Course 11
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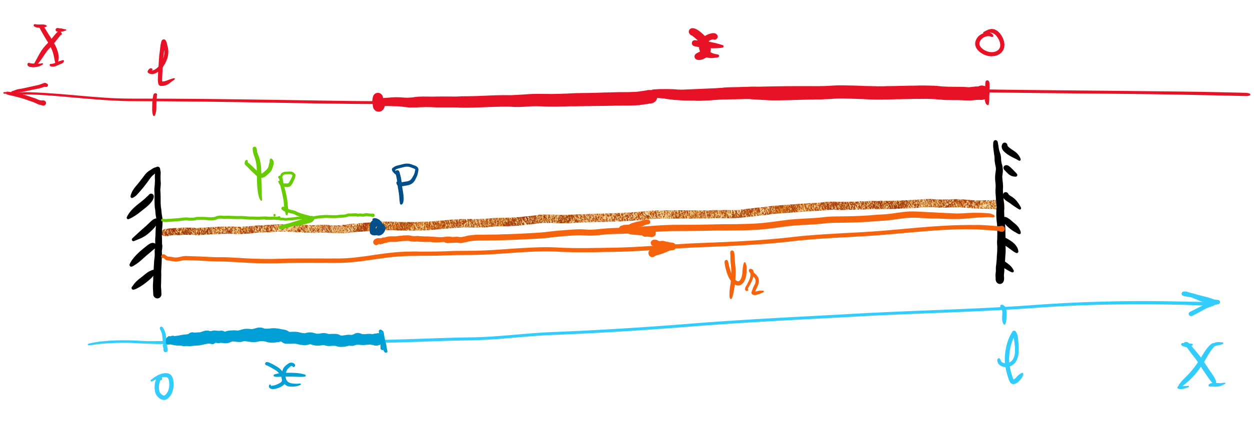
IV Waves

12. Standing waves

12.1.

12.2.



$$\psi_p(t,x) = A\sin(\omega t - k\cdot x)$$

$$\psi_2(t,x) = A\sin\left[\omega t - k(2l - x) - \pi\right]$$

$$\psi(t,x) = \psi_p(t,x) + \psi_2(t,x)$$

$$\psi(t,x) = 2A\cos\left[\frac{\cancel{\omega t} - k\cdot x - \cancel{\omega t} + k(2l-x) + \pi}{2}\right]\sin\left[\frac{\omega t - \cancel{k\cdot x} + \omega t - k(2l - \cancel{x}) - \pi}{2}\right]$$

$$\psi(t,x) = 2A\cos\left[k(l-x) + \frac{\pi}{2}\right]\sin\left[\omega t - k\cdot l - \frac{\pi}{2}\right]$$

$$\psi(t,x) = \underbrace{2A\sin(k\cdot x)}_{\text{the amplitude of a point at } x}\cos(\omega t - k\cdot l)$$

