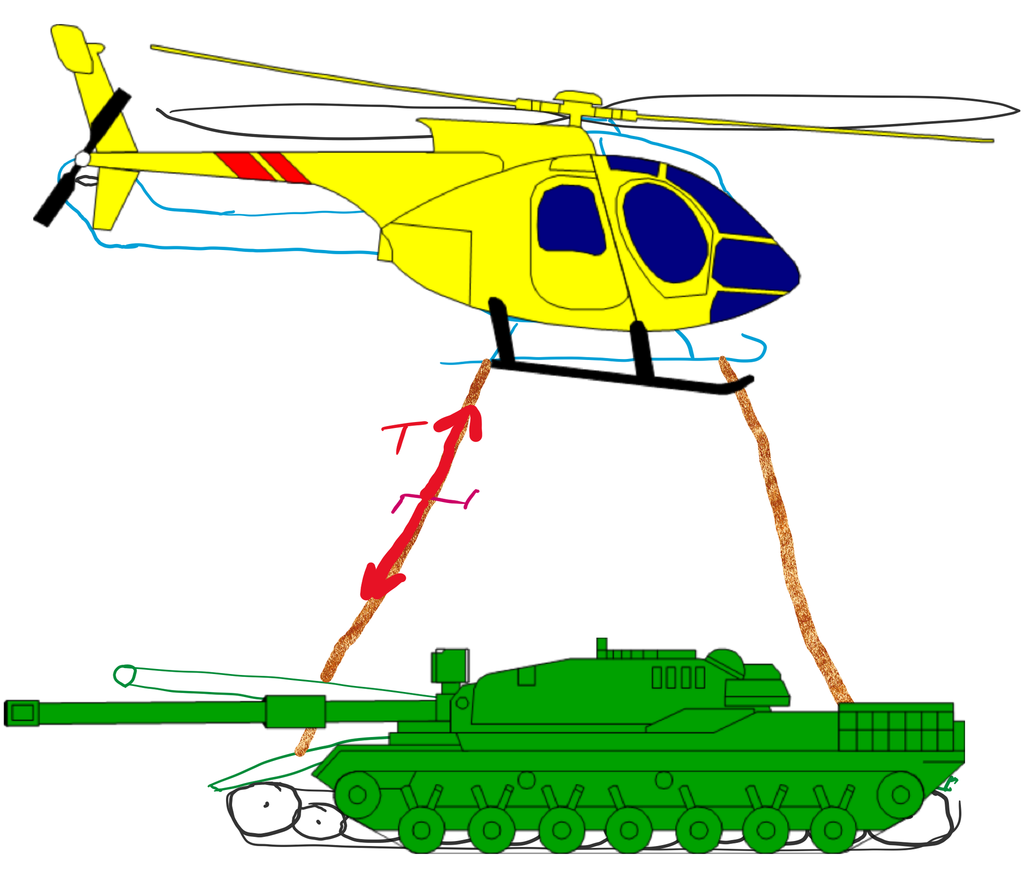
Course 7

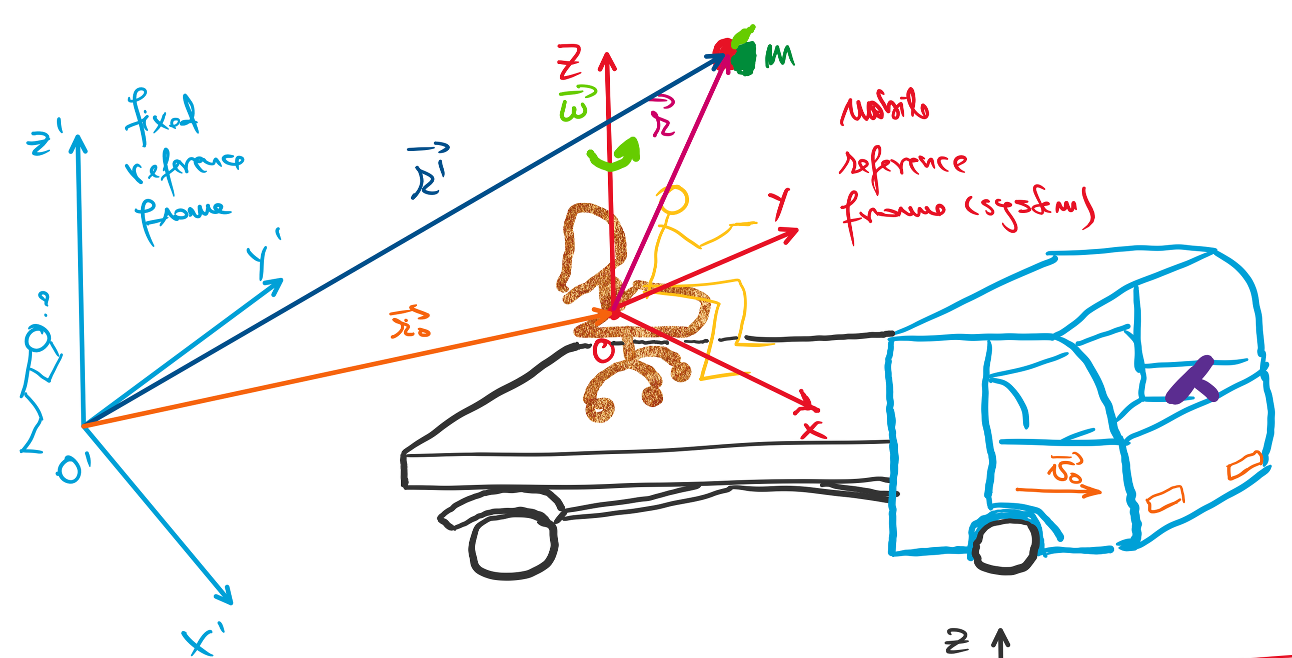
08.11.2021 13:54

II DYNAMICS

8.6 The bonding forces



8.7. The inertial forces



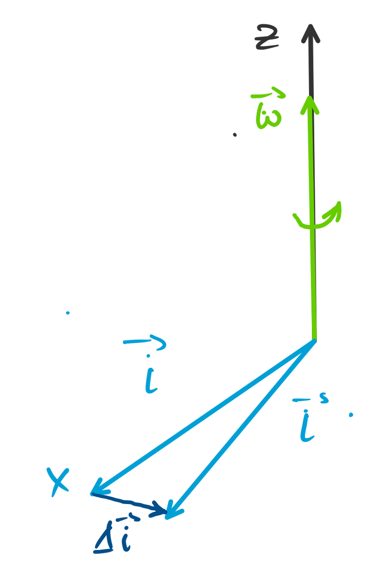
$\vec{r} = x\cdot\vec{i} + y\cdot\vec{j} + z\cdot\vec{k}$

$\Downarrow$

$\vec{v}$

$\Downarrow$

$\vec{a} \cdot m \Rightarrow F$



$$\boxed{\begin{aligned}\frac{d\vec{i}}{dt} &= \vec{\omega}\times\vec{i}\\ \frac{d\vec{j}}{dt} &= \vec{\omega}\times\vec{j}\\ \frac{d\vec{k}}{dt} &= \vec{\omega}\times\vec{k}\end{aligned}}$$

$\vec{r} = x\cdot\vec{i} + y\cdot\vec{j} + z\cdot\vec{k}$

$\vec{r}\,' = \vec{r} + \vec{r}_0$

$$\vec{v}\,' = \frac{d\vec{r}\,'}{dt} = \frac{d}{dt}\left(\vec{r} + \vec{r}_0\right) = \overbrace{\frac{dx}{dt}}^{v_x}\cdot\vec{i} + \overbrace{\frac{dy}{dt}}^{v_y}\cdot\vec{j} + \overbrace{\frac{dz}{dt}}^{v_z}\cdot\vec{k} + \frac{d\vec{r}_0}{dt}$$

$$+\, x\cdot\underbrace{\frac{d\vec{i}}{dt}}_{\vec{\omega}\times\vec{i}} + y\cdot\underbrace{\frac{d\vec{j}}{dt}}_{\vec{\omega}\times\vec{j}} + z\cdot\underbrace{\frac{d\vec{k}}{dt}}_{\vec{\omega}\times\vec{k}}$$

