Electrotechnics
ET

Course 3
Year I-ISA English

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= Course 3 =

1. Energies and Forces in Electrostatic Field

2. Laws Specific to the Electrokinetic Field

1. Energies and Forces in Electrostatic Field

1.1 Electrostatic Energy of a Conductor System

Electrostatic Energy of a Conductor System

In order to establish an electric field in an area of space where it was initially zero it is necessary to transport the electrical charges from the exterior (from infinite) with which the bodies are charged.

The energy of the electrostatic field is equal with the total mechanical work performed for the transport of these charges.

In order to be able to define this way the energy some hypotheses must be made, namely:

- it is considered an isotropic, linear and without permanent polarization environment (medium);
- the operation of storing the charge on the conductors is done very slowly, in order to be able to consider the field as being electrostatic so that there are no irreversible transformations of the mechanical work done in the heat;
- it is considered that the conductor systems are immobile so that no mechanical work is lost for the deformation or displacement of the conductors.

➡ considering this hypothesis, the expression of the electrostatic field energy will be established depending on the charge and on the conductor potential which produces the field.

Suppose we have ***n* spherical conductors,** and we make the following **assumptions**:

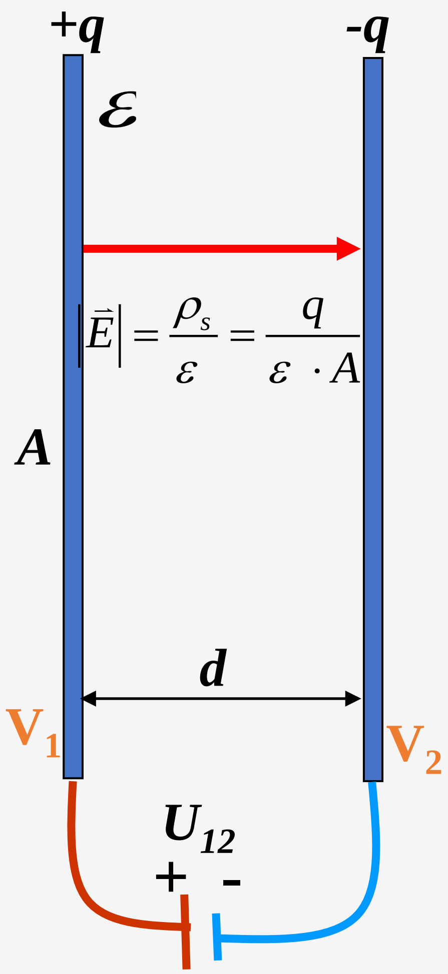
- **all conductors are in identical to zero initial state:**
$$q_i = 0,\quad V_i = 0 \qquad \forall \quad i = 1, 2, .., n$$
- **the final state of the conductors will be given by:**
  - **charges:** $q_1, q_2, \ldots\ldots q_i, \ldots. q_n$
  - **potentials:** $V_1, V_2, \ldots\ldots V_i, \ldots. V_n$
- **an intermediary state will be established proportionally, meaning that it is admitted the existence of the following relationships:**
  - $q_i' = \lambda \cdot q_i$
  - $V_i' = \lambda \cdot V_i$

$$\forall \quad i = 1, 2, .., n \qquad 0<\lambda<1$$

Electrostatic Field Energy:
$$\boxed{W_e = \frac{1}{2} \sum_{i=1}^{n} V_i \cdot q_i}$$

The above relation will give us the expression of the *energy stored in the electric field* of some conductors which have the charges q_i and potentials V_i.

<u>Application:</u>- The energy stored in the electric field of a capacitor:



- a capacitor having the potential difference U_{12} and the loads $+q$ and $-q$ *is considered*. The area of the plates is *A* and the distance between them is *d.*

❖ the energy stored in the capacitor:

$$W_e = \frac{1}{2}\sum_{i=1}^{n} V_i \cdot q_i = \frac{1}{2} V_1 \cdot q - \frac{1}{2} \cdot V_2 \cdot q = \frac{1}{2} U_{12} \cdot q$$

❖ if the charge expression is introduced:

$$C = \frac{q}{U_{12}} \Rightarrow q = C \cdot U_{12}$$

$$W_e = \frac{1}{2} U_{12} q = \frac{1}{2} C\, U_{12}^2 = \frac{1}{2} \frac{q^2}{C}$$

$$C = \frac{q}{U_{12}} \Rightarrow U_{12} = \frac{q}{C}$$

❖ we know that:

$$\left.\begin{aligned} C &= \frac{\varepsilon\, A}{d} \\ U_{12} &= E \cdot d \end{aligned}\right\} \Rightarrow W_e = \frac{1}{2} \cdot \left(\frac{\varepsilon \cdot A}{d}\right) \cdot (E \cdot d)^2 = \frac{1}{2} \cdot \varepsilon \cdot A \cdot d \cdot E^2 = \frac{1}{2} \cdot \varepsilon \cdot V \cdot E^2$$

