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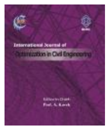

A NOVEL META-HEURISTIC ALGORITHM: TUG OF WAR OPTIMIZATION

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ABSTRACT

This paper presents a novel population-based meta-heuristic algorithm inspired by the game of tug of war. Utilizing a sport metaphor the algorithm, denoted as Tug of War Optimization (TWO), considers each candidate solution as a team participating in a series of rope pulling competitions. The teams exert pulling forces on each other based on the quality of the solutions they represent. The competing teams move to their new positions according to Newtonian laws of mechanics. Unlike many other meta-heuristic methods, the algorithm is formulated in such a way that considers the qualities of both of the interacting solutions. TWO is applicable to global optimization of discontinuous, multimodal, non-smooth, and non-convex functions. Viability of the proposed method is examined using some benchmark mathematical functions and engineering design problems. The numerical results indicate the efficiency of the proposed algorithm compared to some other methods available in literature.

Keywords: tug of war optimization; meta-heuristic algorithm; mathematical functions; optimal design; truss structures.

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1. INTRODUCTION

Nature-inspired meta-heuristic algorithms have been celebrated as powerful global optimization techniques in the last few decades. These methods do not require any gradient information of the involved functions and are generally independent of the starting points. Due to these characteristics, meta-heuristic optimizers are favorable choices when dealing with discontinuous, multimodal, non-smooth, and non-convex functions. This is especially the case when near-global optimum solutions are sought and the intended computational effort is limited.

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Many different meta-heuristic optimization techniques have been presented and successfully applied to different problems. Examples include Genetic Algorithms (GA) [1], Particle Swarm Optimization (PSO) [2], Ant Colony Optimization (ACO) [3], Harmony Search (HS) [4], Big Bang-Big Crunch (BB-BC) [5], Charged System Search (CSS) [6], Magnetic Charged System Search (MCSS) [7], Ray Optimization (RO) [8], Democratic PSO (DPSO) [9], Dolphin Echolocation (DE) [10], Colliding Bodies Optimization (CBO) [11], Water Cycle, Mine Blast and Improved Mine Blast algorithms (WC-MB-IMB) [12], Search Group Algorithm (SGA) [13] and the Ant Lion Optimizer (ALO) [14]. Some applications of meta-heuristics on optimization of structural engineering problems could be found in [15-19].

The aim of the present paper is to introduce a new meta-heuristic algorithm based on the physical phenomenon associated with the game of tug of war. The algorithm, which is called Tug of War Optimization (TWO), is then compared to some of the documented optimization techniques using benchmark problems. The remainder of the paper is organized as follows. The new algorithm together with its physical background is presented in Section 2. In Section 3 some mathematical and engineering design benchmark problems are studied in order to demonstrate the efficiency of the proposed algorithm. The concluding remarks are presented in Section 4.

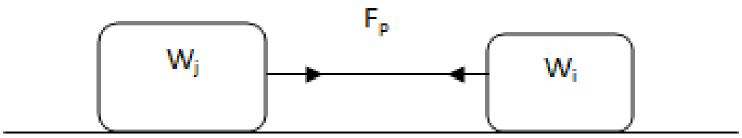
2. TUG OF WAR OPTIMIZATION

2.1 Idealized tug of war framework

Tug of war or rope pulling is a strength contest in which two competing teams pull on the opposite ends of a rope in an attempt to bring the rope in their direction against the pulling force of the opposing team. The activity dates back to ancient times and has continued to exist in different forms ever since. There has been a wide variety of rules and regulations for the game but the essential part has remained almost unaltered. Naturally, as far as both teams sustain their grips of the rope, movement of the rope corresponds to the displacement of the losing team. Fig. 1 shows one of the competing teams in a tug of war.

Figure 1. A competing team in a tug of war

Triumph in a real game of tug of war generally depends on many factors and could be difficult to analyze. However, an idealized framework is utilized in this paper where the two teams having weights W_i and W_j are considered as two objects lying on a smooth surface as shown in Fig. 2.



μ_s : Static coefficient of friction
μ_k : Kinematic coefficient of friction

Figure 2. An idealized tug of war framework

As a result of pulling the rope, the teams experience two equal and opposite forces (F_p) according to Newton's third law. For object *i*, as far as the pulling force is smaller than the maximum static friction force $(W_i \mu_s)$ the object rests in its place. Otherwise the non-zero resultant force can be calculated as:

$$F_r = F_p - W_i \mu_k \tag{1}$$

As a result, the object *i* accelerates towards the object *j* according to the Newton's second law:

$$a = \frac{F_r}{\left(W_i / g \right)} \tag{2}$$

Since the object *i* starts from zero velocity, its new position can be determined as:

$$X_i^{new} = \frac{1}{2} a t^2 + X_i^{old} \tag{3}$$

2.2 Tug of war optimization algorithm

TWO is a population-based meta-heuristic algorithm, which considers each candidate solution $X_i = \{x_{i,j}\}$ as a team engaged in a series of tug of war competitions. The weight of the teams is determined based on the quality of the corresponding solutions, and the amount of pulling force that a team can exert on the rope is assumed to be proportional to its weight. Naturally, the opposing team will have to maintain at least the same amount of force in order to sustain its grip of the rope. The lighter team accelerates toward the heavier team and this forms the convergence operator of the TWO. The algorithm improves the quality of the solutions iteratively by maintaining a proper exploration/exploitation balance using the described convergence operator. The steps of TWO can be stated as follows:

Step 1: Initialization

A population of *N* initial solutions is generated randomly:

$$x_{ij}^{0} = x_{j,min} + rand(x_{j,max} - x_{j,min}) \qquad j = 1,2,\ldots,n \tag{4}$$

where x_{ij}^{0} is the initial value of the jth variable of the ith candidate solution; $x_{j,\max}$ and $x_{j,\min}$ are the maximum and minimum permissible values for the jth variable, respectively; *rand* is a random number from a uniform distribution in the interval [0, 1]; n is the number of optimization variables.

Step 2: Evaluation of candidate designs and weight assignment

The objective function values for the candidate solutions are evaluated. All of the initial solutions are sorted and recorded in a memory denoted as the league. Each solution is considered as a team with the following weight:

$$W_i = \left(\frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}}\right) + 1 \qquad i = 1,2,\ldots,N \tag{5}$$

where $fit(i)$ is the fitness value for the ith particle; The fitness value of the ith team, evaluated as as the penalized objective function value for constrained problems; fit_{best} and fit_{worst} are the fitness values for the best and worst candidate solutions of the current iteration. According to Eq. (5) the weights of the teams range between 1 and 2.

Step 3: Competition and displacement

In TWO each of the teams of the league competes against all the others one at a time to move to its new position in each iteration. The pulling force exerted by a team is assumed to be equal to its static friction force $(W\mu_s)$. Hence the pulling force between teams i and j ($F_{p,ij}$) can be determined as $max\{W_i\mu_s, W_j\mu_s\}$. Such a definition keeps the position of the heavier team unaltered.