$\vec{v}\,' = \vec{v}_0 + \vec{v} + \vec{\omega}\times\vec{r}$

$$\vec{a}\,' = \frac{d\vec{v}\,'}{dt} = \frac{d}{dt}\left(\vec{v}_0 + \vec{v} + \vec{\omega}\times\vec{r}\right) \Rightarrow$$

$$\vec{a}\,' = \frac{d\vec{v}_0}{dt} + \overbrace{\frac{dv_x}{dt}}^{a_x}\cdot\vec{i} + \overbrace{\frac{dv_y}{dt}}^{a_y}\cdot\vec{j} + \overbrace{\frac{dv_z}{dt}}^{a_z}\cdot\vec{k} + \frac{d\vec{\omega}}{dt}\times\vec{r} + \vec{\omega}\times\underbrace{\frac{d\vec{r}}{dt}}_{\vec{v}+\vec{\omega}\times\vec{r}}$$

$$v_x\cdot\underbrace{\frac{d\vec{i}}{dt}}_{\vec{\omega}\times\vec{i}} + v_y\cdot\underbrace{\frac{d\vec{j}}{dt}}_{\vec{\omega}\times\vec{j}} + v_z\cdot\underbrace{\frac{d\vec{k}}{dt}}_{\vec{\omega}\times\vec{k}}$$

$\vec{a}\,' = \vec{a}_0 + \vec{a} + \vec{\varepsilon}\times\vec{r} + \vec{\omega}\times\left(\vec{v} + \vec{\omega}\times\vec{r}\right) + \vec{\omega}\times\vec{v}$

$\vec{a}\,' = \vec{a}_0 + \vec{a} + \vec{\varepsilon}\times\vec{r} + \vec{\omega}\times\left(\vec{\omega}\times\vec{r}\right) + 2\,\vec{\omega}\times\vec{v}$

The force produced by a reaction of a body to an accelerating force equal in magnitude and in opposite direction to the accelerating force is named "inertial force". An inertial force last only as long as the accelerating force does.

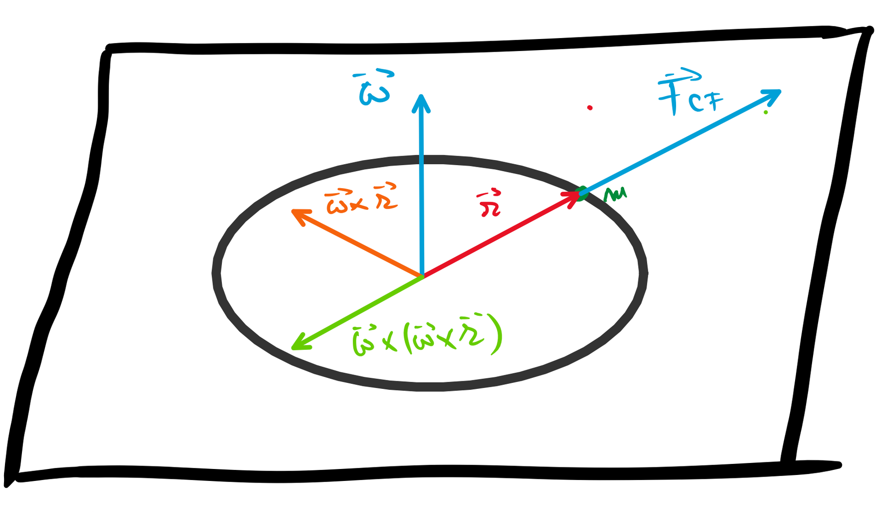
$\vec{F}_i = -m\cdot\vec{a}\,'$

$\vec{F}_s = -m\cdot\vec{a}$ – the self inertial force

$\vec{F}_t = -m\,\vec{a}_0$ – the inertial force at translational motion of the reference frame

