Electrotechnics
ET

Course 2

Year I-ISA English

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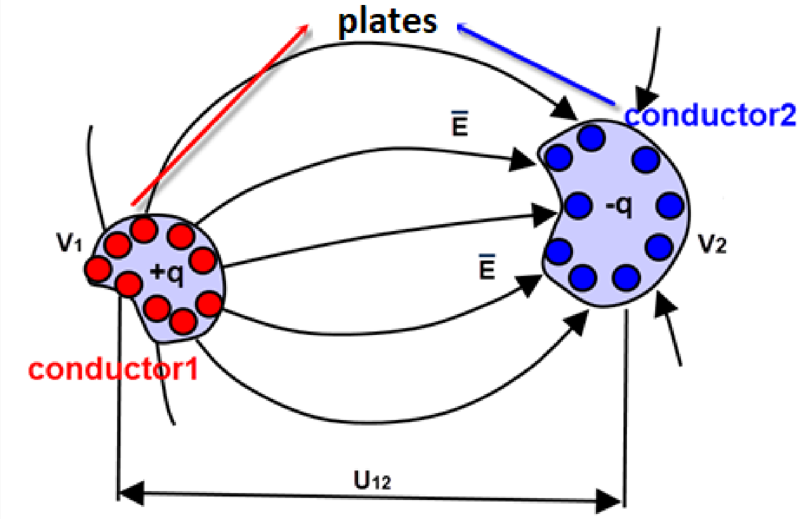
= Course 2 =

1. Electric Capacitor and Capacitance
2. Methods used to calculate the Electric Capacitance

1. Electric Capacitor and Capacitance

Electric Capacitor. Electric Capacitance

- suppose we have 2 conductive bodies charged with electric charges;
- an electric field appears (is generated).



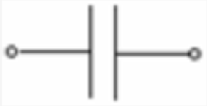
Capacitor = device consisting of 2 homogeneous conductors (called plates), charged with equal electric charges of opposite signs, separated by a dielectric.

Electric capacitance= ratio between the electric charge of one plate and the potential difference between them:

$$\boxed{C = \frac{q}{V_1 - V_2} = \frac{-q}{V_2 - V_1} > 0}$$

$$\boxed{C \stackrel{\text{def}}{=} \frac{q}{U_{12}}}$$

Obs.

- measurement unit: $\boxed{[C]_{SI} - [F]}$, Farad;
- symbol: 

2. Method used to calculate the Electric Capacitance

Direct Method

I. Direct Method

❑ Calculation steps:

1) The capacitor plates are considered to be charged with the electric charges $+q$ and $-q$;

2) Electric field intensity $\vec{E}$ from the dielectric between the two fittings is determined:

⇒ first the electric induction $\vec{D}$ *is determined*, using *Electric Flux Law:* $\oiint_{\Sigma} \vec{D} \cdot d\vec{A} = q_{\Sigma}$

⇒ then out of the relationship $\vec{D} = \varepsilon \vec{E}$, $\vec{E}$ *is calculated*;

3) Voltage U_{12} is calculated with: $U_{12} = \int_{1}^{2} \vec{E} \cdot d\vec{s}$

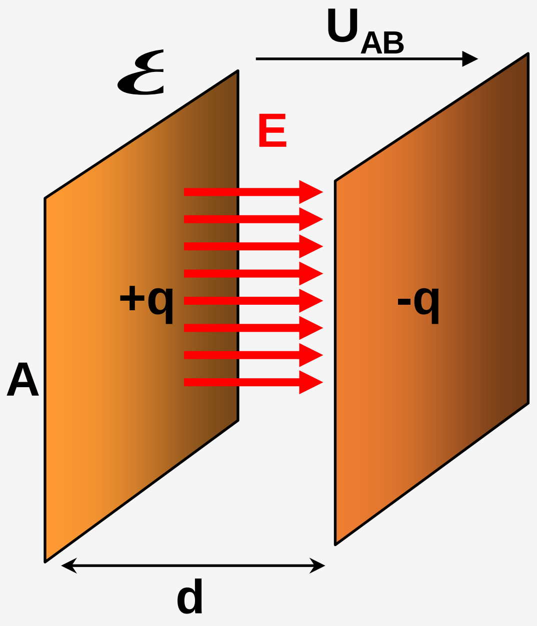
4) Capacitance is determined with the relation: $C = \dfrac{q}{U_{12}}$

Direct Method

Aplications:

Problem 1 = Plane Capacitor Capacitance =

Find the capacitance of the capacitor with parallel plane plates (the capacitor plates are two rectangular metallic plates, with an area of *A*, parallel placed at a distance *d* to each other). The dielectric is linear, homogeneous and isotropic with a constant permittivity Ɛ. The electric charges, equal and of opposite sign, +*q* and –*q*, create in the dielectric an electric field which can be considered uniform.

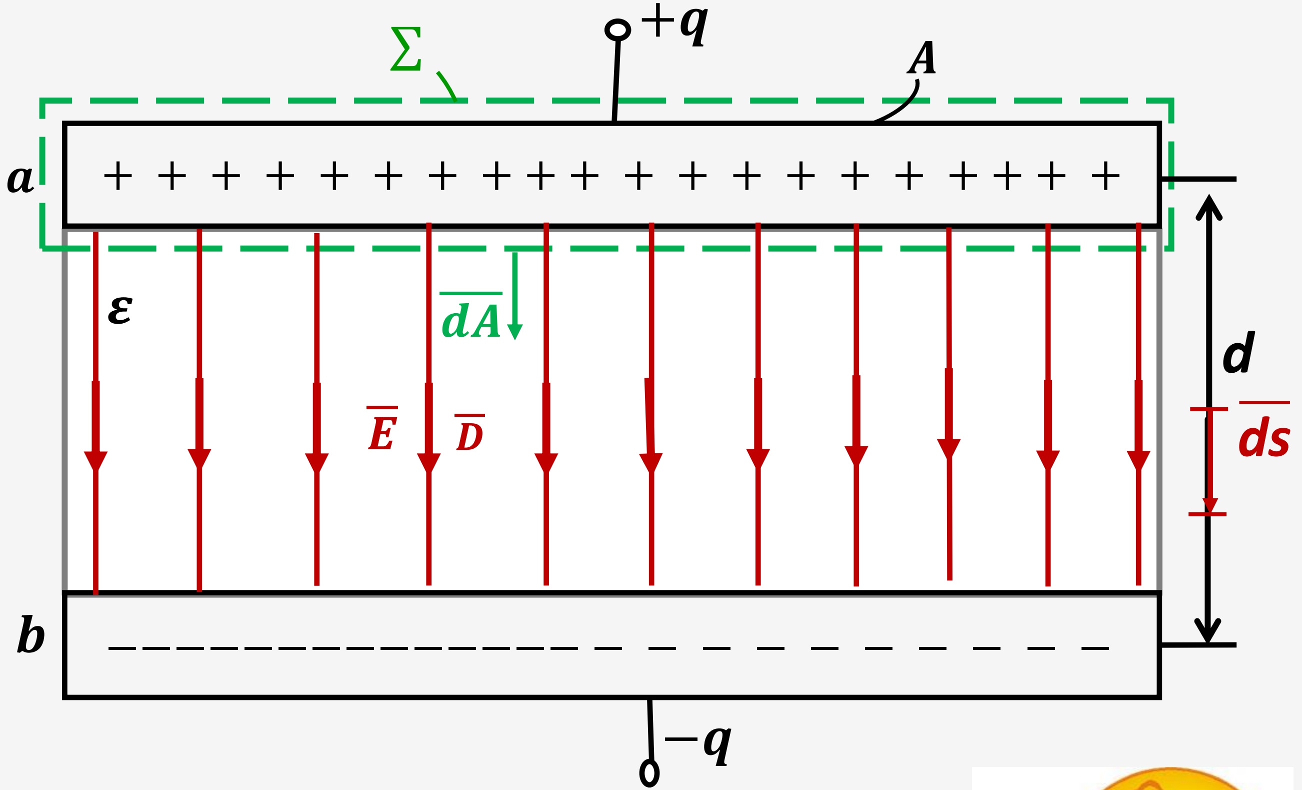


Solution:

Solution:

$M.D.:\ C \leftarrow U_{ab} \leftarrow E\ (D)\ \leftarrow q_\Sigma$

1) we consider the capacitor plates charged with the electric charges $+q$ și $-q$;
2) we find the electric field intensity $\vec{E}$ inside the dielectric formed between the capacitor plates:



- ❑ $\varepsilon \neq \varepsilon_0$:

  ⇨ we apply the Electric Flux Law:

$$\oint_\Sigma \overline{D} \cdot \overline{dA} = q_\Sigma$$

- First, we need to find the electric induction $\vec{D}$:
  - we choose an integration surfaces Σ, so that it includes, obligatorily, only one of the capacitor plates;
  - the scalar product between the two vectors will be: $\overline{D} \cdot \overline{dA} = D \cdot dA \cdot \cos\alpha$

$$\overline{D} \parallel \overline{dA} \quad \Rightarrow \quad \overline{D} \cdot \overline{dA} = D \cdot dA \cdot \cos 0^o = D \cdot dA$$

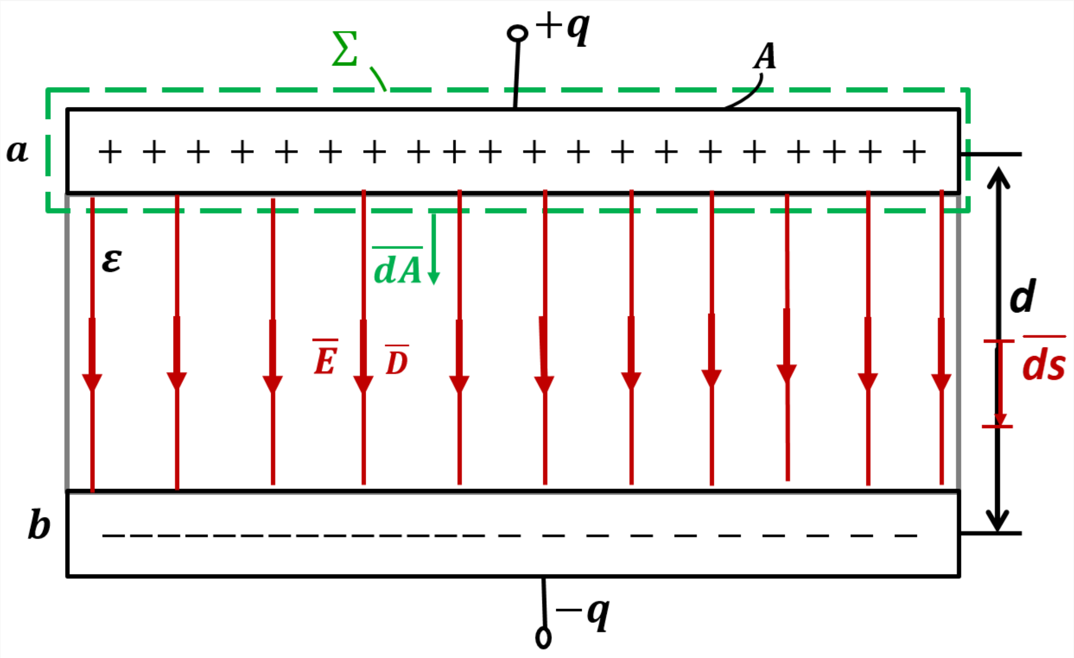
$\boldsymbol{M.D.:\ C \leftarrow U_{ab} \leftarrow E\ (D) \leftarrow q_\Sigma}$

$$\Rightarrow \left.\begin{array}{l} \oint_\Sigma D \cdot dA = q_\Sigma \\ D\ ct\ inside\ \Sigma \end{array}\right\} \Rightarrow D \cdot \oint_\Sigma dA = q$$

$$\Rightarrow D \cdot A = q \quad \Rightarrow D = \frac{q}{A}, \left[\frac{c}{m^2}\right]$$

$$D = \varepsilon \cdot E \quad \Rightarrow \varepsilon \cdot A \cdot E = q$$

$$\Rightarrow E = \frac{q}{\varepsilon \cdot A}, \left[\frac{V}{m}\right]$$

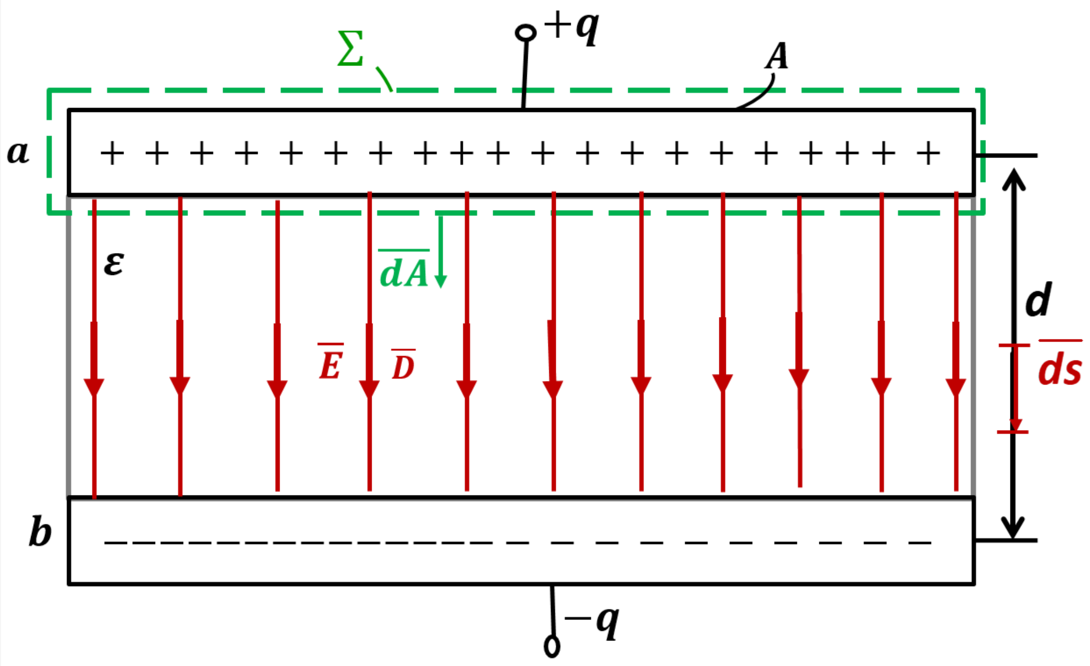


3) we compute the voltage U_{ab}: $$\left.\begin{array}{l} U_{ab} = \int_a^b \overline{E} \cdot \overline{ds} \\ \overline{E} \parallel \overline{ds} \end{array}\right\} \Rightarrow U_{ab} = \int_a^b E \cdot ds$$

