

# Laborator 1

```
// #include <... .h>
// DEFINE const
// def data-type , str
// decl var globale
// functie
// type name ([type] arg1...
//     } var locale
//         ...
//         return var-type
//     }
// }

int main()
{
    ...
    return 0
}
```

Se da o matrice citita dintr-un fisier

Sa se calculeze produsul de pe diagprincipala  
sau laterală în alt fisier și se afișează și numărul

# LAB 6

## Trăsare de algoritmi

RADIX      unitate

4521	0250	3300	0005	0005	✓
0250	3300	0005	0012	0012	✓
3300	4521	0012	0077	0077	✓
1999	0012	4521	0250	0250	✓
0012	0333	0333	3300	0333	✓
0077	0005	0250	0333	1999	✓
0333	0077	0077	4521	3300	✓
0005	1999	1999	1999	4521	✓

## BUBBLE SORT

18 3 29 11 45 27 34 8 25 37 15

$i=0$ ,  $j=0$ ,  $18 > 3$ , T  $\Rightarrow$  swap(18, 3)

3 18 29 11 45 27 34 8 25 37 15

$i=0$ ,  $j=1$ ,  $18 > 29$ , F, 3, 18, 29, 11...

$i=0$ ,  $j=2$ ,  $29 > 11$ , T,  $\Rightarrow$  swap(29, 11)

3 18 11 29 45 27 34 8 25 37 15

$i=0$ ,  $j=3$ ,  $29 > 45$ , F 3, 18, 11, 29, ...

$j=4$ ,  $45 > 27$ , T, swap(45, 27)

3 18 11 29 27 45 34 8 25 37 15

$j=5$ ,  $45 > 34$ , T, swap(45, 34)

3 18 11 29 27 34 45 8 25 37 15

$j=6$ ,  $45 > 8$ , T, swap (45, 8)

3 18 11 29 27 34 8 45 25 37 15

$j=7$ ,  $45 > 25$ , T

3 18 11 29 27 34 8 25 45 37 15

$j=8$ ,  $45 > 32$ , T

3 18 11 29 27 34 8 25 57 45 15

$j=9$ ,  $45 > 15$ , T

3 (8 → 11 29 → 27 34 → 8 → 25 37 → 15 45)

i=1, j=0...9

3, 11, 18, 27, 29, 8, 25, 29, 34, 15, 37, 45

i=2, j=0...8

3, 11, 18, 27, 8, 25, 29, 15, 34, 37, 45

i=3, j=0, 7

3, 11, 18, 8, 25, 15, 29, 34, 37, 45

i=7, j=0, 6

3, 11, 8, 18, 25, 15, 29, 34, 37, 45

i=5, j=0, 5

3, 8, 11, 18, 15, 25, 22 ...

$$i = 6, j = 0, 4$$

3, 8, 11, 15, 18, 25

i = 7, j = 0, 3 FLAG STOP

## SELECTION SORT

0 1 2 3 4 5 6 7 8 9 10  
18, 3, 29, 11, 45, 27, 34, 8, 25, 37, 15

i = 0, min = 18

index\_min = 1

3 18 29 11 45 27 34 8 25 37 15

i = 1 min = 18

index\_min = 7

3 8 29 11 45 27 34 18 25 37 15

i = 2 min = 29

index\_min = 3

3 8 11 29 45 27 34 18 25 37 15

i = 3 min = 29

index\_min

3 8 11 15 45 27 34 18 25 37 29

i = 4 min = 45

index\_min = 7

3 8 11 15 18 27 34 45 25 37 29

$$l = 5 \quad \min = 27$$

index\_min =

3 8 11 15 18 21 34 45 27 37 24

$$l = 6 \quad \min = 31$$

index\_min

3 8 11 15 18 25 27 45 37 32 29

$$l = 7 \quad \min = 45$$

3    27 29 34 37 45 X 3

### INSERTION SORT

18, 3, 29, 11, 45, 27, 34, 8, 25, 37, 15  
*j*  
 $i = 1, 3 < 18 \quad \text{swap}(3, 18)$

