$$= \int_1^2 -26x^2 + 24x - 4\cancel{4} + i \int_1^2 (x^2 - 27x^2 - 12 + 36x + 6x^2 - 4x$$

$$= -26\frac{x^3}{3}\Big|_1^2 + 24\frac{x^2}{2}\Big|_1^2 - 4x\Big|_2^1 + i\int_1^2 -21x^2 \dots -$$

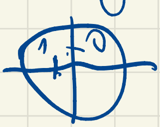
$$= \dots$$

9) $1^i = e^{i\,Log\,1} = e^{i\{\ln 1 + i2k\pi,\ k\in\mathbb{Z}\}}$

10) $i^i = e^{i\,Log\,i} = e^{i\{\ln 1 + i\pi(\frac{1}{2} + 2k),\ k\in\mathbb{Z}\}}$

11) $(-\sqrt{3}+i)^i = e^{i\,Log(-\sqrt{3}+i)} = e^{i\{\ln 2 + i\pi(\frac{\pi}{6} + 2k),\ k\in\mathbb{Z}\}}$ $\quad -\frac{\pi}{6} + \frac{3}{2}\pi$

$$Log(-\sqrt{3}+i) = \left\{\ln\sqrt{3+1} + i\left(\arctan\frac{1}{-\sqrt{3}} + 2k\pi\right)\right\}$$



$$= \left\{\ln 2 + i\left(\arctan -\frac{\sqrt{3}}{3} + 2k\pi\right)\right\}$$

$$= \left\{\ln 2 + i\left(-\frac{\pi}{6} + \frac{6}{6}\pi + 2k\pi\right)\right\}$$

$$= \left\{\ln 2 + i\left(\frac{5\pi}{6} + 2k\pi\right)\right\}$$

1.27

a) $\sin z = \frac{4i}{3}$

$\frac{e^{iz}-e^{-iz}}{\cancel{2i}} = \frac{\cancel{4i}}{3} \Rightarrow e^{iz}-e^{-iz} = \frac{2}{3}$, $e^{iz} = t$

$t - \frac{1}{t} = \frac{2}{3}$

$\frac{t^2-1}{t} = \frac{2}{3}$

$3t^2 - 3 - 2t = 0$

$\Delta = 4 + 32 = 36$

$t_{1,2} = \frac{2+6}{6} = \frac{8}{6} = \frac{4}{3}$

$\frac{2-6}{6} = -\frac{4}{6} = -\frac{2}{3}$

$e^{iz} = \frac{4}{3} \mid Log$

$iz = Log\left(\frac{4}{3}\right) = \left\{ \ln\frac{4}{3} + i2k\pi, k \in \mathbb{Z} \right\} \mid :i$

$z = 2k\pi - i\ln\frac{4}{3}$

$iz = Log\left(-\frac{2}{3}\right) = \{ \ln\frac{2}{3} +$

2) $\cos z = \frac{3i}{4}$

$$\frac{e^{iz}+e^{-iz}}{2} = \frac{3i}{4}$$

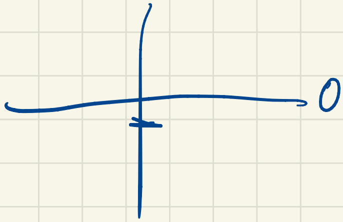
$$\frac{t+\frac{1}{t}}{2} = \frac{3i}{4}$$

$$4\left(t+\frac{1}{t}\right) = 6i$$

$$4\left(\frac{t^2+1}{t}\right) = 6i$$

$$4t^2+4 = 6ti$$

$$4t^2 - 6ti + 4 = 0$$



$(0, -\frac{1}{2})$

$$Log\left(-\frac{i}{2}\right) = \left\{\ln\frac{1}{2} + i\left(\frac{3\pi}{2}+2k\pi\right)\right\}$$

$$-\ln 2 + i\left(\frac{3\pi}{2}+2k\pi\right)\Big\} \,/\cdot i$$

$$i\ln 2 + \frac{3\pi}{2} + 2k\pi \checkmark$$

$$\ln\frac{1}{2} = \ln 2^{-1} = -\ln 2$$

$$\Delta = -36 - 4\cdot 4\cdot 4$$
$$= -36 - 64$$
$$= -100$$

$$t_{1,2}: \quad \frac{6i - i10}{8} = \frac{-4i}{8} = \frac{-i}{2}$$

$$\frac{6i + i10}{8} = \frac{16i}{8} = 2i$$

$$iz = Log\left(-\frac{i}{2}\right) = \left\{\ln\frac{1}{2} + \pi i\left(\frac{3}{2}+2k\right), k\in\mathbb{Z}\right\}$$
$$= \pi\left(\frac{3}{2}+2k\right) - i\ln\frac{1}{2}$$

$$iz = Log(2i) = \left\{\ln 2 + \pi i\left(\frac{1}{2}+2k\right), k\in\mathbb{Z}\right\}$$
$$\pi\left(\frac{1}{2}+2k\right) - i\ln 2$$

$\sin z + \cos z = 1$

$$\frac{e^{iz} - e^{-iz}}{2i} + \frac{e^{iz} + e^{-iz}}{2} = 1$$

$$\frac{t - \frac{1}{t}}{2i} + \frac{t - \frac{1}{t}}{2} = 1 \quad / \cdot 2i$$

$$\frac{\frac{t^2-1}{t} + \left(\frac{t^2-1}{t}\right)i}{2i} = 1$$

$$\frac{t^2-1 + it^2 - i}{t} = 2i$$

$$t^2 - 2ti + it^2 - i - 1 = 0$$

$$t^2 - i(2t + t^2 - 1) - 1 = 0$$

$$t - \frac{1}{t} + i\left(t - \frac{1}{t}\right) - 2i = 0$$

$$t - \frac{1}{t} + it - \frac{i}{t} - 2i = 0 \quad / \cdot t$$

$$t^2 - 1 + it^2 - i - 2it = 0$$

$$t^2(1+i) - 2it - (i+1) = 0$$

SEMINAR 4

$$\int_C f(z)\,dz = \int_C (u+iv)(dx+idy) \quad \text{(linie)}$$

$$\int_C (u\,dx - v\,dy) + i\int_C (v\,dx + u\,dy)$$

if path is a circle $\Rightarrow$ Cauchy's integral formula

$$\boxed{\begin{aligned} &\Rightarrow \int_C f(z)\,dz = 0 \\ &\Rightarrow \int_C \frac{f(z)}{z-z_0}\,dz = 2\pi i f(z_0) \\ &\Rightarrow \int_C \frac{f(z)}{(z-z_0)^{n+1}}\,dz = \frac{2\pi i}{n!} f^{(n)}{}_{z_0} \end{aligned}}$$

1.34 $\int_C (1-2\bar{z})\,dz \qquad C: z_1z_3z_2 \quad z_1=0,\ z_2=1+i,\ z_3=i$

$z_1z_3 \qquad z_1(0,0) \quad z_2(1,1) \quad z_3(0,1)$

$$\int_C (1-2\bar{z})\,dz = \int_{z_1}^{z_3} \big(1-2(x-iy)\big)(dx+idy)$$

$$= \int_{(0,0)}^{(0,1)} \big((1-2x)+2yi\big)(dx+idy)$$

$$= \int_{(0,0)}^{(0,1)} \overbrace{(1-2x)}^{0}dx - 2y\,dy + i\int_{(0,0)}^{(0,1)} \overbrace{(1-2x)}^{idy}dy + \overbrace{2y}^{0}dx$$

$$\begin{array}{ccc} x,y & (0,0) & (0,1) \end{array}$$

$$\begin{vmatrix} 1 & x & y \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{vmatrix} \Rightarrow x=0 \Rightarrow dx=0$$

$$\Rightarrow -\int_0^1 2y\,dy + i\int_0^1 dy$$

$$= -y^2\Big|_0^1 + i \Rightarrow -1+i$$

$$\int_{z_3}^{z_2} (1-2x)dx - \overset{0}{2ydy} + i\int_{z_3}^{z_2} \overset{0}{(1-2x)dy} + \overset{2}{2y}dx$$

$$\begin{vmatrix} 1 & x & y \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} \Rightarrow 0 + y + x - 0 - 1 - x =$$

$$\Rightarrow y = 1$$

$$dy = 0$$

$$\int_0^1 (1-2x)dx + 2i\int_0^1 dx - \frac{2x^2}{2}\Big|_0^1 + 2ix\Big|_0^1$$

$$= 2i$$

$$I = -1 + i + 2i = -1 + 3i$$

2)

$z_1 z_4 z_2$

$z_1 z_4 \quad z_1(0,0) \quad z_4(1,0)$

$$\int_{z_1}^{z_4} (1-2\bar{z})(dx + idy) = \int_{z_1}^{z_4} (1-2(x-iy))(dx + dyi) =$$

$$= \int_{z_1}^{z_2} ((1-2x)+2yi)(dx+idy) =$$

$$= \int_{z_1}^{z_2} \left((1-2x)dx - \overset{0}{\overbrace{2y\,dy}}\right) + i\int_{z_1}^{z_2}\left(\overset{0}{\overbrace{(1-2x)}}dy + \overset{0}{\overbrace{2y\,dx}}\right)$$

$$= \int_{(0,0)}^{(1,0)}$$

$$\begin{vmatrix} 1 & x & y \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0 + y + 0 - 0 - 0 - 0$$

$$\Rightarrow y = 0$$

$$dy = 0$$

$$= \int_0^1 (1-2x)dx = \int_0^1 dx - 2\int_0^1 x\,dx = x\Big|_0^1 - \cancel{2}\frac{x^2}{\cancel{2}}\Big|_0^1$$

$$= 1 - 1 = 0$$

$$= \int_{z_1}^{z_2} \left(\overset{0}{\overbrace{(1-2x)}} dx - 2y\,dy \right) + i \int_{z_1}^{z_2} \left(\overset{-1dy}{\overbrace{(1-2x)}} dy + \overset{0}{\overline{2y\,dx}} \right)$$

$$\begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 1 \cdot y + 0 - y - 0 - x = 0$$

$$1 - x = 0$$

$$x = 1$$

$$dx = 0$$

$$-2\int_0^1 y\,dy - i\int_0^1 dy = -2\frac{y^2}{2}\Big|_0^1 - iy\Big|_0^1$$

$$= -1 - i$$

1.35

a) $\int_C \frac{e^z \sinh z^2}{z^3 - z}\, dz$ $\qquad C: |\overbrace{z - 2i}^{\text{Centru}}| = \underbrace{1}_{\text{Rază}}$

