Electrotechnics ET

Course 4 Year I-ISA English

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= Course 4 =

- 1. Electric Resistance. Methods used to calculate the Resistance
- 2. Resistors Connections
- 3. Specific Laws of the Magnetic Field

1. Electric Resistance. Methods used to calculate the Resistance

Electric Resistor. Electric Resistance

The electrical resistor (electrical resistor) is a passive circuit element that opposes the passage of electric current if an electric voltage is applied to its terminals.

The electric resistance characterizes any electric conductor.

The reciprocal quantity is electric conductance, and is the ease with which an electric current passes.



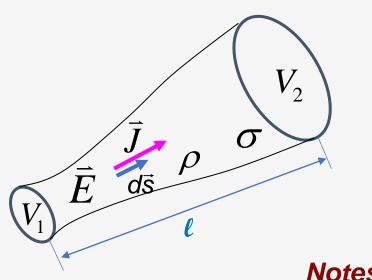






I. Direct Method

For a homogenous conductor ($\vec{E}_i = 0$) and the cross-section area, A constant, the value of the resistance is defined as:



$$R_{12} = \int_{1}^{2} \frac{\rho \, ds}{A} = \frac{\rho}{A} \int_{1}^{2} ds$$



where:

- o ℓ the length of the conductor [m].
- \circ ρ electric resistivity , [Ωm];
- A cross-section area of the conductor, [m²].

Notes

o symbols:



Europe



USA

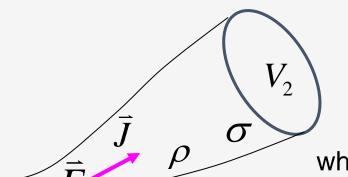


o measuring unit:

$$[R]_{SI}-[\Omega]$$

II. Using Ohm's Law

In an electric circuit, the value of the resistance is calculated with the Ohm's Law, being equal to the ratio between the electric voltage u_{12} , applied to the terminals of the conductor and the intensity i of the current passing through that conductor.



$$I_{12} = R_{12} \cdot i$$

$$R_{12} = \frac{U_{12}}{i} = \frac{V_1 - V_2}{i}$$

$$u_{12} = \int_{1}^{2} \overline{E} \cdot d\overline{s}$$

$$u_{12} = \int_{1}^{2} \overline{E} \cdot d\overline{s}$$
 and: $i = \int_{\Sigma} \overline{J} \cdot d\overline{A} = \int_{\Sigma} \sigma \cdot \overline{E} \cdot d\overline{A}$



o
$$\sigma = \frac{1}{\rho}$$
, $\left[\left(\Omega \cdot m \right)^{-1} \right]$ – electric conductivity.

III. Using the Analogy between the Electrostatic Field and the Electro-kinetic Field

The inverse quantity (or reciprocal quantity) of the electric resistance is called electric conductance, G:

$$G_{12} = \frac{1}{R_{12}}$$

Note
o measuring unit:
$$[G]_{SI} = [\Omega^{-1}] or [S]$$

Analogy: $C \Leftrightarrow G$

$$c \mapsto \sigma$$

Considering the value of the capacitance C known and using the analogy between the electrostatic field and the electro-kinetic field the conductance G is determined

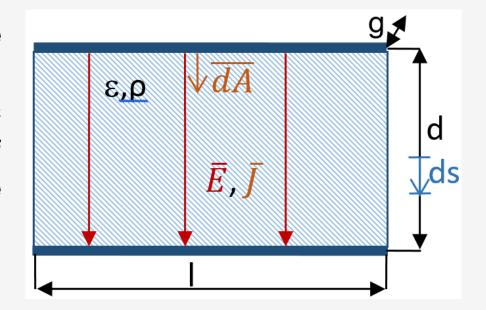
and then resistance R is:

$$R_{12} = \frac{1}{G_{12}}$$

Applications

Problem 1 = The Resistance of the Plane Capacitor with Losses =

Find the electric resistance of the capacitor with plane parallel plates, with losses, using the direct method, Ohm's Law and the analogy between the electrostatic and electro-kinetic fields, knowing that A is the area of the capacitor plate and d is the distance between the plates.

