Basic Electrical Engg – EE102
(Lecture Notes – Single Ph. AC Ckt)

Numerical Examples

- Phasor Representation of Sinusoidals
- Examples of Single Phase Series and Parallel Circuits

Phasor Representation of Sinusoidals

1) Two sinusoidal currents are described as $\boldsymbol{i_1 = 10\sqrt{2}\, sin\, \omega t}$ and $\boldsymbol{i_2 = 20\sqrt{2}\, sin\, (\omega t + 60°)}$. Find the expression for the sum of these currents.

$i_1 = 10\sqrt{2}\, sin\, \omega t = I_{m1} sin\, \omega t \cdots\cdots\cdots (1)$

Where $\boldsymbol{I_{m1}} = 10\sqrt{2} \Rightarrow I_{1_{rms}} = 10$

$i_2 = 20\sqrt{2}\, sin(\omega t + 60°) = I_{m2} sin\, \omega t \cdots\cdots\cdots (2)$

Where $\boldsymbol{I_{m2}} = 20\sqrt{2}(cos\, 60° + j\, sin\, 60°)$

$= 20\sqrt{2}\left(\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = 10\sqrt{2} + j10\sqrt{6}$

$\therefore \boldsymbol{I_m = I_{m1} + I_{m2}} = 10\sqrt{2} + 10\sqrt{2} + j10\sqrt{6} = 20\sqrt{2} + j10\sqrt{6}$

$\boldsymbol{I_m} = I_m \angle \theta \,;\, I_m = \sqrt{\left(20\sqrt{2}\right)^2 + \left(10\sqrt{6}\right)^2} = 37.4 \quad \& \quad \tan^{-1}\frac{10\sqrt{6}}{20\sqrt{2}} = 41°$

$\boldsymbol{I_m} = 37.4\angle 41°$

$\therefore i = 37.4\, sin\, (\omega t + 41°)$

Phasor Representation of Sinusoidals

2) A sinusoidal current having an effective value of $10\angle 0°\ A$ is added to another sinusoidal current of effective value $20\angle 60°\ A$. Find the effective value of the resultant current.

$$\boldsymbol{I = I_1 + I_2} = 10\angle 0° + 20\angle 60° = 10 + 20\ (Cos\ 60° + jSin60°)$$

$$= 20 + j10\sqrt{3} = 26.4\angle 41°$$

$$\therefore i = \sqrt{2}\ (26.4)\ sin\ (\omega t + 41°) = 37.4\ sin\ (\omega t + 41°)$$

Other Examples of Single Phase Series and Parallel Circuits

Example 1

A voltage $(80 + j60)V$ is applied to a series circuit and the current is $(-4 + j10)A$.

(a) Find Z, power factor and active power.

(b) Is the circuit inductive or capacitive ?

(c) Find the parameters of the circuit if f=50Hz.

Example 1

Solution

$$V = 80 + j60 = 100\angle 36.87°\ V$$

$$I = -4 + j10 = 10.77\angle 111.8°$$

a) $\mathrm{Z} = \frac{\mathrm{V}}{\mathrm{I}} = \frac{100\angle 36.87°}{10.77\angle 111.8°} = 9.825\angle -74.93°\ \Omega$

$pf = \cos\theta = \cos(-74.93) = 0.26\ leading$

$P = VI\cos\theta = 100 \times 10.77 \times 0.26 = 280\ W$

b) Current phasor is leading the voltage phasor. The current is capacitive

c) $\mathrm{R} = \mathrm{Z}\cos\theta = 9.825 \times 0.26 = 2.55\ \Omega$

$C = \frac{1}{\omega X_c} = \frac{1}{2\pi \times 50 \times 9.49} = 335.4 \times 10^{-4} F$

Example 2

A parallel RL circuit has a $6\ \Omega$ resistance in one branch and a $0.05\ H$ inductance in the other. It is excited by a $230\ V$, $50\ Hz$ ac supply. Find

a) The circuit current.

b) Z

c) Y

d) Power factor, apparent, active and reactive power and

e) Parameters of the equilent series circuit

Example 2

Solution

a) $\mathrm{I_r} = \frac{230\angle 0°}{6} = 38.33\angle 0°\ \mathrm{A}$

$$I_L = \frac{V}{jX_L} = \frac{230\angle 0°}{j(2\pi \times 50 \times 0.05)} = 14.64\angle -90°\ A$$

$$I = I_r + I_L = 38.33 - j14.66\ A = 41.03\angle -20.9°\ A$$

b) $\mathrm{Z} = \frac{\mathrm{V}}{\mathrm{I}} = \frac{230}{41.03\angle -20.9°} = 5.61\angle 20.9°\ \Omega$

c) $\mathrm{Y} = \frac{1}{\mathrm{Z}} = \frac{1}{5.61\angle 20.9°} = 0.178\angle 20.9°\ siemen$

