Gliwice, 18-11-2024

Mathematical modelling and computer simulations in theory and practice

Documentation of laboratory task no 2

Title: FALLING BODY

Author (Authors): Radosław Jędrzejczyk

Field of studies: Informatics (sem.V)

Project Objective:
We're going to simulate proces of falling body. Air resistence is going to be accounted for. The problem will be considered in 2D space.

Description:
In first step program collects the input data and prepares functions for velocities and positions versus time. Velocity and position functions are found by solving equation:

$$y(0) = 0, \quad y'(0) = v_0, \quad y''(t) + \frac{k}{m}y'(t) = g.$$

Figure 1: Equation for acceleration, given by task instruction

And similar one for x axis:

$$x''(t) = \frac{-k}{m} x'(t), \qquad x'(0) = v_{0x}, \qquad x[0] = 0 \tag{1}$$

After that program is finding time at which object will hit the ground (to stop calculations there) and will find minimum and maximum for every function in order to properly scale plots which are generated afterwards.

Inputs:
1. Initial position on X axis - 'xStart'.
2. Initial position on Y axis - 'yStart' – it's equivalent of height.
3. Initial velocity on X axis - 'vx'
4. Initial velocity on Y axis – 'vy'
5. Object mass – 'm'
6. Air resisntace coefficient – 'k'
7. Gravitational acceleration acting on a body– 'g'
8. Time at which postion and parameters are displayed - 'T'

