K-means

1. choose groups / cluster centers.
2. assign pixels to closet groups.
3. recursion until convergence

Algorithm

Initialize: Pick k random pts as clusters.

Iterate: ① Assign each data pt to closet cluster.

② update cluster to mean of assigned pts.

cost

$$\min_{r} \min_{v} \sum_{i \in D} \sum_{k=1}^{K} \frac{1}{2} r_{ik} \| x^{(i)} - v_k \|_2^2$$

s.t. $\begin{cases} r_{ik} \in \{0,1\} & \forall i, k \\ \sum_{k=1}^{k} r_{ik} = 1 & \forall i \end{cases}$

each data point should be assigned to one and only one cluster.

optimization

Not gradient descent:

① r_{ik} not continuous

② constrained

optimize for r given v

$$\forall i \quad r_{ik} = \begin{cases} 1 & k = \underset{k \in \{1,\dots,K\}}{\operatorname{argmin}} \|x^{(i)} - v_k\|_2^2 \\ 0 & \text{ow.} \end{cases}$$

optimize for v given r.

$$\nabla v_k: \sum_{i \in D} r_{ik}(x^{(i)} - v_k) = 0$$

$$v_k = \frac{\sum_{i \in D} r_{ik} x^{(i)}}{\sum_{i \in D} r_{ik}}$$

properties: local optimum is found

Guaranteed to converge in finite number of iterations