Lecture 8: Frequency Modulation (FM)

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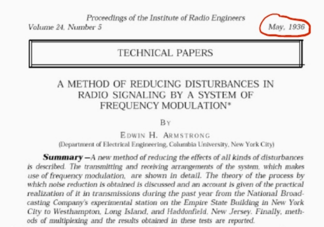
Outline

- FM system: model, bandwidth, etc.
- Demodulation and output SNR
- Reference
  - ✓ [Haykin] Chapter 6

Invention of FM

Edwin Howard Armstrong:

- Invented wideband FM
- Patented the regenerative circuit in 1914
- Presented *A Method of Reducing Disturbances in Radio Signalling by a System of Frequency Modulation* on 6 Nov. 1935
- This is the first paper to describe FM radio before the New York section of the Institute of Radio Engineers (now IEEE)
- Committed suicide in 1954



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TECHNICAL PAPERS

A METHOD OF REDUCING DISTURBANCES IN RADIO SIGNALING BY A SYSTEM OF FREQUENCY MODULATION*

By

EDWIN H. ARMSTRONG
(Department of Electrical Engineering, Columbia University, New York City)

Summary—*A new method of reducing the effects of all kinds of disturbances is described. The transmitting and receiving arrangements of the system, which makes use of frequency modulation, are shown in detail. The theory of the process by which noise reduction is obtained is discussed and an account is given of the practical realization of it in transmissions during the past year from the National Broadcasting Company's experimental station on the Empire State Building in New York City to Westhampton, Long Island, and Haddonfield, New Jersey. Finally, methods of multiplexing and the results obtained in these tests are reported.*

FM vs AM

Fundamental difference:

- AM: information contained in the signal amplitude
  - ✓ additive noise: corrupting the modulated signal directly
- FM: information contained in the signal frequency
  - ✓ additive noise: affecting the signal by changing the frequency of the modulated signal
  - ✓ consequently, affected less by noise than AM

Frequency and Phase

- A carrier waveform

$$s(t) = A\cos \underbrace{\theta_i(t)}_{\text{instantaneous phase angle}}$$

- Constant frequency

$$s(t) = A\cos(2\pi f t + \theta) \Rightarrow \theta_i(t) = 2\pi f t + \theta$$

$$\frac{d\theta_i(t)}{dt} = 2\pi f \Rightarrow f = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$

- Generalization: instantaneous frequency

$$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}, \quad \theta_i(t) = 2\pi\int_0^t f_i(\tau)d\tau + \theta_i(0)$$

- Instantaneous frequency: varied linearly with message

$$f_i(t) = f_c + \underbrace{k_f}_{\text{frequency sensitivity of the modulator}} m(t)$$

- Instantaneous phase angle

$$\theta_i(t) = 2\pi \int_0^t f_i(\tau)d\tau + \theta_i(0) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau, \quad (\theta_i(0) = 0)$$

- FM signal:

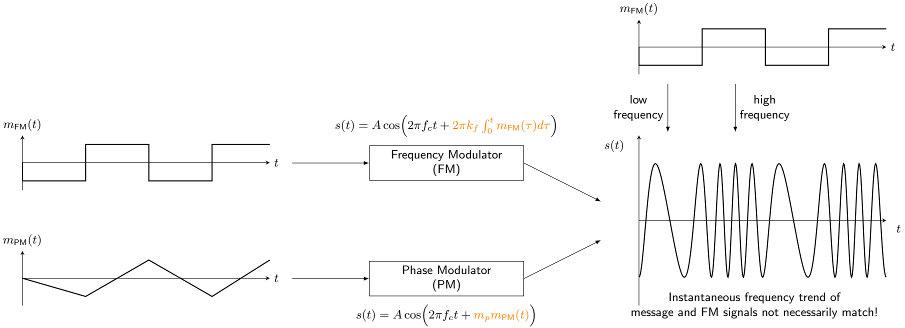
$$s(t) = A\cos\Big(2\pi f_c t + 2\pi k_f \int_0^t m(\tau)d\tau\Big) + \theta_i(0)$$

- Phase modulation (PM) signal:

$$s(t) = A\cos\big(2\pi f_c t + k_p m(t)\big)$$

- FM or PM signals:
  - ✓ constant envelope
  - ✓ non-linear function of the message signal, $m(t)$

PM-FM Equivalence



- FM signal $=$ PM signal with the modulating signal $\int_0^t m(\tau)d\tau$
- Similar properties for PM and FM
- Focusing on FM