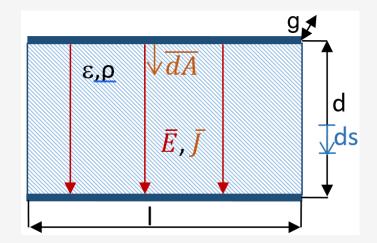


Solution:



Solution:

1) Direct Method



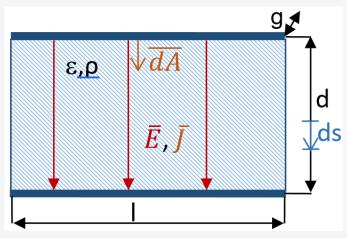
✓ the direct method involves the calculation of the resistance using the definition relation of the resistance:

$$R_{12} = \int_{1}^{2} \frac{\rho \, ds}{A} \iff R = \int_{plate1}^{plate2} \frac{\rho \cdot ds}{A}$$

✓ thus, applying this relation to this application taking into account the data of the problem, we have:

$$R = \frac{\rho}{A} \cdot \int_{1}^{2} ds = \frac{\rho \cdot d}{A}, \qquad \Omega$$

1) Using Ohm's Law



- from Ohm's Law: **u=R**•**i**
- we can find the electric resistance with the relation:

$$R = \frac{u}{i} \quad (1)$$

- from this relation it can be observed that in order to find the electric resistance the electric voltage and the electric current must be found:
- under these conditions we will want to determine the electric voltage the intensity of the electric current; we start with the relation:

$$u = \int_{\text{plate 1}}^{\text{plate 2}} \overline{E} \cdot \overline{ds}$$

 $u = \int_{-\infty}^{\text{plate2}} \overline{E} \cdot \overline{ds}$ under the integral we have the scalar product of two vectors; thus, we need the angle between the two vectors - in order to find out this angle it is necessary to represent vectors \overline{E} and \overline{ds} on the capacitor's representation from the Figure according to the indications from the previous problems (it can be thus noticed that between the two vectors we have a 0 degrees angle):

$$u = \int_1^2 E \cdot ds \cdot \cos 0^\circ \qquad => u = \int_1^2 E \cdot ds \qquad (2)$$

we know that the electric field intensity is respecting the relation:

$$\overline{E} = \boldsymbol{\rho} \cdot \overline{J} \tag{3}$$

the electric current intensity is equal with:

$$i = \int_{\Sigma} \overline{J} \cdot \overline{dA} = i = J \cdot A$$

from here we can find the electric field density:

$$=>J=\frac{i}{A} \qquad \qquad \textbf{(4)}$$

replacing relation (4) in relation (3) the electric field intensity is:

$$=> E = \rho \cdot \frac{i}{A} \tag{5}$$

relation (5) is replaced in relation (2) and the result is:

$$u = \int_{1}^{2} \rho \cdot \frac{i}{A} \cdot ds = \frac{\rho \cdot i}{A} \int_{1}^{2} ds$$
$$= > u = \frac{\rho \cdot i \cdot d}{A} \qquad (6)$$

replacing relation (6) in relation (1) the electric resistance is determined as:

$$=>R = \frac{\frac{\rho \cdot i \cdot d}{A}}{i} = \frac{\rho \cdot i \cdot d}{A} \cdot \frac{1}{i}$$
$$=>R = \frac{\rho \cdot d}{A}, \quad \Omega$$



3) Using the analogy between the electrostatic field and the electro-kinetic field

Electrostatics		Electro-kinetics
$C = \frac{\varepsilon \cdot A}{d} \tag{7}$	\longrightarrow	G
ε	\longrightarrow	σ

- ✓ we begin from the idea that we know the capacitance of the capacitor with plan parallel plates, this value being given by the relation (7)
- \checkmark from the analogy we know that to the electric capacitance, C, from Electrostatics corresponds to the electric conductance, G, from Electro-kinetics and that to the electric permittivity, ε, from Electrostatics corresponds in Electro-kinetics the electric conductivity, σ;
- \checkmark thus, if in relation (7) we replace C with G and ε with σ the electric conductance is equal to:

$$=>G=\frac{\sigma\cdot A}{d}$$

✓ we know that the inverse of the electric conductance is the electric resistance, thus:

$$R = \frac{1}{G} = \frac{d}{\sigma \cdot A}$$
 (8)

✓ also, we know that the inverse of the electric conductivity is called electric resistivity and its notation is ρ:

$$\sigma = \frac{1}{\rho} \quad \Longrightarrow \quad \rho = \frac{1}{\sigma} \tag{9}$$

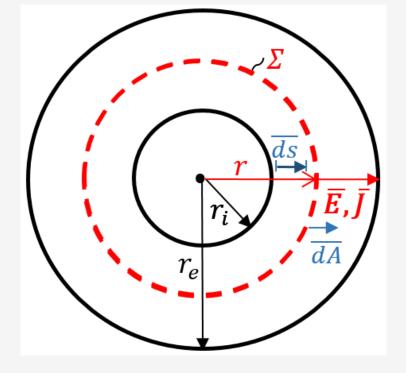
✓ replacing relation (9) in relation (8) we obtain the electric resistance of the capacitor with plan parallel plates, without losses as being:

$$R=\frac{
ho\cdot d}{A},\qquad \Omega$$

Problem 2 = The Resistance of the Spherical Capacitor with Losses =

Find the resistance of the capacitor with spherical plates, with losses, (with radius r_i and r_e) using the direct method, Ohm's Law and the analogy between the electrostatic and electro-kinetic fields.