3 18 29   

$i = 2, 18 < 29 \quad 3 \ 18 \ 29 \ 11 \dots$

$$i = 3, j = 2 \quad 11 < 29$$

$$j = 2 \quad 11 < 18$$

3 11 18 29 45 27 34 8 25 37 15  
*j*      *i*

$$i = 4, j = 3, 45 > 29$$

$$i = 5, j = 4, 27 < 45$$

$$27 < 29$$

$$27 > 18$$

L+18

3 17 18 27 29 45 34 8 25 32 15

i=6, j=5      34 < 45      swap

3 11 18 27 29 34 45 8 25 32 15

i=3, j=6      8 < 45

8 < 34

8 < 29

8 < 27

8 < 18

8 < 11

8 > 3

3 8 11 18 27 29 31 45 25 32 15

i=8, j=7      25 < 45

3 8 11 18 25 27 29 34 45 32 15

i=9, j=8      37 < 45

3 8 11 16 25 27 29 31 37 45 15

i=10, j=9      15 < 45

3 8 11 15 18 20 27 29 34 37 45

Quick SORT

18, 3, 29, 11, 45, 27, 34, 8, 25, 37, 15

pivot = 18

{3, 11, 8, 15} 18 {29, 45, 27, 34, 25, 37}

pivot = 3

pivot = 29

3 {11, 8, 15} 18 {27, 25} 29 {48, 34, 37}

pivot = 11

pivot = 27

pivot = 48

3 {8, 11, 15} 18 {25} 27 29 {34, 37} 48

3 8 11 15 18 25 27 29 34 {37} 48

3 8 11 15 16 25 27 29 34 37 48

## MERGE SORT

18, 3, 29, 11, 45, 27, 34, 8, 25, 37, 15

{18, 3, 29, 11, 45, 27} {34, 8, 25, 37, 15}

{18, 3, 29} {11, 45, 27} {34, 8, 25} {37, 15}

{18, 3} {29} {11, 45} {27} {34, 8} {25} {37} {15}

{18} {3} {29} {11} {45} {27} {34} {8} {25} {37} {15}

DIVIDE

$$\{3, 18\} \quad \{11, 45\} \quad \{8, 34\}$$

$$\{3, 18, 29\} \quad \{11, 27, 45\} \quad \{8, 25, 34\} \quad \{15, 37\}$$

$$\{3, 11, 18, 27, 29, 45\} \quad \{8, 15, 25, 34, 37\}$$

$$\{3, 8, 11, 15, 18, 25, 27, 29, 34, 37, 45\}$$

~~MERGE~~

$$18 \quad 3 \quad 29 \quad 11 \quad 45 \quad 27 \quad 34 \quad 8 \quad 25 \quad 37 \quad 15$$

i      j+1  
8

$$i=0, j=0$$

$$18 > 3$$

$$3 \quad 18 \quad 29 \quad 11 \quad 45 \quad 27 \dots$$

i            j            j+1

$$18 > 29 \quad F$$

$$29 > 11 \quad A$$

$$3 \quad 18 \quad 11 \quad 29 \quad 45 \quad 27 \dots$$

              j            j+1

$29 > 45$  F

$45 > 27$  A

3 18 11 29 27 45 34 8 25 37 15  
 $j \quad j^+$

3 18 11 27 47 34 45 8 25 32 15

$\sum 26 13 86 51 \dots$

$\sum_{i=0}^{j+1} 26 13 86 51 \dots$

$7 > 26$  F

$26 > 13$  A

7 13 26 86 51 ...

$26 > 86$  F

$86 > 51$  A

7 13 26 51 86 5 69 1 3 2 42  
 $\sum_{j=1}^{j+1}$

$86 > 5 A$

$\rightarrow 13 \ 26 \ 51 \ 5 \ 86 \ 69 \ 1 \ 3 \ 2 \ 42$   
 $i \quad i+1$

$86 > 69 A$

$\rightarrow 13 \ 26 \ 51 \ 69 \ 86 \ 1 \ 3 \ 2 \ 42$

$86 > 1 A$

$\rightarrow 13 \ 26 \ 51 \ 69 \ \cancel{17} \ 86 \ 32 \ 42$

$86 > 3 A$

$\rightarrow 13 \ 26 \ 51 \ 69 \ 1 \ 3 \ 86 \ 2 \ 42$

$86 > 2 A$

$\rightarrow 13 \ 26 \ 51 \ 69 \ 1 \ 3 \ 2 \ 86 \ 42$

$86 > 42 A$

$\rightarrow 13 \ 26 \ 51 \ 69 \ 1 \ 3 \ 2 \ \cancel{86} \ 42 \ 86$

$i = 1 \ j = 0, \dots, 9$

~~ANSWER~~

7 13 26 51 1 3 2 6 9 42 86

i = 2    j = 0, ..., 8

7 13 26 1 3 2 51 42 69 86

i = 3    j = 0, .. 7

7 13 1 3 2 20 42 51 69 86

7 26 13 86 51 5 | 69 1 3 2 42

[7, 26, 13 | 86, 51, 5] [69, 1, 3, 2, 42]

[7, 26, 13] (86, 51, 5) [69, 1, 3] [2, 42]

[7, 26] [13] [86, 51] [5] [69, 1] [3] [2] [42]

[7] (26)

(86) [5]

[69] [1]

[7, 26]

[51, 86]

[1, 69]

$$\{7, 13, 26\} \quad \{5, 51, 86\} \quad \{1, 3, 69\} \quad \{2, 42\}$$

$$\{5, 7, 13, 26, 51, 10\} \quad \{1, 2, 3, 42, 69\}$$

$$\{1, 2, 3, 5, 7, 13, 26, 42, 51, 69, 86\}$$

7 26 13 86 51 5 69 1 3 2 42

$$i=0, j=0$$

$$7 > 26 \text{ F}$$

$$26 > 13 \text{ A}$$

7 13 26 86 51 5 69, 1 3 2 42 ✓

$$26 > 86 \text{ F}$$

$$86 > 51 \text{ A}$$

7 13 26 51 86 5 69 1 3 2 42 ✓

$$86 > 5$$

7 13 26 51 5 86 69 1 3 2 42 ✓

$$86 > 69$$

7 13 26 51 5 69 86 1 3 2 42 ✓

86 > 1 > 3 > 2 > 42

7 13 26 51 5 69 1 3 2 42 86 ✓

i = 1 , j = 0, ..., 9

7 13 26 5 51 1 3 2 42 69 86

i = 2 , j = 0, 1, 2

7 13 5 26 1 3 2 42 51 69 86 ✓

i = 3 , j = 0, ..., 2

7 5 13 1 3 2 26 42 51 69 86 ✓

i = 4 , j = 0, ..., 6

7 5 1 3 2 13 26 42 51 69 81

i = 5 , j = 0, ..., 5

7 1 3 2 5 13 26 ..

i = 6 , j = 0, ..., 4

1 3 2 5 7 13 26 ..

i = 7 , j = 0, 1, 2

1 2 3 5 7 13 26 ...

