EXAMPLE 11.1 The amplitude of field vector $\vec{H}$ of an EM-wave is 1 A/m. Find the amplitude of field vector $\vec{E}$ in free space.

Solution: We know that the relation between the amplitude of field vectors $\vec{E}$ and $\vec{H}$ of an EM-wave in free space is given by

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Here, we have

$$H_0 = 1 \text{ A/m}, \mu_0 = 4\pi \times 10^{-7} \text{ weber/A-m}, \varepsilon_0 = 8.85 \times 10^{-12} \text{ C/N-m}^2$$

Hence, $$E_0 = H_0\sqrt{\frac{\mu_0}{\varepsilon_0}} = 1 \times \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} = 560560\ 376.7 \text{ V/m}$$ **Ans.**

EXAMPLE 11.2 The maximum value of field vector $\vec{E}$ of an EM-wave in vacuum is 10^3 N/C. Find the maximum value of magnetic induction vector.

Solution: We know that the relation between the amplitudes of field vectors $\vec{E}$ and $\vec{H}$ of an EM-wave in vacuum is

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

Also we know that

$$B_0 = \mu_0 H_0$$

Eliminating field vector $\vec{H}$ from the above two relations, we have

$$B_0 = \mu_0 E_0 \sqrt{\frac{\varepsilon_0}{\mu_0}} = E_0\sqrt{\varepsilon_0 \mu_0}$$

But $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c =$ speed of light

Hence, $$B_0 = \frac{E_0}{c}$$

Here, $\vec{E} = 10^3$ N/C

Also, $c = 3 \times 10^8$ m/s

$\therefore$ $$B_0 = \frac{10^3}{3 \times 10^8} = 3.33 \times 10^{-6} \text{ weber/A-m}$$ **Ans.**

EXAMPLE 11.3 A conducting medium of conductivity 5 mhos/m and dielectric constant 1 is placed in an external electric field $E = 250 \sin(10^{10} t)$. Find the conduction and displacement current densities in the medium. Show that both the current densities will be equal at frequency $\omega = 5.6 \times 10^{11}$ Hz.

Solution: The conduction and the displacement current densities are expressed as

$$J = \sigma E$$

and $$J_D = \frac{\partial D}{\partial t} = \varepsilon\frac{\partial E}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$$

Here the applied electric field is

$$E = 250 \sin(10^{10} t)$$

Also $\sigma = 5$ mhos/m, $\varepsilon_r = 8.85 \times 10^{-12}$ F/m and $\varepsilon_0 = 1$

Hence conduction and displacement current densities are

$$J = \sigma E = 5 \times 250 \sin(10^{10} t) \text{ A/m}^2$$ **Ans.**

$$J_D = 8.85 \times 10^{-12} \times 1 \times \frac{\partial}{\partial t}[250 \sin(10^{10} t)]$$

$$= 8.85 \times 10^{-12} \times 250 \times 10^{10} \cos(10^{10} t)$$

$$= 2.212 \cos(10^{10} t) \text{ A/m}^2$$ **Ans.**

When the magnitude of both current densities will be equal then we must have

$$J = J_D$$

i.e. $$\sigma E = \varepsilon_0 \varepsilon_r \omega E$$

Hence $$\omega = \frac{\sigma}{\varepsilon_0 \varepsilon} = \frac{5}{8.85 \times 10^{-12} \times 1} = 5.6 \times 10^{11} \text{ Hz}$$

EXAMPLE 11.4 Using Maxwell's first relation, derive the Coulomb's law of electrostatics.