Solution:





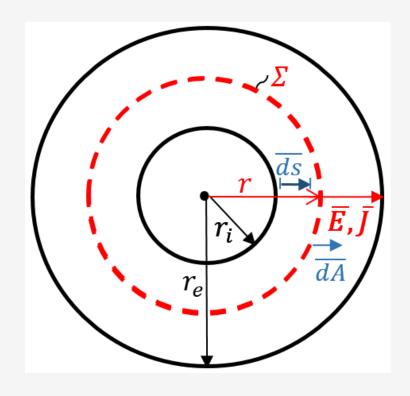
1) Direct Method

✓ the direct method involves the calculation of the resistance using its definition relation:

$$R = \int_{arm1}^{arm2} \frac{\rho \cdot ds}{A}$$

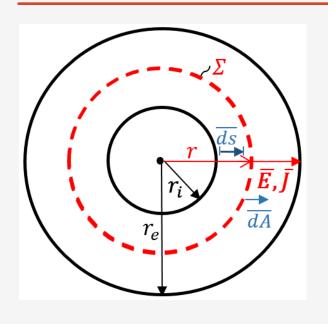
✓ thus, applying this relation to this application taking into account the data of the problem, we have:

$$R = \int_{r_i}^{r_e} \frac{\rho \cdot ds}{A}$$



 \checkmark in this problem, unlike in the previous problem, the area is not constant and equal to the value A anymore, thus it depends on the place we choose for the surface Σ of radius r (which we can choose anywhere between the sphere of radius r_i and the sphere of radius r_e), thus:

$$A = 4 \cdot \pi \cdot r^2$$



$$R = \int_{r_i}^{r_e} \frac{\rho \cdot ds}{A}$$

$$A = 4 \cdot \pi \cdot r^2$$

$$R = \int_{r_i}^{r_e} \frac{\rho}{4 \cdot \pi \cdot r^2} \cdot dr$$

$$R = \frac{\rho}{4 \cdot \pi} \int_{r_i}^{r_e} \frac{1}{r^2} \cdot dr = \frac{\rho}{4 \cdot \pi} \cdot \left(-\frac{1}{r} \right) \Big|_{r_i}^{r_e} = \frac{\rho}{4 \cdot \pi} \left(\frac{1}{r_i} - \frac{1}{r_e} \right)$$

$$=> R = \frac{\rho}{4 \cdot \pi} \cdot \frac{r_e - r_i}{r_i \cdot r_e}, \quad [\Omega]$$

2) Using Ohm's Law

- \checkmark from Ohm's Law: $\mathbf{u} = \mathbf{R} \cdot \mathbf{i}$
 - we can find the electric resistance with the relation : $R = \frac{u}{i}$ (1)
- ✓ from this relation it can be observed that in order to find the electric resistance the electric voltage and the electric current must be found:
 - under these conditions we will want to find the electric voltage and the intensity of the electric current; we start with the relation:

$$u = \int_{r_i}^{r_e} \overline{E} \cdot \overline{dr} = \int_{r_i}^{r_e} E \cdot dr \qquad (2)$$

we know that the electric field intensity is respecting the relation:

$$\overline{E} = \rho \cdot \overline{J} \tag{3}$$

the electric current intensity is:

$$i = \int_{\Sigma} \overline{J} \cdot \overline{dA} = > i = J \cdot A$$
 $i = J \cdot 4 \cdot \pi \cdot r^2$, [A]

from here we can find the electric current density:

$$=> J = \frac{i}{4 \cdot \pi \cdot r^2}$$
, $[A/m^2]$ (4)

relation (4) is replaced in relation (3) the electric field intensity will be:

$$=> E = \frac{\rho \cdot i}{4 \cdot \pi \cdot r^2} , \qquad [V/m] \qquad (5)$$

replacing relation (5) in relation (2) the result is:

$$=> u = \int_{r_i}^{r_e} \frac{\rho \cdot i}{4 \cdot \pi \cdot r^2} \cdot dr = \frac{\rho \cdot i}{4 \cdot \pi} \cdot \left(-\frac{1}{r}\right) \Big|_{r_i}^{r_e} = \frac{\rho \cdot i}{4 \cdot \pi} \left(\frac{1}{r_i} - \frac{1}{r_e}\right)$$