Example 2

Solution (contd…)

d) $$pf = \cos(-20.9°) = 0.934\ lagging$$

$$Apparent\ power = VI = 230 \times 41.03 = 9436.9VA$$

$$Active\ power = VI\cos\theta = 9436.9 \times 0.934 = 8814.06\ W$$

$$Reactive\ power = VI\sin\theta = 9436.9 \times \sin 20.9° = 3366.5\ vars\ (inductive)$$

e) $$Z = 5.61\angle 20.9° = 5.24 + j2\ \Omega$$

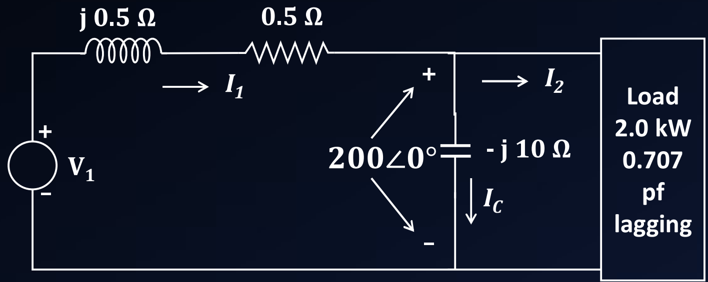
$$R_{eq} = 5.24\ \Omega$$

$$X_{eq} = 2\ \Omega$$

$$L_{eq} = \frac{2}{2\pi \times 50} = 0.00637H$$

Example 3

Determine the load current in the circuit given in Fig. Also, determine the source voltage and load current delivered by the source. Calculate power, reactive power and volt-ampere produced by the source.



Example 3

Solution

$$P = VI\cos\theta$$

$$\therefore I_2 = \frac{2.0\times10^3}{200\times0.707} = 14.14\ A$$

$$I_2 = 14.14\ (\cos\theta\ - \sin\theta)$$

$$= 14.14(0.707 + \sin[\cos^{-1}0.707])$$

$$= \mathbf{10 - j10}$$

The current through the capacitor.

$$I_c = \frac{200\angle0°}{10\angle-90°} = 20\angle90° = \boldsymbol{j20}$$

Example 3

Solution (Contd…)

The supply current

$$I_1 = I_2 + I_c = 10 - j10 + j20 = 10 + j10 = \mathbf{14.14\angle45°}$$

The source voltage

$$V_1 = 200 + (10 + j10)(0.5 + j0.5)$$

$$= 200 + \mathrm{j}10 = \mathbf{200.25\angle2.86°}\boldsymbol{V}$$

The volt-ampere

$$VA = 200.25 \times 14.14 = \mathbf{2831.535}\ \boldsymbol{VA}$$

Example 3

Solution (Contd…)

The power factor angle

$$\theta = 2.86° - 45° = \mathbf{-42.14°}$$

The power

$$P = 200.25 \times 14.14 \cos(-42.14°) = \mathbf{2099.6\,W}$$

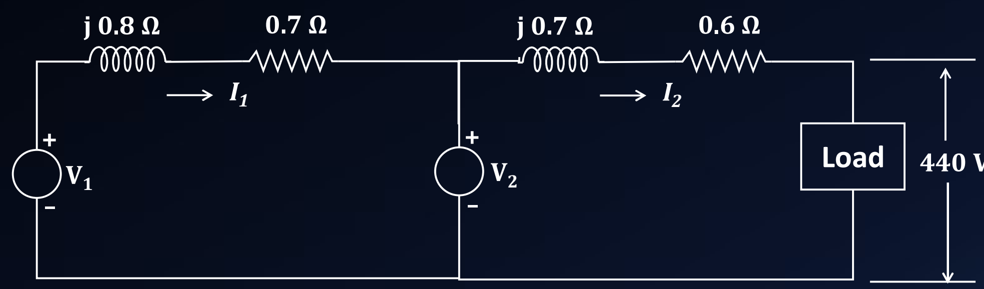
Reactive power

$$Q = 200.25 \times 14.14 \sin(-42.14°) = \mathbf{-1899.9\,VAR}$$

$$= \mathbf{1899.9\,VAR\ (capacitive)}$$

Example 4

For the distribution system shown in Fig., a load of 10 kW at power factor 0.8 lagging is supplied to two sources of voltages V_1 and V_2. The source V_2 supplies 5 kW at 0.6 power factor lagging. Find (a) the two-source voltages V_1 and V_2 and (b) the volt-ampere and power factor of the source V_1 .



