Electrotechnics ET

Course 7

Year I-ISA English

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= Course 7 =

Part II – ELECTRIC CIRCUITS THEORY

ELECTRICAL CIRCUITS IN HARMONIC REGIME

Linear Electric Circuits in Permanent Sinusoidal Regime

Periodic and sinusoidal quantities

Symbolic representations of sinusoidal quantities

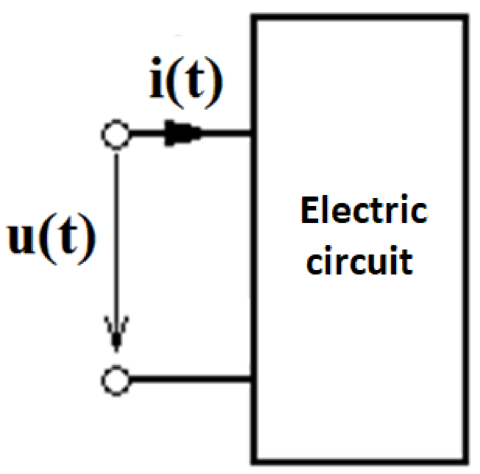
Operations with sinusoidal quantities symbolical represented

Applications

Electrical Circuits in Harmonic Regime (Permanent Sinusoidal Regime)

The permanent sinusoidal regime is the monophase alternative current regime. Is characterized by time variable quantities. The variable quantities can be voltages, electromotive voltages, currents or powers. We name the instantaneous value, that value that the quantity has it at some moment t. By convention, the instantaneous value is denoted with lowercase:

$$u(t);\ e(t);\ i(t);\ p(t)$$



Suppose we have a circuit supplied with a sinusoidal voltage *u(t)* will be passed thought by a sinusoidal current *i(t):*

The maximum value (amplitude) of the variable quantity

$$u(t) = \overbrace{U\sqrt{2}}\sin\underbrace{(\omega t + \gamma_u)}$$

The phase of the variable quantity

$$or\ \ u(t) = U_m \sin(\omega t + \gamma_u)$$

$$or\ \ u(t) = U_{\max} \sin(\omega t + \underbrace{\gamma_u})$$

The phase angle of the variable quantity

$$i(t) = I\sqrt{2}\sin(\omega t + \gamma_i)$$

$$i(t) = I_m \sin(\omega t + \gamma_i)$$

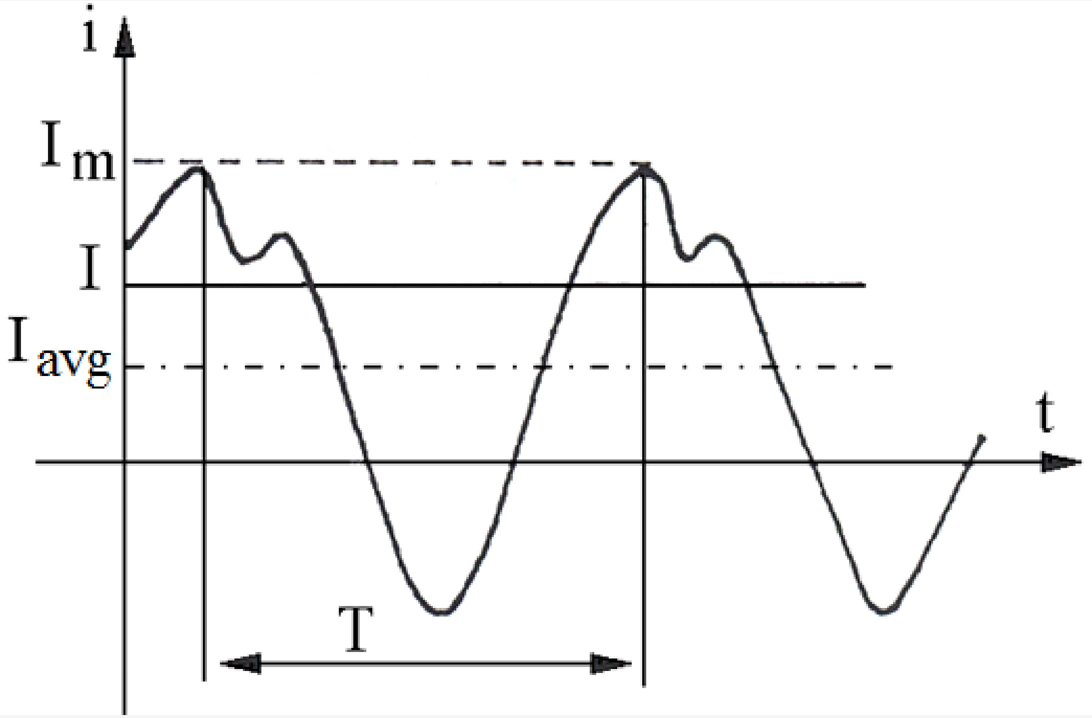
$$i(t) = I_{\max} \sin(\omega t + \gamma_i)$$

Periodic and Sinusoidal Quantities

❑ Periodic Quantities

- variabile parameters whose sequence of values is repeated at equal time intervals;
- they fulfil the relation: $i(t) = i(t + kT)$ $\quad k \in Z$

$$u(t) = u(t + kT)$$



where:

- T – time period;

$$[T]_{S.I.} = 1\ [s]$$

- f – frequency:

$$f = \frac{1}{T}, \quad [Hz]$$

- ω – angular frequency (pulsation):

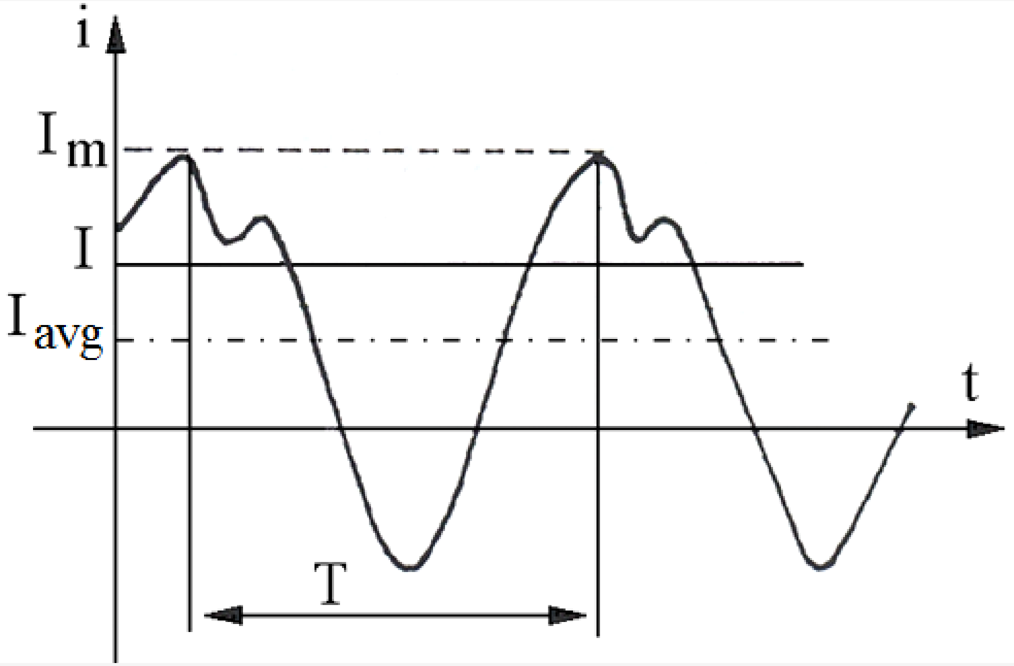
$$\omega = 2\pi f, \quad [rad/s]$$

The periodic quantities is characterized by:

- The maximum value or peak value: $I_m = I\sqrt{2}$
- The average value of the periodical quantity:

$$I_{avg} = \frac{1}{T}\int_0^T i(t)dt \ \geq 0 \ \ or \ < 0$$

- The effective value : $I = \sqrt{\frac{1}{T}\int_0^T i^2(t)dt} \geq 0$



- The peak coefficient: $K_v = \frac{I_m}{I}$;
- The form coefficient: $K_f = \frac{I}{I_{avg}}$
- A periodical quantity whose average value during a period is zero is called alternative quantity.
- A quantity whose instantaneous value keeps the same sign all the time is called pulsating quantity.

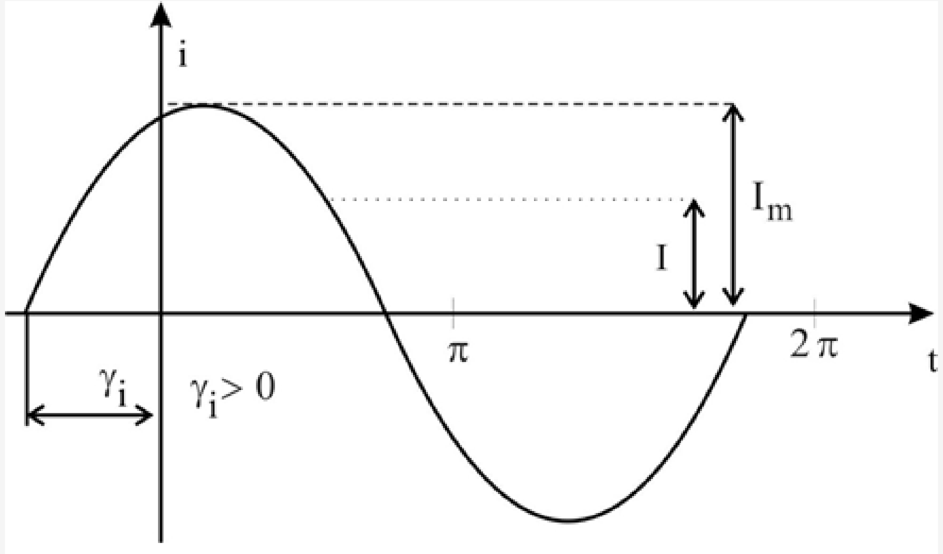
❑ Sinusoidal quantities (or harmonic quantities):

- The alternative quantity which can be put in the form:

$$i(t) = I_m \sin(\omega t + \gamma_i)$$

where:

