- The harmonic oscillations
- Wave reflection / refraction
- Wave interference
- Standing Waves
- Sound pressure
- Sound level
- Columbian force
- Laplace force
- Max Planck's Law
- Schrödinger's equation

EXAMEN

PHYSICS Exam 27.01.2013

S1. Define with your own words the first principle of dynamics. Give an example of a body with the mass between 10 kg and 100 ks. Compare from this potion of view the linear with the circular motion of selected body.

A body continues in it's initial state of relative motion or a relative rest unless an unbalanced force is acting on him

KINEMATICS

$$v = \frac{ds}{dt}$$ rectilinear $$a = \frac{dv}{dt}$$

$s(t) = s(0) + v \cdot t$ - law of motion
$v(t) = v(0) + a \cdot t$ - law of velocity
$s(t) = s(0) + v(0) \cdot t + a \cdot \frac{t^2}{2}$ - law of motion

$v^2 = v_0^2 + 2as$ - Galileo's law

General motion : $\vec{v} = \vec{v}_x \vec{i} + \vec{v}_y \vec{j} + \vec{v}_z \vec{k}$
$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$

$\vec{a}_t$ = change in motion
$\vec{a}_n$ = change of direction

Circular motion: $\omega = \frac{d\alpha}{dt}$ - angular velocity

$\alpha(t) = \alpha(0) + \omega \cdot t$ - law of motion

$\varepsilon = \frac{d\omega}{dt}$ - angular acceleration

$\omega(t) = \omega(0) + \varepsilon t$ - law of velocity

$\alpha(t) = \alpha(0) + \omega(0) t + \varepsilon \cdot \frac{t^2}{2}$ - law of motion

$\boxed{s = R \cdot \alpha}$ - space

T - period
ν - frequency

$\boxed{v = R \cdot \omega}$

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

The vectorial character $\vec{v} = \vec{\omega} \times \vec{R}$

I KINEMATICS

→ velocity : $\vec{v} = \frac{d\vec{r}}{dt} = \frac{ds}{dt}$

→ rectilinear motion with const velocity :

$\vec{v}$ - const → v

$\vec{r} \longrightarrow s$

$$v = \frac{ds}{dt} \Rightarrow ds = v \cdot dt$$

$$\int_{s(0)}^{s(t)} ds = \int_0^t v \cdot dt'$$

$$s \Big|_{s(0)}^{s(t)} = v \Big|_0^t dt'$$

$$s(t) - s(0) = v \cdot t \Big|_0^t$$

$$s(t) = s(0) + v(t-0)$$

$$s(t) = s(0) + v \cdot t$$ law of motion

→ acceleration : $\vec{a} = \frac{\Delta v}{\Delta t} = \frac{d\vec{v}}{dt}$

→ rectilinear motion with const. acceleration

$v(t) = v(0) + a \cdot t$ law of velocity

$s(t) = s(0) + v(0) \cdot t + a \cdot \frac{t^2}{2}$ law of motion

$v^2 = v_0^2 + 2as$ Galileos equation (LAW)

→ curv linear motion : $\vec{v} = v_x \cdot \vec{i} + v_y \cdot \vec{j} + v_z \cdot \vec{k}$

$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \cdot \vec{k}$

→ circular motion: - trajectory : circle
- periodical
- limited in space
- α: Position in circular motion : $s = R \cdot \alpha$

→ $\omega = \frac{d\alpha}{dt}$ - angular velocity

→ circular motion with const. angular velocity:

$$\boxed{\alpha(t) = \alpha(0) + \omega \cdot t}$$

→ angular acceleration

$$\boxed{\varepsilon = \frac{d\omega}{dt}}$$

→ circular motion with const. angular acceleration:

$$\omega(t) = \omega(0) + \varepsilon \cdot t$$

$$\alpha(t) = \alpha(0) + \omega(0) \cdot t + \varepsilon \cdot \frac{t^2}{2}$$ law of motion

→ the relationship between angular and linear quantities:

$$v = R \cdot \omega$$

$$a_n = \frac{v^2}{R} = \omega^2 R = \omega \cdot v$$

$\omega = \frac{2\pi}{T}$ angular velocity

$\nu = \frac{1}{T}$ frequency

$\omega = 2\pi\nu$

→ vectorial characther of circular quantities

$\vec{v} = \vec{\omega} \times \vec{R}$ / $v = \omega \cdot R$

$$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$$

$[\alpha]_{SI} = rad$ $\quad$ $[\omega]_{SI} = \frac{rad}{s}$ $\quad$ $\omega = \frac{dx}{dt}$

$[\varepsilon]_{SI} = \frac{rad}{s^2}$ $\quad$ $\varepsilon = \frac{d\varepsilon}{dt}$

$[\nu]_{SI} = \frac{1}{s} \Rightarrow s^{-1} = Hz$

$[T]_{SI} = s$

$\nu = \frac{1}{T}$; $T = \frac{1}{\nu}$ $\Rightarrow$ $\omega = 2\pi\nu = \frac{2\pi}{T}$

II DYNAMICS

→ Newton's principles

- 1st principle of dynamics: $\boxed{\vec{R} = \sum \vec{F_i} = 0}$ the principle of inertia
- 2nd principle of dynamics: $\boxed{\vec{a} = \frac{\vec{F}}{m}} \Rightarrow \boxed{\vec{F} = m \cdot \vec{a}}$ the fundamental principle of dynamics
- 3rd principle of dynamics: $\boxed{\vec{F}_{AB} = -\vec{F}_{AB}}$ the principle of action and reaction

