$$B = -3$$

$$A = 4$$

$$X(p) = \frac{4}{p-2} - \frac{3}{p-3} \Bigg| \mathcal{L}^{-1}$$

$$x(t) = \mathcal{L}^{-1}\left[\frac{4}{p-2}\right] - \mathcal{L}^{-1}\left(\frac{3}{p-3}\right)$$

$$x(t) = 4e^{2t} - 3e^{3t}$$

② $x''(t) + x(t) = 2\cos t \; \Big| \mathcal{L}, \; x(0) = 0, \; x'(0) = -1$

$$\mathcal{L}[x''(t)](p) + \mathcal{L}[x(t)] = 2\mathcal{L}[\cos t](p)$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$p^2 X(p) - p\underbrace{X(0)}_{1} - \underbrace{X'(0)}_{-1} + X(p) = 2\mathcal{L}[\cos t](p)$$

$$p^2 X(p) - p + 1 + X(p) = 2\mathcal{L}[\cos t](p)$$

$$X(p)(p^2 + 1) = \frac{2p}{p^2+1} - 1$$

$$X(p) = \frac{2p}{(p^2+1)^2} - \frac{1}{p^2+1} \quad \Big| \mathcal{L}^{-1}$$

$$x(t) = \mathcal{L}^{-1}\left[\frac{2p}{(p^2+1)^2}\right] - \mathcal{L}^{-1}\left[\frac{1}{p^2+1}\right]$$

$$x(t) = t\sin t - \sin t$$

$$\left(\frac{1}{p^2+1}\right)' = \frac{1'\cdot(p^2+1) - (p^2+1)'\cdot 1}{(p^2+1)^2} = \frac{-2p}{(p^2+1)^2}$$

$t\sin t$

2.10)

5) $x'''(t) + x'(t) = 1 \quad \big| \mathcal{L}$

$$\mathcal{L}[x'''(t)](p) + \mathcal{L}[x'(t)](p) = \frac{1}{p}$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$p^3 X(p) - \underbrace{p^2 x(0) - p x'(0) - x''(0)}_{0} + pX(p) - \underbrace{x'(0)}_{0}$$

$$X(p)(p^3+p) = \frac{1}{p}$$

$$X(p) = \frac{1}{p^4+p^2} = \frac{1}{p^2(p^2+1)}$$

$$x(t) = \mathcal{L}^{-1}\left[\frac{1}{p^2(p^2+1)}\right] = \frac{p^2+1-p^2}{(p^2+1)p^2}$$

$$= \frac{1}{p^2} - \frac{1}{p^2+1}$$

$$= t - \sin t$$

a) $x''(t) + 4x(t) = \frac{1}{2}(\cos t - \cos 2t)$

$$\mathcal{L}[x''(t)](p) + 4\mathcal{L}[x(t)](p) = \frac{1}{2}\left[\mathcal{L}[\cos t](p) - \mathcal{L}[\cos 2t](p)\right]$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$p^2 X(p) - p\underbrace{x(0)}_{1} - \underbrace{x'(0)}_{0} + 4X(p) = \dots$$

$$X(p)(p^2+4) = \frac{1}{2}\left(\frac{p}{p^2+1}\cdot\frac{p^2+4}{} - \frac{p}{p^2+4}\cdot\frac{p^2+1}{}\right) + p$$

$$X(p)(p^2+4) = \frac{p(p^2+4)-p(p^2+1)}{2(p^2+1)(p^2+4)} + p$$

$$X(p) = \frac{3p}{2(p^2+1)(p^2+4)^2} + \frac{p}{p^2+4}$$

$$x(t) = \mathcal{L}^{-1}\left[\frac{3p}{2(p^2+1)(p^2+4)^2}\right] + \cos 2t$$

ii) $x'''(t) - 3x''(t) + 3x'(t) - x(t) = t^2e^t$

$$\mathcal{L}[x'''(t)](p) - 3\mathcal{L}[x''(t)](p) + 3\mathcal{L}[x'(t)](p) - \mathcal{L}[x(t)](p)$$

iii) $x'(t) + 2x(t) = \sin t \quad , x(0) = 0$

$$\mathcal{L}[x'(t)](p) + 2\mathcal{L}[x(t)](p) = \mathcal{L}[\sin t](p)$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$pX(p) + x(0) + 2X(p) = \frac{1}{p^2+1}$$

$$X(p)(p+2) = \frac{1}{p^2+1}$$

$$X(t) = \frac{1}{(p^2+1)(p+2)}$$

$$x(t) = \mathcal{L}^{-1}\left[\frac{1}{(p^2+1)(p+2)}\right]$$

$$(Ap+B)(p+2) + C(p^2+1)$$

$$Ap^2 + 2Ap + Bp + 2B + Cp^2 + C$$

$$p^2(A+C) + p(2A+B) + 2B + C$$

$$\frac{1}{(p^2+1)(p+2)} = \frac{Ap+B}{p^2+1} + \frac{C}{p+2}$$

$$1 = (p+2)(Ap+B) + C(p^2+1)$$

$$1 = pAp + 2B + 2Ap + pB + p^2C + C$$

$$1 = p^2\underbrace{(A+C)}_{0} + p\underbrace{(2A+B)}_{0} + 2B + C$$

$$2B + C = 1$$

$$2B = 1 - C$$

$$B = \frac{1-C}{2}$$

$$2A + B = 0$$

$$A + C = 0$$

$$A = -C$$

$$\frac{-\frac{1}{5}p + \frac{2}{5}}{p^2+1} + \frac{\frac{1}{5}}{p+2}$$

$$= \frac{-\frac{1}{5}(p-2)}{p^2+1} + \frac{\frac{1}{5}}{(p+2)}$$

$$-\overset{2}{\cancel{2}}C + \frac{1-C}{2} = 0$$

$$-4C + 1 - C = 0$$

$$1 - 5C = 0$$

$$C = \frac{1}{5}$$

$$A = -\frac{1}{5}$$

$$\frac{1}{5}\left[\frac{-p+2}{p^2+1} + \frac{1}{p+2}\right]$$

$$\frac{1}{5}\left[\frac{-p}{p^2+1} + \frac{2}{p^2+1} + \frac{1}{p+2}\right]$$

$$\frac{1}{5}\left\{-\cos t + 2\sin t + e^{-2t}\right\}$$

$$B = \frac{\overset{5}{\cancel{1}} - \frac{1}{5}}{2}$$

$$B = \frac{\frac{4}{5}}{2} = \frac{2}{5}$$

$$\begin{cases} x'(t) = 3x(t) - y(t) \\ y'(t) = -9x(t) + 3y(t) \end{cases} \Bigg| \mathcal{L} \qquad \begin{matrix} x(0) = 1 \\ y(0) = 0 \end{matrix}$$