The resultant force affecting team i due to its interaction with heavier team j in the kth iteration can then be calculated as follows:

$$F_{r,ij}^{k} = F_{p,ij}^{k} - W_i^{k}\mu_k \tag{6}$$

where $F_{p,ij}^{k}$ is the pulling force between teams i and j in the kth iteration and μ_k is coefficient of kinematic friction. Consequently, team i accelerates towards team j:

$$a_{ij}^{k} = \left(\frac{F_{r,ij}^{k}}{W_i^{k}\mu_k}\right) g_{ij}^{k} \tag{7}$$

where a_{ij}^{k} is the acceleration of team i towards team j in the kth iteration; g_{ij}^{k} is the gravitational acceleration constant defined as:

$$g_{ij}^{k} = X_{j}^{k} - X_{i}^{k} \tag{8}$$

where X_j^k and X_i^k are the position vectors for candidate solutions *j* and *i* in the *k*th iteration. Finally, the displacement of the team *i* after competing with team *j* can be derived as:

$$\Delta X_{ij}^{k} = \frac{1}{2} a_{ij}^{k} \Delta t^{2} + \alpha^{k} \beta (X_{\max} - X_{\min}) \circ randn(1, n) \tag{9}$$

The second term of Eq. (9) introduces randomness into the algorithm. This term can be interpreted as the random portion of the search space traveled by team *i* before it stops after the applied force is removed. The role of α^k is to gradually decrease the random portion of the team's movement. For most of the applications α could be considered as a constant chosen from the interval [0.9, 0.99]; bigger values of α decrease the convergence speed of the algorithm and help the candidate solutions explore the search space more thoroughly. β is a scaling factor which can be chosen from the interval (0,1]. This parameter controls the steps of the candidate solutions when moving in the search space. When the search space is supposed to be searched more accurately with smaller steps, smaller values should be chosen for this parameter. For our numerical examples values between 0.01 and 0.05 seem to be appropriate for this parameter; $X_{\max}$ and $X_{\min}$ are the vectors containing the upper and lower bounds of the permissible ranges of the design variables, respectively; $\circ$ denotes element by element multiplication; $randn(1, n)$ is a vector of random numbers drawn from a standard normal distribution.

It should be noted that when team *j* is lighter than team *i*, the corresponding displacement of team *i* will be equal to zero (i.e. ΔX_{ij}^k). Finally, the total displacement of team *i* in iteration *k* is equal to *(i not equal j)*:

$$\Delta X_{i}^{k} = \sum_{j=1}^{N} \Delta X_{ij}^{k} \tag{10}$$

The new position of the team *i* at the end of the *k*th iteration is then calculated as:

$$X_{i}^{k+1} = X_{i}^{k} + \Delta X_{i}^{k} \tag{11}$$

Step 4: Updating the league

Once the teams of the league compete against each other for a complete round, the league should be updated. This is done by comparing the new candidate solutions (the new positions of the teams) to the current teams of the league. That is to say, if the new candidate solution *i* is better than the *Nth* team of the league in terms of objective function value, the *Nth* team is removed from the league and the new solution takes its place.

Step 5: Handling the side constraints

It is possible for the candidate solutions to leave the search space and it is important to deal with such solutions properly. This is especially the case for the solutions corresponding to lighter teams for which the values of ΔX is usually bigger. Different strategies might be used in order to solve this problem. For example, such candidate solutions can be simply brought back to their previous feasible position (flyback strategy) or they can be regenerated randomly. In this paper a new strategy is introduced and incorporated using the global best solution. The new value of the *jth* optimization variable of the *ith* team that violated side constraints in the *kth* iteration is defined as:

$$x_{ij}^{k} = GB_j + (\frac{randn}{k})(GB_j - x_{ij}^{k-1}) \tag{12}$$

where GB_j is the *j*th variable of the global best solution (i.e. the best solution so far); *randn* is a random number drawn form a standard normal distribution. There is a very slight possibility for the newly generated variable to be still outside the search space. In such cases a flyback strategy is used.

The abovementioned strategy is utilized with a certain probability (0.5 in this paper). For the rest of cases the violated limit is taken as the new value of the *jth* optimization variable.

Step 6: Termination

Steps 2 through 5 are repeated until a termination criterion is satisfied. The pseudo code of TWO is presented in Table 1.

Table 1: Pseudo-code of the TWO algorithm developed in this study

```
procedure Tug of War Optimization
begin
Initialize parameters;
    Initialize a population of N random candidate solutions;
    Initialize the league by recording all random candidate solution;
    while (termination condition not met) do
        Evaluate the objective function values for the candidate solutions
        Sort the new solutions and update the league
        Define the weights of the teams of the league Wi based on fit(Xi)
            for each team i
            for each team j
                if (Wi < Wj)
                    Move team i towards team j using Eq. (9);
end if
            end for
Determine the total displacement of team i using Eq. (10)
            Determine the final position of team i using Eq.(11)
            Use the side constraint handling technique to regenerate violating variables
        end for
    end while
end
```

3. TEST PROBLEMS AND OPTIMIZATION RESULTS

In order to evaluate the efficiency of the proposed algorithm, some benchmark test problems are considered from the literature. A set of uni-modal and multi-modal mathematical optimization problems are studied in Section 3.1. In addition, six well-studied engineering design problems are investigated in Section 3.2. Except for the last three examples a population of 20 agents and a maximum number of permitted iterations of 200 are used for all test problems. For the fourth, fifth, and sixth structural design problems the numbers of agents are taken as 30, 40, and 40, respectively while 400 iterations are used for these examples. The coefficient of static friction (μ_s) is taken as unity, while the coefficient of kinematic friction (μ_k) varies linearly from 1 to 0.1. Since smaller values of μ_k let the teams slide more easily towards each other and vice versa, such a parameter selection helps the algorithm's agents to explore the search space at early iterations without being severely affected by each other. As the optimization process proceeds, the values of μ_k gradually decrease allowing for convergence. It was found that using the same value of kinematic friction coefficient for all teams yield the best optimization results for TWO.

3.1 Mathematical optimization problems

In this section the efficiency of the TWO is evaluated by solving the mathematical benchmark problems summarized in Table 2. These benchmark problems are taken from Ref. [20], where some variants of GA were used as the optimization algorithm. The results obtained by TWO are presented in Table 3 along with those of GA variants. Each objective function is optimized 50 times independently starting from different initial populations and the average number of function evaluations required by each algorithm is presented. The numbers in the parentheses indicate the ratio of the successful runs in which the algorithm has located the global minimum with predefined accuracy, which is taken as $\varepsilon = f_{\min} - f_{final} = 10^{-4}$. The absence of the parentheses means that the algorithm has been successful in all independent runs.