→ linear momentum: $\vec{p} = m \cdot \vec{v}$

$$\Delta p = \int_{t_1}^{t_2} F(t)\,dt$$ impulse

$$a = \frac{dv}{dt} \qquad F = m \cdot a$$

$$F = m \cdot \frac{dv}{dt}$$

$$F = \frac{d(m \cdot v)}{dt}$$

$$F = \frac{dp}{dt} \Rightarrow d\vec{p} = \vec{F}(t) \cdot dt$$

$$\int_{p_2}^{p_1} d\vec{p} = \int_{t_1}^{t_2} F(t)\,dt$$

$$p\Big|_{p_2}^{p_1} = \int_{t_1}^{t_2} F(t)\,dt$$

$$\Delta p = \int_{t_1}^{t_2} F(t) \cdot dt$$

→ mechanical work: $W = \vec{F} \cdot \vec{s} = |\vec{F}| \cdot |\vec{s}| \cdot \cos\alpha$ lucrul mecanic al forțelor constante

$$W_{1\to2} = \int_{s_1}^{s_2} F(s)\,d\vec{s}$$ lucrul mecanic al forțelor variabile

→ the power : $\boxed{P = \vec{F} \cdot \vec{v}}$

→ the energy : $\boxed{U = m \cdot g \cdot h}$ gravitational potential energy

$$\left.\begin{aligned} W &= \vec{F} \cdot \vec{s} \\ \vec{F} &= \vec{G} = m \cdot g \\ \vec{s} &= \vec{h} \end{aligned}\right\} \Rightarrow W = m \cdot \vec{g} \cdot \vec{h} \Rightarrow U = m \cdot g \cdot h$$

$E = U + K$ $\boxed{K = \frac{m \cdot v^2}{2}}$ The kinetic energy

$$\left.\begin{aligned} &W = F \cdot s = m \cdot a \cdot s \\ &F = m \cdot a \\ &\left.\begin{aligned} v^2 &= v_0^2 + 2as \\ v_0 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} v^2 &= 2as \\ a \cdot s &= \frac{v^2}{2} \end{aligned} \end{aligned}\right\} W = m \cdot \frac{v^2}{2} \Rightarrow \Rightarrow K = \frac{m \cdot v^2}{2}$$

- the laws of conservation

- variation of the kinetic energy : $W = K_2 - K_1 = \frac{m \cdot v_2^2}{2} - \frac{m \cdot v_1^2}{2} = \Delta K$

- the law of the total energy : $W_{ext} = E_2 - E_1 + Q = \Delta E + Q$

→ the momentum of force : $\vec{M} = \vec{r} \times \vec{F}$

→ the law of conservation of the angular momentum : $\vec{M} = \frac{d\vec{L}}{dt}$

→ the momentum of inertia : $I = \sum_{i=1}^{n} m_i \cdot r_i^2$

$$K = \frac{I \cdot \omega^2}{2}$$

→ Types of forces

- the law of universal attraction : $-\vec{F}_{BA} = \vec{F}_{AB} = k_g \cdot \frac{m_A \cdot m_B}{r_2} \cdot \frac{\vec{r}}{|\vec{r}|}$

- the gravitational force :

$G = m \cdot g \qquad g = 9.81 \ Nm^2/kg$

$g \simeq 10$

- the weight :
- the friction force : $F_{fs} = \mu_s \cdot N$ - at sliding

$F_{fr} = \mu_r \cdot \frac{N}{R}$ - at rolling

- the elastic force : $\frac{F_e}{S} = E \cdot \frac{\Delta \ell}{\ell}$ _ Hooke's law

young's modulus of elasticity

$[E]_{SI} = \frac{N}{m^2}$

$\sigma = \frac{F}{S} \ ; \ \varepsilon = \frac{\Delta \ell}{\ell} \Rightarrow \boxed{\sigma = E \cdot \varepsilon}$ the Hooke's Law

the stress the strain

$\vec{F_e} = -\frac{S \cdot E}{\ell} \cdot \Delta \vec{\ell} \ ; \ \boxed{k_e = \frac{S}{\ell} \cdot E}$

the elastic constant

$\vec{F_e} = -k_e \cdot \Delta \vec{\ell}$

the elastic force

- the inertial forces:

$$\vec{r} = x\cdot\vec{i} + y\cdot\vec{j} + z\cdot\vec{k}$$

$$\Downarrow$$

$$\vec{v}$$

$$\Downarrow$$

$$\vec{a}\cdot m = F$$

$$\frac{d\vec{i}}{dt} = \vec{\omega}\times\vec{i}$$

$$\frac{d\vec{j}}{dt} = \vec{\omega}\times\vec{j}$$

$$\frac{d\vec{k}}{dt} = \vec{\omega}\times\vec{k}$$

$\vec{F_i} = -m\cdot\vec{a}$

$\vec{F_r} = -m\cdot\vec{a}$ the self initial force

$\vec{F_t} = -m\cdot\vec{a_0}$ the initial force at transitional motion of the refrence system

$\vec{F_{rt}} = -m\,\vec{\varepsilon}\times\vec{r}$ the initial force at notational motion of the reference system

$\vec{F_{cf}} = -m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})$ the centrifugal force

$F_c = -2m\vec{\omega}\times\vec{v}$ the CORIOLIS force

the velocity of object

the angular velocity of the refrence frame

the mass of object

III OSCILLATIONS

→ the harmonic oscillations

Taylor series

$$U(x) = U(0) + \frac{1}{1!}\frac{dU(x)}{dx}\Big|_{x=0} \cdot x^1 + \frac{1}{2!}\frac{d^2U(x)}{dx^2}\Big|_{x=0} \cdot x^2 + \dots$$

$$U(0) = 0$$

$$\boxed{U(x) = \frac{1}{2}k \cdot x^2}$$ the elastic potential energy

$$\boxed{\vec{F} = -\nabla U = -\text{grad}\, U} \qquad \nabla = \frac{\partial}{\partial x}\cdot\vec{i} + \frac{\partial}{\partial y}\cdot\vec{j} + \frac{\partial}{\partial z}\cdot\vec{k}$$

$$\vec{F_e} = -k \cdot x$$ an elastic type force

→ the law of motion

$$v = \frac{dx}{dt} = \dot{x}\,;\quad a = \frac{dv}{dt} = \dot{v} = \ddot{x}\,;\quad a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} = \ddot{x}$$

$$\ddot{x} + \omega_0^2 \cdot x = 0$$ second order homogenous diff eq.

$$\omega_0^2 = \frac{k}{m}$$ charac. eq.