- $m_p = \max|m(t)|$: peak message amplitude
- Frequency deviation: deviation of $f_i(t)$ from the carrier frequency, f_c,

$$f_c - k_f m_p \leq f_i(t) \leq f_c + k_f m_p$$

$$\Delta f = k_f m_p$$

- Deviation ratio/Modulation index:

$$\beta = \frac{\Delta f}{W}$$

  ✓ W: message bandwidth

Sinusoidal message $m(t) = A_m \cos(2\pi f_m t)$

Bandwidth of FM

- Deviation ratio/Modulation index:

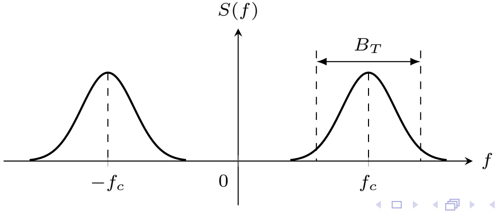
$$\beta = \frac{\Delta f}{W}$$

  - ✓ W: message bandwidth
  - ✓ β small: narrowband FM
  - ✓ β large: wideband FM

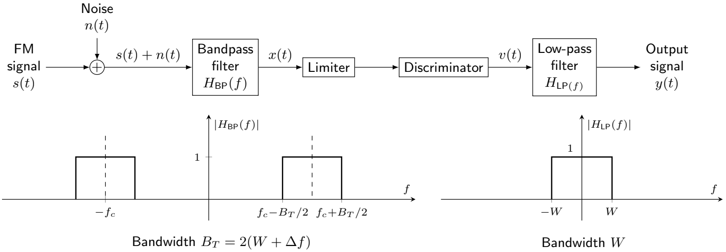
- Carson's rule of thumb: Transmission bandwidth of FM

$$B_T = 2W(\beta + 1) = 2(\Delta f + W)$$

  - ✓ $\beta \ll 1 \Rightarrow B_T \approx 2W$ (as in AM)
  - ✓ $\beta \gg 1 \Rightarrow B_T \approx 2\Delta f$



FM Receiver



- Bandpass filter: removes signals outside bandwidth of $f_c \pm B_T/2$
  - ✓ predetection noise at the receiver is bandpass with a bandwidth of B_T
- FM signal with a *constant envelope*
  - ✓ use a limiter to remove any amplitude variations
- Discriminator: a device with instantaneous amplitude proportional to instantaneous frequency
  - ✓ recovering the message signal
- Final baseband low-pass filter: a bandwidth of W
  - ✓ removing out-of-band noise

Linear Argument at High SNR

- FM: nonlinear modulation and demodulation, no superposition principle
- For *high SNR*, noise and message signals are approximately independent of each other:

$$\text{Output} \approx \text{Message} + \text{Noise (i.e., no other nonlinear terms)}$$
$$y(t) \approx k_f m(t) + n_0(t)$$

(will show)

Phase Noise at High SNR

$$
\begin{aligned}
x(t) &= A\cos\big(2\pi f_c t + \phi(t)\big) + n_I(t)\cos(2\pi f_c t) + n_Q(t)\sin(2\pi f_c t) \\
&= A\cos\big(2\pi f_c t + \phi(t)\big) + r(t)\cos\big(2\pi f_c t + \psi(t)\big)
\end{aligned}
$$

$$
\phi(t) = 2\pi k_f \int_0^t m(\tau)d\tau
$$

- Instantaneous phase of the resultant phasor:

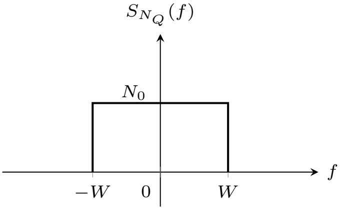
$$
\begin{aligned}
\theta(t) &= \phi(t) + \arctan\left(\frac{r(t)\sin\big(\psi(t)-\phi(t)\big)}{A + r(t)\cos\big(\psi(t)-\phi(t)\big)}\right) \quad \big(\arctan(x) \approx x \text{ if } |x| \ll 1\big) \\
&\approx \phi(t) + \frac{r(t)}{A}\sin\big(\psi(t)-\phi(t)\big)
\end{aligned}
$$

Discriminator output:

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} \approx k_f m(t) + n_d(t)$$

where the *additive* noise term is

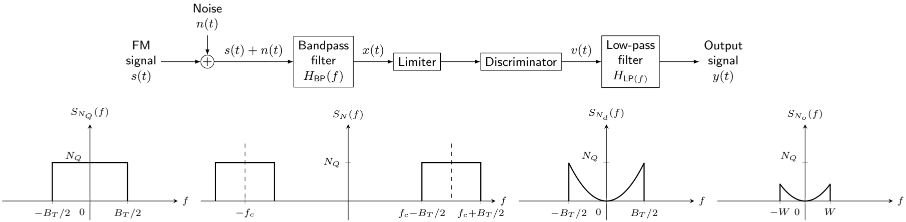
$$\begin{aligned} n_d(t) &= \frac{1}{2\pi A}\frac{d}{dt} r(t)\sin\big(\psi(t) - \phi(t)\big) \quad \text{(HW1.3)} \\ &\approx \frac{1}{2\pi A}\frac{d}{dt} r(t)\sin\big(\psi(t)\big) \\ &= \frac{1}{2\pi A}\frac{d}{dt} n_Q(t) \end{aligned}$$



with PSD

$$S_{N_0}(f) = \left(\frac{1}{2\pi A^2}\right)(2\pi f)^2 S_{N_Q}(f) = \frac{f^2}{A^2} N_0, \quad |f| \leq W \quad \text{(HW1.8)}$$

Noise PSD



- $S_{N_Q}(f)$: PSD of $n_Q(t)$ of narrowband noise $n(t)$
- $S_{N_d}(f)$: PSD of $n_d(t)$ at the discriminator output
- $S_{N_o}(f)$: PSD of $n_o(t)$ at the receiver output

Noise Power

- Average noise power at the receiver output

$$P_N = \int_{-W}^{W} S_{N_0}(f)df = \int_{-W}^{W} \frac{f^2}{A^2} N_0 df = \frac{2N_0W^3}{3A^2}$$

- Average noise power at the output of an FM receiver

$$\propto \frac{1}{\text{carrier power } A^2}$$

- $A \uparrow \Rightarrow$ noise $\downarrow$, called the noise-quieting effect

Output SNR

- $P_S = k_f^2 P$, $P_N = 2N_0 W^3/3A^2$

$$\text{SNR}_{\text{FM}} = \frac{P_S}{P_N} = \frac{3A^2 k_f^2 P}{2N_0 W^3}$$

- For baseband transmission,

$$\text{SNR}_{\text{baseband}} = \frac{P_T}{N_0 W} = \frac{A^2/2}{N_0 W} = \frac{A^2}{2N_0 W}$$

- $P_T = A^2/2$, $\beta = k_f m_p / W$

$$\begin{aligned} \text{SNR}_{\text{FM}} &= \frac{3k_f^2 P}{W^2}\text{SNR}_{\text{baseband}} = 3\beta^2 \frac{P}{m_p^2}\text{SNR}_{\text{baseband}} \\ &\propto \beta^2 \text{SNR}_{\text{baseband}} \text{(could be much higher than AM)} \end{aligned}$$

- Valid for large carrier power
- SNR_{FM}: quadratically increasing with β

Threshold Effect

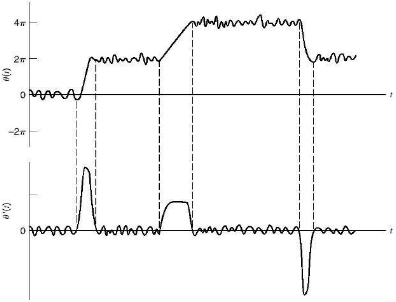
- A more pronounced threshold effect than AM envelope detector
- Threshold point at

$$\text{Carrier-to-noise ratio:} \quad \rho = \frac{A^2}{2N_0 B_T} \approx 10, \quad B_T = 2W(\beta + 1)$$

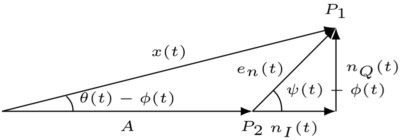
- FM receiver breaks (i.e., significantly deteriorates) at $\rho < 10$
- Analyzed by *S. O. Rice* (very complicated!), the noise in FM receiver is called "click noise" or "Rice noise"

As noise changes randomly, point P_1 wanders around P_2

- High SNR: change of angle is small
- Low SNR: P_1 occasionally sweeps around origin, resulting in changes of 2π in a short time

