Q. Find the general solution for each of the following differential equation.

(1) $y''' - 5y'' + 7y' - 3y = 0$

In operator form;

$$(D^3 - 5D^2 + 7D - 3)y = 0$$

Let $e^{\lambda x}$ be the soln of the differential equation, then the corresponding characteristic equation will be

$$\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$$

By inspection $\lambda = 1$ is the root of above equation hence it becomes

$$(\lambda-1)(\lambda^2 - 4\lambda + 3) = 0$$

$$(\lambda-1)(\lambda^2 - 3\lambda - \lambda + 3) = 0$$

$$(\lambda-1)(\lambda(\lambda-3) - 1(\lambda-3)) = 0$$

$$(\lambda-1)^2(\lambda-3) = 0$$

Roots are 1, 1, 3.

Hence soln functions are $\underbrace{e^x, xe^x}_{\text{due to the repeated root's 1}}, e^{3x}$

Hence complete soln becomes;

$$y = (c_1 + c_2 x)e^x + c_3 e^{3x}$$

(2) $y^{(5)} - 2y^{(4)} + y''' = 0$

In operator form it will be $(D^5 - 2D^4 + D^3)y = 0$

Let $e^{\lambda x}$ be its soln then the characteristic equation will be

$$\lambda^5 - 2\lambda^4 + \lambda^3 = 0$$

$$\lambda^3(\lambda^2 - 2\lambda + 1) = 0$$

$$\lambda^3(\lambda-1)^2 = 0$$

$$\lambda = 0, 0; 0, 1, 1$$

corresponding solution functions are $1, x, x^2, e^x, xe^x$

Hence the general solution becomes

$$y = c_1 + c_2 x + c_3 x^2 + (c_4 + c_3 x) e^x$$

Q(2) Find the general solution using operator method to solve non homogeneous differential equation.

(i) $y'' - 3y' + 2y = 2x^2 + e^x + 2xe^x + 4e^{3x}$

In operator form $(D^2 - 3D + 2)y = 2x^2 + e^x + 2xe^x + 4e^{3x}$
corresponding homogeneous equation will be
$(D^2 - 3D + 2)y = 0$
Let $e^{\lambda x}$ be the solution of the above DE then the characteristic equation becomes

$$\lambda^2 - 3\lambda + 2 = 0$$
$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$
$$\lambda(\lambda - 2) - 1(\lambda - 2) = 0$$
$$(\lambda - 1)(\lambda - 2) = 0$$
$$\lambda = 1, 2.$$

Soln functions e^x, e^{2x}.
Complimentary function (CF) $= c_1 e^x + c_2 e^{2x}$

Particular integral $(y_p) = \frac{1}{F(D)} f(x)$

$$= \frac{1}{(D^2 - 3D + 2)} \left(2x^2 + e^x + 2xe^x + 4e^{3x} \right)$$

$$= \underbrace{\frac{1}{D^2 - 3D + 2}(2x^2)}_{\text{I}} + \underbrace{\frac{1}{(D^2 - 3D + 2)} e^x}_{\text{II}} + \underbrace{\frac{1}{(D^2 - 3D + 2)} 2xe^x}_{\text{III}} + \underbrace{\frac{1}{(D^2 - 3D + 2)} 4e^{3x}}_{\text{IV}}$$

I : $\dfrac{1}{2\left(1+\left(\frac{D^2}{2}-\frac{3D}{2}\right)\right.}$ $(6x^2)$

$= \left[1+\left(\frac{D^2}{2}-\frac{3D}{2}\right)^{-1}\right]x^2$

$= \left[1+(-1)\left(\frac{D^2}{2}-\frac{3D}{2}\right)+\frac{(-1)(-1-1)}{2}\left[\frac{D^2}{2}-\frac{3D}{2}\right]^2\cdots\right]x^2$

$= \left[1-\frac{D^2}{2}+\frac{3D}{2}+\frac{9D^2}{4}+\cdots\right]x^2$

$= \left[1+\frac{3D}{9}+\frac{7D^2}{4}\right]x^2$

$= x^2+3x+7/2$

II ; $\dfrac{1}{(D-1)(D-2)}e^x$

$= \dfrac{1}{(-1)}\dfrac{1}{(D-1)}e^x$

$I = -\dfrac{1}{1!}xe^x$ using formula $\dfrac{1}{(D-a)^r}e^{ax} = \dfrac{x^r e^{ax}}{r!}$