$$x(t) = A\sin(\omega_0 t + \varphi_0)$$ the law of motion for the harmonic oscillations

the elongation

A – Amplitude

$$\omega_0 = \sqrt{\frac{k}{m}}$$

$$\varphi(t) = \omega_0 t + \varphi_0$$ – the phase

→ the energy

$$x(t) = A\sin(\omega_0 t + \varphi_0)$$

$$v = \frac{dx}{dt} = \frac{d[A\sin(\omega_0 t + \varphi_0)]}{dt} \Rightarrow v = A\cdot\omega_0\cdot\cos(\omega_0 + \varphi_0)$$

$\omega_0^2 = \frac{k}{m} \Rightarrow k = m \cdot \omega_0^2$

$a = \frac{dv}{dt} = \frac{d}{dt}\left[\omega_0 A \cdot \cos(\omega_0 t + \varphi_0)\right] = -\omega_0^2 A \sin(\omega_0 t + \varphi_0) \Rightarrow$

$a = -\omega_0^2 x$

$K = \frac{m}{2} \omega_0^2 \cdot A^2 \cos^2(\omega_0 t + \varphi_0)$; $U = \frac{k}{2} A^2 \cdot \sin^2(\omega_0 t + \varphi_0)$

$= \frac{m}{2} \omega_0^2 A \cdot \sin(\omega_0 t + \varphi_0)$

$E = K + U \Rightarrow E = \frac{1}{2} m \omega_0^2 A^2 \Rightarrow \boxed{E = \frac{1}{2} K \cdot A^2}$ constant

→ the damped oscillations :

$F = F_e + F_r$ $\quad x(t) = C_1 \cdot e^{(-\delta - \sqrt{\delta^2 \cdot \omega_0^2}) t} + C_2 \cdot e^{(-\delta + \sqrt{\delta^2 \cdot \omega_0^2}) \cdot t}$

$F = m \cdot a = m \cdot \ddot{x}$

$F_e = -k \cdot x$

$F_r = -r \cdot v = -r \cdot \dot{x}$

the resistant coeff.

the resistance force

$\ddot{x} + \frac{r}{m} \cdot \dot{x} + \frac{k}{m} x = 0$ second order homogenous diff. eq

$\ddot{x} + 2\delta \dot{x} + \omega_0^2 = 0$ $\quad \omega_0^2 = \frac{k}{m}$; $2\delta = \frac{r}{m}$

- large resistance
  - r - large
  - $\delta > \omega_0$; $C_2 = 0$
  - $x(t) = C_1 \cdot e^{(-\delta - \sqrt{\delta^2 \cdot \omega_0^2}) \cdot t}$

- small resistance
  - r - small
  - $\delta < \omega_0$
  - $x(t) = e^{-\delta t}\left[\underbrace{C_1 e^{-i\omega t} + C_2 \cdot e^{+i\omega t}}_{A \sin(\omega t + \varphi_0)}\right]$
  - $x(t) = A(t) \sin(\omega t + \varphi_0)$
  - $A(t) = A \cdot e^{-\delta t}$

- $\Delta = \delta T$ the logarithmic decrement

δ — damping coeff

- the relaxation time (τ)

$$\delta = \frac{1}{\tau}$$ the damping coeff

$$x(t) = A \cdot e^{-\delta t} \sin(\omega t + \varphi_0)$$
$$x(t) = A \cdot e^{-\frac{t}{\tau}} \sin(\omega t + \varphi_0)$$

→ the force oscillations

$$x(t) = X_0(t) + X_1(t)$$
$$X_0(t) = A \cdot e^{-\delta t} \cdot \sin(\omega t + \varphi)$$
$$X_1(t) = A(W_{ext}) \cdot \sin(W_{ext} \cdot t + \varphi_0)$$ the law of motion

$$A(W_{ext}) = \frac{F_0}{m\sqrt{(\omega_0^2 - \omega_{ext}^2) + 4 \cdot \delta^2 \omega_{ext}}}$$ the amplitude of the forced oscillations

$$tg(\varphi_0) = \frac{-2\delta \omega_{ext}}{\omega_0^2 - \omega_{ext}^2}$$ the initial phase

→ the resonance : $\omega_{rez} = \sqrt{\omega_0^2 - 2\delta^2}$

IV WAVES

→ the velocity of waves

- transverse waves : $v_t^s = \sqrt{\frac{T}{\mu}}$ s - solid, t - transverse

- longitudinal waves

$v_l^s = \sqrt{\frac{E}{\rho}}$ s - solid (3D solid), $\rho = \frac{m}{V}$ - density (m^3), E = Young's modulus

$v_l^l = \sqrt{\frac{\chi}{\rho}}$ l - liquid, $\rho - \frac{m}{V}$ d, χ - modulus of compresibility

$v_l^g = \sqrt{\gamma \frac{P}{\rho}}$ $P = \frac{F}{S}$ - pressure

$\gamma = \frac{c_p}{c_v}$ - adiabatic coeff

→ wave reflection

- the law of reflection :- the incident ray, the normal and the reflected ray are in the same plane

$$\hat{i} = \hat{r}$$

→ wave refraction

the law of refraction :- the incident ray, the normal and the refracted ray are in the same plane

$$\frac{\sin \hat{i}}{\sin \hat{r}} = \frac{v_1}{v_2}$$

→ the eq. of a plane wave

$\psi(t,d) = A \sin\left[\omega\left(t - \frac{d}{v}\right)\right]$ the wave function

$\psi(t,d) = A \sin\left[2\pi\left(\frac{t}{T} - \frac{d}{v \cdot T}\right)\right]$ $\omega = \frac{2\pi}{T}$; $\nu = \frac{1}{T}$; $\omega = 2\pi\nu$

$\psi(t,d) = A \sin\left[2\pi\left(\frac{t}{T} - \frac{d}{\lambda}\right)\right]$ $\lambda = v \cdot T$ - the wavelenght