Solution: Suppose that a charge q is distributed uniformly inside a hollow sphere of surface area S. If ρ is the volume charge density of the charge, then from Maxwell's first relation

$$\text{div}\vec{D} = \rho$$

or $$\text{div}\vec{E} = \frac{\rho}{\varepsilon} \quad [\text{since } \vec{D} = \varepsilon\vec{E}]$$

Integrating both sides of the above equation over volume V bounded by the sphere of surface area S, we have

$$\int_V (\text{div}\vec{E})dV = \int_V \frac{\rho}{\varepsilon}dV = \frac{1}{\varepsilon}\int_V \rho dV$$

Using Gauss' divergence theorem

$$\int_S \vec{A}\cdot d\vec{s} = \int_V \text{div}\vec{A}dV$$

on left hand side of the above equation, we get

$$\int_S \vec{E}\cdot d\vec{s} = \frac{1}{\varepsilon}\int_V \rho dV$$

As the charge q is distributed uniformly inside the hollow sphere, we can treat the effect of whole charge from the centre of sphere, and then the electric lines of force will be in the radial direction. In this case, $\vec{E}$ is parallel to $d\vec{s}$. Hence the left hand side of the above equation takes the form

$$\int_S \vec{E}\cdot d\vec{s} = \int_S E\cdot ds\cos 0° = \int_S E\cdot ds = E\int_S ds = E\cdot S$$

If R is the radius of the sphere, then the surface area of the sphere is

$$S = 4\pi R^2$$

$\therefore$ $$\int_S \vec{E}\cdot d\vec{s} = 4\pi R^2 E$$

Also $$\int_V \rho\, dV = q$$

Hence, $$4\pi R^2 E = \frac{q}{\varepsilon}$$

or $$E = \frac{q}{\varepsilon}\frac{1}{4\pi R^2}$$

If a test charge q_0 is situated on the surface of sphere, then the force experienced by the test charge is

$$F = q_0 E = \frac{1}{4\pi\varepsilon}\frac{q q_0}{R^2}$$

i.e. $$F \propto \frac{q q_0}{R^2}$$

Thus, the electrostatic force acting between the two charges q and q_0 is directly proportional to the product of the amount of charges and it is inversely proportional to the square of the distance between the two charges. This is known as Coulomb's law of electrostatics. **Ans.**

EXAMPLE 11.5 In a time-invariant field, the field vector $\bar{H}$ of an EM-wave is expressed as

$$\bar{H} = (x\cos\alpha + y\sin\beta)\,\hat{k}$$

Obtain the expression for current density $\bar{J}$.

Solution: Maxwell's fourth relation is

$$\text{curl}\,\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$$

For time invariant field, we have

$$\frac{\partial \bar{D}}{\partial t} = 0$$

Then Maxwell's fourth relation reduces to

$$\text{curl}\,\bar{H} = \bar{J}$$

Here, we have

$$\bar{H} = (x\cos\alpha + y\sin\beta)\,\hat{k}$$

Hence $$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & (x\cos\alpha + y\sin\beta) \end{vmatrix} = \bar{J}$$

or $$\left[\frac{\partial}{\partial y}(x\cos\alpha + y\sin\beta)\right]\hat{i} - \left[\frac{\partial}{\partial x}(x\cos\alpha + y\sin\beta)\right]\hat{j} = \bar{J}$$

Hence $$\bar{J} = \sin\beta\,\hat{i} - \cos\alpha\,\hat{j}$$ **Ans.**

EXAMPLE 11.6 In free space, the field vector $\bar{E}$ of an EM-wave is expressed as

$$\bar{E} = E_p \sin(\omega t - kz)\,\hat{j}$$

Find the values of $\bar{D}$, $\bar{B}$ and $\bar{H}$. Sketch the field vectors $\bar{E}$ and $\bar{H}$.

