Lecture 4: Baseband and Passband Signals

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Outline

- Energy and power
- Bandwidth
  - ✓ Real-valued signal and one-sided bandwidth
  - ✓ Complex-valued signal and two-sided bandwidth
- Additive white Gaussian noise (AWGN) channel
- Baseband and passband signals
- Upconversion and downconversion
- Representation of passband signals
- Hilbert transform and pre-envelope
- Reference
  - ✓ [Haykin] Chapter 2

Energy and Power

Energy and power: two important concepts in communications

- How much power is needed to transmit a signal?
- How is the signal-to-noise ratio found?
- How much interference do signals create for each other?

Energy: the area under the squared magnitude of a signal

$$E = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(f)|^2 df.$$

Power: time average of energy, evaluated over a period or large interval

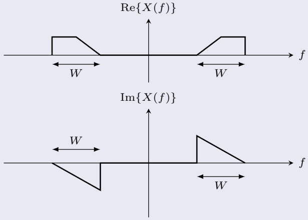
$$P = \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2 dt.$$

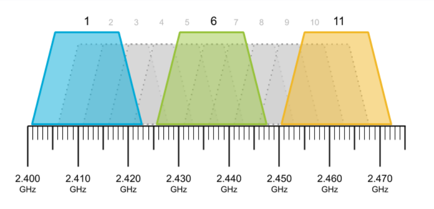
Bandwidth: One-Sided

Bandwidth: the frequencies range of a signal. The way we express this range differs:

One-sided bandwidth → real-valued signals

- Real-valued signals are real in the time domain. Often denoted as "real signals".
- One-sided bandwidth: the range of positive frequencies. For real-valued signals, the negative frequencies are simply mirror images and contain no extra information.
- Physical signals (e.g., light, sound, Wi-Fi) are real-valued.

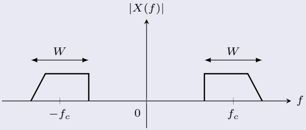




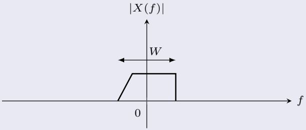
Wi-Fi Bands at 2.4 GHz

Two-sided bandwidth $\rightarrow$ complex-valued signals

- Complex-valued signals can take complex numbers. They describe the *complex envelope* of real-valued *passband* signals.
- Two-sided bandwidth: the range of negative-to-positive frequencies.
- Two-sided bandwidth of a complex-valued signal equals one-sided bandwidth of the corresponding real-valued passband signal.



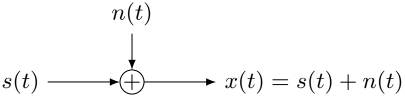
Passband signal



Complex-valued signal in the baseband

Channels

The additive white Gaussian noise (AWGN) channel is considered in this module:



Baseband signals

- Power concentrated in a band around DC

$$U(f) \approx 0, \quad |f| > W$$

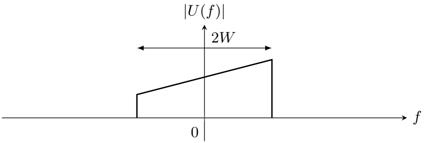
- Complex-valued in general, real-valued in special cases

Passband signals

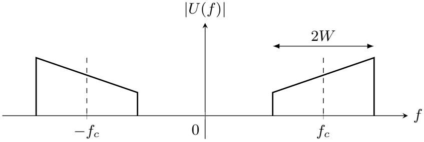
- Power concentrated around carrier frequency f_c that is away from DC

$$U(f) \approx 0, \quad |f \pm f_c| > W, \quad f_c \gg W$$

- Always real-valued



Baseband spectrum: not necessarily symmetric

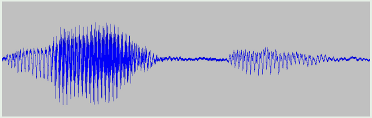


Passband spectrum: symmetric

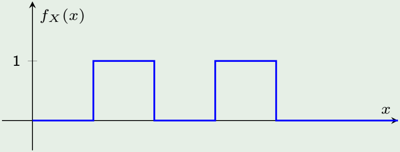
Examples

Real-valued baseband signals

- Speech and audio



- Two-level digital signal



We often want to send such signals over a passband channel (e.g., Wi-Fi channel with 20 MHz bandwidth).

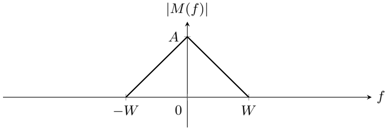
Modulation: Baseband to Passband

Consider a real-valued baseband message signal $m(t)$ with bandwidth W.