  - γ_i – initial phase, for $t=0$;
  - I_m – maximum value (amplitude) of the quantity;
  - ω – angular frequency : $\omega = 2\pi f = \frac{2\pi}{T}, \left[\frac{rad}{s}\right]$



- $t=0$ ➡ $i(0) = I_m \sin\gamma_i$
- The average value: $I_{avg} = \frac{1}{T}\int_0^T i(t)dt = \frac{1}{T}\int_0^T I_m \sin\omega t dt = 0$ ➡ $I_{avg} = 0$
- The effective value: $I = \sqrt{\frac{1}{T}\int_0^T I_m{}^2 \sin^2 \omega t\, dt} = I_m \sqrt{\frac{1}{T}\int_0^T \frac{1}{2}(1-\cos 2\omega t)\, dt} = \frac{I_m}{\sqrt{2}}$ ➡ $I = \frac{I_m}{\sqrt{2}}$

$$\int_0^T \frac{1}{2}(1-\cos 2\omega t)dt = \frac{T}{2}$$

➡ $I_m = I\sqrt{2}$

- The canonical form (usual expressions) of the sinusoidal quantity is:

$$\boxed{i(t) = I\sqrt{2}\sin(\omega t + \gamma_i)}$$

$$\boxed{u(t) = U\sqrt{2}\sin(\omega t + \gamma_u)}$$

- The phase shift angle of the sinusoidal quantity: $\varphi = \gamma_u - \gamma_i$; $\varphi \in (-\pi, \pi]$

- The power factor: $\cos\varphi \in [0,1]$

Notes

– at the socket we have:

- Effective value: *U = 220 V;*
- Maximum value: $U_m = 220\sqrt{2} = 311 V$

Symbolic Representations of Sinusoidal Quantities

The sinusoidal quantity $i(t) = I\sqrt{2}\sin(\omega t + \gamma)$ has its

- Algebric or complex representation :
  - in complex:
    - Puts in correspondence the sinusoidal quantity with the complex number having the module equal with the amplitude of the sinusoidal quantity and the argument equal with the phase:

$$\underline{i} = \sqrt{2}\, I\, e^{j(\omega t + \gamma)}$$

  - in simplified complex :
    - Puts in correspondence the sinusoidal quantity with the complex number having the module equal with the effective value of the sinusoidal quantity and the argument equal with the phase angle:

$$\underline{I} = I\, e^{j\gamma}$$

Euler's theorem: $e^{j\gamma} = \cos\gamma + j\sin\gamma$

$$\Rightarrow \underline{I} = I(\underbrace{\cos\gamma}_{\mathrm{Re}\{\underline{I}\}} + j\underbrace{\sin\gamma}_{\mathrm{Im}\{\underline{I}\}}) \Rightarrow \underline{I} = a + jb$$

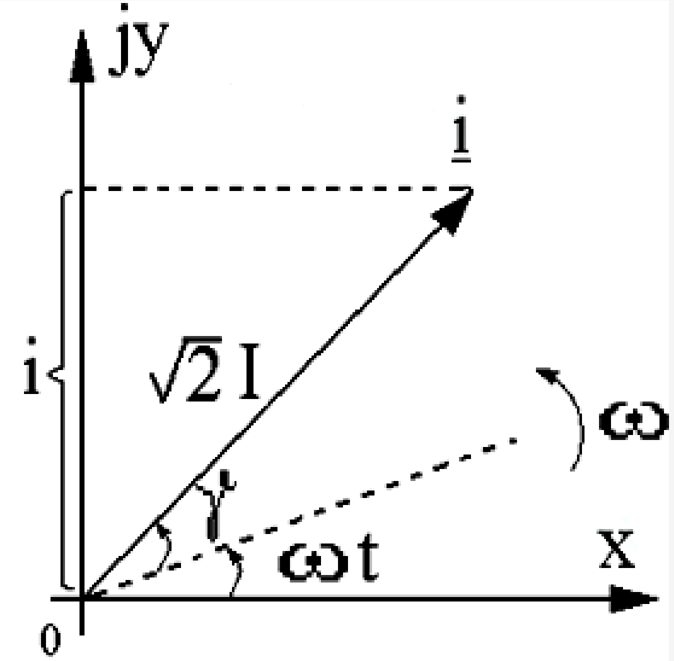
The sinusoidal quantity $i(t) = I\sqrt{2}\sin(\omega t + \gamma)$ has its

❑Geometrical or phasorial representation:

- in complex:

  - Establishes the homographic between the sinusoidal quantity and a rotative vector (phasor) having the length equal with its amplitude and the argument equal with the phase

$$\underline{i} = \sqrt{2}\, I\, e^{j(\omega t+\gamma)}$$

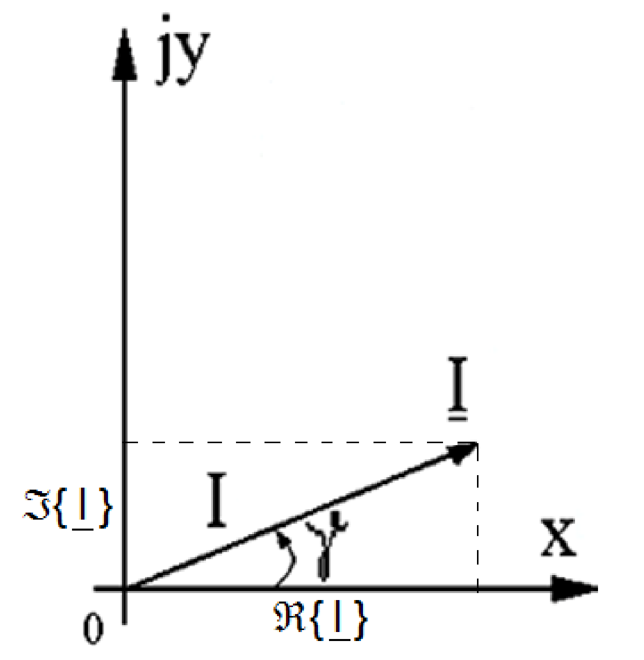


$$i = \mathrm{Im}\{\underline{i}\} = \mathrm{Im}\{\sqrt{2}\, e^{j\omega t}\, \underline{I}\}$$

o in simplified complex :

- Establishes the homographic between the sinusoidal quantity and a rotative vector (phasor) having the length equal with its effective value and the argument equal with the phase angle

$$\underline{I} = I\,e^{j\gamma}$$



- The effective value of the quantity: $I = \sqrt{\Re^2\{\underline{I}\} + \Im^2\{\underline{I}\}}$

$$tg\gamma = \frac{\Im\{\underline{I}\}}{\Re\{\underline{I}\}}$$

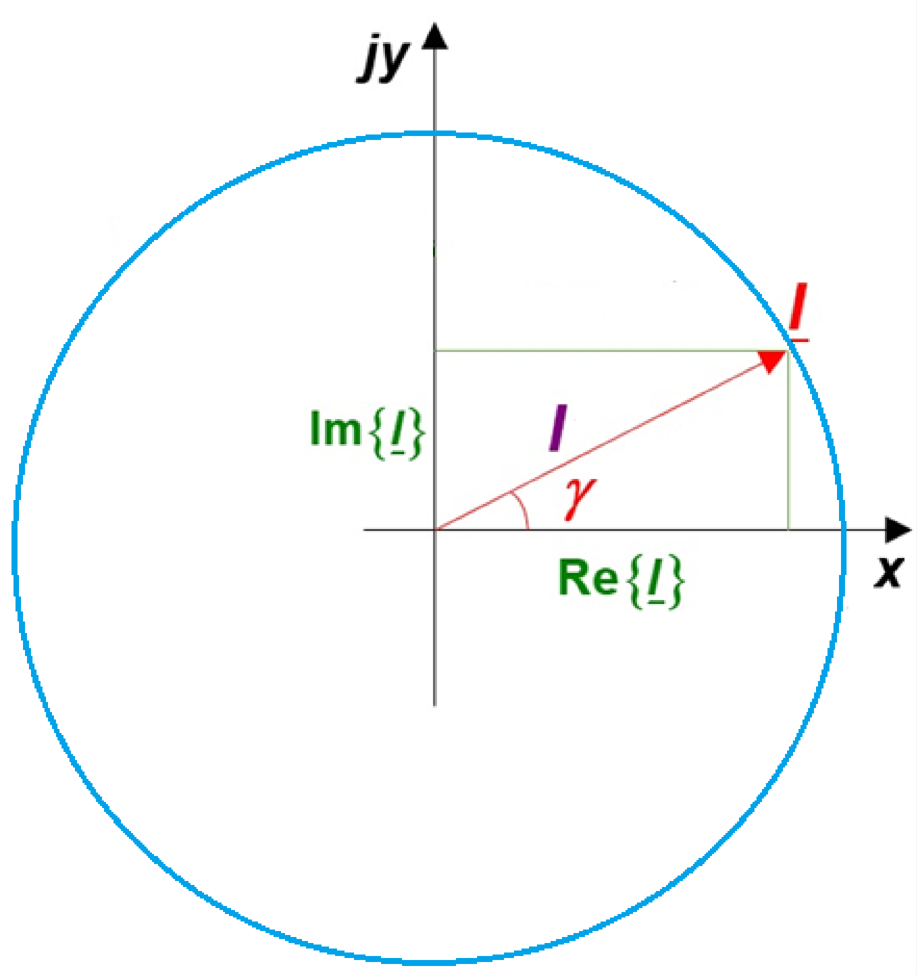
- The phase angle: $\gamma = arctg\,\dfrac{\Im\{\underline{I}\}}{\Re\{\underline{I}\}}$

$$\left.\begin{aligned}\mathrm{Re}\{\underline{I}\} &= I\cdot\cos\gamma\\ \mathrm{Im}\{\underline{I}\} &= I\cdot\sin\gamma\end{aligned}\right\} \Rightarrow \underline{I} = I\cos\gamma + jI\sin\gamma$$

