

Flows in Networks: Slow Example

Daniel Kane

Department of Computer Science and Engineering
University of California, San Diego

Advanced Algorithms and Complexity
Data Structures and Algorithms

Learning Objectives

- Understand why the Ford-Fulkerson algorithm is sometimes slow.
- Provide an example of a potential slow execution of this algorithm.

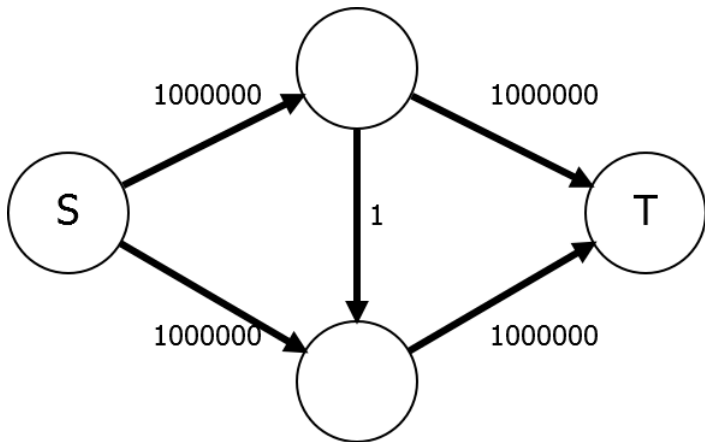
Last Time

Ford-Fulkerson Algorithm for Maxflow:

- Runtime $O(|E||f|)$.
- Potentially bad if $|f|$ is large.
- Does this problem ever actually happen?

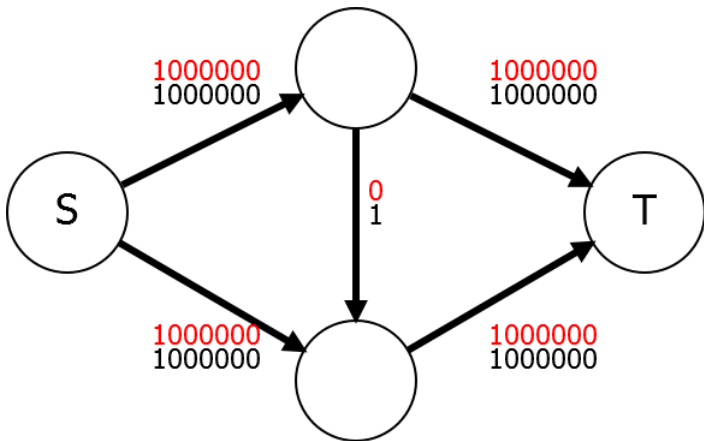
Example

Consider the following example:

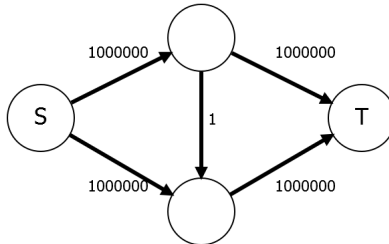
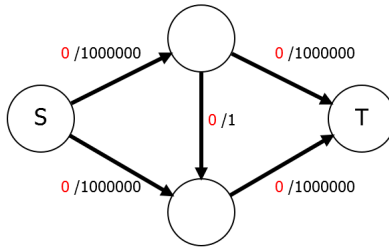


Flow

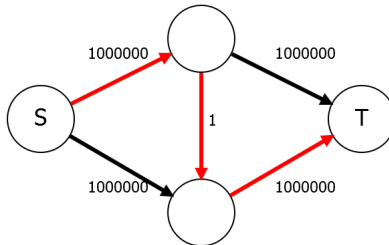
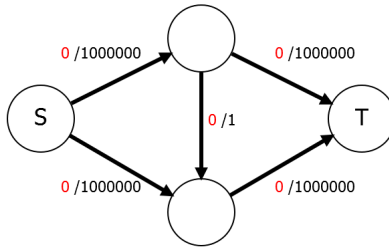
Has following maxflow ($|f| = 2000000$):