III ; $2\dfrac{1}{(D-1)(D-2)}xe^x$ replacing $D \to D+1$ as coefficient of x in e^x is 1

$= 2e^x\dfrac{1}{(D+1-1)(D+1-2)}x$

$= 2e^x\dfrac{1}{(D)(D-1)}x$

$= -2e^x\dfrac{1}{D}(1-D)^{-1}x$

$= -2e^x\dfrac{1}{D}\left[1+D-D^2\right]x$

$= -2e^x \frac{1}{D} [1+x]$

$= -2e^x \int (1+x)\, dx$

$= -2e^x \left[x + \frac{x^2}{2}\right]$

$= -2e^x x - x^2 e^x$

IV : $4 \frac{1}{(D-1)(D-2)} e^{3x}$

$= 4 \frac{1}{(3-1)(3-2)} e^{3x}$

$= \cancel{4} \frac{1}{\cancel{2}\cdot 1} e^{3x} \Rightarrow 2e^{3x}$

Hence PI becomes $x^2 + 3x + 7/2 - 3xe^x - x^2e^x + 2e^{3x}$

$= x^2 + 3x + 7/2 - (3x+x^2)e^x + 2e^{3x}$

Complete soln : CF + y_p

$= c_1 e^x + c_2 e^{2x} + x^2 + 3x + 7/2 - (3x+x^2)e^x + 2e^{3x}$.

(ii) $y'''' + y'' = 3x^2 + 4\sin x - 2\cos x$

in operator form

$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

corresponding homogeneous equation

$(D^4 + D^2)y = 0$

Let $e^{\lambda x}$ be the soln of the above DE then its characteristic equation becomes

$\lambda^4 + \lambda^2$

$\lambda^2(1+$

$\lambda = 0, 0$

soln fun

CF = $c_1 +$

PI = $\frac{1}{F(}$

$= \frac{1}{(D^4}$

$= \frac{1}{(D^4+}$

$= \frac{1}{D^2}$

I =

II =

=

=

=

$\lambda^4 + \lambda^2 = 0$

$\lambda^2(1+\lambda^2) = 0$

$\lambda = 0, 0, \pm i$

soln functions : $1, x, e^{ix}, e^{-ix}$

$CF = c_1 + c_2 x + c_3 \cos x + c_4 \sin x$

[as $c_3' e^{ix} + c_4' e^{-ix} = (c_3' + c_4')\cos x + (c_3' - c_4') i \sin x = c_3 \cos x + c_4 \sin x$]

$$PI = \frac{1}{F(D)} f(x)$$

$$= \frac{1}{(D^4+D^2)} [3x^2 + 4\sin x - 2\cos x]$$

$$= \frac{1}{(D^4+D^2)} 3x^2 + \frac{1}{(D^4+D^2)} 4\sin x - 2\frac{1}{(D^4+D^2)} \cos x$$

$$= \underset{\text{I}}{\frac{1}{D^2}(3x^2)} - \underset{\text{II}}{\frac{1}{(D^2+1)}(3x^2)} + \underset{\text{III}}{\frac{1}{(D^4+D^2)} 4\sin x} - \underset{\text{IV}}{2\frac{1}{(D^4+D^2)} \cos x}$$

$$\text{I} = \iint 3x^2 \Rightarrow \int \cancel{3}\frac{x^3}{\cancel{3}} \Rightarrow \frac{x^4}{4}$$