$k = |\vec{k}| = \frac{2\pi}{\lambda}$ the wave vector; $\psi(t,d) = A\sin(\omega t - k d)$

$$\left.\begin{array}{l}\vec{r} = x\cdot\vec{i} + y\vec{j} + z\cdot\vec{k} \\ k = k\cdot\vec{i}\end{array}\right\} \Rightarrow \vec{k}\cdot\vec{r} = k\cdot x$$

$\psi(t,x) = A\cdot\sin(\omega t - k\cdot x)$ the eq of a plane wave

→ the wave interference

$$\left.\begin{array}{l}\bullet\ \varphi_1(t,x) = \omega_1 t - \varphi_{01} \\ \varphi_2(t,x) = \omega_2 t - \varphi_{02}\end{array}\right\} \Delta\varphi = \varphi_1 - \varphi_2$$

$$= (\omega_1 - \omega_2)t - (\varphi_{01} - \varphi_{02})$$

$$\omega_1 = \omega_2 = \omega$$

$$2\pi\nu_1 = 2\pi\nu_2 \Rightarrow \nu_1 = \nu_2 = \nu$$

$$\frac{1}{\nu_1} = \frac{1}{\nu_2} \Rightarrow T_1 = T_2 = T$$

$$v_1 = v_2 = v$$

$$v_1 T = v_2 T \Rightarrow \lambda_1 = \lambda_2 = \lambda$$

$$\frac{2\pi}{\lambda_1} = \frac{2\pi}{\lambda_2} \Rightarrow k_1 = k_2 = k$$

$$\psi(t,\delta) = 2A\cos\left(\frac{k\delta}{2}\right)\cdot\sin\left(\omega t - k\cdot\frac{d_1+d_2}{2}\right) ; \quad \delta = d_2 - d_1$$

- minimum

$$\cos\left(\frac{k\delta}{2}\right) = 0$$

$$\frac{k\delta}{2} = 1\cdot\frac{\pi}{2} ; 3\cdot\frac{\pi}{2} ; 5\cdot\frac{\pi}{2}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \frac{2\pi\delta^{(n)}}{\lambda 2} = (2n+1)\frac{\pi}{2}$$

$$\delta_{min}^{(n)} = (2n+1)\frac{\lambda}{2} + n, \quad n = 0,1,2..$$

• maximum

$$\cos\left(\frac{k\delta}{2}\right) = \pm 1$$

$$\frac{k\delta}{2} = 0, \pi, 2\pi, 3\pi$$

$$K = \frac{2\pi}{\lambda} \Rightarrow \frac{2\pi\delta^{(u)}}{\lambda \cdot 2} = \frac{2u\pi}{2} \quad \delta^{(u)}_{max} = 2u\frac{\lambda}{2} + u \quad , u = 0, 1, 2 \ldots$$

→ standing waves

• fixed wall : reflection on a fixed wall is with lost of:

- in space : $\frac{\lambda}{2}$
- in time : $\frac{T}{2}$
- in phase : π

• mobile wall: no lost

→ the amplitude of a point to x : $\Psi(t, x) = 2A \cdot \sin(K \cdot x) \cdot \cos(wt - k \cdot e)$

• maximum

$$\sin(k \cdot x) = \pm 1$$

$$\frac{2\pi}{\lambda} \cdot x = 1 \cdot \frac{\pi}{2} ; 3 \cdot \frac{\pi}{2} ; 5 \cdot \frac{\pi}{2}$$

$$K = \frac{2\pi}{\lambda}$$

$$x_n^{max} = (2n+1)\frac{\lambda}{n} \quad , n = 0, 1, 2 \ldots$$

• minimum

$$\sin(k \cdot x) = 0$$

$$\frac{2\pi}{\lambda} \cdot x = 0 \cdot \pi, 1 \cdot \pi, 2 \cdot \pi, 3 \cdot \pi$$

$$x_n^{min} = 2n \cdot \frac{\lambda}{n} \quad , n = 0, 1, 2 \ldots$$

$$i = x_{n+1}^{min} - x_n^{min} \Rightarrow i = \frac{\lambda}{2} \Rightarrow \lambda = 2 \cdot i$$

EXAMEN

$s(t) = t^3 - 2t + 3\sin(2\pi \cdot t)$

$s(0) = 0$

$v = \frac{ds}{dt} = \frac{d(t^3 - 2t + 3\sin(t))}{dt}$

$= 3t^2 - 2 + 3\cos t$

$a = \frac{dv}{dt} = \frac{d(3t^2 \quad + 3\cos t)}{dt} = 6t - 3\sin t$

♡

$v(t) = 120t^3 - 90t + 25 \left[\frac{m}{s}\right]$

$v = \frac{d}{t}$

$a(t), s(t) = ?$

$s(0) = 50m$

$s(t) = s(0) + vt$

$s(t) = s(0) + v(0) \cdot t + \frac{a \cdot t^2}{2}$

$v(t) = v(0) + a \cdot t$

$v(0) = 25 \frac{m}{s}$

$120t^3 - 90t + \cancel{25} = \cancel{25} + a \cdot t \quad / : t$

$120t^2 - 90 = a(t)$

$s(t) = 50 + 25t + \frac{(120t^2 - 90)t^2}{2}$

$s(t) = 50 + 25t + 60t^4 - 45t^2$

$v(t) = 120\sqrt{t^3} - 90\sqrt{t^2} + 20 \ \frac{dm}{s}$

$\frac{m}{dm}$

$= 120\sqrt{t^3} - 90\sqrt{t^2} + 20 \cdot 10^{-1} \frac{m}{s} =$

$= 12t^{\frac{3}{2}} - 9t + 2$

$a(t)$, v_0, t_0

$s(t)$

$a(t) = \frac{dv}{dt} = \frac{d(12t^{\frac{3}{2}} - 9t + 2)}{dt} =$

$= 12 \cdot \frac{3}{2}\sqrt{t} - 9 = 18\sqrt{t} - 9$

$a(t) = 18\sqrt{t} - 9$

$v(t) = v_0 + a \cdot t$

$v(t) = \frac{ds}{dt}$

$s(t) = s_0 + vt$

$ds = v(t)dt$

$a = \frac{dv}{dt}$

$ds = (12t^{\frac{3}{2}} - 9t + 2) \cdot 1$

$ds = 12t^{\frac{3}{2}} - 9t + 2$

$s(t) = \int 12t^{\frac{3}{2}} - 9t + 2$

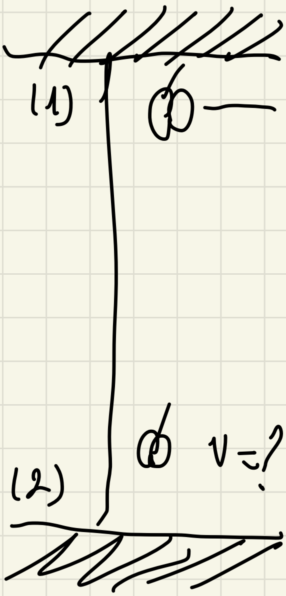
$= \frac{12t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{9t^2}{2} + 2t$