Table 2: Details of the benchmark mathematical problems solved in this study

Function name	Side constraints	Function	Global minimum
Aluffi-Pentiny	$X \in [-10,10]^2$	$f(X) = \frac{1}{4}x_1^4 - \frac{1}{2}x_1^2 + \frac{1}{10}x_1 + \frac{1}{2}x_2^2$	−0.352386
Bohachevsky 1	$X \in [-100,100]^2$	$f(X) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$	0.0
Bohachevsky 2	$X \in [-50,50]^2$	$f(X) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$	0.0
Becker and Lago	$X \in [-10,10]^2$	$f(X) = (\lvert x_1 \rvert - 5)^2 + (\lvert x_2 \rvert - 5)^2$	0.0
Branin	$0 \le x_2 \le 15$ $-5 \le x_1 \le 10$	$f(X) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1)^2 + 10(1 - \frac{1}{8\pi})\cos(x_1) + 10$	0.397887
Camel	$X \in [-5,5]^2$	$f(X) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	−1.0316

Cb3	$X \in [-5,5]^2$	$f(X)=2x_1^2-1.05x_1^4+\frac{1}{6}x_1^6+x_1x_2+x_2^2$	0.0
Cosine mixture	$n=4$, $X \in [-1,1]^n$	$f(X)=\sum_{i=1}^{n}x_i^2-\frac{1}{10}\sum_{i=1}^{n}\cos(5\pi x_i)$	−0.4
DeJoung	$X \in [-5.12,5.12]^3$	$f(X)=x_1^2+x_2^2+x_3^2$	0.0
Exponential	$n=2,4,8$, $X \in [-1,1]^n$	$f(X)=-\exp\left(-0.5\sum_{i=1}^{n}x_i^2\right)$	−1
Goldstein and price	$X \in [-2,2]^2$	$f(\mathbf{X})=[1+(x_1+x_2+1)^2(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2)] \times[30+(2x_1-3x_2)^2(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2)]$	3.0
Griewank	$X \in [-100,100]^2$	$f(\mathbf{X})=1+\frac{1}{200}\sum_{i=1}^{2}x_i^2-\prod_{i=1}^{2}\cos(\frac{x_i}{\sqrt{i}})$	0.0
Hartman 3	$X \in [0,1]^3$	$f(X)=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{3}a_{ij}(x_j-p_{ij})^2\right)$ $a=\begin{bmatrix}3 & 10 & 30\\ 0.1 & 10 & 35\\ 3 & 10 & 30\\ 0.1 & 10 & 35\end{bmatrix}$, $c=\begin{bmatrix}1\\ 1.2\\ 3\\ 3.2\end{bmatrix}$ and $p=\begin{bmatrix}0.3689 & 0.117 & 0.2673\\ 0.4699 & 0.4387 & 0.747\\ 0.1091 & 0.8732 & 0.5547\\ 0.03815 & 0.5743 & 0.8828\end{bmatrix}$.	−3.862782
Hartman 6	$X \in [0,1]^6$	$f(\mathbf{X})=-\sum_{i=1}^{4}c_i\exp\left(-\sum_{j=1}^{6}a_{ij}(x_j-p_{ij})^2\right)$ $a=\begin{bmatrix}10 & 3 & 17 & 3.5 & 1.7 & 8\\ 0.05 & 10 & 17 & 0.1 & 8 & 14\\ 3 & 3.5 & 17 & 10 & 17 & 8\\ 17 & 8 & 0.05 & 10 & 0.1 & 14\end{bmatrix}$, $c=\begin{bmatrix}1\\ 1.2\\ 3\\ 3.2\end{bmatrix}$ and $p=\begin{bmatrix}0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886\\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991\\ 0.2348 & 0.1451 & 0.3522 & 0.2883 & 0.3047 & 0.6650\\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381\end{bmatrix}$	−3.322368

Table 3: Performance comparison of TWO and GA variants in the mathematical optimization problems

Function name	GEN	GEN–S	GEN–S–M	GEN–S–M–LS	TWO
AP	1,360 (0.99)	1,360	1,277	1,253	1092
Bf1	3,992	3,356	1,640	1615	1404
Bf2	20,234	3,373	1,676	1636	1232
BL	19,596	2412	2,439	1,436	1216
Branin	1,442	1,418	1,404	1,257	1189
Camel	1,358	1,358	1,336	1,300	1212
Cb3	9,771	2,045	1,163	1,118	992
CM	2,105	2,105	1,743	1,539	1420

DeJoung	9,900	3,040	1,462	1,281	1346
Exp2	938	936	817	807	314
Exp4	3,237	3,237	2,054	1,496	815
Exp8	3,237	3,237	2,054	1,496	1257
Goldstein and Price	1,478	1,478	1,408	1,325	1690
Griewank	18,838 (0.91)	3,111 (0.91)	1,764	1,652 (0.99)	1766
Hartman3	1,350	1,350	1,332	1,274	1026
Harman6	2,562 (0.54)	2,562 (0.54)	2,530 (0.67)	1,865 (0.68)	1601 (0.68)

As it can be seen from Table 3, TWO generally performs better than GA and its variants in the mathematical optimization problems considered in this study.

3.2 Structural design problems

In order to further investigate the efficiency of the TWO, three engineering design problems are considered in this section. These problems have been previously studied using different optimization algorithms. These constrained optimization problems are turned into unconstrained ones using a penalty approach. If the constraints are satisfied, then the amount of penalty will be zero; otherwise its value can be calculated as the ratio of violated constraint to the corresponding allowable limit.

3.2.1 Design of a tension/compression spring

Weight minimization of the tension/compression spring shown in Fig. 3, subject to constraints on shear stress, surge frequency, and minimum deflection is considered as the first engineering design example. This problem was first described by Belegundu [21] and Arora [22]. The design variables are the mean coil diameter $D(x_1)$, the wire diameter $d(x_2)$, and the number of active coils $N(x_3)$.

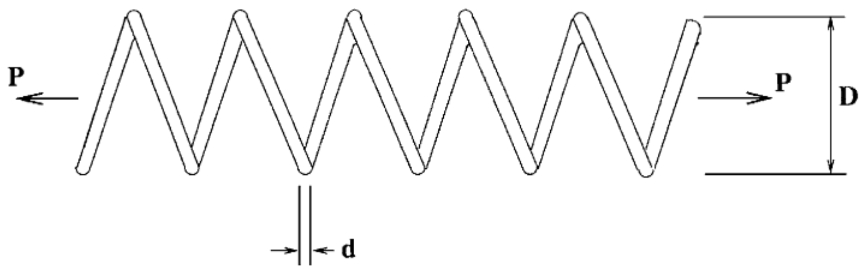


Figure 3. Schematic of the tension/compression spring

The cost function can be stated as:

$$f_{cost}(X)=(x_3+2)x_2x_1^2 \tag{13}$$

while the constraints are:

$$g_1(\mathbf{X}) = 1 - \frac{x_2^3 x_3}{71785 x_1^4} \le 0$$
$$g_2(\mathbf{X}) = \frac{4x_2^2 - x_1 x_2}{12566(x_2 x_1^3 - x_1^4)} + \frac{1}{5108 x_1^2} - 1 \le 0 \tag{14}$$
$$g_3(\mathbf{X}) = 1 - \frac{140.45 x_1}{x_2^2 x_3} \le 0$$
$$g_4(\mathbf{X}) = \frac{x_1 + x_2}{1.5} - 1 \le 0$$

The side constraints are defined as follows:

$$0.05 \le x_1 \le 2$$
$$0.25 \le x_2 \le 1.3 \tag{15}$$
$$2 \le x_3 \le 15$$

This problem has been solved by Belegundu [21] using eight different mathematical optimization techniques. Arora [22] utilized a numerical optimization technique called a constraint correction at the constant cost to investigate the problem. Coello [23] and Coello and Montes [24] used a GA-based algorithm to solve the problem. He and Wang [25] used a co-evolutionary particle swarm optimization (CPSO). Montes and Coello [26] used evolution strategies. Kaveh and Talatahari used improved ant colony optimization [27] and charged system search CSS [6]. Recently, the problem has been solved by Kaveh and Mahdavi [11] using Colliding bodies optimization (CBO).