$$\begin{cases} \mathcal{L}[x'(t)] = 3\mathcal{L}[x(t)] - \mathcal{L}[y(t)] \\ \mathcal{L}[y'(t)] = -9\mathcal{L}[x(t)] + 3\mathcal{L}[y(t)] \end{cases}$$

$$X(p) = \mathcal{L}[x(t)](p)$$

$$Y(p) = \mathcal{L}[y(t)](p)$$

$$\begin{cases} pX(p) - x(0) = 3X(p) - Y(p) \\ pY(p) - y(0) = -9X(p) - 3Y(p) \end{cases}$$

$$\begin{cases} X(p)(p-3) + Y(p) = 1 \\ Y(p)(p+3) + 9X(p) = 0 \end{cases}$$

$$\begin{cases} X(p)(p-3) + Y(p) = 1 \\ X(p)\,9 + Y(p)(p+3) = 0 \end{cases} \qquad \begin{aligned} X(p) &= X \\ Y(p) &= Y \end{aligned}$$

$$\begin{cases} X(p-3) + Y = 1 \quad / \cdot (-(p+3)) \\ X\,9 + \underline{Y(p+3)} = 0 \end{cases}$$

$$\begin{cases} -X(p-3)^2 - Y(p+3) = -(p+3) \\ X\,9 + Y(p+3) = 0 \end{cases}$$

$$X\left(9-(p-3)^2\right) = -(p+3)$$

$$X(p)\left(9-(p-3)^2\right) = -(p-3)$$

$\mathcal{L}^{-1}$

Integral equation

The convultion product

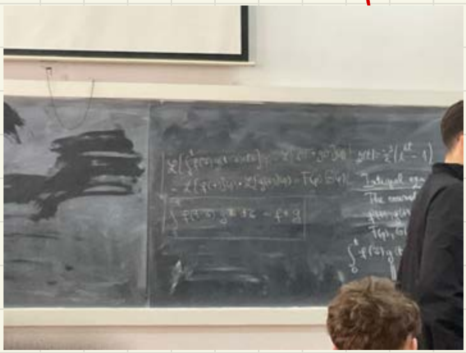
$f(t)$, $g(t)$ originals

$F(p)$, $G(p)$ images

$\int_0^t f(\tau)\, g(t-\tau)\, d\tau$ is the convolution product

$\Vert$

not $f * g$



$$\mathcal{L}\left[\int_0^t f(\tau)\, g(t-\tau)\, d\tau\right](p) = \mathcal{L}[f(t) * g(t)]$$

$$= \mathcal{L}[f(t)](p) \cdot \mathcal{L}[g(t)](p) = F(p)\, G(p)$$

$f(\tau)$ $\cdot g(t-\tau$

① $\int_0^t \tau^3 \cos(t-\tau)\, d\tau =$

$$\mathcal{L}\left[\int_0^t (\underbrace{\tau^3}_{f} * \underbrace{\cos(t-\tau)}_{g})\, d\tau\right] =$$

$$\mathcal{L}[t^3](p)\, \mathcal{L}[\cos t](p)$$

$$\frac{3!}{p^4}\cdot\frac{p}{p^2+1}=\frac{3!\,p}{p^4(p^2+1)}$$

② $\int_0^t e^{2(t-\tau)}\cdot\tau^3\,d\tau=$

$$=\mathcal{L}\left[\int_0^t \left(e^{2(t-\tau)}\cdot\tau^3\right)d\tau\right](p)$$

$e^{g(t)}$ $f(t)$

$$=\mathcal{L}\left[e^{2t} * t^3\right](p)$$

$$=\mathcal{L}\left[e^{2t}\right](p)\,\mathcal{L}\left[t^3\right](p)$$

$\vdots$

③ $\int_0^t \tau^3\,d\tau$

$$\mathcal{L}\left[\int_0^t (t^3+1)\right](p)=$$

(4) $y'(t) + \int_0^t u \cdot y(t-u)\,du = t$

$\mathcal{L}[y'(t)](p) + \mathcal{L}\left[\int_0^t t * y(t)\,du\right](p) = \mathcal{L}[t](p)$

$Y(p) = \mathcal{L}[y(t)](p)$

$p\,Y(p) - y(0) + \mathcal{L}[t](p) \cdot Y(p) = \frac{1}{p^2}$

$p\,Y(p) + 1 + \frac{1}{p^2} \cdot Y(p) = \frac{1}{p^2}$

$Y(p)\left(p + \frac{1}{p^2}\right) + 1 = \frac{1}{p^2}$

$\vdots$

(5) $\int_0^t \sin(t-\tau) * \overset{x(\tau)}{\tau}\,d\tau = \sin^2 t \quad \Big| \, \mathcal{L}$

$\mathcal{L}\left[\int_0^t \sin t * t\,dt\right](p) = \sin^2 t$

$$\mathcal{L}[\sin t](p)\,\mathcal{L}[t](p) = \mathcal{L}[\sin^2 t](p)$$

$$\frac{1}{p^2+1}\cdot\frac{1}{p^2} = \mathcal{L}\left[\frac{1-\cos 2t}{2}\right](p)$$

$$\frac{1}{p^2(p^2+1)} = \frac{1}{2}\mathcal{L}[1](p) - \frac{1}{2}\mathcal{L}[\cos 2t](p)$$

