

8. Circular motion

8.8. The measurement units

$$[K]_{\text{SI}} = \text{rad}$$

$$[\omega]_{\text{SI}} = \frac{\text{rad}}{\text{s}}$$

$$[\varepsilon]_{\text{SI}} = \frac{\text{rad}}{\text{s}^2}$$

$$[T]_{\text{SI}} = \text{s}$$

$$[\gamma]_{\text{SI}} = \frac{1}{\text{s}} = \text{s}^{-1} = \text{Hz}$$

$$\left. \begin{array}{l} \omega = \frac{d\varphi}{dt} \\ \varepsilon = \frac{d\omega}{dt} \\ \gamma = \frac{1}{T} ; T = \frac{1}{\gamma} \\ \omega = 2\pi \gamma = \frac{2\pi}{T} \end{array} \right\}$$

II DYNAMICS

1. Introduction

- Aristotle (384 - 322 b.c.)



- Galileo Galilei (1564 - 1642)

2. Newtonian's Principles

2.1. The 1st Principle of dynamics. The principle of inertia

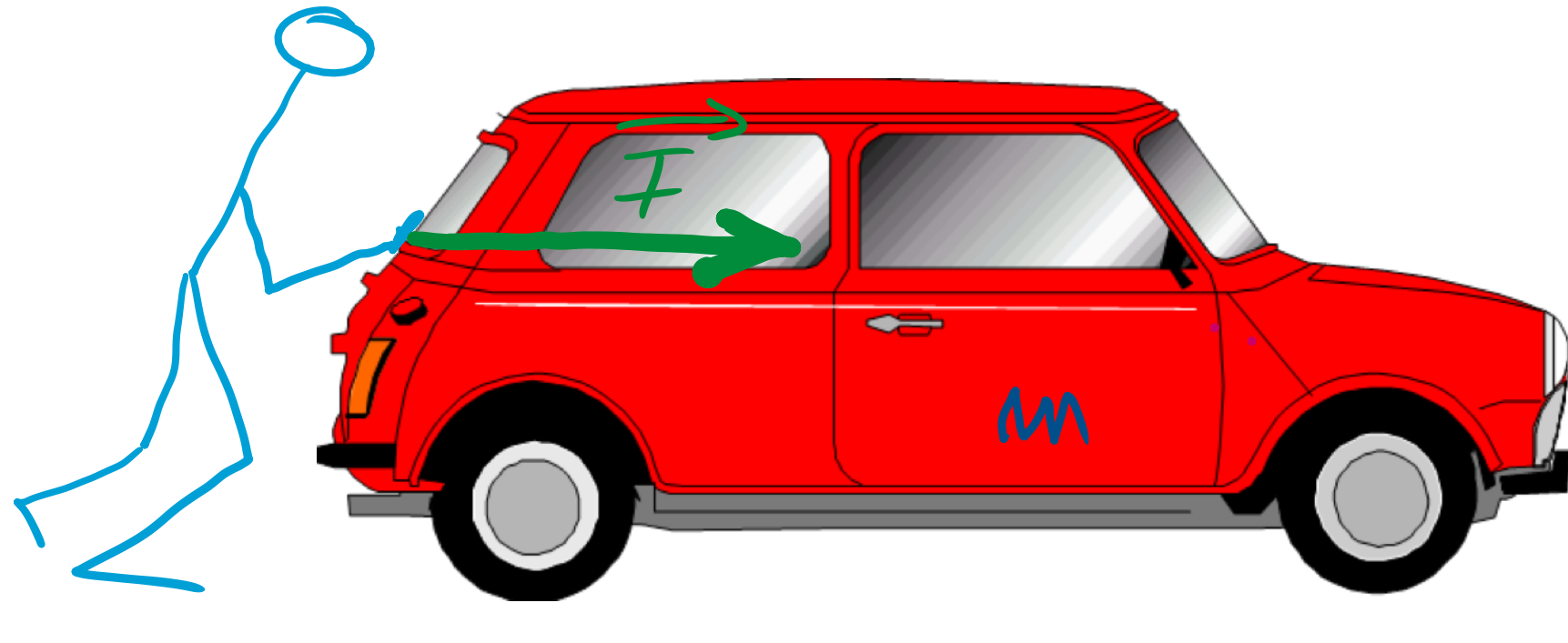
Def: A body continues in its initial state of relative motion or of relative rest unless an unbalanced force is acting on him

