Lecture 6: Noise Performance of Double Sideband (DSB)

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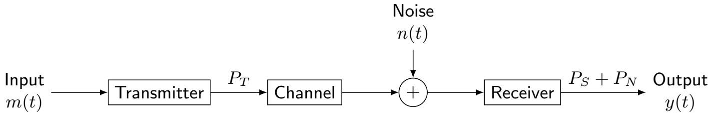
Outline

- SNR of baseband analogue transmission
- Noise in Double Sideband-Suppressed Carrier (DSB-SC)
- SNR of DSB-SC
- Reference
  - ✓ [Haykin] Chapter 6

Noise in Analog Communication Systems

- How do various analog modulation schemes perform in the presence of noise?
- Which scheme performs best?
- How can we measure its performance?



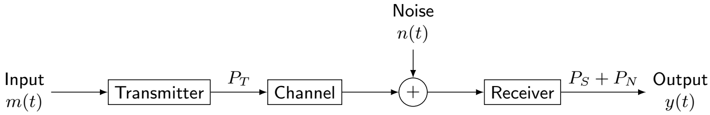
Signal-to-Noise Ratio

Signal-to-noise ratio (SNR) at the output of the receiver:

$$\text{SNR} \triangleq \frac{\text{average power of message signal at the receiver output}}{\text{average power of noise at the receiver output}} = \frac{P_S}{P_N}$$

- Normally expressed in decibels (dB): $\text{SNR}_{\text{dB}} = 10\log_{10}(\text{SNR})$
- Managing the wide range of power levels

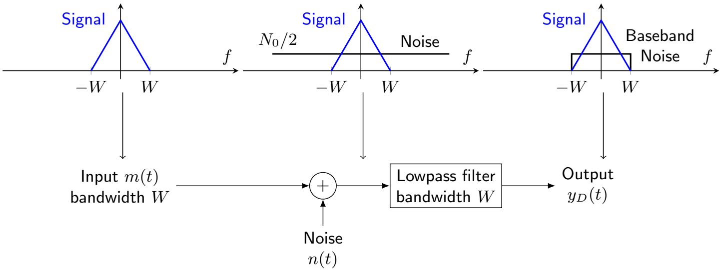
Transmitted Power



- Higher transmitted power $P_T \Rightarrow$ higher received power $P_S \Rightarrow$ higher SNR
- Limited by: equipment capability, battery life, cost, government restrictions, interference with other channels, ...
- For a fair comparison between different modulation schemes, P_T should be the same & so is PSD of noise
- Baseband SNR, $\text{SNR}_{\text{baseband}}$: calibrate and compare the SNR values we obtain

A Baseband Communication System

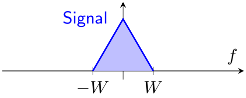
- No modulation
- Suitable for transmission over wires
- Transmit power P_T = message power P
- Unit channel gain or no propagation loss $P_S = P_T = P$
- Results can be extended to bandpass systems



Baseband SNR

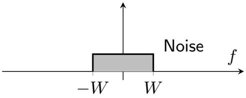
- Average signal (= message) power

$$P = \text{area under the triangular curve}$$



- Noise power
  - ✓ AWGN power spectral density

$$\text{PSD} = N_0/2$$



  - ✓ average noise power at receiver

$$P_N = \text{area under the straight line} = 2WN_0/2 = WN_0$$

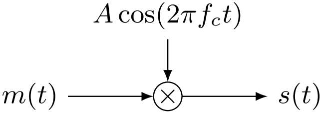
- SNR at receiver output:

$$\text{SNR}_{\text{baseband}} = \frac{P_T}{N_0 W}$$

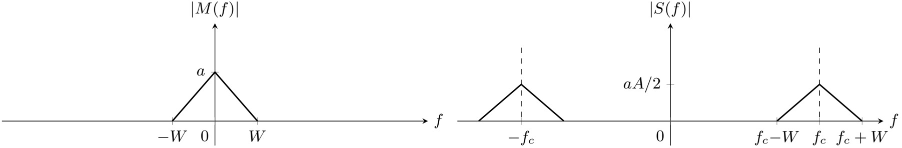
- Improve SNR by:
  - ✓ increasing the transmitted power $P_T \uparrow$
  - ✓ making the channel/receiver less noisy $N_0 \downarrow$

