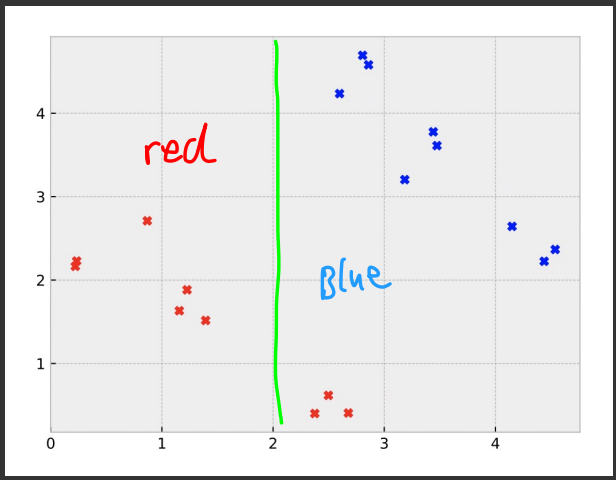
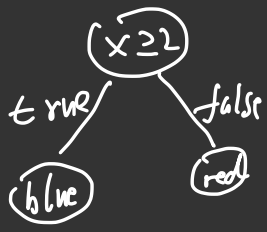
HW 3

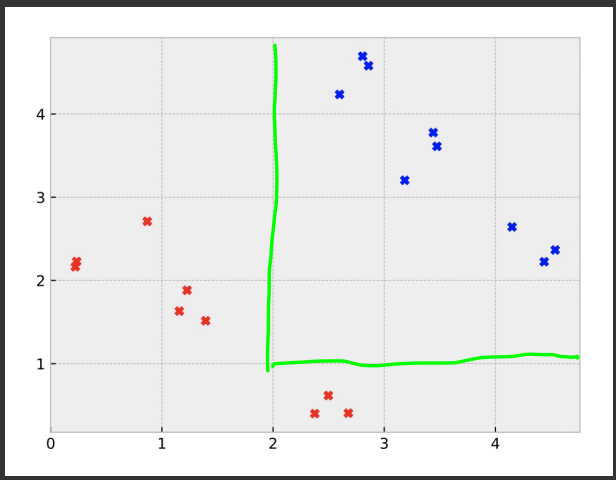
2. Decision trees

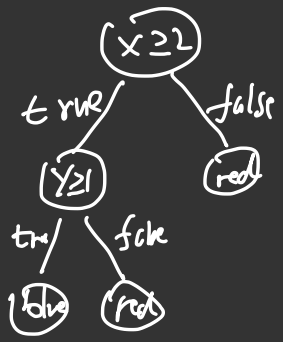
(a)

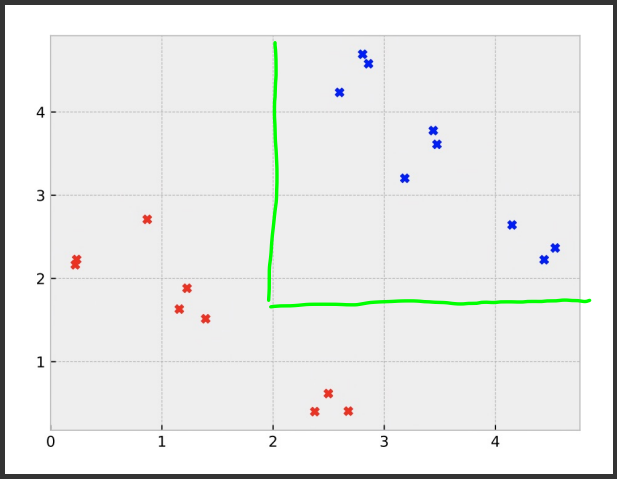


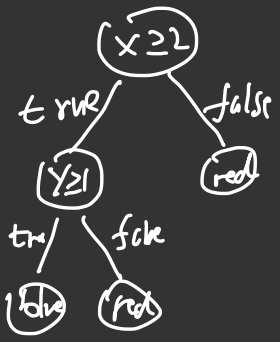


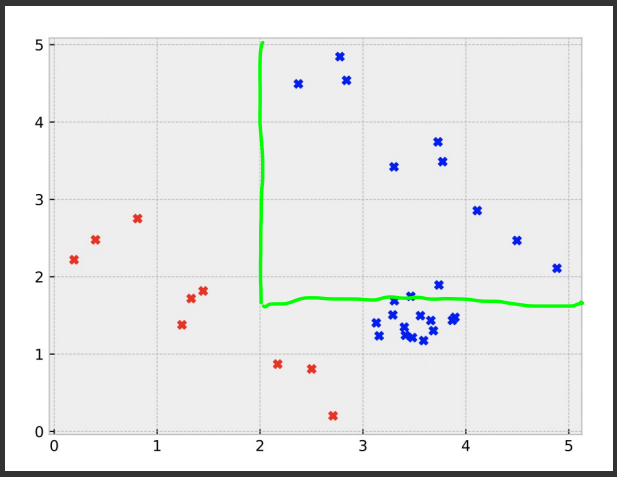
(b)