5 16 9 2 7 26 21 45 19 35 4  
 $i = 1$   $j = 1$   
 $j = 1$

5 < 16  $\Rightarrow i$

16 > 9  $\Rightarrow j$

~~8~~ 5 9 16 2 7 26 21 45 14 ...

16 > 2

16 > 7

5 9 27 16 21 45 19 ...

16 < 2 1

2 1 < 45

45 > 19 +

5 9 27 16 21 14 45 35 4

5 16 9 2 7 26 21 45 19 35 17  
5, 16, 9, 2, 7 } 17 } 26, 21, 45, 19, 35 }  
15, 2 } 7 } 16, 9 } 17 } 26, 21, 19 } 35 } 45 }

2 } 5 } 7 9 } 16 } 17 19 } 26, 21 } 35 45

2 5 7 9 16 17 19 21 } 26 } 35 45

2 5 7 9 16 17 14 21 26 35 45

### BUBBLE SORT

18 3 29 11 45 27 34 8 25 37 15

i

j

i=0 , j=0

18 > 3 T

3 18 29 11 45 27 34 8 25 37 15  
i i+1

18 > 29 F

29 > 11 T

3 18 11 29 45 27 34 8 25 37 15

29 > 45 F

45 > 27 T

3 18 11 25 27 45 34 8 25 37 15

$45 > 34$  A

3 18 11 29 27 34 45 8 25 37 15

$75 > 8$  A

3 18 11 29 27 34 8 45 25 37 45

$45 > 25$

3 18 11 29 27 37 8 25 75 37 15

$45 > 37$

3 18 11 29 27 34 8 25 37 45 15

$45 > 15$

3  $\overbrace{18}^l$  11  $\overbrace{29}^j$  27  $\overbrace{34}^{k_1}$   $\overbrace{8}^{k_2}$  25  $\overbrace{37}^{k_3}$   $\overbrace{15}^{k_4}$  45  
 $i=1, j=0, g$

3 11 18 27  $\overbrace{29}^l$   $\overbrace{8}^j$   $\overbrace{25}^k$   $\overbrace{34}^{m_1}$   $\overbrace{15}^{m_2}$  37 45  
 $i=2, j=0, g$

3 11  $\overbrace{18}^l$   $\overbrace{3}^j$  25  $\overbrace{27}^k$   $\overbrace{15}^{m_1}$  29 34 37 45  
 $i=3, j=0, g$

$i=4, j=0, g$

3 11  $\overbrace{18}^l$   $\overbrace{3}^j$  25  $\overbrace{27}^k$   $\overbrace{15}^{m_1}$  29 34 37 45

$i=5, j=0, g$

3 11 8 18 0 5 15 27 29 34 37 45

$$i=5, j=0,5$$

3 8 11 18 15 25 27 29 34 37 45

$$i=6, j=0,4$$

3 8 11 15 18 25 27 24 34 37 45

## SELECTION SORT

18 3 29 11 45 27 34 8 25 37 15

$$i=0$$

$$\min = 18$$

3 18 29 11 45 27 34 8 25 37 15

$$i=1$$

$$\min = 18$$

3 8 29 11 45 27 34 18 25 37 15

$$i=2$$

$$\min = 29$$

3 8 11 29 45 27 34 18 25 37 15

$$i=3$$

$$\min = 29$$

3 8 11 15 45 27 34 18 25 37 29

$$i=4$$

$$\min = 45$$

3 8 11 15 18 27 34 45 25 37 29

$$i = 5$$

$$\min = 25$$

3 8 11 15 18 25 34 45 27 37 29

$$i = 6$$

$$\min = 34$$

3 8 11 15 18 25 27 45 34 29

- - - - - 27 29 34 45

## INSERTION SORT

18 3 29 11 45 27 34 8 25 37 15  
 $i=1$   $j=1$

$$18 < 3$$

3 18 29 11 45 27 34 8 25 37 15  
 $i=2$   $j=1$

$$18 < 29, \quad 3 > 29 \quad \underline{\hspace{1cm}} \quad \underline{\hspace{1cm}}$$

