ASSIGNMENT - 3

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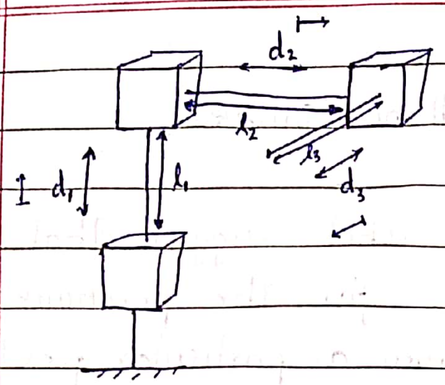
1) A singular configuration in a robot suggests that we have multiple solutions or possibilities for the positions or torques of the robot joints & links, given a particular position of the end effector. Due to this, certain directions of motion or resistance to forces in certain directions of motion may not be attainable by the robot. Mathematically, singularities may correspond to unbounded joint velocities & torques for bounded values of end effector velocities, forces & torques. Singularities generally occur at the boundaries of the manipulator workspace or at points which can be unreachable under small link parameter perturbation. Due to multiple possibilites for a joint / link in a singular configuration, the inverse kinematics will not have a unique solution & will have no solutions / infinitely many solutions.

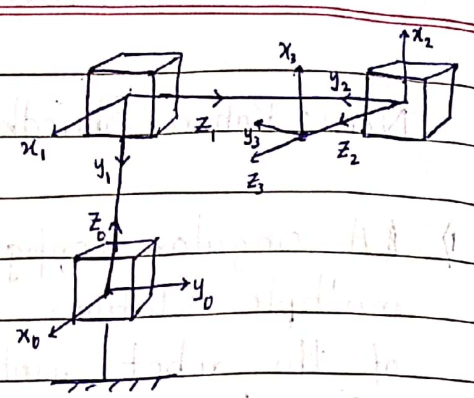
A singularity can be typically found when the rank of Jacobian $J(q)$ becomes less than its maximum value. or when the above stated effects are observed. When a singular configuration occurs, the determinant of the Manipulator Jacobian becomes zero. i.e.

$$\det(J(q)) = 0.$$

Hence, when the determinant value of the Manipulator Jacobian comes close to zero, we can say that the particular configuration is approaching a singular configuration state.

5)





Three link Cartesian Robot (P P P) — Axis choice for Robot DH parameters.

∴ Based on the axis choice above, we construct the D-H parameter table as shown below:-

Link	θ_i	d_i	a_i	α_i
1	0°	$l_1+d_1^*$	0	-90°
2	-90°	$l_2+d_2^*$	0	-90°
3	0	$l_3+d_3^*$	0	0

∴ Writing the transformation matrices:-

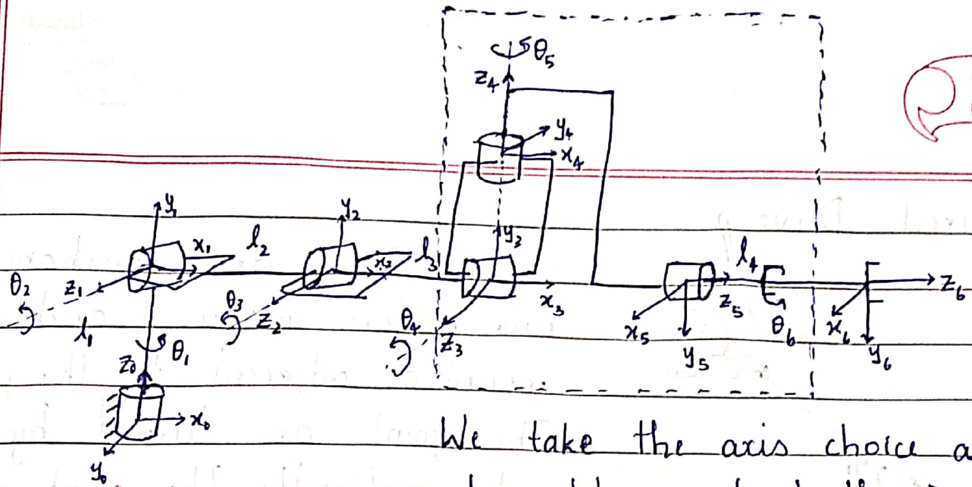
$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & l_1+d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & l_2+d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_3+d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

∴ Writing the forward kinematic equations, we get in matrix form:-

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = A_1 A_2 A_3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

6)



We take the axis choice as shown above. We construct the D-H parameter table as shown below:-

Link	θ_i	d_i	a_i	α_i
1	θ_1^*	l_1	0	90°
2	θ_2^*	0	l_2	0°
3	θ_3^*	0	l_3	0°
4	θ_4^*	0	0	-90°
5	θ_5^*	0	0	-90°
6	θ_6^*	l_4	0	0

$\therefore$ Writing the transformation matrices, we get;