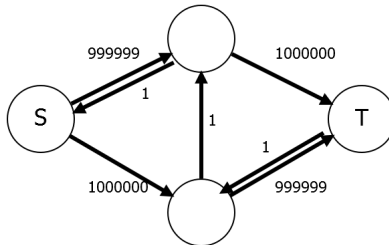
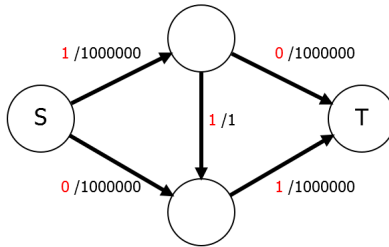
Possible Execution



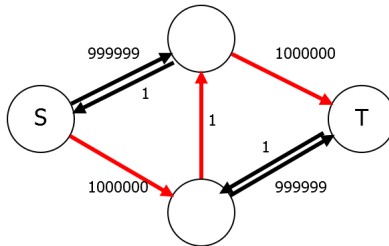
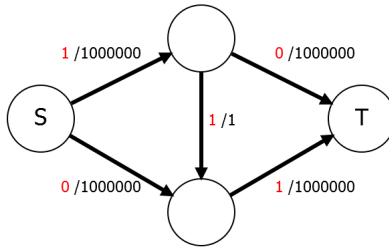
Possible Execution



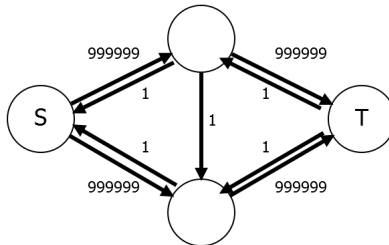
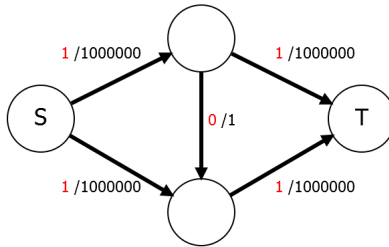
Possible Execution



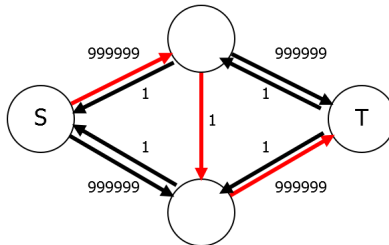
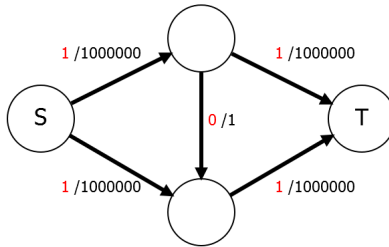
Possible Execution



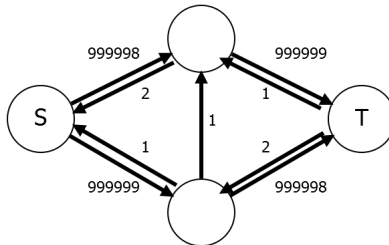
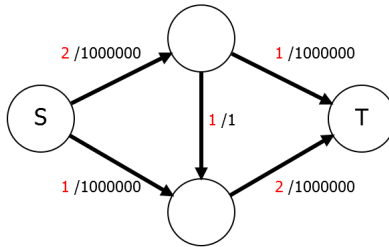
Possible Execution



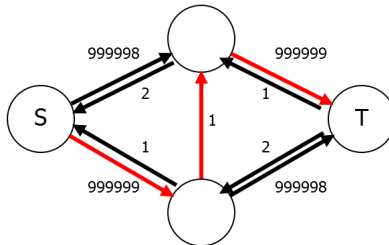
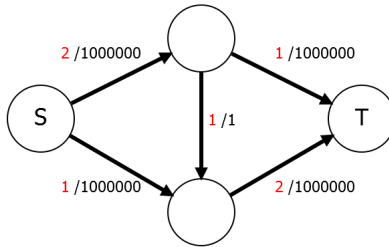
Possible Execution