$$\text{II} = \frac{1}{(D^2+1)} 3x^2$$

$$= (1+D^2)^{-1} 3x^2$$

$$= [1 + (-1)D^2 + \dots] 3x^2$$

$$= [1 - D^2 \dots] 3x^2$$

$$= 3x^2 - 6$$

$$\text{III} : 4\frac{1}{D^2(D^2+1)} \sin x$$

$$= 4\,\text{Im}\left[\frac{1}{(D^2)}\frac{1}{(1+D^2)} e^{ix}\right]$$

$= 4\, Im \frac{1}{(D^2)(D+i)(D-i)} e^{ix}$

$= 4\, Im \frac{1}{(i^2)(2i)} \frac{1}{(D-i)} e^{ix}$

$= 4\, Im \left[\frac{1}{2} \frac{x e^{ix}}{1!}\right]$

$= \frac{4}{2} Im \left[i (x) [\cos x + i \sin x] \right]$

$= 2\, Im [i x \cos x - x \sin x]$

$= 2x \cos x$

$IV = 2 \frac{1}{(D^2)(1+D^2)} \cos x$

$= 2\, Re \left[\frac{1}{(D^2)(D+i)(D-i)} e^{ix} \cos x\right]$

$= 2\, Re \left[\frac{1}{i^2} \frac{1}{2i} \frac{1}{(D-i)} e^{ix}\right]$

$= Re \left[\frac{i x e^{ix}}{1!}\right]$

$= Re (i x [\cos x + i \sin x])$

$= Re (-x \sin x + i x \cos x)$

$IV = -x \sin x$

Hence PI becomes : $\frac{x^4}{4} - 3x^2 + 6 + 2x \cos x + x \sin x$.

Complete solution : CF + PI

$= c_1 + c_2 x + c_3 \cos x + c_4 \sin x + \frac{x^4}{4} - 3x^2 + x (2 \cos x + \sin x)$

Q2) Solve using variation of parameters

(i) $y'' + y = \tan x$

corresponding homogeneous equation; $y'' + y = 0$

Let $e^{\lambda x}$ be the soln of above DE then the characteristic equation becomes

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm i$$

Soln function $= e^{ix}$ & e^{-ix}

CF $= c_1 \cos x + c_2 \sin x$

Now using variation of parameters

PI $= u(x) y_1 + v(x) y_2$ here $y_1 = \cos x$ & $y_2 = \sin x$

$$u(x) = \int \frac{W_1(x)}{W(x)} dx \quad \& \quad v(x) = \int \frac{W_2(x)}{W(x)} dx$$

where $W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$ $\quad W_1(x) = \begin{vmatrix} 0 & y_2 \\ g(x) & y_2' \end{vmatrix}$

$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & g(x) \end{vmatrix}$; $g(x) = \dfrac{f(x)}{a_0(x)} = \dfrac{\tan x}{1} = \tan x.$

So,

$$W(x) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$W_1(x) = \begin{vmatrix} 0 & \sin x \\ \tan x & \cos x \end{vmatrix} = -\frac{\sin^2 x}{\cos x}.$$

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$$W_2(x) = \begin{vmatrix} \cos x & 0 \\ -\sin x & \tan x \end{vmatrix}$$

$$= \sin x$$

$$u(x) = \int \frac{W_1(x)}{W(x)}\,dx \Rightarrow -\int \frac{\sin^2 x}{1\cdot\cos x}\,.dx$$

$$= -\int \frac{1-\cos^2 x}{\cos x}\,dx$$

$$= -\int \sec x\,dx + \int \cos x\,dx$$

$$= -\log|\sec x + \tan x| + \sin x$$

$$v(x) = \int \frac{W_2(x)}{W_1(x)}\,dx$$

$$= \int \frac{\sin x}{1}\,dx \Rightarrow -\cos x.$$

$$PI = (-\log|\sec x + \tan x| + \sin x)\cos x + (-\cos x)\sin x$$

$$= -\log(|\sec x + \tan x|)\cdot\cos x$$

$$= \cos x\cdot\log|\sec x - \tan x|$$

Now complete solution : PI + CF

$$= c_1\cos x + c_2\sin x + \log(|\sec x + \tan x|)\cos x$$

(ii) $y'' + 4y' + 5y = e^{-2x} \sec x$

The homogeneous equation of the following is

$(D^2 + 4D + 5)y = 0$

Let $e^{\lambda x}$ be the solution of the DE.

The characteristic equation becomes

$\lambda^2 + 4\lambda + 5 = 0$

Solving for λ;

$$\lambda = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$$

Solution functions : $e^{(-2-i)x}$, $e^{(-2+i)x}$

$CF = e^{-2x}\left[c_1' e^{ix} + c_2' e^{-ix}\right]$

$= e^{-2x}\left[c_1 \cos x + c_2 \sin x\right]$

According to variation of parameters

~~PI & $e^{-2x}\cos x$~~

$PI = u(x) y_1 + v(x) y_2$.

