induction 2 weber m^{-2} acting at right angles to the strip, a Hall potential difference 29.7 × 10^{-6} V appears across its edge. Find (i) Hall electric field (ii) the number of charge carriers/m^3 in the strip.*

Solution: (*i*) We have

$$E_H = \frac{V_H}{d} = \frac{29.7 \times 10^{-6}}{4 \times 10^{-2}}$$

(*ii*) Charge carriers/unit volume can be had from the relation

$$n = \frac{JB}{e.E_H} = \frac{IB}{A.e.E_H}$$

where *I* is the current and *A* the area of cross-section of the conductor.

$$= \frac{100 \times 2}{2 \times 10^{-5} \times 1.6 \times 10^{-19} \times 7.425 \times 10^{-4}}$$

$$= \frac{10^{30}}{1.6 \times 7.425}$$

$$= \mathbf{8.4 \times 10^{28} \text{ charge carriers m}^{-3}}.$$

Example 29. *An electric field of 100 v/m is applied to a sample of n-type semiconductor whose Hall coeff. is −0.0125 m³/coulomb. Determine the current density in the sample assuming* $\mu_n = 0.35\ m^2\ V^{-1}\ S^{-1}$.

Solution: Hall coeff. is given by

$$R_H = \frac{1}{n \cdot e}$$

$$\therefore \quad n = \frac{1}{e \cdot R_H}$$

In this case

$$e = 1.6 \times 10^{-19}\ \text{C}$$

$$R_H = -0.0125\ \text{m}^3/\text{C}$$

$$\therefore \quad n = \frac{1}{1.6 \times 10^{-19} \times 0.0125} = 5 \times 10^{20}/\text{m}^3$$

Again we have the relations

$$\sigma = n.e.\mu_n$$

and

$$\sigma = \frac{J}{E}$$

$$\therefore \quad J = \sigma \cdot E = n.e.\mu_n.E$$

Putting the various values

$$J = 5 \times 10^{20} \times 1.6 \times 10^{-19} \times 0.36 \times 100$$

$$= \mathbf{2880\ A/m^2}$$