Course 3

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I Kinematics

1. The elements of motion

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2. The velocity

$$\vec{v} = \frac{d\vec{s}}{dt}$$

3. Rectilinear motion with constant velocity

$$\boxed{s(t) = s(0) + v \cdot t}$$ law of motion

4. The acceleration

$$\vec{a} = \frac{d\vec{v}}{dt}$$

5. The rectilinear motion with constant acceleration

* $v(t) = v(0) + a \cdot t$ – the law of velocity

$$\boxed{s(t) = s(0) + v(0) \cdot t + a\frac{t^2}{2}}$$ the law of motion

6. The Galileo's equation (law)

Note $v(t) \to v$

$v(0) \to v_0$

$s(t) - s(0) \to s$

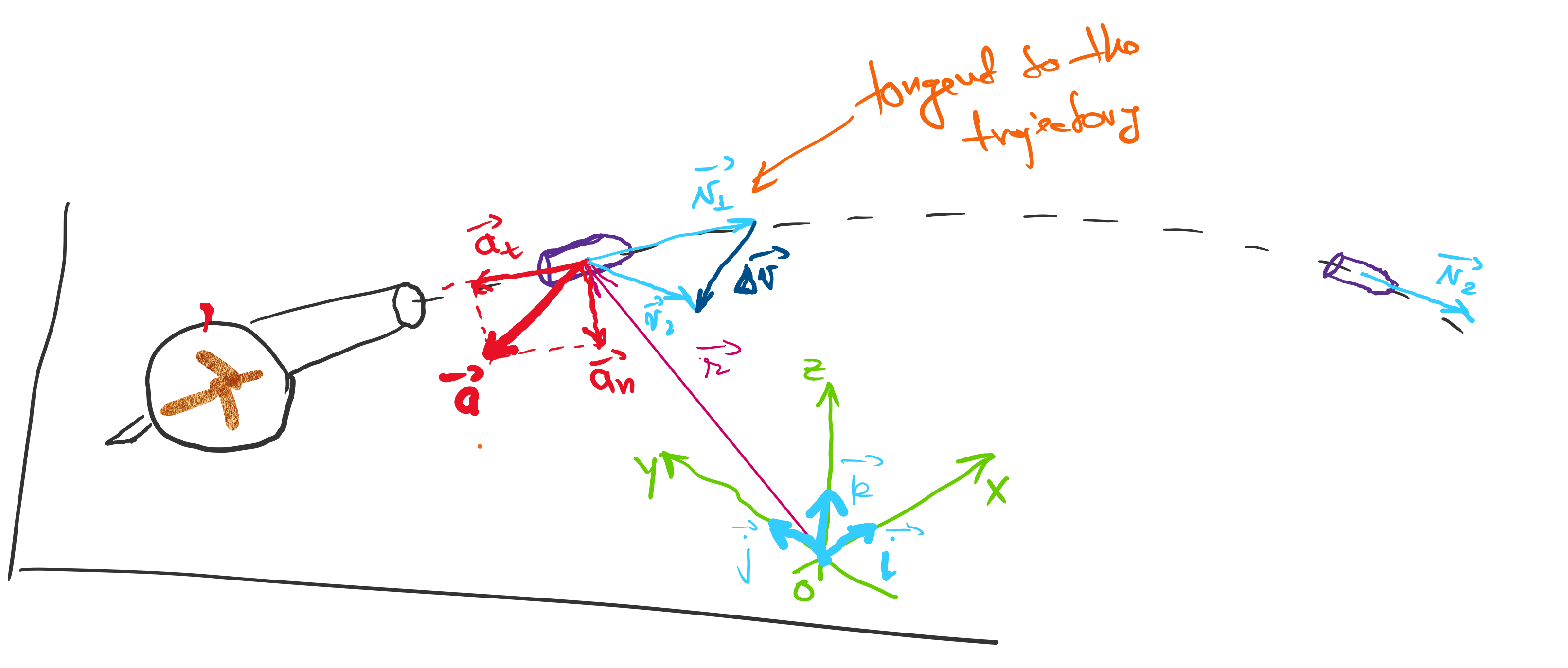
$\Rightarrow t = \frac{v - v_0}{a}$

$$s = v_0 \frac{v - v_0}{a} + \frac{a}{2}\frac{(v - v_0)^2}{a^2}$$

$$s = \frac{\cancel{2v_0 v} - 2v_0^2 + v^2 + v_0^2 - \cancel{2v_0 v}}{2a} \Rightarrow s = \frac{v^2 - v_0^2}{2a} \Rightarrow$$

$$v^2 - v_0^2 = 2as \Rightarrow \boxed{v^2 = v_0^2 + 2a \cdot s}$$ G.e.

7. General motion