$$\frac{1}{p^2(p^2+1)} = \frac{1}{2p} - \frac{1}{2}\,\frac{p}{p^2+4}$$

$$\frac{1}{p^2(p^2+1)} = \frac{1}{2p} - \frac{p}{2(p^2+4)}$$

⑥ $y(t) - 2\int_0^t y(t-u)\sin u\,du = \cos t$

$$\mathcal{L}[y(t)](p) - 2\mathcal{L}[y(t)*\sin t](p) = \mathcal{L}[\cos t](p)$$

$$Y(p) - 2Y(p)\cdot 2\mathcal{L}[\sin t](p) = \mathcal{L}[\cos t](p)$$

$$Y(p)\left(1 - \frac{2}{p^2+1}\right) = \frac{p}{p^2+1}$$

⑦ $\cancel{\int_0^t \sin(t-\tau)\,x(\tau)\,d\tau =}$

9) $x'(t) + 2x(t) + \int_0^t x(\tau)\,d\tau = \sin t$

$$pX(p) - x(0) + 2X(p) + \mathcal{L}[1 * x(t)](p) = \frac{1}{p^2+1}$$

$$X(p)(p+2) + \frac{1}{p}\cdot X(p) = \frac{1}{p^2+1}$$

$$X(p)\left(p+2+\frac{1}{p}\right) = \frac{1}{p^2+1}$$

2) $x''(t) - 5x'(t) + 6x(t) = 0 \quad \Big|\, \mathcal{L}$

$$\mathcal{L}[x''(t)](p) - 5\mathcal{L}[x'(t)](p) + 6\mathcal{L}[x(t)](p) = 0$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$p^2 X(p) - p\overset{1}{x(0)} - \overset{-1}{x'(0)} - 5\left(pX(p) - \overset{1}{x(0)}\right)$$

$$+ 6X(p) = 0$$

$$p^2 X(p) - p + 1 - 5pX(p) + 5 + 6X(p) = 0$$

$$X(p)\left(p^2 - 5p + 6\right) - p + 1 + 5 = 0$$

$$X(p)\left(p^2 - 5p + 6\right) = p - 6$$

$$X(p) = \frac{p-6}{p^2 - 5p + 6}$$

$$X(p) = \frac{p-6}{(p-2)(p-3)}$$

$$X(p) = \frac{A}{p-2} + \frac{B}{p-3}$$

$$p-6 = (p-3)A + (p-2)B$$

$$p-6 = pA - 3A + pB - 2B$$

$$1p-6 = p\underbrace{(A+B)}_{1} - \underbrace{(3A+2B)}_{+6}$$

$$A+B = 1$$

$$3A+2B = 6$$

$$A = 1-B$$

$$3(1-B)+2B =$$

$$B = -3 \quad , A = 4$$

$$X(p)=\frac{4}{p-2}-\frac{3}{p-3}\Big|\mathcal{L}^{-1}$$

$$x(t)=\mathcal{L}^{-1}\left[\frac{4}{p-2}\right]-\mathcal{L}^{-1}\left[\frac{3}{p-3}\right]$$

$$x(t)=4e^{2t}-3e^{3t}$$

① $t\,x''(t)-2t\,x'(t)-2x(t)=0$

$$\mathcal{L}[t\cdot x''(t)](p)-2\mathcal{L}[t\cdot x'(t)]-2\mathcal{L}[x(t)](p)=0$$

$$\mathcal{L}[x(t)](p)=X(p)$$

$$\mathcal{L}[t(p^2X(p)-p\,x(0)-x'(0))](p)-2\mathcal{L}[t\cdot(pX(p)-x(0))]$$

$$-2X(p)=0$$

$$-(p^2X(p)-p\underbrace{x(0)}_{0}-\underbrace{x'(0)}_{2})'+2(pX(p)-\underbrace{x(0)}_{1})'$$

$$-2X(p)=0$$

$$-\left(p^2 X(p) - 1\right)' + 2\left(pX(p) - 1\right)' - 2X(p)$$

$$= -\left(2pX(p) + p^2 X'(p)\right) + 2\left(X(p) + pX'(p)\right) - 2X(p)$$

$$= -2pX(p) - p^2 X'(p) + \cancel{2X(p)} + 2pX'(p) - \cancel{2X(p)}$$

$$= -2pX(p) - p^2 X'(p) + 2pX'(p) = 0$$

Horik

$$\int_0^t f(\tau)\cdot g(t-\tau)\,d\tau \quad \Big|\,\mathcal{L}$$

$$\mathcal{L}\left[f(t) * g(t)\right](p)$$

$$\mathcal{L}\left[f(t)\right](p) \cdot \mathcal{L}\left[g(t)\right](p)$$

① $\int_0^t \tau^3 \cos(t-\tau)\, d\tau \,\Big|\, \mathcal{L}$

$f(t)$ $g(t)$

$\mathcal{L}\left[\int_0^t t^3 * \cos t\right](p)$

$\mathcal{L}[t^3](p) \cdot \mathcal{L}[\cos t](p)$

$\frac{3!}{p^4} \cdot \frac{p}{p^2+1} = \frac{3!\,p}{p^4(p^2+1)}$

② $\int_0^t e^{2(t-\tau)} \cdot \tau^3 \, d\tau \,\Big|\, \mathcal{L} =$

$= \mathcal{L}\left[\int_0^t e^{2t} * t^3 \, dt\right] =$

$= \mathcal{L}[e^{2t}] \cdot \mathcal{L}[t^3] =$

$$= \frac{1}{p-2} \cdot \frac{3!}{p^4} = \frac{6}{p^4(p-2)}$$

④ $$y'(t) + \int_0^t u \cdot y(t-u)\,du = t$$

$$\mathcal{L}[y'(t)](p) + \mathcal{L}\left[\int_0^t t * y(t)\,dt\right](p) = \mathcal{L}[t](p)$$

$$\mathcal{L}[y(t)](p) = Y(p)$$

$$pY(p) - y(0) + \mathcal{L}[t * y(t)](p) = \frac{1}{p^2}$$

$$pY(p) + 1 + \mathcal{L}[t](p) \cdot \mathcal{L}[y(t)](p) = \frac{1}{p^2}$$

$$\underline{pY(p)} + 1 + \underline{\frac{1}{p^2} Y(p)} = \frac{1}{p^2}$$

$$Y(p)\left(\frac{p}{p} + \frac{1}{p^2}\right) = \frac{1}{p^2} - \frac{p^2}{1}$$

$$Y(p)\left(\frac{p^2+1}{p^2}\right) = \frac{1-p^2}{p^2}$$

$$Y(p) = \frac{1-p^2}{p^3+1}$$

$$Y(p) = \frac{(1-p)(1+p)}{(p+1)(p^2-p+1)} \quad \Big| \; \mathcal{L}^{-1}$$

$$\mathcal{L}^{-1}[Y(p)] = \mathcal{L}^{-1}\left[\frac{1-p}{p^2-p+1}\right]$$

$$y(t) = \mathcal{L}^{-1}\left[\frac{1-p}{p^2-p+1}\right]$$

② $x(t) + 4\int_0^t u^2 x(t-u)\,du = 2t^2 \quad \Big|\ \mathcal{L}$

$$\mathcal{L}[x(t)](p) + 4\mathcal{L}\left[\int_0^t t^2 * x(t)\,dt\right](p) = 2\mathcal{L}[t^2](p)$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$X(p) + 4\mathcal{L}[t^2](p)) \cdot X(p) = 2\,\frac{2!}{p^3}$$