!!! V is the volume between the plates and NOT the electric potential

1.2. Location of Energy in the Electric Field

Location of Energy in the Electric Field

$$w_e = \frac{W_e}{V} = \frac{1}{2}\cdot\varepsilon\cdot E^2 = \frac{1}{2}\cdot\varepsilon\cdot E\cdot E = \frac{1}{2}\cdot D\cdot E$$

This relation represent a calculation alternative in relation with the first defining relation of the *electrostatic field energy* (which expresses the energy in relation with the potentials and charges and doesn't specify weather it is located on the conductors or in the dielectric). It is therefore sought to express the energy in relation with $\vec{D}$ and $\vec{E}$.

w_e **is called volume density of the electrostatic energy.**

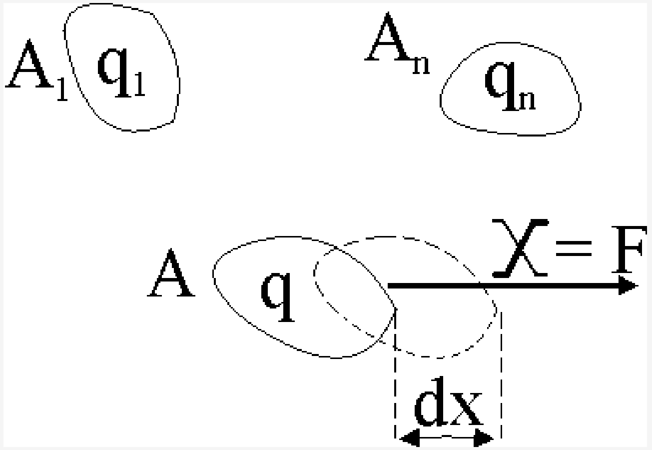
- **in general:** $w_e = \frac{1}{2}\cdot\bar{D}\cdot\bar{E}$
- **the total energy of the electric field is:** $$\boxed{W_e = \iiint_V w_e\, dV = \frac{1}{2}\iiint_V \bar{D}\cdot\bar{E}\, dV}$$

Conclusion: **the electric field energy is located in the dielectric (where there is electric field) and not in the conductive bodies (where the electric field is null).**

1.3. Electrostatic Forces

Electrostatic Forces

A system of *n* conductors charged with electric charges is considered.



The mechanical work performed for the variation with dq_k of the conductor charges must cover the increase of the electric field energy and the mechanical work performed by the external force *X*, which modifies the position of the body:

$$\sum_{k=1}^{n} V_k dq_k = dW_e + X dx$$

Two calculation situations are considered:

a) q_k = const.

- **This condition is fulfilled when the conductors are not connected from exterior sources!**

$$dq_k = 0 \quad \Rightarrow \quad \sum_{k=1}^{n} V_k dq_k = 0$$

$$\left(dW_e\right)_{q_k = ct} = -X\,dx$$

$$\Rightarrow \quad \boxed{X = -\frac{\left(dW_e\right)_{q_k = ct}}{dx} = -\left(\frac{\partial W_e}{\partial x}\right)_{q_k = ct}}$$

b) V_k = const

- **This condition is fulfilled if all conductors are connected at the terminals of some external sources.**

