

Total no. of pages :2

1st SEMESTER

END SEM EXAMINATION

-42-

Old

Roll No. _____

B.Tech (All groups)

NOV 2018

AM – 101 Mathematics-I
(Old Scheme)

Time : 3 hrs

Max. Marks: 70

Note: Attempt all questions selecting two parts from each question. All questions carry equal mark. Assume missing data , if any.

1 (a) Find the necessary condition for convergence of an infinite series of positive terms. Is it sufficient also? Justify your answer.

(b) Discuss the convergence of the series whose nth term is;

$$u_n = \frac{3.6.9 \dots (3n-3)}{7.10.13 \dots (3n+1)} x^{n-1}$$

(c) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{\cos n\pi}{n}$$

2 (a) Compute the value of $\log 1.1$ upto four decimal places using Taylor series.

(b) Find the area common to two cardioids $r = a(1 + \cos\theta)$ and $r = a$.

(c) Find the volume of the solid generated by revolving the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ about the tangent at the vertex.

P.T.O.

3 (a) If $u = e^{xyz}$ then show that

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2 y^2 z^2) e^{xyz}$$

(b) If $u = \tan^{-1} \frac{x^3 - y^3}{x - y}$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.

(c) Determine the points where the function $x^3 + y^3 - 3axy$ has a maximum or minimum and decide its nature.

4. Show by double integration that area between the curves $y^2 =$

(a) $4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{2}$.

(b) Evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dx dy$ by changing the order of integration.

(c) Find by triple integration the volume of the paraboloid of revolution $x^2 + y^2 = 4z$ cut off by the plane $z = 4$.

5. (a) A vector field is given by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$
Show that it is irrotational, and hence find its scalar potential.

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and ϕ is a scalar, then prove that
 $\text{div}(\vec{r} \phi) = 3\phi + \vec{r} \cdot \text{grad } \phi$

(c) Evaluate $\iint_S \vec{A} \cdot d\vec{S}$ where $\vec{A} = 12x^2y\hat{i} - 3yz\hat{j} + 2z\hat{k}$ and S is the part of the plane $x + y + z = 1$ included in the first octant.