$i=3$     29 > 11    18 > 11

3 11 18 29 45 27 34 8 25 37 15  
             $i$      $i$

$i=4$

$i=5$

3 11 17 27 29 45 34 8 25 37 15

$i=6$

3 11 18 27 29 34 45 8 25 37 15

$i=7$

3 8 11 18 27 29 34 45 25 37 15

$i=8$

3 8 11 18 25 27 29 34 45 37 15

$i=9$

3 8 11 18 25 27 29 34 37 45 15

$i=10$

3 8 11 15 18 25 27 29 34 37 45

18 3 29 11 45 27 34 8 25 37 15

$i \quad j$

$\min = 18$

$\max = 0$

$18 > 3$

$\min = 3$   
 $\max = 1$

✓ 3 18 29 11 45 27 34 8 25 37 15  
  |   |  
  X

✓ 3 8 29 11 45 27 34 18 25 37 15  
  |

✓ 3 8 11 29 45 27 34 18 25 37 15  
  |

✓ 3 8 11 15 45 27 34 18 25 37 29  
  |

3 8 11 15 18 27 34 45 25 37 29  
  |

3 8 11 15 18 25 34 45 27 37 29  
  |

3 8 11 15 18 25 27 45 34 37 29  
  |

3 8 11 15 18 25 27 29 34 37 45  
  |

## INSERTION SORT

18 3 29 11 45 27 34 8 25 37 15  
  |   |  
  j i

3 < 18

✓ 3 18 29 11 45 27 34 8 25 37 15  
  |   |  
  j i

29 < 18 F j--

29 < 3 F

✓ 3 18 29 11 45 27 37 8 25 37 15  
j i

11 < 29 A j--  
11 < 18 A j--  
11 < 3 F

✓ 3 11 18 29 45 27 34 8 25 37 15  
j i

✓ ————— 11 ————— j i

✓ 3 11 18 27 29 45 34 8 25 37 15  
j i

✓ 3 11 18 27 29 34 45 8 25 37 15  
j i

3 8 11 18 27 29 34 45 25 37 15  
j

3 8 11 19 25 27 29 34 45 37 15  
j i

3 7 17 18 25 27 29 34 37 45 15

3 8 11 15 18 25 27 29 34 37 45

18 3 29 11 45 27 34 8 25 37 15  
j i

$$3 < 18$$

3 18 29 11 45 27 34 8 25 37 15  
j i

$$\begin{array}{l} 29 < 18 \text{ F } j-- \\ 29 < 3 \text{ F } j-- \end{array}$$

11

$$\begin{array}{l} 11 < 29 \text{ A } j-- \\ 11 < 18 \text{ A } j-- \\ 11 < 3 \text{ F } \end{array}$$

3 11 18 29 45 27 34 8 25 37 15  
j i .

$$\begin{array}{l} 27 < 45 \text{ A } j-- \\ 27 < 25 \text{ A } j-- \\ 27 < 18 \text{ F } \end{array}$$

3 11 18 27 29 45 34 8 25 37 15  
j i

3 11 18 27 29 34 45 8 25 37 15  
j i

3 8 11 18 27 29 34 45 25 37 15  
j i

3 8 11 18 25 27 29 34 45 37 15

### BUBBLE SORT

18 3 29 11 45 27 34 8 25 37 15

$a[j]$   
 $i = 0$   
 $j = 1$

18 > 3 T

3 18 29 11 45 27 34 8 25 37 15

for

j j+1

18 > 29 F

— 11 —————

3 18 11 29 45 27 34 8 25 37 15  
j j+

— 11 —————

$i=0$   $\bar{j}=10$   
3 19 11 29 27 34 8 25 37 15 45

$i=1$   $\bar{j}=0, 9$

3 11 18 27 29 8 25 34 15 37 45

$$i=2, j = \overline{0,1}$$

3 11 18 27 8 25 29 15 34 37 45

$$i=3, j = \overline{0,2}$$

3 11 18 8 25 27 15 29 ...

$$i=4, j = \overline{0,5}$$

3 11 8 18 25 15 27 29 ...

3 8 11 18 15 25 27 29 ...

3 8 11 15 18 25 27 29 ...

18 3 29 11 45 27 34 8 25 37 15  
Min = 18

18 > 3      min = 3

3 18 29 11 45 27 34 8 25 37 15

min = 18      min = 11      min = 8

3 8 29 11 45 27 34 18 25 37 15

3 8 11 29 45 27 34 18 25 37 15

~~3~~ 8 ~~11~~ 15 45 27 34 18 25 37 29

MERGE SORT

18 3 29 11 45 27 | 34 8 25 37 15

[18, 3, 29] | [11, 45, 27] [34, 8, 25] | [37, 15]

[18, 3] [29] [11, 45] [27] [34, 8] [25] [37] [15]

[18] [3] [29] [11] [45] [27] [34] [8] [25] [37] [15]

[18] [3] [11] [45] [34] [8]

DIVIDE

[3, 18] [11, 45] [8, 34]

[3, 18, 29] [11, 27, 45] [8, 25, 34] [10, 37]

[3, 11, 18, 27, 29, 45] [8, 15, 25, 34, 37]

[3, 8, 11, 15, 18, 25, 27, 29, 34, 37, 45]

## QUICK SORT

18 3 29 11 45 27 34 8 25 37 15

pivot = 18

[3, 11, 8, 15] 18 [29, 45, 27, 34, 25, 37]

pivot = 3 pivot = 25

3 [11, 8, 15] 18 [27, 25] 29 [45, 34, 37]

pivot = 11 pivot = 2 = , pivot = 45

3 [8] 11 [15] 18 [25] 27 29 [34, 37] 45

piv = 8, 15, 25, 34

3 8 11 15 18 25 27 27 34 [37] 45

## COUNTING ALGORITHM

$$A = \{5, 4, 6, 8, 1, 2, 8, 8, 5, 5, 2, 4, 1\}$$

$$C = \{ \underbrace{2, 2, 1}_{1}, \underbrace{0, 2, 3}_{2}, \underbrace{1, 0, 3}_{3} \}$$

$$C = \{2, 4, 1, 4, 6, 9, 10, 10, 13\}$$

$$\begin{array}{cccccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 1 & 1 & 2 & 2 & 4 & 4 & 5 & 5 & 5 & 6 & 8 & 8 & 8 \end{array}$$

