Electrotechnics
ET

Course 5

Year I-ISA English

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= Course 5 =

1. Inductances. Methods used to Inductance Calculation

2. Magnetic Circuit Law

1. Inductivities. Methods used to Inductance Calculation

1.1. Inductors. Self Inductance and Mutual Inductance

Inductor (Coil)

Inductor = a reactive electronic circuit component which has as an essential electric parameter **it's self inductance, *L*.**

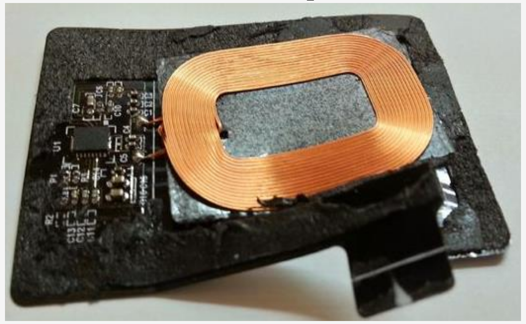
The most important property of an inductor is the fact that it can accumulate magnetic energy.

Images with different inductors (coils) :

Wireless supply system for a mobile phone

Emitter

Receptor



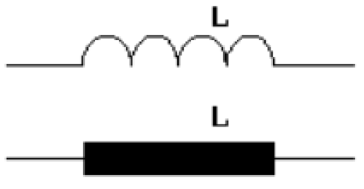
Inductivity and Inductance

Inductivity = represents the extent to which a coil accumulates magnetic energy for a certain value of the current in the circuit.

The inductance of an electric inductor (coil) is defined as a ratio between the total magnetic flux, Ψ, which crosses an area limited by the contour of a circuit and the current i which produces this flux (any other currents are considered null):

$$\boxed{L \overset{\text{def}}{=} \frac{\Psi}{i} \succ 0}$$

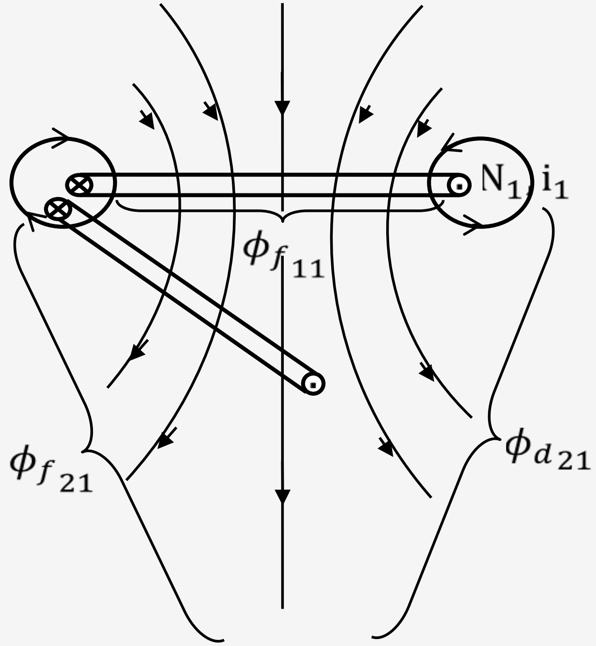
Obs.

- measurement unit: $\boxed{[L]_{SI} - [H]}$, Henry;
- symbol used in electric circuits: 

- if in the proximity of the circuits there are only linear mediums (μ=const.), the inductance depends only on the dimensions and shape of the inductor and on the magnetic permeability of the medium;
- in the case of the ferromagnetic materials (μ dependent on H), inductivity is not constant, but it depends on the current (thus on H).

Self and Mutual Inductances

Two circuits with N_1 and N_2 turns are considered, and it is assumed that only the first circuit is crossed by the current i_1. The fascicular flux produced by current 1 passing through a turn of circuit 1 is named Φ_{f11} and the fascicular flux produced by current 1 passing through a turn of circuit 2 is called Φ_{f21}.



❖ **Self inductance of circuit 1**

(with $i_1 \neq 0$ and $i_2 = 0$):

$$L_{11} = \frac{\Phi_{11}}{i_1} \succ 0$$

where:

- **total flux through circuit 1, Φ_{11} :** $\Phi_{11} = N_1 \cdot \Phi_{f11}$

$$\Rightarrow \quad L_{11} = N_1 \frac{\Phi_{f11}}{i_1} \succ 0$$

It is called self inductance L_{11} of the circuit the positive ratio between the total flux through circuit 1 Φ_{11}, produced by the current of that circuit (in the direction associated after the right-hand thumb rule to the current) and current i_1 producing it.

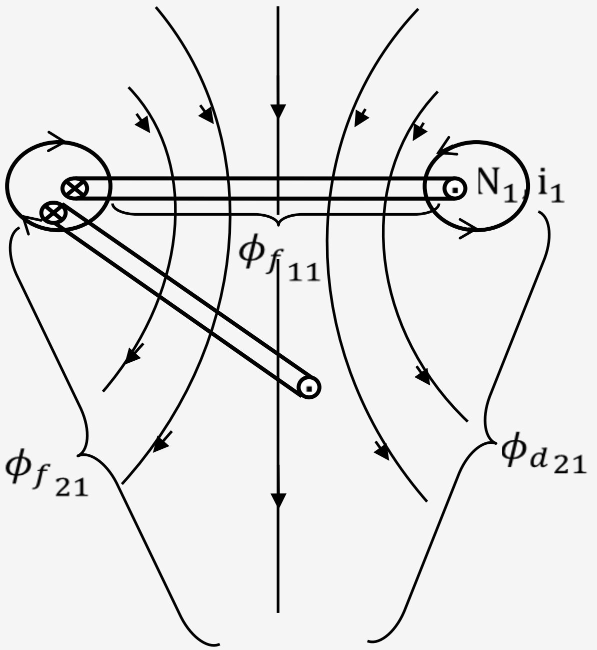
- **Self inductance of circuit 2** (with $i_1 = 0$ and $i_2 \neq 0$) is defined as :

$$\boxed{L_{22} = \frac{\Phi_{22}}{i_2} \succ 0}$$

where:

- **Total flux through circuit 2, Φ_{22} :** $\Phi_{22} = N_2 \cdot \Phi_{f22}$

$$\Rightarrow \quad L_{22} = N_2 \frac{\Phi_{f22}}{i_2} \succ 0$$



- **Mutual inductance L_{21}, between circuit 2 and 1:**

$$\boxed{L_{21} = \frac{\Phi_{21}}{i_1} \prec\succ 0}$$

Is called mutual inductance L_{21} , between circuits 1 and 2, the ratio between the total flux Φ_{21} produced by current 1 passing through circuit 2, and the current i_1 producing it.