The optimization results obtained by TWO are presented in Table 4 along with those of other methods. It can be seen that TWO found the best design overall. It should be noted that some constraints are slightly violated by the designs quoted by Kaveh & Talatahari [6, 26]. Table 5 shows the statistical results obtained for 30 independent optimization runs.

Table 4: Comparison of the optimization results obtained in the tension/compression spring problem

	Optimal design variables			
Methods	$x_1(d)$	$x_2(D)$	$x_3(N)$	f_{cost}
Belegundu [21]	0.050000	0.315900	14.250000	0.0128334
Arora [22]	0.053396	0.399180	9.185400	0.0127303
Coello [23]	0.051480	0.351661	11.632201	0.0127048
Coello & Montes [24]	0.051989	0.363965	10.890522	0.0126810
He and Wang [25]	0.051728	0.357644	11.244543	0.0126747
Montes and Coello [26]	0.051643	0.355360	11.397926	0.012698
Kaveh & Talatahari [27]	0.051865	0.361500	11.000000	0.0126432
Kaveh & Talatahari [6]	0.051744	0.358532	11.165704	0.0126384
Kaveh & Mahdavi [11]	0.051894	0.3616740	11.007846	0.0126697
Present work	0.051592	0.354379	11.428784	0.0126671

Table 5: Comparison of statistical optimization results obtained in the tension/compression spring design problem

Methods	Best	Mean	Worst	Std dev
Belegundu [21]	0.0128334	N/A	N/A	N/A
Arora [22]	0.0127303	N/A	N/A	N/A
Coello [23]	0.0127048	0.012769	0.012822	3.9390e-5
Coello & Montes [24]	0.0126810	0.0127420	0.012973	5.9000e-5
He and Wang [25]	0.0126747	0.012730	0.012924	5.1985e-5
Montes and Coello [26]	0.012698	0.013461	0.016485	9.6600e-4
Kaveh & Talatahari [27]	0.0126432	0.012720	0.012884	3.4888e-5
Kaveh & Talatahari [6]	0.0126384	0.012852	0.013626	8.3564e-5
Kaveh & Mahdavi [11]	0.0126697	0.0127296	0.012881	5.00376e-5
Present work	0.0126671	0.0129709	0.0135213	2.6125e-4

3.2.2 Design of a welded beam

The second test problem regards the design optimization of the welded beam shown in Fig.4. This problem has been used for testing different optimization methods. The aim is to minimize the total manufacturing cost subject to constraints on shear stress (τ), bending stress (σ), buckling load (P_c), and deflection (δ). The four design variables, namely *h* (x_1), *l* (x_2), *t* (x_3), and *b* (x_4), are also shown in the figure.

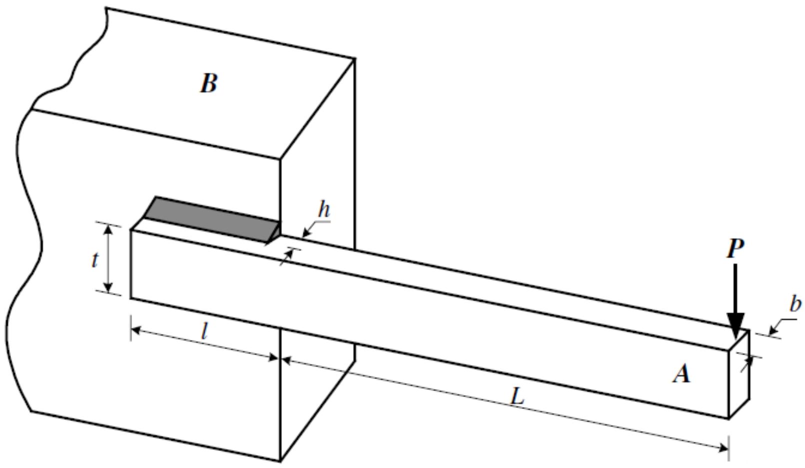


Figure 4. Schematic of the welded beam

The objective function can be mathematically stated as:

$$f_{cost}(X) = 1.10471 x_1^2 x_2 + 0.04811 x_3 x_4 (14.0 + x_2) \tag{16}$$

The optimization constraints are

$$g_1(X) = \tau(\{x\}) - \tau_{max} \leq 0 \tag{17}$$

$$g_2(X) = \sigma(\{x\}) - \sigma_{max} \le 0$$
$$g_3(X) = x_1 - x_4 \le 0$$
$$g_4(X) = 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \le 0$$
$$g_5(X) = 0.125 - x_1 \le 0$$
$$g_6(X) = \delta(\{x\}) - \delta_{max} \le 0$$
$$g_7(X) = P - P_c(\{x\}) \le 0$$

where

$$\tau(\mathbf{X}) = \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2}$$
$$\tau' = \frac{P}{\sqrt{2}x_1x_2}, \tau'' = \frac{MR}{J}$$
$$M = P(L + \frac{x_2}{2}), R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}$$
$$J = 2\left\{\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]\right\}$$
$$\sigma(\mathbf{X}) = \frac{6PL}{x_4x_3^2}, \delta(\mathbf{X}) = \frac{4PL^3}{Ex_3^3x_4} \tag{18}$$
$$P_c(\mathbf{X}) = \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2}\left(1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right)$$
$$P = 6000lb,\ L = 14in,$$
$$E = 30\times10^6\ psi,\ G = 12\times10^6\ psi$$
$$\tau_{\max} = 13.6\times10^3\ psi, \sigma_{\max} = 30\times10^3\ psi$$
$$\delta_{\max} = 0.25in$$

The side constraints can be stated as:

$$0.1 \le x_1 \le 2,\ 0.1 \le x_2 \le 10,\ 0.1 \le x_3 \le 10,\ 0.1 \le x_4 \le 2 \tag{19}$$

Radgsdell and Phillips [28] utilized different optimization methods mainly based on mathematical programming to solve the problem and compared the results. GA-based methods are used by Deb [29], Coello [23], Coello and Montes [24]. This test problem was also solved by He and Wang [25] using CPSO and Montes and Coello [26] using evolution strategies. Kaveh and Talatahari employed ant colony optimization [27] and charged system search [6]. Kaveh and Mahdavi [11] solved the problem using the colliding bodies optimization method.

