Electrotechnics ET

Course 9

Year I-ISA English

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= Course 9 =

ELECTRICAL CIRCUITS IN HARMONIC REGIME

Linear Electric Circuits in Permanent Sinusoidal Regime

Phasors Diagrams. Applications

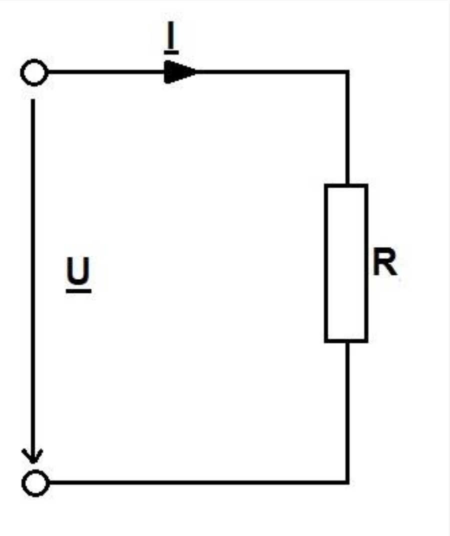
Complex Equivalent Impedances. Applications

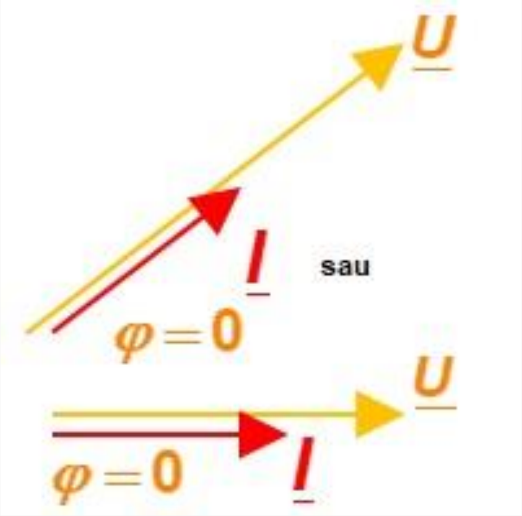
Phasors Diagrams

There exist two types of phasors diagrams:

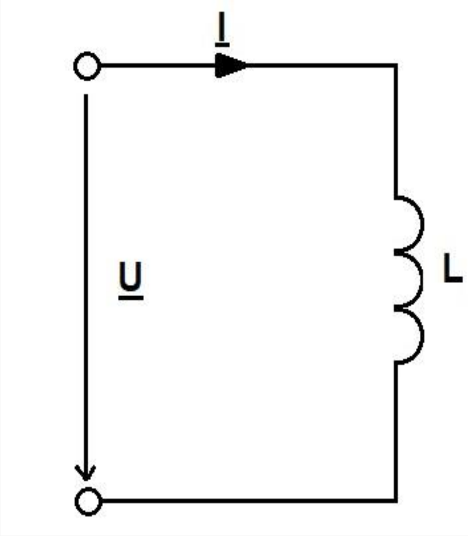
- Phasors Diagrams at Scale:
  - **are drawn respecting the effective values and the phase shift angles of the voltages and currents;**
- Phasors Diagrams of Principle:
  - **are drawn respecting the phase shifts introduced by resistors (0^o), inductors (90^o), capacitors (-90^o) and adding voltages (in series) and currents (in parallel) according to the parallelogram rule;**

❑ Ideal resistor:

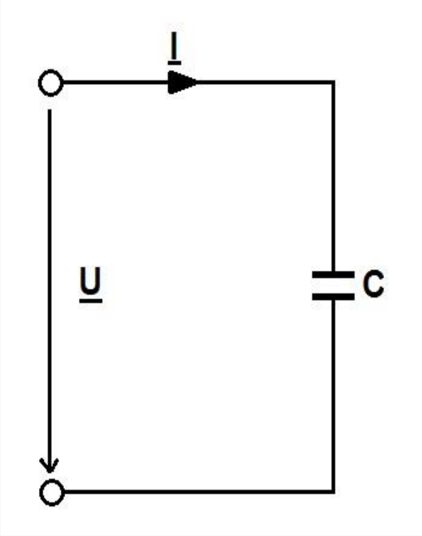


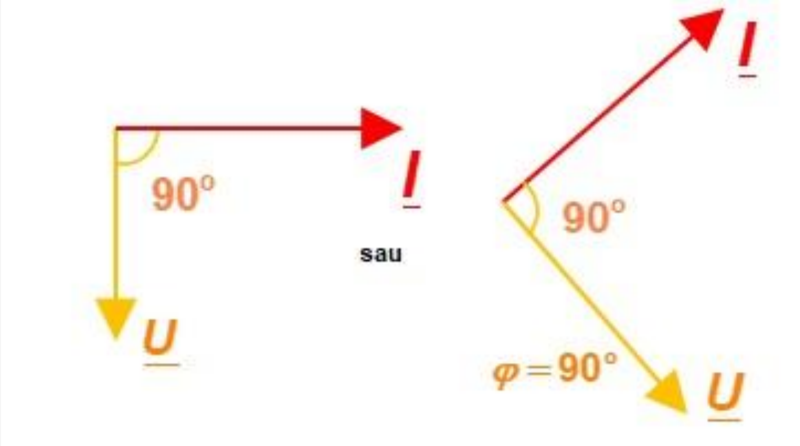


❑ Ideal inductor:



❑Ideal capacitor:

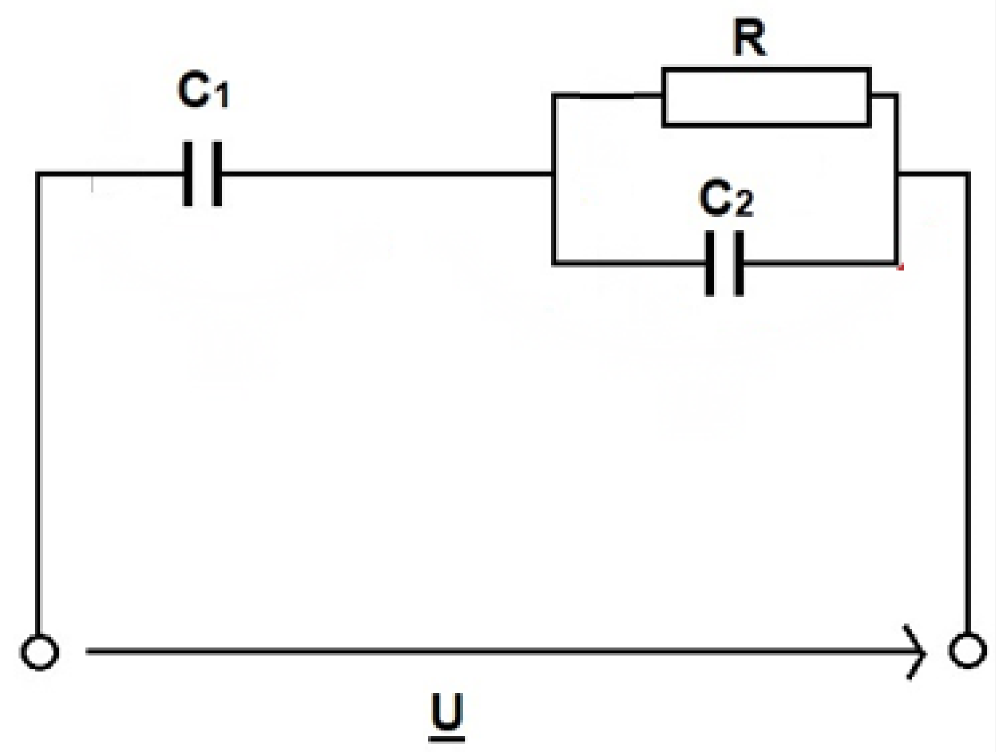




Applications

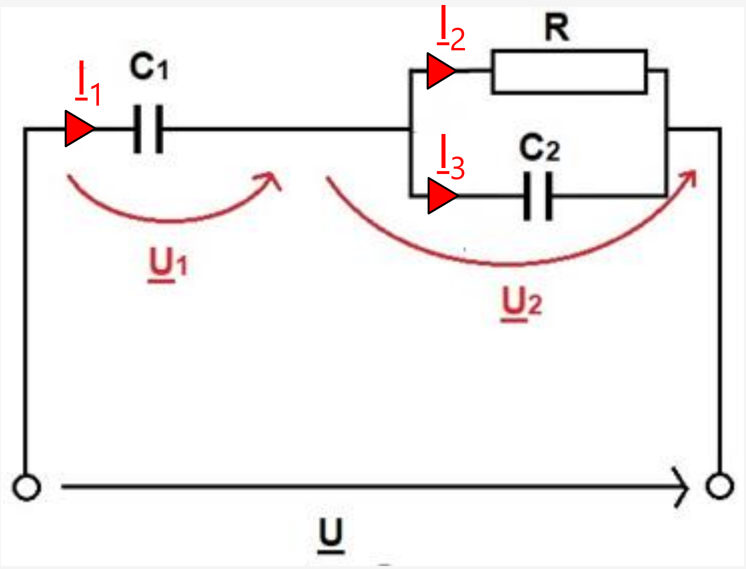
Problem 1

Draw the phasors diagram of principle for to the circuit from the figure:



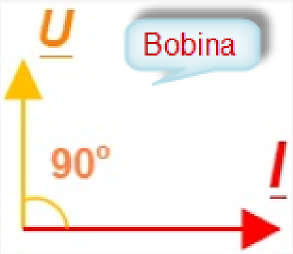
Solution:

Solution:

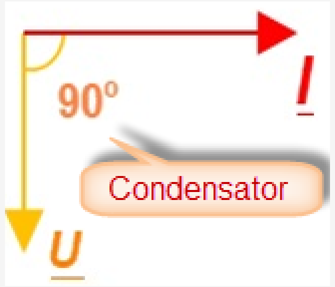


I1
C1
I2
R
I3
C2
U1
U2
U

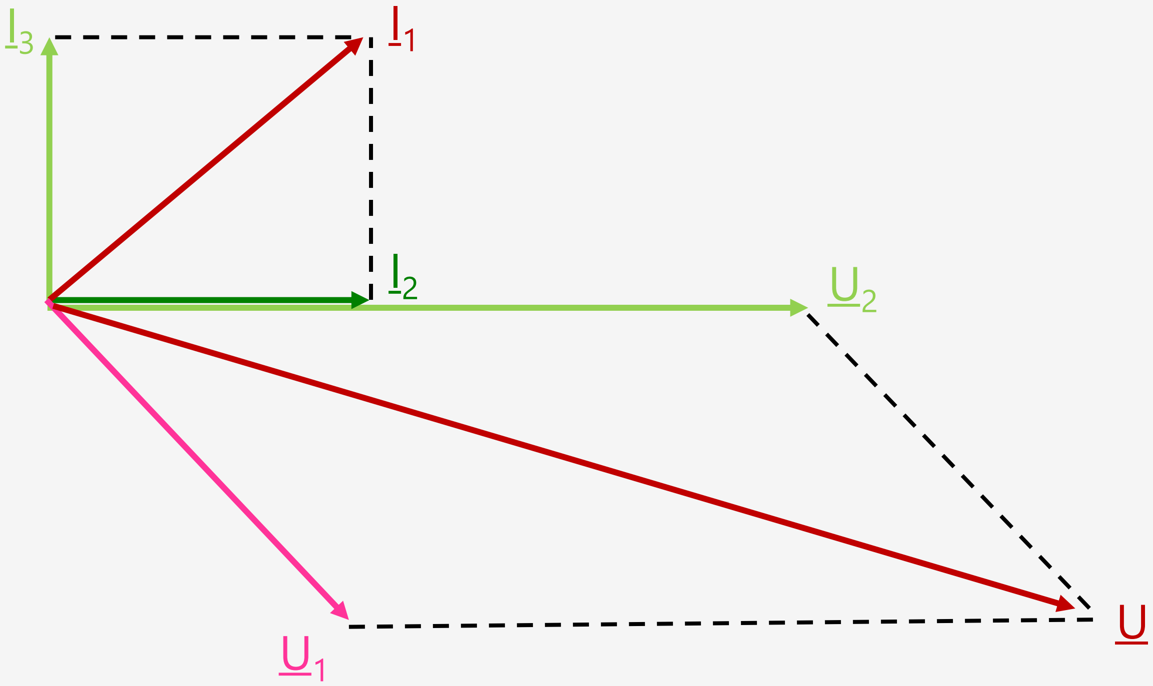
Rezistență
U
φ = 0
I



U
Bobina
90°
I



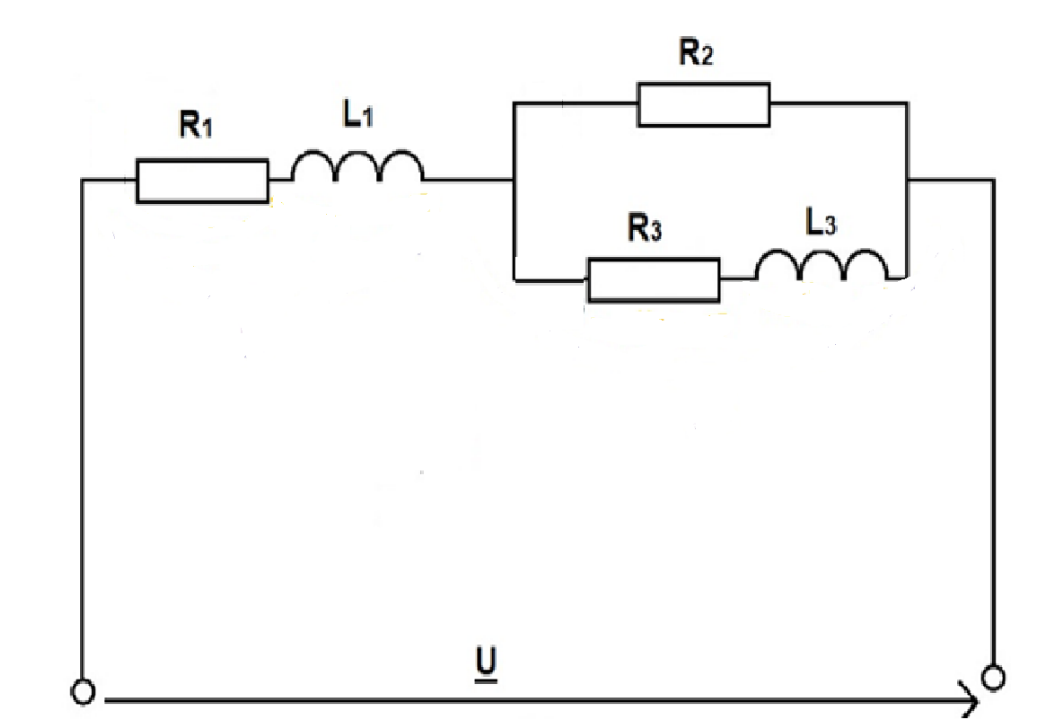
90°
I
Condensator
U



I3
I1
I2
U2
U1
U

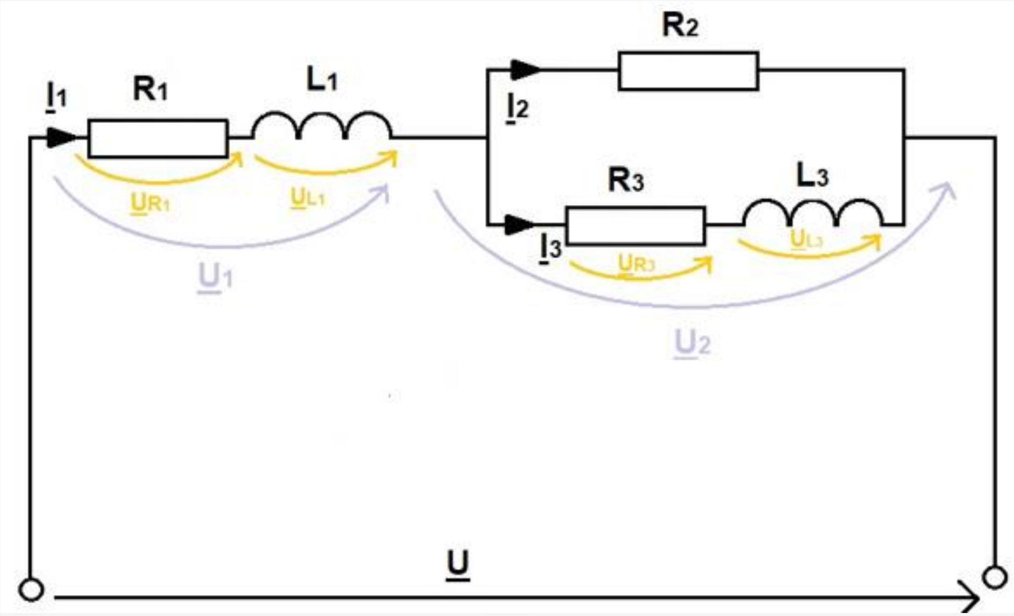
Problem 2

Draw the phasors diagram of principle for the circuit from the figure:



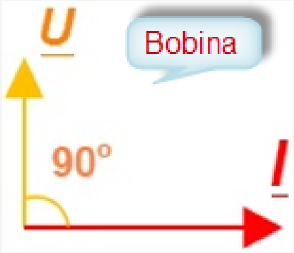
Solution:

Solution:

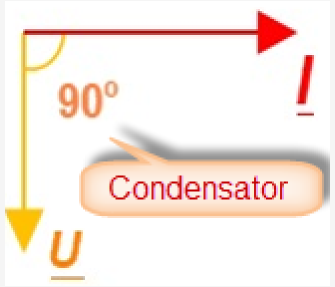


R2
I1
R1
L1
I2
R3
L3
I3
UR1
UL1
UR3
UL3
U1
U2
U

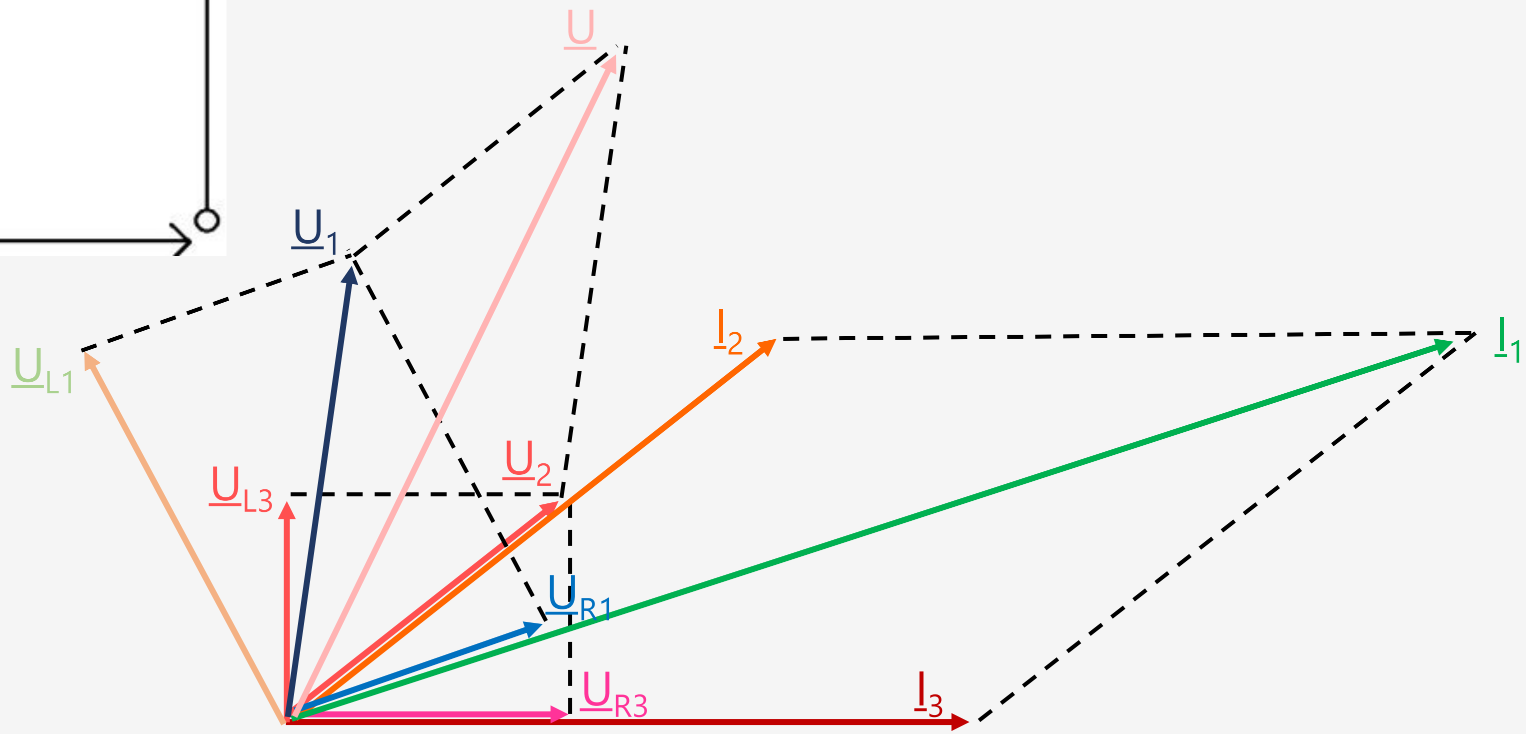
Rezistență
U
φ = 0
I



U
Bobina
90°
I



I
90°
Condensator
U

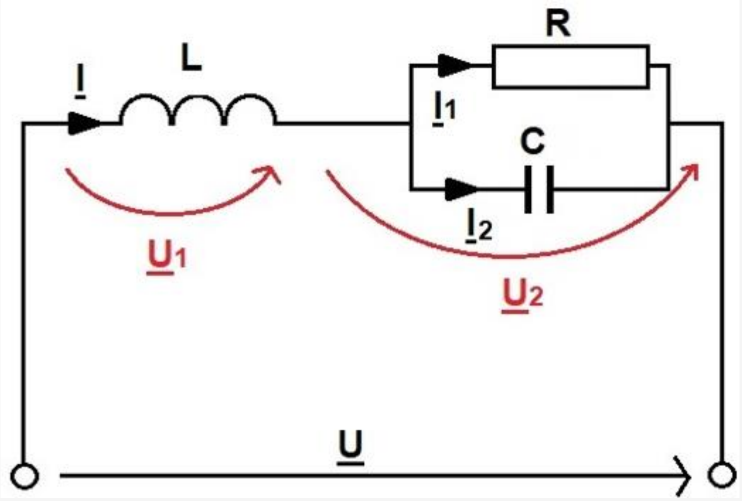


U
U1
I2
I1
UL1
U2
UL3
UR1
UR3
I3

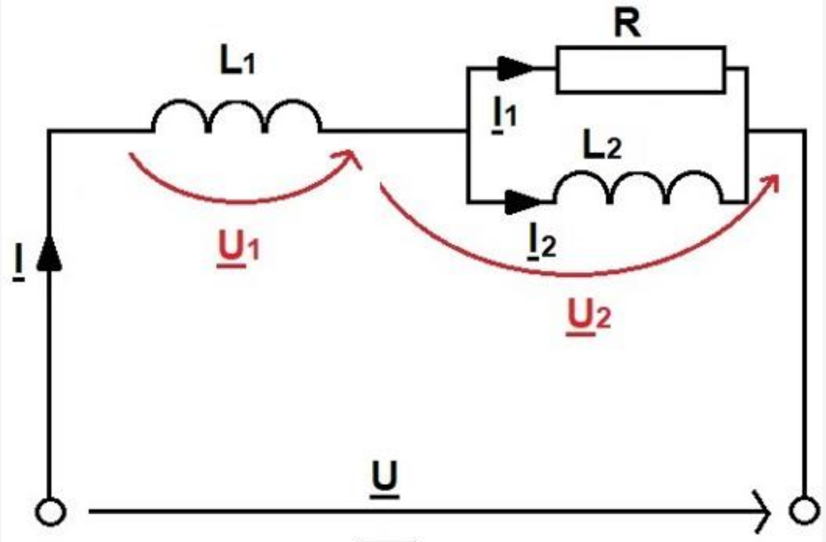
Homework

Draw the phasors diagrams of principle for the circuits from the figures:

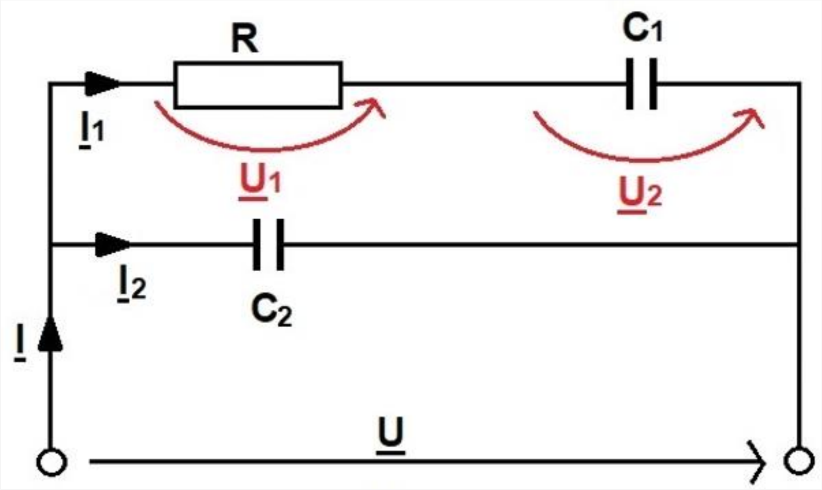
a)



b)

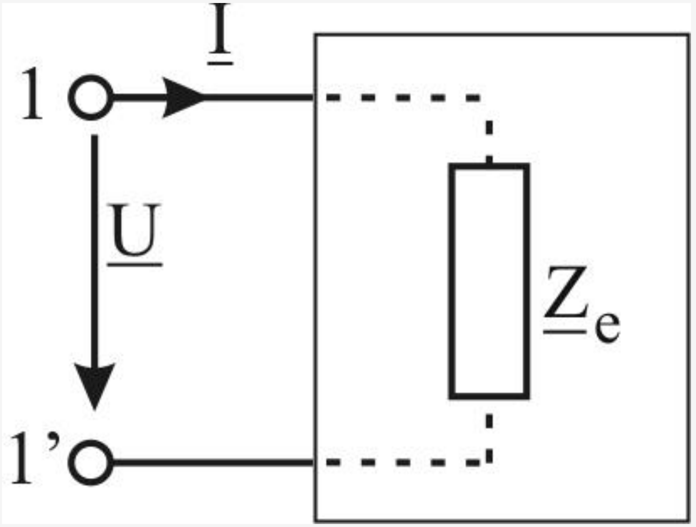


c)



Complex Equivalent Impedances

Equivalent Impedance for Connections without Inductive Coupling



- **Equivalent Complex Impedance:**

$$\underline{Z}_e = \frac{\underline{U}}{\underline{I}} = Z_e e^{j\varphi_e} = R_e + jX_e$$

- **Equivalent Complex Admittance:**

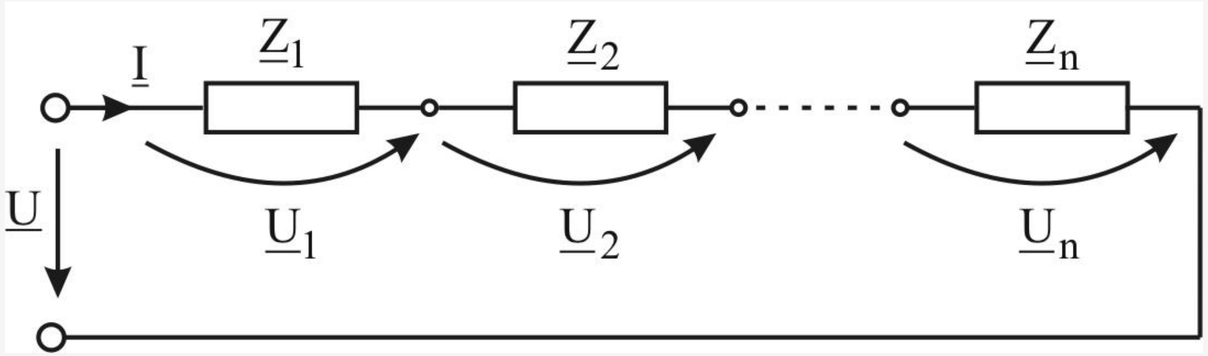
$$\underline{Y}_e = \frac{1}{\underline{Z}_e} = \frac{\underline{I}}{\underline{U}} = Y_e e^{-j\varphi_e} = G_e - jB_e$$

1. Passive Branches without Inductive Coupling

A) Series Circuits (Connections)

$$\underline{U} = \underline{U}_1 + \underline{U}_2 + \ldots + \underline{U}_n$$

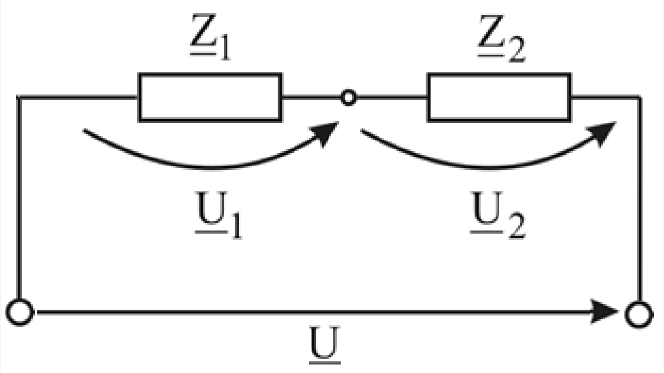
$$\underline{U} = \underline{Z}_1\underline{I} + \underline{Z}_2\underline{I} + \ldots + \underline{Z}_n\underline{I} = \underline{I}\sum_{k=1}^{n}\underline{Z}_k$$



$$\Rightarrow \underline{Z}_e = \frac{\underline{U}}{\underline{I}} = \sum_{k=1}^{n}\underline{Z}_k \Rightarrow \underline{Z}_e = R_e + jX_e \Rightarrow R_e = \sum_{k=1}^{n} R_k > 0 \Rightarrow X_e = \sum_{k=1}^{n} X_k >< 0$$

$$\Rightarrow \sum_{k=1}^{n}\underline{Z}_k = \sum_{k=1}^{n} R_k + j\sum_{k=1}^{n} X_k$$

$$n = 2$$



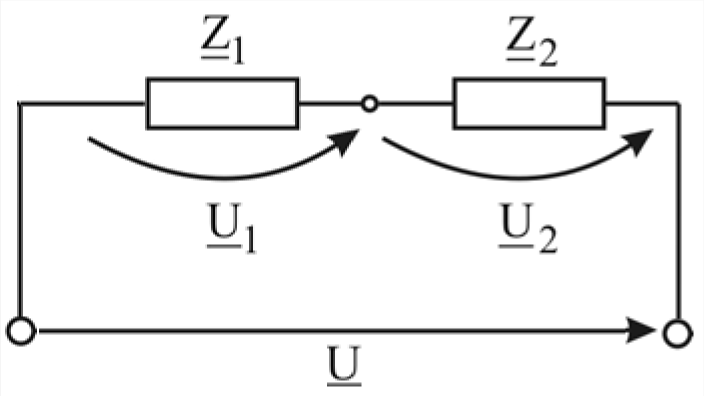
$$\Rightarrow \underline{Z}_e = \underline{Z}_1 + \underline{Z}_2$$