Here $y_1 = e^{-2x}\cos x$ & $y_2 = e^{-2x}\sin x$

$$u(x) = \int \frac{W_1(x)}{W(x)}\,dx \quad \& \quad v(x) = \int \frac{W_2(x)}{W(x)}\,dx$$

$$W_1(x) = \begin{vmatrix} 0 & y_2 \\ g(x) & y_2' \end{vmatrix} \qquad W_2(x) = \begin{vmatrix} y_1 & 0 \\ y_1' & g(x) \end{vmatrix}$$

$$W(x) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

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$$g(x) = \frac{f(x)}{a_0(x)} = \frac{e^{-2x}\sec(x)}{1}$$

$$W = \begin{vmatrix} e^{-2x}\cos x & e^{-2x}\sin x \\ -e^{-2x}\sin x - 2e^{-2x}\cos x & e^{-2x}\cos x - 2e^{-2x}\sin x \end{vmatrix}$$

$$= (e^{-2x})^2 [\cos^2 x - \cos x/\sin x + \sin^2 x + \sin x \cancel{\cos x}]$$

$$= e^{-4x}$$

$$W_1 = \begin{vmatrix} 0 & e^{-2x}\sin x \\ e^{-2x}\sec x & e^{-2x}\cos x - 2e^{-2x}\sin x \end{vmatrix}$$

$$= -e^{-4x}\tan x.$$

$$W_2 = \begin{vmatrix} e^{-2x}\cos x & 0 \\ -e^{-2x}\sin x - 2e^{-2x}\cos x & e^{-2x}\sec x \end{vmatrix}$$

$$W_2 = e^{-4x}$$

$$u(x) = \int \frac{W_1}{W}\,dx = -\int \tan x\,dx$$

$$= \log|\cos x|$$

$$v(x) = \int \frac{W_2}{W}\,dx = \int \frac{\cancel{e^{-4x}}}{\cancel{e^{-4x}}}\,dx = x$$

$$PI = \log|\cos x| \cdot e^{-2x}\cos x + x e^{-2x}\sin x$$

Complete solution : $CF + PI$

$$= e^{-2x}\left[c_1\cos x + c_2\sin x + \cos x \log|\cos x| + x\sin x\right]$$

Q(4) Find the general solution

(i) $x^2y'' - 2xy' + 2y = x^3$

Let $x = e^t$ or $\ln x = t$.

$$\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{dy}{dx}\frac{1}{x} \Rightarrow x\frac{dy}{dx} = \frac{dy}{dt} = \theta t \quad \text{where } \theta \equiv \frac{d}{dt}$$

|||ly

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left[\frac{dy}{dt}\cdot\frac{1}{x}\right]$$

$$= \frac{-1}{x^2}\frac{dy}{dt} + \frac{1}{x^2}\frac{d^2y}{dt^2}.$$

$$\Rightarrow x^2\frac{d^2y}{dx^2} = (\theta)(\theta-1)y.$$

Replacing these in the DE we get.

$$\Rightarrow (\theta)(\theta-1)y - 2\theta y + 2y = e^{3t}.$$

$$\Rightarrow \theta^2 y - \theta y - 2\theta y + 2y = e^{3t}$$

$$\Rightarrow (\theta^2 - 3\theta + 2)y = e^{3t}.$$

Corresponding homogeneous DE will be

$$(\theta^2 - 3\theta + 2)y = 0$$

SSIGN each &

Let $e^{\lambda x}$ be the soln of this DE then, characteristic equation will be

$\lambda^2-3\lambda+2=0$

$(\lambda-1)(\lambda-2)=0$

$\lambda=1,2.$ soln functions are e^t, e^{2t}

$\Rightarrow e^{\ln x}, e^{2\ln x}$

$= x, x^2.$

Complimentary function (CF) $= c_1x+c_2x^2.$

$$PI = \frac{1}{(\theta-1)(\theta-2)} e^{3t}$$

$$= \frac{1}{(3-1)(3-2)} e^{3t} \Rightarrow \frac{e^{3t}}{2}$$

$$= \frac{e^{3\ln x}}{2}$$

$$= \frac{x^3}{2}.$$

Hence complete solution becomes $y = PI + CF$

$$y = c_1x + c_2x^2 + \frac{x^3}{2}$$

(ii) $x^2y'' + 4xy' + 2y = 4\ln x$

Let $x=e^t$

$$x\frac{dy}{dx} = \theta y \qquad \theta \equiv \frac{d}{dt}$$

$$x^2\frac{d^2y}{dx^2} = (\theta)(\theta-1)y$$

replacing these in original DE we get.