$(\underset{x}{0}, \underset{y}{-2})$

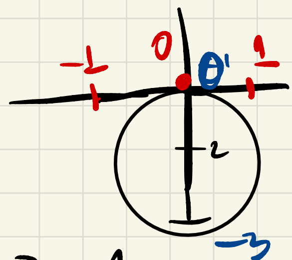
I desenezi cerc

II numitor=0

$z^3 - z = 0$

$z(z^2 - 1) = 0$

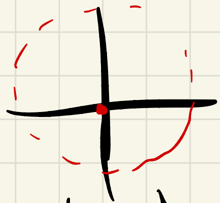
$z = 0$ sau $z = 1$ sau $z = -1$



$\Rightarrow \int_C = 0$

ii) $\int_C z^2 \cos z \sin 2iz\, dz$ $\qquad C: |\underset{0}{z}| = r$

$0=0$



iii) $\displaystyle\int_C \frac{ze^{2z}}{z-1}\,dz$

$C: |z-1|+|z+1|=4$

$\left|(x-1)+yi\right| + \left|(x+1)+yi\right| = 4$

$\sqrt{(x-1)^2+y^2}+\sqrt{(x+1)^2+y^2}=4$

$\sqrt{x^2-2x+1+y^2}+\sqrt{x^2+2x+1+y^2}=4$

$\sqrt{x^2-2x+1+y^2} = 4-\sqrt{x^2+2x+1+y^2} \quad \Big|\,()^2$

$x^2-\cancel{2x}+1+y^2 = 16-8\sqrt{x^2+2x+1+y^2}-x^2\cancel{2x}+\dots$

$2x^2+2+2y^2 = 16-8\sqrt{x^2+2x+1+y^2}$

$z-1=0$

$z=1$

$z \in \mathrm{Int}\,C$

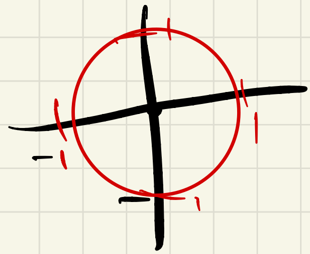
$\displaystyle\int_C \frac{\overbrace{ze^{2z}}^{f(z)}}{\underbrace{z-1}_{z-z_0}} =$

$z_0=1$

$\displaystyle\int_C \frac{f(z)}{z-z_0}\,dz =$

$= 2\pi i f(z_0) = 2\pi i\, e^2$

iv) $\int_C \frac{e^z}{z^2+2z}\,dz$ $|z|=1$



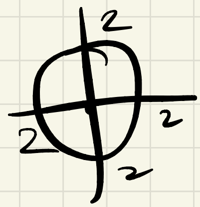
$z^2+2z=0$

$z(z+2)=0$

$z=0\;,\;z=-2$

$$\int_C \frac{\frac{e^z}{z+2}}{z-0}\,dz = 2\pi i f(z) = \frac{2\pi i\, e^0}{2} = \frac{2\pi i}{2} = \pi i$$

v) $\int_C \frac{\sin z}{z^2(z-4)}\,dz =$ $|z|=2$



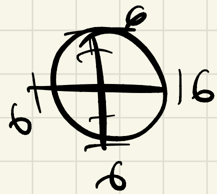
$z^2(z-4)=0$

$z=0\;,\;z=4$

$$\int_C \frac{\frac{\sin z}{z-4}}{z^2}\,dz = \frac{2\pi i}{1!} f^{(1)}(z_0)$$

$$= 2\pi i \left(\frac{\sin z_0}{z_0-4}\right)' =$$

(iii) $\int_C \frac{dz}{z^2+25} dz = \int_C \frac{1}{z^2+25} dz$ C: $|z|=6$



$z^2 = -25$

$z = \pm 5i$

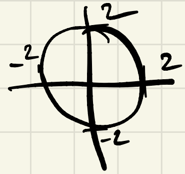
$$\int_C \frac{1}{(z-5i)(z+5i)} dz = \int_C \frac{z-5i-(z+5i)}{(z-5i)(z+5i)} \cdot \left(\frac{-1}{10i}\right)$$

$$-\frac{1}{10i}\left[\int_C \frac{1}{z+5i} dz - \underbrace{\int_C \frac{1}{z-5i} dz}_{2\pi i}\right] = -\frac{1}{10i}\left[2\pi i - 2\pi i\right] = 0$$

$$\int_C \frac{\overbrace{1}^{f(z_0)}}{\underbrace{z+5i}_{z-z_0}} dz = 2\pi i f(z_0) = 2\pi i$$

vii) $\int_C \frac{z \sinh z}{(z^2-1)^2}\, dz$???

$C: |z| = 2$



$(z^2-1)^2 = 0 \; |()\surd$

$z^2 - 1 = 0$

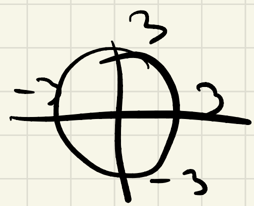
$z^2 = 1 \Rightarrow z = \pm 1$

$$\int_C \frac{z \sinh z}{(z-1)^2(z+1)^2}\, dz = \frac{1}{4z}\int \frac{z \sinh z (z-1)^2 - z \sinh z (z+1)^2}{(z-1)^2(z+1)^2}$$

$$\int_C z \sinh z$$

8) $\int_C \frac{\cos(z+\pi i)}{z(e^z+2)}\, dz$

$C: |z| = 3$



$z(e^z+2) = 0$

$z = 0$

$e^z + 2 = 0$

$e^z = -2$

$z = \mathrm{Log}(-2) = \{\ln 2 + i(\pi + 2k\pi), k \in \mathbb{Z}\}$

$k=0 \Rightarrow \ln 2 + \pi i \notin C$

$$\int_C \frac{\frac{\cos(z+\pi i)}{e^z+2}}{z}\,dz = 2\pi i f(z_0)$$

$f(z)$

$$= \frac{2\pi i \cosh \pi}{3}$$

$$\cos(\pi i) = \frac{e^{\pi i} + e^{-\pi i}}{2} = \cosh \pi$$

ix) $\displaystyle\int_C \frac{\cosh^2 iz}{z^3}\,dz$

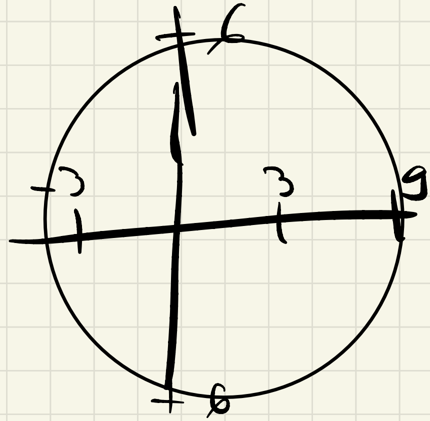
$z^3 = 0$

$z = 0$

$$\int_C \frac{\cosh^2 iz}{z^3}\,dz = \frac{2\pi i}{2!} f^{(2)}(z_0)$$

$$= \frac{8\pi i}{\cancel{2}} \left(\cancel{\cosh iz} \right)^{(2)} = -2\pi i$$

x) $\displaystyle\int_C \frac{z}{(z-2)^3(z+4)}\,dz$



$(z-2)^3 = 0 \Rightarrow z = 2$

$(z+4) = 0 \Rightarrow z = -4$

$$\int_C \frac{\frac{z}{z+4}}{(z-2)^3}\,dz = \frac{2\pi i}{2!}\, f^{(2)}\left(\frac{z}{z+4}\right)$$

$$= \pi i$$

$$\left(\frac{z}{z+4}\right)' = \frac{z'(z+4) - z(z+4)'}{(z+4)^2}$$

$$\frac{z+4-z}{(z+4)^2} = \left(\frac{4}{(z+4)^2}\right)' = \frac{4'(z+4)^2 - 4\left((z+4)^2\right)'}{(z+4)^4}$$

$$-\frac{8\cancel{(z+4)}}{(z+4)^{\cancel{4}}} = \frac{-8}{(z+4)^3}$$

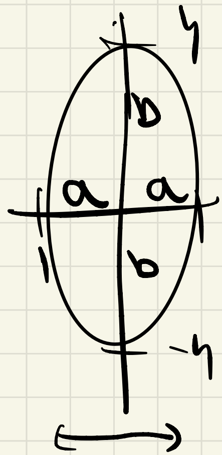
$$\Rightarrow \frac{-\pi i 8}{(z+4)^3} = \frac{-\pi i \cancel{8}}{\cancel{6}^3} = \frac{-\pi i \cdot 8^{2\cdot 2\cdot 2}}{6\cdot 6\cdot 6} = \frac{-\pi i}{3\cdot 3\cdot 3} = \frac{-\pi i}{27}$$

11) $\displaystyle\int_C \frac{z^{100} e^{i\pi z}}{z^2+1}\,dz$

$C: \quad x^2 + \frac{y^2}{4} = 1 \quad |\sqrt{\ }$ $\qquad a=1$, $b=4$

$\sqrt{x^2+\frac{y^2}{4}} = \sqrt{1}$

$|z| = 1$



Elipsă

$z^2+1=0$

$z^2=-1$

$z=\pm i$

$$\int_C \frac{z^{100}e^{i\pi z}}{(z+i)(z-i)}\,dz = \int_C z^{100}e^{i\pi z}\cdot\frac{1}{(z+i)(z-i)}\,dz$$

$$\int_C z^{100} e^{i\pi z} \cdot \frac{(z+i)-(z-i)}{(z+i)(z-i)} dz$$

$$\frac{1}{2i} \int_C \left(\frac{z^{100} e^{i\pi z}}{(z+i)} - \frac{z^{100} e^{i\pi z}}{(z-i)} \right) dz = \frac{1}{2i} \left[2\pi i f(z) - 2\pi i f(z) \right]$$

$$= \frac{1}{2i} \left[2\pi i \left(i^{100} e^{i^2\pi} \right) - 2\pi i \left(i^{100} e^{\pi} \right) \right]$$

$$= \frac{1}{2i} \left[2\pi i e^{-\pi} - 2\pi i e^{\pi} \right]$$

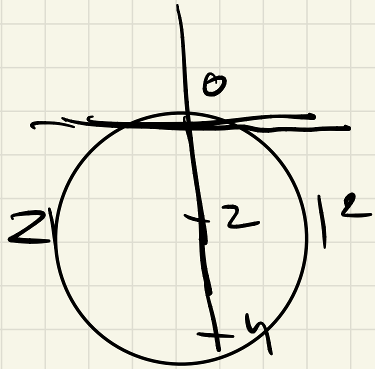
$$= \frac{1}{2i} \left[\frac{2\pi i}{e^{\pi}} - 2\pi i e^{\pi} \right]$$

$$= \pi \left[\frac{1}{e^{\pi}} - e^{\pi} \right]$$

12) $\displaystyle \int_C \frac{\sinh \frac{\pi z}{2}}{(z+i)^{101}} dz \qquad |z+2i| = 2$

$(z+i)^{100} = 0$

$z = -i$



$$\int_C \frac{\sinh \frac{\pi z}{2}}{(z+i)^{100}}\, dz = \frac{2\pi i}{100!} f^{(100)}(z_0)$$

$$= \frac{2\pi i}{100!}\left(\sinh \frac{\pi z_0}{2}\right)^{(100)} =$$

$=$

SEMINAR 5

Power series

$c_i \in \mathbb{C}$ Power series centered in z_0

$$\sum_{n=0}^{\infty} c_n (z-z_0)^n = c_0 + c_1(z+z_0)^1 + c_2(z+z_0)^2 + \dots$$

