4.3.2 The Einstein Relation

If we consider the nonuniformly doped semiconductor represented by the energy-band diagram shown in Fig. 4.14 and assume there are no electrical connections so that the semiconductor is in thermal equilibrium, then the individual electron and hole currents must be zero. We can write

$$J_n = 0 = en\mu_n E_x + eD_n \frac{dn}{dx} \tag{4.31}$$

If we assume quasi-neutrality so that $n \approx N_d(x)$, then we can rewrite Eq. (4.31) as

$$J_n = 0 = e\mu_n N_d(x) E_x + eD_n \frac{dN_d(x)}{dx} \tag{4.32}$$

Substituting the expression for the electric field from Eq. (4.30) into Eq. (4.32), we obtain

$$0 = -e\mu_n N_d(x)\left(\frac{kT}{e}\right)\frac{1}{N_d(x)}\frac{dN_d(x)}{dx} + eD_n \frac{dN_d(x)}{dx} \tag{4.33}$$

Equation (4.33) is valid for the condition

$$\frac{D_n}{\mu_n} = \frac{kT}{e} \tag{4.34a}$$

The hole current must also be zero in the semiconductor. From this condition, we can show that

$$\frac{D_p}{\mu_p} = \frac{kT}{e} \tag{4.34b}$$

Combining Eqs (4.34a) and (4.34b) gives

$$\boxed{\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{e}} \tag{4.35}$$

The diffusion coefficient and mobility are not independent parameters. This relation between the mobility and diffusion coefficient, given by Eq. (4.35), is known as the *Einstein relation.* The diffusion coefficient is approximately 40 times smaller than the mobility at room temperature.

As, $$\frac{D_n}{\mu_n} = \left(\frac{kT}{e}\right) = 0.0259 \approx \frac{1}{40}$$

Table 4.1 *Typical mobility and diffusion coefficient values at $T = 300$ K ($\mu = cm^2/V\text{-}s$ and $D = cm^2/s$)*

	μ_m	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

Table 4.1 shows mobilities of electrons and holes in silicon, gallium arsenide and germanium at
All these values strongly depend upon temperature and doping concentration.

THE HALL EFFECT

The Hall effect is a consequence of the forces that are exerted on moving charges by electric and magnetic fields. The Hall effect is used to distinguish whether a semiconductor is n type or p type[1] and to me the majority carrier concentration and majority carrier mobility. The Hall effect device, as discussed section, is used to experimentally measure semiconductor parameters. However, it is also used extensiv engineering applications as a magnetic probe and in other circuit applications.

The force on a particle having a charge q and moving in a magnetic field is given by

$$F = q\upsilon \times B$$

where the cross product is taken between velocity and magnetic field so that the force vector is perpend to both the velocity and magnetic field.

Figure 4.15 illustrates the Hall effect. A semiconductor with a current I_x is placed in a magnetic perpendicular to the current. In this case, the magnetic field is in the z direction. Electrons and holes flo in the semiconductor will experience a force as indicated in the figure. The force on both electrons and is in the $(-y)$ direction. In a p-type semiconductor ($p_0 > n_0$), there will be a buildup of positive charg the $y = 0$ surface of the semiconductor and, in an n-type semiconductor ($n_0 > p_0$), there will be a build negative charge on the $y = 0$ surface. This net charge induces an electric field in the y-direction as show the figure. In steady state, the magnetic field force will be exactly balanced by the induced electric field. This balance may be written as

