III Oscillations

3. The forced oscillation

3.1.

$F = F_0 + F_{el} + F_{frec}$

$F = m \cdot a = m \cdot \ddot{x}$

$F_{el} = -kx$

$F_{fr} = -\lambda \cdot v = -\lambda \cdot \dot{x}$

$F_{ext} = F_0 \sin(\omega_{ext} t)$ — The amplitude of the force

The external force

$$m\ddot{x} = -k \cdot x - \lambda \cdot \dot{x} + F_0 \sin(\omega_{ext} t) \quad \Big| \frac{1}{m}$$

$$\ddot{x} + \frac{\lambda}{m}\dot{x} + \frac{k}{m}x = \frac{F_0}{m}\sin(\omega_{ext} t)$$

Note

$\omega_0^2 = \frac{k}{m}$; $2\delta = \frac{\lambda}{m}$

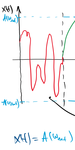
$$\ddot{x} + 2\delta\dot{x} + \omega_0^2 x = \frac{F_0}{m}\sin(\omega_{ext} t)$$ – second order, inhomogeneous differential equation

$$x(t) = x_0(t) + x_1(t)$$

$$x_0(t) = Ae^{-\delta t}\sin(\omega_0 t + \varphi)$$

$$x_1(t) = A(\omega_{ext})\sin(\omega_{ext} t + \varphi_0)$$

$$|x(t)| = x_1(t)$$

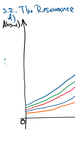


$x(t) = A(\omega_{ext})\sin(\omega_{ext} t + \varphi_0)$ – The law of motion

$A(\omega_{ext}) = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{ext}^2)^2 + 4\delta^2\omega_{ext}^2}}$ – the amplitude of the forced oscillations

$tg(\varphi_0) = -\frac{2\delta\omega_{ext}}{\omega_0^2 - \omega_{ext}^2}$ – the initial phase

3.2. The Resonance



a) $\frac{dA(\omega_{ext})}{d\omega_{ext}}\Big|_{\omega_{ext}=\omega_{res}} = 0$

$$\frac{d}{d\omega_{ext}}\left\{\frac{F_0}{m}\left[(\omega_0^2-\omega_{ext}^2)^2+4\delta^2\omega_{ext}^2\right]^{-\frac{1}{2}}\right\} =$$

$$= \frac{F_0}{m}\left[\ldots\right]^{-\frac{3}{2}}\left(-\frac{1}{2}\right)\left[2(\omega_0^2-\omega_{ext}^2)\cdot(-2\omega_{ext})+4\delta^2\cdot 2\omega_{ext}\right] = 0$$

$$-4\omega_{ext}\left(\omega_0^2-\omega_{ext}^2-2\delta^2\right) = 0$$

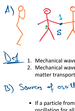
$\omega_{ext} = \omega_{res}$; $\omega_{res} = 0$

$$\omega_0^2 - \omega_{res}^2 - 2\delta^2 \Rightarrow \boxed{\omega_{res} = \sqrt{\omega_0^2 - 2\delta^2}}$$

IV Waves

1. Introduction

A)



Def

1. Mechanical waves represents mechanical oscillations transmitted into an elastic medium.
2. Mechanical waves are characterized by energy transport into an elastic medium and not by matter transport.

B) Sources of oscillations

- If a particle from an elastic medium is forced to oscillate then this became a source of oscillation for all particles from that elastic medium.
- If a particle from an elastic medium start to oscillate all other particles from that medium will oscillate at the further time. And all of these will reproduce the motion of the first particle.
- The propagation (transmission) of an oscillation into an elastic medium is achieved from near to near due to the interaction forces that exists between neighbor particles.
- This transmission take some time and is produced with finite velocity.

2. Classification of waves

2.1. The transverse waves

If the direction of oscillation of a particle into an elastic medium is perpendicular to the direction of propagation off the mechanical wave then this is named a **transverse wave**.

2.2. The longitudinal waves

If the direction of oscillation of a particle into an elastic medium is parallel to the direction of propagation off the mechanical wave then this is named a **longitudinal wave**.

3. Examples

3.1. Transverse waves

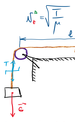
- 1D Solid
- 2D Solids

At the surface of liquids

3.2. Longitudinal waves

- 3D Solids

Bulk liquid



- Gases

4. The velocity of waves

4.1. For transverse waves

a) $v_t^0 = \sqrt{\frac{T}{\mu}}$

b – solid (+3 solid)

$\mu = \frac{m}{l}$ – the linear density (the reduced mass)

4.2. For longitudinal waves

a) $v_l^0 = \sqrt{\frac{E}{\rho}}$

A – solid (+3 solid)

$\rho = \frac{m}{V}$ – the density

E – the Young's modulus of elasticity of S $[E]_{SI} = \frac{N}{m^2}$

B)

$v_l^0 = \sqrt{\frac{\chi}{\rho}}$

L – liquid

$\rho = \frac{m}{V}$ – the density

χ – modulus of compressibility $[\chi]_{SI} = \frac{N}{m^2}$

C)

$v_l^0 = \sqrt{\frac{p}{\rho}}$

$p = \frac{F}{S}$ – the pressure

$\rho = \frac{m}{V}$ – the density

$[p]_{SI} = \frac{N}{m^2}$

$\gamma = \frac{c_p}{c_v}$ – the adiabatic coefficient

where c_p, c_v – the specific heats of constant pressure, volume

5. The Huygens principle

5.1. Elements of a wave

S – the source of a wave

The geometrical place of all of the adjacent points that are oscillating in phase is named **wave surface**.

The most distant wave surface is called the wave-front. The wave-front is a surface which separates the points from an elastic medium which are already in oscillation from the points from that elastic medium which will oscillate further in time (later).

5.2.

Huygens' principle: Each point from the front wave is considered to be a secondary source of a new elementary wave having the same velocity, frequency, wavelength as the initial wave. The new wave front is the envelope of all elementary waves

6. The spherical and plane wave

7. The wave diffraction