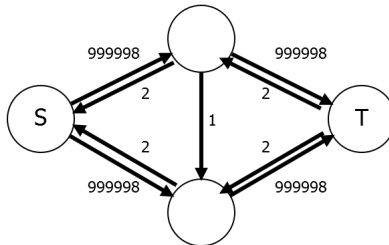
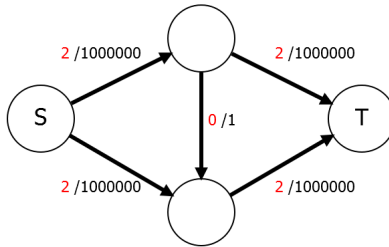
Possible Execution



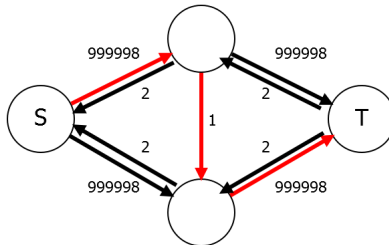
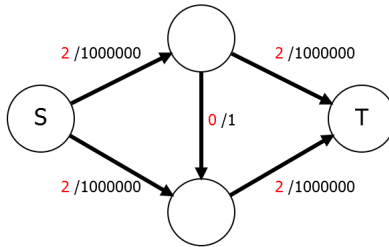
Possible Execution



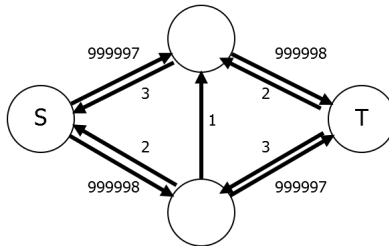
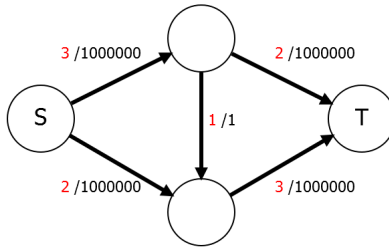
Possible Execution



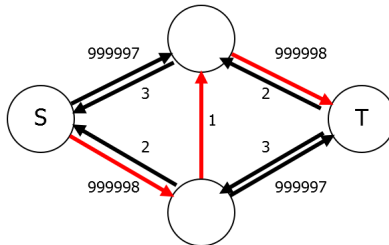
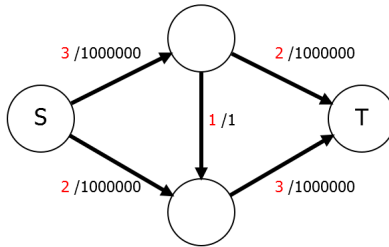
Possible Execution



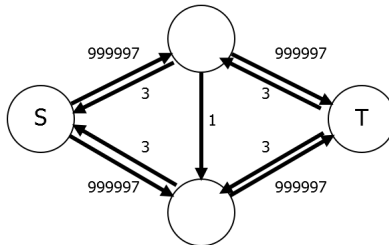
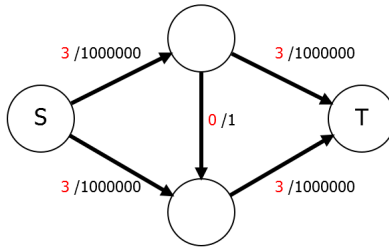
Possible Execution



Possible Execution



Possible Execution



Problem

How many iterations of Ford-Fulkerson will be required to compute a maximum flow, if it keeps choosing augmenting paths in this way?

Time

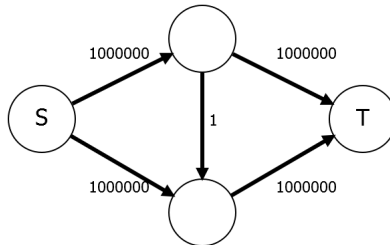
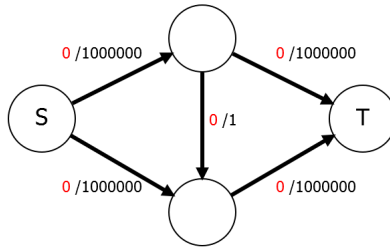
- Each step adds only one unit of flow (limited by middle edge).
- Need 2000000 total units.
- Ford-Fulkerson requires 2000000 iterations.