$\vec{F}_r = -m\,\vec{\varepsilon}\times\vec{r}$ – the inertial force at rotational motion of the reference frame

$\vec{F}_{cf} = -m\,\vec{\omega}\times(\vec{\omega}\times\vec{r})$ – the centrifugal force



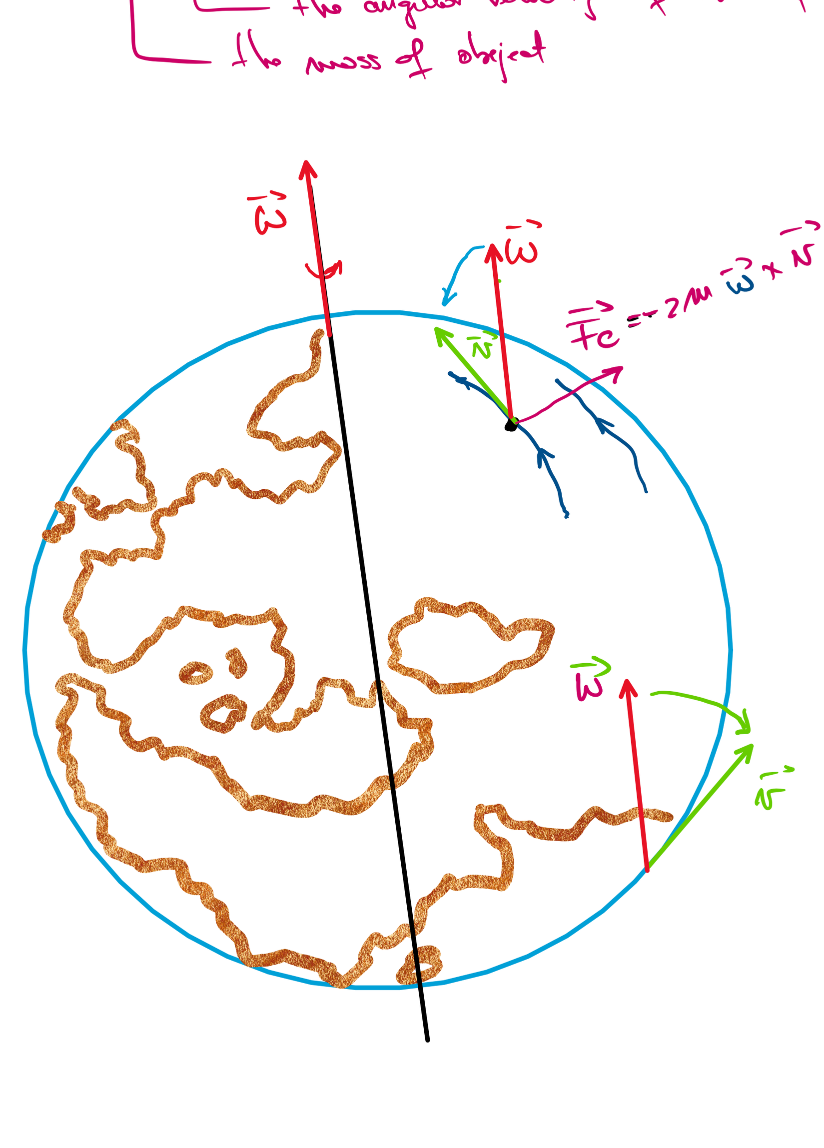
A) $\vec{F}_c = -2m\,\vec{\omega}\times\vec{v}$ – The CORIOLIS force