I Maximum

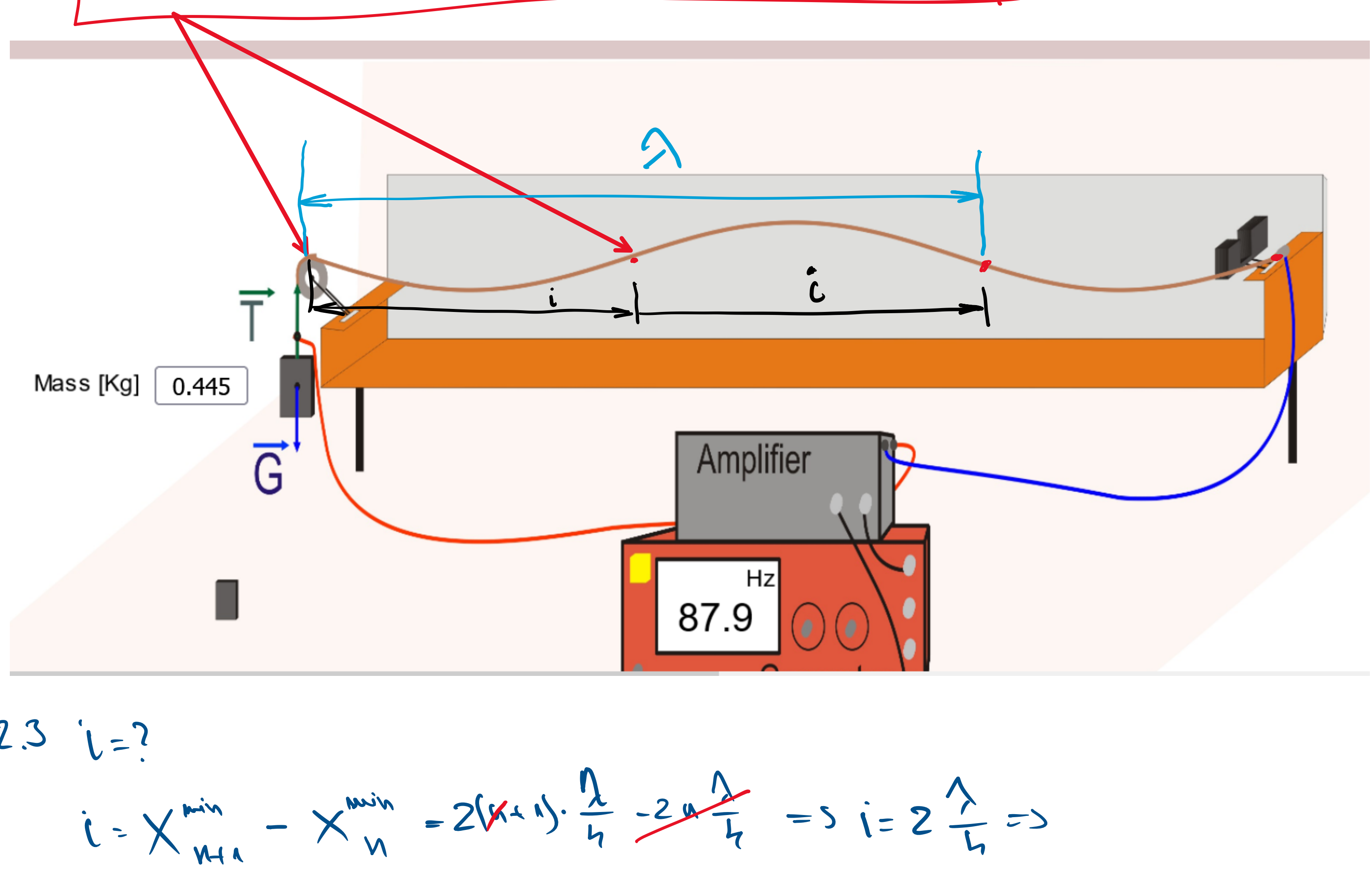
$\sin(k\cdot x) = \pm 1$, $k = \frac{2\pi}{\lambda}$ $\Rightarrow$ $\frac{2\pi}{\lambda}x = \frac{1\cdot\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

$$\boxed{X_n^{(Max)} = (2n+1)\frac{\lambda}{4} \quad \& \quad n = 0,1,2,3,4,\dots}$$

II Minimum

$\sin(k\cdot x) = 0 \Rightarrow \frac{2\pi}{\lambda}x = 0\cdot\pi, 1\cdot\pi, 2\pi, 3\pi, \dots$

$$\boxed{X_n^{(Min)} = 2\cdot n\frac{\lambda}{4} \quad \& \quad n = 0,1,2,3,\dots}$$



12.3 $i = ?$

$i = X_{n+1}^{min} - X_n^{min} = 2(n+1)\cdot\frac{\lambda}{4} - \cancel{2n\frac{\lambda}{4}} \Rightarrow i = 2\frac{\lambda}{4} \Rightarrow$

$i = \frac{\lambda}{2} \Rightarrow \boxed{\lambda = 2\cdot i}$

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V Acoustics

1° Introduction

Def **Acoustics** it's part of physics that studies the creation, propagation, reception and properties of **Sounds**.

