Gliwice, 16-12-2024

Mathematical modelling and computer simulations in theory and practice

Documentation of laboratory task no 5

Title: TRAJECTORY OF A COMET

Author (Authors): Radosław Jędrzejczyk

Field of studies: Informatics (sem.V)

Project Objective:
Project objective is to visualis trajectory of a comet orbiting the Sun.

Description:
We want to describe an unique orbit of a body orbiting the Sun. We can neglect the mas of the orbiting object, because as long as we're not considering objects size of a planet any difrences in trajectory aren't going to be noticable.

In order to describe unique orbit we will use traditional orbital elements – Keplerian elements, those are:

1. Eccentricity (e) – shape of the elipse (<1), parabole (=1) or hyperbole(>1)
2. Semi-major axis (a) – half the distance between the apoapsis and periapsis.
3. Inclination – tilt of the elipse with respect to reference plane.
4. Longitude of the ascending node – orientation of the ascending node (point at which orbit is crossing reference plane, while body is moving up).
5. Argument of periapsis – angle from the ascending node to the periapsis.
6. True anomaly at given moment – angle from periapsis to the object.

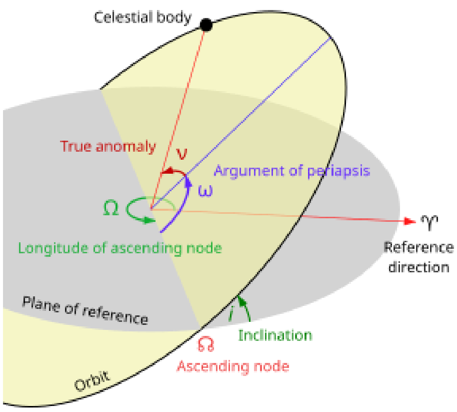

Figure 1: Keplerian orbit elements.

Knowing semi-major axis we can calculate semi-minor axis:

$$\sqrt{a^2(1-e^2)}$$

For parabolic and hyperbolic case we will not use semi-major axis because it's geometrical interpretation is not intuitive, instead we will define minimum altitude (closest approach) A.

Knowing this values we can construct all posbile trajectories around the body. We will start by describing the trajectory on the XY Plane (our reference plane):

- for e <1 (eliptical case) $f(t)=\begin{cases} x=a(\cos(t)-e) \\ y=b\sin(t) \\ z=0 \end{cases}$
- for e = 1(parabolic case) $f(t)=\begin{cases} x=A(t^2-1) \\ y=2At \\ z=0 \end{cases}$
- for e>1 (hyperbolic case) $f(t)=\begin{cases} x=-A\dfrac{\cosh(t)-e}{e-1} \\ y=-A\dfrac{\sinh(t)}{e-1} \\ z=0 \end{cases}$

In next step we're preparing rotation matrice using inclination, longitude of the ascending node and argument of periapsis, what will put orbit in the correct orientation. Knowing true anomaly at a given moment we can point current position of an object on the trajectory.

Inputs:

1. Orbital eccentricity [-].
2. Semi-major axis [AU = Astronomical Units].
3. Minimum altitude [AU] (Used when eccentricity >= 1).
4. Inclination [rad].
5. Longitude of ascending node [rad].
6. Argument of periapsis [rad].
7. True anomaly [rad].

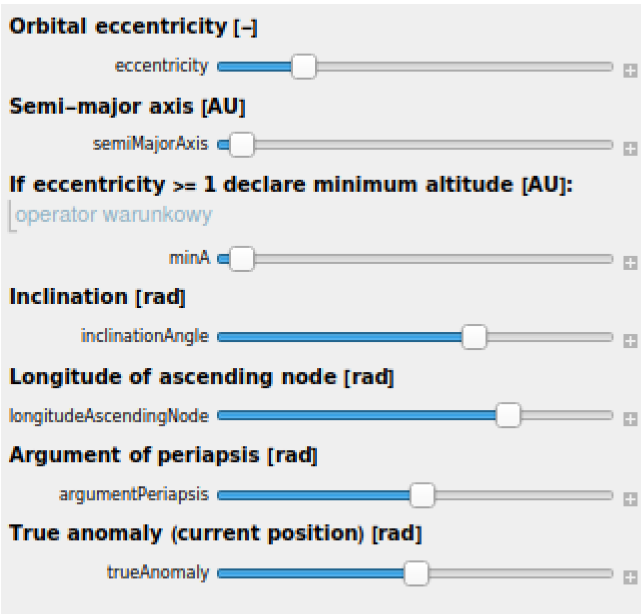

Figure 2: Program input

Outputs:
As an otuput program is displaying visualisation the trajectory in 4 view:

- 3D.
- XY projection.
- XZ projection.
- YZ projection.