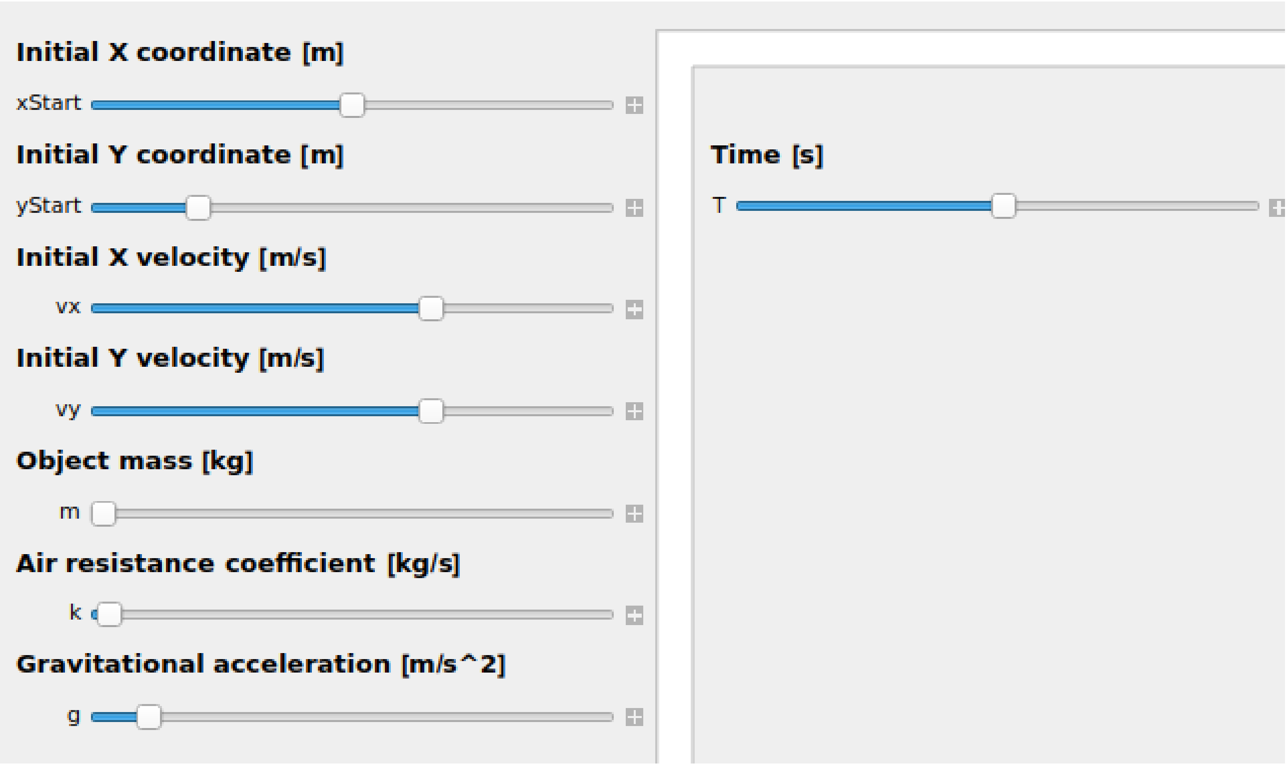

Figure 2: Inputs view.

Outputs:
As an otuput program is providing sets of graphs with data related to the object movement:

1. Position on X-Y plane in current time, and trajectory already traveled.
2. Plots of the changes in x and y coordinates related to time.
3. And plots of velocities for each axis, also in relation to time.

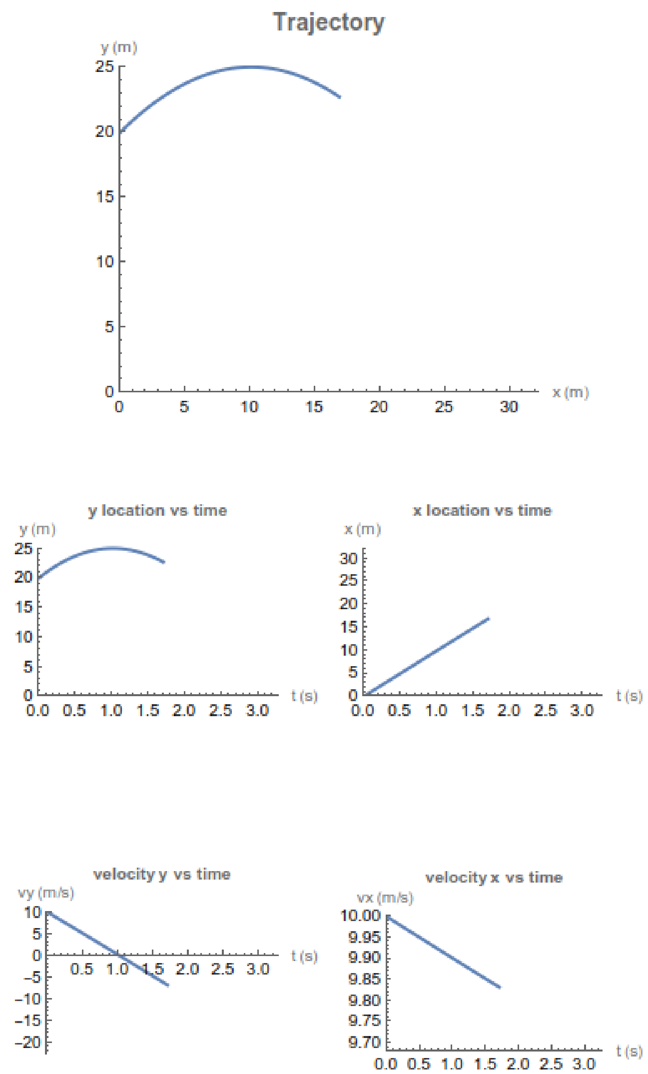

Figure 3: Output view.

Some examples are provided below:

1. Typical example of movement – notice that change of velocity for x is very small – it's depending only on drag.

2. If we enter some extreme values of k than we can observe that deceleration is very rapid for high velocites and plots representing those are more similar to curvers than straigh lines.

3. Similar effect will be observed if we increasy our initial position to some extreme values (on Y axis). We can observe that our object is reaching some critical velocity and than stagnates – it's X velocity is going to reach zero at one point. This example will show that our model isn't perfect tho – in real life drag is modeled in more complex way and our critical velecity is dependend on many diffrent factors such as object area, air density etc. We may consider those values to be hidden inside 'k' constant, although this approach is generaly unpractical.

4. In last example we will set or 'k' constant to 0 – this way we have pure free falling, without interference from mass or air resistance. We can notice that our X velocity remains uchanged.