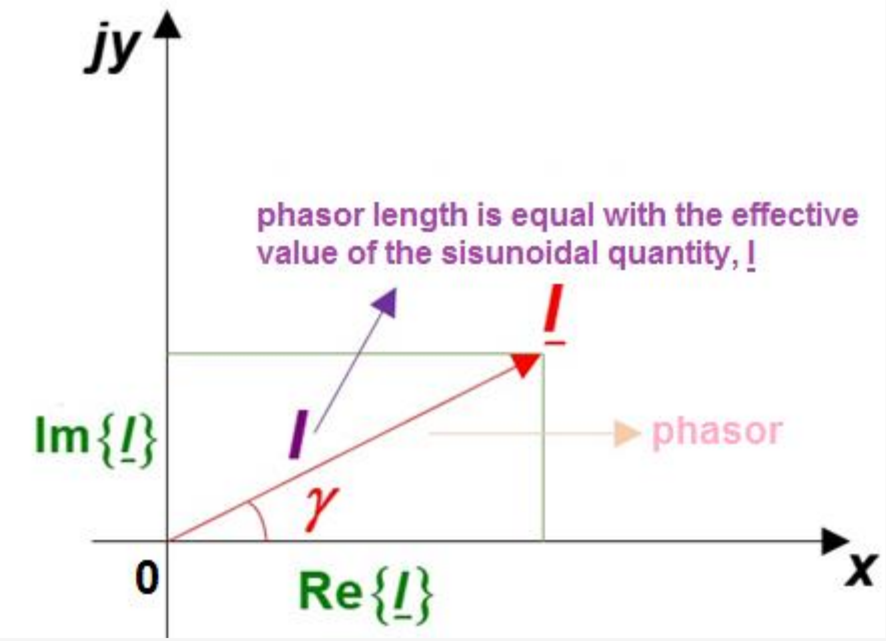
$$\underline{I} = I\angle\gamma$$

■ The phase angle: $\gamma = arctg \dfrac{\Im\{\underline{I}\}}{\Re\{\underline{I}\}}$

$$\gamma = \begin{cases} arctg\dfrac{\text{Im}}{\text{Re}}, & \text{cadranul I; Re, Im >0} \\ \pi + arctg\dfrac{\text{Im}}{\text{Re}}, & \text{cadranele II si III; Re <0, Im<>0} \\ 2\pi + arctg\dfrac{\text{Im}}{\text{Re}}, & \text{cadranul IV, Re >0, Im <0} \end{cases}$$



$\underline{I} = I\angle\gamma$



Operations with Sinusoidal Quantities Symbolical Represented

❑ Sinusoidal Quantities Sum:

$$i_1(t) = I_1\sqrt{2}\sin(\omega t + \gamma_1) \quad , \quad i_2(t) = I_2\sqrt{2}\sin(\omega t + \gamma_2)$$

Leads obvious to a sinusoidal quantity: $$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$$

$$\Updownarrow$$

$$\underline{I} = I_1\cos\gamma_1 + I_2\cos\gamma_2 + j(I_1\sin\gamma_1 + I_2\sin\gamma_2) = \underline{I}_1 + \underline{I}_2$$

- Effective value of the quantity:

$$I = \sqrt{\Re^2\{\underline{I}\} + \Im^2\{\underline{I}\}} \Rightarrow I = \sqrt{I_1^2 + I_2^2 + 2I_1I_2\cos(\gamma_1 - \gamma_2)}$$

- Inițial phase (phase angle):

$$tg\gamma = \frac{\Im\{\underline{I}\}}{\Re\{\underline{I}\}} \Rightarrow tg\gamma = \frac{I_1\sin\gamma_1 + I_2\sin\gamma_2}{I_1\cos\gamma_1 + I_2\cos\gamma_2} \Rightarrow \gamma = arctg\frac{I_1\sin\gamma_1 + I_2\sin\gamma_2}{I_1\cos\gamma_1 + I_2\cos\gamma_2}$$

- Algebraic in complex: $i(t) = i_1(t) + i_2(t) \Leftrightarrow \underline{I} = \underline{I}_1 + \underline{I}_2$

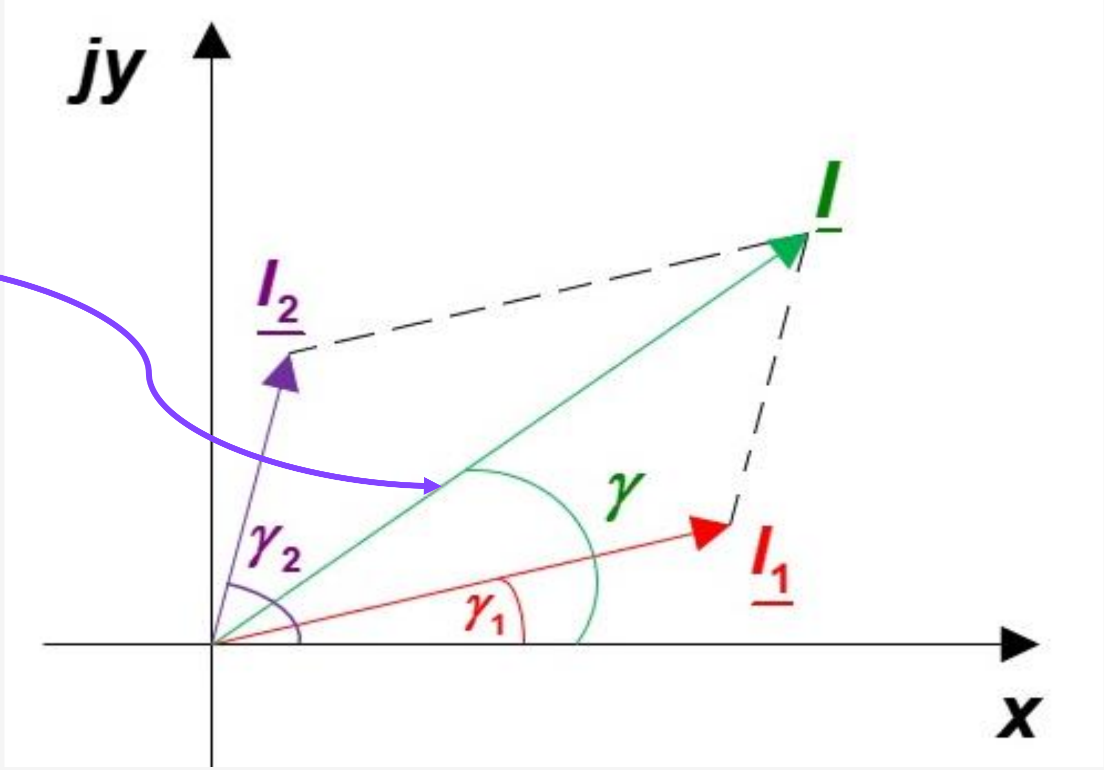
- Geometric as vector or phasors:

Parallelogram rule:

$$\underline{I}_1 = I_1 e^{j\gamma_1} = I_1 \angle \gamma_1$$

$$\underline{I}_2 = I_2 e^{j\gamma_2} = I_2 \angle \gamma_2$$

$$\underline{I} = \underline{I}_1 + \underline{I}_2$$



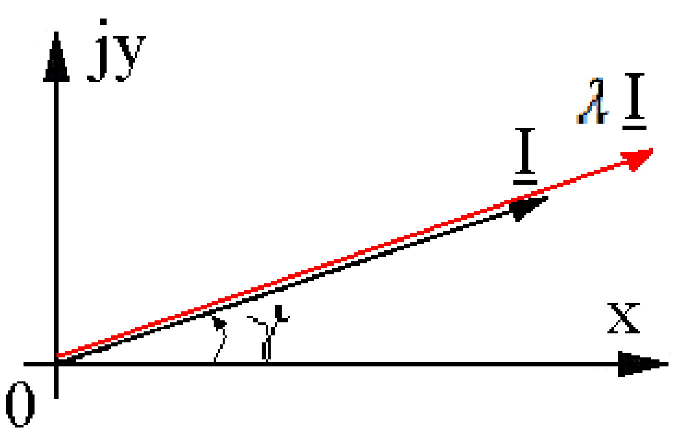
Attention, for effective values:

!!! $I \neq I_1 + I_2$

❑ Multiplying (amplifying) a sinusoidal quantity with a scalar λ

$$i = I\sqrt{2}\sin(\omega t + \gamma)$$

$$\lambda i = \lambda I\sqrt{2}\sin(\omega t + \gamma) \Leftrightarrow \lambda I e^{j\gamma} = \lambda \underline{I}$$

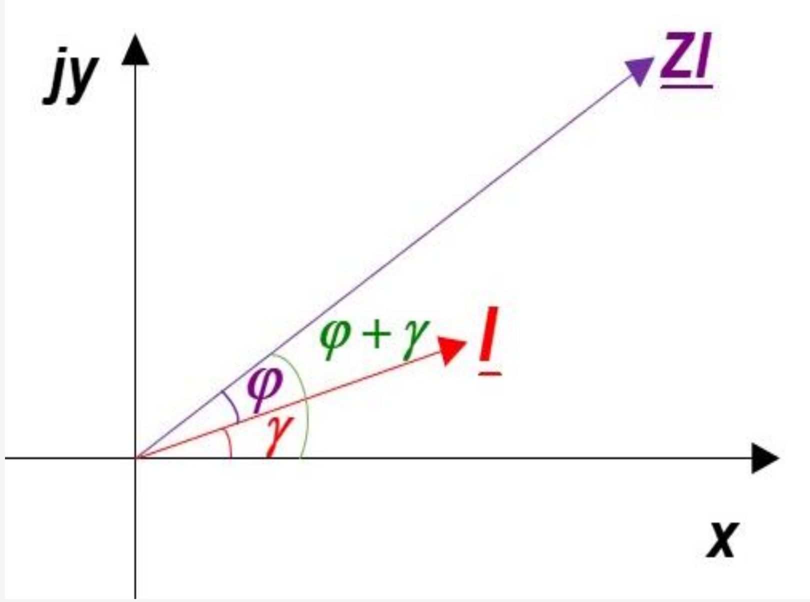


❑ Multiplying $\underline{I}$ with a complex number $\underline{Z}$

$$\underline{I} = I\,e^{j\gamma}$$

$$\underline{Z} = Ze^{j\varphi}$$

$$\Rightarrow \quad \underline{Z}\,\underline{I} = Ze^{j\varphi}\,Ie^{j\gamma} = Z\,I\,e^{j(\varphi+\gamma)}$$