Solution: For free space, the electric displacement vector $\bar{D}$ is given by

$$\bar{D} = \varepsilon_0 \bar{E} = \varepsilon_0 E_p \sin(\omega t - kz)\,\hat{j}$$

Now using the Maxwell's third equation, curl $\bar{E} = -\frac{\partial \bar{B}}{\partial t}$, we have

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_p\sin(\omega t - kz) & 0 \end{vmatrix} = -\frac{\partial \bar{B}}{\partial t}$$

or $$-\frac{\partial \bar{B}}{\partial t} = kE_p\cos(\omega t - kz)\,\hat{i}$$

or $$\bar{B} = -kE_p \int \cos(\omega t - kz)\,dt\,\hat{i}$$

or $$\bar{B} = -\frac{kE_p}{\mu_0\omega}\sin(\omega t - kz)\,\hat{i}$$

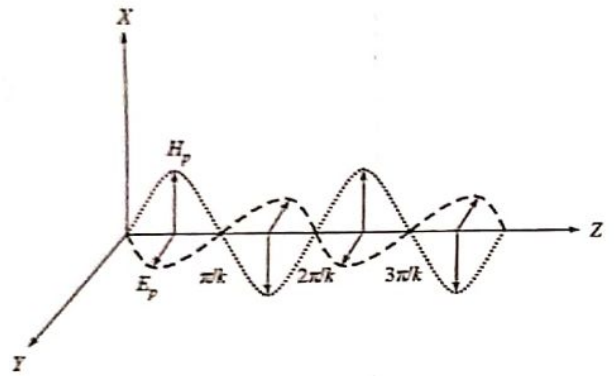
This is the expression for the magnetic induction vector $\bar{B}$. **Ans.**

For free space, the magnetic field strength vector $\bar{H}$ is given by

$$\bar{H} = \frac{\bar{B}}{\mu_0} = -\frac{kE_p}{\mu_0\omega}\sin(\omega t - kz)\,\hat{i}$$

At $t = 0$, $\sin(\omega t - kz) = -\sin(kz)$

The field vectors $\bar{E}$ and $\bar{H}$ are shown in the following figure with the assumption that $E_p > 0$ and $k > 0$. Ans.



Poynting Vector

EXAMPLE 11.7 Show that time average of Poynting vector for a conducting medium can be expressed as

$$<\bar{S}>_t = \frac{1}{2}\text{Re}(\bar{E}\times\bar{H}^*)$$

Solution: The field vectors $\bar{E}$ and $\bar{H}$ for a conducting medium can be expressed as

$$\bar{E} = (\bar{E}_{01} + i\,\bar{E}_{02})e^{i\omega t}$$

and

$$\bar{H} = (\bar{H}_{01} + i\,\bar{H}_{02})e^{i\omega t}$$

Then Poynting vector $\bar{S}$ is given by

$$\bar{S} = \text{Re}\,\bar{E} \times \text{Re}\,\bar{H}$$

or $$\bar{S} = \text{Re}[(\bar{E}_{01} + i\,\bar{E}_{02})e^{i\omega t}] \times \text{Re}[(\bar{H}_{01} + i\,\bar{H}_{02})e^{i\omega t}]$$

or $$\bar{S} = \text{Re}[(\bar{E}_{01} + i\,\bar{E}_{02})(\cos\omega t + i\sin\omega t)] \times \text{Re}[(\bar{H}_{01} + i\,\bar{H}_{02})(\cos\omega t + i\sin\omega t)]$$

or $$\bar{S} = \text{Re}[(\bar{E}_{01}\cos\omega t - \bar{E}_{02}\sin\omega t) + i(\bar{E}_{02}\cos\omega t + \bar{E}_{01}\sin\omega t)]$$
$$\times \text{Re}(\bar{H}_{01}\cos\omega t - \bar{H}_{02}\sin\omega t) + i(\bar{H}_{02}\cos\omega t + \bar{H}_{01}\sin\omega t)$$

or $$\bar{S} = (\bar{E}_{01}\cos\omega t - \bar{E}_{02}\sin\omega t) \times (\bar{H}_{01}\cos\omega t - \bar{H}_{02}\sin\omega t)$$

or $$\bar{S} = (\bar{E}_{01} \times \bar{H}_{01})\cos^2\omega t + (\bar{E}_{02} \times \bar{H}_{02})\sin^2\omega t \times (\bar{E}_{02} \times \bar{H}_{01})\cos\omega t\sin\omega t$$
$$-(\bar{E}_{01} \times \bar{H}_{02})\cos\omega t\sin\omega t$$

Now the time average of Poynting vector $\bar{S}$ is

$$<\bar{S}>_t = \frac{1}{T}\int_0^T \bar{S}\,dt$$

Here $T = \frac{2\pi}{\omega}$ is the time-period of electric and magnetic vibrations.