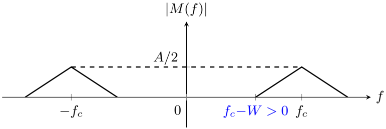
Modulation: Translating to passband by multiplying a sinusoid at frequency $f_c \gg W$

$$u_p(t) = m(t)\cos 2\pi f_c t \longrightarrow U_p(f) = \frac{1}{2}\big(M(f-f_c) + M(f+f_c)\big)$$

$$v_p(t) = m(t)\sin 2\pi f_c t \longrightarrow V_p(f) = \frac{1}{2j}\big(M(f-f_c) - M(f+f_c)\big)$$



Modulation
$\Rightarrow$



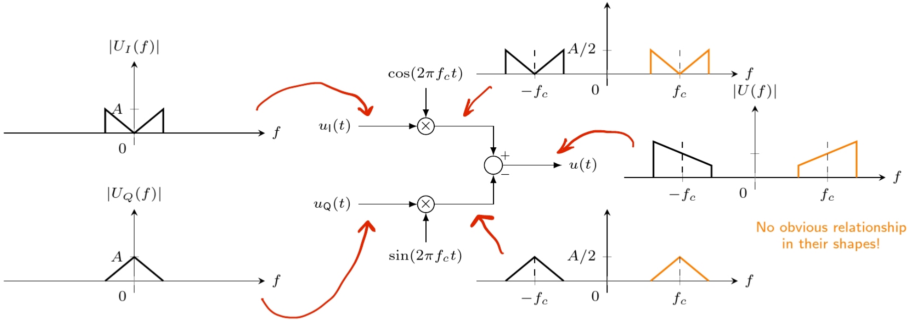
Can we modulate separately using cosine and sine carriers? Yes.

Sinusoids are rapidly varying but predictable (contain no info)

$$\underbrace{u(t)}_{\text{Passband signal}} = \underbrace{u_{\mathsf{I}}(t)}_{\text{In-phase (I) component}} \cos(2\pi f_c t) - \underbrace{u_{\mathsf{Q}}(t)}_{\text{Quadrature (Q) component}} \sin(2\pi f_c t)$$

Real baseband signals (contain all the info)

- $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ are rapidly varying but predictable (contain no information)
- $u_{\mathsf{I}}(t)$ and $u_{\mathsf{Q}}(t)$ are real baseband signals (contain all the information)
- How do we get back the I and Q components from the passband signal?
- Can any passband signal be decomposed into I and Q components?

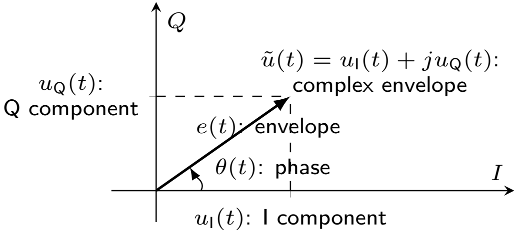


Since $u_I(t)$ and $u_Q(t)$ are real, their spectra are conjugate symmetric

- Block diagram follows directly from the equation defining the modulated signal
- Happens at the transmitter

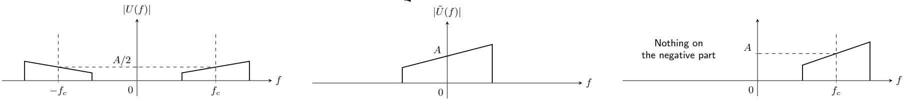
Baseband Signals

- Passband signal can be mapped to a pair of real baseband signals
- That is, passband modulation is two-dimensional
- We can also plot it on the complex plane



Complex Envelope and Passband Signal

$$\underbrace{u(t)}_{\substack{\text{real-valued}\\ \text{passband signal}}} = \mathrm{Re}\{ \underbrace{\tilde{u}(t)}_{\substack{\text{complex envelope}\\ \text{baseband signal}}} \exp(j2\pi f_c t)\}$$



$\mathcal{F}\{u(t)\}$ $\mathcal{F}\{\tilde{u}(t)\}$ $\mathcal{F}\{\tilde{u}(t)\exp(j2\pi f_c t)\}$

All information in a passband signal is contained in its complex envelope.

Time Domain Expressions for a Passband Signal

Passband signal expressions

- In I and Q components

$$u(t) = u_{\mathsf{I}}(t)\cos(2\pi f_c t) - u_{\mathsf{Q}}(t)\sin(2\pi f_c t)$$

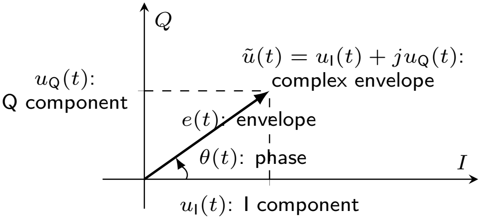
- In envelope and phase

$$u(t) = e(t)\cos\big(2\pi f_c t + \theta(t)\big)$$

- In complex envelope

$$u(t) = \mathrm{Re}\big\{\tilde{u}(t)\exp(j2\pi f_c t)\big\}$$

Starting from one representation, we can derive the rest based on the relations depicted in the figure.

