Course 5

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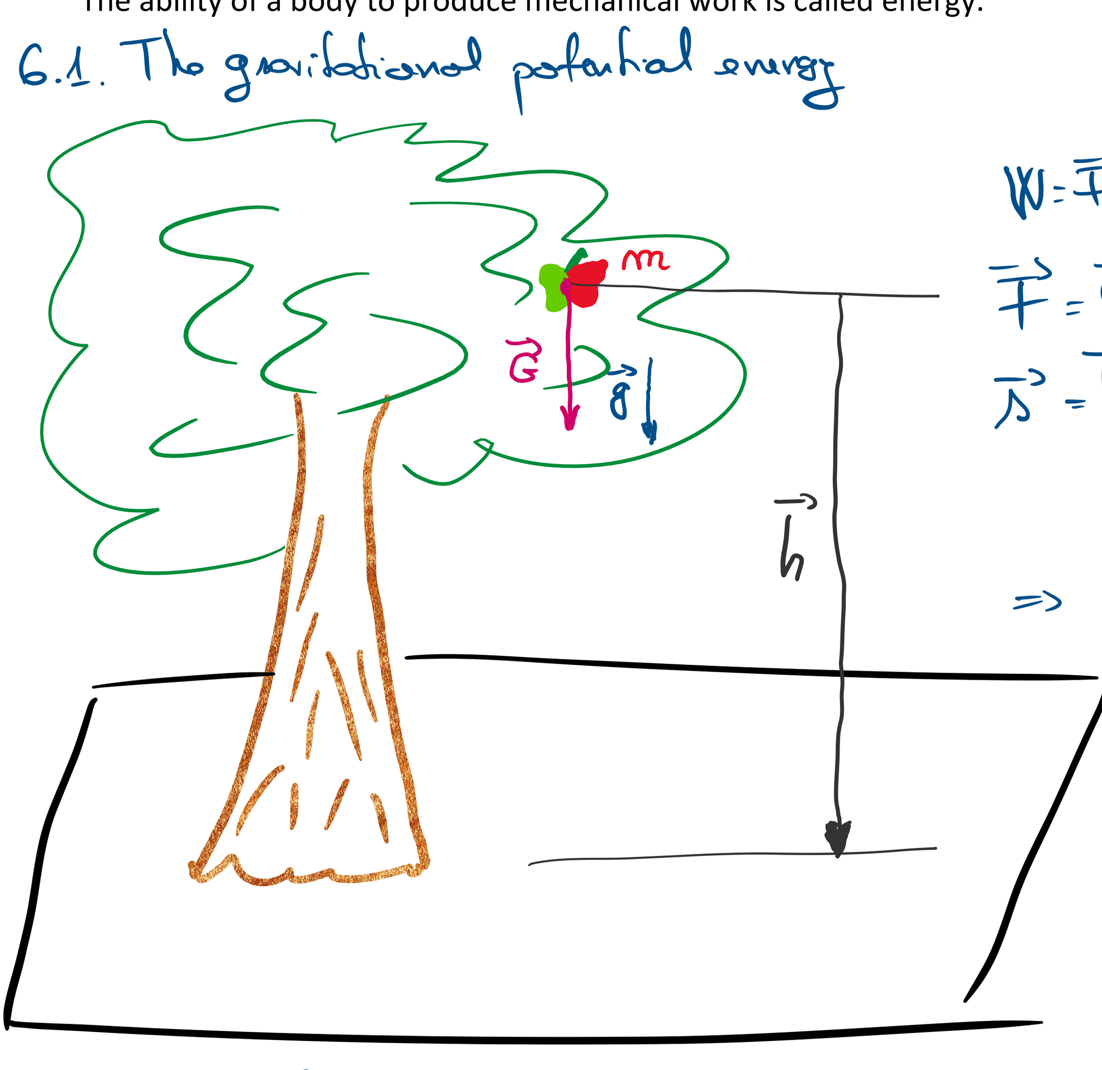
II DYNAMICS

6. The energy

The ability of a body to produce mechanical work is called energy.

