Lecture 8: Noise Performance of Single Sideband (SSB) and Conventional Amplitude Modulation (AM)

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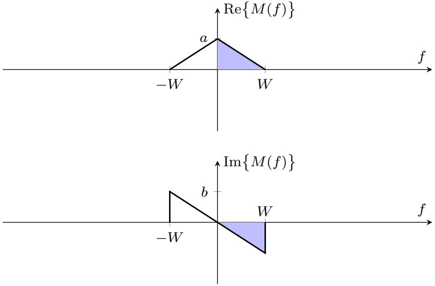
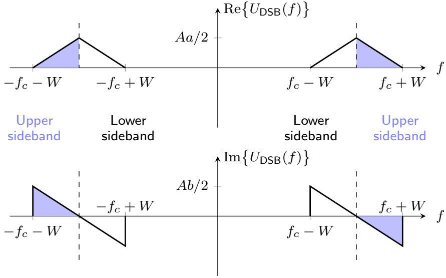
Outline

- Noise in SSB
- Noise in standard AM
  - ✓ coherent detection (of theoretic interest only)
  - ✓ envelope detection
- SNR of SSB and AM
- Reference
  - ✓ [Haykin] Chapter 6

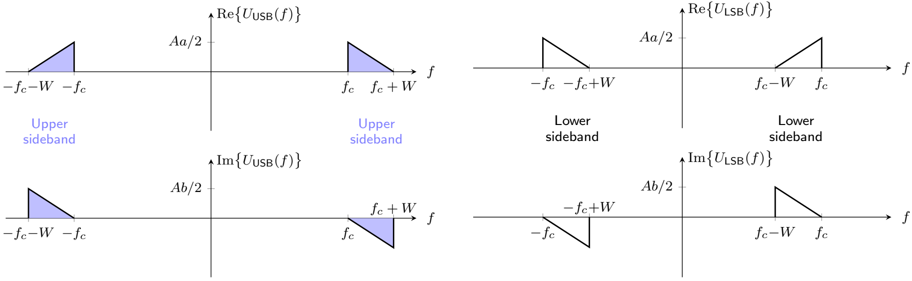


Real message signal has conjugate symmetric spectrum



DSB signal spectrum

One side band is enough to reconstruct the message!

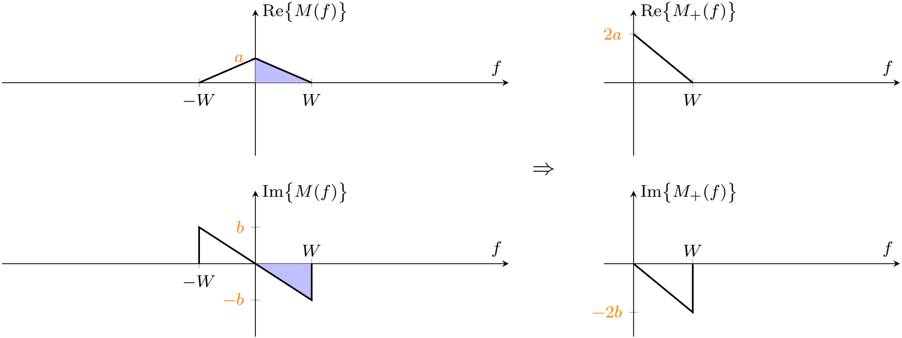


Upper sideband signaling (high freq. side)

Lower sideband signaling (lower freq. side)

Message can be recovered by moving SSB components left and right by f_c, and low pass filtering (just like DSB).

Upper Sideband SSB Modulated Signal: I

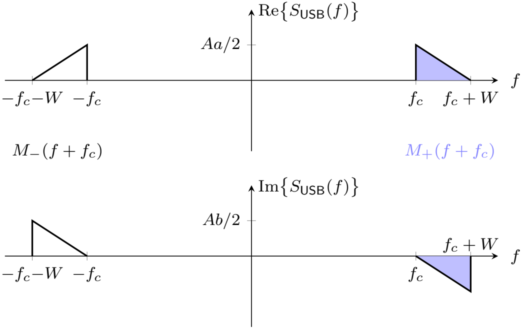


$$m(t) \Longleftrightarrow M(f), \quad m_+(t) = m(t) + j\hat{m}(t) \Longleftrightarrow M_+(f) = \begin{cases} 2M(f), & f > 0, \\ M(0), & f = 0, \\ 0, & f < 0 \end{cases}$$

Pre-envelope removes the negative frequency components.