3. One apple starts to fall from an apple tree without friction from a 3.2 m height. Use the conservation law of energy to calculate the velocity at the root of the apple tree. Consider $g = 10\ m/s^2$

E_c, E_p

$$E_c = \frac{m \cdot v^2}{2}$$

$$E_p = m \cdot g \cdot h$$



$E_c = 0$

$E_p = 0$

$$E_{p1} = E_{c(2)}$$

$$m \cdot g \cdot h = \frac{m \cdot v^2}{2} \quad / : m$$

$$g \cdot h = \frac{v^2}{2} \iff v^2 = 2gh$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 3{,}2 \cdot 10} = 8 \left[\frac{m}{s}\right]$$

$$v(t) = 0{,}5t^2 - 2t^{\frac{1}{2}} + 20\,\frac{dm}{s} = 0{,}05t^2 - 0{,}2t^{\frac{1}{2}} + 2\,\frac{m}{s}$$

$a(t) = ?$

$a(t_0) = ?\quad t_0 = 1$

v_m

$$v(t) = v(0) + a \cdot t \qquad a = \frac{dv}{dt}$$

$$\frac{v(t) - v(0)}{t} = a$$

$$a = \frac{d(0{,}05t^2 - 0{,}2t^{\frac{1}{2}} + 2)}{dt}$$

$$a = \frac{0{,}1t - 0{,}1t^{\frac{3}{2}}}{1} \Rightarrow a(t) = 0{,}1t - 0{,}1t^{\frac{3}{2}}$$

$$a(1) = 0{,}1 - 0{,}1 = 0$$

$$V = \frac{1}{10}\int_{10}^{20} 0{,}05 - 0{,}2t^{\frac{1}{2}} + 2 = \frac{1}{10}\left(0{,}05t - \frac{\cancel{0{,}2}\,0{,}1\,t^{\frac{3}{2}}}{\frac{3}{\cancel{2}}} + 2t\right)\Bigg|_{10}^{20}$$

$$= \frac{1}{10}\left(0{,}05 \cdot 10 - \frac{0{,}1(20-10)^{\frac{3}{2}}}{3} + 2 \cdot 10\right) =$$

$$= \frac{1}{10}\left(0{,}5 - \frac{0{,}1}{3} \cdot 10^{\frac{3}{2}} + 20\right)$$

1. $s(t) = 3t^2 + 2 + \sin(2\pi t)$

$$v(t) = \frac{ds}{dt} = \frac{d(3t^2 + 2 + \sin t)}{dt} = 6t + \cos t$$

$$a(t) = \frac{dv}{dt} = \frac{d(6t + \cos t)}{dt} = 6 - \sin t$$

$f(x)$

$M(t) = 1200t^4 - 100t^2$

$M = \frac{dL}{dt}$ $\quad$ $M = dL/dt$ $\quad$ $dL = M\,dt$

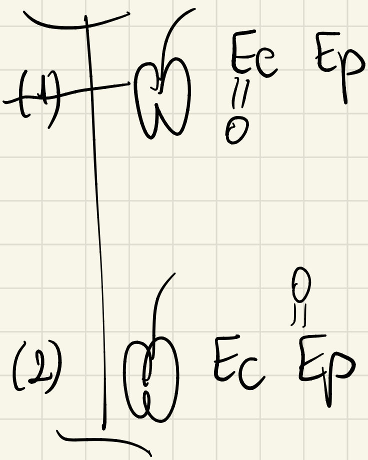
$dt \cdot 1200t^4 - 100t^2 = dL$ $\quad$ $dL = 1200t^4 - 100t^2$

$dL = 1200t^4 - 100t^2$ $\quad$ $L = \int_0^t$

$$dL = \int_0^t 1200t^4 - 100t^2 =$$

$$L = 100\int_0^t 12t^4 - t^2 = 100\left(12\frac{t^5}{5} - \frac{t^3}{3}\right)$$

3. $h = 3{,}2\,m$ $\quad$ $g = 10\,g/m^2$



$$E_c = \frac{m \cdot v^2}{2}$$

$$E_p = m \cdot g \cdot h$$

$$E_p = E_c$$

$$\cancel{m}gh = \frac{\cancel{m}v^2}{2}$$

$$v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

$\Rightarrow \Delta = \ln\left(e^{\cancel{\ \ }\cancel{\ \ }+\delta T}\right) \Rightarrow \boxed{\Delta = \delta T}$

b) The relaxation time, τ

$A(t+\tau) = \frac{A(t)}{e} \Rightarrow Ae^{-\delta(t+\tau)} = \frac{Ae^{-\delta t}}{e^{1}} \Rightarrow$

$\Rightarrow e^{-\delta t-\delta\tau} = e^{-\delta t-1} \Rightarrow \delta\tau = 1 \Rightarrow \tau = \frac{1}{\delta}$

$\delta = \frac{1}{\tau}$ – the damping coefficient

$x(t) = Ae^{-\delta t}\sin(\omega t+\varphi_0)$

$x(t) = Ae^{-\frac{t}{\tau}}\sin(\omega t+\varphi_0)$

3. The forced oscillations

$F = F_e + F_r + F_{ext}$

$x(t) = Ae^{-\delta t}\sin(\omega t+\varphi_0)$

$x(t) = Ae^{-\frac{t}{\tau}}\sin(\omega t+\varphi_0)$

$\boxed{\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar^2}(E-U)\psi_{(\vec{r})} = 0}$ The Schrödinger equation