Voltage Divider

$$\underline{I} = \frac{\underline{U}_k}{\underline{Z}_k} = \frac{\underline{U}}{\underline{Z}_e} = \frac{\underline{U}}{\sum_{k=1}^{n} \underline{Z}_k}$$

$$\underline{U}_k = \frac{\underline{Z}_k}{\sum_{k=1}^{n} \underline{Z}_k}\underline{U}$$

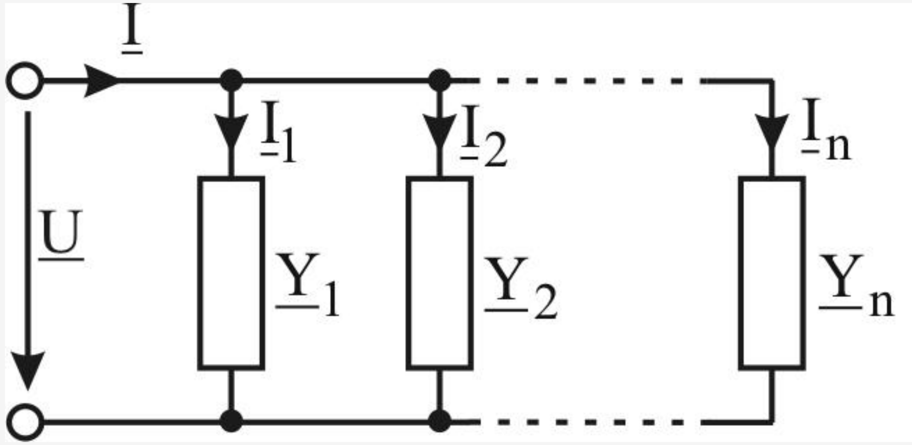
$n = 2$



$$\underline{U}_1 = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2}\underline{U} = \frac{\underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2}\underline{U}$$

$$\underline{U}_2 = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}\underline{U} = \frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2}\underline{U}$$

b) Parallel Circuits (Connections)



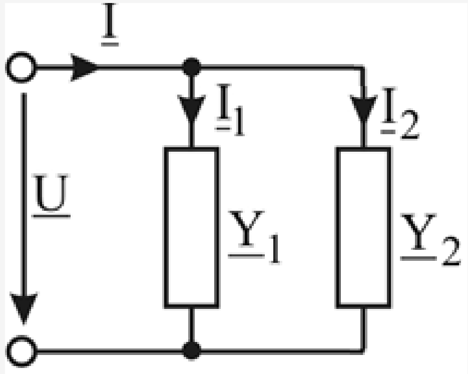
$$\underline{I} = \underline{I}_1 + \underline{I}_2 + \ldots + \underline{I}_n$$

$$\underline{I} = \underline{Y}_1\underline{U} + \underline{Y}_2\underline{U} + \ldots + \underline{Y}_n\underline{U}$$

$$\underline{I} = \underline{U}\left(\underline{Y}_1 + \underline{Y}_2 + \ldots + \underline{Y}_n\right) = \underline{U}\sum_{k=1}^{n}\underline{Y}_k$$

$$\underline{Y}_e = \frac{\underline{I}}{\underline{U}} = \sum_{k=1}^{n}\underline{Y}_k \quad \Rightarrow \quad \underline{Y}_e = G_e - jB_e \quad \Rightarrow \quad G_e = \sum_{k=1}^{n} G_k > 0 \quad \Rightarrow \quad B_e = \sum_{k=1}^{n} B_k >< 0$$

$$n = 2$$

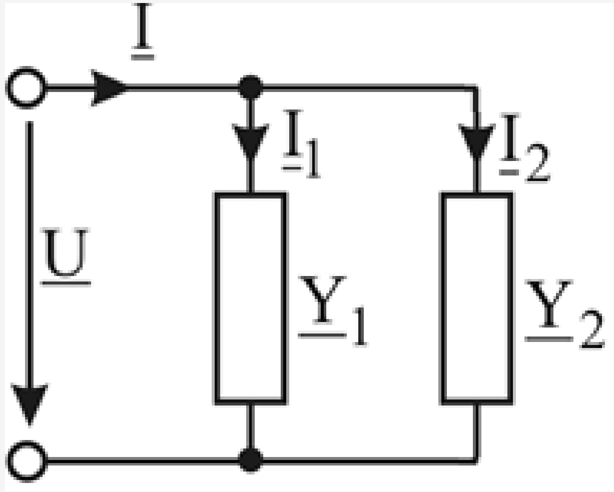


$$\underline{Z}_e = \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2}$$

Current Divider

$$\underline{U} = \frac{\underline{I}_k}{\underline{Y}_k} = \frac{\underline{I}}{\underline{Y}_e} = \frac{\underline{I}}{\sum_{k=1}^{n} \underline{Y}_k} \Rightarrow \underline{I}_k = \frac{\underline{Y}_k}{\sum_{k=1}^{n} \underline{Y}_k} \underline{I}$$

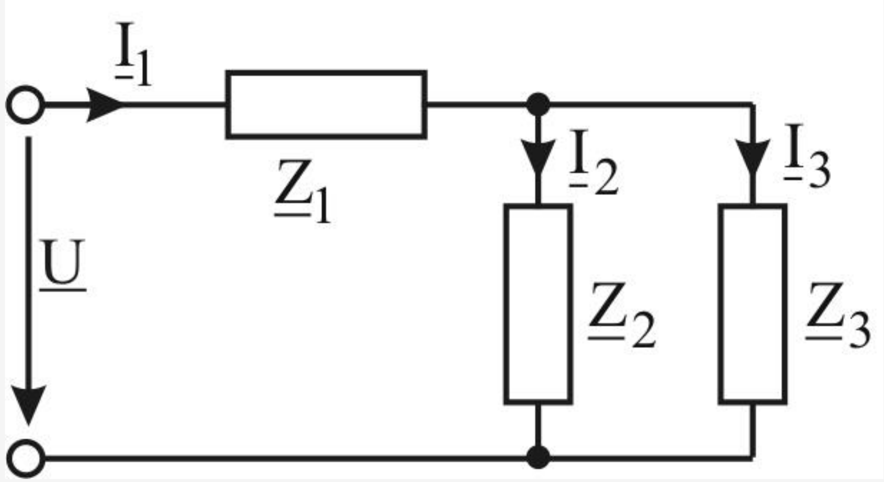
$n = 2$



$$\underline{I}_1 = \frac{\underline{Y}_1}{\underline{Y}_1 + \underline{Y}_2} \underline{I} = \frac{\underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \underline{I}$$

$$\underline{I}_2 = \frac{\underline{Y}_2}{\underline{Y}_1 + \underline{Y}_2} \underline{I} = \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \underline{I}$$

C) Mixt Connection

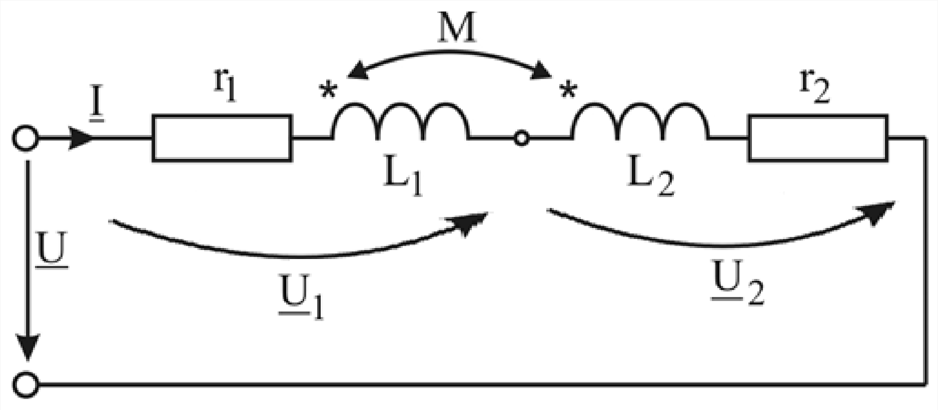


$$\underline{Z}_{2,3} = \frac{1}{\dfrac{1}{\underline{Z}_2} + \dfrac{1}{\underline{Z}_3}} = \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}$$

$$\underline{Z}_e = \underline{Z}_1 + \underline{Z}_{2,3} = \underline{Z}_1 + \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3} = \frac{\underline{Z}_1\underline{Z}_2 + \underline{Z}_2\underline{Z}_3 + \underline{Z}_1\underline{Z}_3}{\underline{Z}_2 + \underline{Z}_3}$$

2. Passive Branches with Inductive Couplings

A) Series Connection



- coupling coefficient is:

$$k = \frac{|M|}{\sqrt{L_1 L_2}} \leq 1$$

- Applying Ohm's Law:

$$\underline{U}_1 = r_1\underline{I} + j\omega L_1\underline{I} + j\omega M\underline{I}$$

$$\underline{U}_2 = r_2\underline{I} + j\omega L_2\underline{I} + j\omega M\underline{I}$$

$$\Rightarrow \underline{U} = \underline{I}\left[(r_1 + r_2) + j\omega(L_1 + L_2 + 2M)\right]$$

Equivalent impedance: $\underline{Z}_e = (r_1 + r_2) + j\omega(L_1 + L_2 + 2M)$

$$\underline{Z}_e = R_e + jX_e \qquad R_e = r_1 + r_2 \qquad L_e = L_1 + L_2 + 2M$$

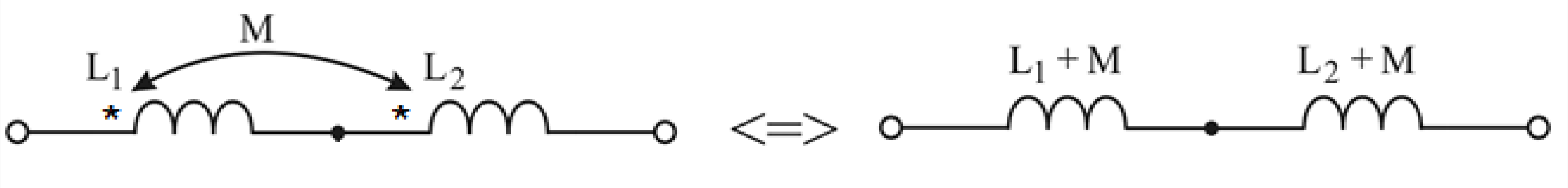
- if $M < 0$: $\Rightarrow L_e^{'} = L_1 + L_2 - 2M$