$R \to$ radius of convergence

$$R = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| \quad \text{or} \quad \lim \frac{1}{\sqrt[n]{|c_n|}}$$

$\{ z \in \mathbb{C} \mid |z-z_0| < R \}$ disk of convergence

Taylor series

Basic Maclaurin series

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$\sin z = \frac{z}{1!} - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

$$\frac{1}{1-z} = 1+z+z^2+\dots+z^n = \sum z^n \quad , \quad |z|<1$$

$$\frac{1}{1+z} = 1-z+z^2-\dots = \sum (-1)^n z^n \quad , \quad |z|<1$$

1.38

i) $\displaystyle\sum_{n=0}^{\infty} (\cos in) z^n =$

$c_n = \cos in$

$(z-z_0)^n \Rightarrow z_0 = 0$

$$R = \lim_{n\to\infty} \left| \frac{\cos in}{\cos i(n+1)} \right| = \lim_{n\to\infty} \left| \frac{e^{i^2 n} + e^{-i^2 n}}{2} \cdot \frac{2}{e^{i^2(n+1)} + e^{-i^2(n+1)}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{e^{-n} + e^{n}}{2} \cdot \frac{2}{e^{-(n+1)} + e^{(n+1)}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{e^{-n}+e^{n}}{e^{-(n+1)}+e^{(n+1)}} \right| = \lim_{n\to\infty} \left| \frac{\overset{e^n}{\frac{1}{e^n}} + e^n}{\overset{e^{n+1}}{\frac{1}{e^{n+1}}} + e^{n+1}} \right| =$$

$$= \lim_{n\to\infty} \left| \frac{1+e^{2n}}{\cancel{e^n}} \cdot \frac{\overset{e}{\cancel{e^{n+1}}}}{1+e^{2(n+1)}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{e(1+e^{2n})}{1+e^{2(n+1)}} \right| = \left| \frac{e \cdot e^{2n}\left(\frac{1}{e^{2n}} + 1\right)}{e^{2n+2}\left(\frac{1}{e^{2n+2}} + 1\right)} \right|$$

$\to 0$ $\to 0$

$$= e^{\cancel{2n}+1-\cancel{2n}-2} = e^{-1} = \frac{1}{e}$$

ii) $$\sum_{n=0}^{\infty} i^n z^n = \lim_{n\to\infty} \left| \frac{i^n}{i^{(n+1)}} \right| = \lim_{n\to\infty} \left| \frac{1}{i} \right| = \lim_{n\to\infty} \frac{1}{i} = -i$$

iii) $$\sum_{n=0}^{\infty} \left(\frac{z}{n+1}\right)^n = \lim_{n\to\infty} \left| \frac{\left(\frac{1}{n+i}\right)^n}{\left(\frac{1}{(n+1)+i}\right)^{n+1}} \right| =$$

$$= \lim_{n\to\infty} \left| \frac{1}{(n+i)^n} \cdot ((n+1)+1)^{n+1} \right| =$$

$$\lim_{n\to\infty} \frac{1}{\sqrt[n]{\left(\frac{1}{n+i}\right)^n}} = \lim_{n\to\infty} |n+i| =$$

IV) $$\sum_{n=0}^{\infty} z^n \frac{1}{(1+i)^n} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{\left(\frac{1}{1+i}\right)^n}} = \lim_{n\to\infty} |1+i| = 1+i$$

$$r = \sqrt{1+1} = \sqrt{2}$$

V) $\sum_{n=0}^{\infty} e^{in\pi} z^n = \lim_{n\to\infty} \frac{1}{\sqrt[n]{\cos n\pi + i \sin n\pi}} = \frac{1}{\sqrt[n]{(\cos\pi + i\sin\pi)^n}}$

$$\sum_{n=0}^{\infty} e^{in\pi} z^n = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|e^{i\pi n}|}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|\cos n\pi + i \sin n\pi|}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[n]{\sqrt{\cos^2 n\pi + \sin^2 n\pi}}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{\sqrt{(\cos^2\pi + \sin^2\pi)^n}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[n]{1^n}} = \lim \frac{1}{1} = \underline{1}$$

VI) $\sum_{n=0}^{\infty} \left(\cos \frac{i}{n}\right)^n z^n = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|\cos \frac{i}{n}|}}$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[n]{|\cos \frac{i}{n}|}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{\frac{e^{\frac{i \cdot i}{n}} + e^{-\frac{i \cdot i}{n}}}{2}}}$$

$$= \lim_{n\to\infty} \frac{1}{\sqrt[n]{\frac{e^{-\frac{1}{n}}+e^{\frac{1}{n}}}{2}}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{1}} = 1$$

Disk de convergență

1.39

$$i)\ \sum_{n=1}^{\infty} \left(\frac{1}{n}+in\right)(z+1+i)^n \qquad (z-\underbrace{(-1-i)}_{z_0})^n$$

$$\lim_{n\to\infty} \left| \frac{\frac{1}{n}+in}{\frac{1}{n+1}+i(n+1)} \right| = \lim_{n\to\infty} \left| \frac{\frac{1+in^2}{n}}{\frac{1+i(n+1)^2}{n+1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{1+in^2}{n} \cdot \frac{n+1}{1+i(n+1)^2} \right| =$$

$$= \lim_{n\to\infty} \left| \frac{n+in^3+1+in^2}{n+i(n+1)^2 n} \right| = \lim_{n\to\infty} \left| \frac{in^3(\ldots}{in^3(\ldots} \right|$$

$$= 1$$

$$\{z \in \mathbb{C} \mid |z - z_0| < 1\}$$

$$|z - (1+i)| < 1$$

$$\{z \in \mathbb{C} \mid |z - (-1-i)| < 0\}$$

ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(z-i)^n}{2^n}$

$$R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{\frac{1}{2^n}}} = \lim_{n\to\infty} \frac{1}{\sqrt[\infty]{2^{-\infty}}} = \lim \frac{1}{2^{-1}} = 2$$

$$\{z \in \mathbb{C} \mid |z - i| < 2\}$$

1.41

i) $f(z) = e^z \sin z \quad , z_0 = 0$

$$f(z) = e^z \cdot \frac{e^{iz} - e^{-iz}}{2i} = \frac{1}{2i}\left[e^{z(1+i)} - e^{z(1-i)}\right]$$

$$f(z) = \frac{1}{2i}\left(\sum \frac{z^n(1+i)^n}{n!} - \sum \frac{z^n(1-i)^n}{n!}\right)$$

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$\sin z = \frac{z}{1!} - \frac{z^3}{3!} + \frac{z^5}{5!} - \dots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

ii) $f(z) = \dfrac{z-1}{z-2}$, $z_0 = 0$

$$f(z) = \frac{z - 2 + 1}{z-2} = 1 + \frac{1}{z-2} =$$

$$\frac{1}{1-z} = 1 + z + z^2 + \dots + z^n = \sum z^n \ , \ |z| < 1$$

$$\frac{1}{1+z} = 1 - z + z^2 - \dots = \sum (-1)^n z^n \ , \ |z| < 1$$

$$= 1 + \frac{1}{2\left(\frac{z}{2} - 1\right)} = 1 + \frac{1}{2} \cdot \frac{1}{\frac{z}{2} - 1} =$$

$$= 1 - \frac{1}{2} \cdot \frac{1}{1-\frac{z}{2}}$$

$$= 1 - \frac{1}{2}\left(1 + \frac{z}{2} + \frac{z^2}{2^2} + \dots + \frac{z^n}{2^n}\right)$$

$$= 1 - \frac{1}{2} \sum_{n=0}^{\infty} \frac{z^n}{2^n} = 1 - \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$$

$$R = \frac{1}{\sqrt[n]{\frac{1}{2^{n+1}}}} = \lim \frac{1}{\sqrt[n]{2^{-(n+1)}}} = \frac{1}{2^{-1}} = 2$$

$$\left|\frac{z}{2}\right| < 2$$

ii) $f(z) = \dfrac{z-1}{z-2}$, $z_0 = i$

$$f(z) = \frac{z-2+1}{z-2} = 1 + \frac{1}{z-2} = 1 + \frac{1}{i-2+z-i}$$

$$= \overset{i-2}{\cancel{1}} + \frac{1}{i-2} \cdot \frac{1}{1+\frac{z-i}{i-2}} = \frac{i-1}{i-2} + \sum_{n=1}^{\infty} (-1)^n \frac{(z-i)^n}{(i-2)^{n+1}}$$

$$R=\frac{(-1)^n}{(i-2)^{n+1}}$$

$n+2-n-1$

$$R=\lim_{n\to\infty}\left|\frac{\frac{(-1)^n}{(i-2)^{n+1}}}{\frac{(-1)^{n+1}}{(i-2)^{n+2}}}\right| = \lim_{n\to\infty}\left|\cancel{\frac{(-1)^n}{(i-2)^{n+1}}}\cdot\frac{(i-2)^{n+2}}{(-1)^{n+1}}\right|$$

$$=\lim_{n\to\infty}\left|\frac{(i-2)}{(-1)}\right| = |i-2| = \sqrt{5}$$

1.44

i) $f(z)=\cos^2 z \quad , z_0=0$

$$f(z)=\cos^2 z = \frac{1+\cos 2z}{2} = \frac{1}{2}+\frac{\cos 2z}{2}$$

$$=\frac{1}{2}+\frac{1}{2}\cdot\left(1-\frac{(2z)^2}{2!}+\frac{(2z)^4}{4!}-\dots\right)$$

$$=\frac{1}{2}+\frac{1}{2}\sum_{n=0}^{\infty}(-1)^n\frac{(2z)^2}{(2n)!} = \frac{1}{2}+\sum_{n=0}^{\infty}(-1)^n\frac{(2z)^{2n}}{2(2n)!}$$