$$\sum_{k=1}^{n} V_k dq_k = (dW_e)_{V_k} + X dx \neq 0$$

$$(dW_e)_{V_k = ct} = \frac{1}{2}\sum_{k=1}^{n} V_k dq_k$$

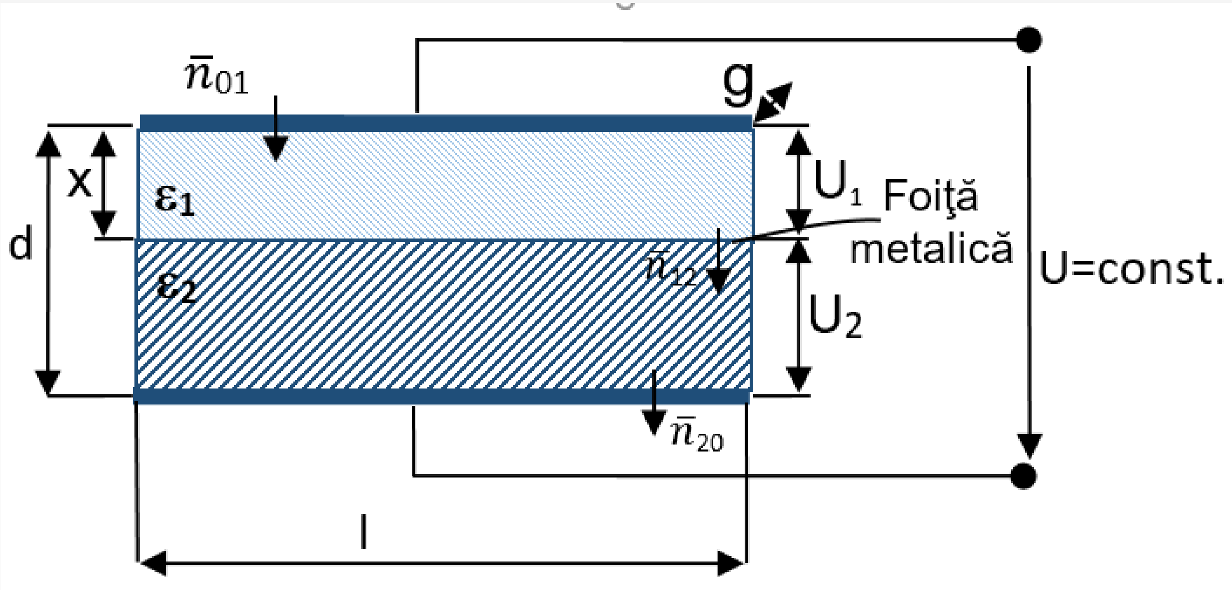
$$X\,dx = (dW_e)_{V_k = ct}$$

$$\boxed{X = \frac{(dW_e)_{V_k = ct}}{dx} = \left(\frac{\partial W_e}{\partial x}\right)_{V_k = ct}}$$

1.4. Applications

Problem 1

A plane capacitor with parallel plates, with the area A, filled with 2 dielectric mediums of permittivity ε_1 (dielectric block 1) and ε_2 (dielectric block 2) separated by a plane very thin metallic foil, as it is presented in the below figure is considered. Knowing that a voltage U is applied to the capacitor, it is required to determine:

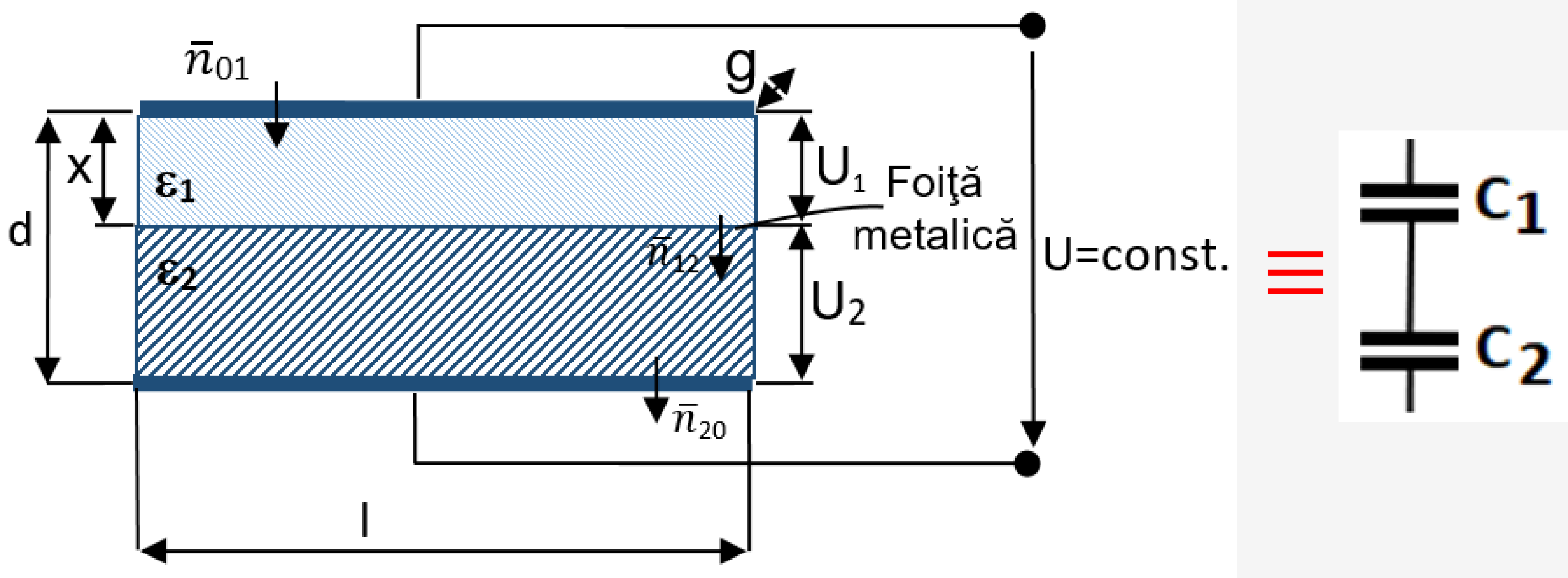


- equivalent capacitance of the system;
- the electrostatic energy;
- the force acting on the dielectric block 1.