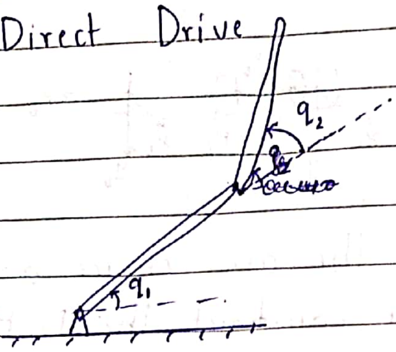
$$A_1 = \begin{bmatrix} C_{\theta_1} & 0 & S_{\theta_1} & 0 \\ S_{\theta_1} & 0 & -C_{\theta_1} & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & l_2 C_{\theta_2} \\ S_{\theta_2} & C_{\theta_2} & 0 & l_2 S_{\theta_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} C_{\theta_3} & -S_{\theta_3} & 0 & l_3 C_{\theta_3} \\ S_{\theta_3} & C_{\theta_3} & 0 & l_3 S_{\theta_3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_{\theta_4} & 0 & -S_{\theta_4} & 0 \\ S_{\theta_4} & 0 & C_{\theta_4} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5 = \begin{bmatrix} C_{\theta_5} & 0 & -S_{\theta_5} & 0 \\ S_{\theta_5} & 0 & C_{\theta_5} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6 = \begin{bmatrix} C_{\theta_6} & -S_{\theta_6} & 0 & 0 \\ S_{\theta_6} & C_{\theta_6} & 0 & 0 \\ 0 & 0 & 1 & l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Writing the forward kinematic equations in matrix form, we get;

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = A_1 A_2 A_3 A_4 A_5 A_6 \begin{bmatrix} P_6 \\ 1 \end{bmatrix}$$

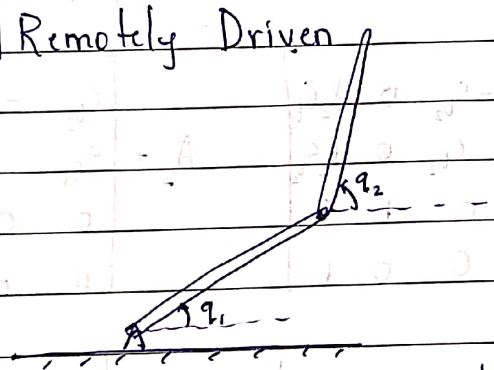
7) A] Direct Drive



In direct drive mechanism, two motors are used and are mounted adjecent to the joints. The joints are turned by the motors directly. The joint angle is taken w.r.t. the position of the previous link as shown.

Advantages:
① As motors are independently actuated, it improves the reliability of the manipulator.
② Adjacent motor configuration is capable of more increases positional speed compared to remotely driven joints. ~~mechanical.~~
③ There is no loss of motion, no hysteresis, no backlash

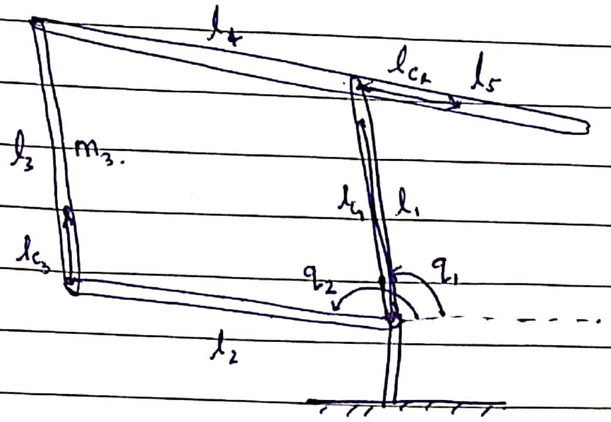
B] Remotely Driven



In comparison to direct drive, absolute angles are considered in remotely driven configuration. In remotely-driven drives, ~~the first link is~~ the joint motors are on the base & the joints are actuated using a timing belt or gear mechanism; starting from the second joint.

Advantages:
① Coriolis forces are avoided in Remotely-driven configuration & hence the robot-dynamics is simplified.
② The inertia of the links is reduced since the motors are mounted at the base.

C] 5 - bar parallelogram arrangement



In the above configuration, there are only four links, however, we consider ground as the fifth link. Although l_1 & l_3 are equal in length, their centers of gravity are not at equal lengths. The two joints at the lowest point are actuated using motors. The equations of motion get decoupled when we have:-

$$m_3 l_2 l_{c_3} = m_4 l_1 l_{c_4}$$

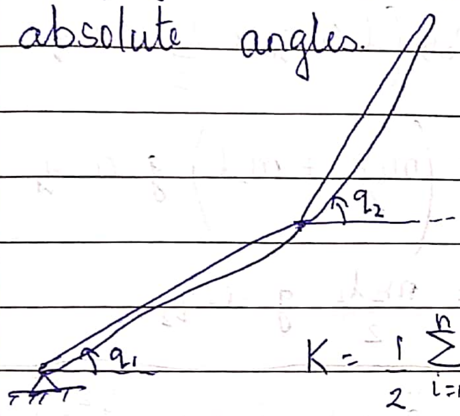
Advantages:

① The dynamics of the robot are easier since the decoupled equation result in no coriolis & centripetal forces.

② The interactions between joints variables can be ignored ~~can be igo~~ since the equations are decoupled

However, due to the closed chain mechanism, the Jacobian derivation differs from the conventional approach used in direct & remote drives. & a different approach must be used.