$M.D.: \ C \leftarrow U_{ab} \leftarrow E\,(D) \leftarrow q_\Sigma$

$$\Rightarrow \quad U_{ab} = \frac{q}{\varepsilon \cdot A} \cdot \int_a^b ds$$

$$\Rightarrow \quad U_{ab} = \frac{q \cdot d}{\varepsilon \cdot A}, [V]$$



4) we determine now the capacitance using the relation:

$$C = \frac{q}{U_{ab}}$$

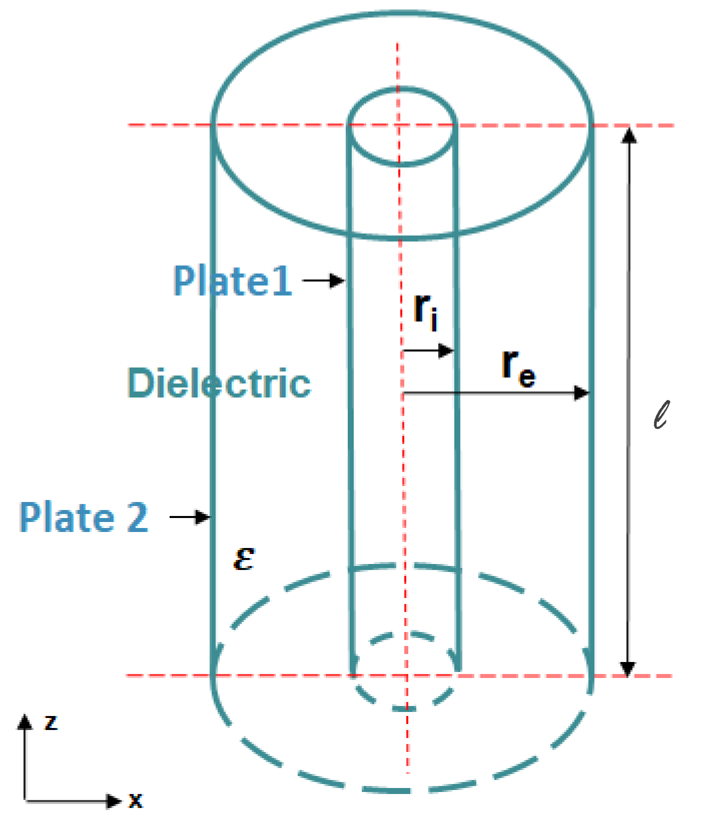
$$\Rightarrow \quad C = \frac{q}{1} \cdot \frac{\varepsilon \cdot A}{q \cdot d}$$

$$\Rightarrow \quad \boxed{C = \frac{\varepsilon \cdot A}{d}, [F]}$$

Direct Method

Problem 2 = Cylindrical Capacitor Capacitance =

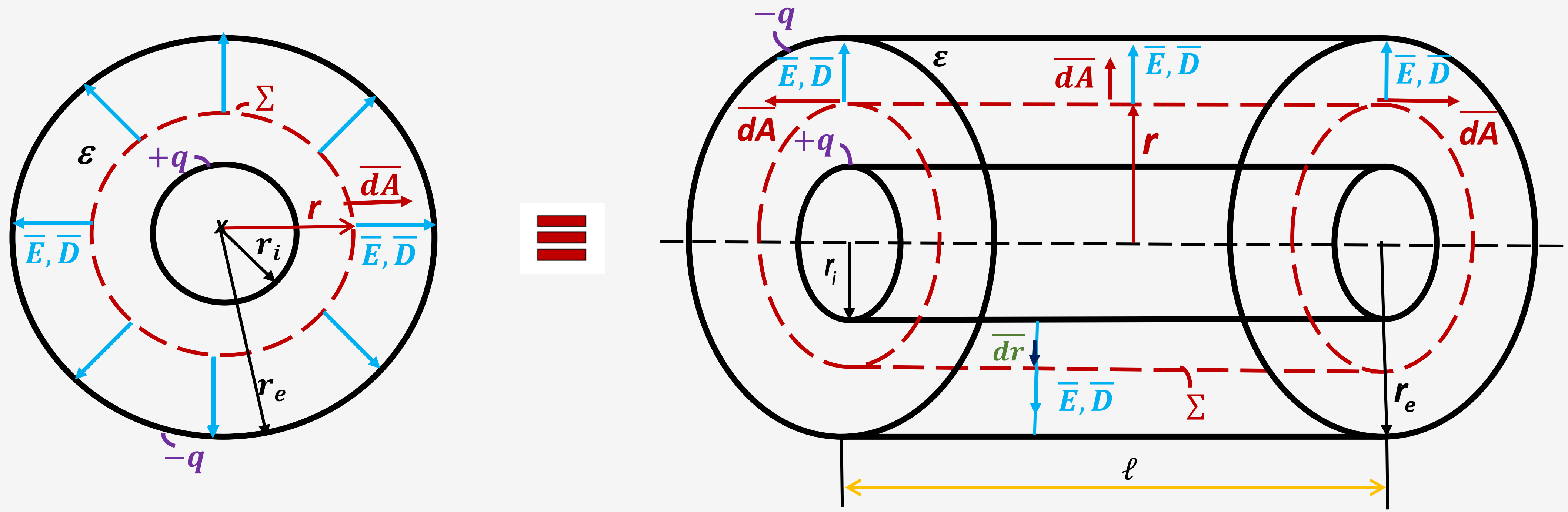
Find the cylindrical capacitor capacitance (The capacitor plates are 2 coaxial cylinders of radius r_e and r_i, and length ℓ, between which a dielectric medium exists with a constant permittivity $\mathcal{E}$).



Solution:

Solution:

$M.D.: \ C \leftarrow U_{12} \leftarrow E\,(D) \ \leftarrow q_\Sigma$



1) we suppose that in the inner plate we have the electrical charge $+q$, and on the outer one we have the electrical charge $-q$;

2) we find the electric field intensity $\vec{E}$ inside the dielectric formed between the two plates of the cylindrical capacitor

❑ $\boldsymbol{\varepsilon \neq \varepsilon_0}$: ⇨ we apply the electric flux low: $\oint_\Sigma \overline{D} \cdot \overline{dA} = q_\Sigma$ (1)

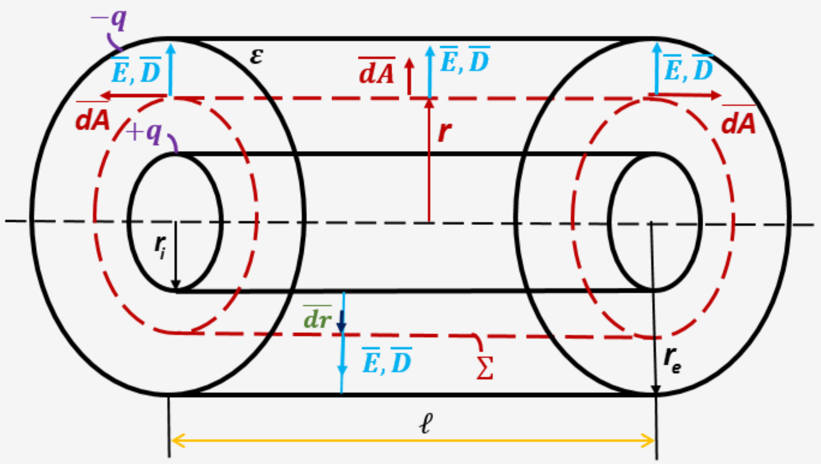
- we choose the integration surface Σ, as a cylinder, coaxial with the two plates of the capacitor, of radius r and length l;

$$M.D.: \; C \leftarrow U_{12} \leftarrow E\,(D) \;\leftarrow q_{\Sigma}$$

- on the cylinder base areas A_{b1} and A_{b2}: $\overline{D} \perp \overline{dA} \Rightarrow \alpha = 90^o$
- on the side area of the cylinder, A_l: $\overline{D} \parallel \overline{dA} \Rightarrow \alpha = 0^o$

$$\Rightarrow \oint_{\Sigma} \overline{D} \cdot \overline{dA} = \int_{A_\ell} D \cdot dA \cdot \cos 0^o + \int_{A_{b1}} D \cdot dA \cdot \cos 90^o + \int_{A_{b2}} D \cdot dA \cdot \cos 90^o$$

$$\Rightarrow \left.\begin{array}{l} \oint_{\Sigma} \overline{D} \cdot \overline{dA} = \int_{A_\ell} D \cdot dA + 0 + 0 = \int_{A_\ell} D \cdot dA \\ D\ ct\ inside\ \ A_\ell \end{array}\right\} \Rightarrow \oint_{\Sigma} \overline{D} \cdot \overline{dA} = D \cdot \int_{A_\ell} dA = D \cdot 2\pi \cdot r \cdot \ell \quad (2)$$