$\circledast$

$$\sum_{n=0}^{\infty} \frac{(-z)^n}{n!} = 1 - \frac{z}{1!} + \frac{z^2}{2!}$$

ii) $f(z) = \cosh z \cos z \quad , \quad z_0 = 0$

$$f(z) = \cosh z \cos z = \frac{e^z + e^{-z}}{2} \cos z =$$

$$= \frac{\left(1 + \cancel{\frac{z}{1!}} + \frac{z^2}{2!} \cancel{+} \dots\right) + \left(1 - \cancel{\frac{z}{1!}} + \frac{z^2}{2!} \cancel{-} \dots\right)}{2} \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)$$

$$= \frac{2 + \frac{2z^2}{2!} + \frac{2z^4}{4!} + \dots}{2} \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)$$

$$= \frac{\cancel{2}\left(1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots\right)}{\cancel{2}} \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)$$

$$= \left(1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots\right)\left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)$$

$$\sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!} \cdot \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$e^{-z} = \sum_{n=0}^{\infty} \frac{(-z)^n}{n!}$$

$$f(z) = \cosh z \cos z = \frac{e^{z}+e^{-z}}{2}\cos z$$

$$= \frac{1}{2}\left(1 + \cancel{\frac{z}{1!}} + \frac{z^2}{2!} + \cancel{\frac{z^3}{3!}} + \cancel{\dots} + 1 - \cancel{\frac{z}{1!}} + \frac{z^2}{2!} - \cancel{\frac{z^3}{3!}}\cancel{\dots}\right)\left(1 - \frac{z^2}{2!} + \frac{z^4}{4!}\dots\right)$$

$$= \frac{1}{2}\left(2 + 2\cdot\frac{z^2}{2!} + \dots\right)\left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} + \dots\right)$$

$$= \underbrace{\left(1 + \frac{z^2}{2!} + \dots\right)}_{S}\left(1 - \frac{z^2}{2!} + \frac{z^4}{4!}\dots\right)$$

$$\sum (-1)^m \frac{z^{2m}}{(2m)!}$$

$$\sum \frac{z^{2n}}{(2n)!}$$

b) $f(z) = \dfrac{1}{3z+5}$ $\quad z_0 = -1$

$$f(z) = \frac{1}{3(z+1)-3+5} = \frac{1}{2+3(z+1)} =$$

$3z+3-3+5$

$$= \frac{1}{2\left(1 + \frac{3}{2}(z+1)\right)} = \frac{1}{2}\cdot\frac{1}{1+\frac{3}{2}(z+1)} = \frac{1}{2}\sum_{m=0}^{\infty}(-1)^m\left(\frac{3}{2}(z+1)\right)^m$$

$$= \frac{1}{2}\sum_{m=0}^{\infty}(-1)^m\cdot\left(\frac{3}{2}\right)^m(z+1)^m$$

iv) $f(z) = \frac{1}{z^2+i} = \frac{1}{i\left(1+\frac{z^2}{i}\right)} = \frac{1}{i} \cdot \frac{1}{1+\frac{z^2}{i}}$

$= \frac{1}{i} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z^2}{i}\right)^n = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{i^{2n+1}}$

vi) $f(z) = \frac{1}{z^2-2z+2} = \frac{1}{(z-1)^2+1} =$

$= \frac{1}{(z-1+i)(z-1-i)} = \frac{z-1+i-(z-1-i)}{(z-1+i)(z-1-i)} \cdot \frac{1}{2i}$

$= \left[\frac{1}{z-1-i} - \frac{1}{z-1+i}\right] \cdot \frac{1}{2i}$

$\left[\frac{1}{z-(1+i)} - \frac{1}{z-(1-i)}\right] \frac{1}{2i} =$

$\frac{1}{-(1+i)} \cdot \frac{1}{1-\frac{z}{(1+i)}} = \frac{1}{-(1+i)} \cdot \sum_{n=0}^{\infty} \left(\frac{z}{(i+1)}\right)^n$

$\frac{1}{z-(1-i)} = \frac{1}{-(1-i)} \cdot \frac{1}{1-\frac{z}{(1-i)}} = \frac{1}{-(1-i)} \sum_{n=0}^{\infty} \left(\frac{z}{1-i}\right)^n$

$\Rightarrow \frac{1}{2i}\left[-\sum_{n=0}^{\infty} \frac{z^n}{(i+1)^{n+1}} + \sum_{n=0}^{\infty} \frac{z^n}{(1-i)^{n+1}}\right]$

$$\Rightarrow \frac{1}{2i}\sum_{n=0}^{\infty} z^n\left(\frac{1}{(1-i)^{n+1}}-\frac{1}{(1+i)^{n+1}}\right)$$

$\underline{\text{vi)}}$ $f(z)=\frac{1}{z^2-2z+2}$

$$z^2-2z+2=0$$

$$\Delta=4-8$$
$$=-4$$

$$z_{1,2}\left\langle\begin{array}{l}\frac{2+i2}{2}=\frac{2(1+i)}{2}=1+i\\[4pt] \frac{2-i2}{2}=1-i\end{array}\right.$$

$$f(z)=\frac{1}{(z-(1+i))(z-(1-i))}$$

$$\cancel{z}-(1+i)-\cancel{z}+(1-i)$$
$$-\cancel{1}-i+\cancel{1}-i$$

$$f(z)=\frac{(z-(1+i))-(z-(1-i))}{(z-(1+i))(z-(1-i))}\cdot\frac{1}{-2i}$$

$$f(z)=\left[\frac{1}{(z-(1-i))}-\frac{1}{(z-(1+i))}\right]\cdot\frac{1}{2i}$$

$$f(z)=\frac{1}{2i}\left[\frac{1}{z-(1+i)}-\frac{1}{z-(1-i)}\right]$$

$$f(z) = \frac{1}{2i}\left[\frac{1}{(1+i)\left(\frac{z}{1+i}-1\right)} - \frac{1}{(1-i)\left(\frac{z}{1-i}-1\right)}\right]$$

$$= \frac{1}{2i}\left[-\frac{1}{(1+i)}\cdot\frac{1}{\left(1-\frac{z}{1+i}\right)} + \frac{1}{(1-i)}\cdot\frac{1}{\left(1-\frac{z}{1-i}\right)}\right]$$

$$= \frac{1}{2i}\left[-\frac{1}{(1+i)}\cdot\sum_{n=0}^{\infty}\frac{z^n}{(1+i)^n} + \frac{1}{(1-i)}\cdot\sum_{n=0}^{\infty}\frac{z^n}{(1-i)^n}\right]$$

$$= \frac{1}{2i}\left[\frac{1}{(1-i)}\sum_{n=0}^{\infty}\frac{z^n}{(1-i)^n} - \frac{1}{(1+i)}\sum_{n=0}^{\infty}\frac{z^n}{(1+i)^n}\right]$$

$$= \frac{1}{2i}\left[\left(\sum_{n=0}^{\infty}\frac{1}{(1-i)^{n+1}} - \sum_{n=0}^{\infty}\frac{1}{(1+i)^{n+1}}\right)z^n\right]$$

V) $f(z) = \dfrac{1}{3z+5}$, $z_0 = -1$

$$= \frac{1}{3z+1-1+5} = \frac{1}{3z-3+3+5} = \frac{1}{3(z+1)-3+5} =$$

$$= \frac{1}{2+3(z-1)} = \frac{1}{2\left(1+\frac{3}{2}(z-1)\right)} = \frac{1}{2}\cdot\frac{1}{1+\frac{3}{2}(z-1)}$$

$$= \frac{1}{2} \cdot \sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{2}\right)^n (z-1)^n =$$

$$R = \lim \frac{1}{\sqrt[n]{\left|(-1)^n \left(\frac{3}{2}\right)^n\right|}} = \frac{1}{\left|-\frac{3}{2}\right|} \Rightarrow R = \frac{2}{3}$$

IV) $\frac{1}{z^2+i}$, $z_0 = 0$

$$f(z) = \frac{1}{z^2+i} = \frac{1}{i\left(\frac{z^2}{i}+1\right)} = \frac{1}{i} \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{i^n}$$

$$R = \lim \frac{1}{\sqrt[n]{\left|\frac{(-1)^n}{i^n}\right|}} = \frac{1}{\frac{1}{i}} = i \Rightarrow r = 1$$

SEM 6

1.45

i) $f(z) = z^3 e^{\frac{1}{z}}$, $z_0 = 0$, $0 < |z| < +\infty$

$$f(z) = z^3 e^{\frac{1}{z}} = z^3 \left(1 + \frac{1}{1! z} + \frac{1}{2! z^2} + \frac{1}{3! z^3} + \frac{1}{4! z^4} \dots\right)$$

$$= z^3 + z^2 + \frac{1}{2}z + \frac{1}{6} + \frac{1}{4!z} + \frac{1}{5!z^2} + \dots$$

$$= z^3 + z^2 + \frac{1}{2}z + \frac{1}{6} + \sum_{n=1}^{\infty} \frac{1}{(n+3)!} \cdot \frac{1}{z^n}$$

0 is essential singularity point

ii) $f(z) = \frac{2\sin^2 z}{z^5}$, $z_0 = 0$, $0 < |z| < +\infty$

$$f(z) = \frac{2}{z^5}\sin^2 z = \frac{1}{z^5}(1 - \cos 2z) =$$

$$= \frac{}{z^5}\left(1 - \left(1 - \frac{(2z)^2}{2!} + \frac{(2z)^4}{4!} - \dots\right)\right)$$

$$= \frac{1}{z^5}\left(\frac{(2z)^2}{2!} - \frac{(2z)^4}{4!} + \dots\right)$$

$$= \frac{2}{z^3} - \frac{2^4}{4!z} + \frac{2^6 z}{6!} - \dots$$

z_0 pole of order 3

iii) $f(z) = ze^{\frac{1}{z+i}}$, $z_0 = -i$, $0 < |z-i| < +\infty$

$$f(z) = ((z+i)-i)e^{\frac{1}{z+i}} = (z+i)e^{\frac{1}{z+i}} - ie^{\frac{1}{z+i}}$$

$$= (z+i)\left(1 + \frac{1}{1!(z+i)} + \frac{1}{2!(z+i)^2} + \dots\right) - i\left(1 + \frac{1}{1!(z+i)} + \frac{1}{2!(z+i)^2} + \dots\right)$$

$$= z+i - i + 1 - \frac{i}{1!(z+i)} + \frac{1}{2!(z+i)} - \frac{i}{2!(z+i)^2} +$$

$$= (z+i) + 1 - i + \frac{1}{(z+i)}\left(\frac{1}{2!} - \frac{i}{1!}\right) + \dots$$

z_0 esential sing point

4) $f(z) = \frac{1-e^{-z}}{z}$, $z_0 = 0$

$$= \frac{1}{z} - \frac{e^{-z}}{z} = \frac{1}{z} - \frac{1}{z}\left(1 - \frac{z}{1!} + \frac{z^2}{2!} - \frac{z^3}{3!} + \dots\right)$$

$$- \cancel{\frac{1}{z}} - \cancel{\frac{1}{z}} + 1 + \frac{z}{2!} - \frac{z^2}{3!} + \dots$$

$$= 1 + \frac{z}{2!} - \frac{z^2}{3!}$$

z_0 remarable

4) $f(z) = \dfrac{1-e^{-z}}{z} = \dfrac{1}{z}\left(1-e^{-z}\right)$

$$e^{z} = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

$$e^{-z} = 1 - \frac{z}{1!} + \frac{z^2}{2!} - \frac{z^3}{3!}$$

$$= \frac{1}{z}\left(1 - \left(1 - \frac{z}{1!} + \frac{z^2}{2!} - \frac{z^3}{3!} + \dots\right)\right)$$

$$= \frac{1}{z}\left(\cancel{1} - \cancel{1} + \frac{z}{1!} - \frac{z^2}{2!} + \frac{z^3}{3!} - \dots\right)$$