$$X(p)\left(\overset{p^3}{\cancel{1}} + \frac{8}{p^3}\right) = \frac{4}{p^3}$$

$$X(p)\left(\frac{p^3+8}{\cancel{p^3}}\right) = \frac{4}{\cancel{p^3}}$$

$$X(p) = \frac{4}{p^3+8} \quad \Big|\ \mathcal{L}^{-1}$$

$$x(t) = 4\mathcal{L}^{-1}\left[\frac{1}{p^3+2^3}\right] =$$

$$p^3+2^3 = (p+2)(p^2-2p+4)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$\frac{1}{(p+3)(p^2-2p+4)} = \frac{A}{p+3} + \frac{Bx+C}{p^2-2p+4}$$

3 $\displaystyle\int_0^t (t-u)\,x'(u)\,du = \sin t \;\Big|\; \mathcal{L},\ x(0) = 2$

$$\mathcal{L}\left[\int_0^t t * x'(t)\,dt\right](p) = \frac{1}{p^2+1}$$

$$\mathcal{L}[t](p)\,\mathcal{L}[x'(t)](p) = \frac{1}{p^2+1}$$

$$\frac{1}{p^2}\left(pX(p) - \underbrace{x(0)}_{2}\right) = \frac{1}{p^2+1}$$

$$\frac{pX(p)}{p^2} - \frac{2}{p^2} = \frac{1}{p^2+1}$$

$$p^2 \qquad\qquad p^2+1$$

$$\frac{X(p)}{p} = \frac{1}{p^2+1} + \frac{2}{p^2}$$

$$\frac{X(p)}{p} = \frac{p^2 + p^2 + 1}{p^2(p^2+1)}$$

$$X(p) = \frac{(2p^2+1)\cancel{p}}{p^2(p^2+1)} = \frac{2p^2+1}{p(p^2+1)} =$$

$$X(t) = 2\mathcal{L}^{-1}\left[\frac{p}{p^2+1}\right] + \mathcal{L}^{-1}\left[\frac{1}{p(p^2+1)}\right]$$

$$= 2\sin t +$$

$$\frac{1}{p(p^2+1)} = \frac{A}{p} + \frac{Bx+C}{p^2+1}$$

$$1 = (p^2+1)A + p(Bx+C)$$

③ $\mathcal{L}^{-1}\left[\frac{p}{2p^2-4p+7}\right]$

$$\boxed{\begin{array}{l} 2p^2-4p+7=0 \\ \Delta = 16-56 \\ \quad = -40 \\ \\ a\left(x-\frac{b}{2a}\right)^2 + \frac{-\Delta}{4a} \\ \\ 2\left(x+\frac{4}{4}\right)^2 + \frac{40}{8} \quad = \frac{\cancel{10}}{\cancel{4}} = 5 \\ \\ \underline{2\left(x+1\right)^2 + 5} \end{array}}$$

$$\mathcal{L}^{-1}\left[\frac{p}{2(p+1)^2+5}\right] = \frac{1}{2}\mathcal{L}^{-1}\left[\frac{p+1-1}{(p+1)^2+\left(\sqrt{\frac{5}{2}}\right)^2}\right]$$

$$\frac{1}{2}\mathcal{L}^{-1}\left[\frac{p+1}{(p+1)^2+\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2} - \frac{1}{(p+1)^2+\left(\frac{\sqrt{5}}{2}\right)^2}\right]$$

$$\frac{1}{2}e^{-t}\cos\frac{\sqrt{5}}{2}t - \frac{1}{2}e^{-t}\sin\frac{\sqrt{5}}{2}$$

$$2p^2-4p+7$$
$$2(p^2-2p+1)+5$$

$$\Delta = 36-4\cdot5\cdot3$$
$$=36-60$$
$$=24$$

$$2p^2-4p+7$$
$$3p^2+6p+5$$
$$3\left(x-\frac{6}{6}\right)^2+\frac{24}{6}$$
$$3(x-1)^2+4$$

$$\frac{p}{3(p-1)^2+4}$$

$$\frac{1}{3}\quad\frac{p}{(p-1)^2+\left(\frac{2}{\sqrt{3}}\right)^2}$$

$$\frac{1}{3}\quad\frac{p-1+1}{(p-1)^2+\left(\frac{2}{\sqrt{3}}\right)^2}$$

Subiect A

① $\displaystyle\int_C \frac{dz}{(z-1)^2(z^2+1)}$, $C: |z-1-i|=2$

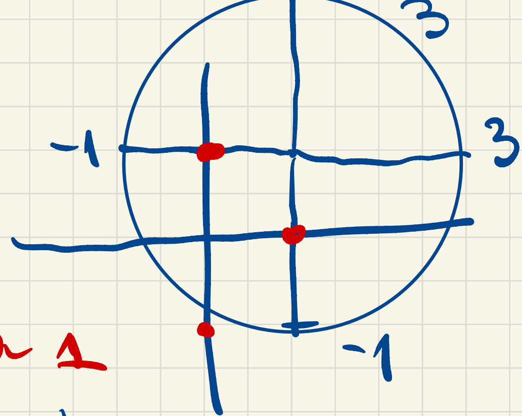
$|z-(1+i)|=2$

$(z-1)^2 = 0$

$z = 1$ pole order 2

$z^2 + 1 = 0$

$z = \pm i$ pole of order 1



2 Residues $(1, i)$

$$I = 2\pi i \left(\operatorname{Res}_{z=1} f(z) + \operatorname{Res}_{z=i} f(z) \right)$$

$$\operatorname{Res}_{z_0=1} f(z) = \frac{1}{1!} \lim_{z\to 1} \left(\cancel{(z-1)^2} \frac{1}{(z^2+1)\cancel{(z-1)^2}} \right)^{(1)}$$

$$= \lim_{z\to 1} \left(\frac{1}{z^2+1} \right)' = \lim_{z\to 1} \frac{\cancel{1(z^2+1)} - (z^2+1)' 1}{(z^2+1)^2}$$

$$= -\lim \frac{2z}{(z^2+1)^2} = -\frac{2}{4} = -\frac{1}{2}$$

$$\operatorname{Res}_{z=i} f(z) = \frac{\left(\frac{1}{(z-i)(z+i)}\right)}{(z-i)'} \Bigg|_{z=i} = \frac{1}{(z^2+1)(z+i)} = \frac{1}{(z-i)^2(z+i)} \Bigg|_{z}$$

$$= \frac{1}{(i-1)^2 2i} = \frac{1}{(-1-2i+1)2i} = \frac{1}{4}$$

$$I = 2\pi i\left(-\frac{1}{2} + \frac{1}{4}\right) = 2\pi i \frac{1}{4} = \frac{\pi i}{2}$$