$$\Phi_{21} = N_2 \cdot \Phi_{f21} \quad \Rightarrow \quad L_{21} = N_2 \frac{\Phi_{f21}}{i_1}$$

- **Mutual inductance L_{12}, between circuits 1 and 2 is defined as:**

$$\boxed{L_{12} = \frac{\Phi_{12}}{i_2} \prec\succ 0}$$

$$\Phi_{12} = N_1 \cdot \Phi_{f12} \quad \Rightarrow \quad L_{12} = N_1 \frac{\Phi_{f12}}{i_2}$$

- **The reciprocity relation:**
  - **Between the two mutual inductances L_{21} and L_{12} the reciprocity relation is followed, namely:**

$$\boxed{L_{12} = L_{21}}$$

❖ **<u>Magnetic coupling coefficient</u> *k* of two circuits (inductors):**

$$0 \le k = \sqrt{\frac{L_{21} L_{12}}{L_{11} L_{22}}} = \frac{|L_{12}|}{\sqrt{L_{11} L_{22}}} = \frac{M}{\sqrt{L_1 L_2}} \le 1$$

❖ **<u>Disspersion coefficient</u> σ:**

$$\sigma = 1 - k^2 = \frac{L_{11} L_{22} - L_{21} L_{12}}{L_{11} L_{22}} = \frac{L_1 L_2 - M^2}{L_1 L_2}$$

Obs.

- **the possible notations for the self inductance are:** ***L_{11}, L_{22}, L_{kk}, L_1, L_2, L_k;***
- **the possible notations for the mutual inductance are:** ***L_{12}, L_{21}, L_{kj}, L_{jk}, M;***

1.2. Methods used to Inductance Calculation

I. Direct method

Calculation steps:

1) **It is supposed that there is a circuit (inductor) passed by the current *i* ;**

2) **The magnetic induction, *B*, is determined in different points:**

first the magnetic field intensity is determined, mostly by using *Ampere's Theorem*:

$$\oint_{\Gamma} \bar{H} \cdot d\bar{s} = \theta$$

where the current linkage (solenația), θ: $\theta = N \cdot i$

where N= inductor's number of turns

then from the relation $\bar{B} = \mu \bar{H}$**, *B* is calculated;**

3) **We determine the fascicular magnetic flux:** $\Phi = \int_{A} \bar{B} \cdot d\bar{A}$

and then the total magnetic flux: $\Psi = N \cdot \Phi$

4) **The inductance is determined with the relation:** $L = \dfrac{\Psi}{i}$

II. Inductance calculation with the help of the reluctance

❖ The ratio between the magnetic voltage U_m and the fascicular magnetic flux Φ_f is called **reluctance** or **magnetic resistance**:

$$R_m = \frac{U_m}{\Phi_f}$$

❖ **The magnetic reluctance (resistance)** can also be determined with the relation:

$$R_m = \int_1^2 \frac{ds}{\mu A}$$

where:

- **μ – the absolute permeability of the medium;**
- **A – the cross-section area of the conductor;**

$$\Rightarrow \quad R_m = \frac{l}{\mu A}$$

where:

- **l – the length of the conductor**

- **The self inductance** is equal with the ratio between the square of the number of turns of the inductor and the total (equivalent) reluctance of the magnetic circuit:

$$\boxed{L = \frac{N^2}{R_m}}$$

III. Inductance calculation with the help of the magnetic energy

- **The self inductance** of a circuit (inductor) passed by the current i is:

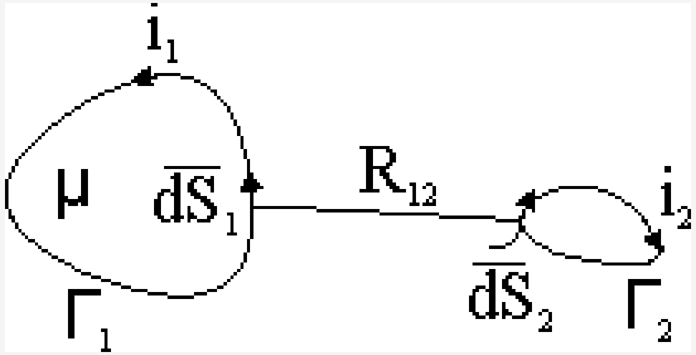
$$\boxed{L = \frac{2W_m}{i^2}}$$

where:

- W_m – magnetic energy.

IV. Neumann's Formula

- **using this formula, we can determine the mutual inductance between 2 circuits: 1 and 2 described by the contours Γ_1 and Γ_2.**
- **considering 2 filiform circuits with the contours Γ_1 and Γ_2 the mutual inductance between the 2 circuits will be given by the relation:**



$$\boxed{L_{12} = \frac{\Phi_{12}}{i_1} = \frac{\mu}{4\pi} \oint_{\Gamma_1} \oint_{\Gamma_2} \frac{d\bar{s}_1\, d\bar{s}_2}{R_{12}}}$$

where:

- Φ_{12} – the magnetic flux produced by the current i_1 which passes through circuit 2;
- μ – the absolut permeability of the medium
- R_{12} – the distance between the two contour elements $\overline{dS}_1$ and $\overline{dS}_2$