$\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar^2}(E-V)\psi_{(\vec{r})} = 0$

$x(t) = Ae^{-\delta t}\sin(\omega t+\varphi_0)$

$x(t) = Ae^{-\frac{t}{\tau}}\sin(\omega t+\varphi_0)$

$\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar^2}(E+U)\psi(\vec{r}) = 0$

$x(t) = Ae^{-\delta t}\sin(\omega t+\varphi_0)$

$x(t) = Ae^{-\frac{t}{\tau}}\sin(\omega t+\varphi_0)$

$\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$

$\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar}(E+U)\psi(\vec{r}) = 0$ $\hbar^2$ $(E+U)\psi_{(\vec{r})} = 0$

$x(t) = Ae^{-\delta t\ \sin\ -\frac{t}{\tau}}\ \sin(\omega t+\varphi_0)$

$\nabla^2\psi_{(\vec{r})} + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$ Ecuația lui Schrödinger

$\nabla^2\psi_{(\vec{r})} + \frac{2m}{\hbar^2}(E+U)\psi(\vec{r}) = 0$

$\nabla^2\psi(\vec{r}) + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$

$$x(t) = Ae^{-\delta t}\sin(\omega t + \varphi_0)$$

$$\nabla^2\psi_{(\vec{r})} + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$$

$$\nabla^2\psi_{(\vec{r})} + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$$

$$\nabla^2\psi_{(\vec{r})} + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$$

$$x(t) = Ae^{-\delta t}\sin(\omega t + \varphi_0)$$

1. The Maxwell's equations

1.1 The integral form

$$\begin{cases} \oint_\Sigma \vec{E}\cdot d\vec{s} = \frac{q_i}{\varepsilon_0} & \text{- the Gauss law for the electric field} \\ \oint_\Sigma \vec{B}\cdot d\vec{s} = 0 & \text{- the Gauss law for the magnetic field} \\ \oint_\Gamma \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s} & \text{- the Faraday's law} \\ \oint_\Gamma \vec{B}\cdot d\vec{l} = \int_S \mu_0(\vec{j}_i + \vec{j}_d) d\vec{s} & \text{- the Ampère's law} \end{cases}$$

$$\oint_\Sigma \vec{E}\cdot d\vec{s} = \frac{q_i}{\varepsilon_0}$$

$$\oint_\Sigma \vec{B}\cdot d\vec{s} = 0$$

$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}$$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \int_S \mu_0(\vec{j}_i + \vec{j}_d) d\vec{s}$$

$$\oint_\Sigma \vec{E}\cdot d\vec{s} = \frac{q_i}{\varepsilon_0}$$

$$\oint_\Sigma \vec{B}\cdot d\vec{s} = 0$$

$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}$$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \int_S \mu_0(\vec{j}_i - \vec{j}_d) d\vec{s}$$

$$\oint_\Sigma \vec{E}\cdot d\vec{s} = \frac{q_i}{\varepsilon_0}$$

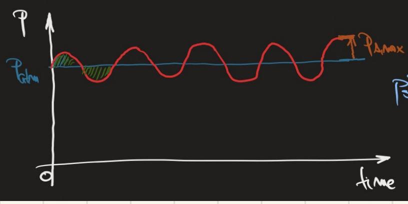
$$\oint_\Sigma \vec{B}\cdot d\vec{s} = 0$$

$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}$$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \int_S \mu_0(\vec{j}_i - \vec{j}_d) d\vec{s}$$

5.1. The sound pressure

$$p_s(t) = p_{s,max} \sin(\omega t)$$



$$p_{s,av}^2 = \frac{1}{T-0}\int_0^T p_{s,max}^2 \sin^2(\omega t)\, dt$$

$$p_{s,av}^2 = \frac{1}{T-0}\int_0^T p_{s,max}^2 \sin^2(\omega t)\, dt$$

$$p_{s,av} = p_{s,max} \cdot \sin(\omega t)$$

$$p_{s,av}^2 = \frac{1}{T-0}\int_0^T p_{s,max}^2 \cdot \sin^2(\omega t)\, dt$$

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$p_{s,av}^2 = \frac{1}{T-0}\int_0^T p_{s,max}^2 \cdot \sin^2(\omega t)\, dt$$

$$s(t) = s(0) + v \cdot t$$
$$v(t) = v(0) + a \cdot t$$
$$s(t) = s(0) + v(0) \cdot t + a \frac{t^2}{2}$$

$v = \dfrac{ds}{dt}$ rectilinear $\qquad a = \dfrac{dv}{dt}$

$s(t) = s(0) + v \cdot t$ - law of motion

$v(t) = v(0) + a \cdot t$ - law of velocity

$s(t) = s(0) + v(0) \cdot t + a \cdot \dfrac{t^2}{2}$ - law of motion

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$s(t) = s(0) + v \cdot t$$
$$v(t) = v(0) + a \cdot t$$
$$s(t) = s(0) + v(0) \cdot t + a \frac{t^2}{2}$$

$$s(t) = s(0) + v \cdot t$$
$$v(t) = v(0) + a \cdot t$$
$$s(t) = s(0) + v(0) \cdot t + a \cdot \frac{t^2}{2}$$

$$x(t) = Ae^{-\delta t}\sin(\omega t + \varphi_0)$$
$$x(t) = Ae^{-\frac{t}{\delta}}\sin(\omega t + \varphi_0)$$

$$\nabla^2 \psi_{(\vec{r})} + \frac{2m}{\hbar^2}(E+U)\psi_{(\vec{r})} = 0$$

$$P^2_{S,ON} = \frac{1}{T-0}\int_0^T P^2_{S,max}\sin^2(\omega t)\,dt$$

$$v = \frac{ds}{dt} \qquad a = \frac{dv}{dt}$$

$$s(t) = s(0) + v.t$$
$$v(t) = v(0) + a.t$$
$$s(t) = s(0) + v(0).t + a\frac{t^2}{2}$$