Table 6 compares the best results obtained by the different optimization algorithms considered in this study. The statistical results for 30 independent runs are provided in Table 7. It can be seen from Table 6 that optimum design found by TWO is about 0.01 percent heavier than that found by CBO which was the best design quoted so far in literature.

Table 6: Comparison of the optimization results obtained in the welded beam problem

Methods	Optimal design variables				
	$x_1(h)$	$x_2(l)$	$x_3(t)$	$x_4(b)$	f_{cost}
Radgsdell and Phillips [28]	0.2444	6.2189	8.2915	0.2444	2.3815
Deb [29]	0.248900	6.173000	8.178900	0.253300	2.433116
Coello [23]	0.208800	3.420500	8.997500	0.210000	1.748309
Coello and Montes [24]	0.205986	3.471328	9.020224	0.206480	1.728226
He and Wang [25]	0.202369	3.544214	9.048210	0.205723	1.728024
Montes and Coello [26]	0.199742	3.612060	9.037500	0.206082	1.737300
Kaveh & Talatahari [27]	0.205700	3.471131	9.036683	0.205731	1.724918
Kaveh & Talatahari [6]	0.205820	3.468109	9.038024	0.205723	1.724866
Kaveh & Mahdavi [11]	0.205722	3.47041	9.037276	0.205735	1.724663
Present work	0.205728	3.47052	9.036631	0.205730	1.724855

Table 7: Comparison of statistical optimization results obtained in the welded beam problem

Methods	Best	Mean	Worst	Std Dev
Radgsdell and Phillips [28]	2.3815	N/A	N/A	N/A
Deb [29]	2.433116	N/A	N/A	N/A
Coello [23]	1.748309	1.771973	1.785835	0.011220
Coello and Montes [24]	1.728226	1.792654	1.993408	0.074713
He and Wang [25]	1.728024	1.748831	1.782143	0.012926
Montes and Coello [26]	1.737300	1.813290	1.994651	0.070500
Kaveh & Talatahari [27]	1.724918	1.729752	1.775961	0.009200
Kaveh & Talatahari [6]	1.724866	1.739654	1.759479	0.008064
Kaveh & Mahdavi [11]	1.724662	1.725707	1.725059	0.0002437
Present work	1.724855	1.726016	1.729970	0.0009951

3.2.3 Design of a planar 10-bar truss structure subject to frequency constraints

The sizing optimization of a planar 10-bar truss subject to frequency constraints is selected as the third test case. The configuration of the structure is depicted in Fig. 5. This is a well-known benchmark problem in the field of frequency constraint structural optimization. Each of the members' cross-sectional area is assumed to be an independent variable. A non-structural mass of 454.0 kg is attached to all free nodes. Table 8 summarizes the material properties, variable bounds, and frequency constraints for this example. This problem has been investigated by different researchers: Grandhi and Venkayya [30] using an optimality algorithm, Sedaghati et al. [31] using a sequential quadratic programming and finite element force method, Wang et al. [32] using an evolutionary node shift method, Lingyun et al. [33] utilizing a niche hybrid genetic algorithm, Gomes [34] employing the standard particle swarm optimization algorithm and Kaveh and Zolghadr utilizing democratic PSO [9], and a hybridized PSRO algorithm [35] among others.

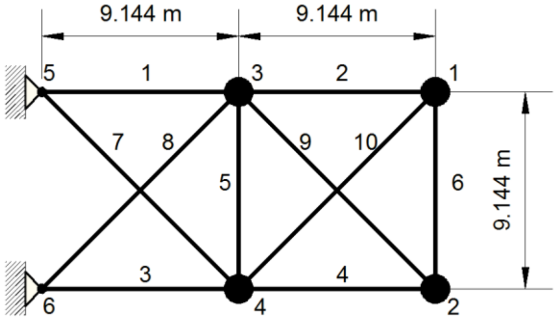


Figure 5. Schematic of the planar 10-bar truss structure

Table 8: Material properties, variable bounds and frequency constraints for the 10-bar truss structure

Property/unit	Value
E (Modulus of elasticity)/ N/m^2	6.89×10^{10}
ρ (Material density)/ kg/m^3	2770.0
Added mass/kg	454.0
Design variable lower bound/m^2	0.645×10^{-4}
Design variable upper bound/m^2	50×10^{-4}
L (Main bar's dimension)/m	9.144
Constraints on first three frequencies/Hz	$\omega_1 \geq 7$, $\omega_2 \geq 15$, $\omega_3 \geq 20$

Table 9 summarizes the design vectors of the optimal structures found by different methods.

Table 9: Optimized designs (cm^2) obtained for the planar 10-bar truss problem (the optimized weight does not include the added masses)

Element number	Grandhi & Venkayya [30]	Sedaghati et al. [31]	Wang et al. [32]	Lingyun et al. [33]	Gomes [34]	Kaveh & Zolghadr		
						DPSO [9]	PSRO [35]	Present work
1	36.584	38.245	32.456	42.23	37.712	35.944	37.075	35.198
2	24.658	9.916	16.577	18.555	9.959	15.530	15.334	14.311
3	36.584	38.619	32.456	38.851	40.265	35.285	33.665	35.305
4	24.658	18.232	16.577	11.222	16.788	15.385	14.849	14.833
5	4.167	4.419	2.115	4.783	11.576	0.648	0.645	0.645
6	2.070	4.419	4.467	4.451	3.955	4.583	4.643	4.671
7	27.032	20.097	22.810	21.049	25.308	23.610	24.528	23.806
8	27.032	24.097	22.810	20.949	21.613	23.599	23.188	24.894
9	10.346	13.890	17.490	10.257	11.576	13.135	12.436	12.843
10	10.346	11.452	17.490	14.342	11.186	12.357	13.500	12.803
Weight (kg)	594.0	537.01	553.8	542.75	537.98	532.39	532.85	532.17

According to Table 9, the weight of the optimal structure found by TWO is 532.17 kg which is slightly better than that of DPSO, the best design quoted so far in the literature. The mean value and the standard deviation of 30 independent runs of TWO on this problem are 537.43 and 3.43 kg, respectively. Table 10 shows the natural frequencies of the optimized structures obtained by different methods. It can be seen that all of the constraints are satisfied.

3.2.4 Design of a spatial 25-bar truss structure

Weight minimization of the spatial 25-bar truss schematized in Fig. 6 is considered as the fourth design example. The material density and modulus of elasticity are 0.1 lb/in^3 and 10000 ksi, respectively. Table 11 shows the two independent loading conditions applied to the structure. The twenty five bars of the truss are classified into eight groups, as follows:

(1) A_1, *(2)* A_2-A_5, *(3)* A_6-A_9, *(4)* A_{10}-A_{11}, *(5)* A_{12}-A_{13}, *(6)* A_{14}-A_{17}, *(7)* A_{18}-A_{21}, *and (8)* A_{22}-A_{25}.