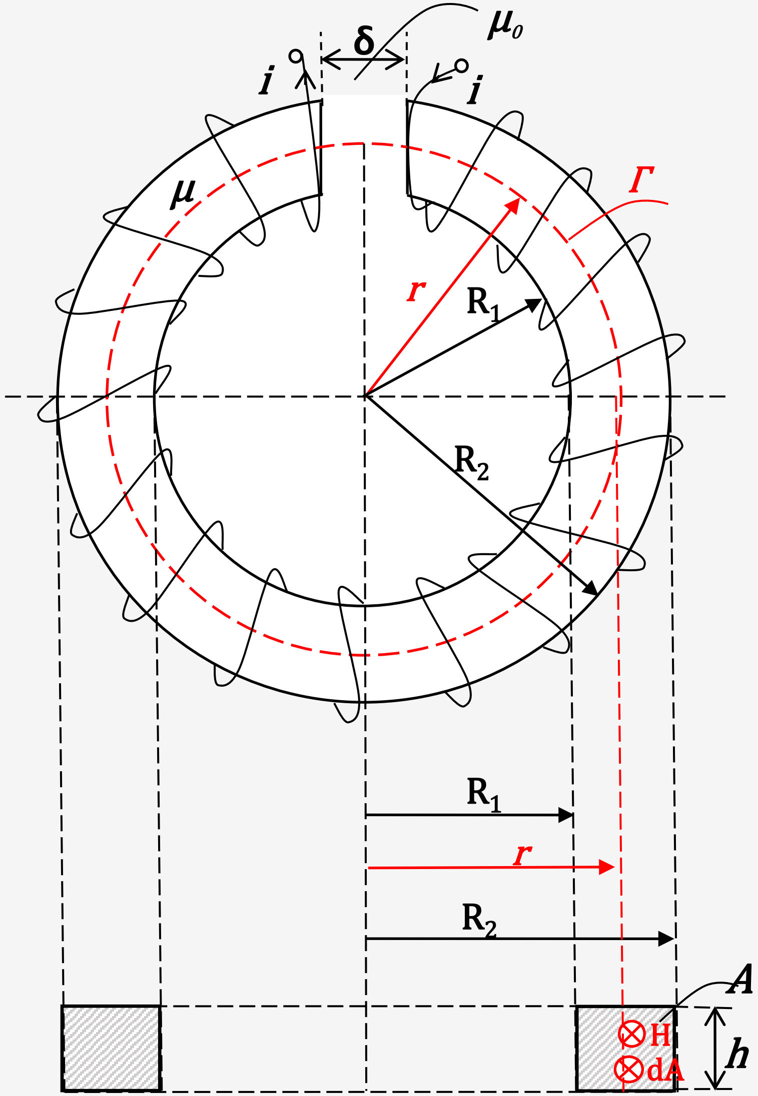
1.3. Applications

Direct Method

Problem 1

Calculate the self inductance of a toroidal inductor, with a square cross-section of area *A*, height *h* (where h = $R_2 - R_1$) having the air gap δ and permeability μ much higher than the vacuum permeability μ_0. The inductor has *N* turns and is passed by the current *i*.

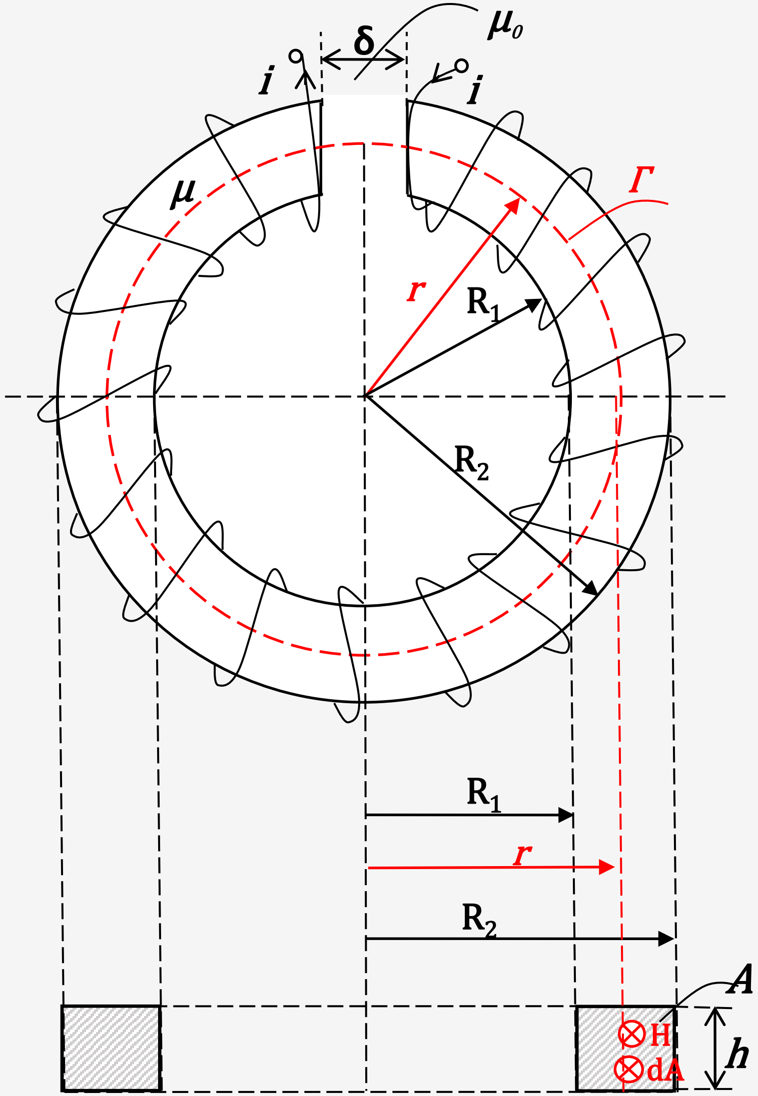
Solution:



Solution:

- in our problem it is said that we have a toroidal inductor with a square cross-section, thus it is like we have had a square and we rotate it after a circle, obtaining this toroid with square cross-section shape;
- in order to calculate the self inductance of the toroidal inductor with air gap, we apply the direct method in order to calculate;
- this method involves completing the following 4 steps:

  1) an inductor **is considered** and a **current *i*** is passing through it (in this problem's data it is said that through the inductor a current i is passing, but the solution for this problem is given for the general case);



2) the magnetic induction, *B*, is determined in different points in space:

- ✓ we know that the magnetic induction *B* is equal with:

$$\overline{\boldsymbol{B}} = \boldsymbol{\mu} \cdot \overline{\boldsymbol{H}} \tag{1}$$

- ✓ thus, from relation (1) we observe that in order to determine the magnetic induction first we have to determine the ***magnetic field intensity***;

- in order to determine the *magnetic field intensity,* we will use *Ampere's Theorem*:

$$\oint_{\boldsymbol{\Gamma}} \overline{\boldsymbol{H}} \cdot \overline{\boldsymbol{ds}} = \boldsymbol{\theta}$$