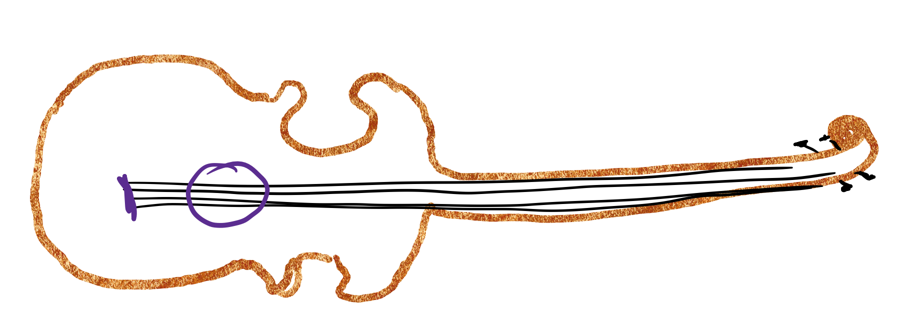
Def The mechanical oscillations capable to impress the human hearing organ (the ear) it's called **sound**.

2° The sound properties

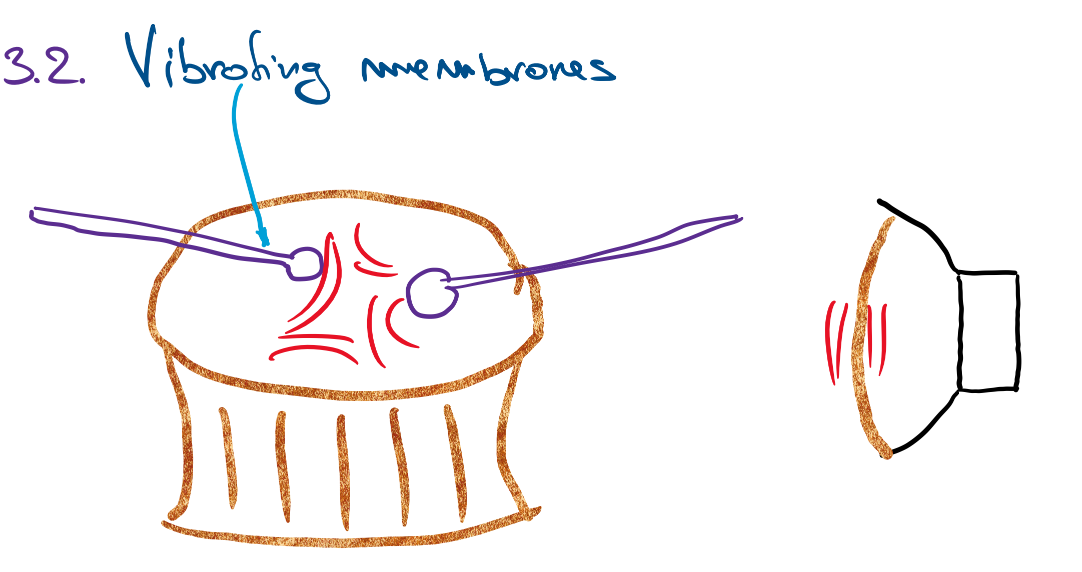
P1. The sound should be produced by a sound source;
P2. An elastic medium must exist between the source and the receiver;
P3. The oscillation frequency must be between 16 Hz and 20 kHz; ($16\,Hz \le \nu \le 20\,kHz$)
P4. The sound power must be high enough to produce an auditive sensation. The sound must have an intensity higher then a threshold $I_0 = 10^{-12}$ W/m^2; ($I_0 \le I \le 10^2\,W/m^2$)
P5. The duration of a sound must be higher then a minimal duration necessary for the human ear to receive the sound ($t > \tau = 0.05\ s = 50\ ms$).