1.3. The differential form

$$\int_V \underbrace{\nabla\cdot\vec{E}}\cdot dV = \int_V \underbrace{\frac{\rho_i}{\varepsilon_0}}\,dV$$

$$\int_V \underbrace{\nabla\cdot\vec{B}}\,dV = \int_V \underbrace{0}\,dV$$

$$\int_S \underbrace{\nabla\times\vec{E}}\,d\vec{s} = \int_S \underbrace{-\frac{\partial\vec{B}}{\partial t}}\,d\vec{s}$$

$$\int_S \underbrace{\nabla\times\vec{B}}\,d\vec{s} = \int_S \underbrace{\mu_0(\vec{j}_i + \vec{j}_d)}\,d\vec{s}$$

$$\begin{cases} \nabla\vec{E} = \frac{\rho_i}{\varepsilon_0} & \text{- Gauss law for } \vec{E} \\ \nabla\vec{B} = 0 & \text{- Gauss law for } \vec{B} \\ \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} & \text{- Faraday's law} \\ \nabla\times\vec{B} = \mu_0\left(\vec{j}_i + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right) & \text{- the Ampére's law} \end{cases}$$

& $C = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$

$$\oint_\Sigma \vec{E}\cdot d\vec{s} = \frac{q_i}{\varepsilon_0}$$

$$\oint_\Sigma \vec{B}\cdot d\vec{s} = 0$$

$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial\vec{B}}{\partial t}\,d\vec{s}$$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \int_S \mu_0(\vec{j}_i + \vec{j}_d)\,d\vec{s}$$

$$\int_V \nabla\cdot\vec{E}\cdot dv = \int_V \frac{\rho_i}{\varepsilon_0}$$

$$\int_V \nabla\cdot\vec{B}\cdot d\vec{v} = \int_V 0\,dv$$

$$\int_S \nabla\times\vec{E}\,d\vec{s} = -\int \frac{\partial\vec{B}}{\partial t}\,d\vec{s}$$

$$\int_S \nabla\times\vec{B}\,d\vec{s} = \int_S \mu_0(\vec{j}_i + \vec{j}_d)\,d\vec{s}$$

$$\int_V \nabla\vec{E}\,dv = \int_V \frac{\rho_i}{\varepsilon_0}$$

$$\int_V \nabla\vec{B}\,dv = \int 0\,dv$$

$$\int_V \nabla\times\vec{E}\cdot d\vec{s} = -\int \frac{\partial\vec{B}}{\partial t}\,d\vec{s}$$

$$\int_V \nabla\cdot\vec{B}\cdot d\vec{s} = \int_S \mu_0(\vec{j}_i + \vec{j}_d)\,d\vec{s}$$

$$\oint_\Sigma \vec{E}\cdot d\vec{s} = \frac{\rho_i}{\varepsilon_0}$$

$$\oint_\Sigma \vec{B}\cdot d\vec{s} = 0$$

$$\oint_\Gamma \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}$$

$$\oint_\Gamma \vec{B}\cdot d\vec{l} = \int_S \mu_0(\vec{j_i} + \vec{j_d}) d\vec{s}$$

$$\int_V \nabla\vec{E}\, dv = \int_V \frac{q_i}{\varepsilon_0}$$

$$\int_V \nabla\vec{B}\, dv = \int_V 0\, dv$$

$$\int_S \nabla\times\vec{E}\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}$$

$$\int_S \nabla\times\vec{B}\cdot d\vec{s} = \int_S \mu_0(\vec{j_i} + \vec{j_d}) d\vec{s}$$

$$\int_V \nabla\vec{E}\cdot dv = \int_V \frac{q_i}{\varepsilon_0}$$

$$\int_V \nabla\vec{B}\cdot dv = \int_V 0\, dv$$

$$\int_S \nabla\times\vec{E}\cdot d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} d\vec{s}$$

$$\int_S \nabla\times\vec{B}\cdot d\vec{s} = \int_S \mu_0(\vec{j_i} + \vec{j_0}) d\vec{s}$$

1.3. The differential form

$$\int_V \underbrace{\nabla\cdot\vec{E}}\cdot dV = \int_V \underbrace{\frac{\rho_i}{\varepsilon_0}} dv$$

$$\int_V \underbrace{\nabla\cdot\vec{B}}\, dV = \int_V \underbrace{0}\, dv$$

$$\int_S \underbrace{\nabla\times\vec{E}}\, d\vec{s} = \int_S \underbrace{-\frac{\partial \vec{B}}{\partial t}} d\vec{s}$$

$$\int_S \underbrace{\nabla\times\vec{B}}\, d\vec{s} = \int_S \underbrace{\mu_0(\vec{j_i} + \vec{j_d})} d\vec{s}$$

$$\begin{cases} \nabla\vec{E} = \frac{\rho_i}{\varepsilon_0} & \text{– Gauss law for } \vec{E} \\ \nabla\vec{B} = 0 & \text{– Gauss law for } \vec{B} \\ \nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} & \text{– Faraday's law} \\ \nabla\times\vec{B} = \mu_0\left(\vec{j_i} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) & \text{– the Ampére's law} \end{cases}$$

& $c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$

$$\nabla\vec{E} = \frac{\rho_i}{\varepsilon_0}$$

$$\nabla\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\left(\vec{j_i} - \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\nabla\vec{E} = \frac{q_i}{\varepsilon_0}$$

$$\nabla\vec{B} = 0$$

$$\nabla\times\vec{E} = \frac{-\partial \vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\left(\vec{j_i} + \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