the velocity of object

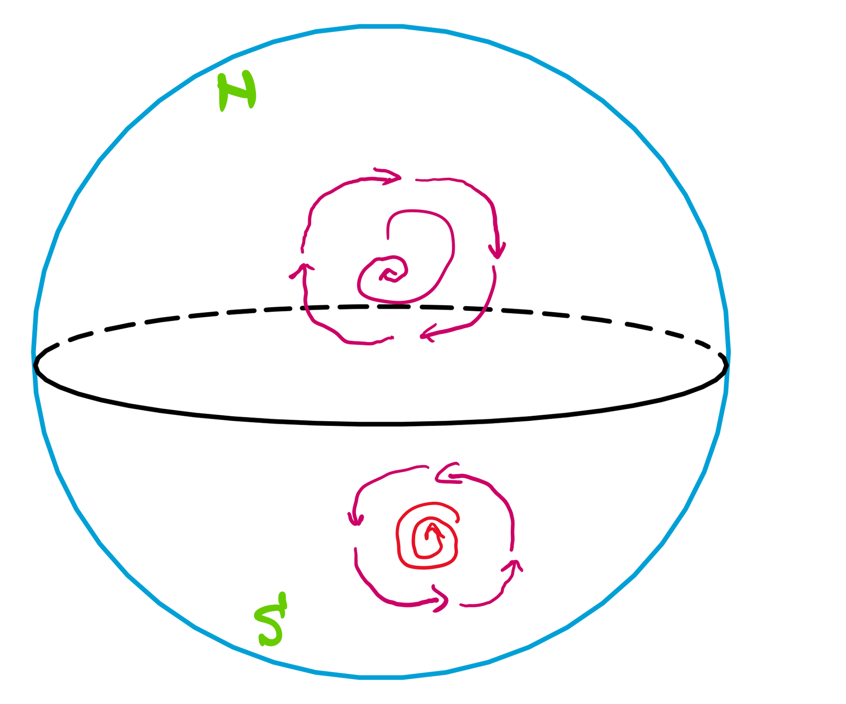
the angular velocity of the reference frame

the mass of object

B)



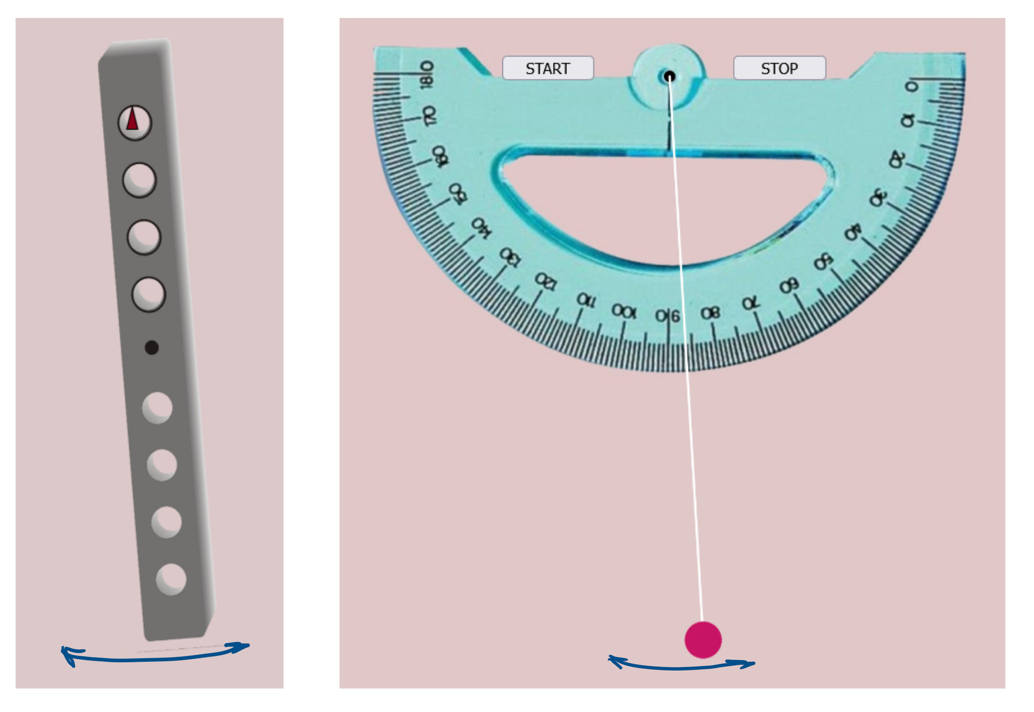
c)