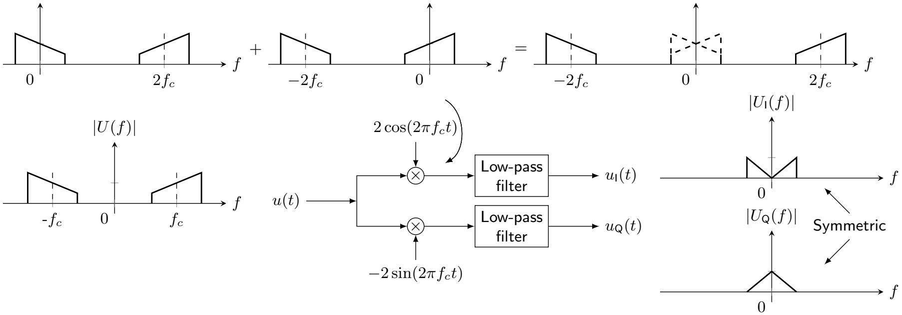


$$e(t) = \sqrt{u_{\mathsf{I}}^2(t) + u_{\mathsf{Q}}^2(t)},$$

$$\theta(t) = \arg(u_{\mathsf{I}}(t) + j u_{\mathsf{Q}}(t))$$

Complex baseband signal

$$\tilde{u}(t) = u_{\mathsf{I}}(t) + j u_{\mathsf{Q}}(t) = e(t)\exp\big(j\theta(t)\big)$$



Receiver needs to be coherent: same phase and frequency of the copy of the carrier at the receiver as those of the incoming signal

Hilbert Transform (HT)

- Hilbert transform of a signal $g(t)$: a linear transformation, defined as

$$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau} d\tau = g(t) * \frac{1}{\pi t}$$

- Inverse Hilbert transform

$$g(t) = -\frac{1}{\pi}\int_{\infty}^{\infty} \frac{\hat{g}(\tau)}{t-\tau} d\tau = -\hat{g}(t) * \frac{1}{\pi t}$$

- HT of $\hat{g}(t)$ is $-g(t)$: $\hat{\hat{g}}(t) = -g(t)$

- In the frequency domain, we have

$$\mathcal{F}\left\{\frac{1}{\pi t}\right\} = -j\mathrm{sgn}(f) = \begin{cases} -j, & f > 0, \\ 0, & f = 0, \\ j, & f < 0 \end{cases}$$

$$\hat{G}(f) = \mathcal{F}\left(\frac{1}{\pi t}\right) G(f) = -j\mathrm{sgn}(f) G(f), \quad \mathrm{sgn}(f) = \begin{cases} +1, & f > 0, \\ 0, & f = 0, \\ -1, & f < 0 \end{cases}$$

- HT introduces a phase shift of -90 degrees for all positive frequencies of the input signal, and +90 degrees for all negative frequencies.

Pre-Envelope

- Define the pre-envelope of a real signal $u(t)$ as the complex-valued function

$$u_+(t) = u(t) + j \underbrace{\hat{u}(t)}_{\mathcal{F}^{-1}\{-j\text{sgn}(f)U(f)\}}$$

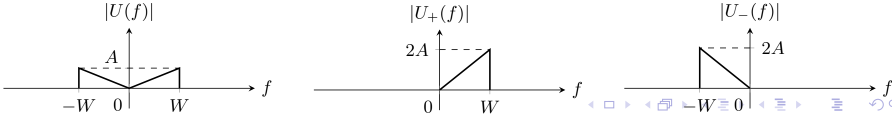
- Its Fourier transform:

$$U_+(f) = U(f) + \text{sgn}(f)U(f) = \begin{cases} 2U(f), & f > 0, \\ U(0), & f = 0, \\ 0, & f < 0 \end{cases}$$

- Pre-envelope removes the negative frequency components
- Similarly define the pre-envelope for negative frequencies

$$u_-(t) = u(t) - j\hat{u}(t)$$

$$U_-(f) = U(f) - \text{sgn}(f)U(f) = \begin{cases} 0, & f > 0, \\ U(0), & f = 0, \\ 2U(f), & f < 0 \end{cases}$$



Example

Consider arbitrary complex-valued baseband signal $\tilde{u}(t)$, whose spectrum is limited to $[-W, +W]$. Define

$$u(t) = \mathrm{Re}\{\tilde{u}(t)\exp(j2\pi f_c t)\}$$

Show that u(t) is a real-valued passband signal concentrated around $\pm f_c$.

Note