$$\vec{r} = x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}) = \underbrace{\frac{dx}{dt}}_{v_x} \cdot \vec{i} + \underbrace{\frac{dy}{dt}}_{v_y} \cdot \vec{j} + \underbrace{\frac{dz}{dt}}_{v_z} \cdot \vec{k} + x\cancel{\frac{d\vec{i}}{dt}} + y\cancel{\frac{d\vec{j}}{dt}} + z \cdot \cancel{\frac{d\vec{k}}{dt}}$$

$$\vec{v} = v_x \cdot \vec{i} + v_y \cdot \vec{j} + v_z \cdot \vec{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x \cdot \vec{i} + v_y \cdot \vec{j} + v_z \cdot \vec{k}) = \underbrace{\frac{dv_x}{dt}}_{a_x} \cdot \vec{i} + \underbrace{\frac{dv_y}{dt}}_{a_y} \cdot \vec{j} + \underbrace{\frac{dv_z}{dt}}_{a_z} \cdot \vec{k} + v_x \cancel{\frac{d\vec{i}}{dt}} + v_y \cancel{\frac{d\vec{j}}{dt}} + v_z \cancel{\frac{d\vec{k}}{dt}}$$

$$\vec{a} = a_x \cdot \vec{i} + a_y \cdot \vec{j} + a_z \cdot \vec{k}$$

$$\vec{a} = \vec{a}_t + \vec{a}_n$$

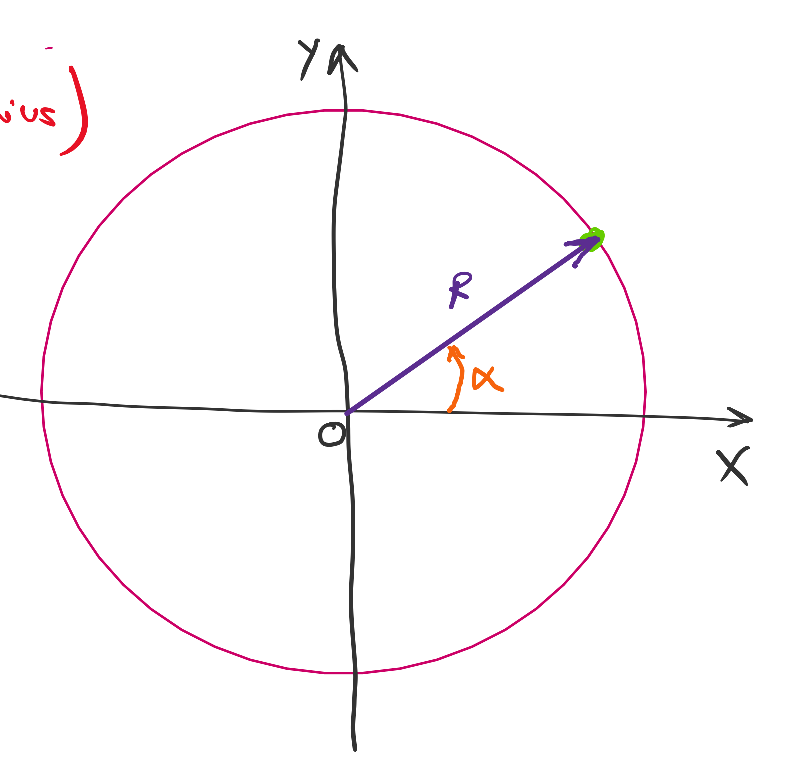
$\vec{a}_t$ The tangent component of the acceleration is responsible with the change in the modulus of the velocity