BUBBLE SORT

$$14 \quad 9 \quad 35 \quad 17 \quad 3 \quad 11 \quad 8 \quad 10 \quad 25 \quad 2 \quad 4$$

i=0    j=0

$$9 \quad 14 \quad 35 \quad 17 \quad 3 \quad 11 \quad 8 \quad 10 \quad 25 \quad 2 \quad 4$$

j=1

---

                                ||

j=2

$$9 \quad 14 \quad 17 \quad 35 \quad 3 \quad 11 \quad 8 \quad 10 \quad 25 \quad 2 \quad 4$$

j=3

9 14 17 3 35 11 8 10 25 2 4

9 14 13 3 11 35 8 10 25 2 4

9 14 13 3 11 8 35 10 25 2 4

9 14 13 3 11 8 10 35 25 2 4

9 14 13 3 11 8 10 25 35 2 4

9 14 13 3 11 8 10 25 2 35 4

9 14 13 3 11 8 10 25 2 4 35

i=1, j=0..9

9 13 3 11 8 10 14 2 4 25 35

i=2, j=0..6

9 3 11 8 10 13 2 4 14 25 35

i=3, j=0..2

3 9 8 10 11 2 4 13 14 25 35

i=4, j=0..6

3 8 9 10 2 4 11 13 ...

i=5, j=0..5

3 8 9 2 4 10 11 13 ...

$i=6 \quad j=0, 1$

3 8 2 4 9 10 ...

3 2 4 8 5 10 11 ...

2 3 4 8 9 10 11 ...

$O(n^2)$  - Worst Case

$O(n)$  - Best case

Elementary method

$O(1)$  - Complexity space

14 9 35 17 3 11 8 10 25 2 4

14 9 35 17 3

min = 3

3 9 35 17 14

3 9 35 17 14

3 9 14 17 35

3 9 14 17 35

$O(1)$  complexity space

$O(\tilde{n})$  - worst, average

ln 9 35 17 5  
j i

$O(n)$  complexity spur

9 ln 35 17 3  
j i

$O(n^2) \sim w$

9 ln 35 17 3  
j i

$O(n) \sim b$

9 ln 17 35 3  
j i

3 9 ln 17 35

ln 9 35 17 3 20 8 10

[ln, 9, 35, 17] [3, 20, 8, 10]

[ln, 9] [35, 17] [3, 20] [8, 10]

[ln] [9] [35] [17] [3] [20] [8] [10]

(9) (ln)

(17) (35)

[3] [20] [8] [10]

(9, ln)

(17, 35)

[3, 20] [8, 10]

(9, ln, 17, 35)

[3, 8, 10, 20]

$\{3, 8, 9, 10, 11, 17, 20, 35\}$

$O(n \log n)$  - W, B, A

$O(n)$  aux

in 9 35 17 3 20 8 10

[9, 15, 8, 10] 14 [35, 17, 20]

[3, 8] 9 [10] 14 [17, 20] 35

3 [8] 9 10 11 17 [20] 35

3 8 9 10 14 17 20 35

$O(n \log n)$

$O(n \log n)$  - B, A

$O(n^2)$

W  $O(n^2)$

B, A  $O(n \log n)$

$O(n \log n)$  - aux

$$\begin{array}{r}
 \cancel{1} \cancel{4} \cancel{9} \cancel{3} \cancel{5} \cancel{1} \cancel{7} \cancel{3} \cancel{2} \cancel{0} \cancel{8} \cancel{1} \cancel{0} \\
 \hline
 \cancel{\infty} \quad X
 \end{array}$$

$$n = 3$$

$$\underbrace{\{1, 3\}}_{\{3, 1\}} \setminus \{9, 20\} \setminus \{35, 10\} \setminus \{17, 7\}$$

$$\{3, 1\} \setminus \{9, 20\} \quad \{10, 35\} \setminus \{8, 17\}$$

$$\{3, 9, 11, 20\} \setminus \{8, 10, 17, 35\}$$

$$\{3, 8, 9, 10, 11, 17, 20, 35\}$$

$O(n \log n)$  - Best

$O(n^k n)$  - Worst

$O(n \log n^2)$  - Average

$O(n)$  - avg

~~3 8 5 2 9 3 7 1 4 8~~

1 2 3 4 5 6 7 8 9

C 1 1 2 1 1 0 1 2 1

C' 1 2 4 5 6 6 7 9 10  
 | | | | | | | | |  
 1 2 3 4 5 6 7 8 9

Spur O(n)  
 Tum O(n)

B 1 2 3 3 4 5 7 8 8 9  
 | | | | | | | | |  
 1 2 3 4 5 6 7 8 9 10

~~3 5 4 8 9 6 1 2 3 5 5 8 7~~

C 2 1 2 0 3 1 1 2 1  
 | | | | | | | | |  
 1 2 3 4 5 6 7 8 9

C' 2 3 5 5 8 9 10 12 13  
 | | | | | | | | |  
 1 2 3 4 5 6 7 8 9

B 1 1 2 3 3 5 5 5 6 7 8 8 9  
 | | | | | | | | | | | |  
 1 2 3 4 5 6 7 8 9 10 11 12 13

3 15 8 7 21 18 1 4  
 h=3

{3, 21} {15, 18} {8, 13} {7, 17}

{3, 21} {15, 18} {11, 13} {4, 7}

{3, 15, 10, 21} {11, 14, 17, 16}

$\{1, 3, 4, 7, 8, 15, 18, 21\}$

$$W = O(n^*n)$$

$$A = O(n \log n^2)$$

$$B = O(n \log n)$$

$$O(1) - \text{space}$$

0.780	0.010	0.005
0.325	0.005	0.010
0.010	0.011	0.111
0.005	0.025	0.325
0.111	0.180	0.209
0.209	0.209	0.780