Substituting the value of $\bar{S}$ in the above relation and using the definite integrals

$$\frac{1}{T}\int_0^T \cos^2\omega t\,dt = \frac{1}{T}\int_0^T \sin^2\omega t\,dt = \frac{1}{2}$$

and $$\int_0^T \cos\omega t\sin\omega t\,dt = 0$$

we get

$$<\bar{S}>_t = \frac{1}{2}[(\bar{E}_{01} \times \bar{H}_{01}) + (\bar{E}_{02} \times \bar{H}_{02})] \quad \text{(i)}$$

The complex conjugate of field vector $\bar{H}$ is given by

$$\bar{H}^* = (\bar{H}_{01} - i\bar{H}_{02})e^{-i\omega t}$$

Hence $$(\bar{E} \times \bar{H}^*) = (\bar{E}_{01} + i\bar{E}_{02})e^{i\omega t} \times (\bar{H}_{01} - i\bar{H}_{02})e^{-i\omega t}$$

or $$(\bar{E} \times \bar{H}^*) = (\bar{E}_{01} + i\bar{E}_{02}) \times (\bar{H}_{01} - i\bar{H}_{02})$$

or $$(\bar{E} \times \bar{H}^*) = (\bar{E}_{01} \times \bar{H}_{01}) + (\bar{E}_{02} \times \bar{H}_{02}) \times i[(\bar{E}_{02} \times \bar{H}_{01}) - (\bar{E}_{01} \times \bar{H}_{02})]$$

Then $$\text{Re}\,(\bar{E} \times \bar{H}^*) = (\bar{E}_{01} \times \bar{H}_{01}) + (\bar{E}_{02} \times \bar{H}_{02}) \quad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$<\bar{S}>_t = \frac{1}{2}\text{Re}(\bar{E} \times \bar{H}^*)$$

This is the required result. Ans.

EXAMPLE 11.8 Assuming that all the energy from a 1000 watt lamp is radiated uniformly, calculate the average value of the intensity of electric field of radiation at a distance of 2 m from the lamp.

Solution: If P is the energy radiated per second from the given lamp in all directions, then the magnitude of Poynting vector $\bar{S}$ is given by

$$S = \frac{P}{4\pi r^2} \quad \text{(i)}$$

where $4\pi r^2$ is the area of a sphere of radius r whose centre lies on lamp.

Here, we have

$$P = 1000 \text{ watt}, \quad r = 2 \text{ m}$$

Hence, $$S = \frac{P}{4\pi r^2} = \frac{1000}{4 \times \pi \times (2)^2} = 19.9 \text{ W-m}^{-2}$$

Also, we know that

$$\frac{E}{H} = 377\ \Omega \quad \text{(ii)}$$

where E and H are the strengths of electrostatic and magnetostatic fields.

Poynting vector $\vec{S}$ in terms of E and H can be expressed as

$$\vec{S} = (\vec{E} \times \vec{H})$$

In scalar form, this relation takes the form

$$S = EH \sin 90° = EH \quad \text{(iii)}$$

Eliminating H from Eqs. (ii) and (iii), we get

$$E \times \frac{E}{377} = S$$

Here, $$S = 19.9 \text{ W/m}^2$$

$\therefore$ $\frac{E^2}{377} = 19.9$ or $E^2 = 19.9 \times 377 = 7502.3$ or $E = \sqrt{7502.3} = 86.6 \text{ V-m}^{-1}$ **Ans.**

EXAMPLE 11.9 If the earth receives 2 cal-min^{-1}-cm^{-2} solar energy, what are the amplitudes of electric and magnetic fields of radiation?

