Department of Applied Mathematics
Delhi Technological University, Delhi

ASSIGNMENT 1

2019-2020

.

Subject Code : **MA-102** Course Title : **Mathematics-II**

Instructions

Write your name and roll number on each page of your assignment. Assignment should be legibly handwritten and on both sides of the paper.

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1. Define rank of a matrix. Hence, find the rank of the following matrices

(a). $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 5 \\ -1 & 2 & 6 & -7 \end{bmatrix}$ (b). $A = \begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$.

2. Reduce the following matrices into (a) row-echelon form and (b) normal form. Hence, find the rank.

(a). $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ (b). $A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}$.

3. Find non-singular matrices P and Q such that PAQ is the normal form where

$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 4 & 2 & -1 & 2 \\ 2 & 2 & -2 & 0 \end{bmatrix}$.

4. Find the inverse of $\begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$ using elementary row transformations.

5. Check the consistency of the following system of linear nonhomogeneous equations and find the solution, if exists:

(a). $2a-2b+4c+3d = 9$; $a-b+2c+2d = 6$; $2a-2b+c+2d = 3$; $a-b+d = 2$.

(b). $4a - b = 12$; $-a + 5b - 2c = 0$; $-2b + 4c = -8$.

6. For what values of λ and μ the equations $a+b+c=6$, $a+2b+3c=10$ and $a+2b+\lambda c=\mu$ has (i) no solution (ii) unique solution (iii) an infinite number of solutions.

7. Show that the vectors $\vec{a}=[2,3,1,-1]$, $\vec{b}=[2,3,1,-2]$ and $\vec{c}=[4,6,2,-3]$ are linearly independent.

8. Examine the vectors $\vec{a}=[1,0,2,1]$, $\vec{b}=[3,1,2,1]$, $\vec{c}=[4,6,2,-4]$ and $\vec{d}=[-6,0,-3,-4]$ for linear independence and dependence. Also, find the relation between them if it exists.

9. Find the characteristic equation of the matrix $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$. Also find the eigen values and eigen vectors of this matrix.

10. Which of the following matrix is diagonalizable
(a). $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (b). $\begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$.

11. Find the characteristic equation of the matrix $\begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -2 \\ 1 & 2 & 1 \end{bmatrix}$. Verify Cayley-Hamilton theorem and hence find A^{-1}.

12. Diagonalise the matrix $A=\begin{bmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{bmatrix}$ and hence find A^5.

13. Let $A=\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP$ is diagonal.

14. If $A=\begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$. Find the eigen values of A^{-2} and A^3.

15. State Cayley-Hamilton theorem. Use it to express $2A^5-3A^4+A^2-4I$ as a linear polynimial in A where $A=\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.