6.1. The gravitational potential energy



$W = \vec{F} \cdot \vec{s}$

$\vec{F} = \vec{G} = m \cdot \vec{g}$

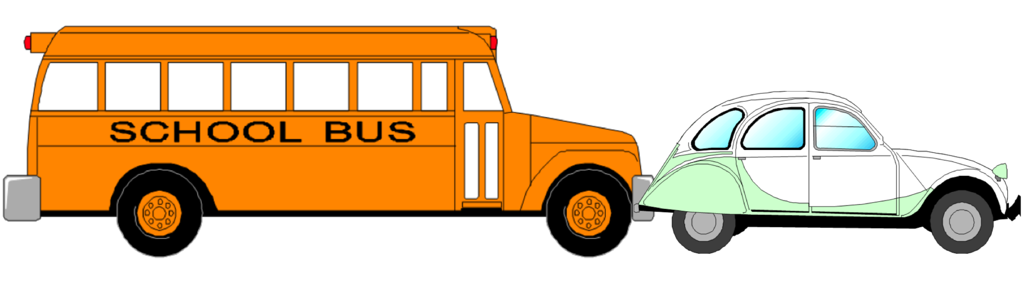
$\vec{s} = \vec{h}$

$\Rightarrow$

$\Rightarrow W = m \cdot \vec{g} \cdot \vec{h} \Rightarrow$

$\boxed{U = m \cdot g \cdot h}$ the gravitational potential energy

6.2. The kinetic energy



$W = \vec{F} \cdot \vec{s} = F \cdot s = m \cdot a \cdot s$

$F = m \cdot a$

$v^2 = v_0^2 + 2\, s \cdot a$

$v_0 = 0$

$\Rightarrow v^2 = 2 a \cdot s$

$a \cdot s = \frac{v^2}{2}$

$\Rightarrow W = m \cdot \frac{v^2}{2} \Rightarrow$

$\boxed{K = \frac{m \cdot v^2}{2}}$ the kinetic energy

7. The laws of conservation

7.1. The theorem of variation of the kinetic energy

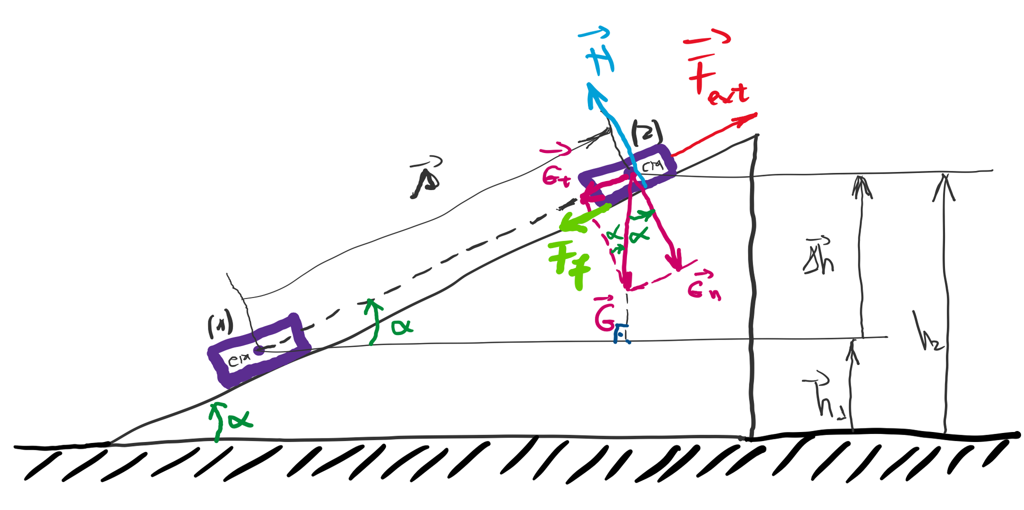
$dW = \vec{F} \cdot d\vec{s} \;\Big|\; \int \Rightarrow \int dW = \int_{\vec{s}_1}^{\vec{s}_2} \vec{F} \cdot d\vec{s} = \int_{\vec{s}_1}^{\vec{s}_2} m \vec{a} \cdot d\vec{s} = \int_{(1)}^{(2)} m \frac{d\vec{v}}{dt} \cdot d\vec{s} \Rightarrow$

$W = m \int_{(1)}^{(2)} d\vec{v} \frac{d\vec{s}}{dt} = m \int_{\vec{v}_1}^{\vec{v}_2} \vec{v}\, d\vec{v} \Rightarrow W = m \frac{\vec{v}^{\,2}}{2} \Big|_{\vec{v}_1}^{\vec{v}_2} = \frac{m v_2^2}{2} - \frac{m v_1^2}{2} \Rightarrow$

$K = \frac{m v^2}{2}$ $\;\;W = K_2 - K_1 \Rightarrow$

$\boxed{\Delta K = W}$

7.2. The law of conservation of the total energy



$\Delta K = W$

$W = \vec{F} \cdot \vec{s} \Rightarrow$

$W = (\vec{F}_{ext} + \underbrace{\vec{G}}_{\vec{G}_t + \vec{G}_n} + \vec{F}_f + \vec{N}) \cdot \vec{s}$