Upper Sideband SSB Modulated Signal: II

Move positive frequency part $m_+(t)$ to f_c



Move negative frequency part $m_-(t)$ to $-f_c$

$$s(t) = \frac{A}{4} m_+(t) \exp(j2\pi f_c t) + \frac{A}{4} m_-(t) \exp(-j2\pi f_c t) \quad \text{(is it real?)}$$

$$S(f) = \frac{A}{4} M_+(f - f_c) + \frac{A}{4} M_-(f + f_c)$$

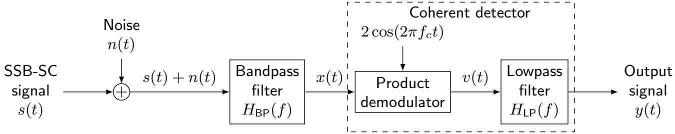
Upper Sideband SSB Modulation

$$\begin{aligned} s(t) &= \frac{A}{4} m_+(t) \exp(j2\pi f_c t) + \frac{A}{4} m_-(t) \exp(-j2\pi f_c t) \\ &= \frac{A}{4}\big(m(t) + j\hat{m}(t)\big) \exp(j2\pi f_c t) + \frac{A}{4}\big(m(t) - j\hat{m}(t)\big) \exp(-j2\pi f_c t) \\ &= \frac{A}{2} m(t) \cos(2\pi f_c t) - \frac{A}{2} \hat{m}(t) \sin(2\pi f_c t) \quad \text{(it is real!)} \end{aligned}$$

- I component: the message $m(t)$
- Q component: its Hilbert transform $\hat{m}(t)$
- $m(t)$ and $\hat{m}(t)$ have the same power P (why?)
- Transmission (signal) power: $P_T = A^2 P/4$, $P = \mathbb{E}\{m^2(t)\}$

Noise in Upper Sideband SSB

- Received signal $x(t) = s(t) + n(t)$, apply a band-pass filter on the upper sideband
- Still denote by $n_I(t)$ the upper-sideband in-phase noise (different from the double-sideband noise in DSB)

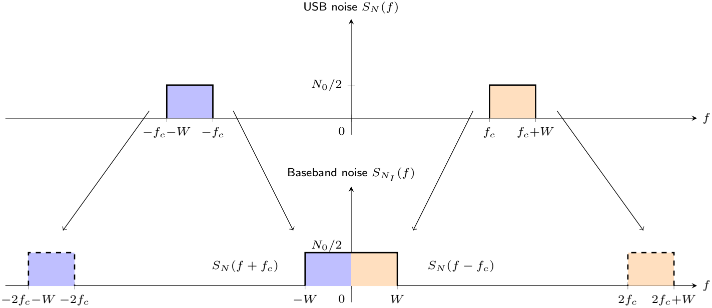


- Demodulation output (after low-pass filtering)

$$y(t) = \left(\frac{A}{2}m(t) + n_I(t)\right)$$

Noise Power

Noise power of $n_I(t)$ = power of band-pass noise $= N_0 W$ (halved compared to DSB)



$$S_{N_I}(f) = \begin{cases} S_N(f+f_c) + S_N(f-f_c) = N_0/2, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

$$P_N = \int_{-W}^{W} \frac{N_0}{2} df = N_0 W \quad \text{(halved compared to DSB)}$$

Output SNR

- Signal power at the receiver output:

$$P_S = \mathbb{E}\{(A/2)^2 m^2(t)\} = \frac{A^2}{4}\mathbb{E}\{m^2(t)\} = \frac{A^2 P}{4} \quad \text{(1/4 of DSB)}$$

- SNR at the receiver output:

$$\text{SNR}_{\text{SSB}} = \frac{P_S}{P_N} = \frac{A^2 P}{4N_0 W}$$

- Transmit power: $P_T = A^2 P/4$ (halved compared to DSB)

- Conclusion: SSB has the same SNR performance as DSB-SC and baseband systems, but only requires half the bandwidth!

DSB, SSB, and standard AM

- DSB:

$$s_{\mathsf{DSB}}(t) = m(t)A\cos(2\pi f_c t)$$

- SSB:

$$s_{\mathsf{SSB}}(t) = \frac{A}{2}m(t)\cos(2\pi f_c t) - \frac{A}{2}\hat{m}(t)\sin(2\pi f_c t)$$

- Standard AM:

$$s_{\mathsf{AM}}(t) = \big(A + m(t)\big)\cos(2\pi f_c t)$$

To ensure non-coherent demodulation,

$$A \geq m_p = \max|m(t)| \quad \text{or} \quad \text{modulation index } \mu = \frac{m_p}{A} \leq 1$$

Coherent detection is sometimes referred to as synchronous recovery.