Table 10: Natural frequencies (Hz) evaluated at the optimized designs for the planar 10-bar truss

Frequency number	Grandhi &Venkaya [30]	Sedaghati et al. [31]	Wang et al. [32]	Lingyun et al. [33]	Gomes [34]	Kaveh and Zolghadr		
						DPSO [9]	PSRO [35]	Present work
1	7.059	6.992	7.011	7.008	7.000	7.000	7.000	7.000
2	15.895	17.599	17.302	18.148	17.786	16.187	16.143	16.127
3	20.425	19.973	20.001	20.000	20.000	20.000	20.000	20.000
4	21.528	19.977	20.100	20.508	20.063	20.021	20.032	20.002
5	28.978	28. 173	30.869	27.797	27.776	28.470	28.469	28.699
6	30.189	31.029	32.666	31.281	30.939	29.243	29.485	29.068
7	54.286	47.628	48.282	48.304	47.297	48.769	48.440	48.280
8	56.546	52.292	52.306	53.306	52.286	51.389	51.257	50.822

Table 11: Independent loading conditions acting on the spatial 25-bar truss

Node	Case 1			Case 2		
	P_x kips	P_y kips	P_z kips	P_x kips	P_y kips	P_z kips
1	0.0	20.0	-5.0	1.0	10.0	-5.0
2	0.0	-20.0	-5.0	0.0	10.0	-5.0
3	0.0	0.0	0.0	0.5	0.0	0.0
6	0.0	0.0	0.0	0.5	0.0	0.0

Maximum displacement limitations of 0.35 in are imposed on all nodes in all directions. The axial stress constraints, which are different for each group, are shown in Table 12. The cross-sectional areas vary continuously from 0.01 to 3.4 in^2 for all members.

Table 12: Member stress limits for the 25-bar spatial truss

Element group	Compressive stress limits ksi (MPa)	Tensile stress limit ksi (MPa)
1	35.092 (241.96)	40.0 (275.80)
2	11.590 (79.913)	40.0 (275.80)
3	17.305 (119.31)	40.0 (275.80)
4	35.092 (241.96)	40.0 (275.80)
5	35.092 (241.96)	40.0 (275.80)
6	6.759 (46.603)	40.0 (275.80)
7	6.959 (47.982)	40.0 (275.80)
8	11.082 (76.410)	40.0 (275.80)

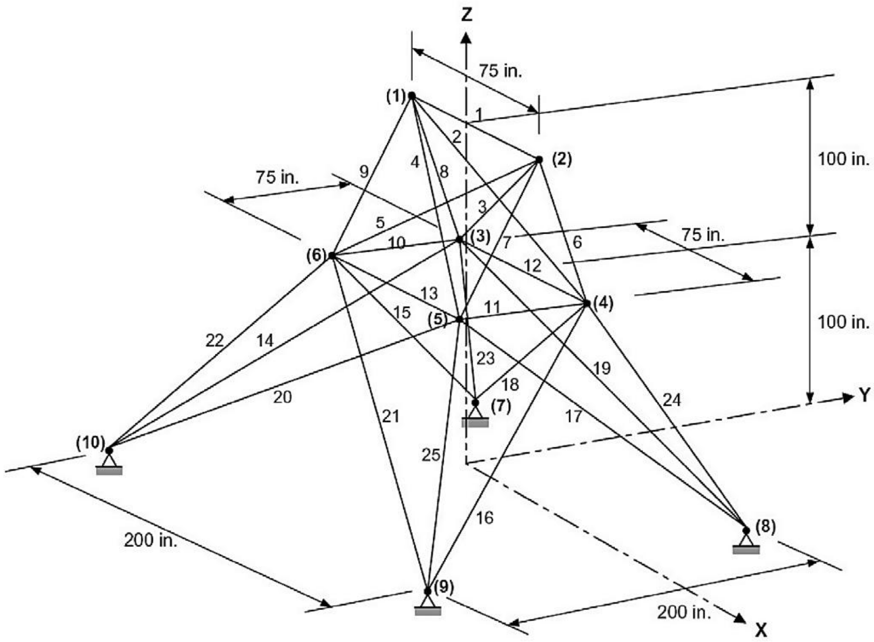


Figure 6. Schematic of the spatial 25-bar truss structure

Table 13 shows that the different optimization methods converged almost to the same structural weight. It seems that the results obtained by different methods are very close together for this example. The best result obtained by TWO is 544.42 kg which is only slightly heavier than that of HS. However, it should be noted that according to our codes the optimum design of HS slightly violates displacement constraints. Moreover TWO requires 12000 analyses to complete the optimization process, while HS has used 15000 analyses. The mean value and standard deviation of 30 independent optimization runs by TWO are 544.53 and 0.211 lb, respectively. Small differences in weight may originate from the level of precision in the implementation of the optimization problem. For example, the displacement limit of 0.350in set for TWO resulted in feasible optimized design quoted in Table 13 yielding a maximum displacement of 0.3504in which can be rounded to the above mentioned limit.

3.2.5 Design of a spatial 72-bar truss structure

A spatial 72-bar truss structure shown in Fig. 7 is considered as the fifth design example. The elements are grouped to form 16 design variables according to Table 15. The material density and the modulus of elasticity are taken as 0.1 lb/in^3 and 10,000 ksi, respectively. All members are subjected to a stress limitations of ±25 ksi. The displacements of the uppermost nodes along x and y axes are limited to ±0.25 in. Cross-sectional areas of bars can vary between 0.10 and 4.00 in^2. The two loading conditions acting on the structure are summarized in Table 14.

This problem has been studied by Erbatur et al. [39] using Genetic Algorithms, Camp and Bichon [40] using Ant Colony Optimization, Perez and Behdinan [41] using Particle Swarm Optimization, Camp [42] using Big Bang-Big Crunch algorithm, and Kaveh and Khayatazad [8] using Ray Optimization among others.

Table 13: Comparison of the optimization results obtained in the spatial 25-bar truss problem

Element group		Optimal cross-sectional areas (in^2)			
		Rajeev and Krishnamoorthy, GA [36]	Schutte and Groenwold, PSO [37]	Lee and Geem, HS [38]	Present work
1	A_1	0.10	0.010	0.047	0.010
2	A_2-A_5	1.80	2.121	2.022	1.979
3	A_6-A_9	2.30	2.893	2.950	2.993
4	A_{10}-A_{11}	0.20	0.010	0.010	0.010
5	A_{12}-A_{13}	0.10	0.010	0.014	0.010
6	A_{14}-A_{17}	0.80	0.671	0.688	0.684
7	A_{18}-A_{21}	1.80	1.611	1.657	1.678
8	A_{22}-A_{25}	3.0	2.717	2.663	2.656
	Best weight (lb)	546	545.21	544.38	544.42
	Average weight (lb)	N/A	546.84	N/A	544.53
	Std Dev (lb)	N/A	1.478	N/A	0.211
	No. of analyses	N/A	9596	15000	12000