$$\vec{R} = \sum_i \vec{F}_i \quad \vec{R} = 0$$

$$m - \text{mass} \quad [m]_{\text{SI}} = \text{kg}$$

2.2. The 2nd Principle of dynamics. The fundamental principle

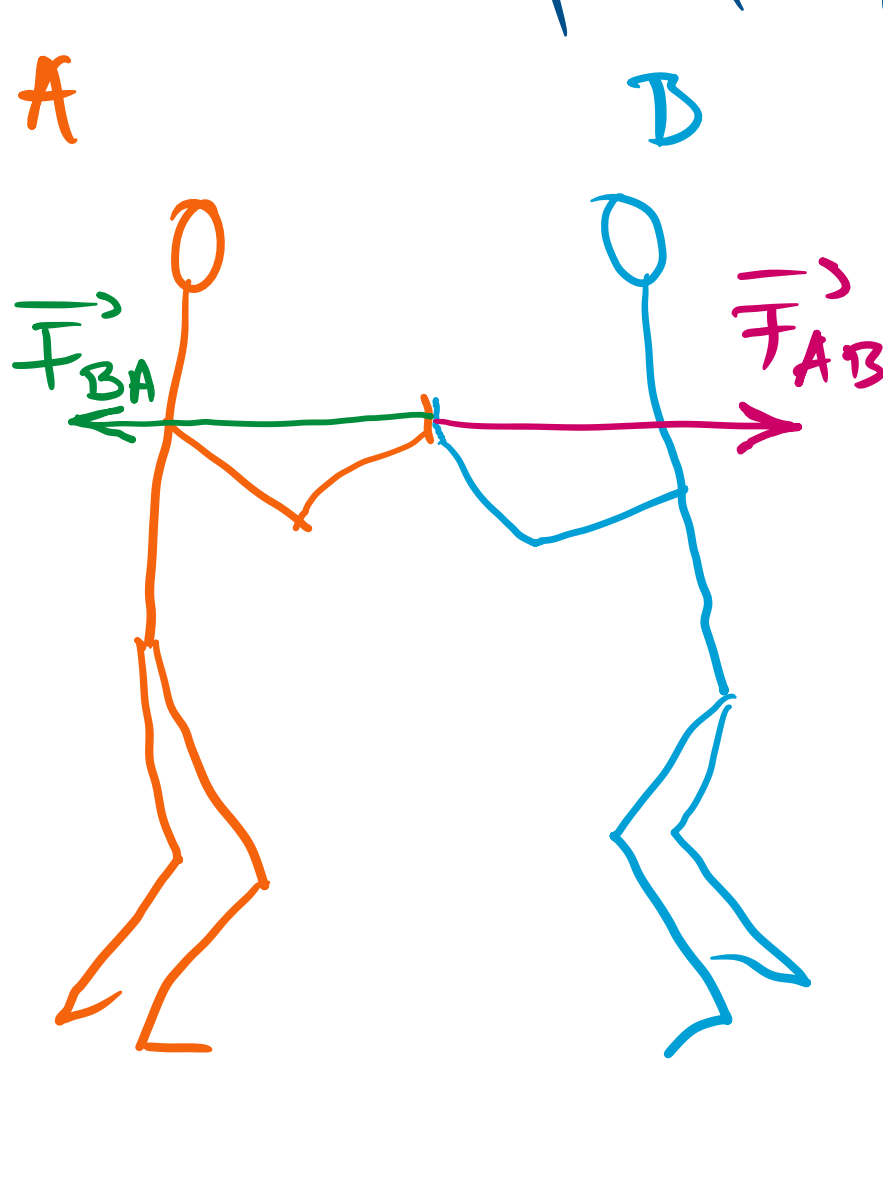
$$\vec{a} = \frac{\vec{F}}{m}$$



Def: The acceleration of a body is directly proportional with the force that act on him and inversely proportional with his mass.