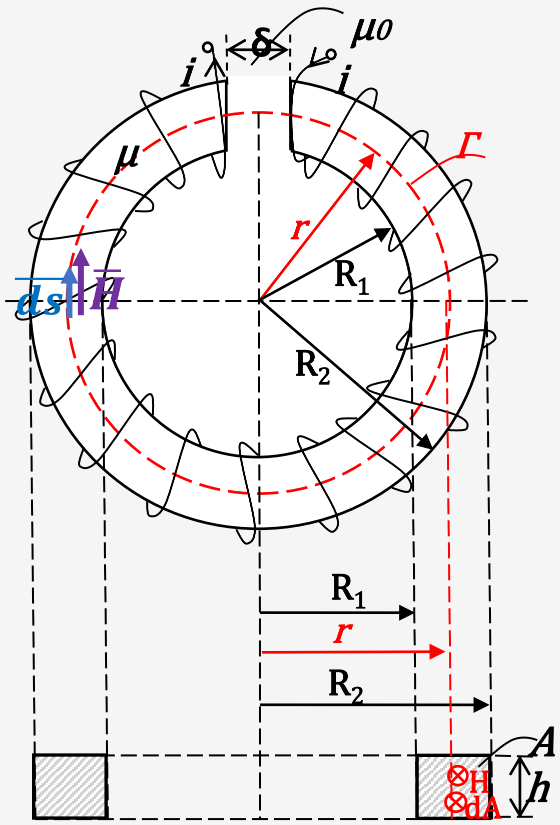
- where the current linkage, $\theta = N \cdot i$, it results:

$$\oint_{\Gamma} \overline{\boldsymbol{H}} \cdot \overline{\boldsymbol{ds}} = \boldsymbol{N} \cdot \boldsymbol{i} \tag{2}$$

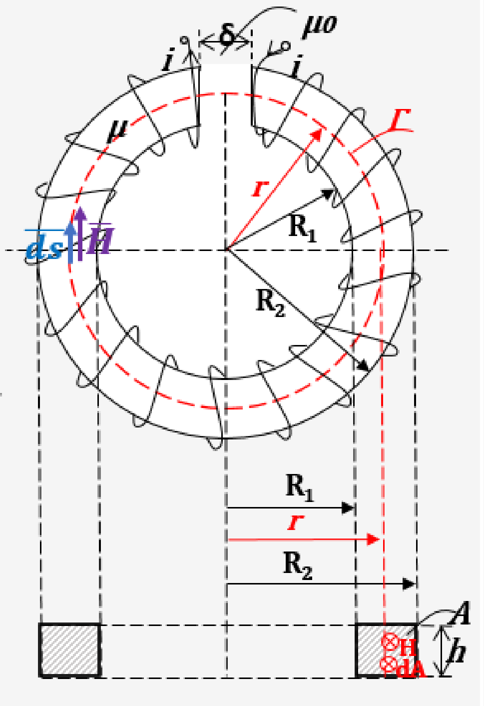
- we observe that under the integral we have the scalar product of two vectors, thus we present them in the *Figure* in order to determine the angle between them:
  - starting from the idea that the magnetic field lines form circles around the conductors passed by an electric current, we can observe in *Figure* a magnetic field line placed at a distance *r* from the center, as a red dashed line, (inside the tor we have an infinity of magnetic field lines, we have drawn only one in order not to overload the drawing);
  - we consider that the *Γ* contour which appears in *Ampere's Theorem* is the magnetic field line of radius *r*, thus the length element $\overline{ds}$ represents a piece from the length of the magnetic field line or of the *Γ contour*, thus resulting that:

$$\overline{H} \parallel \overline{ds}$$

$$=> \oint_{\Gamma} H \cdot ds = N \cdot i$$



- we observe that, the Γ curve passes through an area with permeability μ, when we are inside the tor, and through an area with permeability μ_0, when we are in the air gap, thus we decompose the contour Γ after the two areas and we have:



$$\Gamma = \Gamma_{Fe} + \Gamma_{\delta}$$

$$=> \oint_{\Gamma} H \cdot ds = \oint_{\Gamma} (H_{Fe} + H_{\delta}) \cdot ds = N \cdot i$$

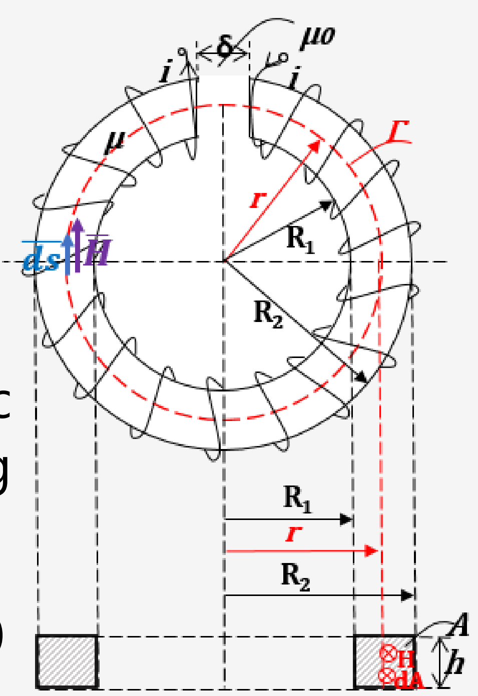
$$=> \oint_{\Gamma_{Fe}} H_{Fe} \cdot ds + \oint_{\Gamma_{\delta}} H_{\delta} \cdot ds = N \cdot i$$

- due to the symmetry of the problem inside the tor at a certain distance r from the center the magnetic field intensity is constant and equal with H_{Fe}, and in the air gap also constant and equal with H_{δ}:

$$=> H_{Fe} \oint_{\Gamma_{Fe}} ds + H_{\delta} \oint_{\Gamma_{\delta}} ds = N \cdot i => \boldsymbol{H_{Fe}}\,(\boldsymbol{2 \cdot \pi \cdot r - \delta}) + \boldsymbol{H_{\delta} \cdot \delta = N \cdot i} \qquad (3)$$

From where do we have $(2 \cdot \pi \cdot r - \delta)$? : 

- $H_{Fe} \oint_{\Gamma_{Fe}} ds = H_{Fe} \cdot the\ length\ of\ the\ curve\ \Gamma_{Fe}$, thus, if we had a complete tor, meaning if we didn’t have the air gap, this length will be practically equal to the length of the circle, $2 \cdot \pi \cdot r$ (this is the case of the homework problem), but because in this problem we have the air gap δ from the length of the circle we must subtrat the length of the air gap, namely δ, and we have $(2 \cdot \pi \cdot r - \delta)$;



✓ from the fact that the magnetic field lines are circular and perpendicular on the separation surfaces between the two mediums (inductor evenly wrapped), it results that:

$$=> \quad B_{Fe} = B_{\delta} = B \tag{4}$$

✓ using *relation (1)*: $B = \mu \cdot H$ and taking into account the fact that the magnetic permeability is $\mu = \mu_0 \cdot \mu_r$ and the vacuum relative permeability is 1, meaning that in our problem $\mu_{r\ airgap} = 1$, we obtain the equality:

$$\mu \cdot H_{Fe} = \mu_0 \cdot H_{\delta} => \quad H_{Fe} = \frac{\mu_0}{\mu} \cdot H_{\delta} \tag{5}$$