Miniproject Elbow Manipulator with remotely driven links & using absolute angles.



$$V_{c_1} = \begin{bmatrix} -\frac{l_1}{2} \sin q_1 \\ \frac{l_1}{2} \cos q_1 \\ 0 \end{bmatrix} \dot{q}_1 \qquad V_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2} \sin q_2 \\ l_1 \cos q_1 & \frac{l_2}{2} \cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = \dot{q}_2 \hat{k}$$

$$K = \frac{1}{2} \sum_{i=1}^{n} m_i v_{c_i}^T v_{c_i} + \frac{1}{2} \sum_{i=1}^{n} \omega_i^T I_i \omega_i$$

$$v_{ci} = J_{v_{ci}}(q) \dot{q} \qquad \omega_i = R_i^T J_{\omega i}(q) \dot{q}$$

$$K = \frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \left[m_i J_{v_{ci}}(q)^T J_{v_{ci}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i q^T J_{\omega_i}(q) \right] \dot{q}$$

$$= \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

In our example;

$$D(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Computing the Christoffel symbols:-

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right] \qquad C_{ijk} = C_{jik}$$

$$C_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\left[\frac{\partial d_{12}}{\partial q_1} + \frac{\partial d_{11}}{\partial q_2} - \frac{\partial d_{11}}{\partial q_1}\right] = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = \frac{\partial d_{12}}{\partial q_2} = -m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = + m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1).$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0.$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0.$$

Potential energy $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$

$$\phi_1 = \frac{\partial V}{\partial q_1} \qquad \phi_2 = \frac{\partial V}{\partial q_2} \Rightarrow \phi_1 = \left(\frac{m_1 l_1}{2} + m_2 l_1\right) g \cos q_1$$

$$\phi_2 = \frac{m_2 l_2}{2} g \cos q_2$$

$\Rightarrow$ Final eq^ns are:-

$$\boxed{d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_1^2 + \phi_1 = \tau_1 \quad \& \quad d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2}$$

When we compare the derived formulae above & the formula derived in miniproject; we observe that the miniproject had extra terms of $\dot{q}_1 \dot{q}_2$. Since the model above uses directly driven links & the miniproject model used remotely driven links, the miniproject model has extra terms of coriolis force which are in the form $\dot{q}_1 \dot{q}_2$ as observed.

10) When we are provided $D(q)$ & $V(q)$, we can derive the equations of motions as follows:-

① The q-matrix representing the joint variables will be taken as

$$q = [q_1, q_2, \dots q_n]^T$$

n = number of links.

② We then calculate the Christoffel symbols by:-

$$c_{i,j,k} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

③ We then calculate ϕ_k as:

$$\phi_k(q) = \frac{\partial V}{\partial q_k}$$

④ Hence, once we have the Christoffel symbols & ϕ_k we can write the Euler-Lagranges equations as:-

$$\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\,\dot{q}_j + \phi_k(q) = \tau_k \quad \dots\ k = 1, 2, \dots, n$$

These are the key steps to be followed while deriving the equations of motion given $D(q)$ & $V(q)$.

4) We derive the transformation matrix for SCARA & Stanford manipulators based on the results of the previous assignment & the textbook.

A] SCARA

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2c_2 \\ s_2 & -c_2 & 0 & a_2s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1A_2A_3 = \begin{bmatrix} c_{12} & s_{12} & 0 & a_2c_{12}+a_1c_1 \\ -s_{12} & -c_{12} & 0 & a_2s_{12}+a_1s_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

B] Stanford.

$$A_1 = \begin{bmatrix} c_1 & 0 & -s_1 & 0 \\ s_1 & 0 & c_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} c_2 & 0 & s_2 & 0 \\ s_2 & 0 & -c_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 A_2 A_3 = \begin{bmatrix} c_1 c_2 & -s_1 & c_1 s_2 & d_3 c_1 s_2 - d_2 s_1 \\ s_1 c_2 & c_1 & s_1 s_2 & d_3 s_1 s_2 + d_2 c_1 \\ -s_2 & 0 & c_2 & d_3 c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~∴ We take the following values & verify the code~~

~~SCARA :~~

~~Link~~	~~θ_i~~	~~d_i~~	~~a_i~~	~~α_i~~
~~1~~	~~60°~~	~~0~~	~~100~~	~~0~~
~~2~~	~~30°~~	~~0~~	~~100~~	~~0~~
~~3~~	~~60°~~	~~0~~	~~100~~	~~0~~

~~Stanford:~~

~~Link~~	~~θ_i~~	~~d_i~~	~~a_i~~	~~α_i~~
~~1~~				
~~2~~				
~~3~~				

∴ We take the following values & verify the code:

~~SCARA~~ : Stanford.

Link	θ_i	d_i	a_i	α_i
1	30	0	0	-90
2	30	50	0	90
3	0	50	0	0

SCARA:

Link	θ_i	d_i	a_i	α_i
1	30	0	50	0
2	30	0	50	180
3	0	50	0	0

The code & the transformation matrix gives the same value
∴ Verified.