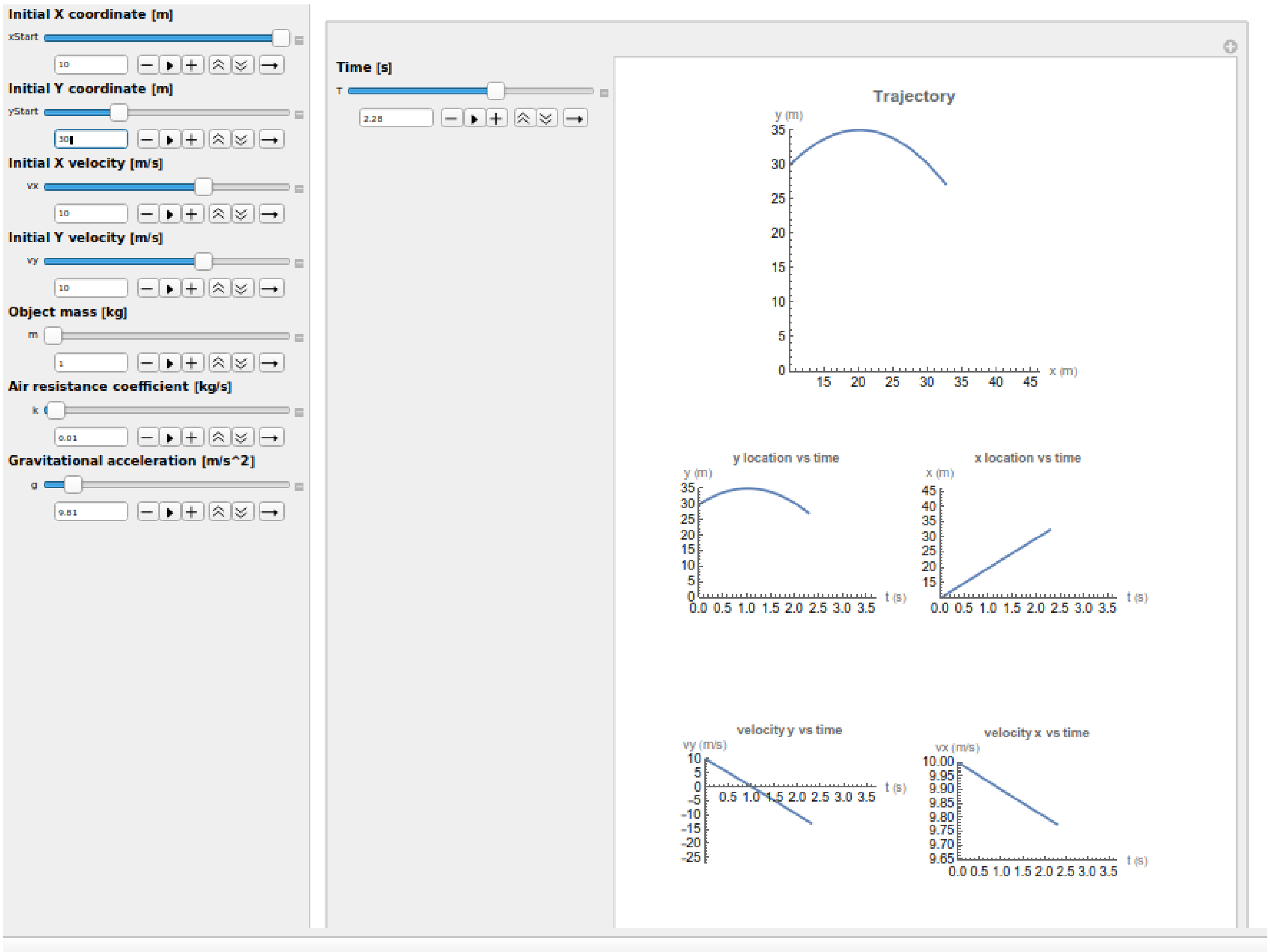

Figure 4: Example 1 - typical movement