$$= 1 - \frac{z}{2!} + \frac{z^2}{3!} - \dots$$

$$= 1 -$$

1) $f(z) = \cos\frac{1}{z}$, $z_0 =$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots$$

$$\cos\frac{1}{z} = 1 + \frac{1}{z^2 2!} - \frac{1}{z^4 4!} + \dots$$

z_0 essential sing

2) $$\frac{\cos z - 1}{z^2} = \frac{1}{z^2}\left(-1 + \cos z\right)$$

$$= \frac{1}{z^2}\left(-1 + \left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)\right)$$

$$= \frac{1}{z^2}\left(-\frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)$$

$$= -\frac{1}{2!} + \frac{z^2}{4!} - \dots$$

z_0 - removable

3) $$f(z) = \frac{\cos z}{z^4} = \frac{1}{z^4}\left(1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \dots\right)$$

$$\frac{1}{z^4} - \frac{1}{2!\,z^2} + \frac{1}{4!} - \dots$$

z_0 pole of order 4

d) $f(z) = \frac{1}{(z-1)(z-4)}$, $z_0 = 1$ $\quad |z-1| < 3$

$$\frac{|z-1|}{3} < 1$$

$$= \frac{1}{z-1} \cdot \frac{1}{(z-1)-3} \qquad (z-z_0)$$

$z - z_0 \qquad (z-z_0)-3$

$$= \frac{1}{z-1} \cdot \frac{1}{3\left(\frac{z-1}{3}-1\right)} \quad \left(\text{or } \frac{1}{(z-1)\left(1-\frac{3}{z-1}\right)} \right)$$

$$= -\frac{1}{z-1} \cdot \frac{1}{3} \frac{1}{\left(1-\frac{z-1}{3}\right)} \qquad \frac{1}{1-z}$$

$$= -\frac{1}{z-1} \cdot \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{z-1}{3}\right)^n$$

$$= -\frac{1}{z-1} \cdot \frac{1}{3} \sum_{n=0}^{\infty} \frac{(z-1)^n}{3^n}$$

$$= -\frac{1}{z-1}\sum_{n=0}^{\infty}\frac{(z-1)^n}{3^{n+1}}$$

$$= -(z-1)^{-1}\sum\frac{(z-1)^n}{3^{n+1}}$$

$$= -\sum\frac{(z-1)^{n-1}}{3^{n+1}}$$

z_0 - pole of order 1

$$f(z) = \frac{dz}{z^2\sin z} = \frac{1}{z^2\left(\frac{z}{1!}-\frac{z^3}{3!}+\frac{z^5}{5!}-\dots\right)} = \frac{1}{z^3 -}$$

SEMINAR 7

Res $\Rightarrow$ z_0 pole of order 1 $\Rightarrow$ $\dfrac{g(z)}{h'(z)}$

Res $\Rightarrow$ z_0 pole of order $n \geq 1$ =

$$= \text{Res}(z_0) = \frac{1}{(n-1)!} \lim_{z \to z_0} \left((z-z_0)^n f(z_0) \right)^{(n-1)}$$

1.49

i) $f(z) = \dfrac{z^2}{(1+z)^3}$

$z_0 = -1$

pole of order 3

$$\text{Res}(z_0) = \frac{1}{2!} \lim_{z \to -1} \left(\cancel{(z+1)^3} \, z^2 \right)^2$$

$$= \frac{1}{2!} \left(z^2 \right)'' = \frac{1}{2} \left(2z \right)' = \frac{1}{2} \cdot 2 = 1$$

ii) $f(z) = \dfrac{1}{1+e^z}$

$1 + e^z = 0$

$e^z = -1 \Rightarrow z_0 = \text{Log}(-1) = \{ \ln 1 + i(\pi + 2k\pi) \}_{k \in \mathbb{Z}}$

$z_0 =$ pole of order 1

$h(z) = 1$

$g'(z) = (1+e^z)' = e^z$

$$\operatorname*{Res}_{z=i(\pi+2k\pi)} f(z) = \frac{1}{(1+e^z)'} = \frac{1}{e^z}\bigg|_{z=i(\pi+2k\pi)}$$

$$= \frac{1}{e^{i(\pi+2k\pi)}} = \frac{1}{\cos(\pi+2k\pi)+i\sin(\pi+2k\pi)} = \frac{1}{-1} = -1$$

iii) $f(z) = \dfrac{1}{\sin \pi z}$

z_0 pole of order 1

$\sin \pi z = 0 \Rightarrow \pi z = \pi k \Rightarrow z = k \quad k \in \mathbb{Z}$

$$\operatorname*{Res}_{z=k} f(z) = \frac{1}{(\sin \pi z)'} = \frac{1}{\pi \cos \pi z}\bigg|_{z=k} = \frac{1}{\pi \cos \pi k} =$$

iv) $f(z) = \dfrac{\cos z}{(z-1)^2}$

$z-1 = 0 \Rightarrow z_0 = -1$

z_0 pole of order 2

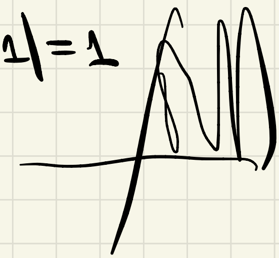
$$\operatorname*{Res}_{z=-1} f(z) = \frac{1}{1!} \lim_{z\to 1}\left(\left((z-1)^2 f(z_0)\right)\right)^{(n-1)}$$

$$= 1 \cdot \lim\left((z-1)^2 \cdot \frac{\cos z}{(z-1)^2}\right)' = \lim_{z\to 1}(\cos z)'$$

$$= -\lim_{z\to 1} \sin z = -\sin 1$$

1.50

i) $\displaystyle\int_C \frac{1}{z^4+1}\,dz \qquad C: \quad |z-1| = 1$



$z^4+1=0$

$z^4=-1 \Rightarrow -1 = \cos\pi + i\sin\pi$

$z_k = \cos\frac{\pi+2k\pi}{4} + i\sin\frac{\pi+2k\pi}{4}, \quad k\in\overline{0,3}$

$z_0 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$z_1 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4} = -\frac{\sqrt{2}}{2} + \frac{i\sqrt{2}}{2}$

$z_2 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2} - \frac{i\sqrt{2}}{2}$

$$z_3 = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4} =$$

pole of order 1

$$\operatorname*{Res}_{z=z_0} f(z) = \frac{1}{4z^3}\bigg|_{z=z_0} = \frac{1}{4\left(\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}\right)^3} = \frac{1}{4\frac{2\sqrt{2}}{2}(1-3i+3i^2-i^3)}$$

$$= \frac{1}{2\sqrt{2}(-1+i)}$$

$$\operatorname*{Res}_{z=z_3} f(z) = \dots = \frac{1}{2\sqrt{2}(-1-i)}$$

$$I = 2\pi i\left(\operatorname*{Res}_{z=z_0} f(z) + \operatorname*{Res}_{z=z_3} f(z)\right)$$

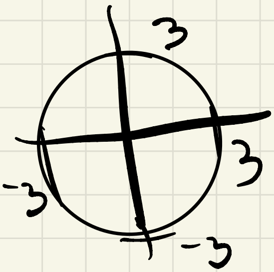
$$I = 2\pi i\left(\frac{1}{2\sqrt{2}(-1+i)} + \frac{1}{2\sqrt{2}(-1-i)}\right)$$

iii) $\int_C \frac{1}{z\cos z^2}\,dz$ $\quad C: \; x^2 + \frac{y^2}{4} = 1$

$$z\cos z^2 = 0$$

$z = 0$ or $z = \pm\sqrt{\frac{\pi}{2} + k\pi}$

b) $\displaystyle\int_C \frac{e^{\frac{\pi}{z-2i}}}{(z-1)(z-2i)}\,dz$, $C: |z|=3$



$(z-1)(z-2i)=0$

$z_1 = 1 \in \text{int}\,C$ pole of order I

$z_2 = 2i \in \text{int}\,C$ pole of order I

$$I = 2\pi i\left(\operatorname*{Res}_{z=z_1} f(z) + \operatorname*{Res}_{z=z_2} f(z)\right)$$

$$\operatorname*{Res}_{z=z_1} f(z) = \left.\frac{e^{\frac{\pi}{z-2i}}}{\left((z-1)(z-2i)\right)'}\right|_{z=1} \Rightarrow \frac{e^{\frac{\pi}{1-2i}}}{1-2i}$$

$$\left((z-1)(z-2i)\right)' = (z-1)'(z-2i) + (z-1)(z-2i)'$$

$$= (z-2i) + (z-1)\Big|_{z=1}$$

$$= 1-2i+1-1$$

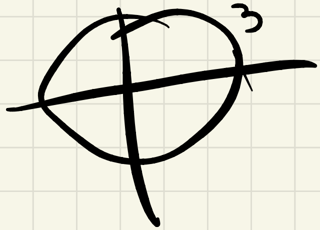
$$= 1-2i$$

$$\operatorname{Res}_{z=2i} f(z) = \frac{e^{\frac{\pi}{z-2i}}}{(z-1)(z-2i)}\Bigg|_{z=2i} = \frac{e^{\frac{\pi}{z-2i}}}{2i-1}$$

$$I = 2\pi i\left(\frac{1}{1-2i} + \frac{1}{2i-1}\right) e^{\frac{\pi}{z-2i}}$$

$$= 2\pi i\left(\cancel{\frac{1}{1-2i}} - \cancel{\frac{1}{1-2i}}\right) e^{\frac{\pi}{z-2i}} = 0$$

c) $$\int_C \frac{e^{\frac{\pi}{z-1}}}{z^2-3z+2}\,dz$$



$$z^2-3z+2=0$$

$$z_{1,2} = \begin{cases} 1 & \in C \\ 2 & \in C \end{cases}$$

$$I = 2\pi i\left(\operatorname{Res}_{z=1} f(z) + \operatorname{Res}_{z=2} f(z)\right)$$

$$\operatorname{Res}_{z=1} f(z) = \frac{e^{\frac{\pi}{z-1}}}{(z^2-3z+2)'}\Bigg|_{z=1} = \frac{-''-}{2z-3}\Bigg|_{z=1} = -e^{\frac{\pi}{z-1}}$$

$$\operatorname{Res}_{z=2} f(z) = e^{\frac{\pi}{z-1}}$$

$$I = 0$$

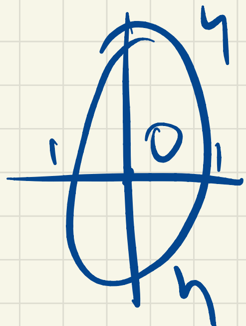
- $f(z) = \frac{1}{1-z} \sin \frac{1}{z}$

$1-z=0 \Rightarrow z=1$ pole of order 1

$$\operatorname{Res}_{z=1} f(z) = \frac{\sin \frac{1}{z}}{(1-z)'} = -\sin \frac{1}{z} = -\sin 1$$

SEM 8.