$$\vec{F} = m \cdot \vec{a}$$

2.3. The 3rd Principle of dynamics. The principle of action and reaction



$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Def: Forces always occur in pairs. If a body A exert a force on a body B called action then the body B will act with an equal force but in opposite direction on body A called reaction.

3. The linear momentum

$$\vec{a} = \frac{\vec{F}}{m} \Rightarrow \vec{F} = m \cdot \vec{a} \quad \left| \Rightarrow \vec{F} = m \cdot \frac{d\vec{v}}{dt} \Rightarrow \vec{F} = \frac{d(m \cdot \vec{v})}{dt} \right| \Rightarrow$$

Note: $\vec{p} = m \cdot \vec{v}$ - the linear momentum

$$\Rightarrow \boxed{\vec{F} = \frac{d\vec{p}}{dt}} \Rightarrow d\vec{p} = \vec{F}(t) dt \quad \left| \int \Rightarrow \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \int_{t_1}^{t_2} \vec{F}(t) dt = \right.$$

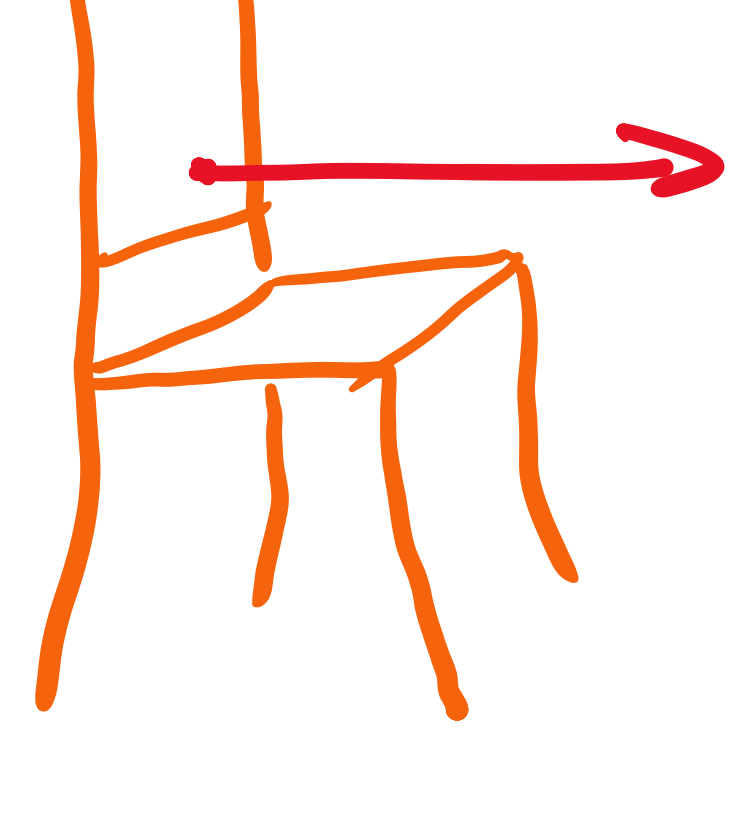
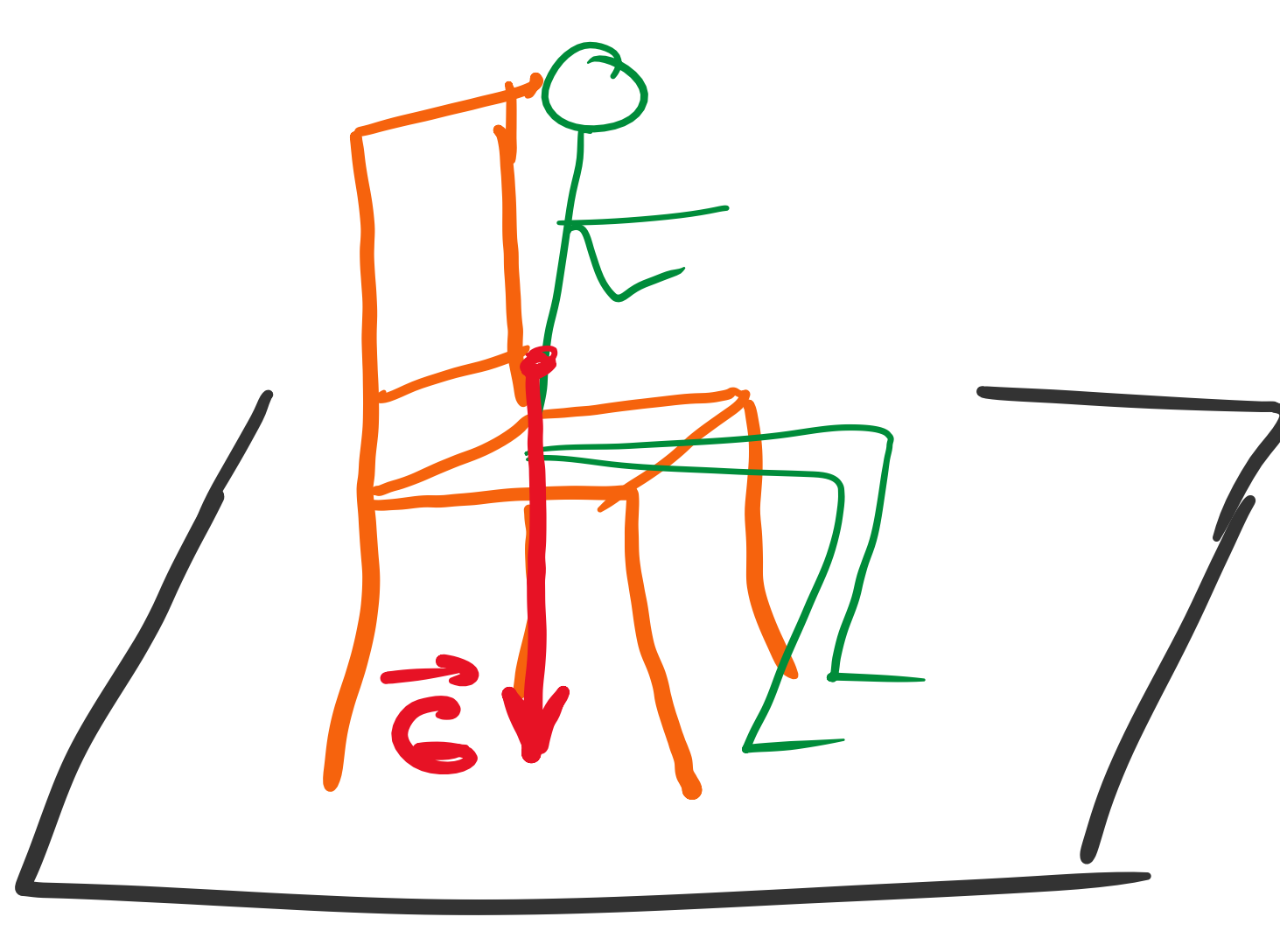
$$\vec{p} \Big|_{\vec{p}_1}^{\vec{p}_2} = \int_{t_1}^{t_2} \vec{F}(t) dt \Rightarrow \vec{p}_2 - \vec{p}_1 = \Delta \vec{p} = \int_{t_1}^{t_2} \vec{F}(t) dt - \text{the impulse}$$