- Example

77, 55, 65, 44, 304, 22, 100, 13

077	100	100
055	022	304
065	013	013
044	044	022
304	304	044
022	055	055
100	065	065
013	077	077

14 9 35 17 3 11 8 10 25 2 4

pivot = 14

9 3 11 8 10 2 4 14 35 17 25  
pivot = 9 pivot = 14

3 8 2 4 9 11 10 14 17 25 35  
pivot = 3 pivot = 11 pivot = 12

2 3 8 4 9 10 11 14 17 25 35  
pivot = 2 pivot = 8 pivot = 10 pivot = 14

2 3 4 8 9 10 11 14 17 25 35

O(log n) - t  
O(n log n) - s

divide

1

14 9 35 17 3 11 8 10 25 2 4

min = 14

3 9 35 17 14 11 8 10

mine = 3

3 8 35 17 14 11 9 10

min = 3

2 8 9 17 14 11 35 10

$$mn = 17$$

3 8 9 10 (h 11)  $\rightarrow$  17

$$m \times n = 17$$

3 8 9 10 11 (h 12)  $\rightarrow$  17

$$m \times n = 17$$

3 8 9 10 11 12  $\rightarrow$  17

$$m \times n = 17$$

$O(n^2)$   
 $O(1)$

3 8 9 10 11 12 35

14 9 35 17 3 11 8 10 25

j i  
14 > 9 swap

9 14 35 17 3 11 8 10 25

j i  
35 > 17 swap

9 14 35 17 3 11 8 10 25

j i  
35 > 17 swap

9 14 17 35 3 11 8 10 25

j i  
35 > 17 swap

9 14 17 35 3 11 8 10 25

j i

$3 < 3\pi < 17, < 17, < 9$   
swap 3, 9

3 9 17 + 3\pi 11 8 10 2\pi  
j c  
 $1 < 3\pi, < 17, < 15$

3 9 11 17 3\pi 8 10 2\pi  
j c  
 $1 < 3\pi, < 17, < 15, < 11, < 9$

3 8 9 11 14 17 3\pi 10 2\pi  
j c  
 $10 < 3\pi, < 17 < 14 < 11$   $O(n^2)$   
 $O(n)$

3 8 9 10 11 14 17 3\pi 2\pi  
j c

$2\pi < 3\pi$

3 8 9 10 11 14 17 2\pi 3\pi

Quick  $O(n \log n)$   
 $O(n \log n)$

Mem  $O(n \log n)$   
 $O(n)$

ims

14 9 35 17 3 11 8 10 25  
j i

9 < 14 swap

9 14 35 17 3 11 8 10 25  
j i  
35 > 14 no swap

9 14 35 17 3 11 8 10 25  
j i

17 < 35, 17 > 9

9 14 17 35 3 11 8 10 25  
j i

3 < 35, < 17, < 11, < 9

$O(n^2)$   
 $O(1)$

3 9 11 14 17 35 11 8 10 25  
j i

3 9 11 14 17 35 8 10 25  
j i

3 8 9 11 14 17 35 10 25

3 8 9 10 11 14 17 35 25

3 8 9 10 11 14 17 25 35

14 9 35 17 3 11 8 10 25

pivot in

{9 3 11 8 10} 14 35 17 25

piv = 9 piv = 35

{3, 8} 9 {11, 10} 14 {17, 25} 35

p=3 p=11 p=17

3 {8} 9 {10} 11 14 17 {25} 35

p=8 p=10 p=25

3 8 9 10 11 14 17 25 35

O(logn)  
O(logn)  
Divide et Impera

14 9 35 17 3 | 11 8 10 25

{14, 9, 35, 17, 3} |

31 27 3 15 8 | 23 44 19 11

(3 1 2 3 3 | 15 8) (2 3 4 4 | 19 11)