$$(\theta^2+3\theta+2)y = 4t.$$

corresponding homogeneous DE will be

$$(\theta^2+3\theta+2)y=0$$

Let $e^{\lambda x}$ be the solution of the above DE then, characteristic equation becomes

$$(\lambda^2+3\lambda+2)y=0$$

$$(\lambda+1)(\lambda+2)=0$$

$$\lambda=-1,-2.$$

soln functions : e^{-t}, e^{-2t} or $\frac{1}{x}, \frac{1}{x^2}$.

$$CF = \frac{c_1}{x} + \frac{c_2}{x^2}.$$

$$PI = \frac{1}{(\theta+1)(\theta+2)} 4t.$$

$$= \frac{1}{(2)\left[1+\frac{3\theta}{2}+\frac{\theta^2}{2}\right]} 4t.$$

$$= \frac{1}{2}\left[1-\left[\frac{3\theta}{2}+\frac{\theta^2}{2}\right]..\right] 4t$$

$$= \frac{1}{2}\left[4t-6-0\right]$$

$$= 2t-3.$$

$$= 2\ln x-3$$

Complete soln : $CF+PI$

$$= \frac{c_1}{x}+\frac{c_2}{x^2}+2\ln x-3$$

Q5) Solve the following system, where x & y are dependent variables and t is independent variable.

(i) $2x' - 2y' - 3x = t$

$2x' + 2y' + 3x + 8y = 2$

in operator form

$D \equiv \frac{d}{dt}$.

$2Dx - 3x - 2Dy = t$

$2Dx + 2Dy + 3x + 8y = 2$

$\Rightarrow (2D-3)x - 2Dy = t$ ... ①

$(2D+3)y + (2D+8)y = 2$ ... ②.

Operating $2D+3$ on ① and $2D-3$ on ②

$(2D+3)(2D-3)x - (2D+3)(2D)y = (2D+3)t$ ... ③

$(2D-3)(2D+3)y + (2D-3)(2D+8)y = (2D+8)2$ ... ④

④ − ③ we get.

$[8D^2 + 16D - 24]y = -8 - 3t$.

$(D^2 + 2D - 3)y = -1 - \frac{3}{8}t$.

Homogeneous DE will be

$(D^2 + 2D - 3)y = 0$

Let $e^{\lambda t}$ be the soln then, characteristic equation becomes

$\lambda^2 + 2\lambda - 3 = 0$

$(\lambda - 1)(\lambda + 3) = 0 \Rightarrow \lambda = 1, -3$.

Soln functions: e^t, e^{-3t}.

$CF = c_1 e^t + c_2 e^{-3t}$

$PI = \frac{1}{(D-1)(D+3)} \left(-e^{0t} - \frac{3}{8}t\right)$

$$= \frac{1}{3} + \frac{3}{8\cdot 3}\left[1 - \left[\frac{2D}{3} + \frac{D^2}{3}\right]\right]^{-1} t$$

$$= \frac{1}{3} + \frac{3}{24}\left[1 + \frac{2D}{3} + \frac{D^2}{3} \cdots\right] t$$

$$= \frac{1}{3} + \frac{\cancel{3}}{\cancel{24}_8}\left[t + \frac{2}{3}\right]$$

$$= \frac{8+2}{24} + \frac{t}{8}$$

$$= \frac{t}{8} + \frac{5}{12}.$$

complete soln for $y = c_1 e^{t} + c_2 e^{-3t} + \frac{t}{8} + \frac{5}{12}.$

For x do ② − ①

$$4y' + 6x + 8y = 2 - t.$$

$$\Rightarrow 4\left(c_1 e^{t} - 3c_2 e^{-3t} + 1/_8\right] + 6x + 8\left[c_1 e^{t} + c_2 e^{-3t} + \frac{t}{8} + \frac{5}{12}\right] = \cdots$$

$$\Rightarrow 12 c_1 e^{t} - 4c_2 e^{-3t} + \frac{23}{6} + t + 6x = 2 - t.$$

$$\Rightarrow x = \frac{4}{6} c_2 e^{-3t} - 2c_1 e^{t} - \frac{t}{3} - \frac{11}{36}$$

$$\Rightarrow x = -2c_1 e^{t} - \frac{2}{3} c_2 e^{-3t} - \frac{t}{3} - \frac{11}{36}$$

$$\Rightarrow x = c_1' e^{t} + c_2' e^{-3t} - \frac{t}{3} - \frac{11}{36}$$

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(ii) $x' + y' - x - 6y = e^{3t}$

$x' + 2y' - 2x - 6y = t$

In operator form

$D \equiv \frac{d}{dt}$

$(D-1)x + (D-6)y = e^{3t}$ ... (1)

$(D-2)x + (2D-6)y = t$ ... (2)

operating $(D-2)$ on 1 and $(D-1)$ on 2 we get.