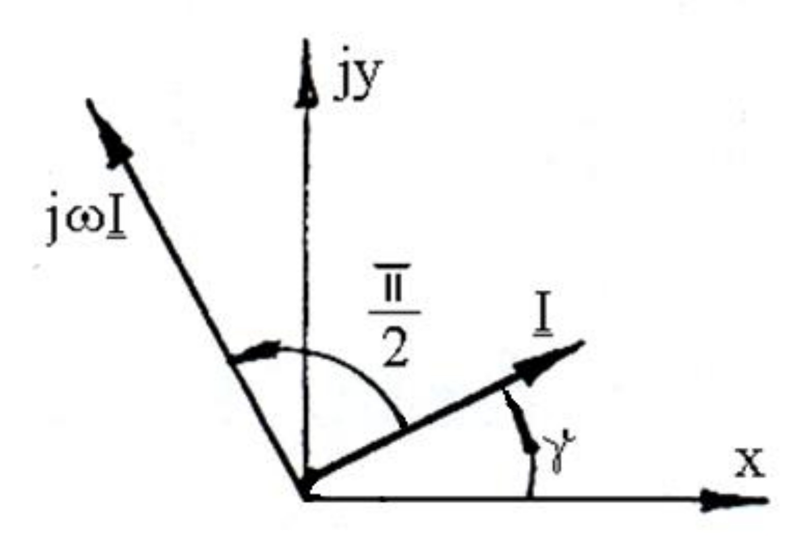


- suppose to increase the phasor length, I, of Z times and to rotate it with the angle φ from its initial position

❑ The derivation of a sinusoidal quantity:

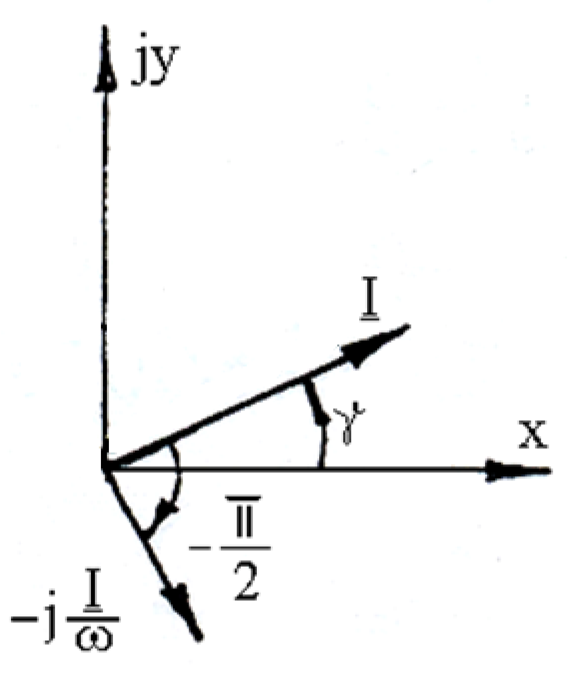
$$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$$

$$\frac{di(t)}{dt} = \sqrt{2}\omega I\cos(\omega t + \gamma) = \sqrt{2}\omega I\sin\left(\omega t + \gamma + \frac{\pi}{2}\right) \Leftrightarrow \omega \underline{I} e^{j\left(\gamma + \frac{\pi}{2}\right)} = j\omega \underline{I}$$



- suppose the increase of the phasor length, I, by ω and its rotation it with the angle $\frac{\pi}{2}$ from its initial position

❑ Integration in time of a sinusoidal quantity



$$i(t) = I\sqrt{2}\sin(\omega t + \gamma)$$

$$\int_0^t i\,dt = -\sqrt{2}\frac{I}{\omega}\cos(\omega t + \gamma) = \sqrt{2}\frac{I}{\omega}\sin\left(\omega t + \gamma - \frac{\pi}{2}\right)$$

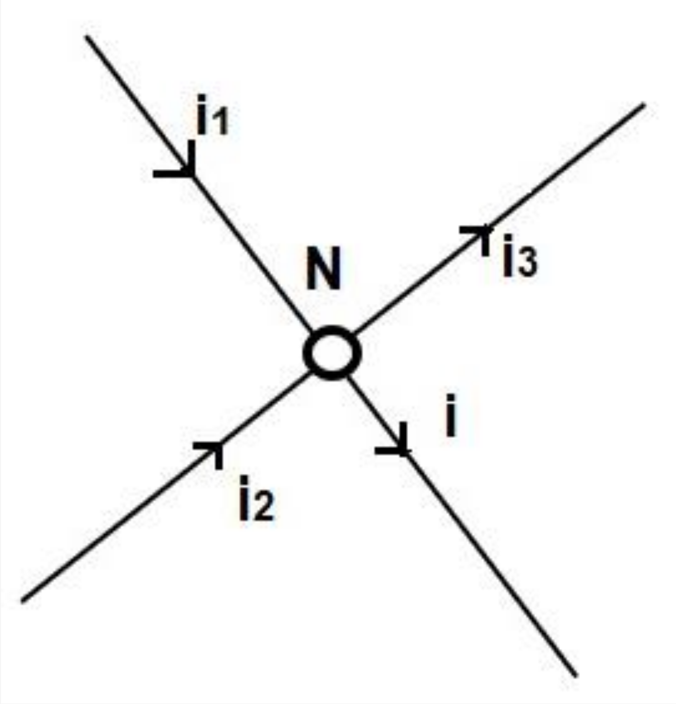
$$\Leftrightarrow \quad \frac{\underline{I}}{\omega}e^{j\left(\gamma - \frac{\pi}{2}\right)} = \frac{\underline{I}}{j\omega} = -j\frac{\underline{I}}{\omega}$$

- suppose the decrease of the phasor length $\underline{I}$ by ω and its rotation with the angle - $\frac{\pi}{2}$ from its initial position

Application

Application 1

Find the current $\boldsymbol{i}$ knowing that:



a) $\boldsymbol{i_1(t) = 5,66 \cdot sin(149^o - \omega t)}$ [A];

b) $\boldsymbol{i_2(t) = -\sqrt{2} \cdot 2 \cdot sin(\omega t - 211^o)}$ [A];

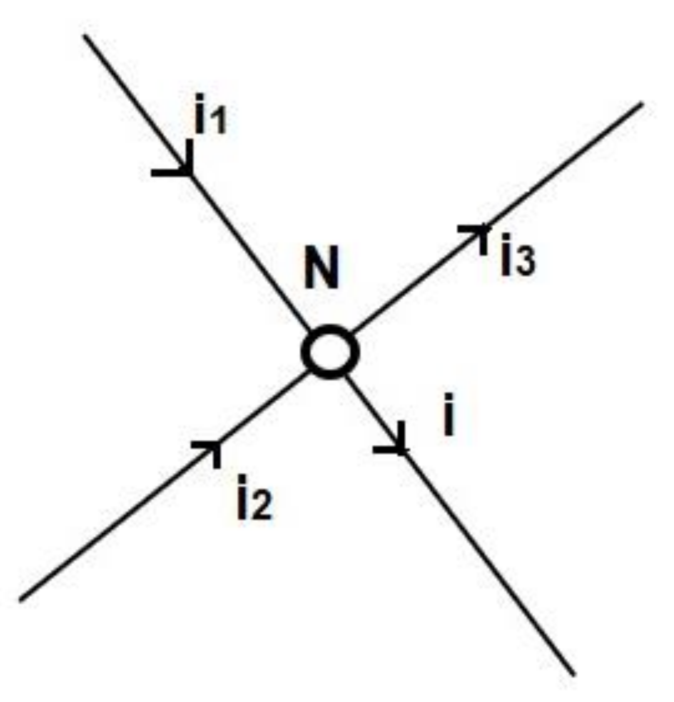
c) $\boldsymbol{i_3(t) = \sqrt{2} \cdot 3} \cdot cos\boldsymbol{(\omega t + 59^o)}$ [A];

and that: $\boldsymbol{i(\mathrm{t}) = i_1(t) + i_2(t) - i_3(t)}$

Solution:

$$i(\mathrm{t}) = i_1(t) + i_2(t) - i_3(t)$$

$$i(\mathrm{t}) = i_1(t) + i_2(t) - i_3(t) \rightleftarrows \underline{I} = \underline{I_1} + \underline{I_2} - \underline{I_3}$$



First, we have to write the three currents in the canonical form $i(t) = I\sqrt{2} \cdot sin(\omega t + \gamma)$, where $\gamma \in (-\pi, \pi]$, using the formulas deduced by the trigonometric circle, and then to represent them as complex numbers:

$$sin\,\alpha = sin(180^o - \alpha)$$

$$sin(-\alpha) = -\,sin\,\alpha$$

$$cos\,\alpha = sin(\alpha + 90^o)$$

$$sin(\alpha + 180^o) = -\,sin\,\alpha$$

$$sin(360^o + \alpha) = sin\,\alpha$$

a) $i_1(t) = 5,66 \cdot sin(149^o - \omega t)$ [A]

$$I_1 = \frac{5,66}{\sqrt{2}} = \frac{5,66}{1,41} \Rightarrow I_1 = 4$$

$$sin(149^o - \omega t) = sin[180^o - (149^o - \omega t)] =$$

$$= sin(180^o - 149^o + \omega t) = sin(31^o + \omega t)$$

$$= sin(\omega \cdot t + 31^o)$$

$$\Rightarrow \quad i_1(t) = 4\sqrt{2} \cdot sin(\omega t + 31^o)$$

- in simplified complex ⇒ $i_1(t) = 4\sqrt{2} \cdot sin(\omega t + 31^o) \rightleftarrows \underline{I_1} = 4 \cdot e^{j31^o}$

- Apply the Euler's Theorem:

$$\underline{I_1} = 4 \cdot (cos\,31^o + j \cdot sin\,31^o) = 4 \cdot \Big(\underbrace{0,85717}_{cos\,31^o} + j \cdot \underbrace{0,51504}_{sin\,31^o}\Big)$$

$$= 3,428 + j \cdot 2,06$$

$$\Rightarrow \quad \underline{I_1} = 3,428 + j \cdot 2,06 \text{ [A]}$$

b) $i_2(t) = -\sqrt{2} \cdot 2\, sin(\omega t - 211^o)\ [A]$

$$\left.\begin{aligned} &I_2 = \frac{\sqrt{2} \cdot 2}{\sqrt{2}} = 2 \\ &sin(\omega t - 211^o) = sin[180^o - (\omega t - 211^o)] = \\ &= sin(180^o - \omega t + 211^o) = sin(-\omega t + 31^o) = \\ &= sin[-(\omega t - 31^o)] = -sin(\omega t - 31^o) \\ &\Rightarrow i_2(t) = -2\sqrt{2} \cdot [-sin(\omega t - 31^o)] \end{aligned}\right\} \Rightarrow i_2(t) = 2\sqrt{2} \cdot sin(\omega t - 31^o)$$