$$=> u = \frac{\rho \cdot i}{4 \cdot \pi} \cdot \frac{r_e - r_i}{r_i \cdot r_e}, \quad [V] \quad (6)$$

replacing relation (6) in relation (1) the electric resistance is determined as:

$$R = \frac{u}{i} = \frac{\rho \cdot i}{4 \cdot \pi} \cdot \frac{r_e - r_i}{r_i \cdot r_e} \cdot \frac{1}{i}$$

$$=> R = \frac{\rho}{4 \cdot \pi} \cdot \frac{r_e - r_i}{r_i \cdot r_e}, \quad [\Omega]$$

3) Using the analogy between the electrostatic field and the electro-kinetic field

Electrostatics		Electro-kinetics
$C = \frac{4 \cdot \pi \cdot \varepsilon \cdot r_e \cdot r_i}{r_e - r_i} \tag{7}$	\longrightarrow	G
ε	\longrightarrow	σ

✓ we begin from the idea that we know the capacitance of the capacitor with spheric plates, this value being given by the relation (7)

$$=>G=\frac{4\cdot\pi\cdot\sigma\cdot r_e\cdot r_i}{r_e-r_i},\qquad \left[\Omega^{-1}\right]or\left[S\right]$$

✓ we know that the inverse of the electric conductance is the electric resistance, thus:

$$G = \frac{1}{R} \implies R = \frac{1}{G}$$

$$\Rightarrow R = \frac{r_e - r_i}{4 \cdot \pi \cdot \sigma \cdot r_e \cdot r_i} \quad (8)$$

 \checkmark also, we know that the inverse of the electric conductivity is called electric resistivity and its notation is ρ:

$$\sigma = \frac{1}{\rho}$$
, $[S/m] = > \rho = \frac{1}{\sigma}$, $[\Omega \cdot m]$ (9)

✓ replacing relation (9) in relation (8) we obtain the electric resistance of the capacitor with spherical plates, without losses as being:

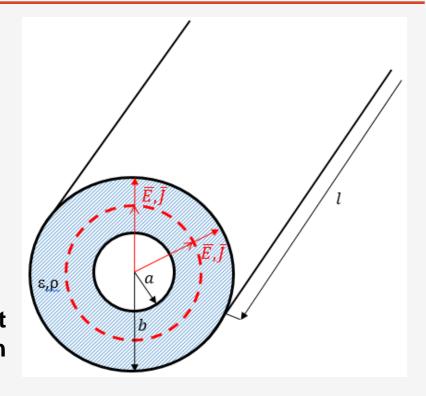
$$=> R = \frac{\rho \cdot (r_e - r_i)}{4 \cdot \pi \cdot r_e \cdot r_i}, \quad [\Omega]$$

Problem 3 = Losses in coaxial cable =

Find the losses in the case of a coaxial cable of length \it{I} , having the permittivity medium ϵ and the resistivity ρ (interior radius a and exterior radius b) using the direct method, Ohm's Law and the analogy between the electrostatic and electro-kinetic fields.

Homework:

Suggestion: the problem is similar with *Problem 2,* meaning it is as you should calculate the resistance of the capacitor with cylindrical plates.

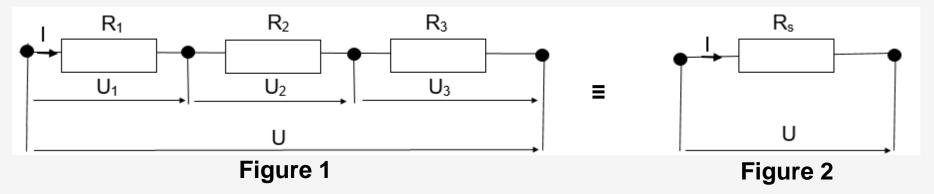




2. Resistors Connections (Networks)

Series Resistors Connection

Suppose we have 3 resistors connected in series (Figure 1) and we want to find the equivalent resistance value (Figure 2) that can replace the whole resistances group in order to not modify the circuit function (meaning that we have to keep the same current I and the same voltage U).