$\vec{a}_n$ The normal component of the acceleration is responsible with the change of the direction of velocity

8. Circular motion

8.1 Characteristics

- the trajectory → Circle (O – the origin; R – radius)
- periodical
- limited in space
- the position → angle α



8.2. The angular velocity

$s, R, \Delta\alpha$

$s = R \cdot \Delta\alpha$

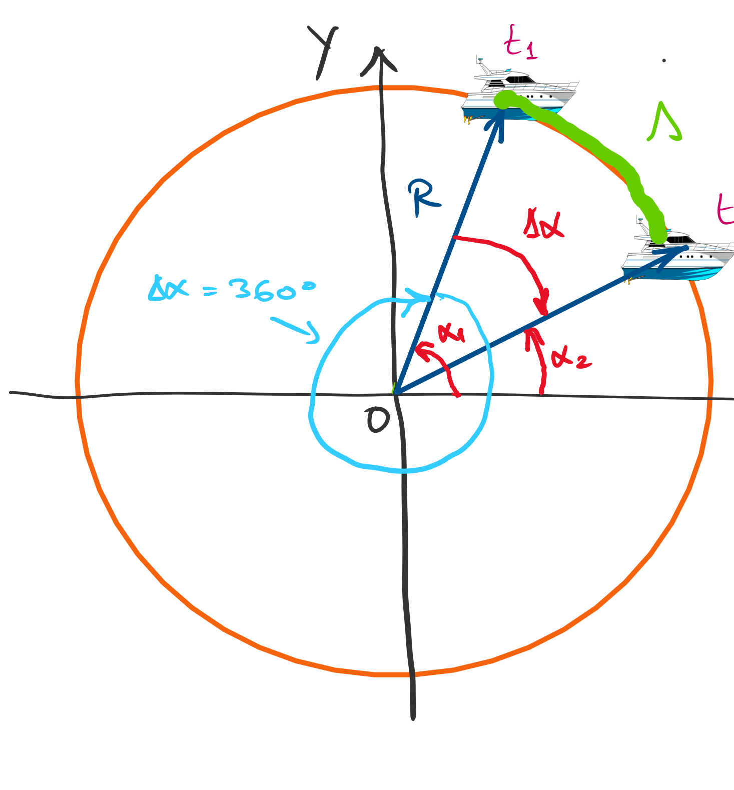
$\ell = 2\pi R$

$\Delta\alpha = \alpha_2 - \alpha_1$

$\Delta t = t_2 - t_1$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\alpha}{\Delta t} = \frac{d\alpha}{dt}$$

$$\boxed{\omega \stackrel{def}{:=} \frac{d\alpha}{dt}}$$ the angular velocity



8.3. The circular motion with constant angular velocity

ω – const $\alpha(t) = ?$

$$\omega = \frac{d\alpha}{dt} \Rightarrow d\alpha = \omega \cdot dt \,\Big|\, \int \Rightarrow \int_{\alpha(0)}^{\alpha(t)} d\alpha = \int_0^t \omega \cdot dt' \Rightarrow$$

$$\alpha \Big|_{\alpha(0)}^{\alpha(t)} = \omega t' \Big|_0^t \Rightarrow \alpha(t) - \alpha(0) = \omega \cdot (t - 0) \Rightarrow$$