② b) $\mathcal{L}^{-1}\left[\frac{p}{3p^2+6p+5}\right](t)$

f.c $\quad a\left(p+\frac{b}{2a}\right)^2 + \frac{-\Delta}{4a}$

$$3p^2+6p+5=0$$
$$\Delta = 36-60$$
$$= -24$$

$$\begin{array}{r} 60 \\ 36 \\ \hline 24 \end{array}$$

$$3\left(p+\frac{6}{6}\right)^2 + \frac{24}{12}$$

$$3(p+1)^2 + 2$$

$$\mathcal{L}^{-1}\left[\frac{p}{3(p+1)^2+2}\right](t) = \frac{1}{3}\mathcal{L}^{-1}\left[\frac{p}{(p+1)^2+\left(\sqrt{\frac{2}{3}}\right)^2}\right](t)$$

$$\frac{1}{3}\mathcal{L}^{-1}\left[\frac{p+1}{(p+1)^2+\left(\sqrt{\frac{2}{3}}\right)^2}\right](t) + \mathcal{L}^{-1}\left[\frac{1}{(p+1)^2+\left(\sqrt{\frac{2}{3}}\right)^2}\right](t)$$

$$\frac{1}{3}e^{-t}\cos\frac{\sqrt{2}}{3}t + \frac{1}{3}\sin\frac{\sqrt{2}}{3}t$$

SUBJECT B

① $$\int_0^{2\pi}\frac{dx}{5+3\cos x} = \int_{|z|=1}\frac{dz}{iz}\cdot\frac{1}{5+3\left(\frac{z^2+1}{2z}\right)}$$

$$\int_{|z|=1}\frac{1}{5+\frac{3z^2+3}{2z}}\frac{dz}{iz} =$$

$$\int_{|z|=1}\frac{2}{3z^2+10z+3}\frac{dz}{i}$$

$$3z^2+10z+3=0$$

$$\Delta=100-36$$

$$=64 \qquad z_1=-3\notin \operatorname{int}\Gamma$$

$$z_2=-\frac{1}{3}\in\operatorname{int}\Gamma$$

$$a(x-x_1)(x-x_2)$$

$$3\left(z+\frac{1}{3}\right)(z+3)$$

$$2\int_{|z|=1} \frac{1}{3\left(z+\frac{1}{3}\right)(z+3)} \frac{dz}{i} =$$

$$\operatorname*{Res}_{z=-\frac{1}{3}} f(z) = \left.\frac{\frac{1}{3(z+3)}}{\left(z+\frac{1}{3}\right)'}\right|_{z=-\frac{1}{3}} = \left.\frac{\frac{1}{3(z+3)}}{1}\right|_{z=-\frac{1}{3}}$$

$$= \frac{1}{3\left(-\frac{1}{3}+3\right)} = \frac{1}{\cancel{3}\left(\frac{8}{\cancel{3}}\right)} = \frac{1}{8}$$

$$I = \frac{4\pi i}{i} \cdot \frac{1}{8} = \frac{\pi}{2}$$

26b $\mathcal{L}\left[\dfrac{e^{2t}-e^{2t}\cos 3t}{t}\right](p)$

$$=\mathcal{L}\left[\frac{e^{2t}}{t}-\frac{e^{2t}\cos 3t}{t}\right](p)$$

$$=\mathcal{L}\left[\frac{e^{2t}}{t}\right](p)-\mathcal{L}\left[\frac{e^{2t}\cos 3t}{t}\right](p)$$

$$=\int_p^\infty \mathcal{L}[e^{2t}](p)-\int_p^\infty \mathcal{L}[e^{2t}\cos 3t](p)$$

$$=\int_p^\infty \frac{1}{p-2}-\int_p \mathcal{L}[\cos 3t](p-2)$$

$$=\int_p^\infty \frac{1}{p-2}-\int_p^\infty \frac{p}{p^2+3^2}\Big|_{p=p-2}$$

$$\ln(p-2)-\frac{1}{2}\int_p^\infty \frac{2(p-2)}{(p-2)^2+3^2}$$

$$\int \frac{u'(x)}{u(x)}=\ln|u(x)|$$

$$\frac{1}{2}\ln(p-2) - \frac{1}{2}\ln((p-2)^2+3^2)$$

$$\frac{1}{2}\left(\ln(p-2)^2 - \ln((p-2)^2+3^2)\right)$$

$$\frac{1}{2}\ln\frac{(p-2)^2}{(p-2)^2+3^2}$$

Subject C

$$\cancel{\frac{z(z-1)}{(\ )(\ )}\cdot\frac{dz}{iz}}$$

① $$\int_{-\infty}^{\infty}\frac{x^2-x}{(x^2+1)(x^2+9)}\,dx$$

$$f(z)=\frac{z^2-z}{(z^2+1)(z^2+9)}$$

$$z^2+1=0 \qquad z^2+9=0$$
$$z=\pm i \qquad z=\pm 3i$$

$$z_0=i \qquad z_0=3i$$

$$\operatorname*{Res}_{z_0=i} f(z) = \left.\frac{\frac{z^2-z}{z^2+9}}{(z^2+1)'}\right|_{z=i} = \left.\frac{\frac{z^2-z}{z^2+9}}{2z}\right|_{z=i}$$

$$= \frac{-1-i}{\frac{-1+9}{2i}} = \frac{-1-i}{16i}$$

$$\frac{\frac{-1-i}{-1+9}}{2i}$$

$$\frac{-1-i}{8} \cdot \frac{1}{2i}$$

$$\operatorname*{Res}_{z_0=3i} \left.\frac{\frac{z^2-z}{z^2+1}}{(z^2+9)'}\right|_{z=3i} = \left.\frac{\frac{z^2-z}{z^2+1}}{2z}\right|_{z=3i} = \frac{-9-3i}{\frac{-9+1}{6i}}$$

