Department of Applied Mathematics

Mathematics-I (MA101)

Assignment-I (Unit-I) **Academic Year: 2019-20**

1. Determine if the following series converges or diverges:

$$1+\frac{1}{2\sqrt[3]{2}}+\frac{1}{3\sqrt[3]{3}}+\frac{1}{4\sqrt[3]{4}}+\cdots$$

Answer: Convergent.

2. State and prove Leibnitz test and hence determine whether the following series converges or diverges:

$$\sum_{n=2}^{\infty}\frac{cos(n\pi)}{\sqrt{n}}$$

Answer: Convergent.

3. Verify whether the infinite series converges of diverges:

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+\cdots$$

Answer: Convergent.

4. Examine the convergence of the following series:

$$\sum_{n=1}^{\infty}\frac{\sqrt{n+1}-\sqrt{n}}{n^p}$$

Answer: Convergent if $p > {}^1/_2$ and divergent if $p \leq {}^1/_2$.

5. Using limit comparison test, examine the convergence of the series:

$$\frac{\sqrt{2}-1}{3^3-1}+\frac{\sqrt{3}-1}{4^3-1}+\frac{\sqrt{4}-1}{5^3-1}+\cdots$$

Answer: Convergent.

6. Discuss the convergence of the series:

(a) $1+1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots\ (p>0)$ Answer: Convergent.

(b) $\sum_{n=1}^{\infty}\frac{n!}{5^n}$ Answer: Divergent.

(c) $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ Answer: Convergent.

7. Show that the series

$$\sum\frac{3.6.9\ldots 3n}{7.10.13\ldots(3n+4)}x^n\ ,x>0$$

Converges for $x \leq 1$, and diverges for $x > 1$.

8. Find the nature of the series:

$$\sum\frac{(n!)^n}{(2n!)}x^n,\qquad (x>0)$$

Answer: Divergent.

9. State and prove Integral test and hence determine all the values of p for which the series $\sum e^{-np}$ converges.

Answer: The series converges for all $p > 0$.

10. Test for the convergence of the series

$$\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \cdots \quad (0 < x < 1).$$

Answer: Convergent.

11. Show that the series given below converges.

$$\frac{1.2}{3^2.4^2} - \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} - \cdots$$

12. Test for absolute convergence the series

$$\sum (-1)^{n-1} \frac{(n)^2}{(n+1)!}$$

Answer: Absolutely Convergent.

13. Test the conditional convergence and absolute convergence of:

$$\sum \frac{(-1)^{n+1}}{\sqrt{n}}$$

Answer: Not Absolutely Convergent but conditionally convergent.