Time

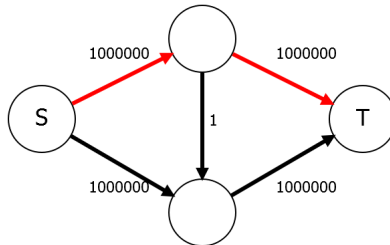
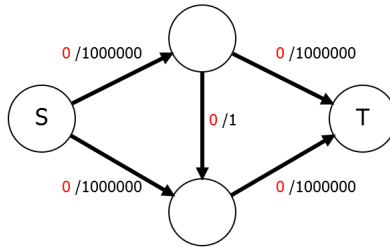
- Each step adds only one unit of flow (limited by middle edge).
- Need 2000000 total units.
- Ford-Fulkerson requires 2000000 iterations.

Doesn't need to be this bad.

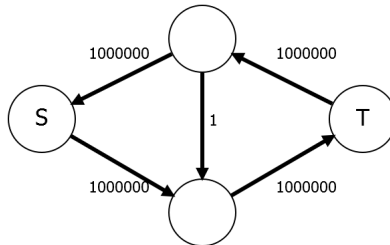
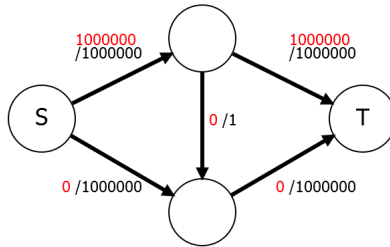
Another Possible Execution



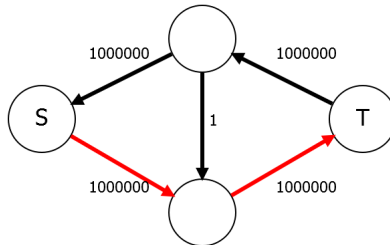
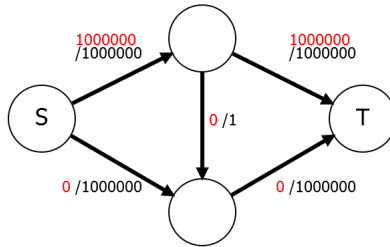
Another Possible Execution



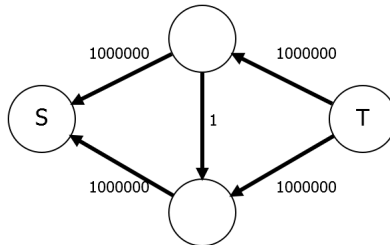
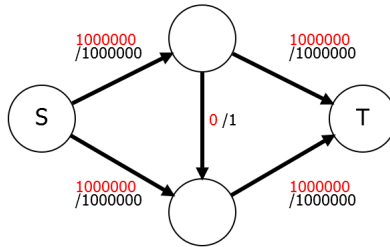
Another Possible Execution



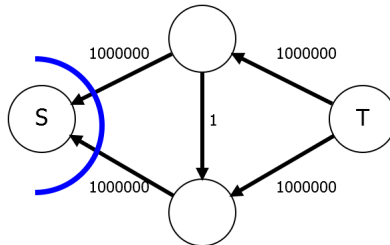
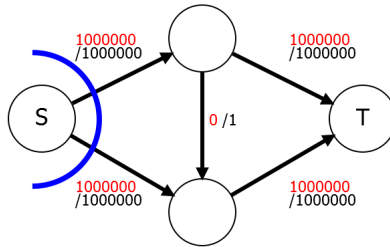
Another Possible Execution



Another Possible Execution



Another Possible Execution



Next Time

We'll find a way to ensure that we never have this type of problem.