$$= \frac{-9-3i}{-8\cdot 6i} = \frac{-9-3i}{-48i} = \frac{9+3i}{48i}$$

$$I = 2\pi i\left(\frac{\overset{3}{\cancel{-1}}-i}{16i} + \frac{9+3i}{48i}\right) = \frac{-3-\cancel{3i}+9+\cancel{3i}}{48i}$$

$$\cancel{2}\pi i \cdot \frac{6}{\cancel{48i}} = \frac{\pi i 6}{24} = \frac{\pi}{4}$$

$$\left((z^2+1)(z^2+9)\right)' = \underline{(z^2+1)'(z^2+9) + (z^2+1)(z^2+9)'}$$

$$2z(z^2+9) + 2z(z^2+1)$$

$$2z^3 + 18z + 2z^3 + 2z$$

$$\frac{z^2+z}{4z^3+20z}$$

$$\frac{-1+i}{-4i+20i} = \frac{-1+i}{-16i}$$

$$\left.\frac{z^2-z}{4z^3+20z}\right|_{z=i} \quad \frac{-1-i}{-4i+20i} = \frac{-1-i}{16i}$$

$$\left.\frac{z^2-z}{4z^3+20z}\right|_{z=3i} = \frac{-9-3i}{-48i}$$

② b

$$s = \sum_{n=0}^{\infty} \frac{n}{2^n} \sin\frac{n\pi}{3}$$

$$s = \sum_{n=0}^{\infty} \frac{n \sin\frac{n\pi}{3}}{2^n}$$

$f(n)$

$$Z[n f(n)] = -z(F(z))'$$

$$Z\left[n \sin\frac{n\pi}{3}\right] = -z\left(Z\left[\sin\frac{n\pi}{3}\right]\right)'(z)$$

$$= -z\left(\frac{z\sin\frac{\pi}{3}}{z^2 - 2z\cos\frac{\pi}{3} + 1}\right)'(z) =$$

$$= -z\left(\frac{z\frac{\sqrt{3}}{2}}{z^2 - 2z\cdot\frac{1}{2} + 1}\right)'(z)$$

$$= -z\left(\frac{z\frac{\sqrt{3}}{2}}{z^2 - z + 1}\right)'(z)$$

$$= -z\left(\frac{\left(z\frac{\sqrt{3}}{2}\right)'(z^2 - z + 1) - \left(z\frac{\sqrt{3}}{2}\right)(z^2 - z + 1)'}{(z^2 - z + 1)^2}\right)$$

$$= -z\left(\frac{\frac{\sqrt{3}}{2}(z^2 - z + 1) - \left(z\frac{\sqrt{3}}{2}\right)(2z - 1)}{(z^2 - z + 1)^2}\right)$$

② c) $x''(t) - 4x'(t) - 5x(t) = 3e^{-t}$

$x(0) = 0 \quad x'(0) = 2$

$$\mathcal{L}[x''(t)](p) - 4\mathcal{L}[x'(t)](p) - 5\mathcal{L}[x(t)](p)$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$p^2X(p) - p\underbrace{x(0)}_{0} - \underbrace{x'(0)}_{2} - 4(pX(p) - \underbrace{x(0)}_{0}) - 5X(p) =$$

$$X(p)(p^2 - 4p - 5) - 2 = 3\mathcal{L}[e^{-t}](p)$$

$$X(p)(p^2 - 4p - 5) = \frac{3}{p+1} + 2^{p+1}$$

$$X(p)(p^2 - 4p - 5) = \frac{3 + 2p + 2}{p+1}$$

$$X(p) = \frac{2p+5}{(p+1)(p^2-4p-5)} = \frac{A}{p+1} + \frac{Bp+C}{p^2-4p-5}$$

$$2p+5 = (p^2 - 4p - 5)A + (p+1)(Bp+C)$$

$$2p+5 = p^2A - 4pA - 5A + p^2B + pC + pB + C$$

$$2p+5 = p^2(A+B) + p(-4A + B + C) - 5A + C$$

$A+B=0 \Rightarrow A=-B \quad |:(-1)$

$-4A+B+C=2$

$4B+B+C=2$

$5B+C=2$

$-5A+C=5$

$C=5(A+1)$

$5B+5(A+1)=2$

$-5A+5A+5=2$

$x(t)=$ Pula-n mă-ta

Subiect D

① $f(z)=\dfrac{1}{z(z-1)(z-2)}$

$z=0\ ,\ z=1 \qquad z=2$

2a) $x(t)+4\int_0^t u^2\,x(t-u)\,du=2t^2$

$$\mathcal{L}[x(t)](p) + 4\mathcal{L}\left[\int_0^t t^2 * x(t)\,dt\right](p) = 2\mathcal{L}[t^2](p)$$

$$X(p) + 4\mathcal{L}[t^2](p)\,X(p) = 2\,\frac{2!}{p^3}$$

$$X(p) + 4\,\frac{2!}{p^3}\,X(p) = \frac{4}{p^3}$$

$$X(p)\left(\overset{p^3}{\cancel{1}} + \frac{8}{p^3}\right) = \frac{4}{p^3}$$

$$X(p) = \frac{4}{p^3+8} \Rightarrow x(t) = 4\mathcal{L}^{-1}\left[\frac{1}{p^3+2^3}\right]$$

$$a^3+b^3 = (a+b)(a^2-ab+b^2)$$

$$x(t) = 4\mathcal{L}^{-1}\left[\frac{1}{(p+2)(p^2-2p+4)}\right]$$

$$\frac{1}{(p+2)(p^2-2p+4)} = \frac{A}{p+2} + \frac{Bp+C}{p^2-2p+4}$$

$$1 = (p^2-2p+4)A + (p+2)(Bp+C)$$

$$1 = \cancel{p^2A} - 2pA + 4A + \cancel{p^2B} + pC + 2pB + 2C$$

$$1 = p^2\underbrace{(A+B)}_{0} - p\underbrace{(2A - C - 2B)}_{0} \underbrace{+ 4A + 2C}_{0}$$