Solution:

a) Equivalent capacitance of the system:

- ✓ if the separation surface between the two mediums is a very thin metallic foil and it is not charged with electric charge and the device acts like 2 capacitors with plan parallel plates connected in series, as it can be seen in the figure



- ✓ we know that the capacitance for a capacitor with plan parallel plates is (or if we don't know we can calculate – see the problem from *Course 2* from the *Capacitance calculation using the Direct method*)

$$\boldsymbol{C = \frac{\varepsilon \cdot A}{d}} \qquad (1)$$

- the equivalent capacitance of the 2 capacitors connected in series will be:

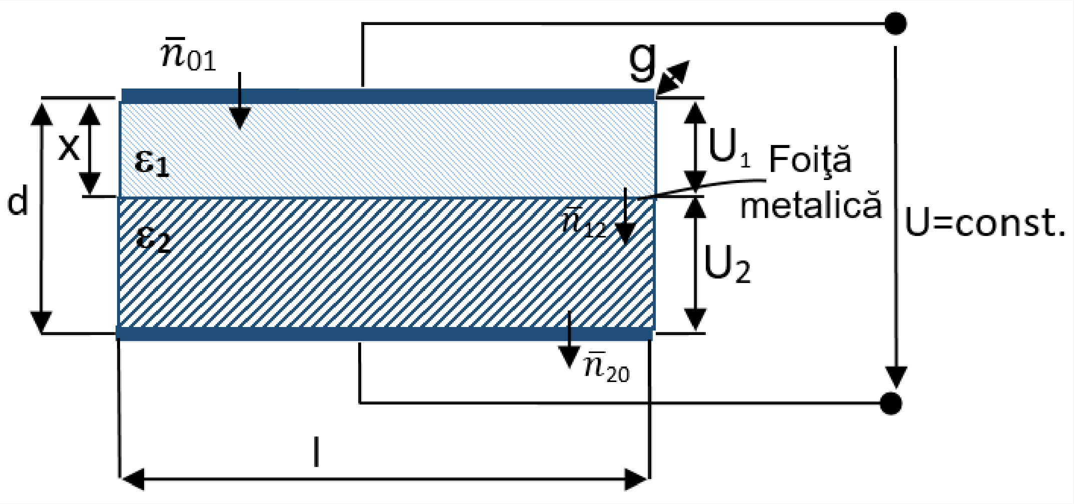
$$\boldsymbol{C_e = \frac{C_1 \cdot C_2}{C_1 + C_2}} \qquad (2)$$

- applying relation (1) for each of the capacitors taking into account the data of the problem we can determine the capacitance of each of the capacitor:
    - the capacitance of the first capacitor:

$$\boldsymbol{C_1 = \frac{\varepsilon_1 \cdot A}{x} = \frac{\varepsilon_1 \cdot l \cdot g}{x}} \qquad (3)$$

    - the capacitance of the second capacitor:

$$\boldsymbol{C_2 = \frac{\varepsilon_2 \cdot A}{d - x} = \frac{\varepsilon_2 \cdot l \cdot g}{d - x}} \qquad (4)$$



- ✓ we introduce the relations (3) and (4) in relation (2) ant the result is the equivalent capacitance of the entire system:

$$\boldsymbol{C_e = \frac{\frac{\varepsilon_1 \cdot A}{x} \cdot \frac{\varepsilon_2 \cdot A}{d-x}}{\frac{\varepsilon_1 \cdot A}{x} + \frac{\varepsilon_2 \cdot A}{d-x}} = \frac{\frac{\varepsilon_1 \cdot l \cdot g}{x} \cdot \frac{\varepsilon_2 \cdot l \cdot g}{d-x}}{\frac{\varepsilon_1 \cdot l \cdot g}{x} + \frac{\varepsilon_2 \cdot l \cdot g}{d-x}} = \frac{\frac{\varepsilon_1 \cdot \varepsilon_2 \cdot (l \cdot g)^2}{x \cdot (d-x)}}{\frac{l \cdot g}{x \cdot (d-x)} \cdot [\varepsilon_1 \cdot (d-x) + \varepsilon_2 \cdot x]}}$$

$$\Rightarrow \quad \boldsymbol{C_e = \frac{\varepsilon_1 \cdot \varepsilon_2 \cdot (l \cdot g)^2}{x \cdot (d-x)} \cdot \frac{x \cdot (d-x)}{l \cdot g \cdot [\varepsilon_1 \cdot (d-x) + \varepsilon_2 \cdot x]}}$$

$$\Rightarrow \boldsymbol{C_e = \frac{\varepsilon_1 \cdot \varepsilon_2 \cdot l \cdot g}{\varepsilon_1 \cdot (d-x) + \varepsilon_2 \cdot x}, [F]} \qquad \textbf{(5)}$$

b) The Electrostatic Energy:

