~ Experimental Study of Transverse and Longitudinal Standing Waves ~

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Summary:

→ the aim of the laboratory is to observe the transverse and longitudinal standing waves, calculate the frequency of fundamental and superior harmonic, while comparing the theoretical values with the experim ones.

→ the waves combine according to the general law of wave interference

→ this study has a lot of applications in music, science.

→ there are certain frequencies for which the interference results in a stationary vibration pattern called standing waves.

Transverse waves

→ transmisible only through solid media

→ the differential equation describing the propagation

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{\mu}{F} \cdot \frac{\partial \Psi}{\partial t^2} = \frac{1}{v} \cdot \frac{\partial \Psi}{\partial t^2} \quad (1)$$

→ $\Psi(x,t)$ = the transverse deformation of the string

→ F, the force which tensions the string

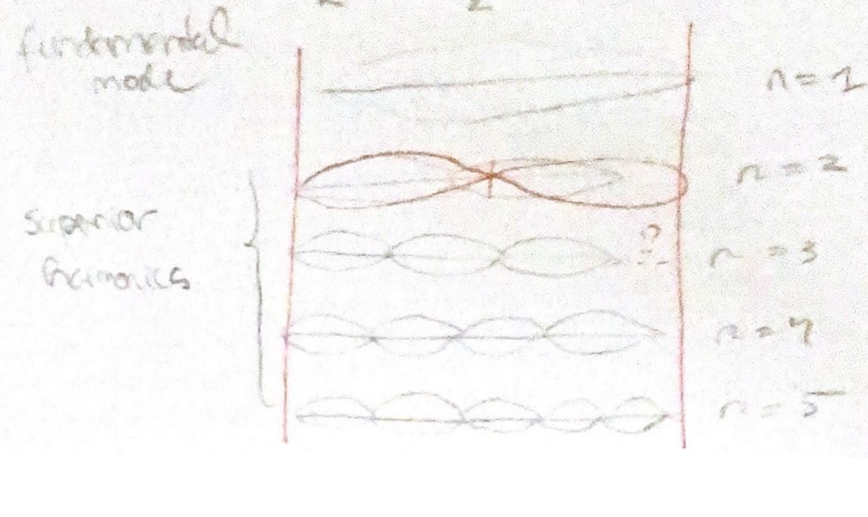
→ μ, the linear density; → v = the speed

→ (1) => $\Psi(x,t) = \Psi_p + \Psi_r = A \cdot \sin(\omega t + x) + A \sin(\omega t + kx + \pi)$ (2)

and: $\Psi(x,t) = 2 \cdot A \sin(kx) \cos(\omega t) = A(x) \cdot \cos(\omega t)$ (2)

→ the amplitude $A(x) = 2A\sin(kx)$ is const in time for a fixed position x along the string and for certain frequencies of mechanical oscillations

→ a minimum value $A(x) = 0$ corresponds to the nodes position for $kx = n\pi$, $x = n \cdot \frac{\pi}{k} = n \cdot \frac{\lambda}{2}$ with $n = (1, 2, \dots)$



→ $A_{max} = 2A$, for $kx = (2n+1)\frac{\pi}{2}$, or $x = (2n+1)\frac{\lambda}{4}$

→ the resonance frequencies ν_n are related to the wave velocity in the string V and to the lenght of the string L

$$L = n \cdot \frac{\lambda_n}{2} \quad (4) \quad \text{and} \quad \nu_n = \frac{v}{\lambda_n}, \; \lambda = \frac{v}{\nu_n} \quad (5)$$

→ knowing $v = \sqrt{F/\mu}$ and $\nu_n = nv/2L$ ⇒

→ the fundamental frequency: $\nu_1 = \frac{v}{2L} = \frac{1}{2L} \cdot \sqrt{\frac{T}{\mu}}$ (6)

where $T \equiv F$ is the tension in the string

Longitudinal waves

→ transmisible in all media; sound is a long mechanical wave that can be transmited in air

→ the velocity of long waves $v = \sqrt{\frac{E}{\rho}}$ E Young's modulus of elasticity, ρ – the mass density

→ equation of the progressive wave $y_p = A\sin\left[\omega\left(t - \frac{x}{v}\right)\right]$

→ the resulte wave $\Psi = y_p + y_r = 2A \cdot \cos\left(\frac{\pi}{2} - \frac{\omega}{v}x\right)\sin\left(\omega t - \frac{\pi}{2}\right)$

$$= 2A\sin\left(\frac{\omega}{v} \cdot x\right)\cos(\omega t) \quad (8)$$

→ amp of standing waves is min if: $2A\sin\left(\frac{\omega}{v} \cdot x\right) = 0 \Leftrightarrow x_{min} = n \cdot \frac{\lambda}{2}$

→ amp of standing waves is max if: $\sin\left(\frac{\omega}{v} \cdot x\right) = \pm 1 \Leftrightarrow x_{max} = (2n+1)\frac{\lambda}{2}$

→ the stretched spring represents a quasi-continious medium of propagation and the Newton's formula for the speed of longitudinal waves in elastic media becomes

$V = \sqrt{\langle E\rangle / \langle \rho\rangle}$, $\langle E\rangle$: Young's module of elasticity for the stretched spring

→ $\langle \rho \rangle$ – the density of the spring viewed as quasi-continous medium of propagation

$$\rightarrow \langle E\rangle = \frac{F}{S} \cdot \frac{L}{\Delta L} = \frac{K \cdot \Delta L}{S} \cdot \frac{L}{\Delta L} = \frac{K \cdot L}{S} \quad (9)$$

$$\rightarrow \langle \rho \rangle = \frac{m}{V} = \frac{m}{S \cdot L} \quad (10)$$

$$\rightarrow \text{then: } \nu_n = \frac{n}{2L} \cdot \sqrt{\frac{\langle E\rangle}{\langle \rho\rangle}} = \frac{n}{2 \cdot L}\sqrt{\frac{K}{m}} \quad (11)$$