1) $\displaystyle\int_C \frac{1}{z\cos z^2}\, dz =$



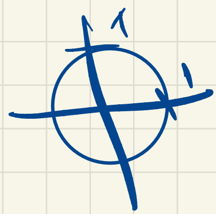
$z \cos z^2 = 0$

$z = 0$

$\cos z^2 = 0$

$z^2 = \frac{\pi}{2} + k\pi \Rightarrow z = \sqrt{\frac{\pi}{2} + k\pi}$

IV) $\displaystyle\int_C (z+1)\, e^{\frac{1}{z}}\, dz$ $\quad C: |z| = 1$



$(z+1)\left(1 + \frac{1}{1!z} + \frac{1}{2!z^2} + \frac{1}{3!z^3} + \cdots\right)$

$(z+1)\displaystyle\sum_{n=0}^{\infty} \frac{1}{n!\,z^n} =$

V) $\displaystyle\int_C \frac{z^2\, dz}{(z+3)(z-2)^2}$, $\quad C: |z-i| = 3$

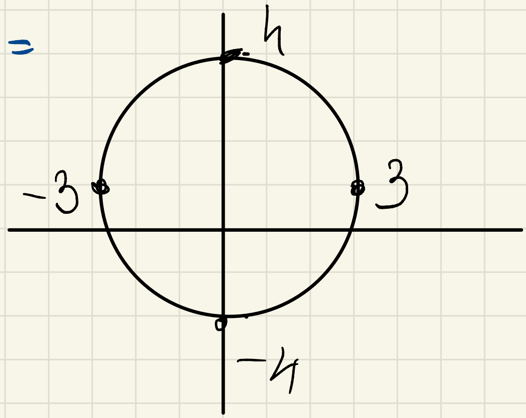
$z_1 = -3$ $\quad z_2 = 2$ — poles of order 2

poles of order 1

$$\operatorname*{Res}_{z=-3} f(x) = \frac{z^2}{\left((z+3)(z-2)^2\right)'} =$$

$$= \frac{z^2}{(z-2)^2 + (z+3)2(z-2)}$$

$$= \frac{z^2}{25} = \frac{9}{25}$$



$$\operatorname*{Res}_{z=2} f(z) = \frac{1}{1!} \lim_{z \to z_0} \left((z-z_0)^2 f(z) \right)'$$

$$= \lim_{z \to 2} \left(\cancel{(z-2)^2} \frac{z^2}{(z+3)\cancel{(z-2)^2}} \right)'$$

$$= \lim_{z \to 2} \left(\frac{z^2}{z+3} \right)' = \lim_{z \to 2} \frac{(z^2)'(z+3) - (z+3)' z^2}{(z+3)^2}$$

$$= \frac{2z(z+3) - z^2}{(z+3)^2} = \frac{4 \cdot 5 - 4}{25}$$

$$I = 2\pi i \Big(\underbrace{\operatorname*{Res}_{z} f(z)}_{\frac{9}{25} + \frac{16}{25} = 1} \Big) = \frac{32\pi i}{25}$$

6) $$\int_{|z|=3} z^2 \sin\frac{1}{z-i}\, dz$$

$z_0 = i$

$$z^2 \sin\left(\frac{1}{z-i}\right)$$

$$\sin\left(\frac{1}{z-i}\right) = \left(\frac{1}{z-1} - \frac{1}{3!(z-1)^3} + \dots\right)$$

$$\sin z = \left(\frac{z}{1!} - \frac{z^3}{3!} + \frac{z^5}{5!}\right.$$

$$-1 + 2i(z-i) + (z-i)^2$$

$$= \cancel{-1} + \cancel{2i}z - 2 + z^2 - \cancel{2iz} + \cancel{1}$$

$$z^2 = \cancel{-1} + \cancel{2iz} + \cancel{2} + z^2 - \cancel{1} - \cancel{2iz}$$

ENUNȚURI

→ determine where the points are monogenic

$$CR \begin{cases} \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x} \end{cases}$$

→ can... imaginary/real of function

$$\Delta u(x,y)=0 \text{ sau } \Delta v(x,y)=0 \Rightarrow \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

→ reconstruct...

→ evaluate the integral

⊛ using Cauchy's Integral formula

grad 1 $\quad \frac{f(z)}{z-z_0} = 2\pi i f(z_0)$

grad ≥2 $\quad \frac{f(z)}{(z-z_0)^n} = \frac{1}{(n-1)!} 2\pi i f(z_0)^n$

→ radius of the .. power series

$$R = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| \text{ sau } R = \lim_{n\to\infty} \frac{1}{\sqrt[n]{|c_n|}}$$

→ disk of convergence

$$K = \{ z \in \mathbb{C} \mid |z - z_0| < r \}$$

→ expand the function ... in Taylor series

!!! serii sub formă de sume

$$e^z = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{z^n}{n!}$$

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \dots = \sum_{n=0}^{\infty} z^n$$

$$\frac{1}{1+z} = 1 - z + z^2 - z^3 + \dots = \sum_{n=0}^{\infty} (-1)^n z^n$$

$$\cos z = 1 - \frac{z^2}{2!} + \frac{z^4}{4!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}$$

$$\sin z = \dots = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!}$$

TAYLOR → RAZA CONV

$f(z) =$ funcție $f(z) = \cos z$; $\frac{z^2}{2}$; ... ISR

AJUNGI la sumă $\sum_{n=0}^{\infty} c_n{}^n z^n$

LAURENT → singular point

removable → no principal part

essential → ∞ principal part

pole of order → puterea cea mai mare

a lui z

ex $\frac{1}{z^{5}} + \cancel{\frac{1}{z^5}}$

pole de ordin 5

$f(z) =$ ceva...

RESIDUE → LAURENT series

(1.49) găsește reziduul în punctul singular

grad 1 – $\operatorname{Res} f(z) = \frac{g(z)}{h'(z)} \Big/_{z=z_0}$ z_0 = singularity point

grad ≥ 1 $\operatorname{Res} f(z) = \frac{1}{(n-1)!} \lim_{z \to z_0} \left((z-z_0)^n f(z)\right)^{(n-1)}$

$f(z) =$ ceva

(1.50) EVALUATE THE INTEGRAL

$$I = 2\pi i \sum_{n=0}^{\infty} \operatorname{Res}_{z=z_k} f(z) \quad k \in \overline{0,n}$$

(1.55) EVALUATE THE INTEGRALS

Pentru $\int_0^{2\pi} dx$

$$d\theta = \frac{dz}{iz}$$

$$\cos z = \frac{z^2+1}{2z}$$

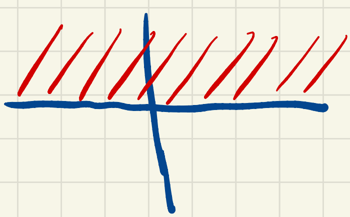
$$\sin z = \frac{z^2-1}{2iz}$$, $I = 2\pi i \sum_{n=0}^{\infty} \operatorname{Res}_{z=z_0} f(z_0)$

Calculezi Residue , $C: |z|=1$

1.57 EVALUATE THE INTEGRALS

Pentru $\int_{-\infty}^{\infty} dx$

calculezi Residue și le iei doar pe cele din



roșu

$$I = 2\pi i \sum_{n=0}^{\infty} \operatorname{Res}_{z=z_0} f(z_0)$$

1.58 EVALUATE THE INTEGRALS

Pentru $\int_{-\infty}^{\infty}$ Ceva $e^{i\lambda x}$,

$\lambda > 0 \Rightarrow$ Residue positiv

$\lambda < 0 \Rightarrow$ Residue negativ

$$d\theta = \frac{dz}{iz}$$

$$\cos\theta = \frac{z^2+1}{2z}$$

$$\sin\theta = \frac{z^2-1}{2iz}$$

PENTRU $\int_0^{2\pi}$

1.55

a) $$\int_0^{2\pi} \frac{dx}{1+3\cos^2 x} = \int_0^{2\pi} \frac{dz}{iz}\,\frac{1}{1+3\left(\frac{z^2+1}{2z}\right)^2}$$

$$= \int_{C:|z|=1} \frac{1}{1+\frac{3(z^4+2z^2+1)}{4z^2}}\,\frac{dz}{iz} = \int_{C:|z|=1} \frac{4z}{3z^4+10z^2+3}\,\frac{dz}{i}$$

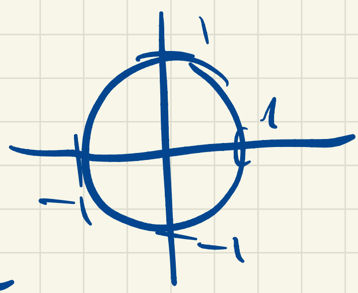
$$3z^4+10z^2+3=0$$

$$3t^2+10t+3=0$$

$$\Delta = 100-36$$
$$= 64$$

$$t_{1,2} = \frac{-10+8}{6} = -\frac{1}{3} \in \text{int}$$

$$\frac{-10-8}{6} = 3 \notin \text{int}$$



$$z^2 = -\frac{1}{3} \Rightarrow z = \pm\sqrt{-\frac{1}{3}} = \pm i \frac{1}{\sqrt{3}}$$

z_0 pole of order 1

$$\operatorname*{Res}_{z=-\frac{i}{\sqrt{3}}} f(z) = \frac{z}{(3z^4+10z^2+3)'}\Bigg|_{z=-\frac{i}{\sqrt{3}}} =$$

$$= \frac{z}{12z^3+20z}\Bigg|_{z=-\frac{i}{\sqrt{3}}} = \frac{-\frac{i}{\sqrt{3}}}{\frac{12i}{3\sqrt{3}} - \frac{{}^{3}\!/\,20i}{\sqrt{3}}}$$

$$= \frac{-\frac{i}{\sqrt{3}}}{-\frac{48i}{3\sqrt{3}}} = \cancel{\frac{i}{\sqrt{3}}} \cdot \frac{3\cancel{\sqrt{3}}}{48\cancel{i}} = \frac{3}{48} = \frac{1}{16}$$

$$\operatorname*{Res}_{z=\frac{i}{\sqrt{3}}} f(z) = \frac{z}{12z^3+20z}\Bigg|_{z=\frac{i}{\sqrt{3}}} = \frac{\frac{i}{\sqrt{3}}}{\frac{-12i}{3\sqrt{3}} + \frac{20i}{\sqrt{3}}}$$

$$= \frac{1}{\cancel{\sqrt{3}}} \cdot \frac{3\cancel{\sqrt{3}}}{48i} = \frac{1}{16}$$

$$I = \frac{4}{i} \cdot 2\pi i \left(\frac{1}{16} + \frac{1}{16} \right) = \pi$$

ii) $$\int_0^{2\pi} \frac{dx}{13 + 12\sin x} = \int_{|z|=1} \frac{dz}{iz} \frac{1}{13 + 12\left(\frac{z^2-1}{2iz}\right)}$$