[3 1 2 7 3} [15 8} [23 4 4} [19 1 8}

[3 1 2 7 3} [3} [15} [8} [2 5} {4 4} [1 9 3} [2 7}

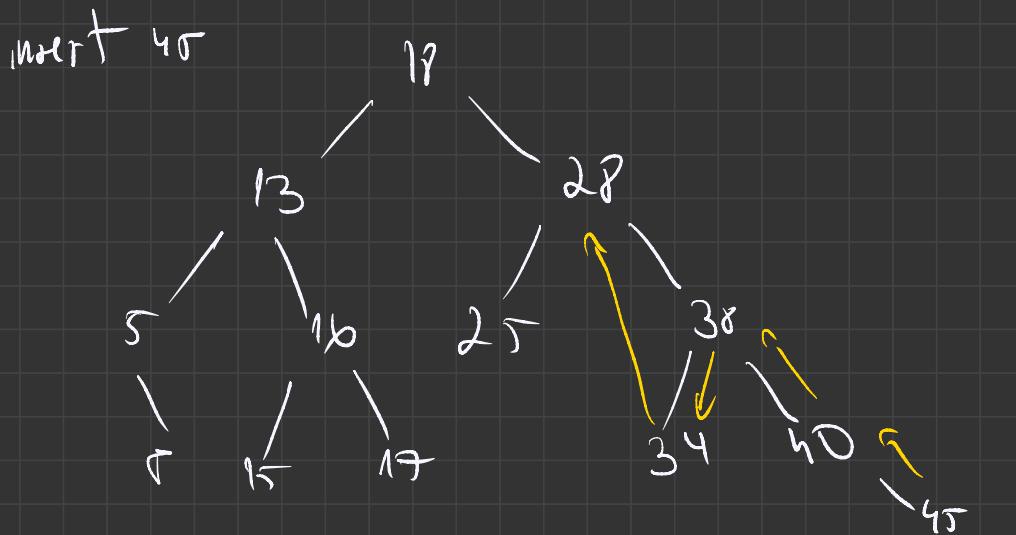
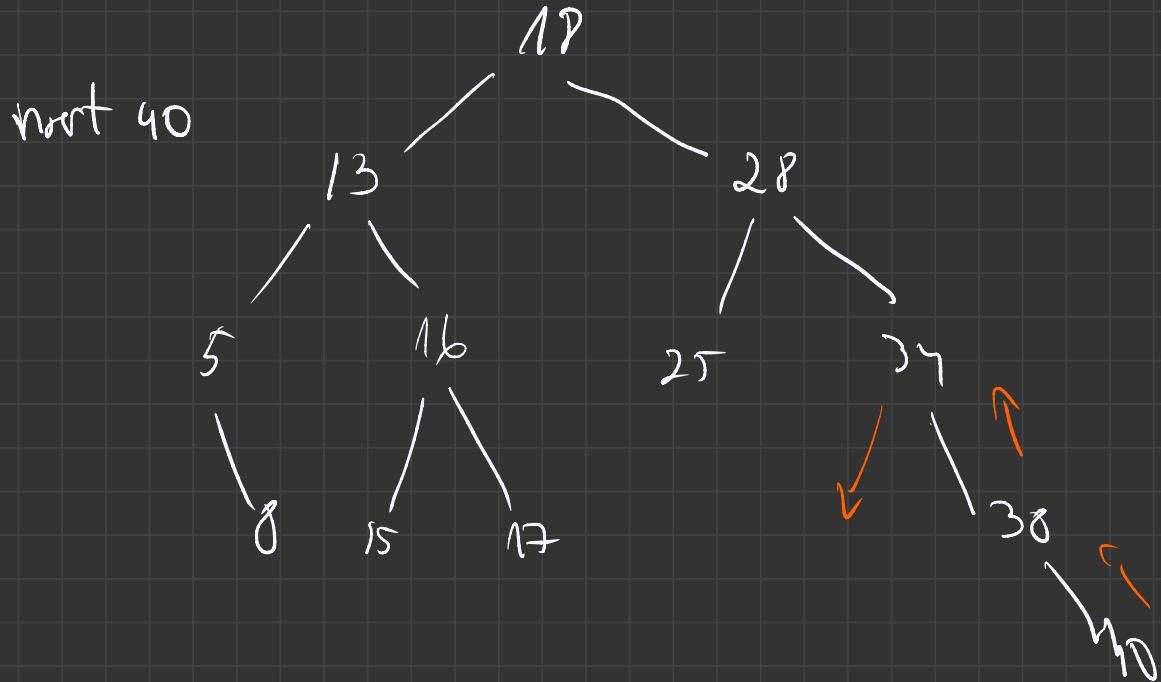
[3 1} [2 2} [3}