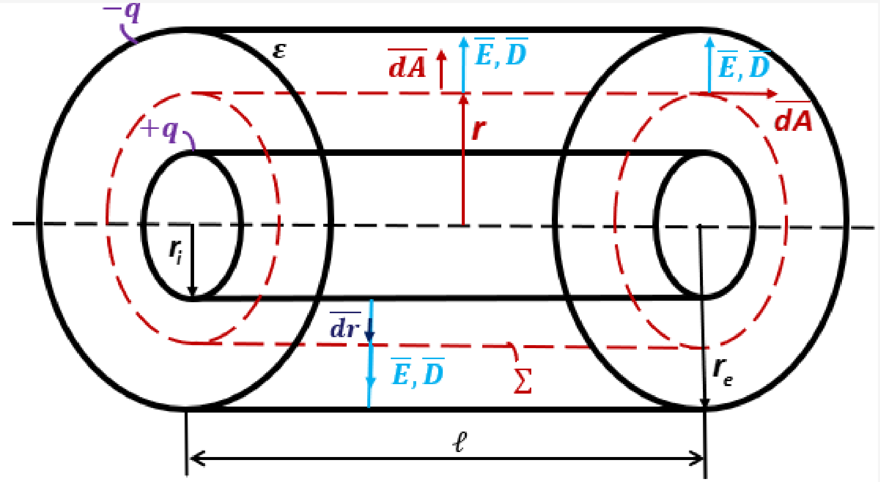


- we replace the integral (1) with the solution (2): $D \cdot 2\pi \cdot r \cdot \ell = q$

$$\Rightarrow \left.\begin{array}{l} D = \dfrac{q}{2\pi \cdot r \cdot \ell} \\ D = \varepsilon \cdot E \end{array}\right\} \Rightarrow E = \frac{q}{2\pi \cdot \varepsilon \cdot r \cdot \ell}, \left[\frac{V}{m}\right]$$

$$M.D.:\ C \leftarrow U_{12} \leftarrow E\ (D)\ \leftarrow q_{\Sigma}$$

3) we compute now the voltage U_{12}: $U_{12} = \int_1^2 \overline{E} \cdot \overline{ds}$



$$\left.\begin{array}{l} U_{12} = \int_{r_i}^{r_e} \overline{E} \cdot \overline{dr} \\ \overline{E} \parallel \overline{dr} \\ E = \dfrac{q}{2\pi \cdot \varepsilon \cdot r \cdot \ell} \end{array}\right\} \Rightarrow U_{12} = \int_{r_i}^{r_e} \frac{q}{2\pi \cdot \varepsilon \cdot r \cdot \ell} \cdot dr$$

$$\Rightarrow U_{12} = \frac{q}{2\pi \cdot \varepsilon \cdot \ell} \cdot \int_{r_i}^{r_e} \frac{1}{r} \cdot dr \Rightarrow U_{12} = \frac{q}{2\pi \cdot \varepsilon \cdot \ell} \cdot \ln r \Big|_{r_i}^{r_e}$$

$$\Rightarrow U_{12} = \frac{q}{2\pi \cdot \varepsilon \cdot \ell} \cdot \ln \frac{r_e}{r_i}$$

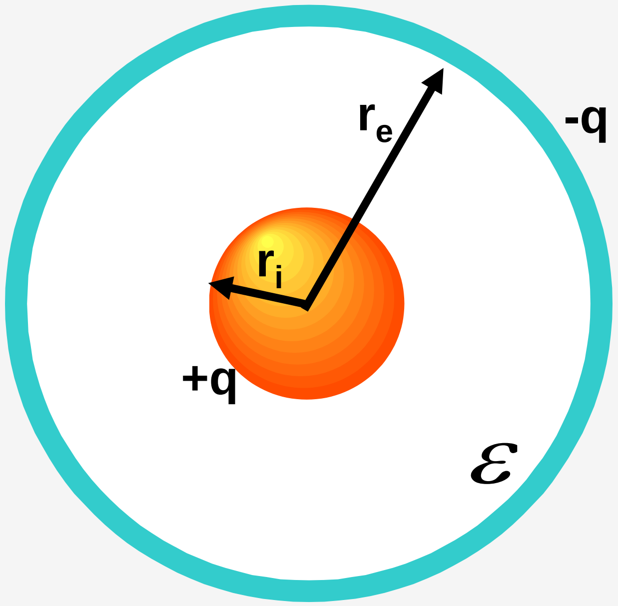
4) we determine the cylinder capacitor capacitance with the relation:

$$C = \frac{q}{U_{12}} \Rightarrow C = \frac{q}{1} \cdot \frac{2 \cdot \pi \cdot \varepsilon \cdot \ell}{q \cdot \ln \frac{r_e}{r_i}} \Rightarrow \boxed{C = \frac{2\pi \cdot \varepsilon \cdot \ell}{\ln \frac{r_e}{r_i}}, [F]}$$

Direct Method

<u>Problem 3</u> = Spherical capacitor capacitance =

Find the spherical capacitor capacitance (the capacitor plates (coats) are two metallic spheres, concentric, of radius r_e and r_i, and the dielectric between the spheres has the constant permittivity $\mathcal{E}$).



Homework:

Field Lines Approximation Method

II. Field Lines Approximation Method

- **suppose the following two assumptions:**

1) The shape of the field line is approximated by circle arcs and/or straight lines, as appropriate;

2) Along a field line, the electric field intensity, is considered constant:

$$U = \int_{C \equiv field\ line} \bar{E} \cdot d\bar{s} \quad \Rightarrow \quad U = \int_{C \equiv field\ line} E \cdot ds$$

$$\Rightarrow \quad U = E \cdot \ell_{field\ line}$$

$$\Rightarrow \quad \boxed{E = \frac{U}{\ell_{field\ line}}}$$

Field Lines Approximation Method

❑ calculation steps:

1) The plates of the capacitor are considered to be supply with the voltage U=ct ;
2) The field lines are approximated by circle arcs and/or straight lines, as appropriate,

➡ The electric field intensity, *E, is approximated* : $E = \dfrac{U}{\ell_{field\ line}}$