$C: |z| = 1$

$$= \int_C \frac{1}{13 + \frac{12z^2 - 12}{2iz}} \frac{dz}{iz} =$$

$$\int_C \frac{2i\cancel{z}}{26iz + 12z^2 - 12} \cdot \frac{dz}{\cancel{iz}} = 2\int_C \frac{1}{26iz + 12z^2 - 12}$$

$$12z^2 + 26iz - 12 = 0 \quad | : 2$$

$$6z^2 + 13iz - 6 = 0$$

$$\Delta = -169 + 144$$

$$= -25$$

$$z_{1,2} \quad \frac{-13i + 5i}{12} = \frac{-8i}{12} = \frac{-2i}{3} \in \operatorname{int} C$$

$$\frac{-13i-5i}{12} = \frac{-\cancel{18}}{\cancel{12}} = \frac{-3i}{2} \notin \text{int } C$$

pole of order 1

$$\operatorname*{Res}_{z=-\frac{2i}{3}} f(z) = \left.\frac{1}{(6z^2+13iz-6)'}\right|_{z=-\frac{2i}{3}} =$$

$$= \left.\frac{1}{12z+13i}\right|_{z=-\frac{2i}{3}} = \frac{1}{\frac{-24i}{3}+\frac{{}^{3}13i}{}} =$$

$$= \frac{1}{\frac{15i}{3}} = \frac{\cancel{3}}{\cancel{15}i} = \frac{1}{5i}$$

$$I = 2\pi i \frac{1}{5i} = \frac{2\pi}{5}$$

V) $$\int_0^{2\pi} e^{2\cos x}\,dx = \int_C e^{\frac{\cancel{2}(z^2+1)}{\cancel{2}z}} \frac{dz}{iz}$$

$$= \int_C e^{\frac{z^2+1}{z}} \frac{dz}{iz} = \frac{1}{i}\int e^{\frac{z^2}{z}+\frac{1}{z}} \frac{dz}{z}$$

$$\frac{1}{i}\int_C \frac{1}{z} e^{z+\frac{1}{z}}\, dz = \frac{1}{i} \operatorname*{Res}_{z=0} f(z)$$

$$f(z) = \frac{1}{z} e^{z+\frac{1}{z}} = \frac{1}{z}\left(e^{z} \cdot e^{\frac{1}{z}}\right)$$

$$= \frac{1}{z}\left(1 + \frac{z}{1!} + \frac{z^2}{2!} + \dots\right)\left(1 + \frac{1}{z1!} + \frac{1}{2!z^2} + \dots\right)$$

$$= 2\pi \sum_{n=0}^{\infty} \frac{1}{(n!)^2}$$

PENTRU $\int_{-\infty}^{+\infty}$

1.57 $$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)(x^2+9)}\, dx$$

$$f(z) = \frac{z^2}{(z^2+1)(z^2+9)}$$

$$(z^2+1)(z^2+9)=0$$

$$z^2+1=0 \Rightarrow z=\pm i \Rightarrow z_0=i$$

$$z^2+9=0 \Rightarrow z=\pm 3i \Rightarrow z_0=3i$$

$$\operatorname*{Res}_{z=z_0} f(z) = \operatorname*{Res}_{z=i} \frac{\frac{z^2}{z^2+9}}{(z^2+1)}\Bigg|_{z=i}$$

$$= \operatorname*{Res}_{z=i} \frac{-\frac{1}{8}}{2i} = -\frac{1}{16i}$$

$$\operatorname*{Res}_{z=z_0} f(z) = \operatorname*{Res}_{z=z_0} \frac{\frac{z^2}{z^2+1}}{2z}\Bigg|_{z=3i} = \operatorname*{Res}_{z=3i} \frac{\frac{-9}{-8}}{6i}$$

$$= \frac{9}{48i} = \frac{3}{16i}$$

$$I = 2\pi i\left(-\frac{1}{16i}+\frac{3}{16i}\right) = \cancel{2}\pi\cancel{i}\cdot\frac{\cancel{2}}{\cancel{16}\cancel{i}} = \frac{\pi}{4}$$

b) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{x^4+1}\,dx$

$$f(z) = \frac{z^2}{z^4+1}$$

$$z^4 + 1 = 0$$

$$z^4 = -1$$

$z_0 = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ pole of order 1

$$\operatorname*{Res}_{z=z_0} f(z) = \operatorname*{Res}_{z=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}} \left.\frac{z^2}{(z^4+1)'}\right|_{z=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}}$$

$$= \operatorname{Res} \left.\frac{z^2}{4z^3}\right|_{z=\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}}$$

$$= \operatorname{Res} \frac{\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^2}{4\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^3} = \frac{\cancel{\frac{\sqrt{2}}{2}}\cancel{(1+i)^2}}{4\cancel{\frac{\sqrt{2}}{2}}(1+i)^{\cancel{3}}} = \frac{1}{2\sqrt{2}(1+i)}$$

$$I = \cancel{2}\pi i \cdot \frac{1}{\cancel{4}(1+i)} = \frac{\pi i}{1+i} = \frac{\pi \cancel{i}}{\cancel{i}\left(\frac{1}{i}+1\right)} = \frac{\pi}{i^{-1}+1}$$

$z_0 = -\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$ pole order 1

$$\operatorname*{Res}_{z=z_0} f(t) = \operatorname{Res} \frac{\cancel{\left(-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)^2}}{4\left(-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)^{\cancel{3}}}\Bigg|_{z=}$$

$$= \operatorname{Res} \frac{1}{4\left(-\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)} = \frac{1}{-\cancel{4}\cdot\frac{\sqrt{2}}{\cancel{2}}(1-i)} =$$

$$= \frac{1}{-2\sqrt{2}(1-i)}$$

$$I = 2\pi i \left(\frac{\overset{(1-i)}{1}}{2\sqrt{2}(1+i)} - \frac{\overset{(1+i)}{1}}{2\sqrt{2}(1-i)} \right)$$

$$= 2\pi i \left(\frac{(1-i)-(1+i)}{2\sqrt{2}(1+i)(1-i)} \right)$$

$$= 2\pi i \left(\frac{\cancel{1}-i-\cancel{1}-i}{2\sqrt{2}(1-\cancel{i}+\cancel{i}+1)} \right)$$

$$= 2\pi i \left(\frac{-2i}{4\cdot\sqrt{2}} \right) = \frac{4\pi}{\cancel{4}\sqrt{2}} = \frac{\pi}{\sqrt{2}}$$

c) $\int_{-\infty}^{\infty} \frac{1}{x^6+1}\,dx$

$f(z) = \frac{1}{z^6+1}$

$z^6 = -1$

$z_k = \cos\frac{\pi+2k\pi}{6} + i\sin\frac{\pi+2k\pi}{6}$

poli buni

$z_0 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} \iff \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$z_1 = \cos\frac{3\pi}{6} + i\sin\frac{3\pi}{6} \iff \frac{\sqrt{3}}{2} + \frac{1}{2}i$

$z_2 = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} \iff -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

3 poles of order 1 $\iff$ 3 Residues

$I = 2\pi i\left(\text{Res}1 + \text{Res}2 + \text{Res}3\right) = \frac{2\pi}{3}$

o) $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)^3}\,dx$

$$f(z) = \frac{1}{(z^2+1)^3} = \frac{1}{[(z-i)(z+i)]^3}$$

2 poles of order 3 $\Rightarrow$ $z_0 = i$

$z_0 = -i$ Mai bun

$$\operatorname{Res}_{z=i} f(z) = \frac{1}{2!}\lim_{z\to i}\left[(z-i)^2 f(z_0)\right]^{(2)}$$

$$= \frac{1}{2}\lim\left[\cancel{(z-i)^3}\cdot\frac{1}{(z+i)^3\cancel{(z-i)^3}}\right]^{(2)}$$

$$= \frac{1}{2}\lim_{z\to i}\left(\frac{1}{(z+i)^3}\right)'' = \frac{1}{2}\lim_{z\to i}\frac{1'(z+i)^3-((z+i)^3)'}{(z+i)^5}$$

$$= \frac{1}{2}\lim\left(\frac{-3(z+i)^2}{(z+i)^5}\right)' = \frac{1}{2}\lim\frac{(-3(z+i)^2)'(z+i)^5-(-3(z+i)^2)(z+i)\ldots}{(z+i)^7}$$

$$= \frac{1}{2}\lim\frac{-6(z+i)(z+i)^5-(-3(z+i)^2)5(z+i)^4}{(z+i)^7}$$

$$=\frac{1}{2}\lim \frac{-6(z+i)^6+3(z+i)^2\cdot 5(z+i)^4}{(z+i)^7}$$

$$=\frac{1}{2}\lim \frac{-6(z+i)^6+15(z+i)^6}{(z+i)^7}$$

$$=\frac{1}{2}\lim \frac{9\cancel{(z+i)^6}}{\cancel{(z+i)^7}}=\frac{1}{2}\lim \frac{9}{(z+i)}$$

$$=\frac{1}{2}\cdot\frac{9}{2i}=\frac{9}{4i}$$

$$I=2\pi i\cdot\frac{9}{4i}=\frac{\cancel{18}\pi}{\cancel{4}}=\frac{9\pi}{2}$$

$$=\int_{-\infty}^{\infty}\frac{1}{(z-i)^3(z+i)^3}\,dz$$

$$=2\pi i\cdot \operatorname{Res}\frac{1}{(z-i)^3(z+i)^3}$$

$$=2\pi i\cdot\frac{1}{2!}\cdot\left.\left(\frac{1}{(z+i)^3}\right)''\right|_{z=i}=\pi i\frac{12}{(2i)^5}=\pi i\frac{12}{32i}=\frac{12\pi}{32}=\frac{3\pi}{8}$$

$$\left(\frac{1}{(z+i)^3}\right)' = \frac{1'(z+i)^3-((z+i)^3)'}{(z+i)^6} = \frac{-3\cancel{(z+i)^2}}{(z+i)^6} = \left(\frac{-3}{(z+i)^4}\right)'$$

$$\left(\frac{-3}{(z+i)^4}\right)' = \frac{(-3)'(z+i)^4-(-3)((z+i)^4)'}{(z+i)^8} = \frac{-(-3)4(z+i)^3}{(z+i)^8}$$

$$= \frac{12\cancel{(z+i)^3}}{(z+i)^{\cancel{8}}} = \frac{12}{(z+i)^5} = \frac{12}{(2i)^5}$$