Current position is marked as red point, trajectory as blue line and the Sun as yellow point. Additionaly axes are marked (X - green, Y - red, Z – purple) and direction to ascending node is marked as blue dashed line.

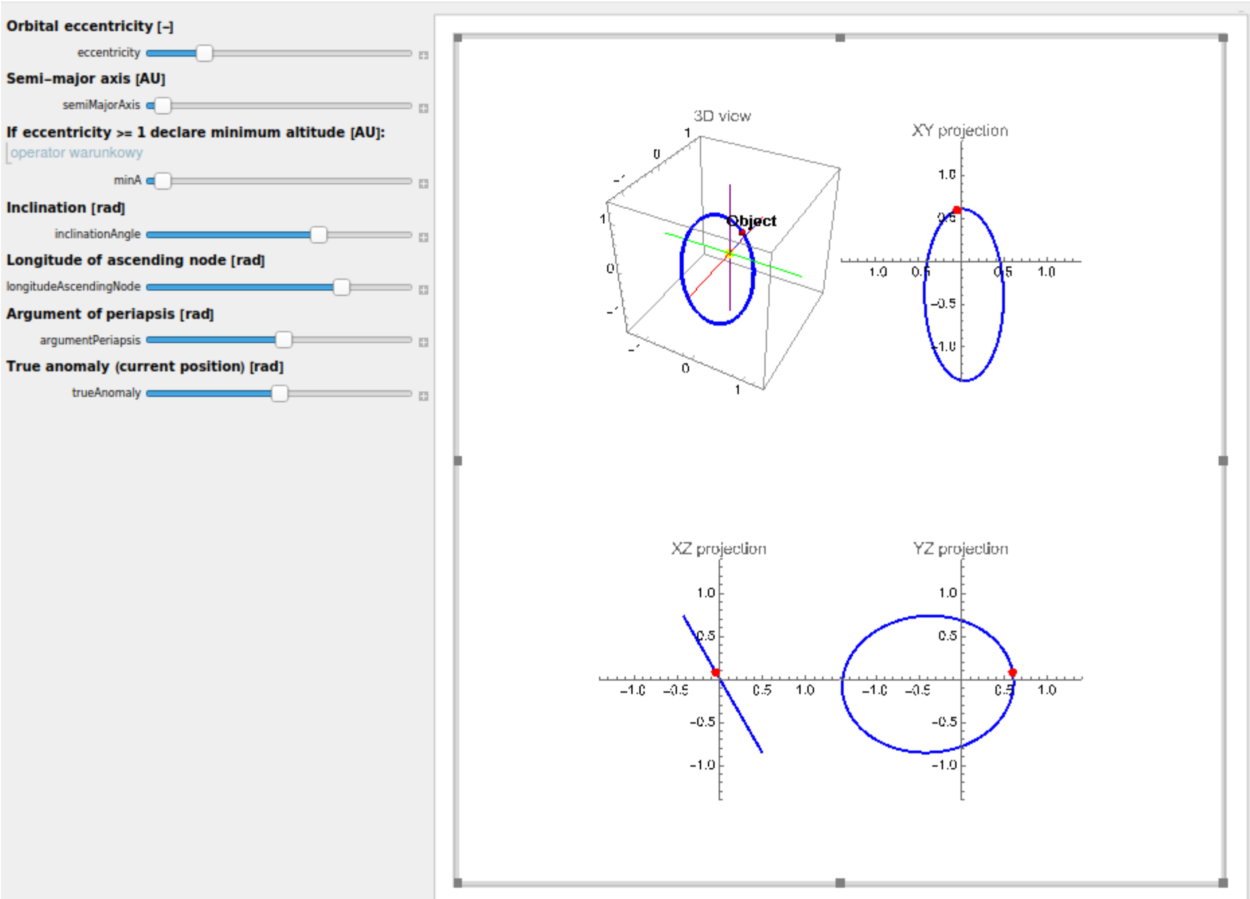

Figure 3: Program output - eliptical case

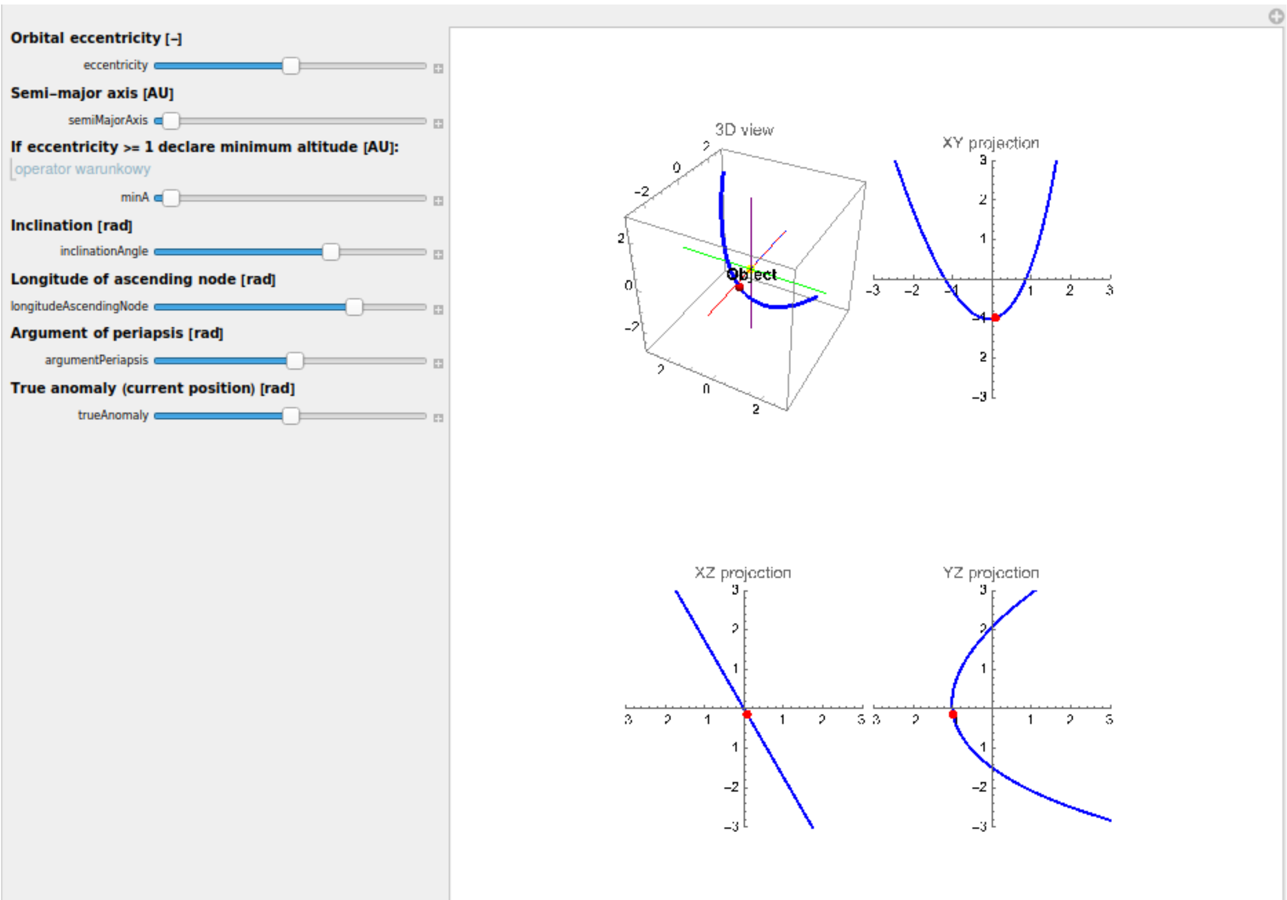

Figure 4: Program output - parabolic case.