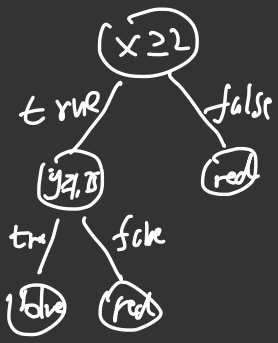




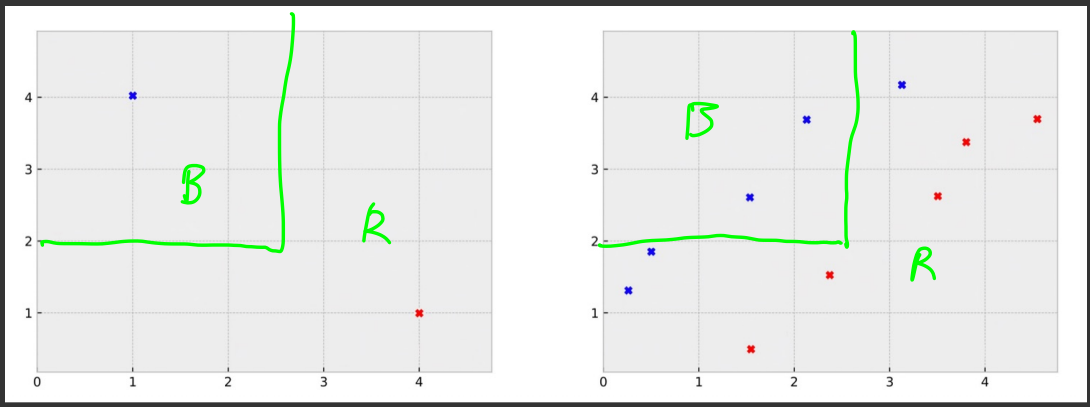

0
1
2
3
4
1
2
3
4


x≥2
true
false
y≥1
red
true
false
blue
red


0
1
2
3
4
5
1
2
3
4
5

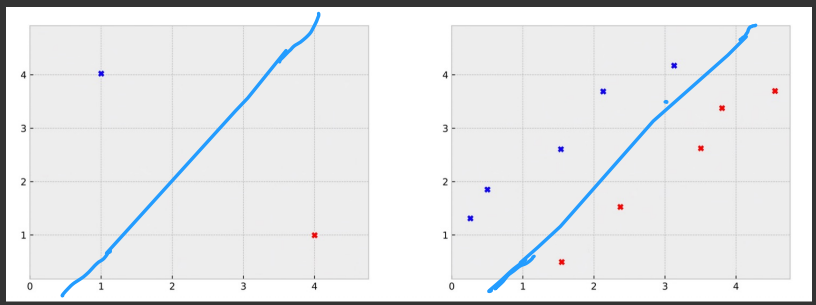

x≥2
true
false
y≥1,8
red
true
false
blue
red

(d)



test_err = 3/10 = 0.3

(e)



4. (a) $\Pr[Maj(x) \neq y]$ $\quad 1/3 \cdot \frac{6}{5}$

$\leq \Pr[\sum_{i=1}^{2n/3} z_i \geq \frac{n}{2}]$

$= \Pr[\sum_{i=1}^{2n/3} z_i \geq \frac{2n}{6}(1-p) - \frac{n}{3} + \frac{5}{6}np]$

$= \Pr[\sum_{i=1}^{2n/3} z_i \geq \frac{5n}{6}E[z_1] + \frac{5}{6}n(p-\frac{2}{5})]$

$-\frac{1}{2} \cdot \frac{3n}{5} \cdot \frac{1}{36} + \frac{1}{6} \leq \exp(-2 \cdot \frac{5}{6}n(p-\frac{2}{5})^2)$

$-2 \cdot \frac{5}{6} \cdot n(\frac{2}{5})^2 = -\frac{1}{2} \cdot \frac{5}{6} \cdot n \cdot \frac{4}{25}$

$\frac{5}{6} - \frac{2}{3}$

$= \exp(-4n/15) < \exp(-n/15)$

So The

(b) $\Pr[Maj(x) = y]$ $\quad W_i := I[f_i(x) = y]$

$\leq \Pr[\sum_{i=1}^{3n/5} W_i \geq \frac{n}{2}]$ $\quad EW_i = np$

$= \Pr[\sum_{i=1}^{3n/5} W_i \geq \frac{3n}{5}p - \frac{3np}{5} + \frac{n}{2}]$

$= \frac{3np}{5} + \frac{3n}{5}(\frac{5}{6} - p)$

$= \exp(-2\frac{3n}{5} \cdot (\frac{5}{6} - p)^2) = \exp(-\frac{1}{30})$