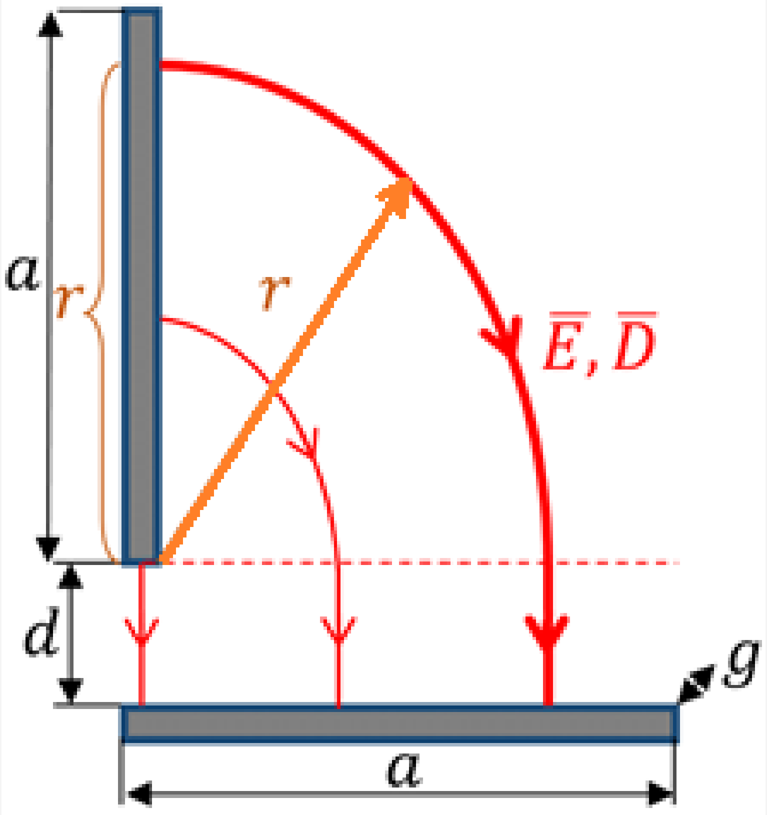
3) Electric induction is determined, *D*: $\bar{D} = \varepsilon \bar{E}$
4) The surface charge density, ρ_S , is determined:
   = on the surface of a conductor environment the value of the surface charge density in a point is equal with the value of the electric induction in that point $\rho_S = D$
5) The electric charge, *q,* is calculated : $q = \int_A \rho_S \, dA$
6) The capacitance, C, is calculated: $C = \dfrac{q}{U}$

Field Line Approximation Method

Applications:

Problem 1

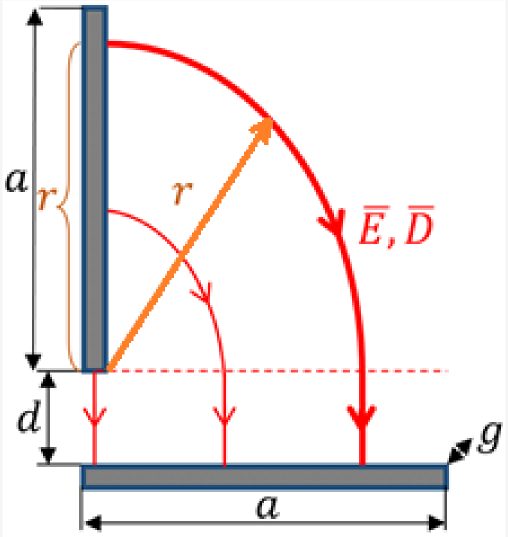
Calculate the capacitance of the capacitor present in the picture using field line approximation method.



Solution:

Note: *the field line approximation method* is applied when the capacitor plates are non-parallel, and the *direct method* can not be applied*!*

Solution:



✓ we start from the capacitance definition relation:

$$C = \frac{q}{U}$$

✓ To solve a problem using *field approximation method*, we go to (follow) the steps:

⑥ ⟸ ⑤ ⟸ ④ ⟸ ③ ⟸ ② ⟸ ①

$$\mathbf{C} \Leftarrow \mathbf{q} \Leftarrow \boldsymbol{\rho}_{\mathbf{s}} \Leftarrow \mathbf{D} \Leftarrow \mathbf{E} \Leftarrow \mathbf{U}$$

1) we consider the capacitor plates supply with a ***voltage, U*** constant;

2) we find the electric field intensity, *E:*

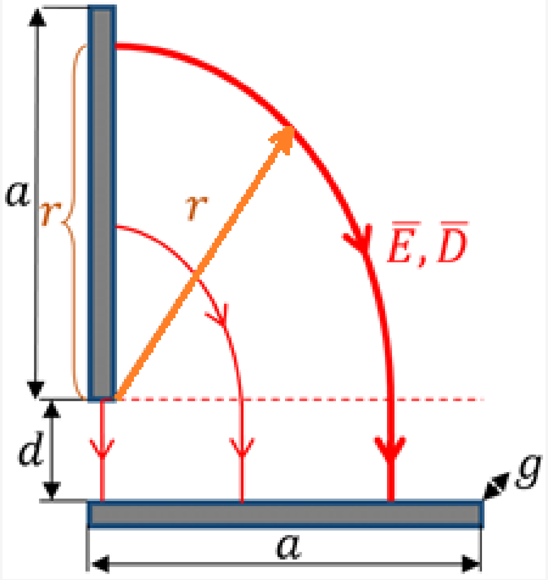
✓ the electric field lines between the capacitor plates are approximated to have the shapes of circle sectors (circular arches) or straight lines;

✓ between the capacitor plates, there exists, an infinity of filed lines

✓ we choose one field line (draw in bold line in the figure) to find the problem solution, we draw also the distance from the center to this line, denoted with *r* , and then by integration we find the solution inside the dielectric between the capacitor plates;

How we approximate the field lines inside the dielectric from our capacitor?

- we know that the electric field lines come out perpendicularly from one conductor and enter perpendicularly on the other conductor;
- note that the small side of the first capacitor plate (its width) is parallel with the second capacitor plate, so that the above write condition is fulfilled, and on d distance we approximate the field line to be a straight line;
- from first capacitor plate to the dotted red line, we approximate the field line as an arc of a circle;
- we know also that the electric field lines are parallel between them, and in this way, we can draw the electric field lines spectrum , as that from figure.



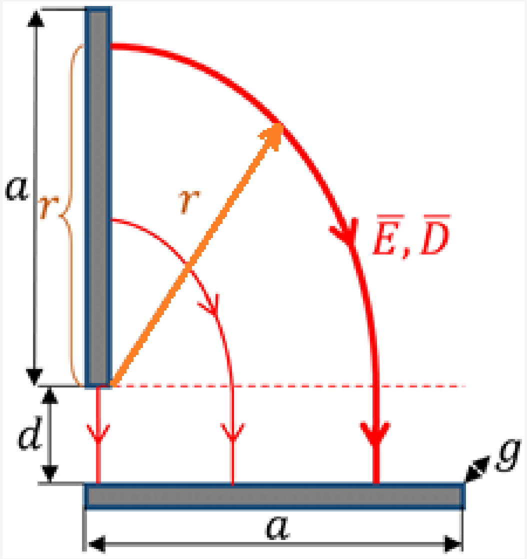
✓ we approximate the electric field intensity E with the relation:

$$E = \frac{U}{l_{field\ line}}$$

✓ the length of the field line for this capacitor is:

$$\boldsymbol{l_{fieldline}} = \frac{\pi}{2} \cdot r + d$$

- the arc length of a circle is equal with cu $\boldsymbol{\alpha} \cdot \boldsymbol{r}$, (where $\boldsymbol{\alpha}$ is the central angle of the arc and $\boldsymbol{r}$ is the arc radius) and in our geometry the central angle is $\alpha = \frac{\pi}{2}$



$$=> \quad E = \frac{U}{\frac{\pi}{2} \cdot r + d} = \frac{2 \cdot U}{\pi \cdot r + 2 \cdot d}$$

3) we find the electric induction *D*:

$$D = \varepsilon \cdot E$$

$$=> \quad D = \frac{2 \cdot \varepsilon \cdot U}{\pi \cdot r + 2 \cdot d}$$

4) we find the surfaces charge density (distribution) – we know that on the surface of a conductor medium the value of the surface charge density in a point is equal with the value of the electric induction in the same point – so:

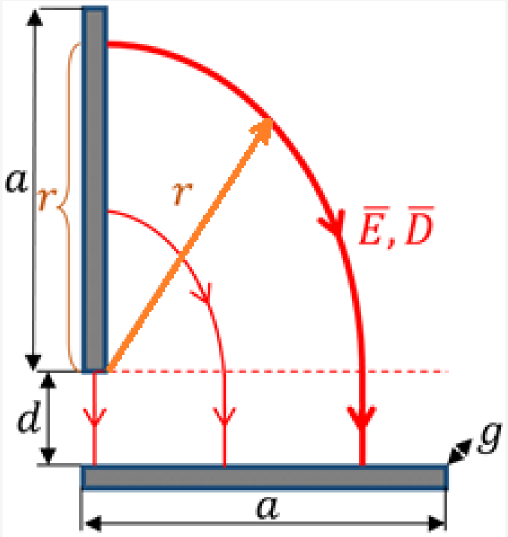
$$\rho_S = D$$

$$=> \rho_S = \frac{2 \cdot \varepsilon \cdot U}{\pi \cdot r + 2 \cdot d}$$

5) we find the electric charge q:

✓ we know that we can compute the electric charge with the relation:

$$q = \int_A \rho_S \cdot dA$$



✓ in this application we observed that we have no variation of the capacitor plate area, and only one side of it can vary (more exactly the side *a* of the capacitor plate varies with the radius *r*, meaning we arbitrary choose a field line to find the problem solution on it and we said that this line is placed at a variable distance *r*, *r* being able to be anywhere along the length *a* of the capacitor plate), so that we have to do the change of the variable, to change the surface integral to line integral; and so we have to express the area in terms of radius *r* (we use the formula used to calculate the rectangle, because the capacitor plate is a rectangle in this problem), and the integration limits are choose from *0 to a*, because we can select the field line on which we compute anywhere in that area (meaning on the plate area):

$$dA = g \cdot dr \qquad => \qquad q = \int_0^a \rho_S \cdot g \cdot dr$$

$$q = \int_0^a \frac{2 \cdot \varepsilon \cdot U \cdot g}{\pi r + 2d} \cdot dr = 2 \cdot \varepsilon \cdot U \cdot g \cdot \int_0^a \frac{1}{\pi r + 2d} \cdot dr$$

- form mathematic we know that:

$$\int \frac{1}{a \cdot x + b} dx = \frac{1}{a} \cdot \ln |a \cdot x + b|$$

$$=> q = \frac{2 \cdot \varepsilon \cdot U \cdot g}{\pi} \cdot ln \frac{\pi \cdot a + 2 \cdot d}{2 \cdot d}$$

6) we can now find the electric capacitance *C*:

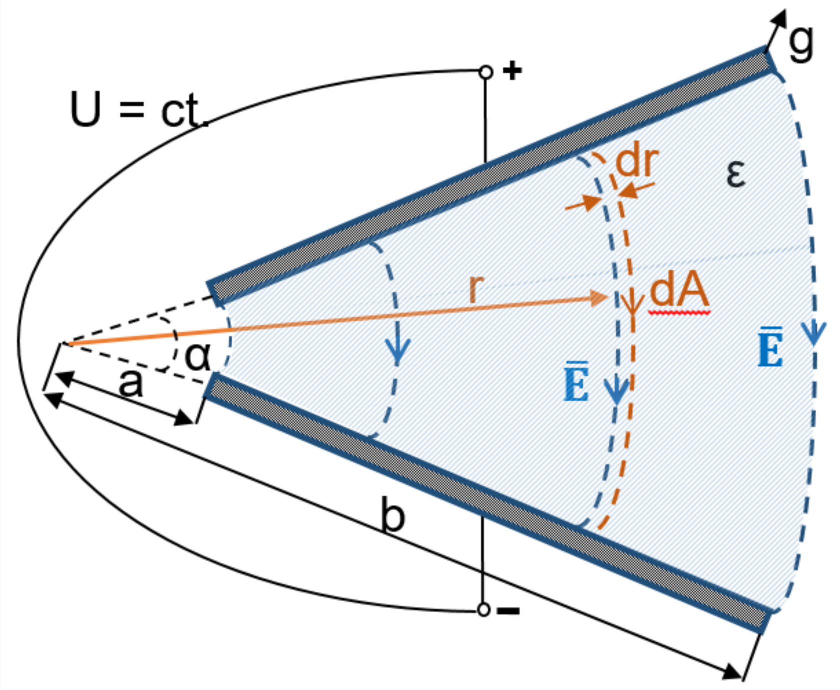
$$C = \frac{q}{U} = \frac{1}{U} \cdot \frac{2 \cdot \varepsilon \cdot U \cdot g}{\pi} ln \frac{\pi \cdot a + 2 \cdot d}{2 \cdot d}$$

$$C = \frac{2 \cdot \varepsilon \cdot g}{\pi} \cdot ln \frac{a \cdot \pi + 2 \cdot d}{2 \cdot d}, [F]$$

Field Line Approximation Method

Problem 2

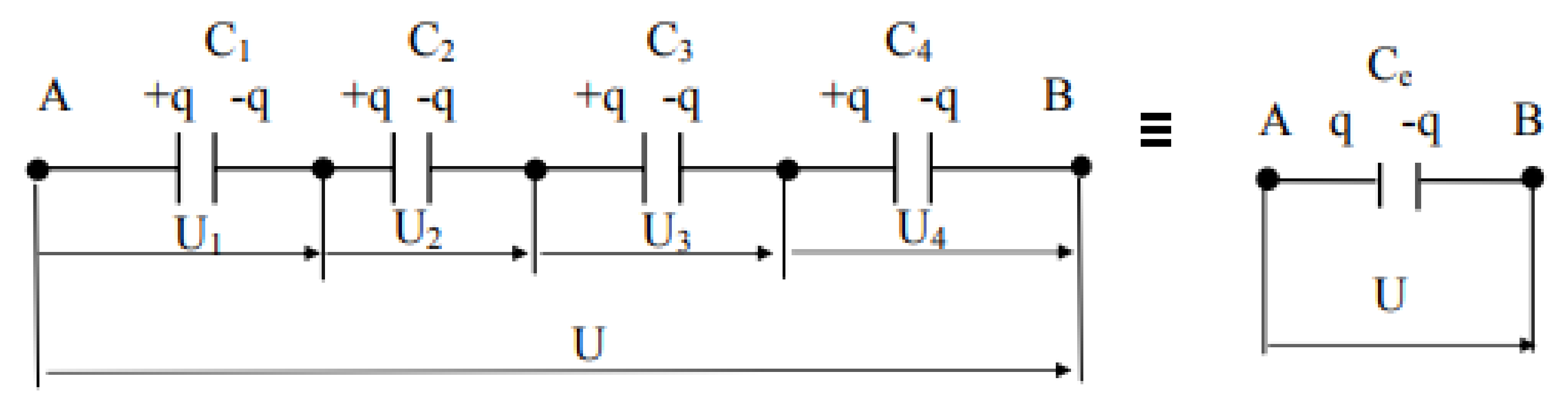
Calculate the capacitance of the plan capacitor with plates that are not parallel presented in the figure.