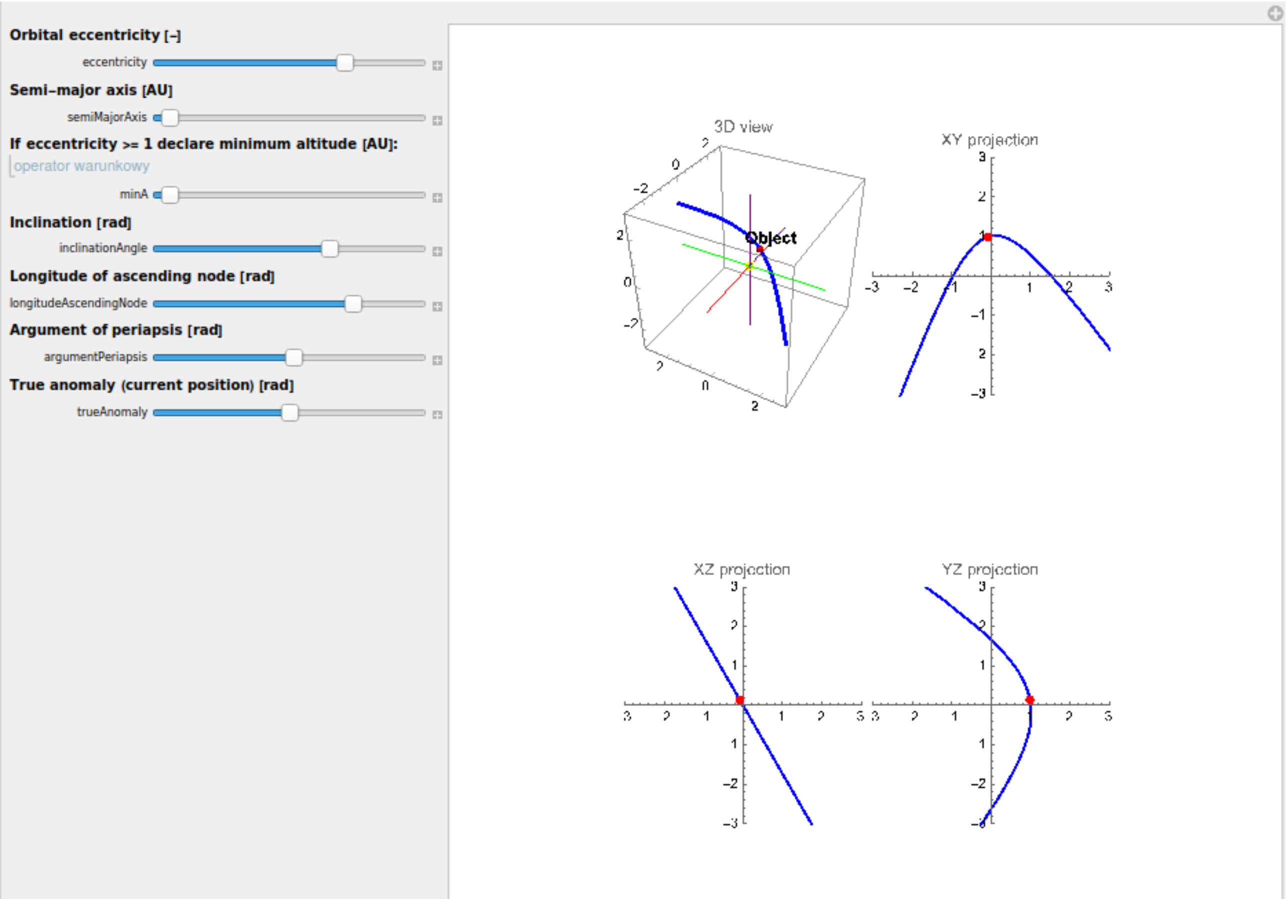

Figure 5: Program output - hyperbolic case.

Enclosures:

- ❑ File with the program (Jędrzejczyk_Radosław_proj_5)