Double Sideband-Suppressed Carrier (DSB-SC) Modulation

$$s(t) = m(t)A\cos(2\pi f_c t)$$

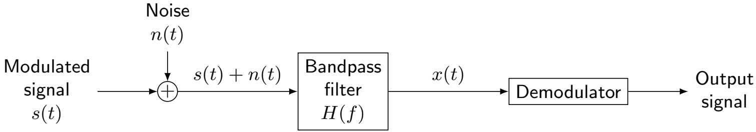


- A: amplitude of the carrier
- f_c: carrier frequency
- $m(t)$: message signal with $\mathbb{E}\{|m(t)|^2\} = P$

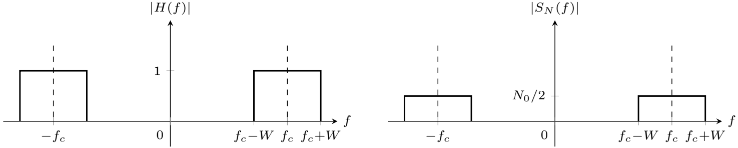


DSB-SC Receiver with Noise

- Receiver



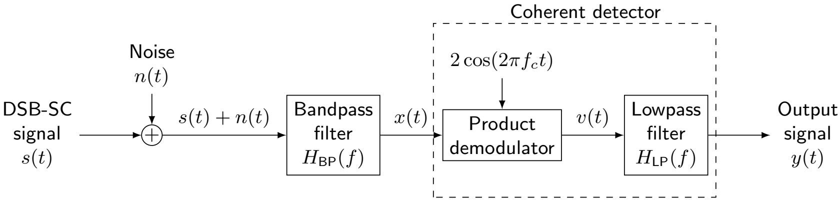
- Received signal
  - ✓ Bandpass filter



  - ✓ Received and filtered noisy signal:

$$
\begin{aligned}
x(t) &= s(t) + n(t) \\
&= Am(t)\cos(2\pi f_c t) + n_{\mathsf{I}}(t)\cos(2\pi f_c t) - n_{\mathsf{Q}}(t)\sin(2\pi f_c t) \\
&= \big(Am(t) + n_{\mathsf{I}}(t)\big)\cos(2\pi f_c t) - n_{\mathsf{Q}}(t)\sin(2\pi f_c t)
\end{aligned}
$$

- Synchronous detection = Product detection = Coherent detection
- "Detection" and "demodulation" are used interchangeably

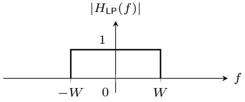


- Multiply $x(t)$ with $2\cos(2\pi f_c t)$:

$$\begin{aligned} v(t) &= 2\cos(2\pi f_c t)\cdot x(t) \\ &= \big(Am(t)+n_{\mathsf{I}}(t)\big)\big(\cos(4\pi f_c t)+1\big) - n_{\mathsf{Q}}(t)\sin(4\pi f_c t) \\ &= \big(Am(t)+n_{\mathsf{I}}(t)\big) + \underbrace{\big(Am(t)+n_{\mathsf{I}}(t)\big)\cos(4\pi f_c t) - n_{\mathsf{Q}}(t)\sin(4\pi f_c t)}_{\text{High frequency around } \pm 2f_c} \end{aligned}$$

- LP filter output

$$y(t) = Am(t) + n_{\mathsf{I}}(t)$$



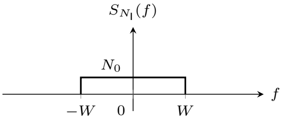
- Signal power at the receiver output

$$P_S = \mathbb{E}\{A^2 m^2(t)\} = A^2 \mathbb{E}\{m^2(t)\} = A^2 P$$

- Noise power

$$S_{N_{\mathsf{I}}}(f) = \begin{cases} S_N(f+f_c) + S_N(f-f_c) = N_0, & |f| \le W \\ 0, & \text{otherwise} \end{cases}$$

$$P_N = \int_{-W}^{W} N_0 df = 2N_0 W$$



Comparison

- SNR at the receiver output:

$$\mathsf{SNR}_{\text{DSB-SC}} = \frac{P_S}{P_N} = \frac{A^2 P}{2N_0 W}$$

- Transmit power:

$$P_T = \mathbb{E}\{A^2 m^2(t)\cos^2(2\pi f_c t)\} = \frac{1}{T}\int_0^T A^2\cos^2(2\pi f_c t)\mathbb{E}\{m^2(t)\}dt$$
$$= \frac{1}{T}\int_0^T A^2\cos^2(2\pi f_c t) P dt = \frac{A^2 P}{2}$$

- Compare with baseband transmission $\mathsf{SNR}_{\text{DSB-SC}} = \frac{A^2 P}{2N_0 W} = \frac{P_T}{N_0 W} = \mathsf{SNR}_{\text{baseband}}$

- Conclusion: DSB-SC system has the same SNR performance as a baseband system

Note