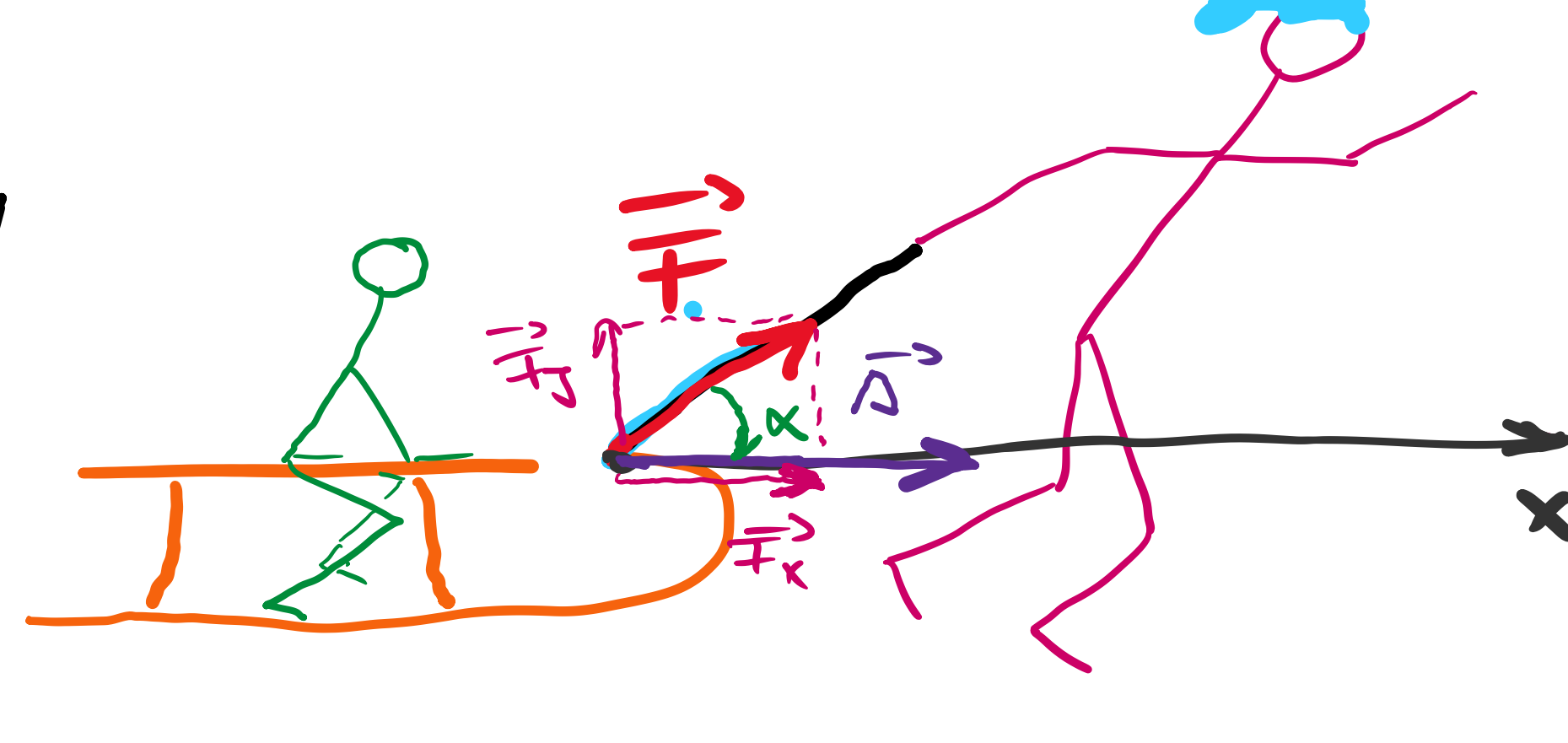
In particular
if $v_1 = 0 \Rightarrow \vec{p}_1 = 0$
 $\Delta \vec{p} = \vec{p}_2 = m \cdot \vec{v}_2$
the impulse

4. The mechanical work

A)