Table 14: Independent loading conditions acting on the spatial 72-bar truss

node	Case 1			Case 2		
	P_x kips (kN)	P_y kips (kN)	P_z kips (kN)	P_x kips(kN)	P_y kips(kN)	P_z kips (kN)
1	5	5	-5	_	_	-5
2	_	_	_	_	_	-5
3	_	_	_	_	_	-5
4	_	_	_	_	_	-5

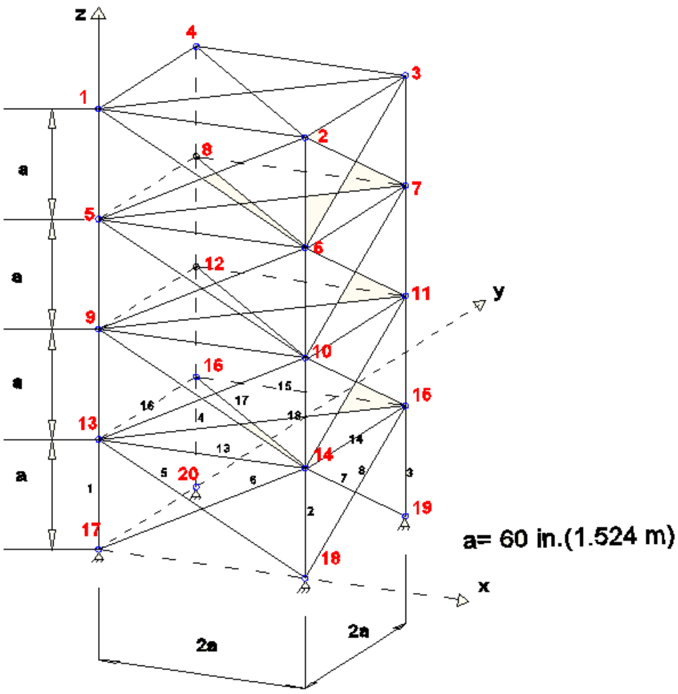


Figure 7. Schematic of the spatial 72-bar truss structure

Table 15 compares the results obtained by the present method to those previously reported in the literature. The weight of the best result obtained by TWO is 379.846 lb which is the best among the compared methods. Moreover, the mean weight of the results for 20 independent optimization runs of TWO is 381.98 lb which is less than all other methods. Also, TWO requires only 16000 structural analyses while ACO, BB-BC and RO require 18,500, 19,621, and 19,084, respectively.

Table 15: Comparison of the optimization results obtained in the spatial 72-bar truss problem

Element group	Optimal cross-sectional areas (in^2)					
	Erbatur et al. [39]	Camp and Bichon [40]	Perez and Behdinan [41]	Camp [42]	Kaveh and Khayatazad [8]	Present work
1–4	1.755	1.948	1.7427	1.8577	1.8365	1.9961
5–12	0.505	0.508	0.5185	0.5059	0.5021	0.5100
13–16	0.105	0.101	0.1000	0.1000	0.1000	0.1000
17–18	0.155	0.102	0.1000	0.1000	0.1004	0.1000
19–22	1.155	1.303	1.3079	1.2476	1.2522	1.2434
23–30	0.585	0.511	0.5193	0.5269	0.5033	0.5184
31–34	0.100	0.101	0.1000	0.1000	0.1002	0.1000
35–36	0.100	0.100	0.1000	0.1012	0.1001	0.1001
37–40	0.460	0.561	0.5142	0.5209	0.5730	0.5211
41–48	0.530	0.492	0.5464	0.5172	0.5499	0.5098
49–52	0.120	0.100	0.1000	0.1004	0.1004	0.1000

53–54	0.165	0.107	0.1095	0.1005	0.1001	0.1000
55–58	0.155	0.156	0.1615	0.1565	0.1576	0.1569
59–66	0.535	0.550	0.5092	0.5507	0.5222	0.5346
67–70	0.480	0.390	0.4967	0.3922	0.4356	0.3959
71–72	0.520	0.592	0.5619	0.5922	0.5971	0.5821
Best weight (lb)	385.76	380.24	381.91	379.85	380.458	379.846
Mean weight (lb)	N/A	383.16	N/A	382.08	382.553	381.976
Standard deviation (lb)	N/A	3.66	N/A	1.912	1.221	3.161
Number of analyses	N/A	18,500	N/A	19621	19,084	16000

3.2.6 Design of a 120-bar dome truss

The last test problem is the weight minimization of a 120-bar dome truss shown in Fig. 8. This structure was considered by Soh and Yang [43] as a configuration optimization problem. It has been solved by Lee and Geem [38], Kaveh and Talatahari [44], Kaveh et al. [45], Kaveh and Khayatazad [8], and Kaveh and Mahdavi [11] as a sizing optimization problem. The members of the structure are divided into seven groups as shown in Fig. 8.

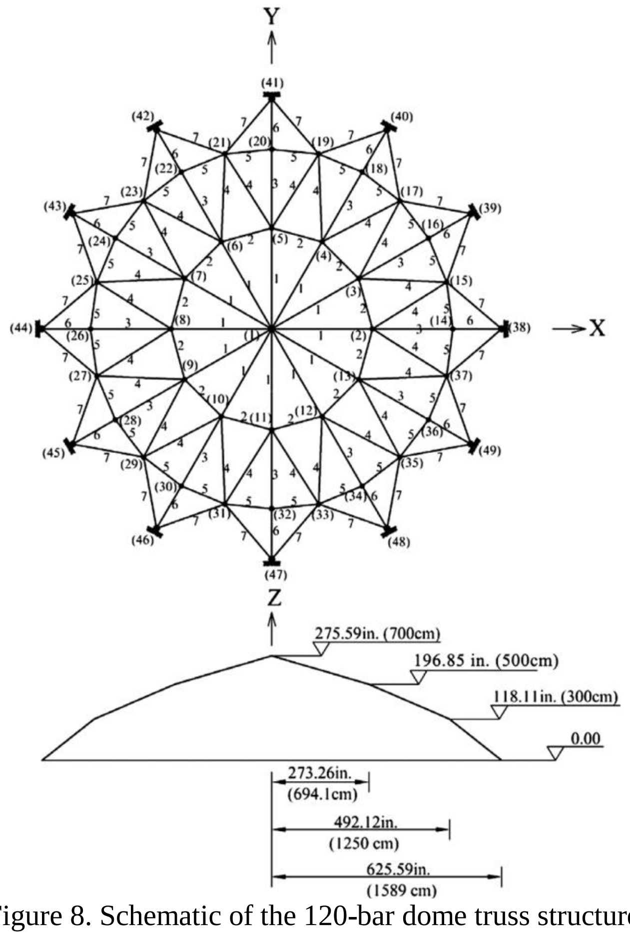


Figure 8. Schematic of the 120-bar dome truss structure

The allowable tensile and compressive stresses are set according to the AISC-ASD [46] provisions as follows:

$$\begin{cases} \sigma_i^+ = 0.6F_y & for \quad \sigma_i \geq 0 \\ \sigma_i^- & for \quad \sigma_i \leq 0 \end{cases} \tag{20}$$