- ✓ replacing relation (5) in relation (3) the result is:

$$=> \frac{\mu_0}{\mu} \cdot H_\delta \cdot (2 \cdot \pi \cdot r - \delta) + H_\delta \cdot \delta = N \cdot i$$

$$H_\delta \cdot \left[\frac{\mu_0}{\mu} \cdot (2 \cdot \pi \cdot r - \delta) + \delta\right] = N \cdot i$$

$$=> H_\delta = \frac{N \cdot i}{\frac{\mu_0}{\mu} \cdot H_\delta \cdot (2 \cdot \pi \cdot r - \delta) + \delta}, \left[\frac{A}{m}\right] \tag{6}$$

- ✓ replacing expression (6) obtained for H_δ in relation (5) we obtain:

$$H_{Fe} = \frac{\mu_0}{\mu} \cdot \frac{N \cdot i}{\frac{\mu_0}{\mu} \cdot (2 \cdot \pi \cdot r - \delta) + \delta}$$

$$=> H_{Fe} = \frac{\mu_0}{\mu} \cdot \frac{N \cdot i \cdot \mu}{\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta}$$

$$=> H_{Fe} = \frac{N \cdot i \cdot \mu_0}{\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta}, \left[\frac{A}{m}\right]$$

- ✓ it was not necessary to calculate H_{Fe}, because, as it can be seen in relation (4) we can determine B knowing only H_δ, thus it is sufficient to know H_{Fe} or H_δ because we have shown that in this problem:

$$B = \mu_0 \cdot H_\delta \text{ or } B = \mu \cdot H_{Fe}$$

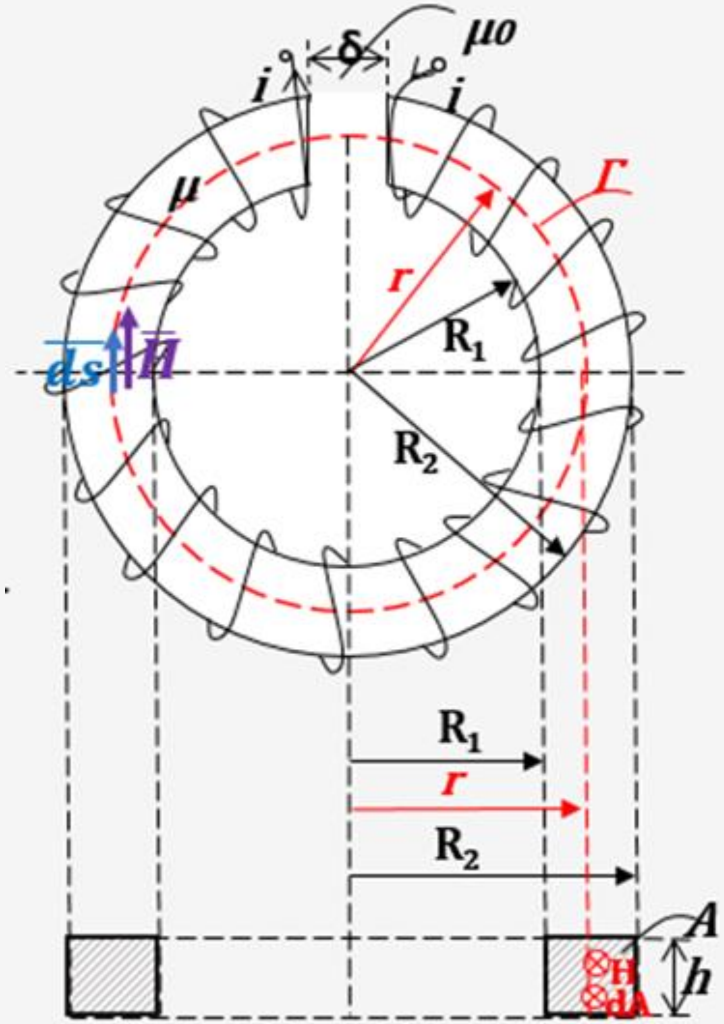
- ✓ with these observations we obtain the expression of the magnetic induction as:

$$B = \frac{\boldsymbol{\mu \cdot \mu_0 \cdot N \cdot i}}{\boldsymbol{\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta}} \text{, [T]} \tag{7}$$

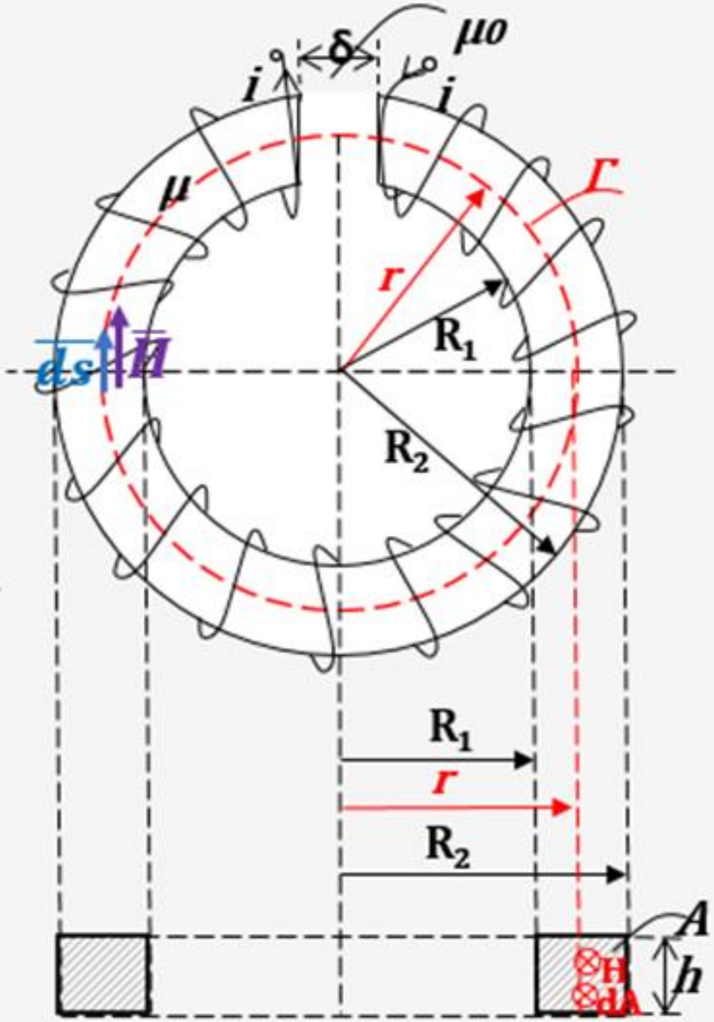
3) the fascicular magnetic flux, *Φ*, is determined and then the total magnetic flux, *ψ*, through the tor section *A*:

- ✓ from the *Magnetic Flux Law,* we know that ***the fascicular magnetic flux*** respects the relation:

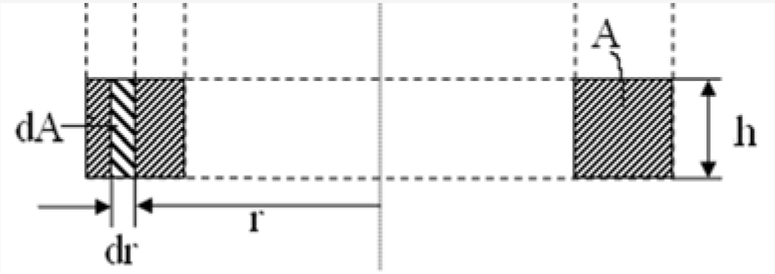
$$\Phi = \int_A \bar{B} \cdot \overline{dA} \tag{8}$$



- The area A is the cross-section area of the inductor, that we know that is a square as can be seen in the figure:
  - the magnetic field line are circular inside the toroidal inductor, so that in the square cross-section we observed that the vector of the magnetic field intensity is perpendicular on the A surface;
  - we know that the area element vector $\overline{dA}$ always perpendicular on the surface, and we convenient choose it to enter in that surface, as magnetic field intensity vector, as can be seen in the Figure (as denote, if the vector goes in the surface is used this symbol ⊗ , if goes out of the surface is symbolized with ⊙
  - result that the two vectors, $\bar{B}$ and $\overline{dA}$, are parallels and both enter in the A surface, so the angle between the two vectors is 0 degrees;



- We note that not all the area is variable, only one side of it varies, we have chosen an aleatory filed line to work on, and we consider it to be somewhere inside the toroidal inductor, at r distance from its center and can be anywhere between R_1 și R_2, so we have to change the variable and to express the area element *dA* in terms of length element *dr,* so it is considered to be:



$$dA = h \cdot dr \tag{9}$$

- We introduce (7) and (9) relations in (8) relation and considering the above notes we obtain the fascicular magnetic flux as being:

$$\Phi = \int_{R_1}^{R_2} \frac{\mu \cdot \mu_0 \cdot N \cdot i}{\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta} \cdot h \cdot dr$$

$$\Phi = \int_{R_1}^{R_2} \frac{\mu \cdot \mu_0 \cdot N \cdot i}{\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta} \cdot h \cdot dr \Rightarrow \Phi = \frac{\mu \cdot \mu_0 \cdot N \cdot i \cdot h}{1} \int_{R_1}^{R_2} \frac{1}{\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta} \cdot dr$$

- We know from math that:

$$\int \frac{1}{a \cdot x + b} dx = \frac{1}{a} \cdot \ln|a \cdot x + b| \Rightarrow \Phi = \frac{\mu \cdot \mu_0 \cdot N \cdot i \cdot h}{2 \cdot \pi \cdot \mu_0} \cdot \ln|\mu_0 \cdot (2 \cdot \pi \cdot r - \delta) + \mu \cdot \delta|\Big|_{R_2}^{R_1}$$

$$\Rightarrow \Phi = \frac{\mu \cdot N \cdot i \cdot h}{2 \cdot \pi} \cdot \ln \frac{\mu_0 \cdot (2 \cdot \pi \cdot R_2 - \delta) + \mu \cdot \delta}{\mu_0 \cdot (2 \cdot \pi \cdot R_1 - \delta) + \mu \cdot \delta}$$

- we know that $h = R_2 - R_1$ so we can express R_2 in terms of R_1 *and* h, resulting that $R_2 = h + R_1$, so the magnetic flux becomes:

$$\Phi = \frac{\mu \cdot N \cdot i \cdot h}{2 \cdot \pi} \cdot \ln \frac{\mu_0 \cdot (2 \cdot \pi \cdot h + 2 \cdot \pi \cdot R_1 - \delta) + \mu \cdot \delta}{\mu_0 \cdot (2 \cdot \pi \cdot R_1 - \delta) + \mu \cdot \delta} \quad , [Wb]$$

✓ The total magnetic flux is:

$$\boldsymbol{\Psi} = \boldsymbol{N} \cdot \boldsymbol{\Phi}$$

$$=> \boldsymbol{\Psi} = \frac{\boldsymbol{\mu} \cdot N^2 \cdot \boldsymbol{i} \cdot \boldsymbol{h}}{2 \cdot \boldsymbol{\pi}} \cdot \ln \frac{\boldsymbol{\mu}_0 \cdot (2 \cdot \boldsymbol{\pi} \cdot \boldsymbol{h} + 2 \cdot \boldsymbol{\pi} \cdot \boldsymbol{R}_1 - \boldsymbol{\delta}) + \boldsymbol{\mu} \cdot \boldsymbol{\delta}}{\boldsymbol{\mu}_0 \cdot (2 \cdot \boldsymbol{\pi} \cdot \boldsymbol{R}_1 - \boldsymbol{\delta}) + \boldsymbol{\mu} \cdot \boldsymbol{\delta}}, [\boldsymbol{Wb}] \quad (10)$$

4) We can find the inductance with the relation:

$$\boldsymbol{L} = \frac{\boldsymbol{\Psi}}{\boldsymbol{i}} \quad (11)$$

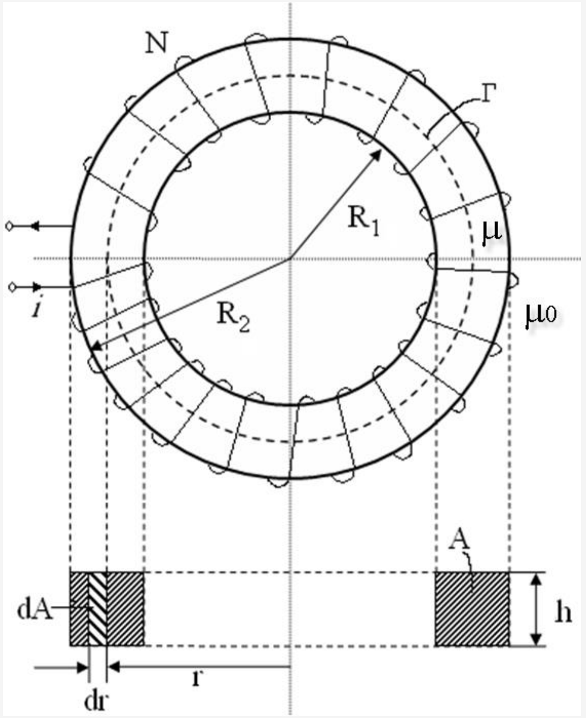
✓ we replace the total magnetic flux expression in the inductance expression and we obtain the inductance of the toroidal inductor with air gap from *Figure 1*:

$$\boldsymbol{L} = \frac{\boldsymbol{\mu} \cdot N^2 \cdot \boldsymbol{h}}{2 \cdot \boldsymbol{\pi}} \cdot \ln \frac{\boldsymbol{\mu}_0 \cdot (2 \cdot \boldsymbol{\pi} \cdot \boldsymbol{h} + 2 \cdot \boldsymbol{\pi} \cdot \boldsymbol{R}_1 - \boldsymbol{\delta}) + \boldsymbol{\mu} \cdot \boldsymbol{\delta}}{\boldsymbol{\mu}_0 \cdot (2 \cdot \boldsymbol{\pi} \cdot \boldsymbol{R}_1 - \boldsymbol{\delta}) + \boldsymbol{\mu} \cdot \boldsymbol{\delta}}, [\boldsymbol{H}]$$

Verification :

✓ if we has a toroidal inductor without an air gap, meaning $\delta = 0$, then:

$$\text{for } \delta = 0 \quad => L = \frac{\mu \cdot N^2 \cdot h}{2 \cdot \pi} \cdot \ln \frac{\mu_0 \cdot (2 \cdot \pi \cdot h + 2 \cdot \pi \cdot R_1)}{\mu_0 \cdot 2 \cdot \pi \cdot R_1}$$

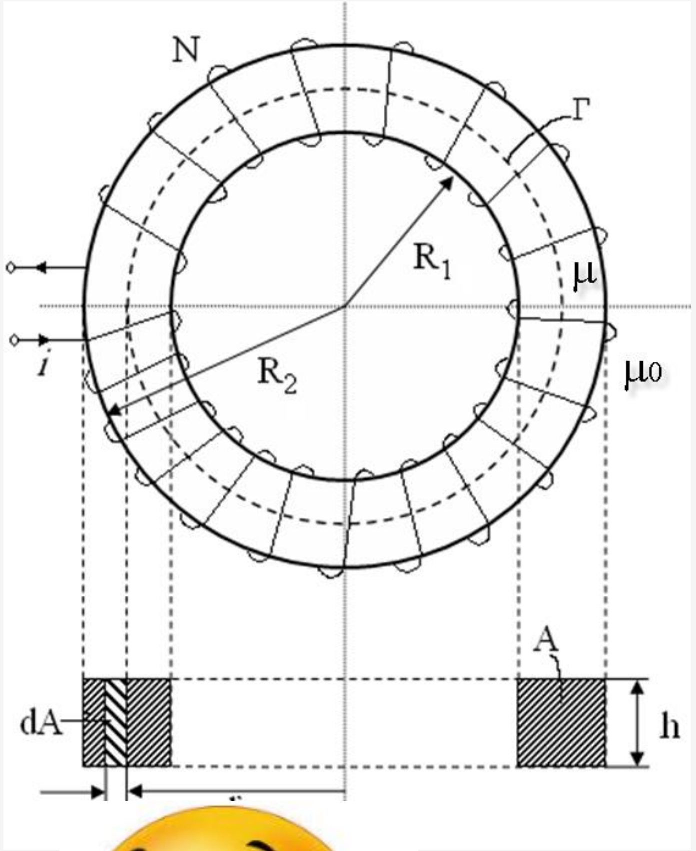


$$=> \boldsymbol{L} = \frac{\boldsymbol{\mu} \cdot \boldsymbol{N}^2 \cdot \boldsymbol{h}}{2 \cdot \boldsymbol{\pi}} \ln \frac{\boldsymbol{h} + \boldsymbol{R}_1}{\boldsymbol{R}_1}, [\boldsymbol{H}]$$

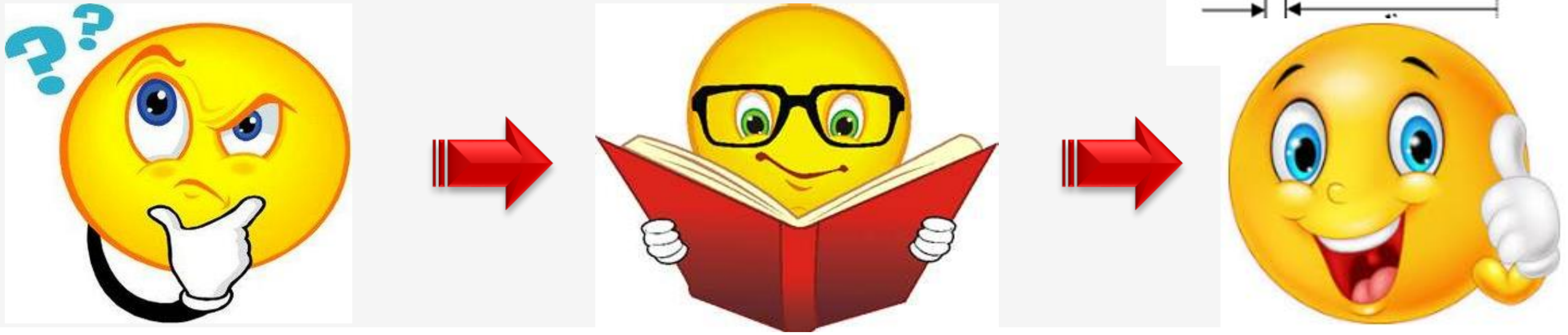
Direct Method

<u>Problem 2</u>

Find the self inductance of a toroidal inductor, with a square cross section of area *A*, height *h* (where h = $R_2 - R_1$) and permeability μ much higher than the vacuum permeability μ_0. The inductor has *N* turns and is passed by the current *i*.