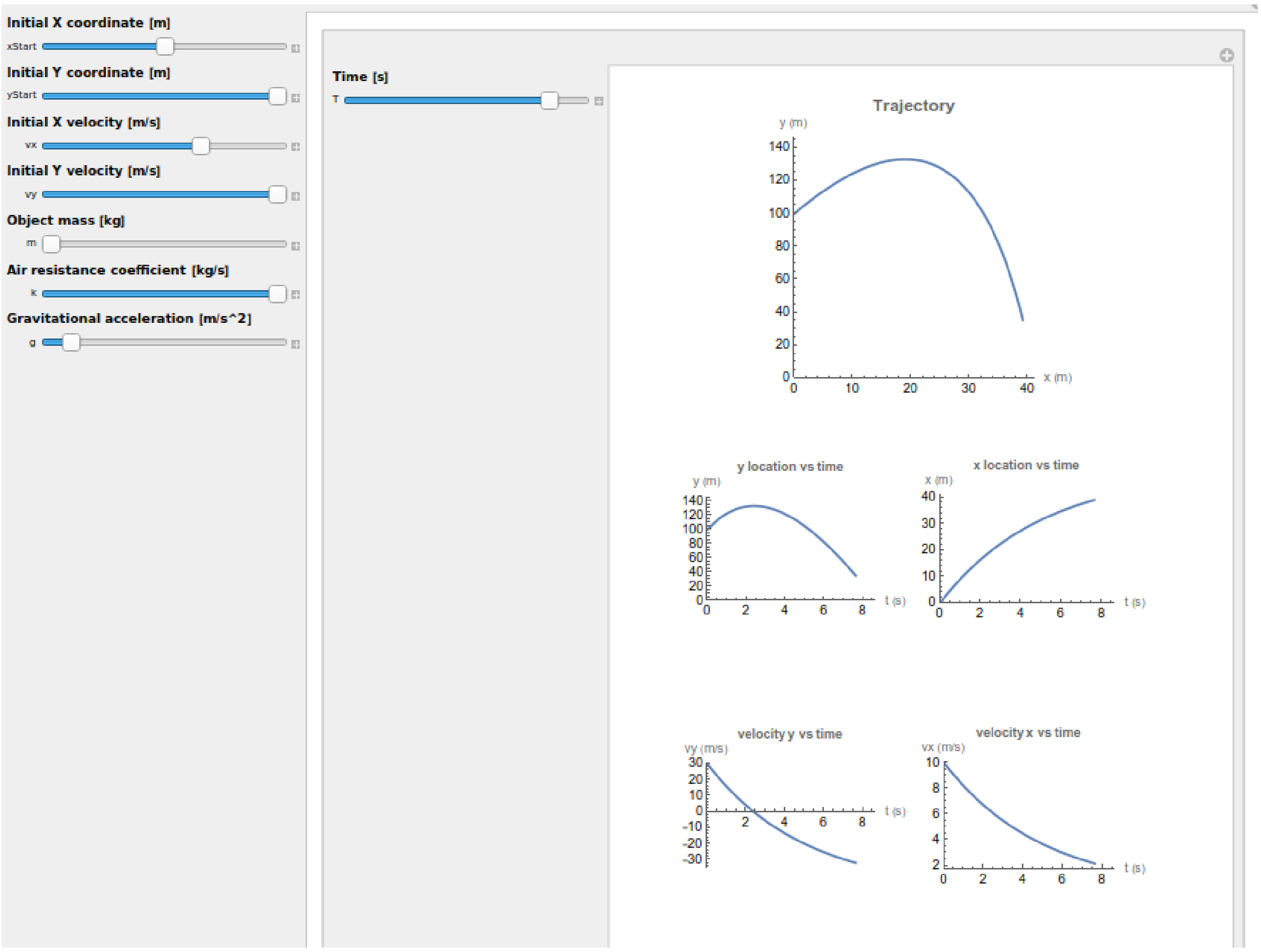

Figure 5: Example 2 - extreme drag coefficient