where σ_i^- is the compressive allowable stress and depends on the elements slenderness ratio

$$\sigma_i^- = \begin{cases} \left[\left(1-\dfrac{\lambda_i^2}{2C_c^{\,2}}\right)F_y\right] \Big/ \left(\dfrac{5}{3}+\dfrac{3\lambda_i}{8C_c}-\dfrac{\lambda_i^3}{8C_c^3}\right) & for \quad \lambda_i \leq C_c \\ \dfrac{12\pi^2 E}{23\lambda_i^2} & for \quad \lambda_i \geq C_c \end{cases} \tag{21}$$

where E is the modulus of elasticity, F_y is the yield stress of the material, λ_i is the slenderness ratio ($\lambda_i = \dfrac{K_i L_i}{r_i}$), K_i is the effective length factor, L_i is the member length and r_i is the radius of gyration. C_c is the critical slenderness ratio separating elastic and inelastic buckling regions ($C_c = \sqrt{2\pi^2 E / F_y}$).

The modulus of elasticity and the material density are taken as 30,450 ksi (210 GPa) and 0.288 lb/in^3, respectively. The yield stress is taken as 58.0 ksi (400 MPa). The radius of gyration is expressed in terms of cross-sectional areas of the members as $r_i = aA_i^b$ [47]. Constants a and b depend on the types of sections adopted for the members such as pipes, angles, etc. In this example pipe sections are used for the bars for which a=0.4993 and b=0.6777. The dome is considered to be subjected to vertical loads at all unsupported nodes. These vertical loads are taken as -13.49 kips (60 kN) at node 1, -6.744 kips (30 kN) at nodes 2 through 14, and -2.248 kips (10 kN) at the other nodes. Four different problem variants are considered for this structure: with stress constraints and no displacement constraints (Case 1), with stress constraints and displacement limitations of ±0.1969 in (5 mm) imposed on all nodes in x and y directions (Case 2), no stress constraints and displacement limitations of ±0.1969 in (5 mm) imposed on all nodes in z direction (Case 3), and all the abovementioned constraints imposed together (Case 4). For cases 1 and 2 the maximum cross-sectional area is taken as 5.0 in^2 (32.26 cm^2) while for cases 3 and 4 it is taken as 20 in^2 (129.03 cm^2). The minimum cross-sectional is taken as 0.775 in^2 (5 cm^2) for all cases.

Table 16 compares the results obtained by different optimization techniques for this example. It can be seen that the results found by TWO are comparable to those of other methods. In case 1 the best result obtained by TWO is the same as that of CBO which is the best result so far. In cases 3 and 4 the results obtained by TWO are better than those of RO, IRO, and CBO and are only slightly heavier than that of HPSACO (0.01 and 0.004 percent in cases 3 and 4, respectively). The average number of structural analyses required by the RO and IRO algorithms were reported as 19,900 and 18,300, respectively, while TWO uses

40 candidate solutions and 400 iterations like CBO resulting in a maximum number of analyses of 16000. HPSACO and HS, respectively, performed 10000 and 35000 structural analyses to obtain their optimal results. It should be noted that HPSACO is a hybrid method which combines good features of PSO, ACO and HS.

Table 16: Comparison of the optimization results obtained in the 120-bar dome problem

Element group	Optimal cross-sectional areas (in^2)									
	Case 1					Case 2				
	HS [38]	HPSACO [44]	RO [8]	CBO [11]	Present work	HS [38]	HPSACO [44]	RO [8]	CBO [11]	Present work
1	3.295	3.311	3.128	3.1229	3.1229	3.296	3.779	3.084	3.0832	3.0831
2	3.396	3.438	3.357	3.3538	3.3538	2.789	3.377	3.360	3.3526	3.3526
3	3.874	4.147	4.114	4.1120	4.1120	3.872	4.125	4.093	4.0928	4.0928
4	2.571	2.831	2.783	2.7822	2.7822	2.570	2.734	2.762	2.7613	2.7613
5	1.150	0.775	0.775	0.7750	0.7750	1.149	1.609	1.593	1.5918	1.5923
6	3.331	3.474	3.302	3.3005	3.3005	3.331	3.533	3.294	3.2927	3.2927
7	2.784	2.551	2.453	2.4458	2.4458	2.781	2.539	2.434	2.4336	2.4336
Best weight (Ib)	19707.77	19491.3	19476.193	19454.7	19454.67	19893.34	20078.0	20071.9	20064.5	20064.86
Average weight (Ib)	-	-	-	19466.0	19454.98	-	-	-	20098.3	20106.85
Std (Ib)	-	-	33.966	7.02	1.17	-	-	112.135	26.17	116.15
	Case 3					Case 4				
	HPSACO [44]	RO [8]	CBO [11]	Present work		HPSACO [44]	RO [8]	IRO [45]	CBO [11]	Present work
1	2.034	2.044	2.0660	1.9667		3.095	3.030	3.0252	3.0273	3.0247
2	15.151	15.665	15.9200	15.3920		14.405	14.806	14.8354	15.1724	14.7261
3	5.901	5.848	5.6785	5.7127		5.020	5.440	5.1139	5.2342	5.1338
4	2.254	2.290	2.2987	2.1960		3.352	3.124	3.1305	3.119	3.1369
5	9.369	9.001	9.0581	9.5439		8.631	8.021	8.4037	8.1038	8.4545
6	3.744	3.673	3.6365	3.6688		3.432	3.614	3.3315	3.4166	3.2946
7	2.104	1.971	1.9320	1.9351		2.499	2.487	2.4968	2.4918	2.4956
Best weight (Ib)	31670.0	31733.2	31724.1	31673.62		33248.9	33317.8	33256.48	33286.3	33250.31
Average weight (Ib)	-	-	32162.4	31680.34		-	-	-	33398.5	33282.64
Std (Ib)	-	274.991	240.22	6.15		-	354.333	-	67.09	25.38

4. CONCLUSIONS

In this paper a novel population-based meta-heuristic algorithm based on a game of tug of war is introduced. The algorithm, denoted as Tug of War Optimization (TWO), considers each candidate solution as a team participating in a series of tug of war competitions. Like other meta-heuristic optimization algorithms TWO uses a combination of randomness and exploitation of previously obtained favorable results to perform global optimization. The quality of the candidate solutions improves iteratively as the optimization process proceeds. Unlike many other meta-heuristic methods, the algorithm is formulated in such a way that considers the qualities of both of the interacting teams. In other words, since the weights of the teams are involved in the convergence operator, two losing teams are not treated the same by a particular winning team. The team with less weight would probably move more drastically.

As a meta-heuristic optimization algorithm, TWO does not require information on the derivatives of the objective function and the constraints. This makes the algorithm applicable to a wide range of optimization problems from different fields. Some classical mathematical and structural optimization problems are solved to evaluate the efficiency of the proposed algorithm. Results indicate the superiority of TWO compared to some other state-of-the-art optimization algorithms.

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