- in simplified complex $\Rightarrow$ $i_2(t) = 2\sqrt{2} \cdot sin(\omega t - 31^o) \rightleftarrows \underline{I_2} = 2 \cdot e^{-j31^o}$

- Apply the Euler's Theorem:

$$\underline{I_2} = 2 \cdot e^{j(-31^o)} = 2 \cdot (cos\, 31^o - j \cdot sin\, 31^o) = 2 \cdot (0,85717 - j \cdot 0,51504)$$

$$\Rightarrow \underline{I_2} = 1,714 - j \cdot 1,03\ [A]$$

c) $i_3(t) = \sqrt{2} \cdot 3\, cos(\omega t + 59^o)\ [A]$

$$\left.\begin{array}{r} I_3 = \dfrac{\sqrt{2} \cdot 3}{\sqrt{2}} = 3 \\ cos(\omega \cdot t + 59^o) = sin(\omega t + 59^o + 90^o) = \\ = sin(\omega t + 149^o) \end{array}\right\} \Rightarrow i_3(t) = 3\sqrt{2} \cdot sin(\omega t + 149^o)$$

- in simplified complex $\quad i_3(t) = 3\sqrt{2} \cdot sin(\omega t + 149^o) \rightleftarrows \underline{I_3} = 3 \cdot e^{j \cdot 149^o}$

- Apply the Euler's Theorem:

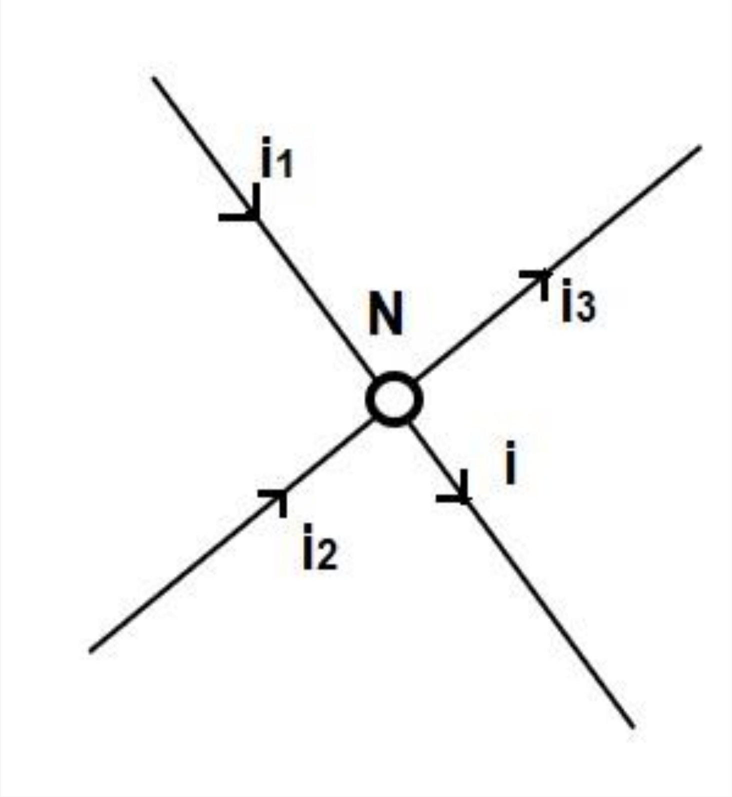
$$\Rightarrow \underline{I_3} = 3 \cdot (cos\, 149^o + j \cdot sin\, 149^o)$$

$$= 3 \cdot (-0,85717 + j \cdot 0,51504) = -2,571 + j \cdot 1,545$$

$$\Rightarrow \underline{I_3} = -2,571 + j \cdot 1,545\ [A]$$

The current i is:

$$\boldsymbol{i}(\mathbf{t}) = \boldsymbol{i}_1(\boldsymbol{t}) + \boldsymbol{i}_2(\boldsymbol{t}) - \boldsymbol{i}_3(\boldsymbol{t}) \rightleftarrows \underline{\boldsymbol{I}} = \underline{\boldsymbol{I}_1} + \underline{\boldsymbol{I}_2} - \underline{\boldsymbol{I}_3}$$



$$\underline{\boldsymbol{I}_1} = \mathbf{3,428} + \boldsymbol{j} \cdot \mathbf{2,06}\ [\mathrm{A}]$$

$$\underline{\boldsymbol{I}_2} = \mathbf{1,714} - \boldsymbol{j} \cdot \mathbf{1,03}\ [\boldsymbol{A}]$$

$$\underline{\boldsymbol{I}_3} = -\mathbf{2,571} + \boldsymbol{j} \cdot \mathbf{1,545}\ [\boldsymbol{A}]$$

Knowing these complex numbers for the three currents, now we can solve this algebraic (is complex) or geometric (as phasors) :

Method I - Algebraic (in complex)

$$\underline{I} = \underline{I_1} + \underline{I_2} - \underline{I_3} = 3,428 + j \cdot 2,06 + 1,714 - j \cdot 1,03 + 2,571 - j \cdot 1,545$$

$$\Rightarrow \boxed{\underline{I} = 7,713 - j \cdot 0,515}$$

- We know now the complex of $\underline{I}$, by we need the sinusoidal form, so that we must find the effective value and the phase angle to write the i(t):

$$I = \sqrt{Re^2\left\{\underline{I}\right\} + Im^2\left\{\underline{I}\right\}} = \sqrt{(7,713)^2 + (0,515)^2} = \sqrt{59,49 + 0,26} = \sqrt{59,75}$$

- The effective value is: $\boxed{I = 7,73[A]}$

$$\gamma = arctg\frac{Im\left\{\underline{I}\right\}}{Re\left\{\underline{I}\right\}} = arctg\left(\frac{-0,515}{7,713}\right) = arctg(-0,0667) = -3,50 = -3^o50`$$

- The phase angle is: $\boxed{\gamma = -3^o50`}$

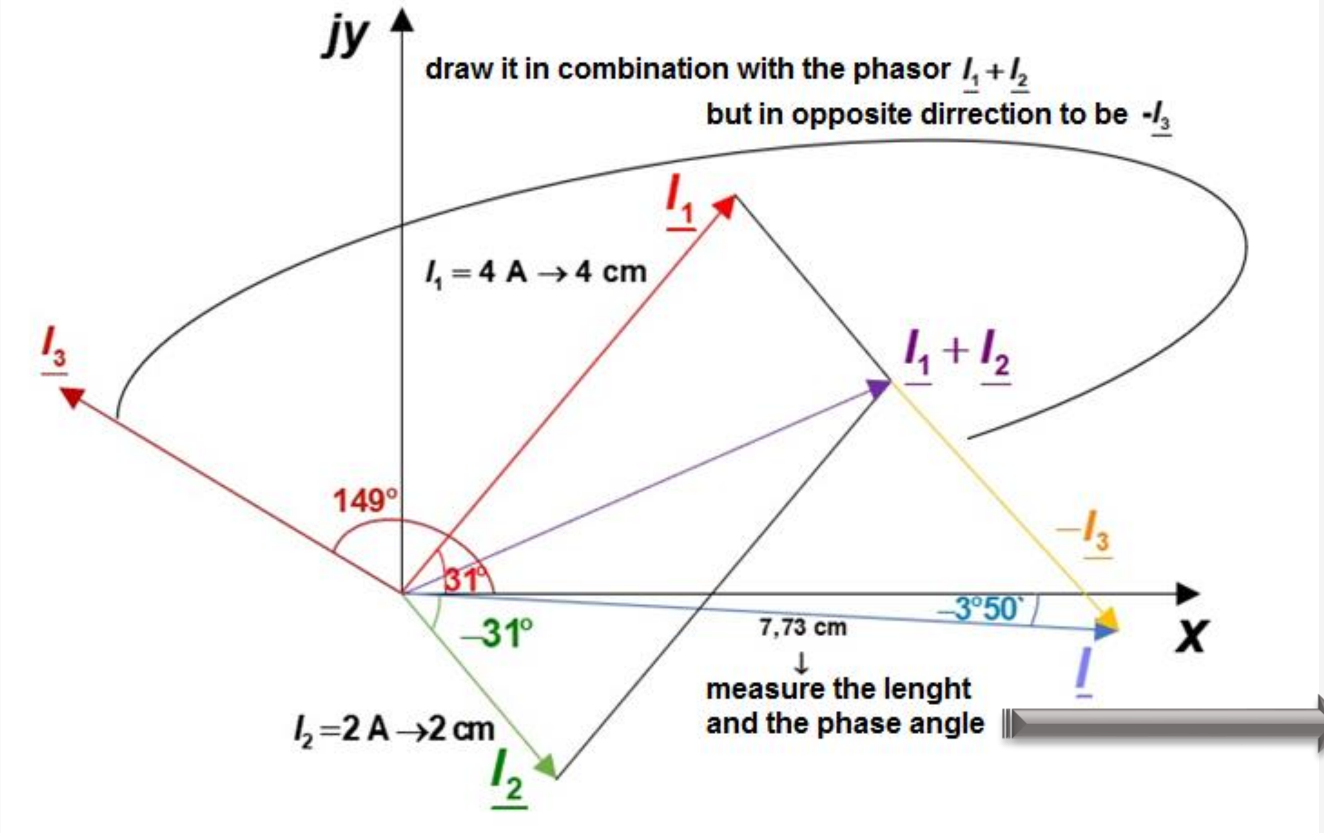
$$\Rightarrow \boxed{i(t) = 7,73 \cdot \sqrt{2}\, sin(\omega \cdot t - 3^o50`)[A]}$$

Method II - Geometric (as phasors)

$$\left.\begin{aligned}\underline{I_1} &= 4\angle 31^o\\ \underline{I_2} &= 2\angle -31^o\\ \underline{I_3} &= 3\angle 149^o\end{aligned}\right\}$$

We draw these currents, considering the scale: 1 cm→1 A

$$i(\mathrm{t}) = i_1(t) + i_2(t) - i_3(t) \rightleftarrows \underline{I} = \underline{I_1} + \underline{I_2} - \underline{I_3}$$



$$\left.\begin{aligned}&\boxed{I = 7,73[A]}\\ &\boxed{\gamma = -3^o50`}\end{aligned}\right\}$$

$$\boxed{i(t) = 7,73 \cdot \sqrt{2}\, sin(\omega t - 3^o50`)[A]}$$

Application 2

To symbolic representation is simplified complex $\underline{I} = I\angle\gamma_i$ correspond the geometric representation of phasor $\underline{I}$. Find the phasor $\underline{Z}\underline{I}$, corresponding to the multiplication with the complex number $\underline{Z} = Z\angle\varphi$.