$W = F_{ext} \cdot s - G_t \cdot s + 0 - F_f \cdot s + 0 = \Delta K$

$W_{ext} = \Delta K + G_t \cdot s + F_f \cdot s$

$\sin\alpha = \frac{\Delta h}{s} = \frac{G_t}{G} \Rightarrow G_t \cdot s = G \cdot \Delta h = m g (h_2 - h_1)$

$Q = F_f \cdot s$ - the heat

$W_{ext} = \Delta K + mg(h_2 - h_1) + Q$

$W_{ext} = \frac{m v_2^2}{2} + m g h_2 + Q$

$- \left(\frac{m v_1^2}{2} + m g h_1 \right)$

Note

$E = \frac{m v^2}{2} + m g h = K + U$

$W_{ext} = E_2 - E_1 + Q$

$\boxed{W_{ext} = \Delta E + Q}$

For an isolated system with no friction the total energy is conserved

- Isolated system

  $F_{ext} = 0 \Rightarrow W_{ext} = 0$

- No friction

  $F_f = 0 \Rightarrow Q = 0$

$\Rightarrow 0 = \Delta E + 0 \Rightarrow$

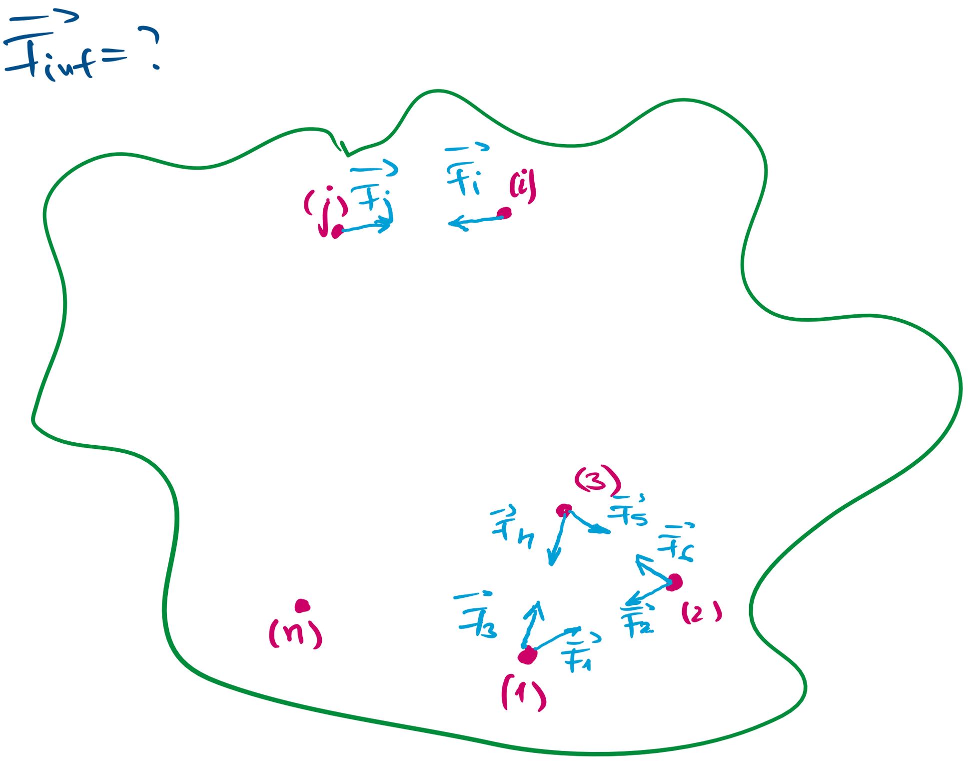
$\Delta E = 0 \Rightarrow$

$E = E_1 = E_2$ - const

7.3. The law of conservation of the linear momentum

A) $\vec{F} = \frac{d\vec{p}}{dt}$ $\qquad \vec{F} = \vec{F}_{ext} + \vec{F}_{int}$

B) $\vec{F}_{int} = ?$



n - number of particles

N - number of forces

$N = n \cdot (n-1)$

n	N
1	0
2	2
3	6
. .	. .

n - large

$N \sim n^2$

$(n \sim 10^{20} \rightarrow N \sim 10^{40}$

$N = 10^{40} - 10^{20})$

$\vec{F}_{int} = \sum_{i=1}^{N} \vec{F}_i = \frac{1}{2} \sum_{i=1}^{N} (\vec{F}_i + \vec{F}_i) \Rightarrow$

$\vec{F}_{int} = \frac{1}{2} \left[\sum_{i=1}^{N} \vec{F}_i + \sum_{j=1}^{N} \vec{F}_j \right] = \frac{1}{2} \sum_{i,j=1}^{N} (\vec{F}_i + \vec{F}_j)$