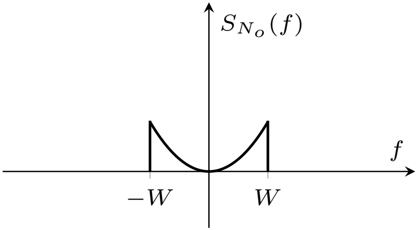


Phase noise

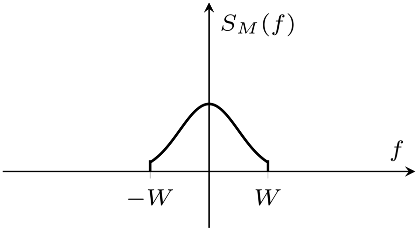


Phasor diagram of the FM carrier and noise signals

Improve Output SNR



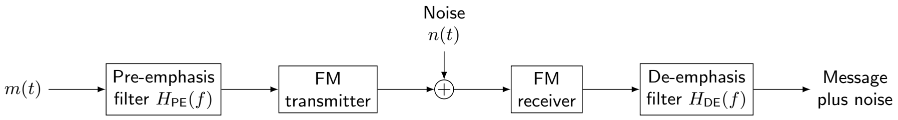
Noise PSD



Message PSD

- Noise PSD at detector output $\propto$ square of frequency
- Message PSD typically decays towards the ends of its band

Pre-emphasis and De-emphasis



- $H_{\text{PE}}(f)$: artificially emphasizes high frequency components of the message prior to modulation (before noise is introduced)
- $H_{\text{DE}}(f)$: de-emphasizes high frequency components at the receiver, and restore the original PSD of the message
- In theory, $H_{\text{PE}}(f) \propto f$, $H_{\text{DE}}(f) \propto 1/f$
- This can improve output SNR by around 13 dB

- Assumptions:
  - ✓ single-tone modulation $m(t) = A_m \cos(2\pi f_m t)$
  - ✓ message bandwidth $W = f_m$
  - ✓ for AM system, modulation index $\mu = m_p/A = A_m/A = 1$, $m_p = \max|m(t)| = A_m$
  - ✓ for FM system, modulation index $\beta = \Delta f/W = 5$, $\Delta f = k_f m_p = k_f A_m$ (used in commercial FM transmission with $\Delta f = 75$ kHz and $W = 15$ kHz)

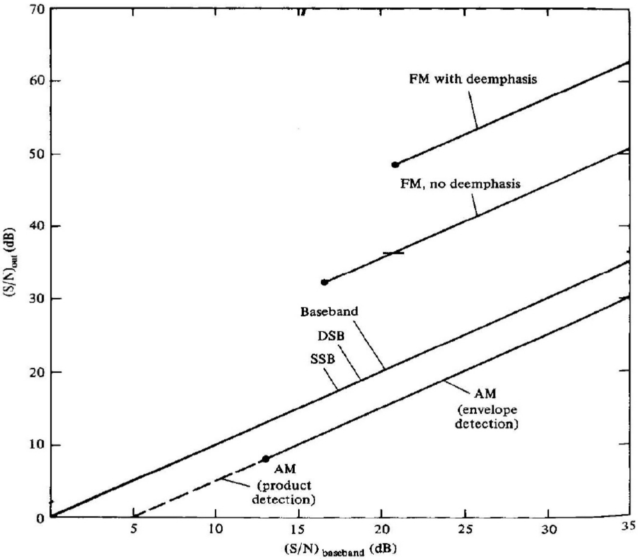
- SNR expressions for various modulation schemes

$$\mathsf{SNR}_{\mathsf{DSB\text{-}SC}} = \mathsf{SNR}_{\mathsf{baseband}} = \mathsf{SNR}_{\mathsf{SSB}}$$

$$\mathsf{SNR}_{\mathsf{AM}} = \frac{P}{A^2 + P}\mathsf{SNR}_{\mathsf{baseband}} = \frac{\mu^2}{2+\mu^2}\mathsf{SNR}_{\mathsf{baseband}} \leq \frac{1}{3}\mathsf{SNR}_{\mathsf{baseband}}$$

$$\mathsf{SNR}_{\mathsf{FM}} = \frac{3\beta^2}{2}\mathsf{SNR}_{\mathsf{baseband}} = \underbrace{\frac{75}{2}}_{15.7\text{dB}}\mathsf{SNR}_{\mathsf{baseband}} \quad \text{(without pre/de-emphasis)}$$

Performance of Analog Systems



Conclusions

- (Full) AM:
  - ✓ SNR: 4.8 dB worse than a baseband system
  - ✓ transmission bandwidth: $B_T = 2W$
- DSB:
  - ✓ SNR: identical to a baseband system
  - ✓ transmission bandwidth: $B_T = 2W$
- SSB:
  - ✓ SNR: again identical
  - ✓ transmission bandwidth: $B_T = W$
- FM:
  - ✓ SNR: 15.7 dB better than a baseband system
  - ✓ transmission bandwidth: $B_T = 2(\beta + 1)W = 12W$
  - ✓ with pre- and de-emphasis, SNR is increased by 13 dB with the same bandwidth

Note