- Pre-detection signal:

$$
\begin{aligned}
x(t) &= \big(A + m(t)\big)\cos(2\pi f_c t) + n(t) \\
&= \big(A + m(t)\big)\cos(2\pi f_c t) + n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t) \\
&= \big(A + m(t) + n_I(t)\big)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)
\end{aligned}
$$

- Multiply with $2\cos(2\pi f_c t)$:

$$
y(t) = \big(A + m(t) + n_I(t)\big)\big(1 + \cos(4\pi f_c t)\big) - n_Q(t)\sin(4\pi f_c t)
$$

- LPF

$$
\tilde{y} = A + m(t) + n_I(t)
$$

Output SNR

- Signal power at the receiver output:

$$P_S = \mathbb{E}\{m^2(t)\} = P$$

- Noise power:

$$P_N = 2N_0W \quad \text{(same as DSB)}$$

- SNR at the receiver output:

$$\mathsf{SNR}_{\mathsf{AM}} = \frac{P}{2N_0W}$$

- Transmitted power

$$P_T = \frac{A^2}{2} + \frac{P}{2} = \frac{A^2 + P}{2} \quad \text{(with } A^2 \text{ term!)}$$

Comparison

- SNR of a baseband system with the same transmitted power:

$$\text{SNR}_{\text{baseband}} = \frac{P_T}{N_0 W} = \frac{A^2 + P}{2N_0 W}$$

- Thus

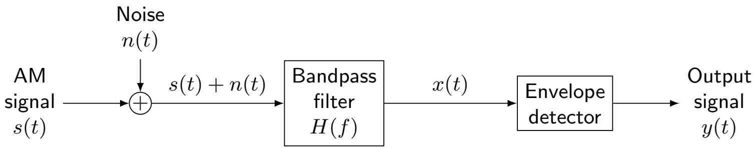
$$\text{SNR}_{\text{AM}} = \frac{P}{2N_0 W} = \frac{P}{A^2 + P}\frac{A^2 + P}{2N_0 W} = \frac{P}{A^2 + P}\text{SNR}_{\text{baseband}}$$

- Note

$$\frac{P}{A^2 + P} < 1$$

- Conclusion: performance of standard AM with coherent detection is worse than a baseband system.

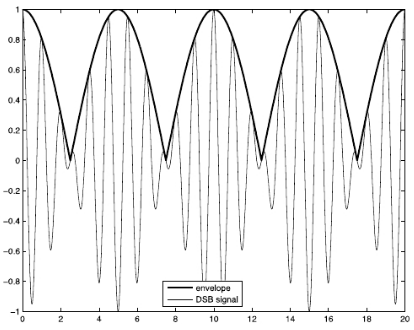
- The envelope (or magnitude of complex envelope) does not depend on carrier phase
- Extracting the envelope of a passband signal does not require carrier sync
- Can we recover the message from the envelope?



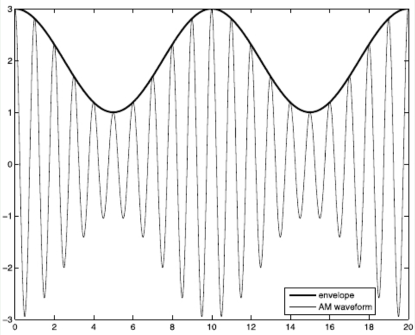
Model of AM envelope detector

What does the envelope tell us?

Example: sinusoidal message waveform



DSB modulated signal



DSB signal + strong carrier component

- Envelope = message magnitude
- Envelope detection loses message sign

- Envelope = message + DC
- Envelope detector + DC block recovers message info

Threshold Effect

- Small noise case:
  - ✓ Almost same performance as coherent detection (assume $\mu \leq 1$ or $m_P \leq A$)
- Large noise case:
  - ✓ Information is lost!
  - ✓ Threshold effect: below some carrier-to-noise ratio level (very low A), performance of envelope detector deteriorates very rapidly (not the case in coherent detection)

Summary

(De-) modulation format	Output SNR	Transmitted power	Baseband reference SNR	Output SNR / reference SNR
AM coherent detection	$\frac{P}{2N_0W}$	$\frac{A^2+P}{2}$	$\frac{A^2+P}{2N_0W}$	$\frac{P}{A^2+P} < 1$
DSB-SC coherent detection	$\frac{A^2P}{2N_0W}$	$\frac{A^2P}{2}$	$\frac{A^2P}{2N_0W}$	1
SSB coherent detection	$\frac{A^2P}{4N_0W}$	$\frac{A^2P}{4}$	$\frac{A^2P}{4N_0W}$	1
AM envelope detection (small noise)	$\frac{P}{2N_0W}$	$\frac{A^2+P}{2}$	$\frac{A^2+P}{2N_0W}$	$\frac{P}{A^2+P} < 1$
AM envelope detection (large noise)	Poor	$\frac{A^2+P}{2}$	$\frac{A^2+P}{2N_0W}$	Poor

- A: carrier amplitude
- P: power of message signal
- N_0: single-sided PSD of noise
- W: message bandwidth

Note