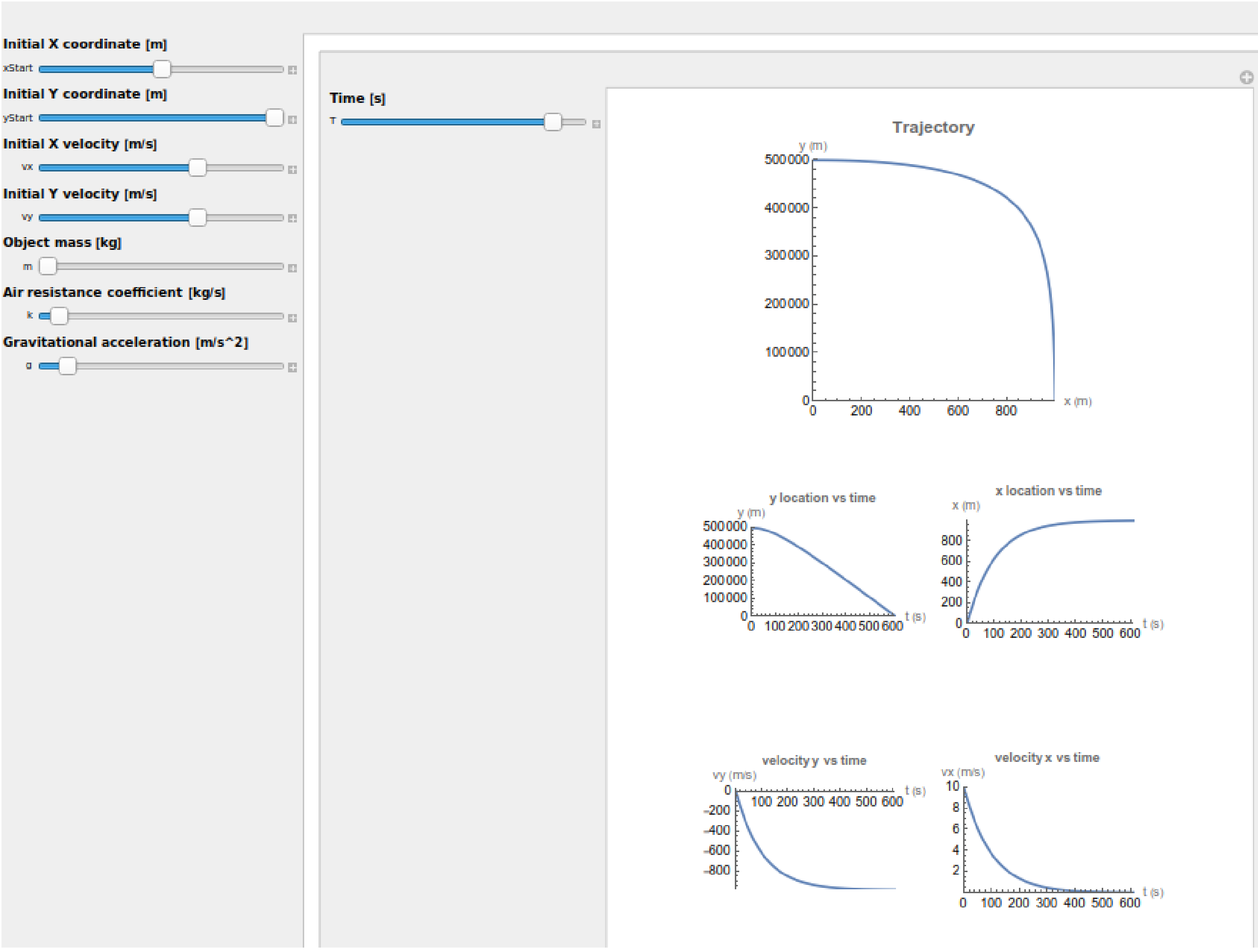

Figure 6: Example 3 - Extreme height (inital Y coordinate)

