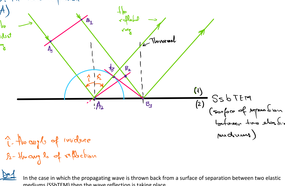
11 Waves

8. The wave reflection

A)



$\hat{i}$ - the angle of incidence
$\hat{r}$ - the angle of reflection

Def In the case in which the propagating wave is thrown back from a surface of separation between two elastic mediums (SSbTEM) then the wave reflection is taking place.

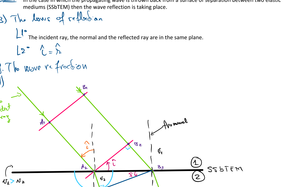
B) The laws of reflection

L^1 The incident ray, the normal and the reflected ray are in the same plane.

L^2 $\hat{i} = \hat{r}$

9. The wave refraction

A)



$\hat{i}$ - the angle of incidence
$\hat{r}$ - the angle of refraction

Def The turning on banding of a wave when it passes from one medium into another medium is named wave refraction.

B) Laws of refraction

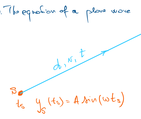
L^1 The incident ray, the normal and the refracted ray are in the same plane.

L^2 $\Delta A_1B_1B_2$: $\sin\hat{i} = \frac{B_1B_2}{A_1B_2}$; $\Delta A_1A_2B_2$: $\sin\hat{r} = \frac{A_1B_2}{A_1B_2}$ $\Rightarrow$ $\frac{\sin\hat{i}}{\sin\hat{r}} = \frac{\frac{B_1B_2}{A_1B_2}}{\frac{A_1B_2}{A_1B_2}} = \frac{B_1B_2}{A_1B_2} \Rightarrow$

$\frac{\sin\hat{i}}{\sin\hat{r}} = \frac{n_2 \cdot c}{n_1 \cdot c} \Rightarrow$ $\boxed{\frac{\sin\hat{i}}{\sin\hat{r}} = \frac{n_2}{n_1} = n_{12}}$ L2 of refraction

n_{12} - indices of refraction

10. The equation of a plane wave



$y_P(t_P) = A\sin(\omega t_P)$

t_B, t_P, t_P

$t_B = t_P + t$

$v = \frac{d}{t} \Rightarrow t = \frac{d}{v}$

$t_P = t_B - \frac{d}{v}$

$y_P(t_P) = A\sin[\omega(t_B - \frac{d}{v})]$

$$\boxed{\psi(t,d) = A\sin\left[\omega\left(t - \frac{d}{v}\right)\right]}$$

↳ the wave function

Note $\omega = \frac{2\pi}{T}$; $\nu = \frac{1}{T}$ $\omega = 2\pi\nu$

T - the period
ν - the frequency

$\psi(t,d) = A\sin\left[2\pi\left(\frac{t}{T} - \frac{d}{vT}\right)\right]$

Note

$\lambda = v \cdot T$ - the wavelength

$\psi(t,d) = A\sin\left[2\pi\left(\frac{t}{T} - \frac{d}{\lambda}\right)\right]$

$\psi(t,d) = A\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}d\right)$

Note

$k = |\vec{k}| = \frac{2\pi}{\lambda}$ - the wave number

$$\boxed{\psi(t,d) = A\sin(\omega t - kd)}$$

$\psi(t,\vec{r}) = A\sin(\omega t - \vec{k}\vec{r})$ - the equation of spherical waves

$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

if $\vec{k} = k\vec{i}$ | $\vec{k}\vec{r} = kx$

$$\boxed{\psi(t,x) = A\sin(\omega t - kx)}$$ - the equation of a plane wave

11. The waves interference

A) **Def**

The **wave interference** represents the **superposition** of two or more coherent waves resulting into a **new wave pattern of minimum and maximum.**

Def **Coherence** is the attribute of two or more waves or part of the same wave to have the relative phase constant in time.