SEM 10
LA PLACE

23 Find the images of the following integrals

a) $f(t) = t^2 \cos t$

$$\mathcal{L}[t^2 \cdot \cos t](p) = (-1)^2 \left(\mathcal{L}[\cos t](p)\right)^{(2)}$$

$$= (-1)^2 \left(\frac{p}{p^2+1}\right)^{(2)}$$

$$= \left(\frac{p}{p^2+1}\right)' = \frac{p'(p^2+1) - p(p^2+1)'}{(p^2+1)^2}$$

$$= \frac{p^2+1-2p^2}{(p^2+1)^2} = \frac{1-p^2}{(p^2+1)^2} =$$

$$= \frac{(1-p^2)'(p^2+1)^2 - (1-p^2)\left((p^2+1)^2\right)'}{(p^2+1)^4}$$

$$= \frac{-2p(p^2+1)^{\cancel{2}} - 2\left(2p\,(\cancel{p^2+1})(1-p^2)\right)}{(p^2+1)^{\cancel{4}}}$$

$$= \frac{-2p(p^2+1) - 4p(1-p^2)}{(p^2+1)^3}$$

$$= \frac{-2p^3 - 2p - 4p + 4p^3}{(p^2+1)^3}$$

$$= \frac{2p^3 - 6p}{(p^2+1)^3}$$

ii) $f(t) = t(e^t + \cosh t)$

$$f(t) = te^t + t\cosh t$$

$$\mathcal{L}[te^t + t\cosh t](p)$$

$$\mathcal{L}[te^t](p) + \mathcal{L}[t\cosh t](p)$$

$$= (-1)\mathcal{L}[e^t]'(p) + (-1)\mathcal{L}[\cosh t]'(p)$$

$$= -\left(\frac{1}{p-1}\right)' - \left(\frac{p}{p^2-1}\right)'$$

$$= -\left(\frac{\overset{0}{1'(p-1)} - 1\cdot(p-1)'}{(p-1)^2}\right) - \left(\frac{p'(p^2-1) - p(p^2-1)'}{(p^2-1)^2}\right)$$

$$= + \frac{1}{(p-1)^2} - \left(\frac{p^2-1-2p^2}{(p^2-1)^2}\right)$$

$$= + \frac{\overset{(p+1)^2}{1}}{(p-1)^2} + \frac{p^2+1}{(p-1)^2(p+1)^2}$$

$$= \frac{2p^2+2p+2}{(p^2-1)^2}$$

(ii) $f(t) = (t+1)\sin 2t$

$$\mathcal{L}[(t+1)\sin 2t](p)$$

$$\mathcal{L}[t\sin 2t + \sin 2t](p) = \mathcal{L}[t\sin 2t](p) + \mathcal{L}[\sin 2t]$$

$$(-1)\mathcal{L}[\sin 2t]'(p) + \frac{2}{p^2+4}$$

$$-\left(\frac{2}{p^2+4}\right)' + \frac{2}{p^2+4}$$

$$= -\left(\frac{2'(p^2+4) - 2(p^2+4)'}{(p^2+4)^2}\right) + \frac{2}{p^2+4}$$

$$= \frac{4p}{(p^2+4)^2} + \frac{\overset{p^2+4}{2}}{p^2+4}$$

$$= \frac{4p + 2p^2 + 8}{(p^2+4)^2} = \frac{2p^2 + 4p + 8}{(p^2+4)^2}$$

4) $f(t) = e^{2t}\sin t$

$$\mathcal{L}[e^{2t}\sin t](p)$$

$$\mathcal{L}[\sin t](p-2) =$$

$$= \frac{1}{p^2+1}\Big|_{p=p-2} = \frac{1}{(p-2)^2+1}$$

5) $f(t) = e^t \cos nt$

$$\mathcal{L}[e^t \cos nt]$$

$$\mathcal{L}[\cos nt](p-1)$$

$$\frac{p}{p^2+n^2}\Big|_{p=p-1} = \frac{p-1}{(p-1)^2+n^2}$$

6) $f(t) = e^t \sinh t$

$$\mathcal{L}[e^t \sinh t]$$

$$\mathcal{L}[\sinh t](p-1) = \frac{1}{p^2-1}\Big|_{p=p-1}$$

$$= \frac{1}{(p-1)^2-1}$$

7) $f(t) = e^{3t}\sin^2 t$

$$\mathcal{L}[e^{3t}\sin^2 t](p) = \mathcal{L}[\sin^2 t](p-3)$$

$\sin^2 t$

$$\cos 2t = \cos^2 t - \sin^2 t$$
$$= 1 - 2\sin^2 t \; / -1$$
$$\cos 2t - 1 = -2\sin^2 t \; / : (-2)$$
$$\frac{1-\cos 2t}{2} = \sin^2 t$$

$$\frac{1}{2}\mathcal{L}[1-\cos 2t](p-3) = \frac{1}{2}\mathcal{L}[1](p-3) - \frac{1}{2}\mathcal{L}(\cos 2t)(p-3)$$

$$\frac{1}{2}\cdot\frac{1}{p-3} - \frac{1}{2}\frac{p}{p^2+4}\Big|_{p=p-3}$$

$$\frac{1}{2p-6} - \frac{1}{2}\cdot\frac{p-3}{(p-3)^2+4}$$

$$\frac{1}{2}\left(\frac{1}{p-3} - \frac{p-3}{(p-3)^2+4}\right)$$

8) $f(t) = t^{-1} \sin^2 2t$

$$\mathcal{L}\left[\frac{\sin^2 2t}{t}\right](p)$$

$$= \int_p^\infty \mathcal{L}[\sin^2 2t](y)\, dy$$

$$= \int_p^\infty \mathcal{L}\left[\frac{1-\cos 4t}{2}\right](y)\, dy$$

$$= \frac{1}{2}\int_p^\infty \left(\mathcal{L}[1] - \mathcal{L}[\cos 4t]\right)(y)\, dy$$

$$= \frac{1}{2}\int_p^\infty \left(\frac{1}{y} - \frac{y}{-y^2+16}\right) dy$$

$$= \frac{1}{2}\left(\int_p^\infty y^{-1}\, dy + \cancel{\int_p^\infty \frac{y}{16-y^2}\, dy}\right)$$

$$= \frac{1}{2}\left(\ln y + \cancel{\int_p^\infty \frac{y}{(4-y)(4+y)}\, dy}\right)$$

$$= \frac{1}{2}\left(\ln y + \int_p^{\infty} \frac{4-4+y}{(4-y)(4+y)}\, dy\right)$$

$$= \frac{1}{2}\left(\ln y + \int_p^{\infty} \frac{4}{4-y}\, dy + \int_p^{\infty} \frac{1}{4+y}\, dy\right)$$

$$\frac{2}{2}\,\frac{2y}{y^2+16} \qquad \frac{1}{2}\left(\ln y - \frac{1}{2}\ln y^2+16\right)\Big/_p^{\infty}$$

$$(y^2+16)' = 2y \qquad \frac{2}{5}\ \frac{2}{2}\ \frac{1}{2}\ln p - \frac{1}{4}\ln p^2+16$$

$$\ln a - \ln b = \ln\frac{a}{b} \qquad \frac{1}{3}\left(\ln p^2 - \ln(p^2+16)\right)$$

$$= \frac{1}{3}\ln\frac{p^2}{p^2+16}$$

9) $f(t) = t - n \quad , \ n \le t \le n+1$

$$\mathcal{L}[t-n](p) = \mathcal{L}[t](p) - \mathcal{L}[n](p)$$

$$= \frac{1}{p^2} - \frac{p/n}{p} = \frac{1-pn}{p^2}$$

9, 10 ??

2.4

Find the originals $f(t)$ corresponding

$p-2 \quad p-3$

i) $F(p) = \dfrac{p}{p^2-5p+6} = \dfrac{p}{(p-3)(p-2)} = \dfrac{A}{p-3} + \dfrac{B}{p-2}$

$p^2-5p+6=0$

$p_{1,2}$ = 2, 3

$p = A(p-2)+B(p-3)$

$1p = p\underbrace{(A+B)}_{1} \underbrace{-2A-3B}_{0}$

$\begin{cases} A+B=1 \\ -2A-3B=0 \end{cases}$

$A = 1-B$

$-2(1-B)-3B=0$

$-2+2B-3B$

$-2-B=0$

$-B=2$

$\Rightarrow A=3$

$B=-2$

$\dfrac{3}{p-3} - \dfrac{2}{p-2}$

$\Rightarrow 3e^{3t}-2e^{2t}$

ii) $F(p) = \dfrac{1}{p^2-7p+25} = \dfrac{1}{\quad}$

$$\Delta = 81 - 100$$
$$= -19$$
$$\left(x - \frac{9}{2}\right)^2 + \frac{19}{4}$$

$$F(p) = \frac{1}{p^2 - 6p + 25} = \frac{1}{p^2 - 6p + 9 + 16}$$
$$= \frac{1}{4} \frac{4}{(p-3)^2 + 4^2} = \frac{1}{4} e^{3t} \sin 4t$$

3) $$F(p) = \frac{p}{(p^2+1)^2} =$$

4) $$F(p) = \frac{1}{(p^2+4)^2}$$
$$\mathcal{L}^{-1}\left[\frac{1}{p^2+4} \cdot \frac{1}{p^2+4}\right] \Leftrightarrow \mathcal{L}^{-1}\left[\frac{1}{p^2+4}\right]$$

$$\mathcal{L}^{-1}\left[\frac{1}{p^2+4}\right] = \left(\frac{1}{p^2+4}\right)' = \frac{-2p}{(p^2+4)^2}$$

$$-\frac{1}{2}\left(\frac{p}{(p^2+4)^2}\right) =$$

$$+\frac{1}{2}\left(\frac{2}{p^2+2^2}\right)'$$

$$\frac{1}{2}\sin 2t$$

SEM 11

$$\mathcal{L}[f(t)](p) = F(p)$$

$$\mathcal{L}[f'(t)](p) = pF(p) - f(0)$$

$$\mathcal{L}[f''(t)](p) = p^2 F(p) - pf(0) - f'(0)$$

1) $x''(t) - 5x'(t) + 6x(t) = 0 \quad \Big| \mathcal{L}$ $\quad x(0)=1$, $x'(0)=1$

$$\mathcal{L}[x''(t)](p) - 5\mathcal{L}[x'(t)](p) + 6\mathcal{L}[x(t)](p) = 0$$

$$\mathcal{L}[x(t)](p) = X(p)$$

$$p^2 X(p) - p\underbrace{X(0)}_{1} - \underbrace{X'(0)}_{-1} - 5\left[pX(p) - \underbrace{X(0)}_{1}\right] + 6X(p) = 0$$

$$p^2 X(p) - p + 1 - 5pX(p) + 5 + 6X(p) = 0$$

$$X(p)(p^2 - 5p + 6) = p - 1 - 5$$

$$X(p) = \frac{p-6}{p^2-5p+6} = \frac{A}{p-2} + \frac{B}{p-3}$$

$$\Rightarrow p - 6 = (p-3)A + (p-2)B$$

$$p - 6 = pA - 3A + pB - 2B$$

$$p - 6 = p(A+B) - 3A - 2B$$

$$\left.\begin{aligned} A + B &= 1 \\ -3B - 2B &= -6 \end{aligned}\right\} \Rightarrow \begin{cases} A + B = 1 \\ 3A + 2B = 6 \end{cases}$$

$$A = 1 - B$$

$$3(1-B) + 2B = 6$$

$$3 - 3B + 2B = 6$$

$$3 - B = 6$$