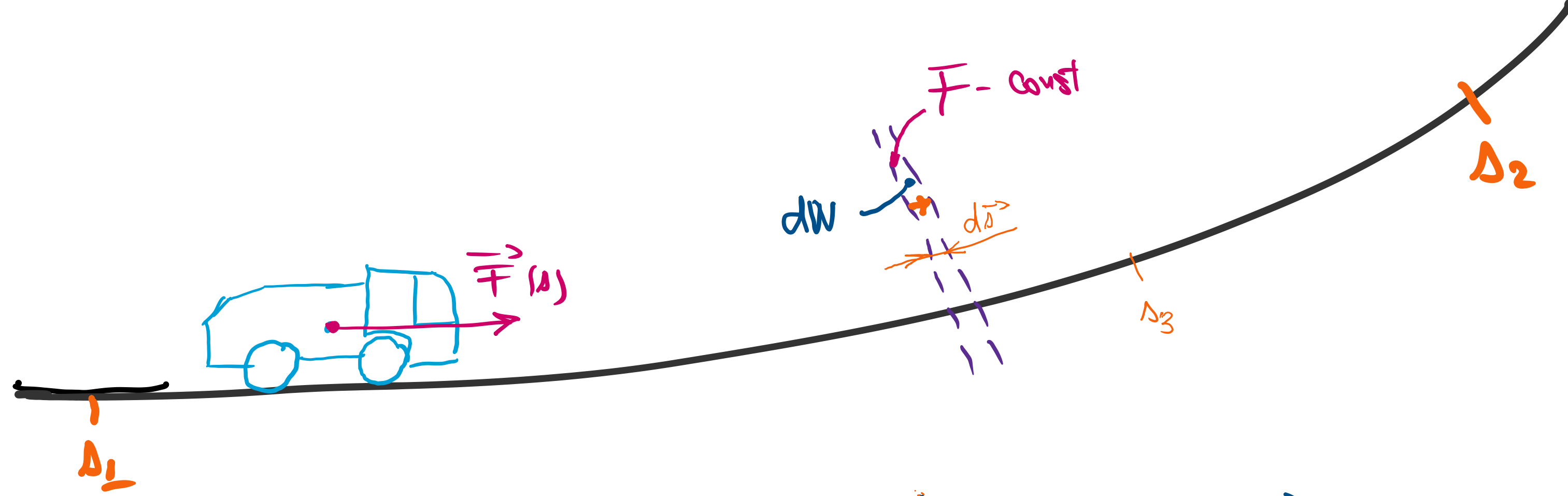


$$W = \vec{F} \cdot \vec{s} \quad [W]_{\text{SI}} = \text{J}$$



$$W = \vec{F} \cdot \vec{s} = |\vec{F}| |\vec{s}| \cdot \cos \alpha \Rightarrow W = F_x \cdot s$$

B)



$$W = \vec{F} \cdot \vec{s} \quad \left| d \Rightarrow dW = d(\vec{F} \cdot \vec{s}) = d\vec{F} \cdot \vec{s} + \vec{F} \cdot d\vec{s} \Rightarrow \right.$$

$$\boxed{dW = \vec{F} \cdot d\vec{s}} \quad \left| \int \Rightarrow \int_{\vec{s}_1}^{\vec{s}_2} dW = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F}(s) \cdot d\vec{s} = \right.$$

No limits

[Stepan Mihail 1]

$$W_{(s_1 \rightarrow s_2)} = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F}(s) \cdot d\vec{s}$$

5. The Power

$$P = \frac{dW}{dt} \Rightarrow dW = P(t) \cdot dt \quad \left| \int \Rightarrow \int_{t_1}^{t_2} dW = \int_{t_1}^{t_2} P(t) dt \Rightarrow \right.$$

$$W = \int_{t_1}^{t_2} P(t) dt$$

$$P = \frac{dW}{dt} \quad \left| \Rightarrow P = \frac{d(\vec{F} \cdot \vec{s})}{dt} \xrightarrow{\vec{F} \text{ const}} \vec{F} \cdot \frac{d\vec{s}}{dt} \Rightarrow \boxed{P = \vec{F} \cdot \vec{v}}$$

6. The energy