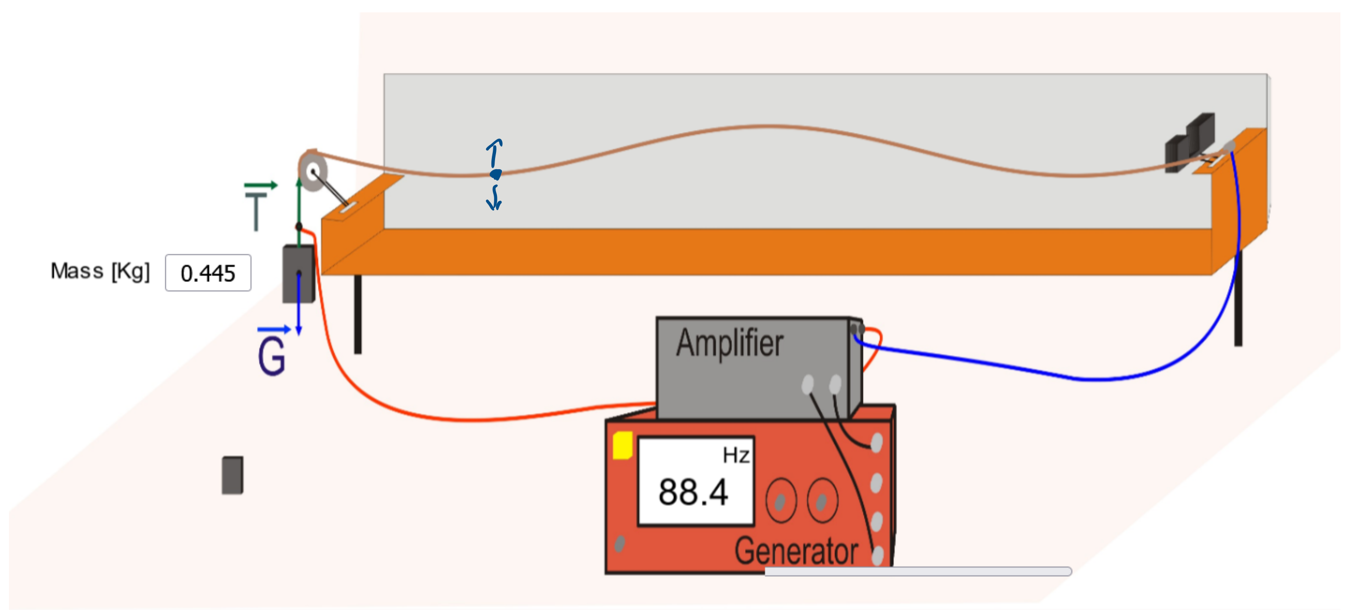


III Oscillations

1° Introduction

A) Ex:





B) The characteristics of oscillatory motion

– It is periodical (T)