$$\boxed{\alpha(t) = \alpha(0) + \omega \cdot t}$$ – the law of motion

8.4. The angular acceleration

$$\boxed{\varepsilon \stackrel{def}{:=} \frac{d\omega}{dt}}$$

8.5. The circular motion with constant angular acceleration

ε – const

$$\varepsilon = \frac{d\omega}{dt} \Rightarrow d\omega = \varepsilon \cdot dt \,\Big|\, \int \Rightarrow \int_{\omega(0)}^{\omega(t)} d\omega = \int_0^t \varepsilon \cdot dt' \Rightarrow$$

$$\omega \Big|_{\omega(0)}^{\omega(t)} = \varepsilon t' \Big|_0^t \Rightarrow \boxed{\omega(t) = \omega(0) + \varepsilon \cdot t}$$ the law of velocity

$$\omega = \frac{d\alpha}{dt} \Rightarrow d\alpha = \omega(t)\, dt \,\Big|\, \int \Rightarrow \int_{\alpha(0)}^{\alpha(t)} d\alpha = \int_0^t [\omega(0) + \varepsilon t'] dt' \Rightarrow$$

$$\alpha \Big|_{\alpha(0)}^{\alpha(t)} = \omega(0) \int_0^t dt' + \varepsilon \int_0^t t' dt' \Rightarrow$$

$$\alpha(t) - \alpha(0) = \omega(0)\, t' \Big|_0^t + \varepsilon \frac{t'^2}{2} \Big|_0^t \Rightarrow$$

$$\boxed{\alpha(t) = \alpha(0) + \omega(0) \cdot t + \varepsilon \frac{t^2}{2}}$$ the law of motion

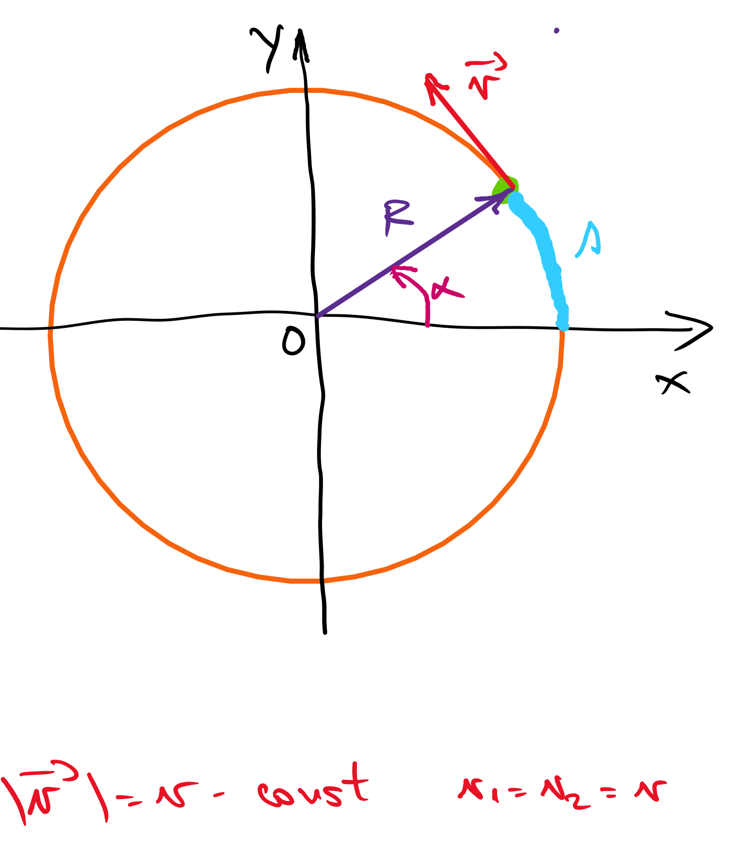
8.6 The relationship between angular (circular) and linear physical quantities