- ❖ In the case of the series connection the total electrical voltage U at the group terminals is divided on the 3 resistors, thus having the voltages U₁, U₂, U₃, and the electrical current remains the same, I, to all the resistors connected in series
- ❖ We apply the generalized Ohm's Law at the terminals of the three resistors

$$U = U_1 + U_2 + U_3$$
 (1)

• We apply Ohm's Law at the terminals of each of the resistors:

$$U_1 = R_1 \cdot I;$$
 $U_2 = R_2 \cdot I;$ $U_3 = R_3 \cdot I;$ $U = R_s \cdot I$ (2)

Including relation (2) in relation (1):

$$R_s \cdot I = R_1 \cdot I + R_2 \cdot I + R_3 \cdot I$$

The equivalent resistance of the 3 resistors connected in series

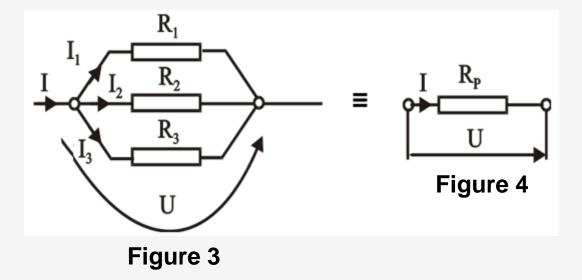
$$R_s = R_1 + R_2 + R_3$$

❖ We can extend the relation to *n* resistors series connected:

$$R_s = \sum_{k=1}^n R_k$$

Parallel Resistors Connection

Suppose we have 3 resistors connected in parallel and we want to find what value should have an equivalent resistance which could replace the entire resistor group without modifying the circuit initial function (meaning that we should keep the same current, I and the same voltage, U).



- In the case of the parallel connection the total electric current I is divided on the 3 resistors, thus having the currents I_1 , I_2 , I_3 and the electric voltage remains the same, U
- ❖ We apply the *First Kirchhoff Law* in the first circuit (Figure 3):

$$I = I_1 + I_2 + I_3$$
 (3)

* We apply Ohm's law at the terminals of each different resistor:

$$I_{1} = \frac{U}{R_{1}}; \qquad I_{2} = \frac{U}{R_{2}}; \qquad I_{3} = \frac{U}{R_{3}}; \qquad I = \frac{U}{R_{p}}$$
 (4)

• Including relation (2) in relation (1): $\frac{U}{R_p} = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3}$

The equivalent resistance of the 3 resistors connected in parallel:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

❖ We can extend the relation to *n resistors* parallel connected:

$$\frac{1}{R_p} = \sum_{k=1}^n \frac{1}{R_k}$$

❖ It can be demonstrated that for 2 parallel connected resistors :

$$R_p = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Applications

Problem 1

Find the value of the equivalent resistance between the terminals AD and CB of the network presented, knowing that:

$$R_1=25 \Omega$$
;

$$R_2=50 \Omega$$
;

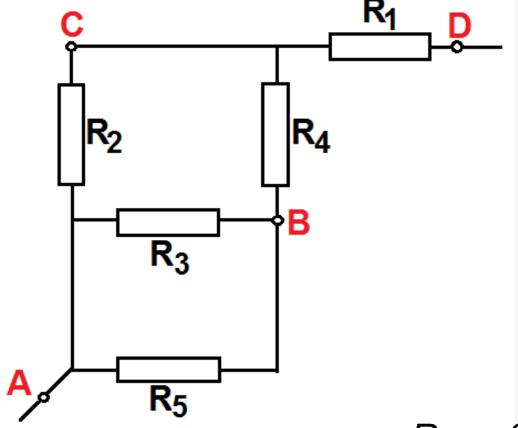
$$R_3 = 60 \Omega;$$

$$R_4 = 160 \Omega;$$

$$R_5=120 \Omega$$
.

Solution:



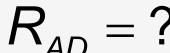


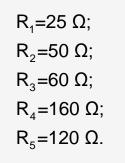
$$R_{AD} = ?$$

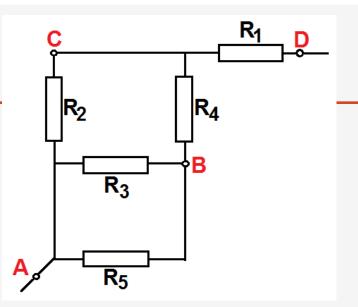
 $R_{CR} = ?$

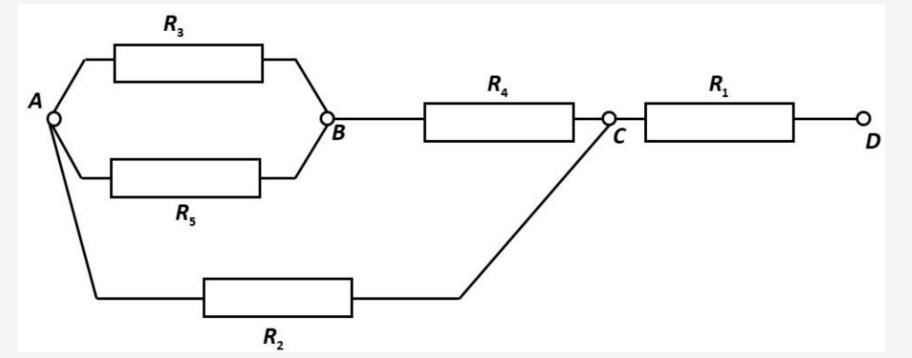
$$R_{CB} = ?$$











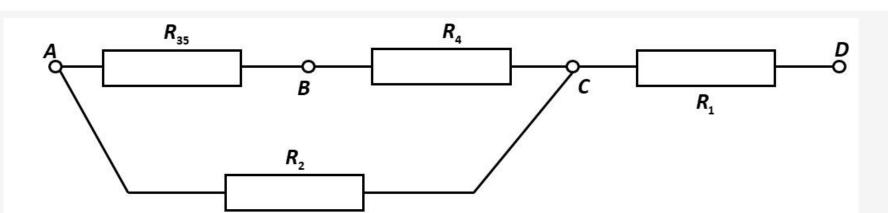
$$R_3 \parallel R_5$$
:

$$R_{35} = \frac{R_3 \cdot R_5}{R_3 + R_5}$$

$$R_{35} = \frac{60 \cdot 120}{60 + 120}$$

$$R_{35} = \frac{\cancel{60} \cdot \cancel{120}^{40}}{\cancel{180}_{\cancel{3}}}$$

$$R_{35} = 40 \ \Omega$$



$$R_1=25 \Omega$$
;

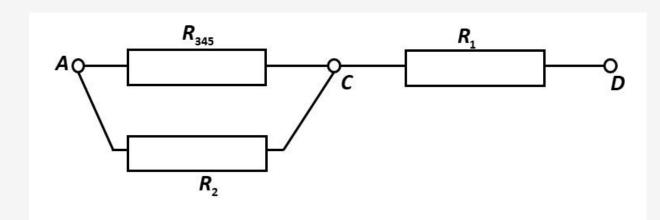
$$R_2=50 \Omega$$
;

$$R_3=60 \Omega$$
;

$$R_4=160 \Omega$$
;

$$R_5=120 \Omega$$
.

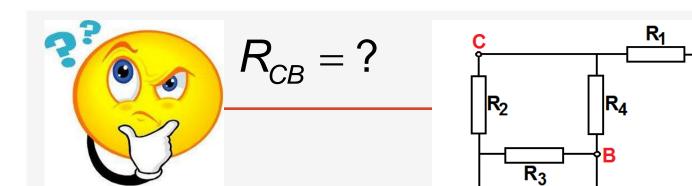
$$R_{345} = R_{35} + R_4 = 40 + 160 = 200 \Omega$$

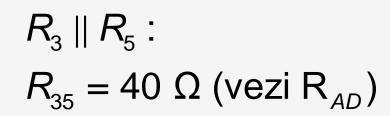


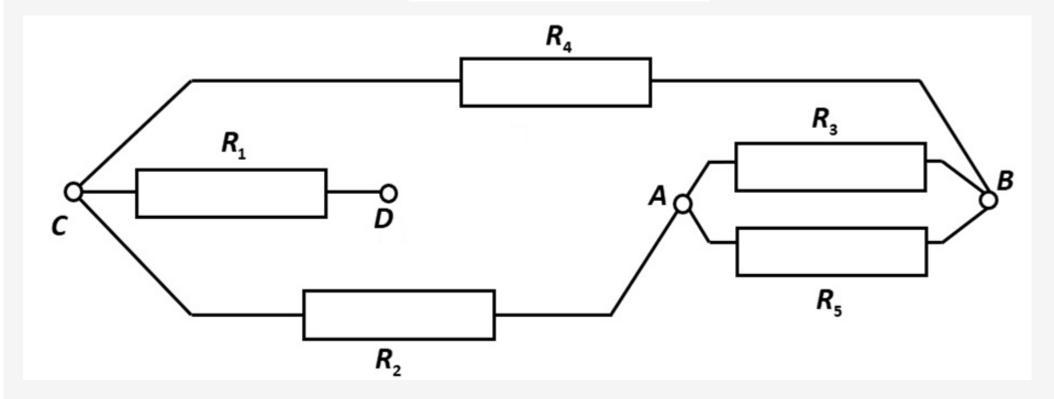
$$R_{2345} = \frac{R_{345} \cdot R_2}{R_{345} + R_2} = \frac{200 \cdot 50}{200 + 50}$$