$\vec{F}_{int} = \frac{1}{2} 2 \sum_{i \neq j} (\vec{F}_i + \vec{F}_j) \Longrightarrow \vec{F}_{int} = \sum_{i<j} (\vec{F}_i + \vec{F}_j) \xrightarrow{\vec{F}_j = -\vec{F}_i}$

$\vec{F}_{int} = \sum_{i<j} (\underbrace{\vec{F}_i - \vec{F}_i}_{0}) = 0$

A) $\vec{F} = \vec{F}_{ext} + 0 = \vec{F}_{ext}$

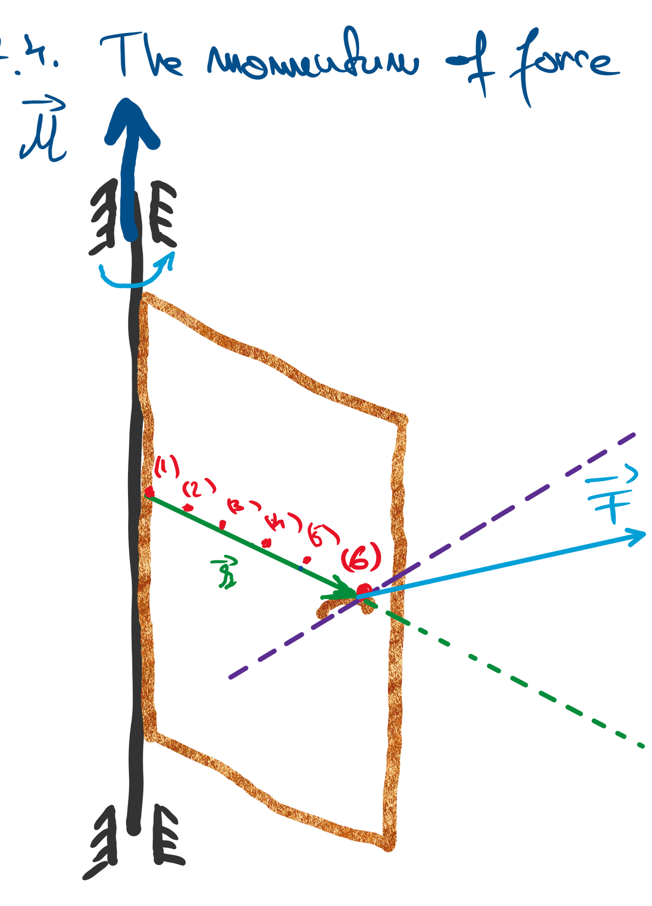
$\vec{F} = \frac{d\vec{p}}{dt}$

For an isolated system the linear momentum is conserved!

Isolated system

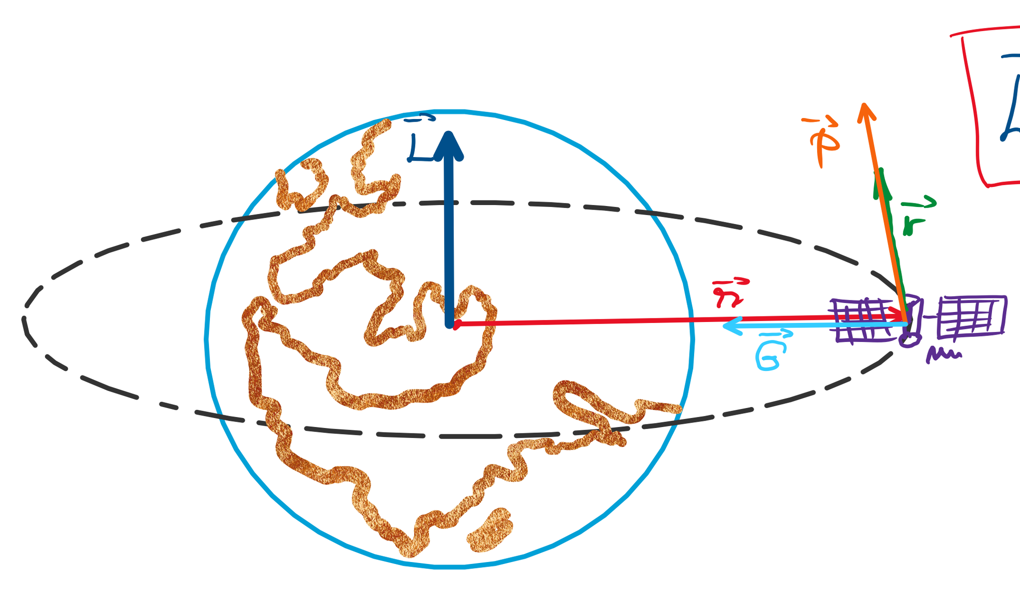
$\vec{F}_{ext} = \vec{F} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \Rightarrow \vec{p}$ - const