$(D-2)(D-1)x + (D-2)(D-6)y = (D-2)e^{3t}$ ... (3)

$(D-2)(D-1)x + (D-1)(2D-6)y = (D-1)t$ ... (4)

(3) − (4)

$(-D^2+6)y = e^{3t} + t - 1$

$(D^2-6)y = 1 - t - e^{3t}$.

Homogeneous DE for the above will be

$(D^2-6)y = 0$

Let $e^{\lambda x}$ be the solution of this DE then its characteristic equation

$\lambda^2 - 6 = 0$

$\lambda = \pm\sqrt{6}$

Soln function $= e^{\sqrt{6}x}, e^{-\sqrt{6}x}$

$CF = c_1 e^{\sqrt{6}x} + c_2 e^{-\sqrt{6}x}$

$PI = \frac{1}{(D^2-6)} e^{0t} - \frac{1}{(D^2-6)} t - \frac{1}{(D^2-6)} e^{3t}$

$= \frac{-1}{6} + \frac{1}{6\left(1-\frac{D^2}{6}\right)} t - \frac{e^{3t}}{3}$

$= \frac{-1}{6} + \frac{1}{6}\left(1-\frac{D^2}{6}\right)^{-1} t - \frac{e^{3t}}{3}$

$= \frac{-1}{6} + \frac{1}{6}\left[1 + \frac{D^2}{6} + \cdots\right]t - \frac{e^{3t}}{3}$

$= \frac{-1}{6} + \frac{t}{6} - \frac{e^{3t}}{3}$

$= \frac{t}{6} - \frac{e^{3t}}{3} - \frac{1}{6}$

complete solution for y becomes PI + CF

$$y = c_1 e^{\sqrt{6}x} + c_2 e^{-\sqrt{6}x} + \frac{t}{6} - \frac{e^{3t}}{3} - \frac{1}{6}$$

Now for x subtracting 2 from 1

$x - y' = e^{3t} - t.$

$x = y' + e^{3t} - t$

$x = \sqrt{6}c_1 e^{\sqrt{6}x} - \sqrt{6}c_2 e^{-\sqrt{6}x} + \frac{1}{6} - \cancel{e^{3t}} + \cancel{e^{3t}} - t$

$x = c_1' e^{\sqrt{6}x} + c_2' e^{-\sqrt{6}x} + 1/6 - t$

Q6) Check whether the following set of functions are linearly independent or not :-

in interval (0,1)

(i) $\sin 2x$ and $\cos 2x$

(i) $\sin 2x$ and $\cos 2x$

To check for linear independence the wronskian with $y_1 = \sin 2x$

$y_2 = \cos 2x$

$$W = \begin{vmatrix} \sin 2x & \cos 2x \\ 2\cos 2x & -2\sin 2x \end{vmatrix} = -2[\sin^2 2x + \cos^2 2x] = -2.$$

Since $W \neq 0$ for each $x \in (0,1)$ & independent of x^3 hence the set of functions are linearly independent.

(ii) e^{3x} , e^{-3x}

$y_1 = e^{3x}$ & $y_2 = e^{-3x}$

$$W = \begin{vmatrix} e^{3x} & e^{-2x} \\ 3e^{3x} & -2e^{-2x} \end{vmatrix}$$

$W = -2e^{x} - 3e^{x}$

$= -5e^{x}$

Since $W \neq 0$ for any interval I this set is linearly independent

(iii) x , x^2 , x^3

$y_1 = x$, $y_2 = x^2$, $y_3 = x^3$

$$W = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$$

$W = (x)(12x^2 - 6x^2) - x^2(6x - 0) + x^3(2 - 0)$

$= (x)(6x^2) - 6x^3 + 2x^3$

$W = 2x^3$

Since $W = 0$ for $x = 0$ hence this set of functions is linearly independent for the interval

$(-\infty, 0) \cup (0, \infty)$