- **Equivalent inductivity is always a positive parameter**

The elimination rule (desfacere) of the magnetic coupling between 2 inductors

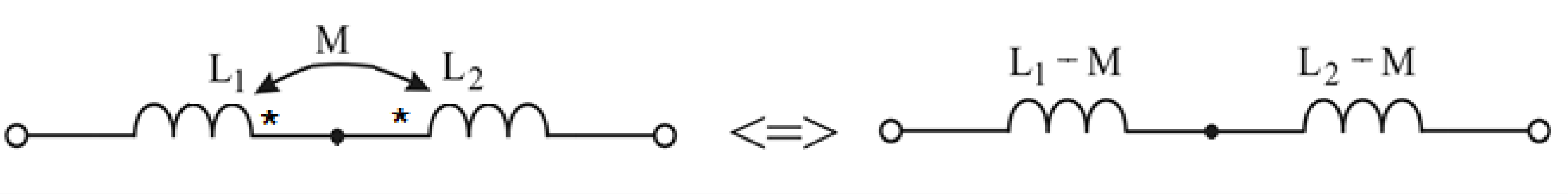
❖ **Additional Coupling**

- **the marked terminals are asymmetric to the common point**

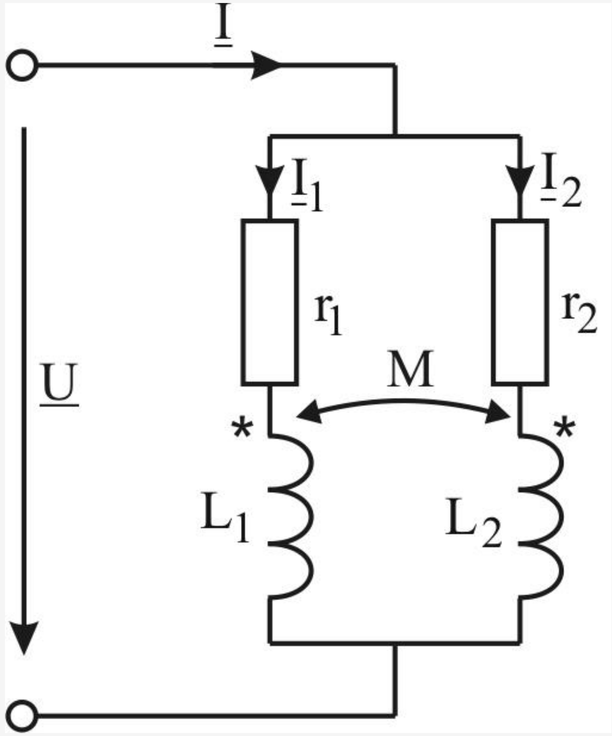


❖ **Differential Coupling**

- **the marked terminals are symmetric to the common point**



b) Parallel connection



$$\underline{U} = (r_1 + j\omega L_1)\underline{I}_1 + j\omega M\underline{I}_2 = \underline{Z}_1\underline{I}_1 + \underline{Z}_M\underline{I}_2$$

$$\underline{U} = (r_2 + j\omega L_2)\underline{I}_2 + j\omega M\underline{I}_1 = \underline{Z}_M\underline{I}_1 + \underline{Z}_2\underline{I}_2$$

$$\underline{Z}_1 = r_1 + j\omega L_1$$

$$\underline{Z}_2 = r_2 + j\omega L_2$$

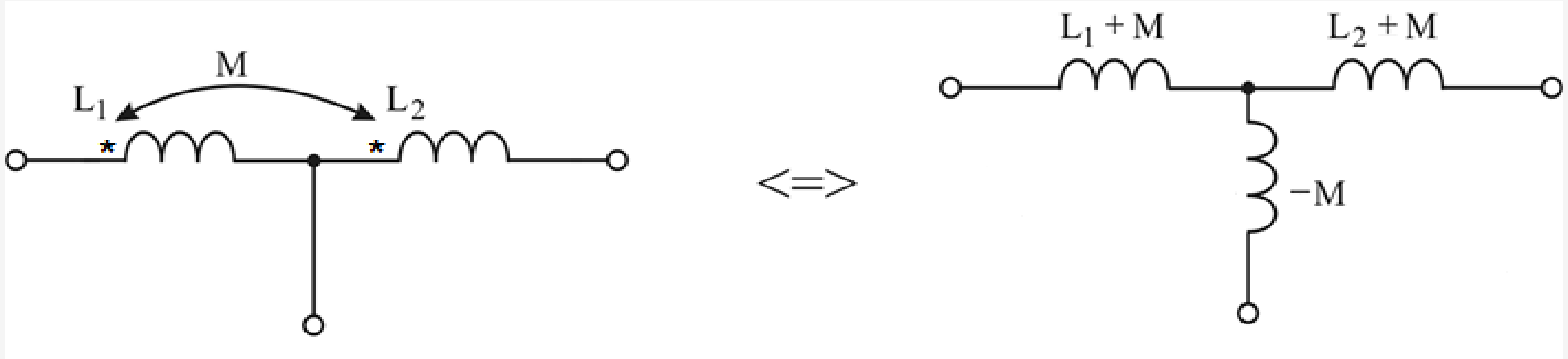
$$\underline{Z}_M = j\omega M$$

$$\underline{Z}_e = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U}}{\underline{I}_1 + \underline{I}_2} \Rightarrow \boxed{\underline{Z}_e = \frac{\underline{Z}_1\underline{Z}_2 - \underline{Z}_M{}^2}{\underline{Z}_1 + \underline{Z}_2 \mp 2\underline{Z}_M}}$$

The elimination rule (desfacere) of the magnetic coupling between 2 inductors

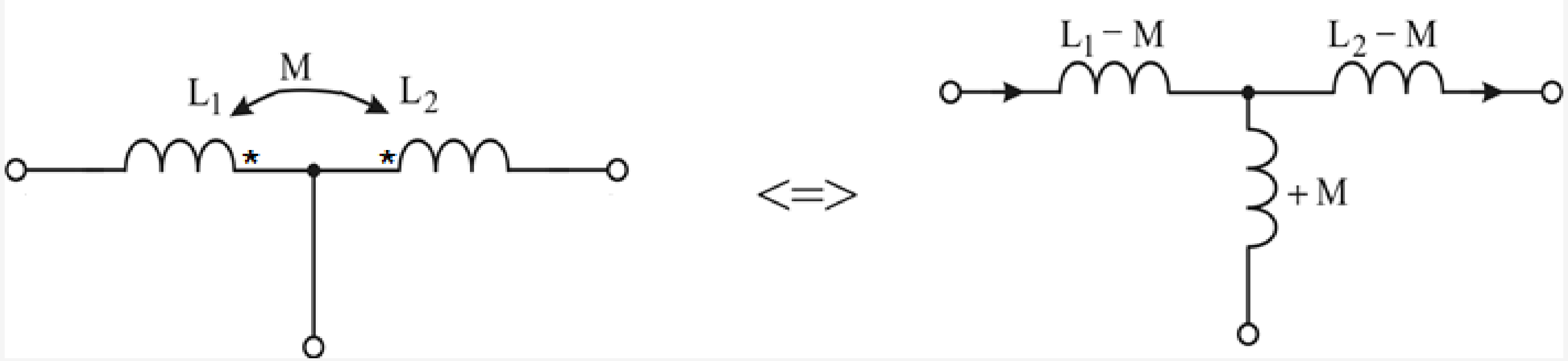
❖ Additional Coupling

- the marked terminals are asymmetric to the common point



❖ Differential Coupling

- the marked terminals are symmetric to the common point



Applications

Problem 3

Find the currents, voltages and powers and draw the phasors diagram for the circuit, knowing that:

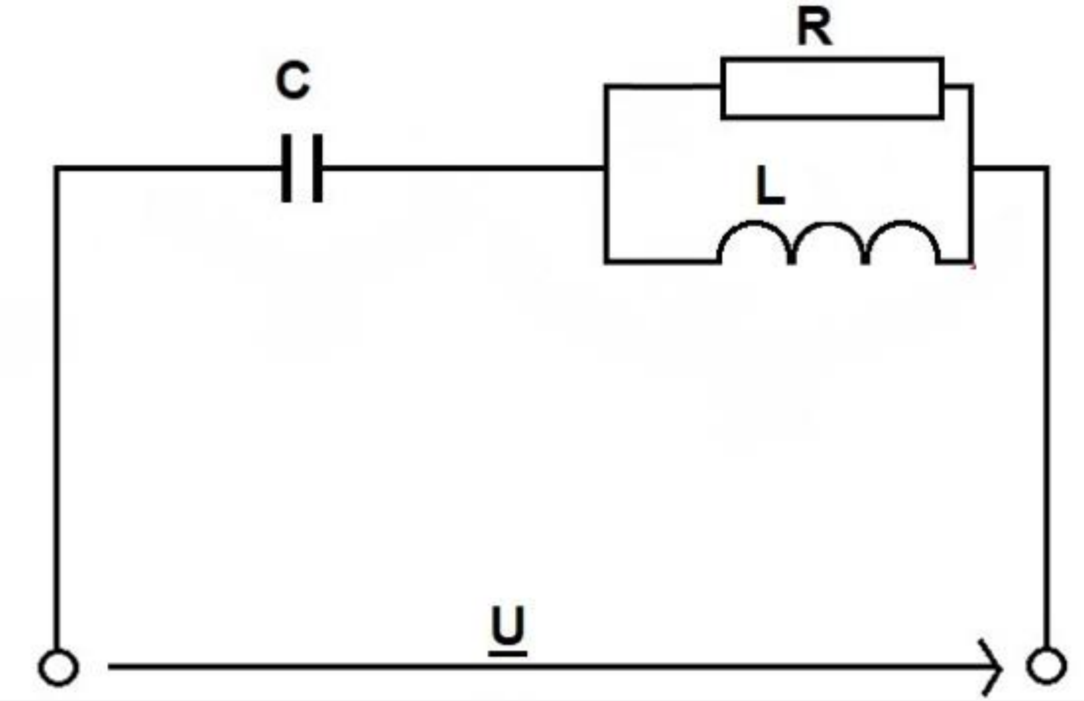
$$u(t) = 220\sqrt{2}\, sin(\omega \cdot t + 0^o)\ [V];$$

***f* = *50* [Hz];**

***R* = *60* [Ω];**

***L* = *0.2* [H];**

***C* = *60* [μF].**



Solution:

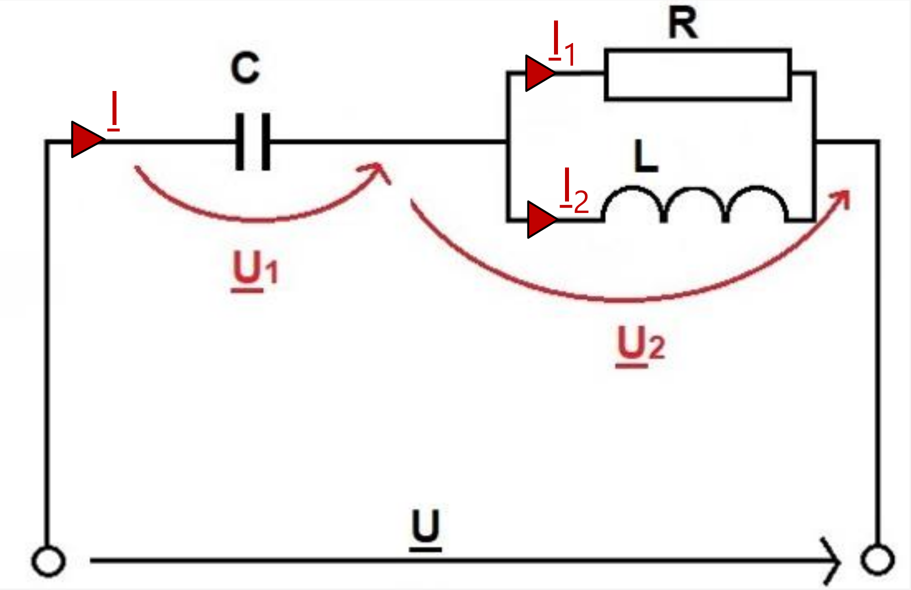
Solution:

- the supply voltage is:

$$u(t) = 220\sqrt{2}\, sin(\omega \cdot t + 0^o)\ [V]$$

  - we write it in simplified complex:

$$\left.\begin{array}{l} U = \dfrac{220\sqrt{2}}{\sqrt{2}} = 220 \\ \gamma_u = 0^o \end{array}\right\} \Rightarrow \underline{U} = U \cdot e^{j \cdot \gamma_u} = U\angle\gamma_u$$



$$\Rightarrow \underline{U} = 220 \cdot e^{j \cdot 0^o} = 220 \cdot (cos\, 0^o + j \cdot sin\, 0^o) \Rightarrow \boxed{\underline{U} = 220, [\mathrm{V}]}$$

- pulsation :

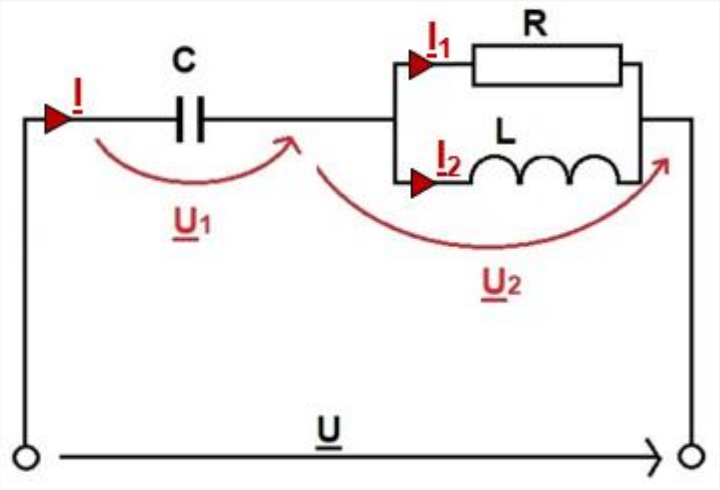
$$\omega = 2 \cdot \pi \cdot f \Rightarrow \omega = 2 \cdot \pi \cdot 50 = 2 \cdot 3,14 \cdot 50$$

$$\Rightarrow \boxed{\omega = 314, \left[\frac{rad}{s}\right]}$$

- we calculate the impedances of each component of the circuit:

$$\left.\begin{array}{l} \underline{Z}_R = R \\ R = 60\ [\Omega] \end{array}\right\} \Rightarrow \boxed{\underline{Z}_R = 60, [\Omega]}$$

$$\left.\begin{array}{l} \underline{Z}_L = j \cdot \omega \cdot L \\ L = 0.2\ [\mathrm{H}] \\ \omega = 314 \left[\frac{rad}{s}\right] \end{array}\right\} \Rightarrow \underline{Z}_L = j \cdot 314 \cdot 0,2 \Rightarrow \boxed{\underline{Z}_L = 62,8 \cdot j, [\Omega]}$$

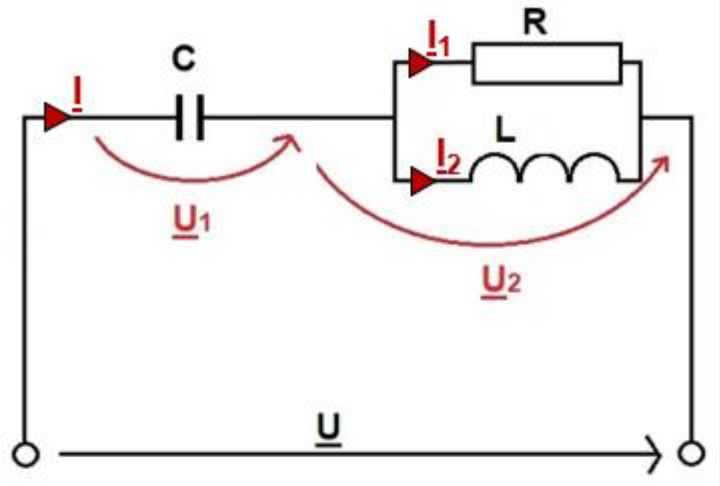


$$\underline{Z}_C = \frac{1}{j \cdot \omega \cdot C} \Rightarrow \left.\begin{array}{l} \underline{Z}_C = -j \cdot \dfrac{1}{\omega \cdot C} \\ C = 60\ [\mu\mathrm{F}] \\ \omega = 314 \left[\frac{rad}{s}\right] \end{array}\right\} \Rightarrow \underline{Z}_C = -j \cdot \frac{1}{314 \cdot 60 \cdot 10^{-6}} \Rightarrow \boxed{\underline{Z}_C = -53,08 \cdot j, [\Omega]}$$

we calculate the equivalent impedance of the circuit:

$$\left.\begin{aligned}
&\underline{Z_e} = \underline{Z_C} + \frac{\underline{Z_R} \cdot \underline{Z_L}}{\underline{Z_R} + \underline{Z_L}} \\
&\underline{Z_R} = 60\ [\Omega] \\
&\underline{Z_L} = 62,8 \cdot j\ [\Omega] \\
&\underline{Z_C} = -53,08 \cdot j\ [\Omega]
\end{aligned}\right\} \Rightarrow \underline{Z_e} = -53,08 \cdot j + \frac{60 \cdot 62,8 \cdot j}{60 + 62,8 \cdot j}$$

$$\Rightarrow \boxed{\underline{Z_e} = 31,37 - 23,11 \cdot j, [\Omega]}$$

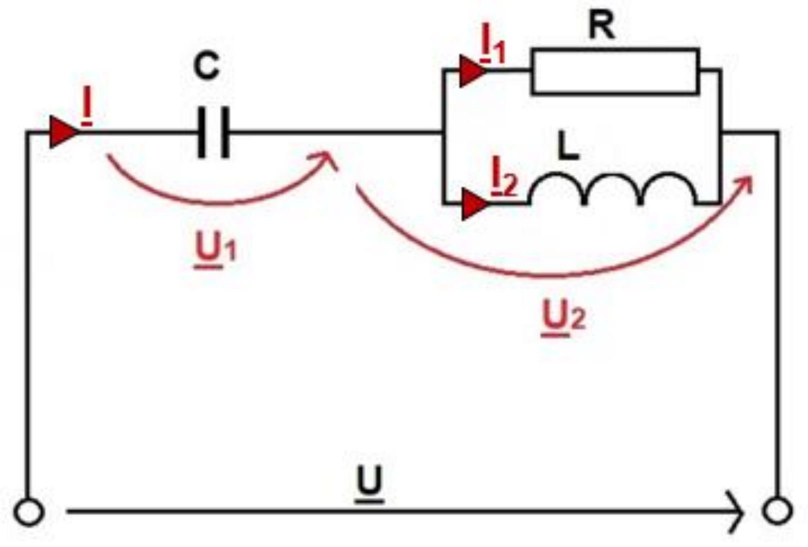


- we calculate the effective value (module) of this complex equivalent impedance: $Z_e = \sqrt{31,37^2 + 23,11^2} = 38,96$

- we calculate the phase angle: $\varphi = arctg\left(-\frac{23,11}{31,37}\right) = -36^o24`$

$$\Rightarrow \boxed{\underline{Z_e} = 38,96\angle - 36^o24`, [\Omega]}$$

we calculate the **total current** from the circuit:



- *Ohm's Law:* $\underline{U} = \underline{Z}_e \underline{I} \Rightarrow \underline{I} = \frac{\underline{U}}{\underline{Z}_e}$

$$\Rightarrow \underline{I} = \frac{220\angle 0^o}{38,96\angle -36^o24`}$$

  - the effective values are divided, and the phase angles are subtracted:

$$\Rightarrow \underline{I} = 5,65\angle\left(0^o - (-36^o24`)\right) \Rightarrow \boxed{\underline{I} = 5,65\angle 36^o24`, [A]}$$

- or:

$$\underline{I} = \frac{220}{31,37 - 23,11 \cdot j} \Rightarrow \boxed{\underline{I} = 4,55 + 3,35 \cdot j, [A]}$$

$$I = \sqrt{4,55^2 + 3,35^2} = 5,65 \qquad \gamma_i = arctg\frac{3,35}{4,55} = 36^o24`$$

$$\Rightarrow \mathrm{i}(t) = 5{,}65\sqrt{2}\, sin(314 \cdot t + 36^o24')\ [A]$$

- one of the currents is found with the Current Divider Theorem:

$$\underline{I}_2 = \frac{\underline{Z}_R}{\underline{Z}_R + \underline{Z}_L} \cdot \underline{I}$$