$$4A + 2C = 1$$

$$A + B = 0 \Rightarrow A = -B$$

$$2A - C - 2B = 0$$

$$\begin{cases} A + B = 0 \\ 2A - C - 2B = 0 \\ 4A + 2C = 1 \end{cases}$$

$$2(2A + C) = 1$$

$$2A + C = \frac{1}{2}$$

$$C = \frac{1}{2} - 2A$$

$$A = -B$$

$$\begin{cases} -2B - 2B - C = 0 \\ -4B + 2C = 1 \end{cases}$$

$$\begin{cases} -4B - C = 0 \\ -4B + 2C = 1 \end{cases}$$

$$-3C = 1$$

$$C = -\frac{1}{3} \Rightarrow -\frac{4}{12}$$

$$A = -B$$

$$B = -A$$

$$4A - \frac{2}{3} = 1$$

$$4A = {}^{3}1 + \frac{2}{3}$$

$$4A = \frac{5}{3} \Rightarrow A = \frac{5}{12} \Rightarrow B = -\frac{5}{12}$$

$$4\,\mathcal{L}^{-1}\left[\frac{\frac{5}{12}}{p+2}\right]$$

$$4\cdot\frac{5}{12}e^{-2t}+4\,\mathcal{L}^{-1}\left[\frac{-\frac{5}{12}p-\frac{4}{12}}{p^2-2p+4}\right]$$

2b $$s=\sum_{n=0}^{\infty}\overbrace{\frac{n\cos\frac{\pi n}{2}}{3^n}}^{f(n)}$$

$$Z\left[n\cos\frac{\pi n}{2}\right]=-z\left(Z\left[\cos\frac{\pi n}{2}\right]\right)'$$

$$=-z\left(\frac{z(z-\overbrace{\cos\tfrac{\pi}{2}}^{0})}{z^2-\underbrace{2z\cos\tfrac{\pi}{2}}+1}\right)$$

$$=-z\left(\frac{z^2}{z^2+1}\right)'_{(3)}$$

$$=-z\left(\frac{(z^2)'(z^2+1)-(z^2+1)'z^2}{(z^2+1)^2}\right)$$

$$=-3\left(\frac{2z(z^2+1)-2z\cdot z^2}{(z^2+1)^2}\right)$$

$$= -3\left(\frac{6\cdot 10 - 6\cdot 9}{10^2}\right)$$

$$= -3\,\frac{60-54}{100} = -3\,\frac{6}{100}$$

$$= -\frac{9}{50}$$

① $\cancel{1<}\,|z|<2$ $\qquad 1 < |z| < 2$

$\frac{|z|}{2} < 1 \qquad 1 < |z| \Rightarrow$

$\frac{1}{|z|} < 1 \qquad \Rightarrow \left|\frac{1}{|z|}\right| < 1$

$$\frac{A}{z} + \frac{B}{z-1} + \frac{C}{z-2} = 1 \qquad \left(\frac{|z|}{2}\right) < 1$$

$$(z-2)(z-1)A + z(z-2)B + (z-1)zC = 1$$

$$A(z^2-3z+2) + B(z^2-2z) + (z^2-z)C = 1$$

$$Az^2 - \underline{3Az} + 2A + Bz^2 - \underline{2Bz} + Cz^2 - \underline{zC} = 1$$

$$2A = 1$$

$$A = \frac{1}{2}$$

$$z^2(A + B + C) + z(-3A - 2B - C)$$

$$A + B + C = 0$$

$$-3A - 2B - C = 0$$

$$-2A - B = 0$$

$$\frac{-2}{2} - B = 0$$

$$-1 - B =$$

$$-B = 1$$

$$B = -1$$

$$\frac{1}{2} - 1 + C = 0$$

$$C = \overset{2}{1} - \frac{1}{2}$$

$$C = \frac{1}{2}$$

$$\frac{\frac{1}{2}}{z} - \frac{1}{z-1} + \frac{\frac{1}{2}}{z-2} =$$

$$\frac{1}{2z} - \frac{1}{z-1} + \frac{1}{2(z-2)}$$

$$\frac{1}{2z} - \frac{1}{-(1-z)} + \frac{1}{-2(2-z)}$$

$$\frac{1}{2z} + \cancel{\frac{1}{1-z}} - \frac{1}{4\left(1-\frac{z}{2}\right)}$$

$$\frac{1}{2z} + \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n - \frac{1}{4}\sum\left(\frac{z}{2}\right)^n$$

$$\frac{1}{1-z} = \frac{1}{z\left(\frac{1}{z}-1\right)}$$

$$= \frac{1}{-z\left(1-\frac{1}{z}\right)}$$

$X(n)$

$$9x_n - x_{n+2} = 3^n \quad / \mathcal{Z}$$

$$9\,\mathcal{Z}[x(n)] - \mathcal{Z}[x(n+2)] = \mathcal{Z}[3^n]$$

$$9\,\mathcal{Z}[f(n+0)] - \mathcal{Z}[f(n+2)] = \frac{z}{z-3}$$

$$9(z^0\,\mathcal{Z}[f(n)]$$

$$x_{n+1} - 2x_n = n \;/\; Z \qquad x_0 = 0$$

$$Z[x(n+1)] - 2Z[x(n)] = Z[n]$$

$$= \frac{z}{(z-1)^2}$$

$$Z[x(n+1)] = z^1 Z[x(n)] - z\,x(0)$$

$p=1 \downarrow 2$

$$= z\,F(z) - z\,\underbrace{x(0)}_{0}$$

$$= z\,F(z)$$

$$Z[x(n)] = F(z)$$

$p=0 \Downarrow 1$

$$z\,F(z) - 2F(z) = \frac{z}{(z-1)^2}$$

$$F(z)(z-2) = \frac{z}{(z-1)^2}$$

$$F(z) = \frac{z}{(z-1)^2(z-2)}$$

$$F(z) = \frac{z}{(z-1)^2(z-2)} \quad / \cdot \frac{1}{z}$$

$$\frac{F(z)}{z} = \frac{1}{(z-1)^2(z-2)}$$

$\downarrow$ desparte polinoamele

$$\frac{F(z)}{z} = \frac{A}{} + \frac{B}{} + \frac{C}{} \quad / \cdot z$$

Aplica Z^{-1}

$$F(z) = Z[x(n)] \quad / \; Z^{-1}$$

$$x(n)$$

② $x_{n+2} + 4x_{n+1} + 3x_n = 3^n \quad | \; Z$

$$\mathcal{Z}[x(n+2)](z) + \mathcal{Z}[x(n+1)](z) + 3[x(n)](z) = \mathcal{Z}[3^n](z)$$