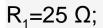
$$R_{2345} = \frac{\overset{40}{200} \cdot 50}{250_{5}} = 40 \ \Omega$$

$$R_{AD} = R_{2345} + R_1 = 40 + 25 = 65 \Omega$$







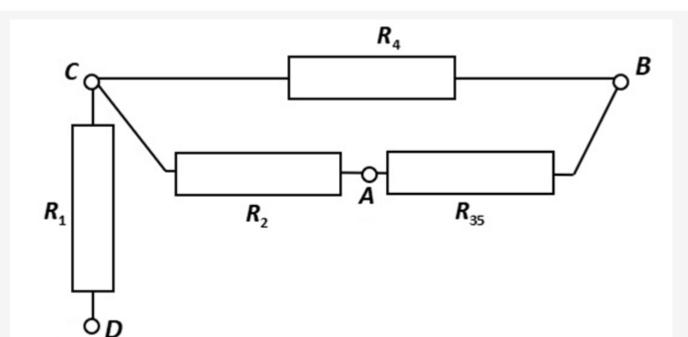


$$R_2=50 \Omega$$
;

$$R_3=60 \Omega$$
;

$$R_4 = 160 \Omega;$$

$$R_5$$
=120 Ω .



$$R_{235} = R_{35} + R_2$$

 $R_{235} = 40 + 50$
 $R_{235} = 90 \Omega$

$$R_4$$
 R_{235}

$$R_{235} \parallel R_4$$

$$R_{CB} = R_{2345} = \frac{R_{235} \cdot R_4}{R_{235} + R_4} = \frac{160 \cdot 90}{160 + 90}$$

$$R_{CB} = R_{2345} = \frac{R_{235} \cdot R_4}{R_{235} + R_4} = \frac{160 \cdot 90}{160 + 90}$$

$$R_{CB} = \frac{{}^{32}160 \cdot 90}{250_5} = \frac{288}{5} = 57,6 \Omega$$

Problem 2

For the resistance connections represented in the figure, find the equivalent resistance relative to the terminals: 1) AB;

- 2) CD;
- 3) CB.

The following values are known:

$$R_1=1 \Omega$$
;

 $R_2=2 \Omega$;

 $R_3=3 \Omega$;

 $R_4=4 \Omega;$

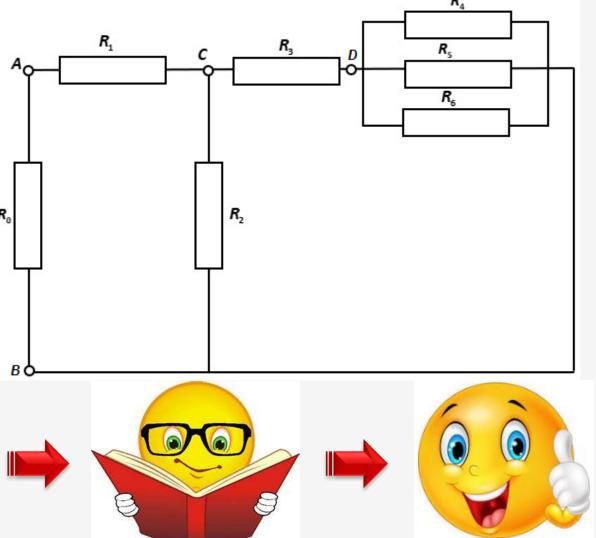
 $R_5=5 \Omega;$

 $R_6=6 \Omega$;

 $R_0=10 \Omega$.

Homework:





3. Specific Laws of the Magnetic Field

3.1. Temporary Magnetization Law (TML)

Magnetization

permanent component, \overline{M}_{D} , independent from the value \overline{H} ;

temporary component, M_t, dependent by the value H.

$$\overline{M} = \overline{M}_p + \overline{M}_t$$

• where \overline{H} represents the magnetic field intensity $[H]_{si} - [A/m]$

$$[H]_{SI} - [A/m]$$

In each point in a body and in every moment the temporary magnetization is depending on the magnetic field intensity H:

$$\overline{M}_{t} = f(\overline{H})$$

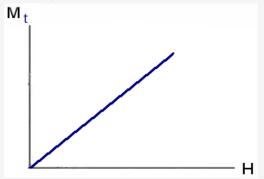
in linear isotropic magnetic materials, without permanent magnetization:

$$\overline{M}_t = \chi_m \overline{H}$$
 where:

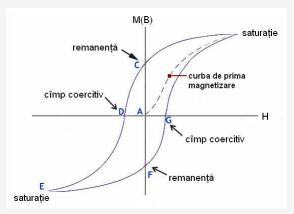
o χ_m - magnetic susceptibility.