Example 4

Solution

The load current

$$I_L = \frac{10\times100}{440\times0.8} = \mathbf{28.41\ A}$$

In complex form,

$$I_L = 28.41 \times 0.8 - j28.41 \times 0.6 = 22.73 - j17.05$$

Reactive power of the load

$$Q_L = \frac{P_L}{0.8} \times 0.6 = \frac{10}{0.8} \times 0.6 = \mathbf{7.5\ kVAR}$$

Example 4

Solution (Contd...)

The voltage $V_2 = 440 + j0 + (22.73 - j17.05)(0.6 + j0.7)$

$$= \mathbf{465.57 + j5.68 = 465.6\angle 0.7°}$$

The current supplied by $\boldsymbol{V_2}$, $I_2 = \frac{5\times1000}{465.6\times0.6} = \mathbf{17.9\ A}$

The power factor angle of $\boldsymbol{V_2}$, $\theta_2 = \cos^{-1} 0.6 = 53.13°$

So, the phasor angle of $\boldsymbol{I_2}$, $\theta_2 = -(53.13 - 0.7) = -52.43°$

$$I_2 = 17.9\ cos\ (-52.43°) + j17.9\ sin\ (-52.43°)$$

$$I_2 = 10.9 - j14.19$$

Hence, $\boldsymbol{I_1 = I_L - I_2 = 22.73 - j17.05 - 10.9 + j14.19}$

$$\boldsymbol{I_1 = 11.83\ - j2.86 = 12.17\angle - 13.6°}$$

Example 4

Solution (Contd...)

The total power and reactive power supplied by the two sources are given by

$$P = P_L + I_L^2 \times 0.6 + I_1^2 \times 0.7 = \mathbf{10.588\ kW}$$

$$Q = \mathbf{7.5 + \left[28.41^2 \times 0.7 + 12.7^2 \times 0.8\right] \times 10^{-3} = 8.183\ kVAR}$$

Voltage of the source V_1,

$$V_1 = V_2 + I_1 \times (0.7 + j0.8) = \mathbf{476.32\ \angle 1.58°}$$

The power, reactive power and KVA aupplied by source

Example 4

Solution (Contd...)

The power, reactive power and KVA aupplied by source

$$P_{r_1} = P - P_{r_2} = 10.588 - 5 = \mathbf{5.588\ \ kW}$$

$$Q_{r_1} = Q - Q_{r_2}$$

$$Q_{r_2} = \frac{5}{0.6}\sin(\cos^{-1} 0.6) = \frac{5\times 0.8}{0.6} = 6.66$$

$$\therefore Q_{r_1} = 8.183 - 6.66 = \mathbf{1.52\ kVAR}$$

$$kVA = \sqrt{5.588^2 + 1.52^2} = \mathbf{5.79\ kVA}$$

$$\text{Power Factor} = \cos\left(\tan^{-1}\frac{1.52}{5.88}\right) = \mathbf{0.965\ lagging}$$

Example 5

Find the resonance frequency for a series circuit, if $L = 32\ \mu H$ and $C = 450pF$. Determine required value of R for the quality factor $Q = 0.05$. Find lower and upper cut off frequencies and bandwidth.

Solution

The resonance frequency, $f_r = \frac{1}{2\pi\sqrt{LC}} = 1.33\ MHz$

The quality factor, $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

$\therefore R = \sqrt{\frac{L}{CQ^2}} = 5.33\ k\Omega$

Example 5

Solution (Contd...)

The lower cut off frequency, $f_1 = -\frac{f_r}{2Q} + f_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1} = 66.33\ kHz$

The upper cut off frequency, $f_2 = \frac{f_r}{2Q} + f_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1} = 26670\ kHz$

Bandwidth $= f_2 - f_1 = 26603.67$ kHz

Example 6

An inductive coil having resistance of $20\ \Omega$ and inductance of $0.2\ H$ is connected in parallel with a $100\ \mu F$ capacitor. Calculate the frequency at which the circuit acts as a non-reactive resistor R. Find the value of R

Solution

The admittance of the parallel circuit, $Y = \frac{1}{R+j\omega L} + j\omega C = \frac{R-j\omega(L-\omega^2 CL^2 - CR^2)}{R^2+\omega^2 L^2}$

For the circuit to be non inductive, $L - \omega^2 CL^2 - CR^2 = 0$

Accordingly, $\omega = \sqrt{\frac{1}{CL} - \frac{R^2}{L^2}} = 200$

Hence $f_r = \frac{200}{2\pi} = 31.8\ Hz$

Example 6

Solution (Contd…)

$$\therefore Y = \frac{R}{R^2+\omega^2 L^2} \Rightarrow Z = \frac{1}{Y} = R'$$

The non reactive resistance, $R' = \frac{R^2+\omega^2 L^2}{R} = 100\Omega$