Solution: Here, we have

$$S = 2\frac{\text{cal}}{\text{min-cm}^2} = 1400 \text{ J-s}^{-1}\text{-m}^{-2}$$

But, by definition of Ponyting vector, we have

$$\vec{S} = (\vec{E} \times \vec{H})$$

i.e. $$S = EH$$

Thus, $$EH = 1400 \text{ J-s}^{-1}\text{-m}^{-2} \quad \text{(i)}$$

Also, we know that

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.85 \times 10^{-12}}} \approx 377\ \Omega \quad \text{(ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$E^2 = 1400 \times 377$$

or $$E = \sqrt{1400 \times 377} = 726.5 \text{ V-m}^{-1}$$

Then, from Eq. (i), we have

$$H = \frac{E}{377} = \frac{7265}{377} = 1.927 \text{ A-m}^{-1}$$

Hence, the amplitudes of electric and magnetic fields of radiation are

$$E_0 = E\sqrt{2} = 726.5 \times 1.414 = 1027.3 \text{ V-m}^{-1}$$ **Ans.**

and $$H_0 = H\sqrt{2} = 1.927 \times 1.414 = 2.717 \text{ A-m}^{-1}$$ **Ans.**

Propagation of EM-waves

EXAMPLE 11.10 Show that for a good conductor, the phase difference between the field vectors $\vec{E}$ and $\vec{H}$ of an EM-wave is $\pi/4$.

Solution: The propagation constant for a conducting medium can be expressed as

$$K = \sqrt{\varepsilon\mu\omega^2\left[1 + \frac{i\sigma}{\omega\varepsilon}\right]}$$

Suppose that ϕ is the phase difference between the field vectors $\vec{E}$ and $\vec{H}$ of an EM-wave, then we have

$$K = \alpha + i\beta = ke^{i\phi}$$

Hence $$\alpha + i\beta = \sqrt{\varepsilon\mu\omega^2\left[1 + \frac{i\sigma}{\omega\varepsilon}\right]}$$

Equating real and imaginary parts from both sides of above equation, we obtain

$$\alpha^2 - \beta^2 = \varepsilon\mu\omega^2$$

and $$2\alpha\beta = \mu\omega\sigma$$

Solving the above equations, we get

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} + 1\right]^{1/2}$$

and $$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2}$$

For a good conductor $\sigma >> \varepsilon\mu$, then

$$\alpha = \beta = \sqrt{\frac{\sigma\omega\mu}{2}} \quad \text{(i)}$$

Again since $K = \alpha + i\beta = ke^{i\phi}$, so we have

$$\alpha = k\cos\phi$$

and $$\beta = k\sin\phi$$

Then $$\phi = \tan^{-1}\left(\frac{\beta}{\alpha}\right) \qquad \text{(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\phi = \tan^{-1}(1) = \frac{\pi}{4}$$ Ans.

EXAMPLE 11.11 Show that for a good conductor, the wave impedance experienced by EM-wave can be expressed as

$$Z_c = \sqrt{\frac{\mu\omega}{\sigma}}\, e^{-i\pi/4}$$

Solution: Maxwell's third relation for a conducting medium can be expressed as

$$\text{curl}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu\frac{\partial \vec{H}}{\partial t}$$