Homework:

Capacitor Connections (Networks)

a) Capacitors connected in series



$$U = U_1 + U_2 + \ldots + U_4$$

$$q = q_1 = q_2 = \ldots = q_4$$

$$U_1 = \frac{q}{C_1}; \quad U_2 = \frac{q}{C_2}; \quad U_3 = \frac{q}{C_3}; \quad U_4 = \frac{q}{C_4}; \quad U = \frac{q}{C_e}$$

$$\Rightarrow \frac{q}{C_e} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} + \frac{q}{C_4}$$

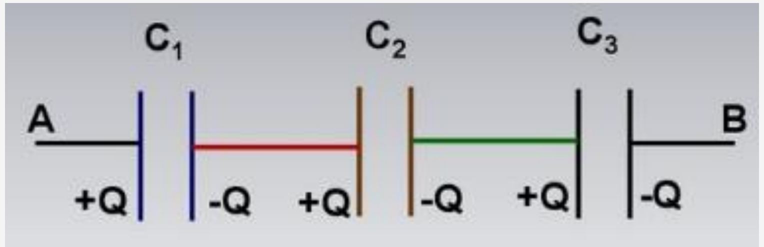
$$\Rightarrow \frac{1}{C_e} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

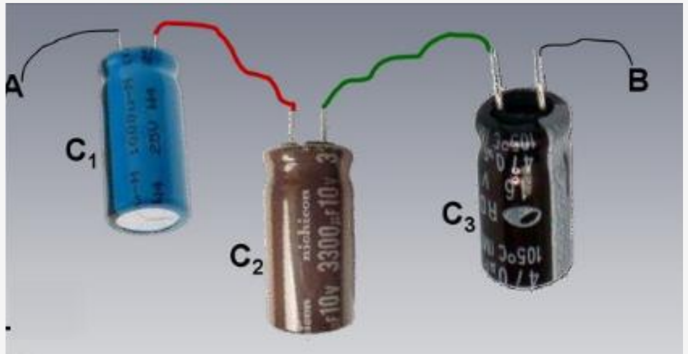
- n capacitors connected in series: $$\boxed{\frac{1}{C_e} = \sum_{k=1}^{n} \frac{1}{C_k}}$$

- *2* capacitors connected in series: $$C_e = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

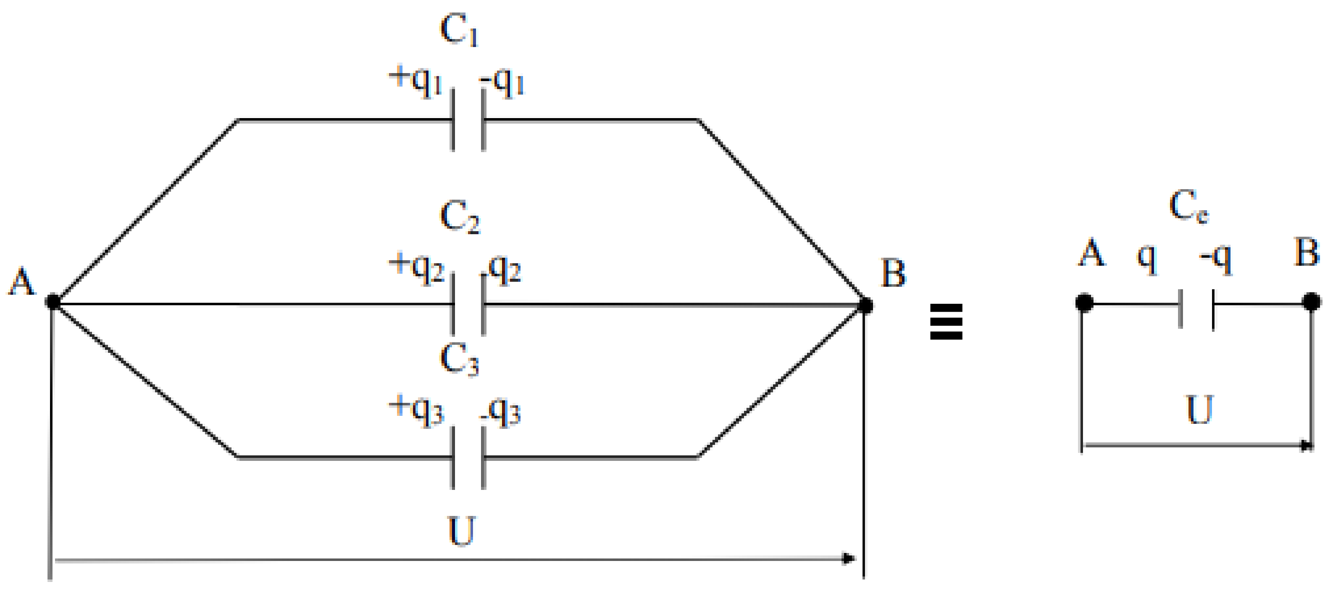
Obs: connection of capacitors in series is useful especially when high voltages are used, voltages that a single capacitor could not support.

Example:





b) Capacitors connected in parallel



$$q = q_1 + q_2 + q_3$$

$$q_1 = C_1 U \qquad q_2 = C_2 U \qquad q_3 = C_3 U$$

$$q = C_e U$$

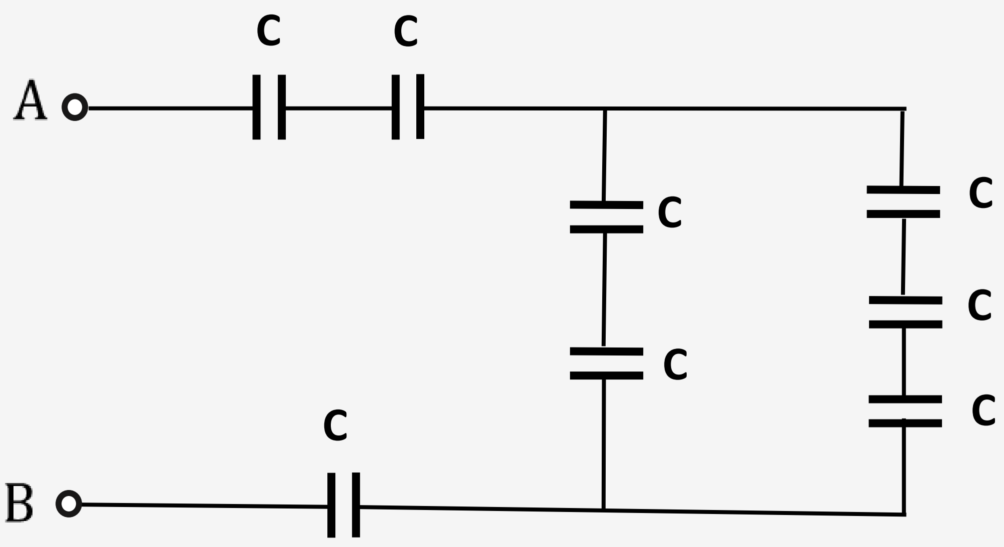
$$C_e U = C_1 U + C_2 U + C_3 U \Rightarrow C_e = C_1 + C_2 + C_3$$

❖ *n* capacitors connected in parallel: $\boxed{C_e = \sum_{k=1}^{n} C_k}$

Applications:

Problem 1

Find the equivalent capacitance for the circuit presented in the figure, relative to AB terminals:



Solution:

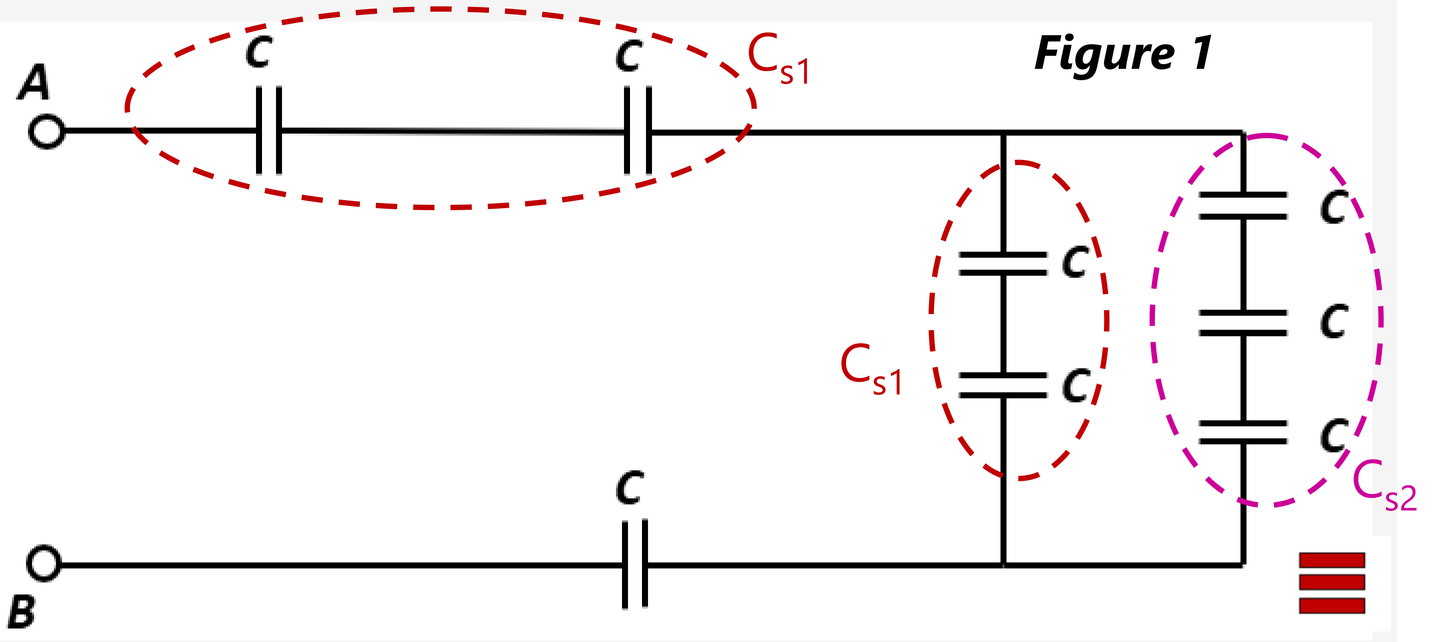
Solution:

- we note that we have two groups each of two capacitors in series connection and we denote their equivalent capacitance C_{s1} :

$$C_{S1} = \frac{C \cdot C}{C + C} = \frac{C^2}{2 \cdot C} = \frac{C}{2}$$

- we denote as C_{s2} the equivalent capacitance between the other three capacitors that are also in series connection:

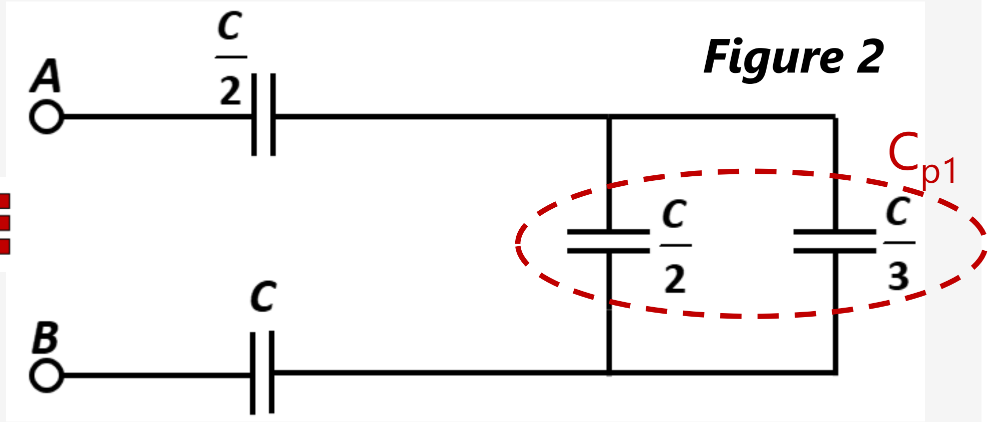
$$\frac{1}{C_{S2}} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \Rightarrow \frac{1}{C_{S2}} = \frac{3}{C} \Rightarrow C_{S2} = \frac{C}{3}$$



✓ we replace their equivalent capacitances in the circuit from Figure 1 and we draw again the equivalent circuit as being the one from Figure 2.

✓ we calculate the Cp1 as the equivalent capacitance between $\frac{C}{2}$ parallel with $\frac{C}{3}$:

$$C_{P1} = \frac{C}{2} + \frac{C}{3} = \frac{5 \cdot C}{6}$$



✓ so, the circuit can be reduced to that one from Figure 3, of which we can find directly the equivalent capacitance seen between the terminals A and B, denoted C_{AB}:

Figure 3

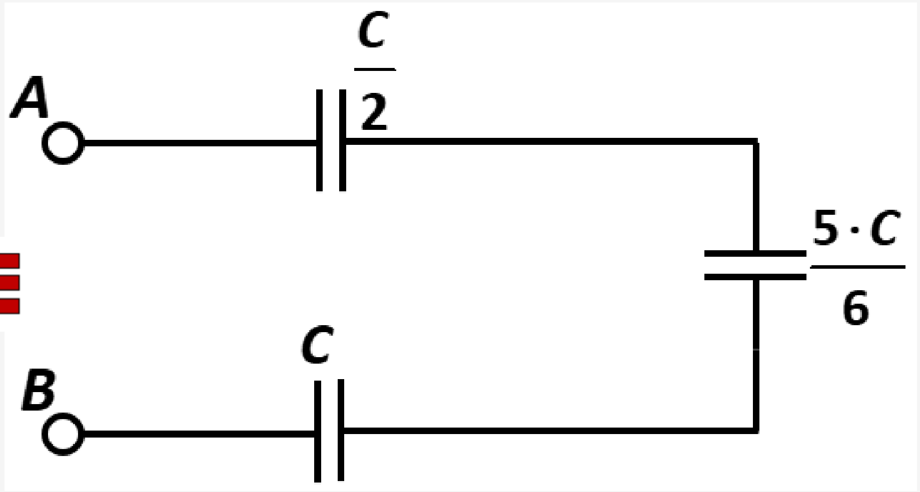


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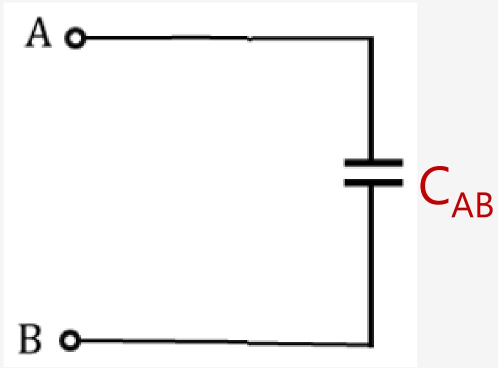
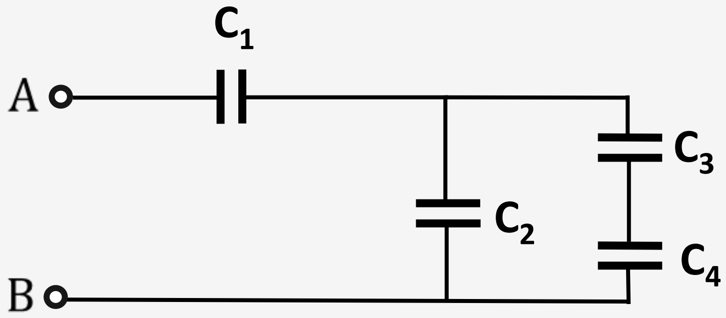
Figure 4



$$\frac{1}{C_{AB}} = \frac{1}{C} + \frac{6}{5 \cdot C} + \frac{2}{C} = \frac{21}{5 \cdot C}$$

The equivalent capacitance between terminals A – B: $\boxed{C_{AB} = \frac{5 \cdot C}{21}, [F]}$

Problem 2



Find the equivalent capacitance of the circuit from the figure, relative to the AB terminals, knowing that:

$$C_1 = 1\,\mu F \; ; \; C_2 = 3\,\mu F \; ;$$

$$C_3 = 6\,\mu F \; ; \; C_4 = 2\,\mu F \, .$$

Homework:

Thank you for your attention!!!

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Questions???