11. The Maxwell's equations

11.1 The integral form

$$\begin{cases} \oint_{\Sigma} \vec{E}\cdot d\vec{s} = \dfrac{q_i}{\varepsilon_0} & \text{- the Gauss law for the electric field} \\ \oint_{\Sigma} \vec{B}\cdot d\vec{s} = 0 & \text{- the Gauss law for the magnetic field} \\ \oint_{\Gamma} \vec{E}\cdot d\vec{l} = -\int_{S} \dfrac{\partial \vec{B}}{\partial t}\, d\vec{s} & \text{- the Faraday's law} \\ \oint_{\Gamma} \vec{B}\cdot d\vec{l} = \int_{S} \mu_0 (\vec{j}_i + \vec{j}_d)\, d\vec{s} & \text{- the Ampère's law} \end{cases}$$

$$\oint_{\Sigma} \vec{E}\, d\vec{s} = \frac{q_i}{\varepsilon_0}$$

$$\oint_{\Sigma} \vec{B}\cdot d\vec{s} = 0$$

$$\oint_{\Gamma} \vec{E}\cdot d\vec{l} = -\int_{S} \frac{\partial \vec{B}}{\partial t}\, d\vec{s}$$

$$\oint_{\Gamma} \vec{B}\cdot d\vec{l} = \int_{S} \mu_0 (\vec{j}_i + \vec{j}_d)\, d\vec{s}$$

$$\int_{V} \nabla\cdot\vec{E}\cdot dV = \int_{V} \frac{q_i}{\varepsilon_0}$$

$$\int_{V} \nabla\cdot\vec{B}\cdot dV = \int_{V} 0\, dV$$

$$\int_{S} \nabla\times\vec{E}\cdot d\vec{s} = -\int_{S} \frac{\partial \vec{B}}{\partial t}\, d\vec{s}$$

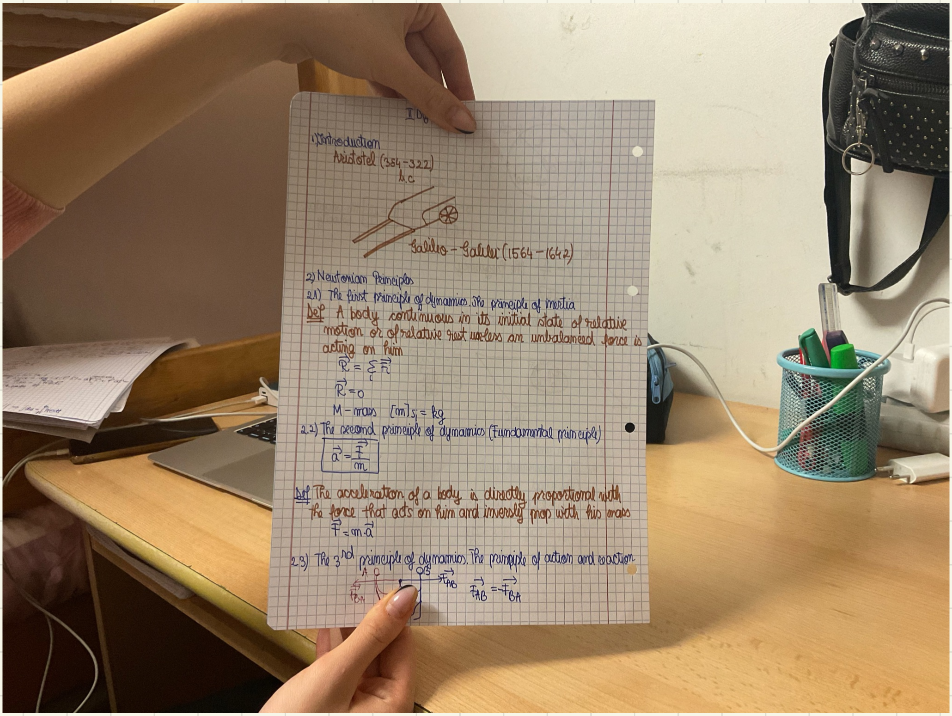
$$\int_{S} \nabla\times\vec{B}\cdot d\vec{s} = \int_{S} \mu_0 (\vec{j}_i + \vec{j}_d)\, d\vec{s}$$

$$\nabla\vec{E} = \frac{q_i}{\varepsilon_0}$$

$$\nabla\vec{B} = 0$$

$$\nabla\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla\times\vec{B} = \mu_0\left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$



PI) A body continuous in it's initial state of relative motion or of relative rest unless an unbalanced force is acting on him

PII) The acceleration of a body is directly proportional with the force that acts on him and inversly prop with his mass

PIII) Principle of action and reaction

$$a = \frac{F}{m}$$

→ produs de o sursă ce scoate sunet
→ frecvența oscilatorie $16\ Hz << 20000\ Hz$
→ un mediu elastic trebuie să existe între sursă și receptor
→ puterea sunetului să fie destul de mare
→ intensitatea sunetului > intensitatea de prag $10^{-12} < I < 10^{2}$
→ durata unui sunet > decât o durată minimă 50 ms

→ produs de o sursă ce scoate sunet
→ între receptor și emițător trebuie să existe un mediu elastic
→ frecvența oscilatorie cuprinsă între $16 Hz <$ sunet $< 20.000 Hz$
→ puterea sunetului să fie destul de mare, intensitatea sunetului > in. de prag
$10^{-2} < I < 10^{2}$
→ durata unui sunet > o durată minimă 50 ms

→ produs de o sursă ce scoate sunet
→ între receptor și emițător trebuie să existe un mediu elastic
→ frecvența oscilatorie cuprinsă între 16 Hz și 20.000 Hz
→ puterea sunetului destul de mare
intensitatea sunet > intensitate prag $10^{-2} < I < 10^{2}$
→ durata unui sunet > o durată minimă 50 ms

→ produs de o sursă ce scoate sunet
→ trebuie să existe un mediu elastic între receptor și emițător
→ frecvența oscilatorie între 16 Hz și 20.000 Hz
→ puterea sunetului destul de mare
$10^{-2} < I < 10^{2}$
→ durata sunetului > durată minimă 50 ms

→ produs de o sursă ce scoate sunet
→ trebuie să existe un mediu elastic între receptor și emițător
→ frecvență oscilatorie 16 Hz și 20.000 Hz
→ puterea sunetului destul de mare, intensitatea > intensitate de prag
$10^{-2} < I < 10^{2}$
→ durata mai mare 50 ms