

Cheatsheet - Comparison and Non-Comparison Sorting Algorithms

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1. About

This cheatsheet provides an overview of some common sorting algorithms.

2. Comparison Sort Overview

Name	Worst case complexity	Best case complexity
Bubble	$\Theta(N^2)$	$\Theta(N)$
Insertion	$\Theta(N^2)$	$\Theta(N)$
Selection	$\Theta(N^2)$	$\Theta(N^2)$
Quicksort	$\Theta(N^2)$	$\Theta(N \times \log N)$
Mergesort	$\Theta(N \times \log N)$	$\Theta(N \times \log N)$

Because comparison sorts must compare pairs of elements, **they cannot** run faster than $N \times \log N$.

3. Bubble Sort

3.1. Pseudocode

```
1. function BubbleSort( $A, N$ )
2.   swapped = true
3.   while (swapped) do
4.     swapped = false
5.     for  $0 \leq i < N - 1$  do
6.       if ( $A[i] > A[i + 1]$ ) then
7.         swap( $A[i], A[i + 1]$ )
8.         swapped = true
9.       end if
10.    end for
11.     $N = N - 1$ 
12.  end while
13.  return  $A$ 
14. end function
```

3.2. Time Complexity

The **best case** for bubble sort is:

$$T(N) = C_0 \times N + C_1$$

Additionally:

- $T(N)$ is $O(N)$, $O(N^2)$ and $O(N^3)$, etc.
- $T(N)$ is $\Omega(N)$, $\Omega(\log N)$ and $\Omega(1)$, etc.
- $T(N)$ is $\Theta(N)$

The **worst case** for bubble sort is:

$$T(N) = C_0 \times N^2 + C_1 \times N + C_2.$$

Additionally:

- $T(N)$ is $O(N^2)$ and $O(N^3)$, etc.
- $T(N)$ is $\Omega(N^2)$, $\Omega(\log N)$ and $\Omega(1)$, etc.
- $T(N)$ is $\Theta(N^2)$

4. Insertion Sort

4.1. Pseudocode

```
1. function InsertionSort( $A, N$ )
2.   for  $1 \leq j \leq N - 1$  do
3.      $\text{ins} = A[j]$ 
4.      $i = j - 1$ 
5.     while ( $i \geq 0$  and  $\text{ins} < A[i]$ ) do
6.        $A[i + 1] = A[i]$ 
7.        $i = i - 1$ 
8.     end while
9.      $A[i + 1] = \text{ins}$ 
10.  end for
11. end function
```

5. Selection Sort

5.1. Pseudocode

```
1. function SelectionSort( $A, N$ )
2.   for  $0 \leq i < N - 1$  do
3.      $\text{min} = \text{pos\_min}(A, i, N - 1)$ 
4.      $\text{swap}(A[i], A[\text{min}])$ 
5.   end for
6. end function
```

The function $\text{pos_min}(A, a, b)$ returns the position of the minimum value between positions a and b (both inclusive) in array A .

6. Quicksort

6.1. Pseudocode

```
1. function Quicksort( $A, \text{low}, \text{high}$ )
2.   if  $\text{low} < \text{high}$  then
3.      $p = \text{partition}(A, \text{low}, \text{high})$ 
4.     Quicksort( $A, \text{low}, p - 1$ )
5.     Quicksort( $A, p + 1, \text{high}$ )
6.   end if
7. end function
```

TODO: This needs further explanation.

The function $\text{partition}(A, \text{low}, \text{high})$ selects a number (the pivot), moves all numbers lower than the pivot to the left part of the array and moves the pivot to its final position.

7. Mergesort

7.1. Pseudocode

```
1. function MergeSort( $A, \text{int } l, \text{int } h$ )
2.   if ( $l < h$ )
3.      $\text{mid} = l + \text{floor}((h - 1) / 2)$ 
4.     MergeSort( $A, l, \text{mid}$ )
5.     MergeSort( $A, \text{mid} + 1, h$ )
6.     Merge( $A, l, \text{mid}, h$ )
7.   end if
8. end function
```

The function Merge creates two arrays of both halves (left and right) and then merges them to produce a single, sorted array.