7.4. The momentum of force



$\boxed{\vec{M} = \vec{r} \times \vec{F}}$

7.5. The angular (orbital, circular) momentum



$\boxed{\vec{L} = \vec{r} \times \vec{p}}$

7.6. The law of conservation of the angular momentum

A) $\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times \vec{p}) = \frac{d\vec{r}}{dt} \times \vec{p} + \vec{r} \times \frac{d\vec{p}}{dt} = \underbrace{\vec{v} \times m \cdot \vec{v}}_{0} + \vec{r} \times \vec{F} \Rightarrow$

$\frac{d\vec{L}}{dt} = \vec{M} \Rightarrow \boxed{\vec{M} = \frac{d\vec{L}}{dt}}$

B)

A body characterized buy a null (zero) momentum of force the angular momentum is conserved

$\vec{M} = 0 \Rightarrow 0 = \frac{d\vec{L}}{dt} \Rightarrow \vec{L}$ - const

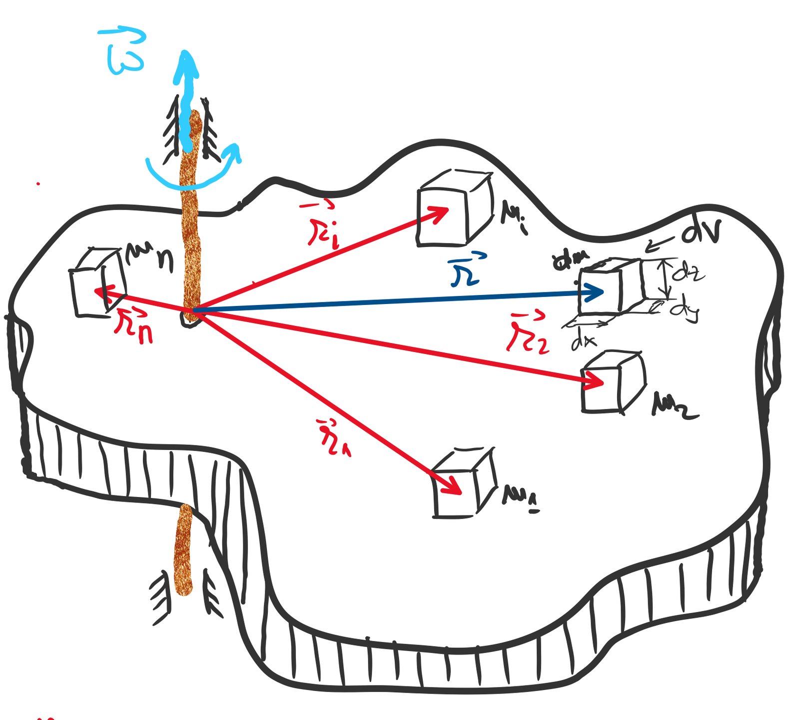
$\vec{M} = 0$

- $\vec{r} = 0$
- $\vec{F} = 0$
- $\vec{r} \parallel \vec{F}$ (- central forces - conservative forces)

/* $\vec{M} = \vec{r} \times \vec{F}$ */

7.7. The momentum of inertia

A)



$K = ?$

$K = \sum_{i=1}^{n} K_i$

$K_i = \frac{m_i v_i^2}{2}$

$v_i = \omega \cdot r_i$

$\Rightarrow$

$K_i = \frac{m_i \omega^2 r_i^2}{2}$

$K = \sum_{i=1}^{n} \frac{m_i r_i^2 \omega^2}{2} \Rightarrow$

$K = \frac{\sum_{i=1}^{n} m_i r_i^2}{2} \omega$

Note

$I = \sum_{i=1}^{n} m_i r_i^2$ - the momentum of inertia

$\boxed{K = \frac{I \omega^2}{2}}$ $\qquad K = \frac{m v^2}{2}$

$I = \int dm\, \vec{r}^{\,2} \Rightarrow I = \int r^2 dm$

Note

$\rho = \frac{dm}{dV}$ - the density $\Rightarrow dm = \rho \cdot dV$

$\Rightarrow$

$I = \int_V r^2 \rho(r)\, dV = \iiint_{z_{min}\, y_{min}\, x_{min}}^{z_{max}\, y_{max}\, x_{max}} r^2 \rho(r)\, dx\, dy\, dz$