Numerical applications: $\underline{I} = 3 + j \cdot 5 \text{ A}$

$$a)\ \underline{Z_1} = 4 - j \cdot 2\ \Omega;$$

$$b)\ \underline{Z_2} = 3\ \Omega;$$

$$c)\ \underline{Z_3} = j\ \Omega;$$

$$d)\ \underline{Z_4} = a = -\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2}\ \Omega;$$

Solution:

To product $\underline{Z}\underline{I} = Z\angle\varphi \cdot I\angle\gamma = Z \cdot I\angle(\gamma + \varphi)$

correspond a phasor obtained from phasor $\underline{I}$

by increase its module by **Z** and its rotation with the angle **φ**

***Method I* - Algebaric (in complex)**

a) $\underline{Z_1} = 4 - j \cdot 2\ \Omega;$

$\underline{I} = 3 + j \cdot 5\ \text{A}$

$$\underline{Z_1}\underline{I} = (4 - j \cdot 2)(3 + j \cdot 5) = 12 + 20 \cdot j - 6 \cdot j - 10 \cdot j^2$$

$$= 12 + 14 \cdot j - 10 \cdot (-1) =$$

$$\underline{Z_1}\underline{I} = 22 + j \cdot 14$$

The effective value of this product:

$$\underline{Z_1}\underline{I} = 22 + 14 \cdot j\ \Omega \quad \Rightarrow Z_1 I = \sqrt{22^2 + 14^2} \quad \Rightarrow Z_1 I = \sqrt{680} \Rightarrow Z_1 I = 26{,}076$$

The phase angle: $(\varphi_1 + \gamma) = arctg\dfrac{14}{22} = arctg\dfrac{7}{11} = arctg 0{,}6363 = 32{,}47$

$$\Rightarrow (\varphi_1 + \gamma) = 32^{o}47`$$

$$\Rightarrow \underline{Z_1}\underline{I} = 22 + j \cdot 14 = 26{,}07\angle 32^{o}47`$$

or

We can find separately the effective value and the phase angle for $\underline{I}$, then for $\underline{Z_1}$ and then to multiple them:

$\underline{Z_1} = 4 - j \cdot 2\ \Omega$

$$\left.\begin{aligned} Z_1 &= \sqrt{4^2 + 2^2} = 4{,}47 \\ \varphi_1 &= arctg\left(-\frac{2}{4}\right) = -26^o56` \end{aligned}\right\} \Rightarrow \underline{Z_1} = 4{,}47\angle -26^o56`$$

$\underline{I} = 3 + j \cdot 5\ \text{A}$

$$\left.\begin{aligned} I &= \sqrt{9 + 25} = 5{,}83 \\ \gamma &= arctg\frac{5}{3} = 59{,}036 = 59^o04` \end{aligned}\right\} \Rightarrow \underline{I} = 5{,}83\angle 59^o04`$$

$$\underline{Z_1}\underline{I} = 4{,}47\angle -26^o56` \cdot 5{,}83\angle 59^o04` =$$

$$= 4{,}47 \cdot 5{,}83\angle\left(-26^o56` + 59^o04`\right) = 26{,}07\angle 32^o47`$$

$$\underline{Z_1}\underline{I} = 26{,}07\angle 32^o47`$$

$\boldsymbol{b)}\ \underline{Z_2} = 3\ \Omega;$

$\underline{I} = 3 + j \cdot 5\ \text{A}$

The multiplication with a constant gives a colinear phasor

$$\underline{Z_2}\underline{I} = 3 \cdot \underline{I} = 3 \cdot (3 + j \cdot 5) = 9 + 15j$$

$$\underline{Z_2}\underline{I} = 9 + 15j$$

$$Z_2 I = \sqrt{9^2 + 15^2} = \sqrt{306} = 17{,}49$$

$$(\varphi_2 + \gamma) = arctg\frac{15}{9} = arctg1{,}66 = 59{,}036 = 59^o04`$$

$$\Rightarrow \underline{Z_2}\underline{I} = 17{,}49\angle 59^o04`$$

c) $\underline{Z_3} = j\ \Omega;$

$$\underline{I} = 3 + j \cdot 5 \text{ A}$$

The imaginary unit can be written $j = 1\angle 90^o$, so, to multiplication by j is corresponding a rotation with 90º

$$\underline{Z_3}\underline{I} = j \cdot \underline{I} = j \cdot (3 + j \cdot 5) = 3 \cdot j + 5 \cdot j^2 = 3 \cdot j + 5 \cdot (-1)$$

$$\underline{Z_3}\underline{I} = -5 + 3j$$

$$Z_3 I = \sqrt{(-5)^2 + 3^2} = \sqrt{34} = 5{,}83$$

$$(\varphi_3 + \gamma) = \pi + arctg\left(\frac{3}{-5}\right) = \pi + arctg(-0{,}6) = 149{,}036$$

$$(\varphi_3 + \gamma) = 149^o04`$$

$$\underline{Z_3}\underline{I} = 5{,}83\angle 149^o04`$$

$$d)\ \underline{Z}_4 = a = -\frac{1}{2} + \mathrm{j}\cdot\frac{\sqrt{3}}{2}\ \Omega;$$

$$\underline{I} = 3 + j\cdot 5\ \mathrm{A}$$

$$a = -\frac{1}{2} + j\cdot\frac{\sqrt{2}}{3} = 1\angle 120^{o}$$

$$\downarrow$$

$$\sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1$$

$$\varphi_4 = arctg\frac{\frac{\sqrt{3}}{2}}{\left(-\frac{1}{2}\right)} = \pi + arctg\left(\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{2}{1}\right) = 180 + arctg\left(-\sqrt{3}\right)$$

$$\varphi_4 = 180 - 60 = 120^{o}$$

Is the rotation operator specific to three phases electric circuits, that suppose a rotation with 120° (forward, in strigonometric direction)

$$\underline{Z}_4\underline{I} = a\cdot\underline{I} = 1\angle 120^{o}\cdot 5{,}83\angle 59^{o}04\text{`}$$

$$\underline{Z}_4\underline{I} = 5{,}83\angle 179^{o}04\text{`}$$

***Method II* - Geometric (phazorial)**

We represent them as phasors, by choosing a proper scale:

$$\left.\begin{aligned}\underline{I} &= 5,83\angle 59^{o}04` \\ \underline{Z}_1 &= 4,47\angle -26^{0}56`\end{aligned}\right\}$$

$$\Rightarrow \underline{Z}_1\underline{I} = 26,07\angle 32^{o}47`$$

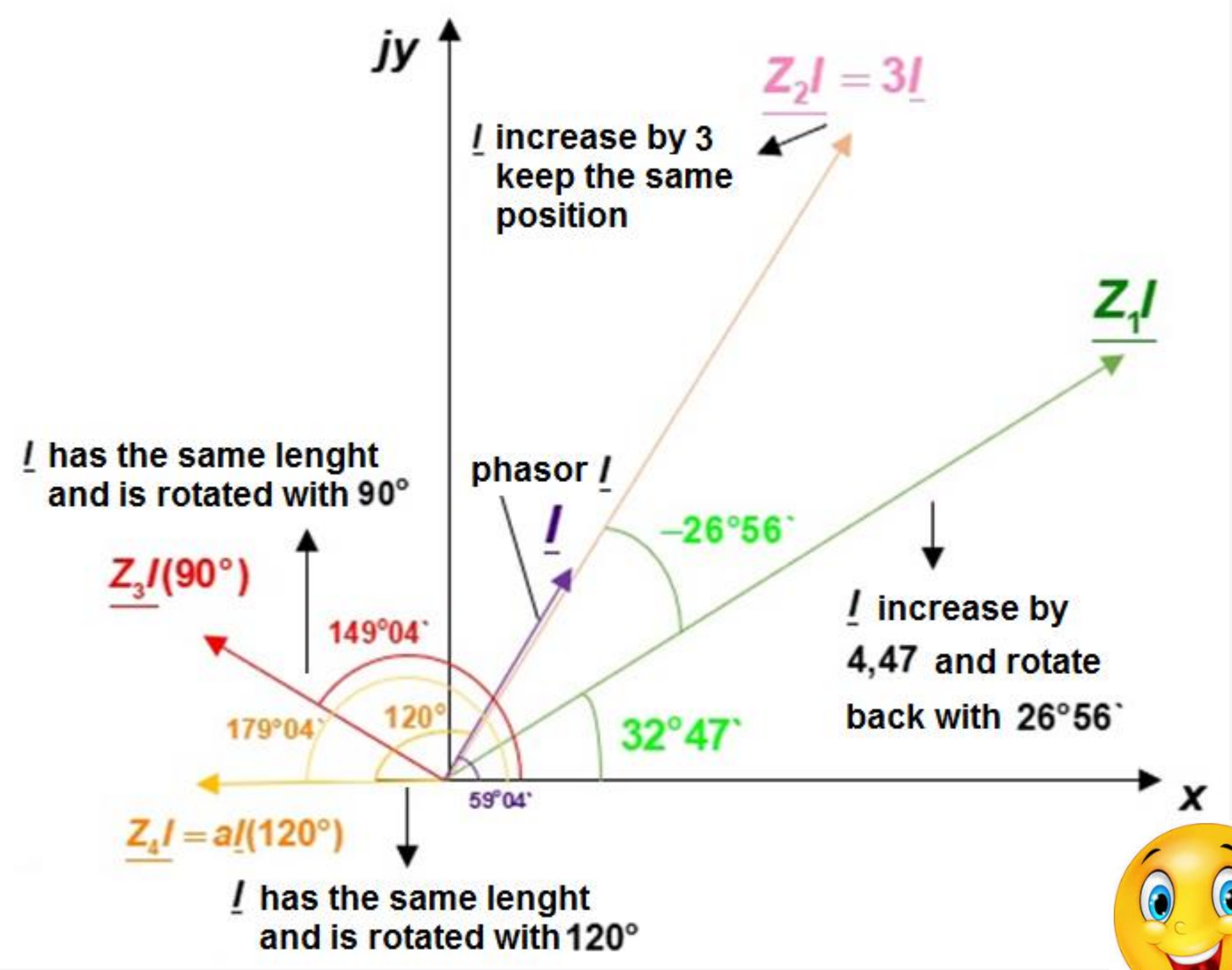
$$\underline{Z}_2\underline{I} = 3\cdot\underline{I} = 17,49\angle 59^{o}04`$$