■ Materials for which:

- χ_m < 0 are called diamagnetic materials;
- χ_m > 0 are called paramagnetic materials.
- Materialele for which:
 - χ_m doesn't depend on \overline{H} are called linear materials;



• χ_m depends on \overline{H} are called nonlinear materials.



3.2. The Law of the Dependence between \overline{B} , \overline{H} and \overline{M} in Magnetic Field

In any point in the field and at any time, the magnetic induction B is equal with the sum between the magnetic field intensity H and the magnetization \overline{M} , multiplied with the vacuum permeability μ_0 .

$$\bar{\mathsf{B}} = \mu_0 \left(\bar{\mathsf{H}} + \bar{\mathsf{M}} \right)$$

where:

o μ_0 – vacuum magnetic permeability: $\mu_0 = 4\pi 10^{-7}$ $\left[\frac{H}{m}\right]$ o μ – absolute permeability of the medium: $\mu = \mu_0 \ \mu_r$

ο μ_r –relative permeability of the medium: $\mu_r = 1 + \chi_m$

$$\overline{M} = \overline{M}_t(\overline{H}) + \overline{M}_p$$

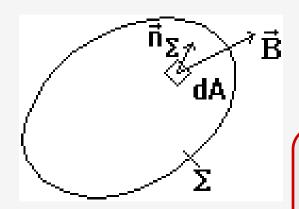
$$\overline{M}_p = 0$$

$$\bar{B} = \mu \bar{H}$$

- Units of measurement in SI: [μ]
 - $[\mu_r] \longrightarrow adimensional$
 - $[\chi_m]$ adimensional

3.3. Magnetic Flux Law

☐ General (global) form of the law:



$$\phi_{\Sigma} = \bigoplus_{\Sigma} \overline{B} \ d\overline{A} = 0$$

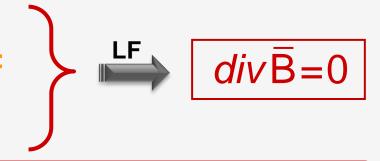
The magnetic flux through a closed surface Σ is equal with the surface integral of the scalar product between the magnetic induction and the surface element and is at all times null.

- **Units of measurement in SI:**
- $[B]_{SI} [T]$, Tesla;
- $[H]_{SI}$ [A/m];
- $\left[\phi\right]_{SI}-\left[Wb\right]$, Weber.

□ Local form (LF) of the law:

- is deduced from the global form:
- Gauss-Ostrogradski transform is applied, and the result is:

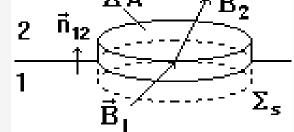
$$\iint_{\Sigma} \overline{\mathbf{B}} \cdot d\overline{\mathbf{A}} = \iiint_{V_{\Sigma}} div \, \overline{\mathbf{B}} \, dV$$



In any point in the field and at any time, the divergence of the magnetic induction is identical zero.

Consequence Conservation of the normal component of magnetic induction on discontinuous surfaces:

$$\operatorname{div}_{S} \overline{B} = \overline{n}_{12} (\overline{B}_{2} - \overline{B}_{1}) = 0$$
 $B_{1n} = B_{2n}$



when passing through a surface of discontinuity the normal component of magnetic induction is preserved.

Attention !!!

o often a field of auxiliary vectors is introduced A, called vector magnetic potential, which is a result from the local form of the law:

$$div\bar{B}=0$$
 \longrightarrow $\bar{B}=rot\bar{A}$

- Obs. The vector field \overline{A} is determined only if its divergence is also known, divergence which can be given by:
 - ✓ Coulomb's Calibration Condition: $div \overline{A} = 0$
 - ✓ Lorentz's Calibration Condition: $\operatorname{div} \overline{A} = -\mu \sigma V \mu \varepsilon \frac{\partial V}{\partial t}$
 - where V is the electric potential , [V].

In stationary magnetic field

Ampere's Theorem

$$\oint \overline{H} \cdot d\overline{s} = i_{S_{\Sigma}} \rightarrow rot \overline{H} = \overline{J}$$

Analogy between the Electric Field and Magnetic Field

Electric Field	Magnetic Field
Scalar Potential - V	Vector Potential - \overline{A}
Electric Field - Ē	Magnetic Field - \bar{H}
Permittivity - ε	Permeability - μ
Volume Charge Density - ρ _ν	Current Density - \overline{J}
Capacitance - C	Inductance - L
Laplace Equation	Laplace Equation
Poisson Equation	Poisson Equation