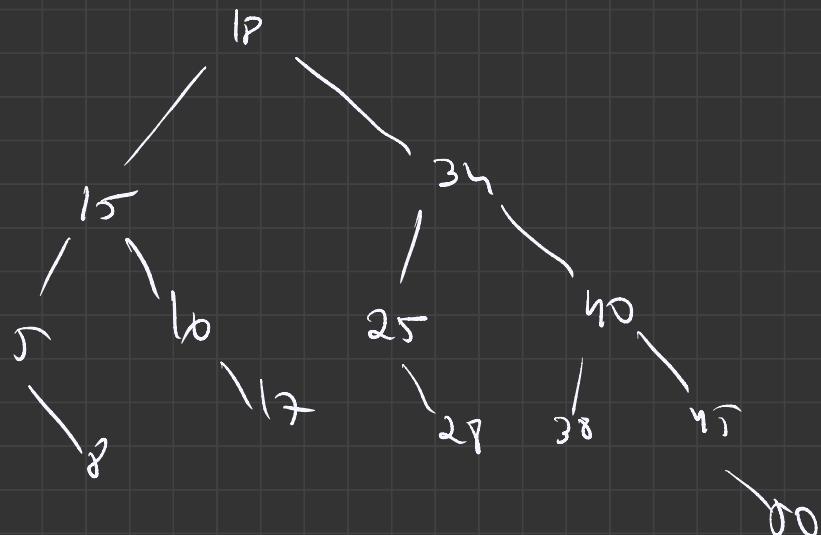
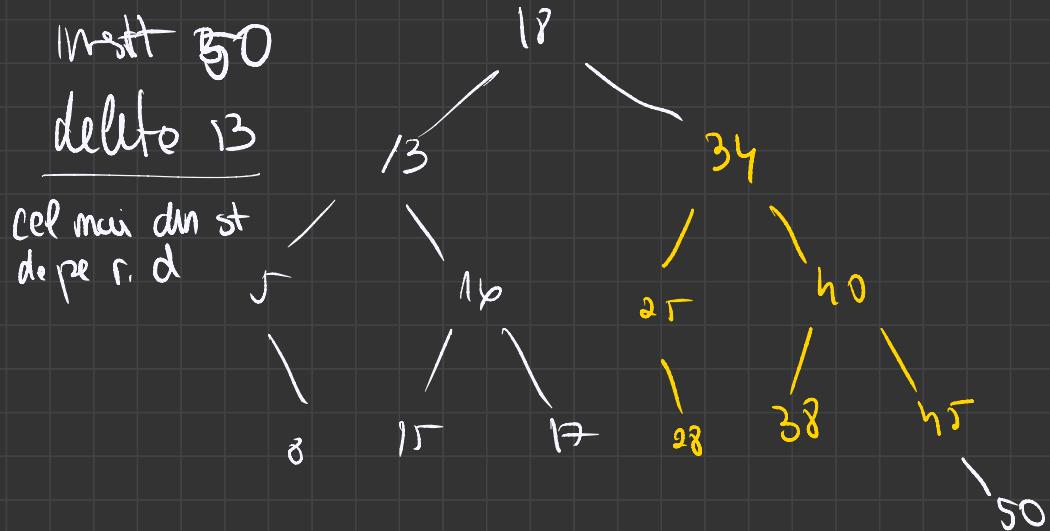
[2 7 3 1} [8 1 5} [2 3 4 4} [1 1, 1 5}

3 2 7 3 1

# ARBORI

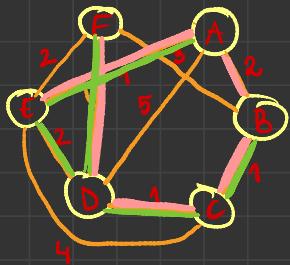
5 8 13 17 18 25 28 34 38





Test: delete, insertion Arbori binar de căutare  
 : exerciții de parcurgere preordere, postordere, în ordere

L A B 10



	A	B	C	D	E	F
A	0	2	$\infty$	5	1	$\infty$
B	2	0	1	$\infty$	$\infty$	3
C	$\infty$	1	0	1	4	$\infty$
D	5	$\infty$	1	0	2	1
E	1	$\infty$	4	2	0	2
F	$\infty$	3	$\infty$	1	2	0

A :  $\{(B, 2), (D, 5), (E, 1)\}$

B :  $\{(A, 2), (C, 1), (F, 3)\}$

C :  $\{(B, 1), (D, 1), (E, 4)\}$

D :  $\{(A, 5), (C, 1), (E, 2), (F, 1)\}$

E :  $\{(A, 1), (C, 4), (D, 2), (F, 2)\}$

F :  $\{(B, 3), (D, 1), (E, 2)\}$



start F    F - B - A - D - C - E



start A    A --> B --> C  
              |  
              |  
              |  
              D  
              |  
              E

start F    F --> B --> C  
              |  
              |  
              |  
              D  
              |  
              E

## KUSKRAL

A - E		B - C		C - D		D - F		A - B		E - F		D - E		B - F		C - E		A - D
1		1		1		1		2		2		2		3		4		5



$$A - E \rightarrow 1$$

$$B - C \rightarrow 1$$

$$D - F \rightarrow 1$$

$$C - D \rightarrow 1$$

$$A - B \rightarrow 2$$

$$CT = 6$$

PRIM



A	B	C	D	E	F
*	*	*	*	*	*

START \*

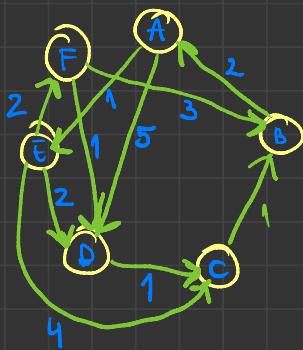
$$Q : \{ (B, 2), (D, 5), (\underline{E}, 1) \} \quad A - E \rightarrow 1$$

$$Q : \{ (C, 1), (\underline{D}, 2), (F, 2), (B, 2) \} \quad E - D \rightarrow 2$$

$$Q : \{ (\underline{C}, 1), (F, 1) \} \quad D - C \rightarrow 1$$

$$Q : \{ (\underline{B}, 1), (F, 1) \} \quad C - B \rightarrow 1$$

$$Q : \{ (\underline{F}, 1) \} \quad \frac{D - F \rightarrow 1}{CT = 6}$$



	A	B	C	D	E	F
A	0	$\infty$	$\infty$	5	1	$\infty$
B	2	0	$\infty$	$\infty$	$\infty$	$\infty$
C	$\infty$	1	0	$\infty$	$\infty$	$\infty$
D	$\infty$	$\infty$	1	0	$\infty$	$\infty$
E	$\infty$	$\infty$	4	2	0	2
F	$\infty$	3	$\infty$	1	$\infty$	0

A:  $\{(D, 5), (E, 1)\}$

B:  $\{(A, 2)\}$

C:  $\{(B, 1)\}$

D:  $\{(C, 1)\}$