$$\underline{Z}_3\underline{I} = j\cdot\underline{I}\left(\text{rotation with } 90^{o}\right)$$

$$= 5,83\angle 149^{o}04`$$

$$\underline{Z}_4\underline{I} = a\cdot\underline{I}\left(\text{rotation with } 120^{o}\right)$$

$$= 5,83\angle 179^{o}04`$$



Homework

Write in the canonical form,

$$\mathbf{i(t)} = \boldsymbol{I\sqrt{2}\,sin(\omega \cdot t + \gamma_i)\,, \gamma_i \in (-\pi, \pi]}$$

$$\mathbf{u(t)} = \boldsymbol{U\sqrt{2}\,sin(\omega \cdot t + \gamma_u)\,, \gamma_u \in (-\pi, \pi]}$$

$$\mathbf{e(t)} = \mathbf{E}\boldsymbol{\sqrt{2}\,sin(\omega \cdot t + \gamma_e)\,, \gamma_e \in (-\pi, \pi]}$$

the above quantities mărimile and find their complex values:

a) $e_1(t) = 20\sin\left(\omega \cdot t - \frac{\pi}{4}\right)$ V;

b) $e_2(t) = 60\sin\left(\omega \cdot t + \frac{\pi}{4}\right)$ V;

c) $e_3(t) = 50\sqrt{2}\cos(\omega \cdot t - \pi)$ V;

d) $e_4(t) = 10\sqrt{2}\sin(\omega \cdot t)$ V;

e) $e_5(t) = 30\sqrt{2}\sin\left(\omega \cdot t + \frac{\pi}{2}\right)$ V;

f) $ig_1(t) = \sqrt{2}\sin\left(\omega \cdot t - \frac{\pi}{2}\right)$ A;

g) $u_1(t) = 200\sqrt{2}\sin \underbrace{100\pi}_{\omega=100\pi} t$ V;

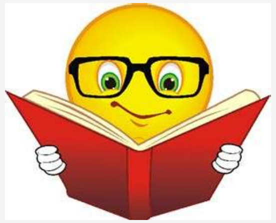
h) $e_6(t) = 120\sin\left(\omega \cdot t + \frac{\pi}{4}\right)$ V;

i) $e_7(t) = 60\sqrt{2}\cos\omega \cdot t$ V;

j) $e_8(t) = 100\sqrt{2}\sin\omega \cdot t$ V;

k) $ig_2(t) = 10\sqrt{2}\sin\omega \cdot t$ A;

Results:



$$\underline{\boldsymbol{E}_1} = \mathbf{10 - 10 \cdot \boldsymbol{j}}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_2} = \mathbf{30 + 30 \cdot \boldsymbol{j}}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_3} = \mathbf{-50 \cdot \boldsymbol{j}}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_4} = \mathbf{10}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_5} = \mathbf{30 \cdot \boldsymbol{j}}, [\mathrm{V}];$$

$$\underline{\boldsymbol{I}_{\boldsymbol{g}_1}} = \mathbf{-\boldsymbol{j}}\,, [\mathrm{A}];$$

$$\underline{\boldsymbol{U}_1} = \mathbf{200}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_6} = \mathbf{60 + 60 \cdot \boldsymbol{j}}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_7} = \mathbf{60 \cdot \boldsymbol{j}}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{E}_8} = \mathbf{100}\,, [\mathrm{V}];$$

$$\underline{\boldsymbol{I}_{\boldsymbol{g}_2}} = \mathbf{10}, [\mathrm{A}].$$

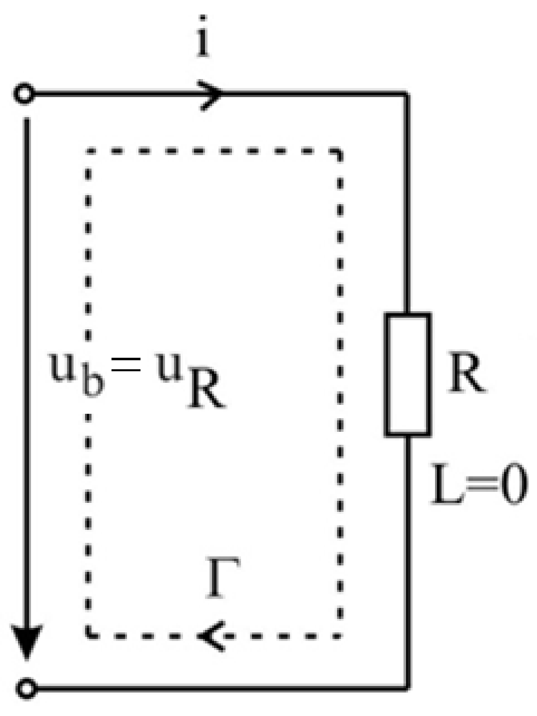
Circuit Elements in Alternative Current

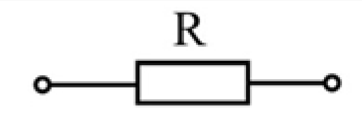
Passive circuit elements

❑ Ideal resistor

$$\oint \overline{E}\overline{ds} = e_\Gamma = u_f - u_b = -\frac{d\Phi}{dt} \cong 0$$

$$\Rightarrow \quad u_b = u_f = R \cdot i \quad \Rightarrow \quad u_R = R \cdot i$$





- **Power delivered by the resistor:**

$$p = u_R \cdot i = R \cdot i^2 > 0$$

$$P = P_{avg} = \frac{1}{T}\int_0^T p \cdot dt = \frac{1}{T}\int_0^T R \cdot 2 \cdot I^2 \sin^2 \omega t dt =$$

$$= \frac{1}{T} R \cdot 2 \cdot I^2 \int_0^T \frac{1}{2}(1 - \cos 2\omega t) dt$$

$$\Rightarrow \quad P = \frac{1}{T} R \cdot 2 \cdot I^2 \frac{T}{2} = R \cdot I^2 \quad [W]$$

- **Active Power:**

$$P = R \cdot I^2 \quad [W]$$

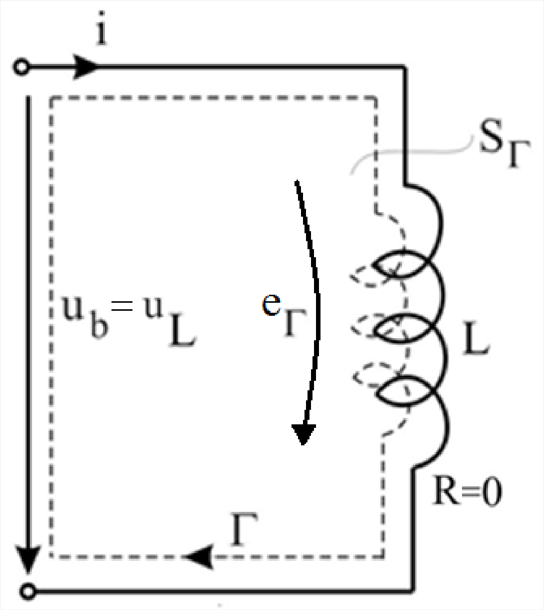
❑ Ideal inductor

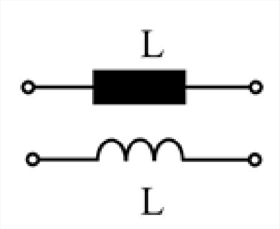
$$\int_{\Gamma} \overline{E} \cdot \overline{ds} = e_{\Gamma} = u_f - u_b = -\frac{d\Phi}{dt}$$

$$\Rightarrow \quad u_f = R \cdot i \cong 0 \quad \Rightarrow \quad u_b = -e_{\Gamma} = \frac{d\Phi}{dt}$$

$$\xrightarrow{\Phi = L \cdot i} \quad u_L = \frac{d\Phi}{dt} = L\frac{di}{dt}$$

$$\Rightarrow \quad p = u_L \cdot i = \frac{d}{dt}\left(\frac{L \cdot i^2}{2}\right) >< 0$$





$$p = u_L \cdot i$$

$$i = I\sqrt{2}\sin\omega t$$

$$u_L = L\frac{di}{dt} = \underbrace{L \cdot I \cdot \sqrt{2}\,\omega}_{U_{max}} \cos\omega t$$

$$\Rightarrow \quad p = u_L \cdot i = L\,I\sqrt{2}\,\omega\cos\omega t\, I\sqrt{2}\sin\omega t$$

$$p = \omega L I^2 2\sin\omega t\cos\omega t$$

$$p = \omega L I^2 \sin 2\omega t$$

The average power: $P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T \omega L I^2 \sin 2\omega t\,dt$

