Department of Applied Mathematics
Delhi Technological University, Delhi

ASSIGNMENT 3
2019-2020

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Subject Code : **MA-102** Course Title : **Mathematics-II**

Instructions

Write your name and roll number on each page of your assignment. Assignment should be legibly handwritten and on both sides of the paper.

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1. Find a series solution in powers of x and $(x-1)$ of Airy's equation

$$y'' - xy = 0, \quad -\infty < x < \infty.$$

2. Classify the singular point of the following equations:
   a. $(1-x^2)y'' + 2xy' + n(n+1)y = 0.$
   b. $x^3(x-2)y'' + x^3y' + 6y = 0.$
   c. $\left(x - \frac{\pi}{2}\right)^2 y'' + \cos(x)y' + \sin(x)y = 0.$

3. Find power series solution of the following differential equations:
   a. $y'' + (x-1)y' + y = 0$ about $x = 2$
   b. $(1-x^2)y'' + 2xy' + y = 0$ about $x = 0$.
   c. $y'' + \cos(x)y = 0$ about $x = 0$.

4. Find a power series solution of $(x^2-1)y''(x) + 3xy'(x) + xy(x) = 0$ subject to
   a. $y(0) = 4$, $y'(0) = 6$ and
   b. $y(2) = 4$, $y'(2) = 6$.

5. Use the method of Frobenius to find solutions of the following differential eqution in some interval $0 < x < R$.
   a. $2x^2y'' - xy' + (x-5)y = 0,$
   b. $2x^2y'' + xy' + (x^2-3)y = 0,$
   c. $x^2y'' - xy' - \left(x^2 + \frac{5}{4}\right)y = 0$
   d. $x^2y'' + (x^2-3x)y' + y = 0.$

6. Show that for $n = 0, 1, 2, 3$ the corresponding Legendre polynomial is given by
$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n.$$

7. Find two linearly independent solutions of the Bessel equation of order $3/4$ for all $x > 0$.

8. Define Legendre polynomial $P_n(x)$. If m and n are non-negative integers then show that $\displaystyle\int_{-1}^{1} P_n(x)P_m(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \dfrac{2}{2n+1} & \text{if } m = n. \end{cases}$

9. Show that $x = 0$ is a regular singular point of hypergeometric differential equation $x(x-1)y'' + (3x-1)y' + y = 0$. Hence, find a power series solution about $x = 0$.

10. Show that Chebyshev's equation $(1 - x^2)y''(x) - xy'(x) + a^2y(x) = 0$ with $a \in (0, \infty)$ has the following linearly independent power series solutions
$$y_1(x) = 1 + \sum_{n=1}^{\infty} \frac{1}{(2n)!} \left[\prod_{k=0}^{n-1} (4k^2 - a^2) \right] x^{2n} \text{ and}$$
$$y_2(x) = x + \sum_{n=1}^{\infty} \frac{1}{(2n+1)!} \left[\prod_{k=0}^{n-1} (4k^2 + 4k + 1 - a^2) \right] x^{2n+1}.$$

11. Given $n \in \mathbb{N}$ and $x \in (0, +\infty)$. Then prove the following
a. $xJ_v'(x) = vJ_v(x) - xJ_{v+1}(x)$.
b. $xJ_v'(x) = -vJ_v(x) + xJ_{v-1}(x)$.
c. $2J_v'(x) = J_{v-1}(x) - J_{v+1}(x)$.
d. $2vJ_v(x) = x[J_{v-1}(x) + J_{v+1}(x)]$.
e. $\dfrac{d}{dx}(x^{-v}J_v) = -x^{-v}J_{v+1}(x)$.
f. $\dfrac{d}{dx}(x^v J_v) = x^v J_{v-1}(x)$.

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