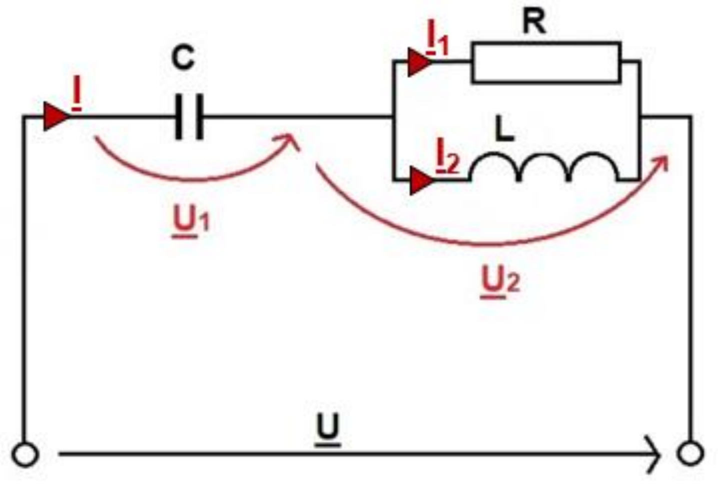
$$\underline{Z}_R = 60\ [\Omega]$$

$$\underline{Z}_L = 62,8 \cdot j\ [\Omega]$$

$$\underline{I} = 4,55 + 3,35 \cdot j\ [A]$$

$$\Rightarrow \underline{I}_2 = \frac{60}{60 + 62,8 \cdot j} \cdot (4,55 + 3,35 \cdot j)$$

$$\Rightarrow \boxed{\underline{I}_2 = 3,84 - 0,67 \cdot j, [A]}$$



$$I_2 = \sqrt{3,84^2 + 0,67^2} = 3,9$$

$$\gamma_{i_2} = arctg\left(-\frac{0,67}{3,84}\right) = -9^o53`$$

$$\Rightarrow \boxed{\underline{I}_2 = 3,9\angle - 9^o53`, [A]}$$

$$\Rightarrow i_2(t) = 3,9\sqrt{2}\, sin(314 \cdot t - 9^o53')\ [A]$$

- $\underline{I}_1$ is the difference between $\underline{I}$ and $\underline{I}_2$:

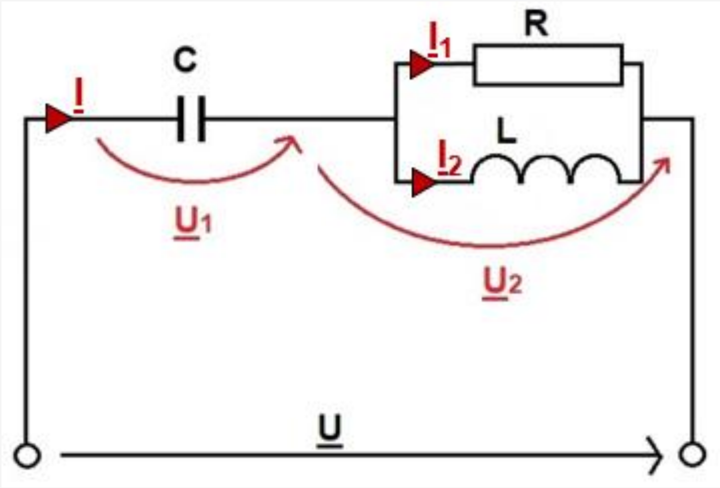
$$\underline{I}_1 = \underline{I} - \underline{I}_2 \Rightarrow \underline{I}_1 = 4,55 + 3,35 \cdot j - 3,84 + 0,67 \cdot j$$

$$\Rightarrow \boxed{\underline{I}_1 = 0,71 + 4,02 \cdot j, [A]}$$

$$\boxed{\underline{I}_1 = 0,71 + 4,02 \cdot j\,, [A]}$$

$$\left.\begin{aligned} I_1 &= \sqrt{0,71^2 + 4,02^2} = 4,08 \\ \gamma_{i_1} &= arctg\frac{4,02}{0,71} = 80^o7` \end{aligned}\right\} \Rightarrow \boxed{\underline{I}_1 = 4,08\angle 80^o7`, [A]}$$

$$\Rightarrow i_1(t) = 4{,}08\sqrt{2}\,sin(314 \cdot t + 80^o7')\,[A]$$



- we calculate the voltages, using Ohm's Law:

$$\left.\begin{aligned} \underline{U}_1 &= \underline{Z}_c \cdot \underline{I} \\ \underline{Z}_C &= -53,08 \cdot j\,[\Omega] \\ \underline{I} &= 4,55 + 3,35 \cdot j\,[A] \end{aligned}\right\} \Rightarrow \underline{U}_1 = (-53,08 \cdot j) \cdot (4,55 + 3,35 \cdot j)$$

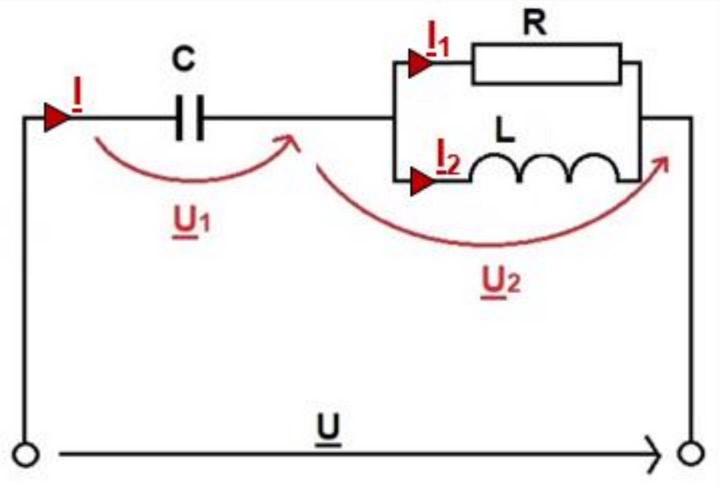
$$\Rightarrow \boxed{\underline{U}_1 = 177 - 241 \cdot j, [V]}$$

$$\left.\begin{aligned} U_1 &= \sqrt{241^2 + 177^2} = 300 \\ \gamma_{u_1} &= arctg\left(-\frac{241}{177}\right) = -53^o36` \end{aligned}\right\} \Rightarrow \boxed{\underline{U}_1 = 300\angle -53^o36`, [V]}$$

$$\Rightarrow u_1(t) = 300\sqrt{2}\,sin(314 \cdot t - 53^o36')\,[V]$$

$$\left.\begin{aligned} &\underline{U_2} = \underline{Z_R} \cdot \underline{I_1} \\ &\underline{Z_R} = 60\ [\Omega] \\ &\underline{I_1} = 0,71 + 4,02 \cdot j\ [A] \end{aligned}\right\} \Rightarrow \quad \underline{U_2} = 60 \cdot (0,71 + j \cdot 4,02)$$

$$\Rightarrow \quad \boxed{\underline{U_2} = 43 + 241 \cdot j, [V]}$$



$$\left.\begin{aligned} &U_2 = \sqrt{43^2 + 241^2} = 245 \\ &\gamma_{u_2} = arctg\frac{241}{43} = 80^o7` \end{aligned}\right\} \Rightarrow \quad \boxed{\underline{U_2} = 245\angle 80^o7`, [V]}$$

$$\Rightarrow \quad u_2(t) = 245\sqrt{2}\, sin(314 \cdot t + 80^o07')\ [V]$$

- **Possible checks:** $\underline{U_2} = \underline{Z_L} \cdot \underline{I_2}$

  $\underline{U_1} + \underline{U_2} = \underline{U}$ • we must obtain the same value.

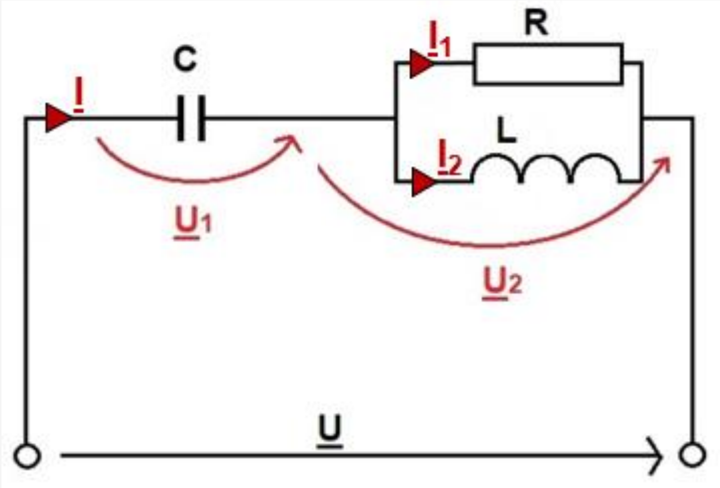
We note that $\underline{U_1} + \underline{U_2} = \underline{U}$ (complex values) although $U_1 > U$ (effective values) and $U_2 > U$ (effective values).

Also $\underline{I_1} + \underline{I_2} = \underline{I}$ $\quad I_1 + I_2 = 7{,}98\ \mathrm{A} \Rightarrow I_1 + I_2 > I$ (as effective values).

We compute the powers

Complex power received to the terminals cand be found in two different ways:

$$\underline{S} = \begin{cases} \underline{U} \cdot \underline{I}^{*} = 220 \cdot (4{,}55 - j \cdot 3{,}35) \\ \underline{Z}_e \cdot I^2 = (31{,}37 - j \cdot 23{,}11) \cdot 5{,}65^2 \end{cases} \Rightarrow \underline{S} = 1001 - j \cdot 738 \text{ VA}$$



We know that:

$$\underline{S} = P + j \cdot Q = 1001 - j \cdot 738 \text{ VA} \Rightarrow$$

$$\Rightarrow P = 1001 \text{ W}$$

$$Q = -738 \text{ VAR}$$

We check now the active power and the reactive power using the other formulas:

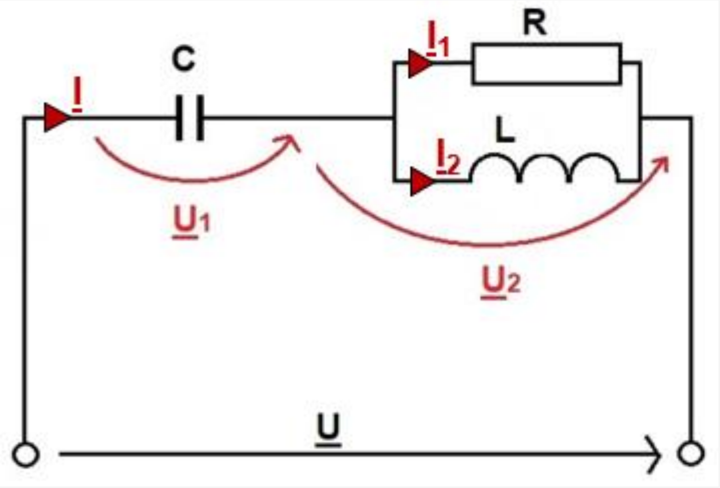
$$P = \begin{cases} R \cdot I_1^2 = 60 \cdot 4{,}08^2 \\ U \cdot I \cdot \cos\varphi = 220 \cdot 5{,}65 \cdot \cos(-36^{\circ}24`) \end{cases} = 1001 \text{ W}$$

$\Rightarrow$ P generated = P consummated

We check also the reactive power:

$$Q = \begin{cases} X \cdot I^2 = X_L \cdot I_L^2 + X_C \cdot I_C^2 \\ U \cdot I \cdot \sin\varphi \end{cases}$$

$$Q = \left.\begin{cases} \omega \cdot L \cdot I_2^2 - \dfrac{1}{\omega \cdot C} \cdot I^2 = 955 - 1693 \\ U \cdot I \cdot \sin\varphi = 220 \cdot 5{,}65 \cdot \sin\left(-36^\circ 24`\right) \end{cases}\right\} = -738 \text{ VAr}$$



- We note that the inductor consumes 955 VAr, but the capacitor „produces” 1693 VAr, covering the inductor consumption
- The excces of reactive power 738 VAr, it is ceded to the terminals, the circuit having a capacitive behavior

The phasors diagrams is a simple draw in the complex coordinate system of the three currents phasors at a convenient scale mm/A. The three voltages phasors have their scale mm/V.