A) $s = R \cdot \alpha$

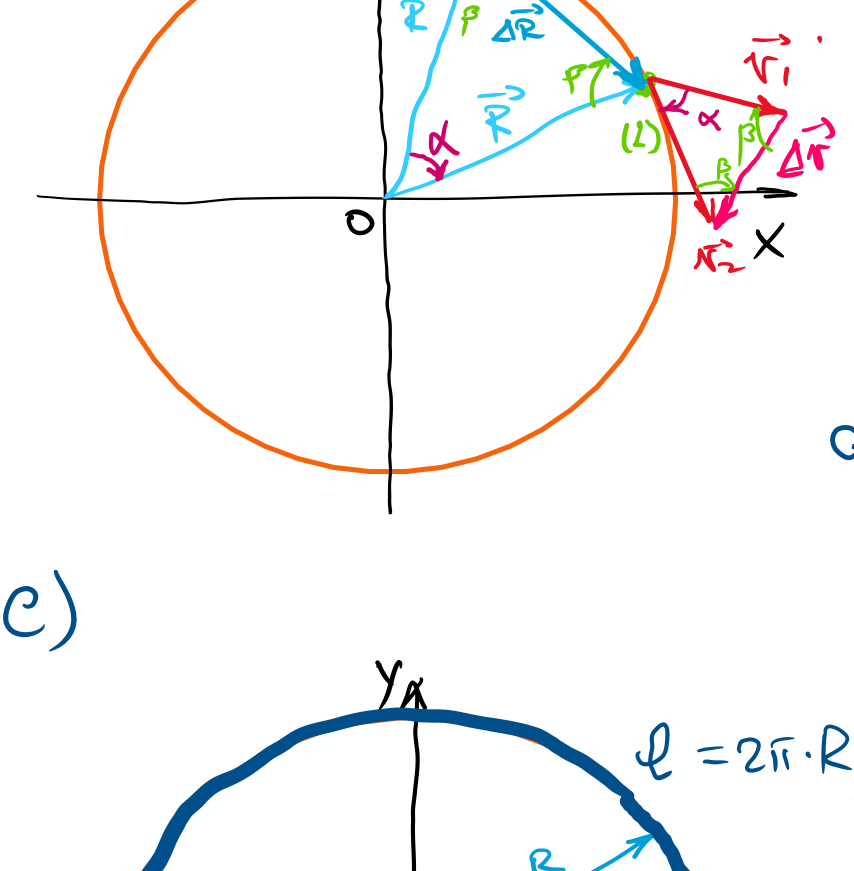
$$v = \frac{ds}{dt} = \frac{d}{dt}(R \cdot \alpha) \Rightarrow$$

$$v = R \cdot \frac{d\alpha}{dt} = R \cdot \omega$$

$$\boxed{v = R \cdot \omega} = \omega \cdot R$$



B)