$$x(1) = 1,\ x(0) = 0$$

$$\underline{\mathcal{Z}[x(n+2)](z)} = z^2\mathcal{Z}[x(n)](z) - \underbrace{z^2 x(0)}_{0} - \underbrace{z\,x(1)}_{0}$$

$$\underline{4\mathcal{Z}[x(n+1)](z)} = z^1\mathcal{Z}[x(n)](z) - \underbrace{z^1 x(0)}_{0}$$

$$\underline{3\mathcal{Z}[x(n+0)](z)} = z^0[x(n)]$$

$$z^2\mathcal{Z}[x(n)](z) - z + 4z\mathcal{Z}[x(n)](z) + 3\mathcal{Z}[x(n)](z)$$

$$z^2F(z) - z + 4zF(z) + 3F(z)$$

$$z^2F(z) - z + 7F(z) = \frac{z}{z-3}$$

$$F(z)(z^2 + 4z + 7) = \frac{z}{z-3} + z$$

$$F(z)(z^2+7) = \frac{z(z-2)}{z-3(z^2+4z+7)}$$

$$F(z) = \frac{z(z-2)}{(z-3)(z^2+4z+7)}$$

A, B, C

$$\frac{F(z)}{z} = \frac{(z-2)}{+ \quad +}$$

$$F(z) = 2 - 1 \cdot z$$

$$F(z) = \mathcal{Z}[x(n)] \; \frac{4}{z}$$

② $9x_n - x_{n+2} = 3^n \Big|_{\mathcal{Z}} \quad x_0 = 0, \; x_1 = 1$

$$9\mathcal{Z}[x(n)](z) - \mathcal{Z}[x(n+2)](z) = \mathcal{Z}[3^n](z)$$

$$\mathcal{Z}[x(n+0)](z) = z^0\mathcal{Z}[x(n)](z) - z^0 x(0)$$
$$= \mathcal{Z}[x(n)](z) = F(z)$$

$$\mathcal{Z}[x(n+2)](z) = z^2\mathcal{Z}[x(n)](z) - z^2x(0) - zx(1)$$

$$= z^2F(z) - z$$

$$9F(z) - z^2F(z) + z = \frac{z}{z-3}$$

$$F(z)\left(9 - z^2\right) = \frac{z}{z-3} - z$$

$$F(z)\left(9-z^2\right) = \frac{z - z^2 + 3z}{(z-3)}$$

$$F(z) = \frac{4z - z^2}{(9-z^2)(z-3)}$$

$$F(z) = \frac{z(4-z)}{(9-z^2)(z-3)}$$

$$F(z) = \frac{(z-4)z}{(z-3)^2(z+3)} \quad \Big| :z$$

$$\frac{F(z)}{z} = \frac{z-4}{(z-3)^2(z+3)}$$

$A, B, C \quad \Big| \cdot z$

$$F(z) = \left(\frac{Az}{z-3} + \frac{Bz}{(z-3)^2} + \frac{Cz}{z+3} \right) \Big| z^{-1}$$

$$x(n) = Z^{-1} A\left[\frac{z}{z-3}\right] + \dots$$

- $2x_{n+1} - x_n = \quad \Big| Z$

$$2Z[x(n+1)](z) - Z[x(n)](z) = \cdots \quad , x(0)=0$$

$$Z[x(n+1)](z) = z^1 Z[x(n)](z) - z \underbrace{x(0)}_{0}$$

$$2zF(z)$$

$$Z[x(n+0)](z) = z^0 Z[x(n)](z) - z^0 \underbrace{x(0)}_{0}$$

$$F(z)$$

$$2zF(z) - F(z)$$

$$F(z)(2z-1) = \dots$$

$$\sum_{n=0}^{\infty} \frac{n^2 \sin\frac{n\pi}{3}}{3^n} \Bigg|_z \qquad f(n)$$

$$\mathcal{Z}\left[n^2 \sin\frac{n\pi}{3}\right]_{(3)} =$$

$$f(n) = n \sin\frac{n\pi}{3}$$

$$\mathcal{Z}[n f(n)]_{(3)} = \mathcal{Z}\left[n\, n \sin\frac{n\pi}{3}\right]$$

$$= -z\left(\mathcal{Z}\left[n \sin\frac{n\pi}{3}\right]\right)'_{(3)}$$

$$= -z\left(-z\left(\mathcal{Z}\left[\sin\frac{n\pi}{3}\right]\right)'\right)'_{(3)}$$

$$= -z\left(-z\left(\frac{z \sin\frac{\pi}{3}}{z^2 - 2z\cos\frac{\pi}{3} + 1}\right)'\right)'_{(3)}$$

$\sin$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{\pi}{3}$
	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$

184 Cum aflu de ex $\cos \frac{8\pi}{3}$ sau $\sin \frac{7\pi}{4}$?

Dacă am ex de ăla cu $z_0, z_1, z_2 \ldots$ și cu $\int$, cum știu pe care z îl aleg?

198 - 202 nu înțeleg :(

Dacă am $\int_{-\infty}^{\infty}$ și am

$z_1 = \frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}$; $z_2 = -\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}$.

... iau care are ambele „+" sau fac modul?

E doar coincidență că nu e niciodată luat și negativ?

• 206 de ce gradul 2 și cum ai aflat așa rapide răspunsul?

• fix înainte de S 10, de unde am știut că $(z^2+1)^3=(z+i)^3(z-i)^3$?

– tot aici de ce e așa:

$$\operatorname*{Res}_{z=i} f = \frac{1}{2}\lim_{z\to i}\left[\frac{1}{(z+i)^3}\right]''$$

$$= \frac{1}{2}\lim_{z\to i}\left[\frac{-3\cancel{(z+i)^2}}{(z+i)^{\cancel{6}\,4}}\right]'$$

$$= -\frac{3}{2}\lim_{z\to i}\frac{-4\cancel{(z+i)^3}}{(z+i)^{\cancel{8}\,5}}$$

$$= \frac{-\cancel{12}^{\,6}}{-\cancel{2}_{\,1}}\cdot\frac{1}{(2i)^5} = \frac{6}{32\cdot i} = \frac{3}{16i}$$

La mulți
ani ION
și MARIA!!
:))))))
<3
<3

$$\int_0^{2\pi} \frac{dx}{1+3\cos^2 x}$$