- ✓ the electrostatic energy stored in the capacitor is determined with the formula:

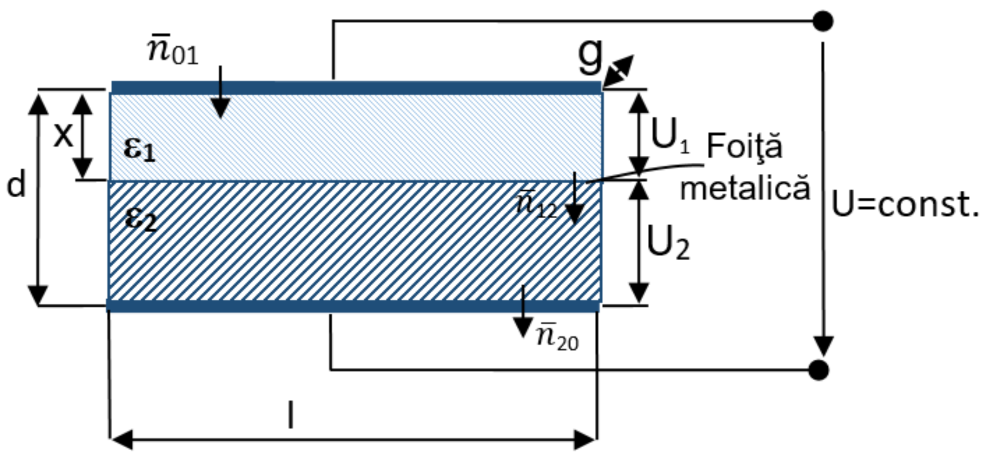
$$W_e = \frac{1}{2} \cdot C \cdot U^2$$

- ✓ taking into account the capacitance of the device we replace C with C_e expression given by relation (5), the electrostatic energy is obtained:

$$W_e = \frac{1}{2} \cdot U^2 \cdot \frac{\varepsilon_1 \cdot \varepsilon_2 \cdot l \cdot g}{\varepsilon_1 \cdot (d - x) + \varepsilon_2 \cdot x}, [J] \qquad (6)$$

- the measurement unit for the electric energy is *"Joule" – [J]*

c) The force that acts on the dielectric block 1



- ✓ in this application we observe that under the action of a force on the dielectric block 1 the dimension x of this block would modify;
- ✓ from the data of the problem, we know that to the capacitor a constant voltage U is applied;
- ✓ thus, taking into account these observations, the calculation relation used in this problem is:

$$\boldsymbol{F} = \left(\frac{\partial \boldsymbol{W_e}}{\partial \boldsymbol{x}}\right)_{\boldsymbol{U=ct}}$$

- ✓ because the energy was determined above (see relation 6), we can determine the force:

$$F = \frac{1}{2} \cdot U^2 \cdot \left(\frac{\partial C_e}{\partial x}\right)_{U=ct} = \frac{1}{2} \cdot U^2 \cdot \frac{\partial}{\partial x}\left(\frac{\varepsilon_1 \cdot \varepsilon_2 \cdot l \cdot g}{\varepsilon_1 \cdot (d-x) + \varepsilon_2 \cdot x}\right)$$

- from mathematics we know that:

$$\left(f^{-1}\right)' = (-1) \cdot \frac{f'}{f^2}$$

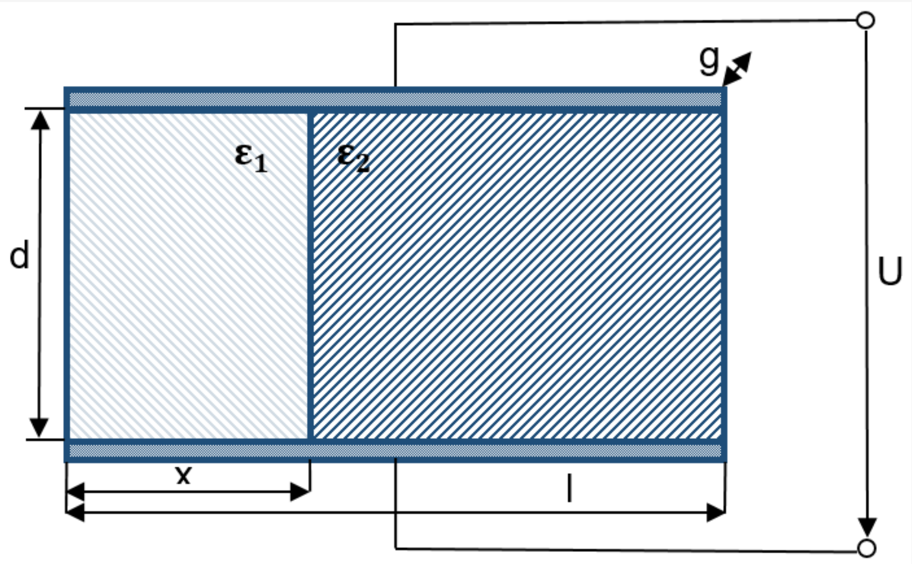
$$=> F = \frac{1}{2} \cdot U^2 \cdot \varepsilon_1 \cdot \varepsilon_2 \cdot l \cdot g \cdot \frac{\varepsilon_1 - \varepsilon_2}{[\varepsilon_1 \cdot (d-x) + \varepsilon_2 \cdot x]^2}, \quad [\mathrm{N}]$$