$\varphi_1(t,x) = \omega t - \varphi_{01}$
$\varphi_2(t,x) = \omega t - \varphi_{02}$ $\Rightarrow$ $\Delta\varphi = \varphi_1 - \varphi_2 = [(\omega t - \varphi_{01}) - (\varphi_{02} - \varphi_{02})]$

$\Downarrow$ constant - time

$\omega_1 = \omega_2 = \omega$

$2\pi\nu_1 = 2\pi\nu_2 \Rightarrow \nu_1 = \nu_2 = \nu$

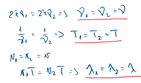
$\frac{1}{T_1} = \frac{1}{T_2} \Rightarrow T_1 = T_2 = T$

$v_1 = v_2 = v$

$v_1 T = v_2 T \Rightarrow \lambda_1 = \lambda_2 = \lambda$

$\frac{2\pi}{\lambda_1} = \frac{2\pi}{\lambda_2} \Rightarrow k_1 = k_2 = k$

B)



$\psi_1(t,d_1) = A\sin(\omega t - kd_1)$

$\psi_2(t,d_2) = A\sin(\omega t - kd_2)$

$\psi(t,d_1,d_2) = \psi_1(t,d_1) + \psi_2(t,d_2)$

$\psi(t,d_1,d_2) = 2A\cos\left[\frac{\omega t - kd_1 - \omega t + kd_2}{2}\right]\sin\left[\frac{\omega t - kd_1 + \omega t - kd_2}{2}\right]$

$\psi(t,d_1,d_2) = 2A\cos\left(k\frac{d_1 - d_2}{2}\right)\sin\left(\omega t - k\frac{d_1 + d_2}{2}\right)$

Note

$\delta = d_2 - d_1$

$\psi(t,\delta) = 2A\cos\left(\frac{k\delta}{2}\right)\sin\left(\omega t - k\frac{d_1 + d_2}{2}\right)$

↳ the amplitude

I. Max/Minimum

$\cos\left(\frac{k\delta}{2}\right) = 0 \Rightarrow \frac{k\delta}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$k = \frac{2\pi}{\lambda} \Rightarrow \frac{\frac{2\pi}{\lambda}\delta}{2\pi} = (2n+1)\frac{\pi}{2}$

$$\boxed{\delta_{min}^{(n)} = (2n+1)\frac{\lambda}{2} \quad + \quad n = 0, 1, 2, 3, \ldots}$$

II. Max/Maximum

$\cos\left(\frac{k\delta}{2}\right) = \pm 1 \Rightarrow \frac{k\delta}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots$

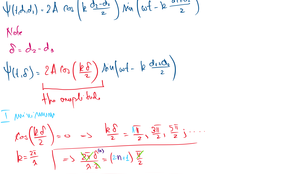
$k = \frac{2\pi}{\lambda} \Rightarrow \frac{\frac{2\pi}{\lambda}\delta}{2\pi} = n\frac{\pi}{\pi} \Rightarrow$

$$\boxed{\delta_{max}^{(n)} = 2n\frac{\lambda}{2} \quad + \quad n = 0, 1, 2, 3, 4, \ldots}$$

12. Standing waves

12.1. The waves reflection

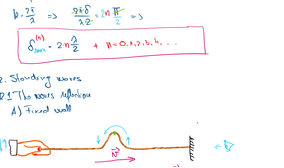
A) Fixed wall



Reflection on a fixed wall is with

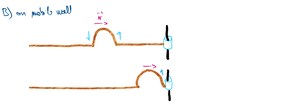
Lost of:
- in space $\frac{\lambda}{2}$
- in time $\frac{T}{2}$
- in phase $\frac{2\pi}{\lambda} \cdot \pi$

B) On mobile wall



No lost

12.1.



$\psi_I(t,x) = A\sin(\omega t - kx)$

$\psi_R(t,x) = -A\sin[\omega t - k(2\ell - x) - \pi]$

$\psi(t,x) = \psi_R(t,x) + \psi_I(t,x)$