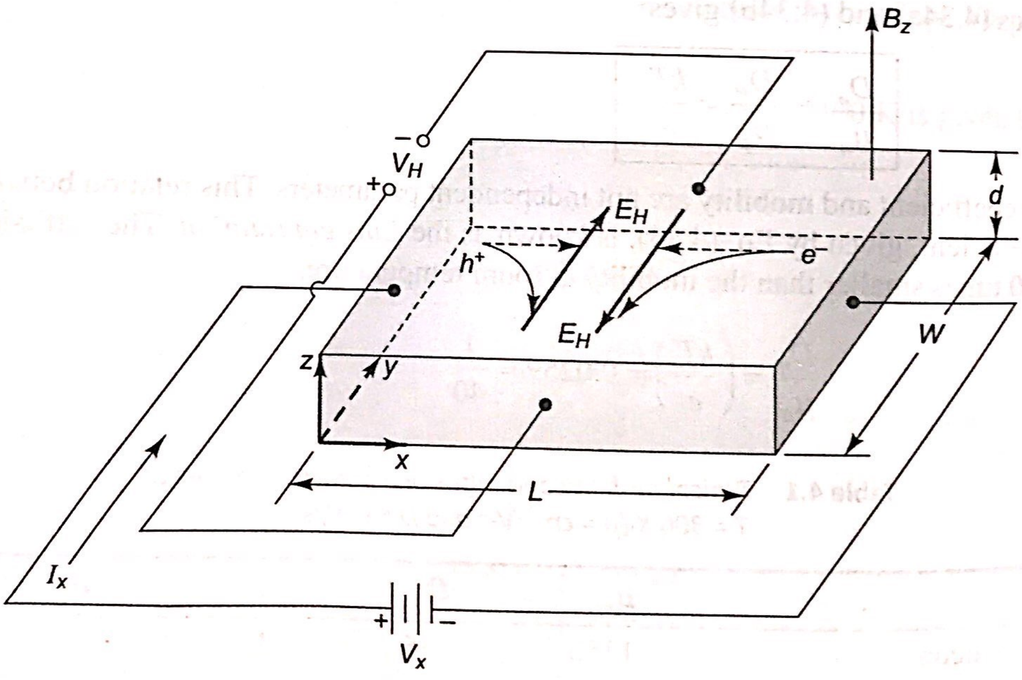

Fig. 4.15 *Geometry for measuring the Hall effect*

[1] We will assume an extrinsic semiconductor material in which the majority carrier concentration is much larger than the mi carrier concentration.

$$F = q\,[E + \upsilon \times B\,] = 0 \tag{4.37a}$$

which becomes

$$qE_y = q\upsilon_x B_z \tag{4.37b}$$

The induced electric field in the y-direction is called the *Hall field*. The Hall field produces a voltage across the semiconductor which is called the *Hall voltage*. We can write

$$V_H = +E_H W \tag{4.38}$$

where E_H is assumed positive in the $+ y$ direction and V_H is positive with the polarity shown.

In a p-type semiconductor, in which holes are the majority carrier, the Hall voltage will be positive as defined in Fig. 4.15. In an n-type semiconductor, in which electrons are the majority carrier, the Hall voltage will have the opposite polarity. The polarity of the Hall voltage is used to determine whether an extrinsic semiconductor is n-type or p-type.

Substituting Eq. (4.38) into Eq. (4.37) gives

$$V_H = \upsilon_x W B_z \tag{4.39}$$

For a p-type semiconductor, the drift velocity of holes can be written as

$$\upsilon_{dx} = \frac{J_x}{ep} = \frac{I_x}{(ep)(Wd)} \tag{4.40}$$

where e is the magnitude of the electronic charge. Combining Eqs (4.40) and (4.39), we have

$$V_H = \frac{I_x B_z}{epd} \tag{4.41}$$

or, solving for the hole concentration, we obtain

$$p = \frac{I_x B_z}{edV_H} \tag{4.42}$$

The majority carrier hole concentration is determined from the current, magnetic field, and Hall voltage. For p-type semiconductor, Hall coefficient is given by $R_H = \dfrac{1}{ep}$

For an n-type semiconductor, the Hall voltage is given by

$$V_H = -\frac{I_x B_z}{ned} \tag{4.43}$$

so that the electron concentration is

$$n = \frac{I_x B_z}{edV_H} \tag{4.44}$$

Note that the Hall voltage is negative for the n-type semiconductor; therefore, the electron concentration determined from Eq. (4.44) is actually a positive quantity. For n-type semiconductor Hall coefficient is given as

$$R_H = -\frac{1}{en}$$

Once the majority carrier concentration has been determined, we can calculate the low-field majority carrier mobility. For a p-type semiconductor, we can write

$$J_x = ep\mu_p E_x \tag{4.45}$$

The current density and electric field can be converted to current and voltage so that Eq. (4.45) becomes

$$\frac{I_x}{Wd} = \frac{ep\mu_p V_x}{L} \tag{4.46}$$

The hole mobility is then given by

$$\mu_p = \frac{I_x L}{epV_x Wd} \tag{4.47}$$

Similarly for an n-type semiconductor, the low-field electron mobility is determined from

$$\mu_n = \frac{I_x L}{enV_x Wd} \tag{4.48}$$

Example 4.6

Objective *To determine the majority carrier concentration and mobility, given Hall effect parameters.*

Consider the geometry shown in Fig. 4.13. Let $L = 10^{-1}$ cm, $W = 10^{-2}$ cm, and $d = 10^{-3}$ cm. Also assume that $I_x = 1.0$ mA, $V_x = 12.5$ V, $B_z = 500$ gauss $= 5 \times 10^{-2}$ tesla, and $V_H = -6.25$ mV.

A negative Hall voltage for this geometry implies that we have an n-type semiconductor. Using Eq. (4.44), we can calculate the electron concentration as

$$n = \frac{-(10^{-3})(5\times10^{-2})}{(1.6\times10^{-19})(10^{-5})(-6.25\times10^{-3})} = 5\times10^{21}\ \text{m}^{-3} = 5\times10^{15}\ \text{cm}^{-3}$$

The electron mobility is then determined from Eq. (4.48) as

$$\mu_n = \frac{(10^{-3})(10^{-3})}{(1.6\times10^{-19})(5\times10^{21})(12.5)(10^{-4})(10^{-5})} = 0.10\ \text{m}^2/\text{V-s}$$

or

$$\mu_n = 1000\ \text{cm}^2/\text{V-s}$$

Comment It is important to note that MKS units must be used consistently in the Hall effect equations to yield correct results.

SUMMARY

- The two basic transport mechanisms are drift, due to an applied electric field, and diffusion, due to a density gradient.
- Carriers reach an average drift velocity in the presence of an applied electric field, due to scattering events. Two scattering processes within a semiconductor are lattice scattering and impurity scattering.