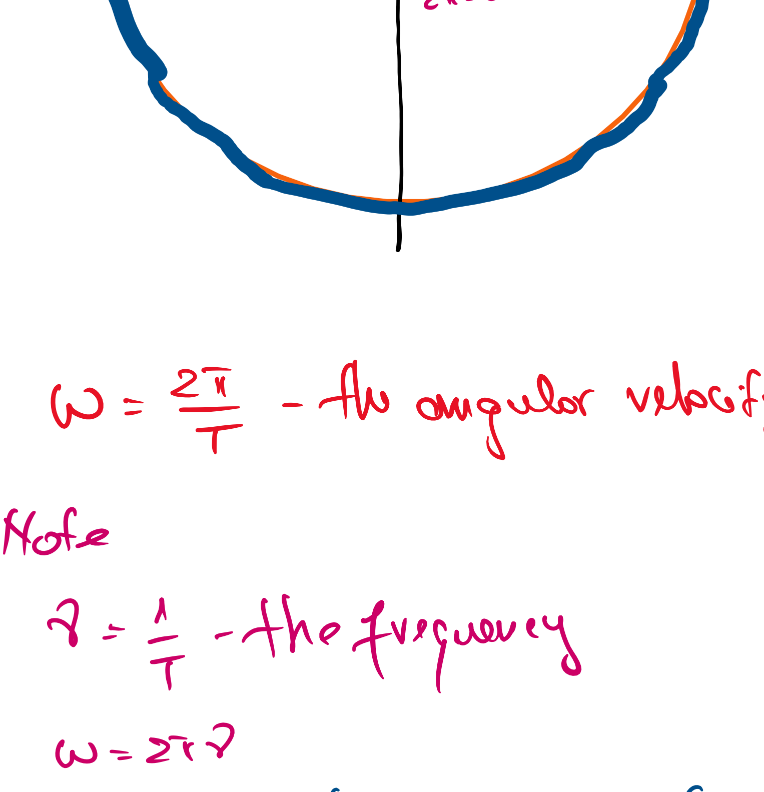
$|\vec{v}| = v$ – const $v_1 = v_2 = v$

$$\frac{\Delta v}{\Delta R} = \frac{v}{R} \,\Big|\, \Delta R \cdot \frac{1}{\Delta t} \,\Big|\, \lim_{\Delta t \to 0}$$

$$\frac{dv}{dt} = \frac{dR}{dt} \cdot \frac{v}{R} \Rightarrow$$

$$a_n = \frac{v^2}{R} = \frac{R^2 \cdot \omega^2}{R} = R \cdot \omega^2 = v \cdot \omega$$

C)



$v = \frac{d}{t}$

$d = \ell = 2\pi R$

$t = T$ – the period

$v = \frac{2\pi \cdot R}{T}$

$v = \frac{2\pi}{T} R$

$v = \omega \cdot R$ $\Rightarrow$

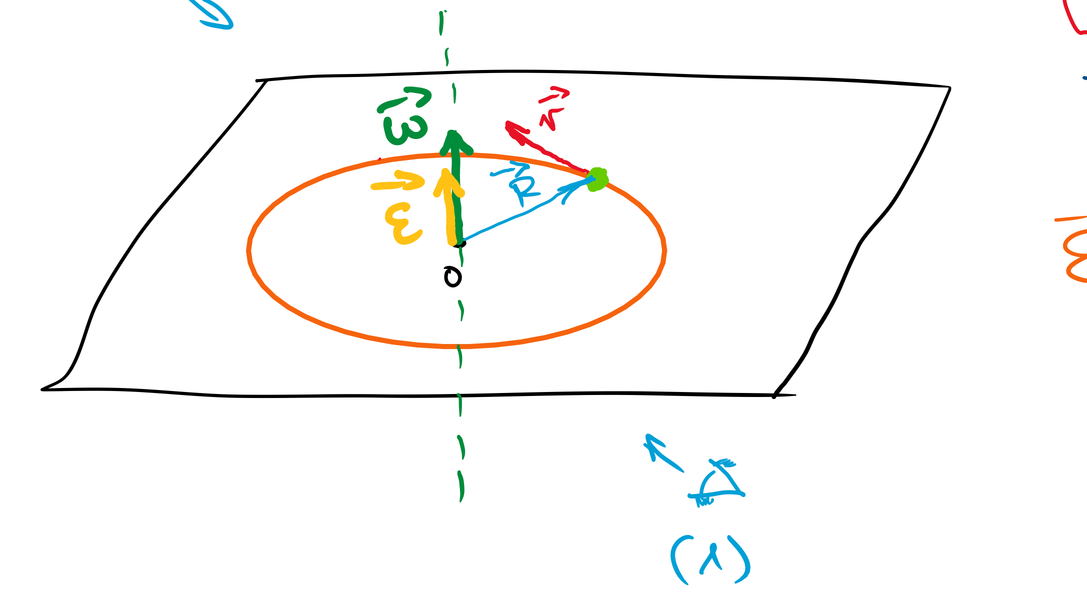
$\omega = \frac{2\pi}{T}$ – the angular velocity

Note

$\nu = \frac{1}{T}$ – the frequency

$\omega = 2\pi\nu$

8.7. The vectorial character of circular quantities



$$\boxed{\vec{v} = \vec{\omega} \times \vec{R}}$$

$$\vec{v} = -\vec{R} \times \vec{\omega}$$

$$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$$