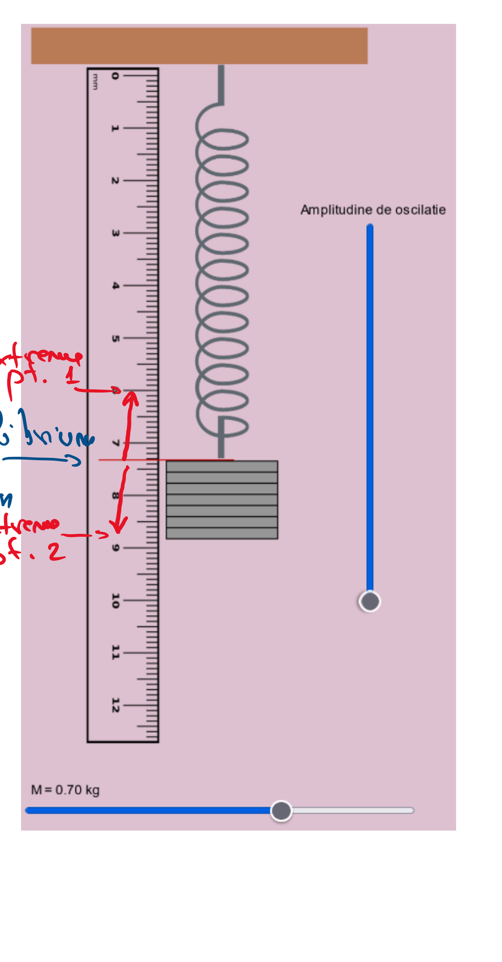
– ∃ of an equilibrium position

– ∃ two extreme points

– symmetrical around the eq. position

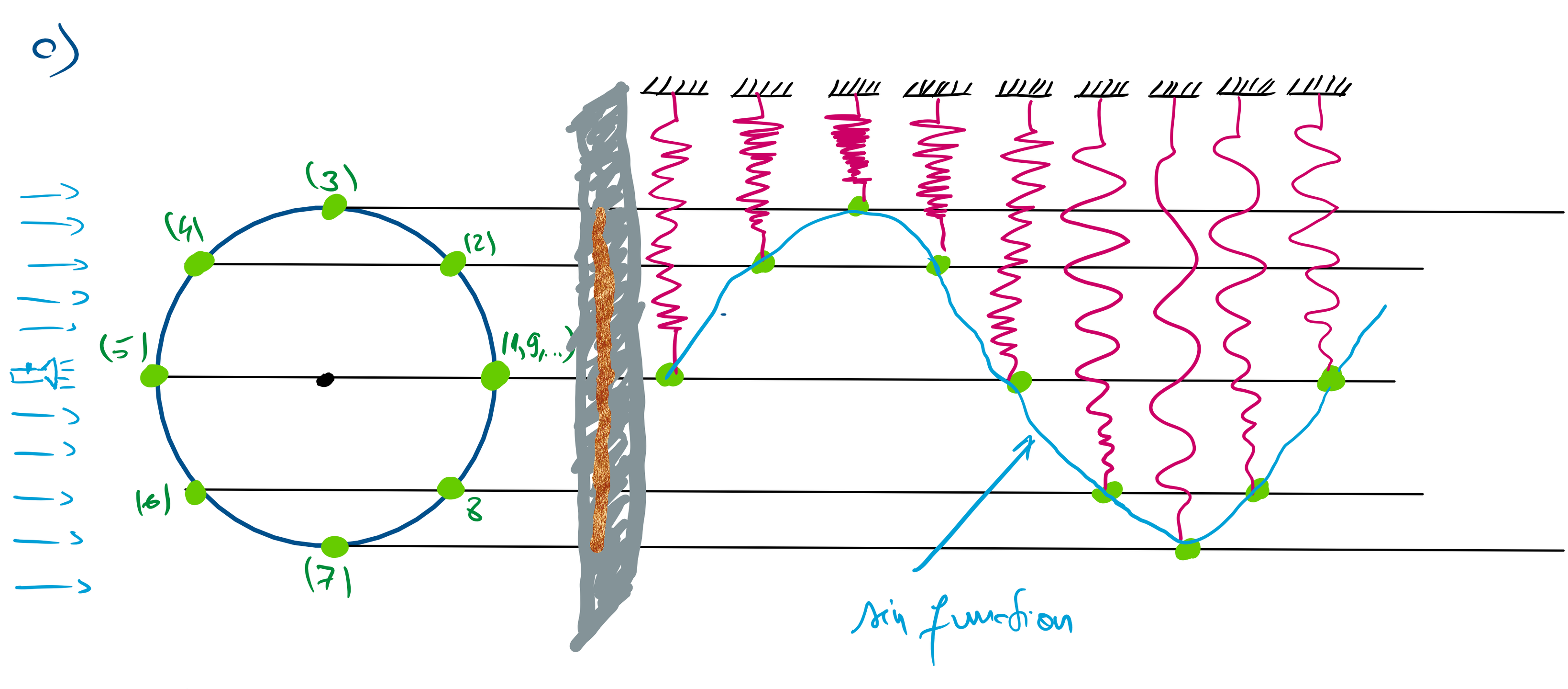
– the velocity in the extreme points is zero

