Equations

Capacity external wall

$$\frac{C_{1o}}{\Delta t} \cdot (T_{ow}(t) - T_{ow}(t-1)) = \frac{T_{eq,air}(t) - T_{ow}(t)}{R_{Rest}} + \frac{T_{owi}(t) - T_{ow}(t)}{R_{1o}} \tag{1}$$

Energy balance inner side of external wall

$$\begin{aligned}\dot{Q}_{SolRadOWI}(t) + \dot{Q}_{LoadOWI}(t) - \min\{A_o; A_i\} \cdot \alpha_{Rad}(t) \cdot (T_{owi}(t) - T_{iwi}(t)) \\ + A_o \cdot \alpha_{owi} \cdot (T_{air}(t) - T_{owi}(t)) + \frac{T_{ow}(t) - T_{owi}(t)}{R_{1o}} + \dot{Q}_{iwHC}(t) = 0\end{aligned} \tag{2}$$

Capacity inner wall

$$\frac{C_{1i}}{\Delta t} \cdot (T_{iw}(t) - T_{iw}(t-1)) = \frac{T_{iwi}(t) - T_{iw}(t)}{R_{1o}} \tag{3}$$

Energy balance inner side of inner wall

$$\begin{aligned}\dot{Q}_{SolRadIWI}(t) + \dot{Q}_{LoadIWI}(t) + \min\{A_o; A_i\} \cdot \alpha_{Rad}(t) \cdot (T_{owi}(t) - T_{iwi}(t)) \\ + A_i \cdot \alpha_{iwi} \cdot (T_{air}(t) - T_{iwi}(t)) + \frac{T_{iw}(t) - T_{iwi}(t)}{R_{1i}} + \dot{Q}_{owHC}(t) = 0\end{aligned} \tag{4}$$

Energy balance air mass

$$\begin{aligned}\dot{Q}_{air}(t) = -A_o \cdot \alpha_{owi} \cdot (T_{air}(t) - T_{owi}(t)) - A_i \cdot \alpha_{iwi} \cdot (T_{air}(t) - T_{iwi}(t)) \\ + \dot{V}_{rate}(t) \cdot \rho_{air} \cdot c_{air} \cdot (T_{env}(t) - T_{air}(t)) \\ + \dot{Q}_{SolConv}(t) + \dot{Q}_{igConv}(t) + \dot{Q}_{airHC}(t)\end{aligned} \tag{5}$$

Capacity of air mass

$$\dot{Q}_{air}(t) = \frac{\rho_{air} \cdot c_{air} \cdot V_{air}}{\Delta t} \cdot (T_{air}(t) - T_{air}(t-1)) \tag{6}$$

Matrix structure

$$\vec{T}_{ow} = \begin{pmatrix} \frac{1}{R_{Rest}} + \frac{C_{1o}}{\Delta t} + \frac{1}{R_{1o}} \\ \frac{1}{R_{1o}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{7}$$

$$\vec{T}_{owi} = \begin{pmatrix} -\frac{1}{R_{1o}} \\ -\min\{A_o; A_i\} \cdot \alpha_{Rad}(t) - A_o \cdot \alpha_{owi} - \frac{1}{R_{1o}} \\ 0 \\ \min\{A_o; A_i\} \cdot \alpha_{Rad}(t) \\ 0 \\ 0 \end{pmatrix} \tag{8}$$

$$\vec{T}_{iw} = \begin{pmatrix} 0 \\ 0 \\ \frac{C_{1i}}{\Delta t} + \frac{1}{R_{1i}} \\ \frac{1}{R_{1i}} \\ 0 \\ 0 \end{pmatrix} \tag{9}$$

$$\vec{T}_{iwi} = \begin{pmatrix} 0 \\ \min\{A_o; A_i\} \cdot \alpha_{Rad}(t) \\ 0 \\ -\min\{A_o; A_i\} \cdot \alpha_{Rad}(t) - A_i \cdot \alpha_{iwi} - \frac{1}{R_{1i}} \\ A_i \cdot \alpha_{iwi} \\ 0 \end{pmatrix} \tag{10}$$

$$\vec{T}_{air} = \begin{pmatrix} 0 \\ A_o \cdot \alpha_{owi} \\ 0 \\ A_i \cdot \alpha_{iwi} \\ -A_o \cdot \alpha_{owi} - A_i \cdot \alpha_{iwi} - \dot{V}_{rate}(t) \cdot \rho_{air} \cdot c_{air} \\ \frac{\rho_{air} \cdot c_{air} \cdot V_{air}}{\Delta t} \end{pmatrix} \tag{11}$$

$$\vec{\dot{Q}}_{air} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ -1 \end{pmatrix} \tag{12}$$

$$\vec{\dot{Q}}_{airHC} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{13}$$

$$\vec{\dot{Q}}_{iwHC} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

$$\vec{\dot{Q}}_{owHC} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{15}$$

Right hand side:

$$\vec{b} = \begin{pmatrix} \frac{C_{1o}}{\Delta t} \cdot T_{ow}(t-1) + \frac{T_{eq,air}(t)}{R_{Rest}} \\ -\dot{Q}_{SolRadOWI}(t) - \dot{Q}_{LoadOWI}(t) \\ \frac{C_{1i}}{\Delta t} \cdot T_{iw}(t-1) \\ -\dot{Q}_{SolRadIWI}(t) - \dot{Q}_{LoadIWI}(t) \\ -\dot{V}_{rate}(t) \cdot \rho_{air} \cdot c_{air} \cdot T_{env}(t) - \dot{Q}_{SolConv}(t) - \dot{Q}_{igConv}(t) \\ \frac{\rho_{air} \cdot c_{air} \cdot V_{air}}{\Delta t} \cdot T_{air}(t-1) \end{pmatrix} \tag{16}$$

Resulting system:

$$\left(\vec{T}_{ow}, \vec{T}_{owi}, \vec{T}_{iw}, \vec{T}_{iwi}, \vec{T}_{air}, \vec{\dot{Q}}_{air}, \vec{\dot{Q}}_{airHC}, \vec{\dot{Q}}_{iwHC}, \vec{\dot{Q}}_{owHC}\right) \cdot \vec{x} = \vec{b} \tag{17}$$

Note: $\vec{x}$ has 9 entries, but only 6 equations are written down. In the case that a free-float temperature is computed, the following three equations are added:

$$\dot{Q}_{airHC} = 0 \tag{18}$$

$$\dot{Q}_{iwHC} = 0 \tag{19}$$

$$\dot{Q}_{owHC} = 0 \tag{20}$$

If the convective heat flow rate affecting the air node for a fixed set temperature as well as fixed heat flows affecting IW and OW is computed, the following three equations are used:

$$T_{air} = T_{set} \tag{21}$$

$$\dot{Q}_{iwHC} = \dot{Q}^{fix}_{iwHC} \tag{22}$$

$$\dot{Q}_{owHC} = \dot{Q}^{fix}_{owHC} \tag{23}$$