Homework:

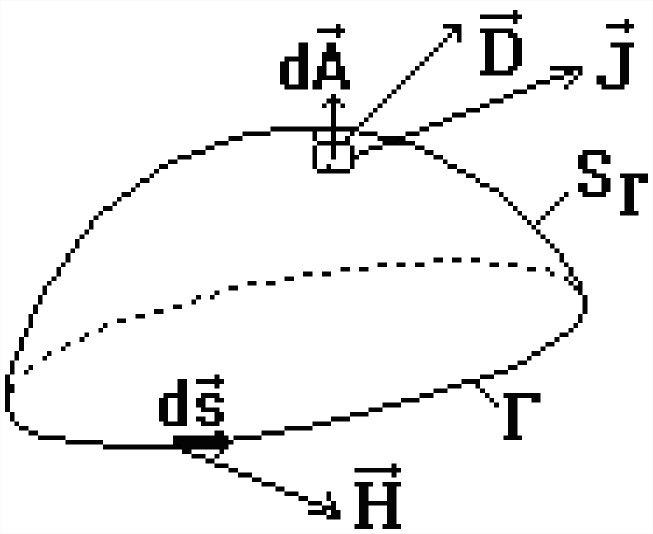


2. Magnetic Circuit Law
(Total Current Law)

❑ General (global) form of the law:

At any moment and in any medium, the magnetomotive voltage, $u_{mm\Gamma}$ along any closed curve Γ is equal with the sum of two terms: the first is the **current linkage** ***$\theta_{S\Gamma}$ corresponding to the conduction currents passing through an open surface S_Γ bordered by the curve Γ and the second term is the electric flux $\Psi_{S\Gamma}$ derived in relation to time through the same surface S_Γ.***

$$u_{mm_\Gamma} = \theta_{S_\Gamma} + \frac{d\Psi_{S_\Gamma}}{dt} \quad \textbf{(1)}$$



where:

- $u_{mm_\Gamma} = \oint_\Gamma \bar{H} d\bar{s}$ **(2)**
- $\theta_{S_\Gamma} = \int_{S_\Gamma} \bar{J} d\bar{A}$ **(3)**
- $\Psi_{S_\Gamma} = \int_{S_\Gamma} \bar{D} d\bar{A}$ **(4)**

we introduce (2), (3), (4) in (1) ⇨ ❑ **Integral form of the law:**

$$\boxed{\oint_\Gamma \bar{H} d\bar{s} = \int_{S_\Gamma} \bar{J} d\bar{A} + \frac{d}{dt} \int_{S_\Gamma} \bar{D} d\bar{A}} \quad \textbf{(5)}$$

where:

- **the first term of the right term of the relation (5) is called the conduction electrical current through S_Γ (current linkage θ):**

$$i = \int_{S_\Gamma} \bar{J} d\bar{A}$$

- **the second term of the right term of the relation (5) is called displacement current (Hertzian current):**

$$i_H = i_D = \frac{d}{dt} \int_{S_\Gamma} \bar{D} d\bar{A}$$

❖ The derivative of a flux integral of a vector $\bar{D}$:

$$\frac{d}{dt}\int_{S_\Gamma}\bar{D}\,d\bar{A}=\int_{S_\Gamma}\left[\frac{\partial\bar{D}}{\partial t}+\bar{v}\,\mathrm{div}\bar{D}+\mathrm{rot}\,\left(\bar{D}\times\bar{v}\right)\right]d\bar{A}$$

- **from the local form of the Electric Flux Law:** $\mathrm{div}\bar{D}=\rho_V$

$$\frac{d}{dt}\int_{S_\Gamma}\bar{D}\,d\bar{A}=\int_{S_\Gamma}\left[\frac{\partial\bar{D}}{\partial t}+\bar{v}\cdot\rho_v+\mathrm{rot}\,\left(\bar{D}\times\bar{v}\right)\right]d\bar{A} \qquad \textbf{(6)}$$

The sum of 3 currents with the following densities:

- The density of the displacement current: $\bar{J}_D=\dfrac{\partial\bar{D}}{\partial t}$
- The density of the convection current: $\bar{J}_v=\bar{v}\,\rho_v$
- The density of the theoretical Roentgen current: $\bar{J}_R=rot\,(\bar{D}\times\bar{v})$

- **The most General Form of the Law:**

$$\boxed{\oint_\Gamma\bar{H}d\bar{s}=\theta+i_D+i_v+i_R}$$

❑ Local form of the law:

- **is deduced from the global form;**
- **(6) is introduced in (5) and the result is:**

$$\oint_{\Gamma} \bar{H}\, d\bar{s} = \int_{S_\Gamma} \bar{J}\, d\bar{A} + \int_{S_\Gamma} \frac{\partial \bar{D}}{\partial t} d\bar{A} + \int_{S_\Gamma} \bar{v} \cdot \rho_v\, d\bar{A} + \int_{S_\Gamma} \mathrm{rot}\,\left(\bar{D} \times \bar{v}\right) d\bar{A}$$

- **Stokes Theorem is applied:** $\oint_{\Gamma} \bar{H}\, d\bar{s} = \int_{S_\Gamma} rot\, \bar{H}\, d\bar{A}$

$$\int_{S_\Gamma} rot\, \bar{H}\, d\bar{A} = \int_{S_\Gamma} \bar{J}\, d\bar{A} + \int_{S_\Gamma} \frac{\partial \bar{D}}{\partial t} d\bar{A} + \int_{S_\Gamma} \bar{v} \cdot \rho_v\, d\bar{A} + \int_{S_\Gamma} \mathrm{rot}\,\left(\bar{D} \times \bar{v}\right) d\bar{A}$$

LF →

$$\boxed{rot\, \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} + \bar{v}\, \mathrm{div} \bar{D} + \mathrm{rot}\,\left(\bar{D} \times \bar{v}\right)}$$

❖ in immobile mediums: $\boxed{rot\, \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}}$ → The First Maxwell Equation

Conclusions

MCL says that the magnetomotive voltage, $u_{mm\Gamma}$, along any closed curve Γ is equal with the sum of four types of independent currents, each of them being obtained in specific conditions and mediums:

- $i = \int_{S_\Gamma} \bar{J}\, d\bar{A} \rightarrow$ **the conduction current due to the displacement of the electrical load under the action of the voltage differences (in mediums where there is σ);**
- $i_D = \int_{S_\Gamma} \frac{\partial \bar{D}}{\partial t} d\bar{A} \rightarrow$ **the displacement current, appears due to the time variation of the vector $\bar{D}$ (in mediums where there is ε);**
- $i_v = \int_{S_\Gamma} \bar{v}\, \rho_v\, d\bar{A} \rightarrow$ **the transport current due to the relative transport speed of the real load ρ_v;**
- $i_R = \int_{S_\Gamma} \mathrm{rot}(\bar{D} \times \bar{v})\, d\bar{A} \rightarrow$ **Roentgen current due to the relative transport speed of the polarization load ρ_{vp}:**

$$\boxed{u_{mm_\Gamma} = i + i_D + i_v + i_R}$$

Thank you for your attention!!!
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Questions???