3. The sound sources

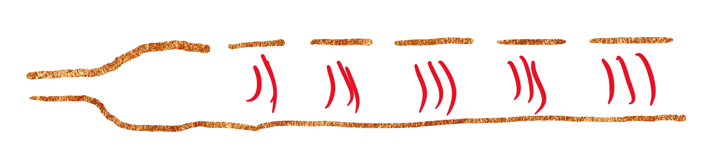
3.1. Vibrating strings



3.2. Vibrating membranes

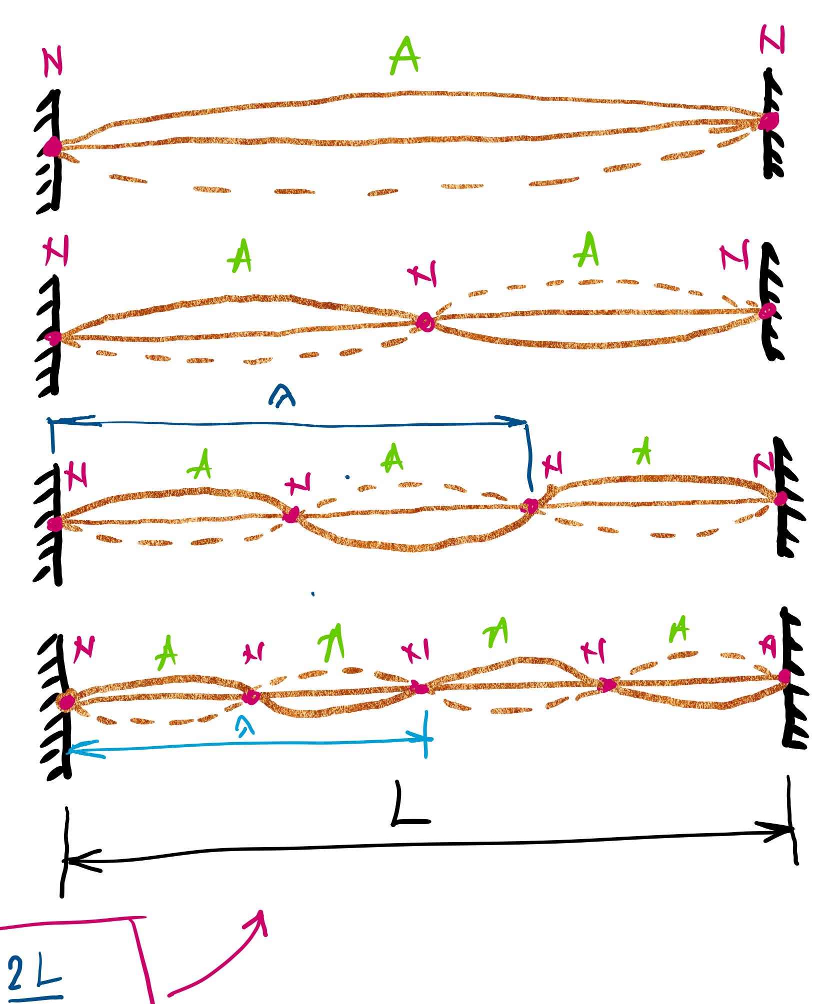


3.3 Vibrating air columns



4. The fundamental sound and the superior harmonics

A)



λ

$\lambda_1 = \frac{2L}{1}$

$\lambda_2 = \frac{2L}{2}$

$\lambda_3 = \frac{2L}{3}$

$\lambda_4 = \frac{2L}{4}$

$$\boxed{\lambda_n = \frac{2L}{n}}$$

ν, λ, v $\quad [\nu]_{IS} = \frac{1}{s}$; $[\lambda]_{IS} = m$; $[v]_{IS} = \frac{m}{s}$

$\frac{m}{s} = m\cdot\frac{1}{s}$, $v = \lambda\cdot\nu$ $\Rightarrow$ $\boxed{\nu = \frac{v}{\lambda}}$