- the measurement unit for the force is *"Newton" – [N]*

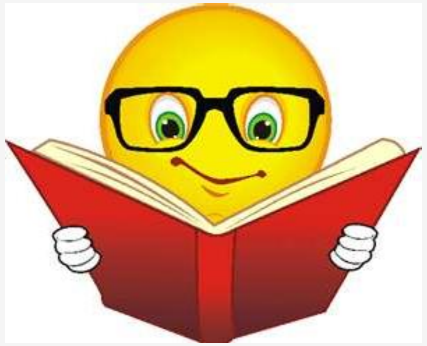
Problem 2

A plane capacitor with parallel plates, filled with 2 dielectric mediums of permittivity ε_1 (dielectric block 1) and ε_2 (dielectric block 2) separated by a plane very thin metallic foil, as it is presented in the below figure is considered. Knowing that a voltage U is applied to the capacitor, it is required to determine:

- equivalent capacitance of the system;
- the electrostatic energy;
- the force acting on the dielectric block 1.



Homework:


2. The Electrokinetic Field

Specific Laws

2.1. Electric Conduction Law (ECL)

1) Electrical Conduction Law (ECL)

❑ Local form of the law:

$$\boxed{\rho \bar{J} = \bar{E} + \bar{E}_i}$$

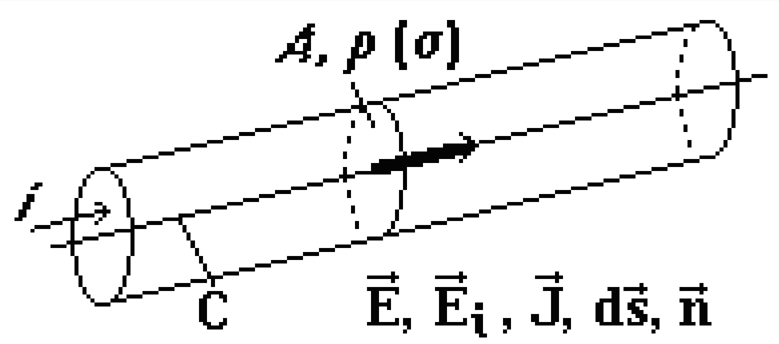
The product between the local resistivity ρ and the instantaneous local density of the conduction electric field is equal to the instantaneous local intensity of the electric field in a broad sense.

$$\boxed{\bar{J} = \sigma\left(\bar{E} + \bar{E}_i\right)}$$

The local and instantaneous density of the current passing through a body, $\vec{J}$, is equal to the product between the conductivity of the body and the local and instantaneous intensity of the electrical field in a broad sense, which is equal to the sum between the local and instantaneous intensity of the electric field $\vec{E}$ and electric imprimated field $\vec{E}_i$.

❑ Integral form of the law:

$$\int_C \rho \cdot \bar{J} \cdot d\bar{s} = \int_C \bar{E} \cdot d\bar{s} + \int_C \bar{E}_i \cdot d\bar{s}$$



Electric Current :

$$i = \int_A \bar{J} d\bar{A}, \ [A] \Rightarrow J = \frac{i}{A}, \left[\frac{A}{m^2}\right] \Rightarrow \int_C \rho \cdot \frac{i}{A} \cdot d\bar{s} = \int_C \bar{E} \cdot d\bar{s} + \int_C \bar{E}_i \cdot d\bar{s}$$

$$\left.\begin{array}{l} \rho \dfrac{i}{A} \displaystyle\int_C d\bar{s} = \int_C \bar{E} \cdot d\bar{s} + \int_C \bar{E}_i \cdot d\bar{s} \\ \displaystyle\int_C \bar{E} \cdot d\bar{s} = u \qquad \int_C \bar{E}_i \cdot d\bar{s} = e \end{array}\right\} \Rightarrow \rho \frac{l}{A} i = u + e$$

$$R = \rho \frac{l}{A} \Rightarrow \boxed{R \cdot i = u + e}$$ **= Ohm's Law or:** $$\boxed{i = G \cdot (u + e)}$$