Considering the EM-wave as plane wave, the del operator (∇) and partial time derivative operator $\partial/\partial t$ can be expressed as

$$\nabla = iK$$

and $$\frac{\partial}{\partial t} = -i\omega$$

Then Maxwell's third relation takes the form

$$i\vec{K} \times \vec{E} = -\mu(-i\omega\vec{H})$$

or $$\vec{K} \times \vec{E} = \mu\omega\vec{H}$$

The above equation in scalar form becomes

$$KE_0 = \mu\omega H_0$$

Then the wave impedance of conducting medium is given by

$$Z_c = \frac{E_0}{H_0} = \frac{\mu\omega}{K} \qquad \text{(i)}$$

For a conducting medium, the propagation constant can be expressed as

$$K = \alpha + i\beta$$

Here $$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2}$$

and $$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$$

For a good conductor $\sigma >> \varepsilon\omega$, the above two expressions reduce to

$$\alpha = \beta = \sqrt{\frac{\sigma\omega\mu}{2}}$$

Hence, for a good conductor

$$K = \alpha + i\beta = \sqrt{\frac{\sigma\omega\mu}{2}} + i\sqrt{\frac{\sigma\omega\mu}{2}}$$

or $$K = \sqrt{\frac{\sigma\omega\mu}{2}}\,(1+i)$$

or $$K = \sqrt{\sigma\omega\mu}\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right)$$

or $$K = \sqrt{\sigma\omega\mu}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

or $$K = \sqrt{\sigma\omega\mu}\, e^{i\pi/4} \qquad \text{(ii)}$$

Substituting the value of K from Eq. (ii) into Eq. (i), we get

$$Z_c = \frac{\mu\omega}{\sqrt{\sigma\omega\mu}\, e^{i\pi/4}}$$

Hence, $$Z_c = \sqrt{\frac{\mu\omega}{\sigma}}\, e^{-i\pi/4}$$ Ans.

This is the wave impedance experienced by an EM-wave in a good conductor.

EXAMPLE 11.12 Calculate the skin depth of an EM-wave in aluminium at the frequency of 71.6 MHz. The electrical conductivity of aluminium is $3.54 \times 10^7\ (\Omega\text{-m})^{-1}$ and permeability is $4\pi \times 10^{-7}$ weber/A-m.

Solution: The expression for skin depth is given by

$$\delta = \sqrt{\frac{2}{\sigma\omega\mu}}$$

Here, we have

$$\sigma = 3.54 \times 10^7\ (\Omega\text{-m})^{-1}, \qquad \mu = 4\pi \times 10^{-7}\ \text{weber/A-m},$$

$$\omega = 2\pi f = 2\pi \times 71.6\ \text{MHz} = 4.5 \times 10^8\ \text{Hz}$$

Hence, $$\delta = \sqrt{\frac{2}{3.54 \times 10^7 \times 4.5 \times 10^8 \times 4\pi \times 10^{-7}}} = 10\ \mu\text{m}$$ Ans.

EXAMPLE 11.13 Show that for a poor conductor, the skin depth can be expressed as

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Solution: We know that for a conducting medium, the propagation constant can be expressed as

$$K = \alpha + i\beta$$

Here,

$$\alpha = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}+1\right]^{1/2}$$

and

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}-1\right]^{1/2}$$

and skin depth is given by

$$\delta = \frac{1}{\beta}$$

For a poor conductor $\sigma << \varepsilon\omega$, so we can approximate the first term in square root bracket of right hand side of expression of β using the binomial theorem as

$$\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} = \left(1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right)^{1/2}$$

$$= 1 + \frac{\sigma^2}{2\varepsilon^2\omega^2} - \frac{\sigma^4}{8\varepsilon^4\omega^4} + \cdots$$

$$= 1 + \frac{\sigma^2}{2\varepsilon^2\omega^2}$$

i.e.

$$\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2} = -1 = \frac{\sigma^2}{2\varepsilon^2\omega^2}$$

The expression of β therefore reduces to

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\frac{\sigma^2}{2\varepsilon^2\omega^2}\right]^{1/2}$$

or

$$\beta = \omega\sqrt{\frac{\varepsilon\mu}{2}}\,\frac{\sigma}{\sqrt{2}\varepsilon\omega}$$

or

$$\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$

Hence, the skin depth for a poor conductor can be expressed as

$$\delta = \frac{1}{\beta} = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$ **Ans.**

EXAMPLE 11.14 For the sea water, $\mu = \mu_0 = 4\pi \times 10^{-7}$ weber/A-m, $\varepsilon = 70\,\varepsilon_0$ and $\sigma = 5\ (\Omega\text{-m})^{-1}$. Calculate the skin depth and attenuation constant of sea water.