$\nu_n = \frac{v}{\lambda_n} \Rightarrow \nu_n = \frac{v}{\frac{2L}{n}} \Rightarrow$

$$\nu_n = \frac{v}{2L}\cdot n$$

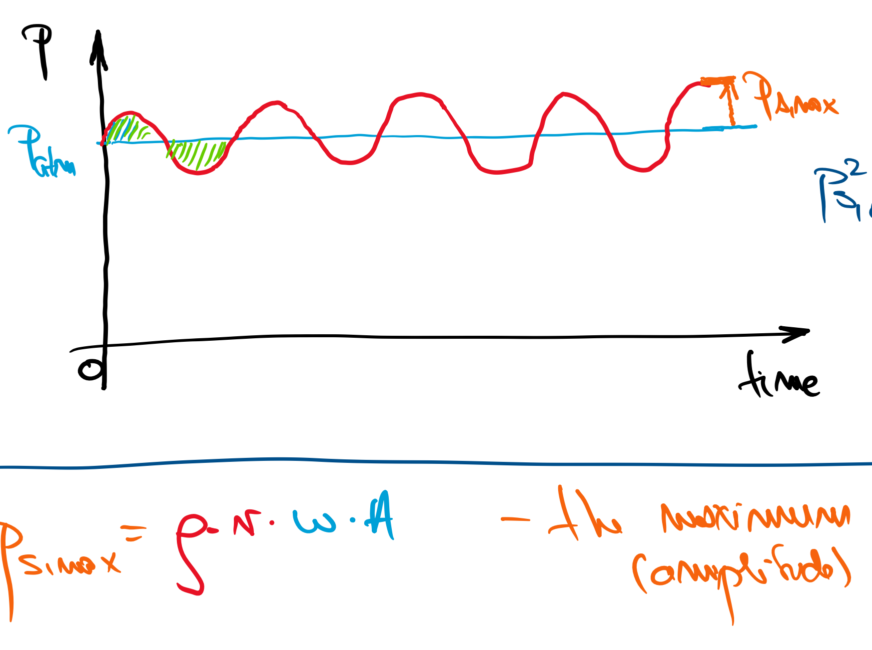
$\boxed{\nu_1 = \frac{v}{2L}}$ – the fundamental frequency

$\boxed{\nu_n = \nu_1\cdot n}$ if $n = 2,3,4,5,6,\dots$

→ superior harmonics

5. The sound characteristics

5.1. The sound pressure



$P_s(t) = P_{s,max}\sin(\omega t)$

$$P_{s,av}^2 = \frac{1}{T-0}\int_0^T P_{s,max}^2\sin^2(\omega t)\,dt$$

$P_{s,max} = \rho\cdot v\cdot\omega\cdot A$ – the maximum sound pressure (amplitude)

ρ – the density, v – the velocity } elastic medium

$\omega = 2\pi\cdot\nu$ – the pulsation, A – the amplitude } stimulus

5.2. The sound intensity, I

$[I]_{IS} = \frac{W}{m^2}$; $\quad I \stackrel{def}{:=} \frac{1}{S}\left\langle\frac{\partial E}{\partial t}\right\rangle_t$

E – the energy of sound
t – the time
S – the surface
$\langle\ \rangle_t$ – the average in time

$$\left\langle\frac{\partial E}{\partial t}\right\rangle_t := \frac{1}{T-0}\int_0^T\frac{\partial E}{\partial t}\cdot dt$$

$$\boxed{I = \frac{1}{2}\rho\cdot v\cdot\omega^2\cdot A^2}$$

$I = \frac{\rho^2\cdot v^2\cdot\omega^2\cdot A^2}{2\rho\cdot v} = \frac{(\rho\cdot v\cdot\omega\cdot A)^2}{2\rho\cdot v}$ $\Rightarrow$ $\boxed{I = \frac{P_{s,max}^2}{2\rho\cdot v}}$

Note
$Z = \rho\cdot v$ – the acoustic impedance

$$\boxed{I = \frac{P_{s,max}^2}{2Z}}$$

Note
$P_{s,eff} = \frac{P_{s,max}}{\sqrt{2}}$ – the effective sound pressure (the measured one) $\Rightarrow$ $\boxed{I = \frac{P_{s,eff}^2}{Z}}$