2.2. Joule – Lenz law

(The Law of Energy Transformation in Conductors)

❑ Local Form of the Law:

$$\boxed{p = \bar{E}\,\bar{J}}$$

The volume density of the electromagnetic power ceded to bodies in the conduction process is equal to the scalar product between the electric field intensity and the density of the conduction electric current.

$$\xrightarrow{\text{ECL}} \quad \rho\,\bar{J} = \bar{E} + \bar{E}_i \;\rightarrow\; \bar{E} = \rho\,\bar{J} - \bar{E}_i$$

$$\Rightarrow \quad \boxed{p = \rho J^2 - \bar{E}_i\,\bar{J} = p_R - p_g}$$

where:

- p_R – volume density of the of the power dissipated through the Joule effect;
- p_g – volume density of the power generated under the influence of the imprinted fields.

❑ Integral (global) Form of the Law:

$$\iiint_V p\,dV = \iiint_V \bar{E}\,\bar{J}\,dV$$

$$\left.\begin{array}{l} \iiint_V p\,dV = \int_C \bar{E}\,d\bar{s} \iint_\Sigma \bar{J}\,d\bar{A} \\ \int_C \bar{E}\,d\bar{s} = u \qquad \iint_\Sigma \bar{J}\,d\bar{A} = i \end{array}\right\} \Rightarrow \boxed{P = u \cdot i}$$

$$\xrightarrow{\text{ECL}} R \cdot i = u + e \Rightarrow u = R \cdot i - e$$

$$\Rightarrow \boxed{P = R \cdot i^2 - e \cdot i}$$

$$\left.\begin{array}{l} P_R = R \cdot i^2 \\ P_G = e \cdot i \end{array}\right\} \Rightarrow$$

$$\Rightarrow P = P_R - P_G$$

where:
- o P – the power receive by the conductor in the conduction process;
- o P_R – the power dissipated through the Joule effect;
- o P_G – the power generated due to the imprinted electromotor voltage (voltage sources).

2.3. The Law of Electrolysis

The Law of Electrolysis

$$\boxed{m = k\,i\,t}$$

Expresses the connection between the mass, *m*, of an element or chemical radical that is deposited on one of the electrodes of the electrolysis bath and the current running through the bath.

where: $k = \dfrac{1}{F_0} \dfrac{A}{\upsilon}$

- $\dfrac{A}{\upsilon}$ – chemical equivalent;
- A – atomic or molecular mass [g/mol];
- υ – the valence of the deposited substance;
- F_0 – Faraday's constant,

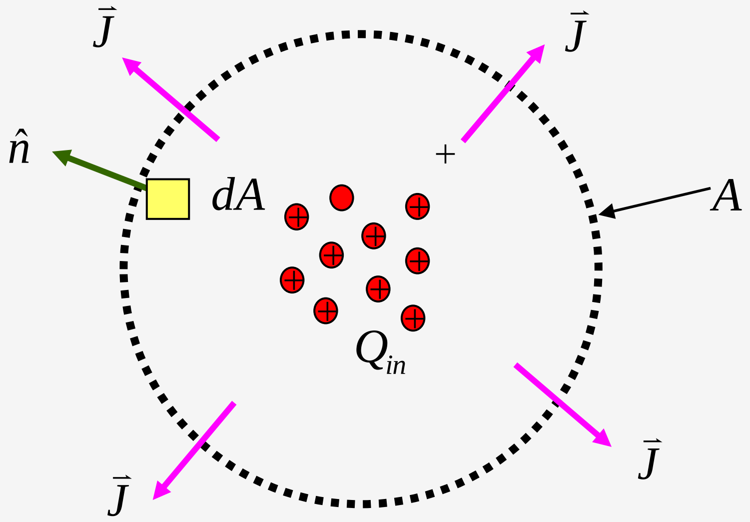
F_0 = 96 490 C/echivgram.

2.4. The Law of Electrical Charge Conservation

4) The Law of Electrical Charge Conservation

❑ Global Form of the Law:

In each moment, the conduction electric current intensity, i_{Σ}, exiting the closed surface Σ, is equal to the decrease in time speed of the electric charge, q_{Σ}, which charges the bodies from the interior of the surface Σ, regardless of their cinematic state.



$$\boxed{i_{\Sigma} = -\frac{dq_{\Sigma}}{dt}}$$

❑ Local Form of the Law:

- it is deduced from the global form:

$$i_{\Sigma} = \oiint_{\Sigma} \bar{J} \cdot d\bar{A} \qquad q_{\Sigma} = \iiint_V \rho_V dV$$

where: o V – a volume leaning on the surface Σ.