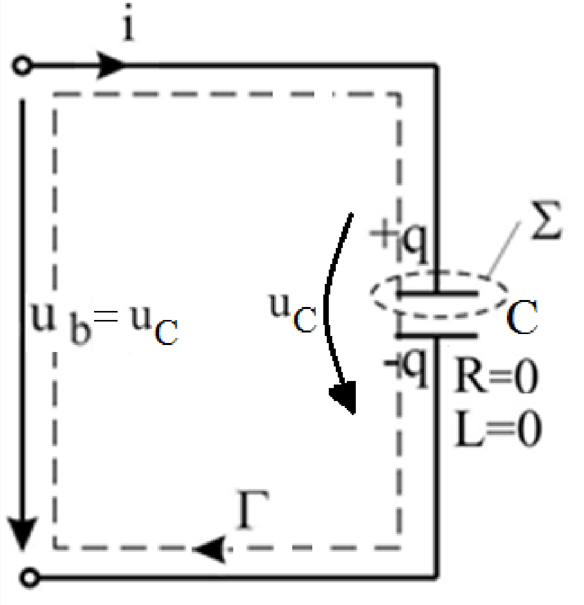
$$\frac{1}{T}\int_0^T I\sqrt{2}\sin 2\omega t = 0$$

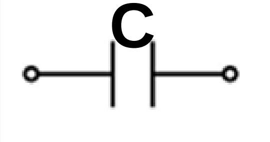
$$\Rightarrow \quad P = 0$$

- **The ratio:** $\frac{U_{max}}{I_{max}} = \frac{\omega L I\sqrt{2}}{I\sqrt{2}} = \omega L$ **is called inductive reactance**

(reactanta iuductiva): $X_L = \omega L \quad [\Omega]$

❑ Ideal capacitor





$$e_\Gamma = \oint_\Gamma \overline{E}\,\overline{ds} = u_f + u_c - u_b = -\frac{d\Phi}{dt}$$

$$\left.\begin{aligned} e_\Gamma &= -\frac{d\Phi}{dt} = -L\frac{di}{dt} \cong 0 \\ u_f &= R \cdot i \cong 0 \end{aligned}\right\} \Rightarrow u_b = u_c = \frac{q}{C}$$

$$i = \frac{dq}{dt} = \frac{d}{dt}(C \cdot u_c) = C\frac{du_c}{dt} \Rightarrow u_c = \frac{1}{C}\int i\,dt$$

$$\Rightarrow p = u_c \cdot i = \frac{d}{dt}\left(\frac{C \cdot u_c^2}{2}\right) >< 0$$

$$p = u_C \cdot i$$

$$u_C = U\sqrt{2}\sin\omega t$$

$$i_C = C\frac{du_c}{dt} \Rightarrow i_C = \underbrace{CU\sqrt{2}\,\omega}_{I_{max}}\cos\omega t$$

$$\Rightarrow p = u_C \cdot i = U\sqrt{2}\sin\omega t\, CU\sqrt{2}\,\omega\cos\omega t$$

$$p = \omega CU^2\, 2\sin\omega t\cos\omega t$$

$$p = \omega CU^2 \sin 2\omega t$$

The average power: $P = \frac{1}{T}\int_0^T p\,dt = \frac{1}{T}\int_0^T \omega CU^2 \sin 2\omega t\,dt$

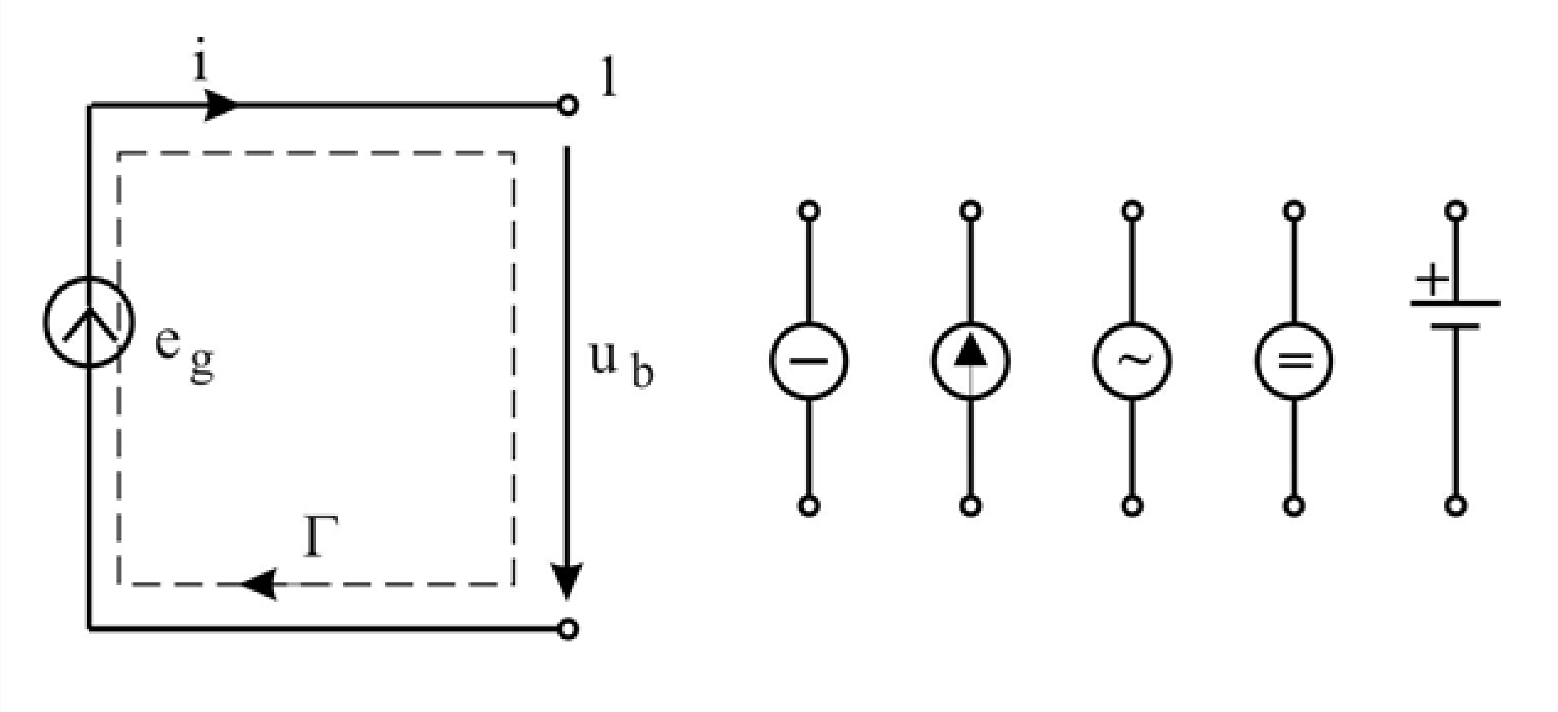
$$\frac{1}{T}\int_0^T U\sqrt{2}\sin 2\omega t = 0$$

$$\Rightarrow P = 0$$

- **The ratio:** $\frac{U_{max}}{I_{max}} = \frac{U\sqrt{2}}{CU\sqrt{2}\,\omega} = \frac{1}{\omega C}$ **is called captance (reactanță capacitivă):** $X_C = \frac{1}{\omega C}\ \ [\Omega]$

Active circuit elements

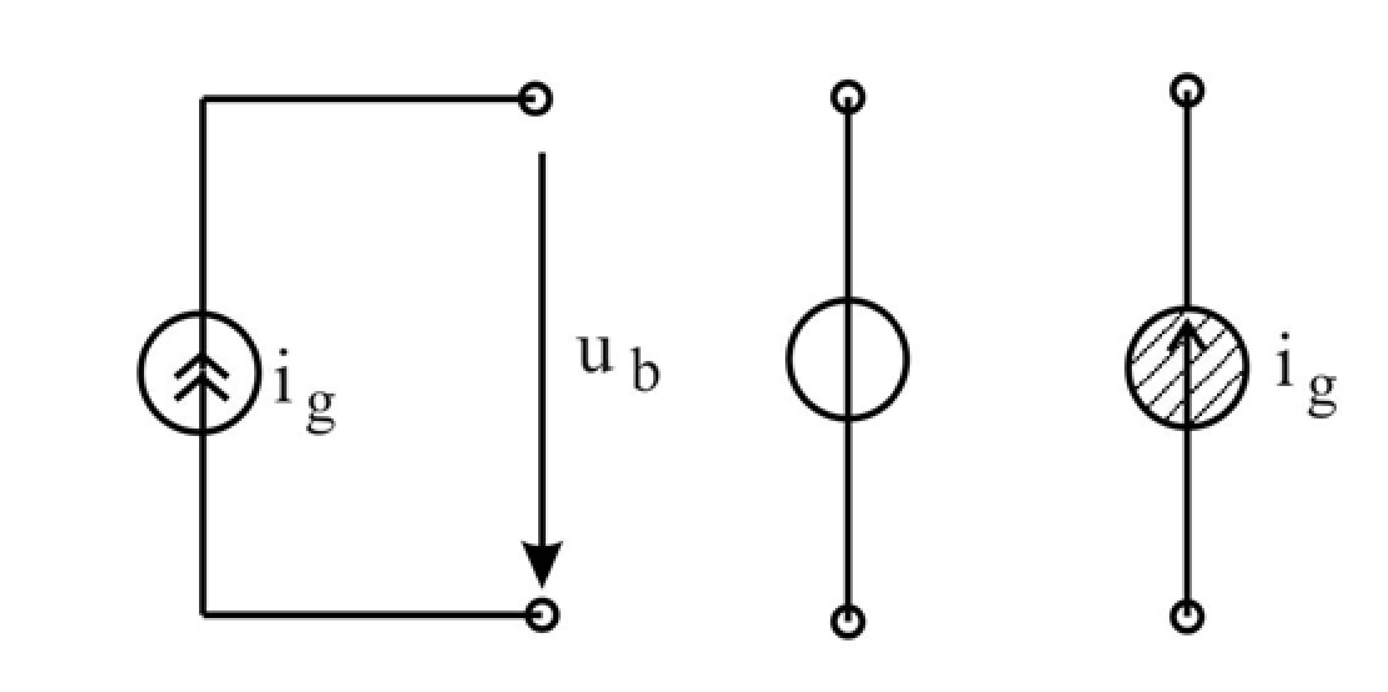
❑ Ideal Voltage Source



$$e_g = u_b$$

$$e_g = e = E\sqrt{2}\sin(\omega t + \gamma_e)$$

❑ Ideal current source



$$i_g = I_g \sqrt{2} \sin(\omega t + \gamma_{i_g})$$

Thank you for your attention!!!

☺☺☺☺☺

Questions???