# ASSIGNMENT 5
## (Fourier Series and Fourier Transforms)
## Mathematics-II (MA102) 2019-20

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1. Find the Fourier series expansion of $f(x) = \pi + x$, $-\pi < x < \pi$. Hence prove that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$$

   Answer : $\pi + 2\sum\left[\frac{(-1)^{n+1}}{n}sin(nx)\right]$.

2. Obtain a fourier series to represent the function $f(x) = |\cos x|$ for $-\pi < x < \pi$.

   Answer : $\frac{2}{\pi} + \frac{4}{\pi}\left[\frac{cos2x}{3} - \frac{cos4x}{15} + ...\right]$.

3. Determine the sine series expansion for the function defined by:

$$f(x) = \begin{cases} 1, & 0 < x < \frac{\pi}{2} \\ 2, & \frac{\pi}{2} < x < \pi. \end{cases}$$

   Answer : $f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\left[cos\frac{n\pi}{2} + 1 - 2(-1)^n\right]sinnx$.

4. In the interval $[0, 2]$, expand the function

$$f(x) = \begin{cases} x, & 0 < x \leq 1 \\ 2 - x, & 1 < x \leq 2. \end{cases}$$

   as a i) series of cosines ii) series of sines.

   Answer : i) $f(x) = \frac{1}{2} - \frac{4}{\pi^2}\sum\frac{cos(2n-1)\pi x}{(2n-1)^2}$ ii) $f(x) = \frac{8}{\pi^2}\sum(-1)^{n-1}\frac{sin(2n-1)\pi x/2}{(2n-1)^2}$.

5. Find the complex fourier series representation of $f(x) = e^{-x}$ on the interval $(-\pi, \pi)$.

   Answer :$f(x) = \frac{sinh\pi}{\pi}\sum_{-\infty}^{\infty}(-1)^n\left(\frac{1-in}{1+n^2}\right)e^{inx}$.

6. Obtain the fourier series expansion of the periodic function $f(x) = e^x, -\pi < x < \pi$ with $f(x + 2\pi) = f(x)$.

   Answer : $\frac{1}{\pi}sinh\pi + \frac{2sinh\pi}{\pi}\sum\left[\frac{(-1)^n}{n^2+1}\{cos(nx) - nsin(nx)\}\right]$.

7. Obtain up to first harmonic, the Fourier series of the function given below in table form:

| x : | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| y : | 9 | 18 | 24 | 28 | 26 | 20 |

   Answer :$y \approx 20.83 - 8.33cos\frac{\pi x}{3} - 1.156sin\frac{\pi x}{3}$.

8. Find the Fourier transform of $e^{-|t|}$.

   Answer : $\sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\omega^2}$

9. Find the Fourier transform of

$$f(t) = \begin{cases} 1 - t^2, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$$

   and hence evaluate $\int_0^\infty \left(\frac{tcost-sint}{t^3}\right) cos\frac{t}{2} dt$.

   Answer: $F(\omega) = -2\sqrt{\frac{2}{\pi}}\left(\frac{\omega cos\omega - sin\omega}{\omega^3}\right)$; the value of integral is $\frac{3\pi}{8}$.

10. Find the inverse fourier transform of the function $F(\omega) = e^{-|\omega|a}$, $a > 0$.

    Answer: $f(t) = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+t^2}$.

11. Find the Fourier sine and Fourier cosine transform of $e^{-t}$.

    Answer: $F_s(\omega) = \sqrt{\frac{2}{\pi}} \cdot \frac{\omega}{1+\omega^2}$ and $F_c(\omega) = \sqrt{\frac{2}{\pi}} \cdot \frac{1}{1+\omega^2}$.

12. Find the solution of the differential equation $y' - y = u_0(t)e^{-t}$, $-\infty < t < \infty$, where $u_0(t)$ represents the unit step function.

    Answer:

$$y(t) = \begin{cases} \frac{-1}{2} e^t, & t < 0 \\ \frac{-1}{2} e^{-t}, & t > 0 \end{cases}$$