- the Gauss-Ostrogradski Transform is applied, and the result is:

$$\oiint_{\Sigma} \bar{J} \cdot d\bar{A} = \iiint_V div\bar{J}\, dV$$

- $$\frac{dq_{\Sigma}}{dt} = \frac{d}{dt}\iiint_V \rho_V dV = \iiint_V \frac{\partial \rho_V}{\partial t} dV$$

$$\Rightarrow \iiint_V div\,\bar{J}\, dV = -\iiint_V \frac{\partial \rho_V}{\partial t} dV$$

$\xrightarrow{LF}$ $\boxed{div\,\bar{J} = -\frac{\partial \rho_V}{\partial t}}$

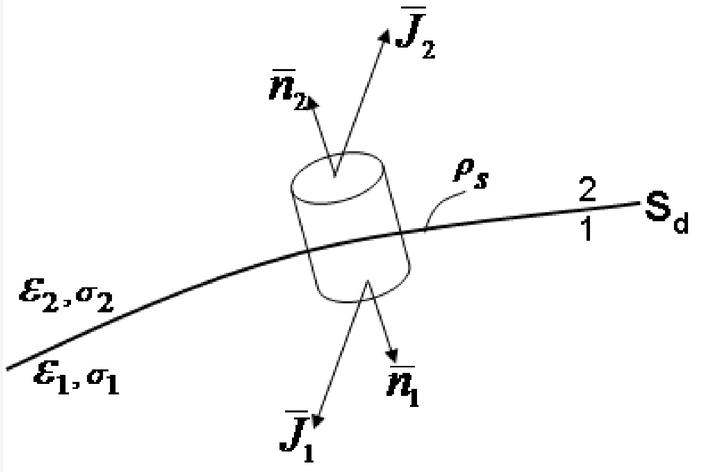
Continuity Law (Field Sources)

- **Case of the stationary electrokinetic regime (ρ_V constant relative to time):**

❑ **Local Form of the Law:** $\boxed{div\,\bar{J} = 0}$

❑ **Global Form of the Law :** $\boxed{\oiint_{\Sigma} \bar{J} \cdot d\bar{A} = 0}$

Consequence **Conservation of the normal component of the electric current density to the discontinuity surfaces:**



Considering ***S_d a sufficiently smooth discontinuity surface of the conduction electric current density*** on which an electric charge with the density ρ_S is distributed. If $\vec{J}_1$ and $\vec{J}_2$ are the electric current densities in the vicinity of S_d (immobile) the equation given by LF of the law written on the cylindrical surface with the height $\Delta s \to 0$ and the areas of the bases ΔA is written:

$$\bar{J}_1\bar{n}_1 + \bar{J}_2\bar{n}_2 = -\frac{\partial \rho_s}{\partial t} \quad \text{or:} \quad J_{2n} - J_{1n} = -\frac{\partial \rho_s}{\partial t}$$

- if $\rho_S = 0$ → $J_{1n} = J_{2n}$
- if S_d is separating a conductor from a dielectric → $J_n = 0$ → J tangential to the conductor's surface
- on the separation surface of 2 conductors → $\sigma_1 E_{1n} = \sigma_2 E_{2n}$

The analogy between the electrostatic regime and the stationary electro-kinetic regime

$$U_{AB} = \int_A^B \bar{E} \cdot d\bar{s}$$

$$u_{AB} = \int_A^B \bar{E} \cdot d\bar{s}$$

$$div\bar{D} = \rho_v$$

$$\oiint_{\Sigma} \bar{D} \cdot d\bar{A} = q_{\Sigma}$$

$$\oiint_{\Sigma} \bar{J} \cdot d\bar{A} = i$$

$$div\bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$$div_S \bar{D} = \rho_s$$

$$div_S \bar{J} = -\frac{\partial \rho_s}{\partial t}$$

$$\bar{D} = \varepsilon \cdot \bar{E}$$

$$\bar{J} = \sigma \cdot \bar{E}$$

$$\bar{D} = \varepsilon \cdot \bar{E} + \bar{P}_p$$

$$\bar{J} = \sigma \cdot \bar{E} + \sigma \cdot \bar{E}_i$$

$$C = \frac{q}{U}$$

$$G = \frac{i}{u}$$

The analogy between the quantities from electrostatic regime and from stationary electro-kinetic regime

$$U_{AB} \Leftrightarrow u_{AB}$$

$$\bar{E} \Leftrightarrow \bar{E}$$

$$\bar{D} \Leftrightarrow \bar{J}$$

$$q \Leftrightarrow i$$

$$\varepsilon \Leftrightarrow \sigma$$

$$\bar{P}_p \Leftrightarrow \sigma \cdot \bar{E}_i$$

$$C \Leftrightarrow G$$

Thank you for your attention!!!

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Questions???