$\underline{U} = 220\angle 0^{o}$ V;

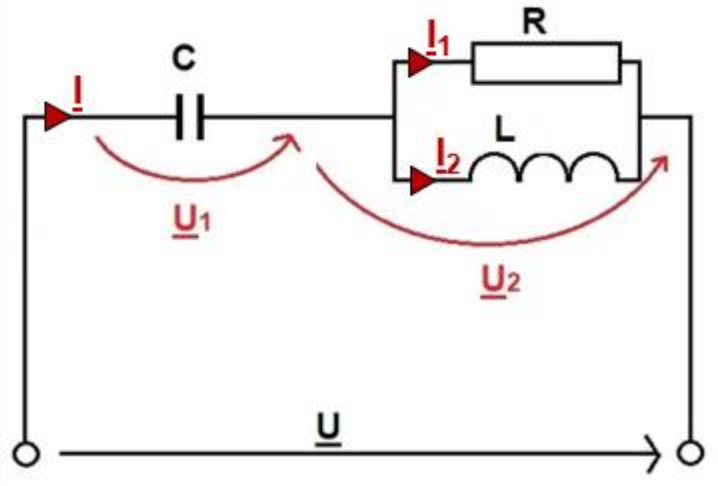
$\underline{U_1} = 300\angle -53^{o}36`$ V;

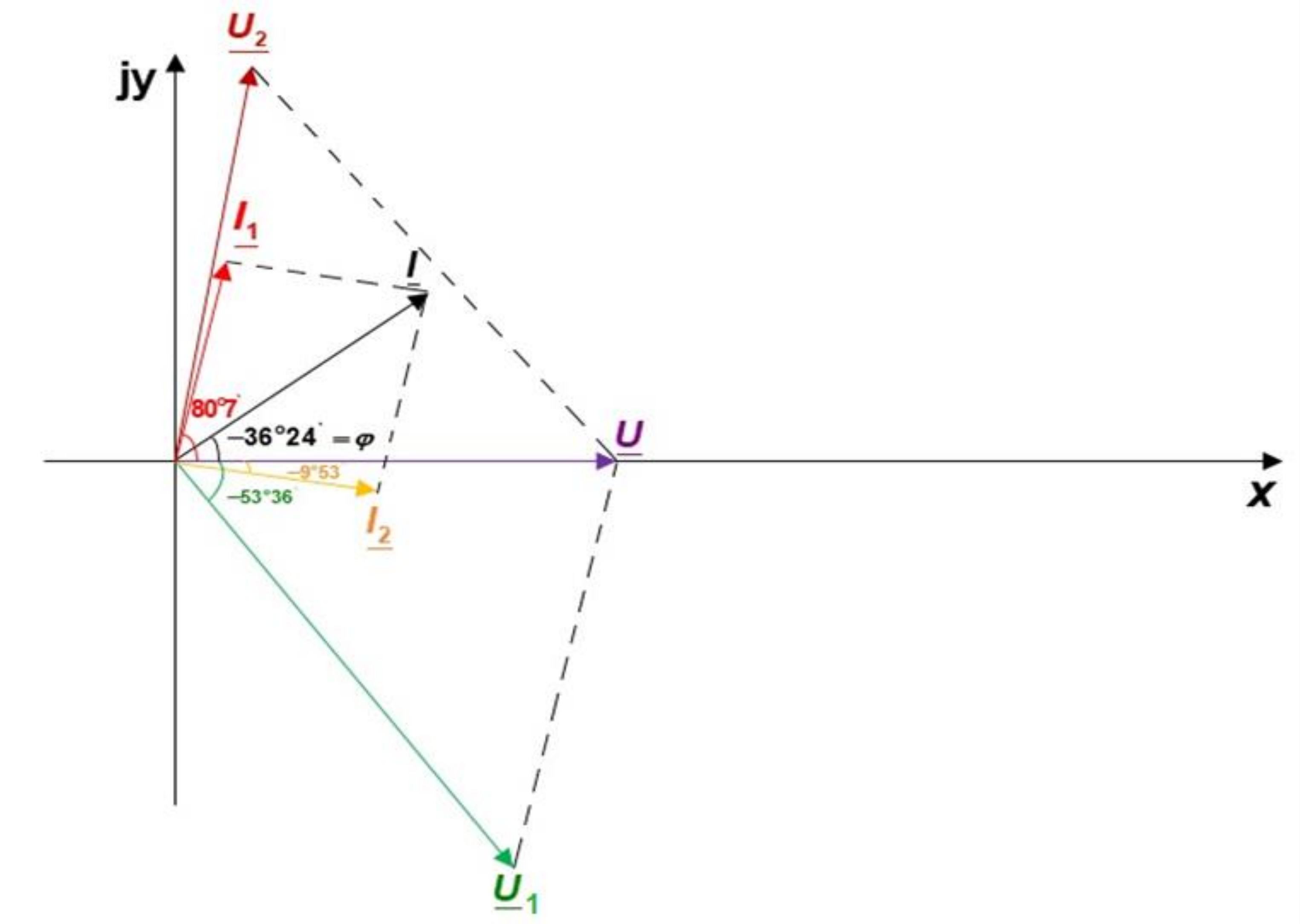
$\underline{U_2} = 245\angle 80^{o}7`$ V;

$\underline{I} = 5{,}65\angle 36^{o}24`$ A;

$\underline{I_1} = 4{,}08\angle 80^{o}7`$ A;

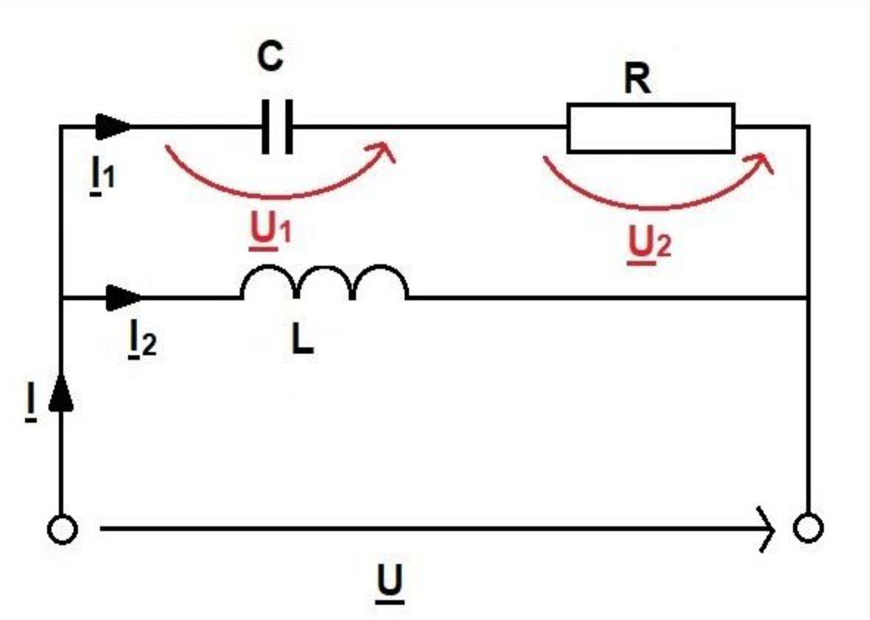
$\underline{I_2} = 3{,}9\angle -9^{o}53`$ A;





Homework

Find the currents, voltages and powers and draw the phasors diagram for the circuit, knowing that:



$$u(t) = 220\sqrt{2}\,sin(\omega \cdot t + 0^o)\,[V];$$

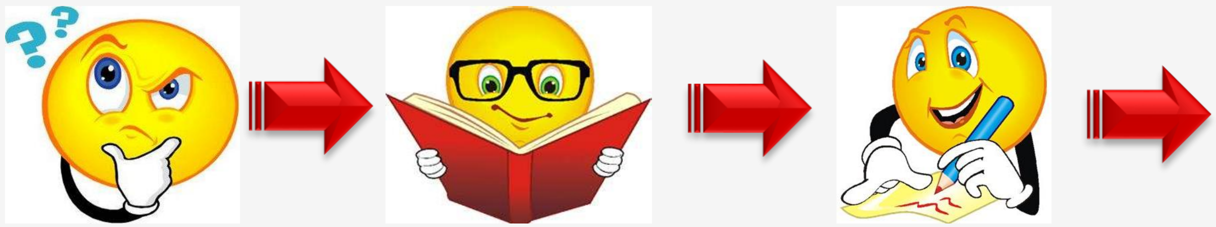
f **= *50* [Hz];**

R **= *60* [Ω];**

L **= *0.2* [H];**

C **= *60* [μF].**

Solution:



$\underline{I}_1 = 2{,}06 + j \cdot 1{,}82 = 2{,}75\angle 41^o30`$ A;

$\underline{I}_2 = -j \cdot 3{,}5 = 3{,}5\angle -90^o$ A;

$\underline{I} = 2{,}06 + j \cdot 1{,}68 = 2{,}66\angle -39^o20`$ A;

$\underline{U}_1 = 145{,}7\angle -48^o30` = 96{,}6 - j \cdot 109{,}34$ V;

$\underline{U}_2 = 123{,}6 + j \cdot 109{,}2 = 164{,}8\angle 41^o30`$ V;

$\underline{S} = 452 + j \cdot 370$ VA;

$P = 452$ V;

$Q = 370$ VAr.

Thank you for your attention!!!

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Questions???