– linear trajectory



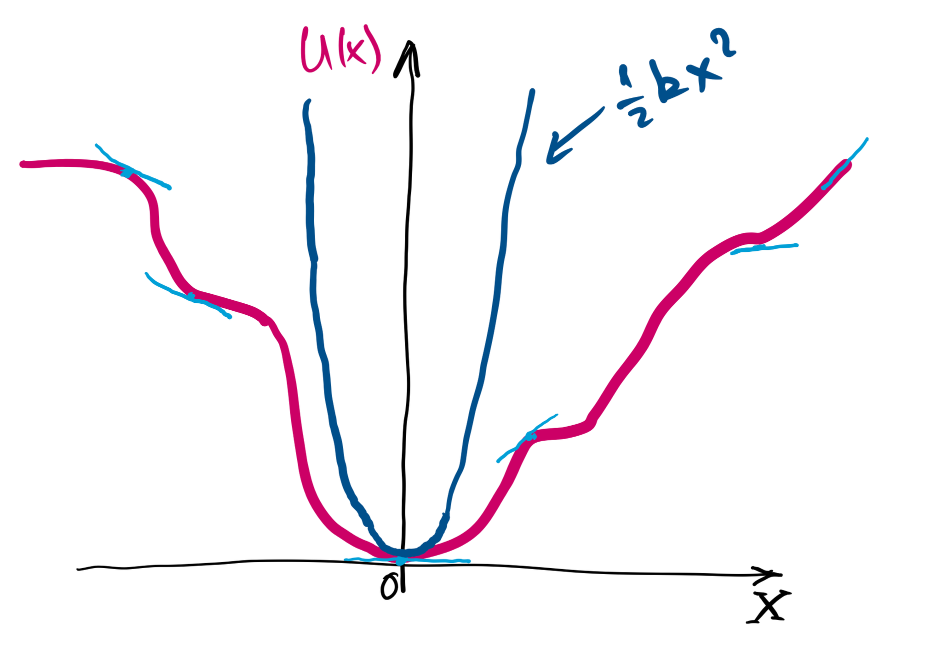
Any motion that repeat itself over the equal time interval fort and back over the same path it's named oscillatory motion

c)



2. The harmonic oscillations

2.1. The energy of an harmonic oscillator



Taylor series

$$U(x) = U(0) + \frac{1}{1!}\left.\frac{dU(x)}{dx}\right|_{x=0}\cdot x^1 + \frac{1}{2!}\left.\frac{d^2U(x)}{dx^2}\right|_{x=0}\cdot x^2 + \ldots$$

$U(0) = 0$

$\left.\dfrac{dU(x)}{dx}\right|_{x=0} = 0$

Note

$\left.\dfrac{d^2U(x)}{dx^2}\right|_{x=0} = k$

$\Rightarrow \boxed{U(x) = \frac{1}{2}kx^2}$ – the elastic potential energy

$\vec{F} = -\nabla U = -\operatorname{grad} U$