Solution: Since the sea water is a poor conductor, therefore, the skin depth is given by

$$\delta = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$$

Here, we have

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ weber/A-m},\ \varepsilon = 70\,\varepsilon_0 = 70 \times 8.85 \times 10^{-12} \text{ C/N-m}^2$$

and

$$\sigma = 5\ (\Omega\text{-m})^{-1}.$$

Hence,

$$\delta = \frac{2}{5}\sqrt{\frac{70 \times 8.85 \times 10^{-12}}{4 \times 3.14 \times 10^{-7}}} = 0.4 \times \sqrt{4.934 \times 10^{-4}} = 0.4 \times 2.22 \times 10^{-2} = 0.0089 \text{ m}$$ **Ans.**

Also, the attenuation constant of sea water is

$$\beta = \frac{1}{\delta} = \frac{1}{0.0089} = 112.36 \text{ m}^{-1}$$ **Ans.**

EXAMPLE 11.15 Show that for a good conductor the refractive index can be expressed as

$$n = \sqrt{\frac{\mu_r\sigma}{\omega\varepsilon_0}}\,e^{i\pi/4}$$

Solution: If Z_0 and Z_c are the wave impedances of free space and a good conductor, then we have

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \quad \text{and} \quad Z_c = \sqrt{\frac{\mu\omega}{\sigma}}\,e^{-i\pi/4}$$

If n is the refractive index of the conductor, then we have

$$n = \frac{c}{v}$$

or

$$n = \frac{\mu c}{\mu v} = \frac{\mu_r\mu_0 c}{\mu v}$$

or

$$n = \frac{\mu_r Z_0}{Z_c}, \quad [\text{since } Z_c = \mu v \quad \text{and} \quad Z_0 = \mu_0 c]$$

or

$$n = \frac{\mu_r\sqrt{\frac{\mu_0}{\varepsilon_0}}}{\sqrt{\frac{\mu\omega}{\sigma}}\,e^{-i\pi/4}} = \frac{\mu_r\sqrt{\frac{\mu_0}{\varepsilon_0}}}{\sqrt{\frac{\mu_r\mu_0\omega}{\sigma}}\,e^{-i\pi/4}}$$

Hence, the refractive index for a good conductor can be expressed as

$$n = \sqrt{\frac{\mu_r\sigma}{\omega\varepsilon_0}}\,e^{i\pi/4}$$ **Ans.**

29. An EM-wave of frequency 3.0 MHz passes from vacuum into a non-magnetic medium with permittivity 4.0. Calculate the change in its frequency.

30. Treating ocean water as a non-magnetic dielectric medium of dielectric constant 80 and conductivity 4.3 mho/m, calculate the frequency for which the penetration depth will be 10 cm. Also show that for frequencies less than 10^8 Hz, it will behave as a good conductor.

31. For silver, $\mu_r = 1$, $\sigma = 3 \times 10^7$ mho/m. Calculate the skin depth at 10^8 Hz frequency.

32. Calculate the skin depth and wave velocity at a frequency of 1.6 MHz in aluminium for which $\sigma = 38.2 \times 10^6$ mho/m and $\mu_r = 1$.

33. For silver, $\sigma = 3.0 \times 10^6$ mho/m, calculate the frequency at which the depth of penetration is 1 mm.

29. It is reduced by 50 Hz

30. 6×10^{6} Hz

31. 9.0×10^{-4} cm

32. 6.44×10^{-5} m, 647 m/s

33. 84.4 Hz