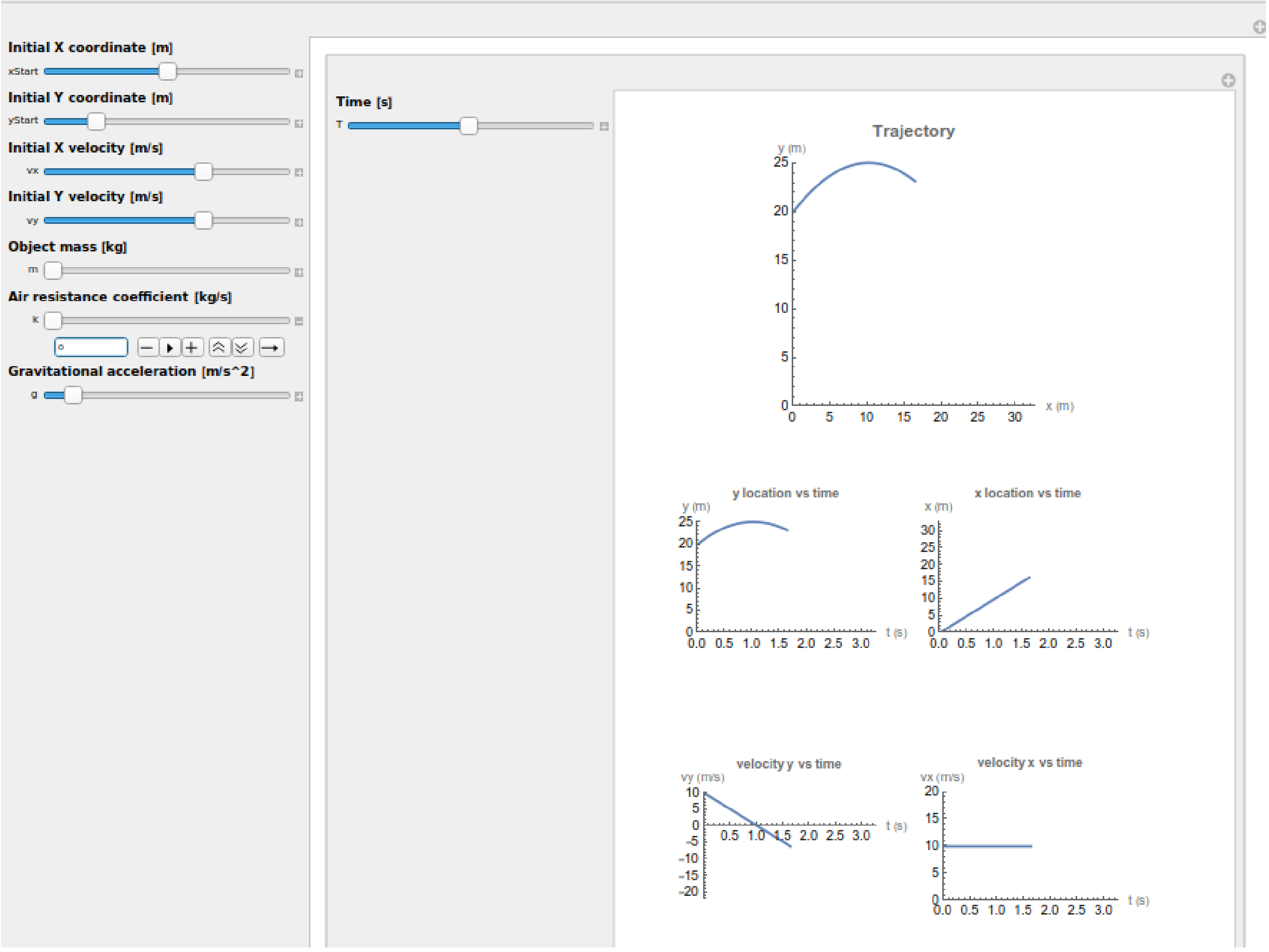

Figure 7: Example 4 - pure free falling.

We could theoriticaly perform many more operations – throw object downwards, don't throw it at all, analyse different gravitational pulls etc.

Program is performing multiple numerical operations, which can be sensitive to extremly low and high number, be careful changing ranges in the program as it may destabilise the solutions. If you want to do this anyway you need to edit this part of the code:

```
}], Style["Time [s]", 10, Bold],
    styl                pogrubiony

{{T, groundHitTime/2}, 0.0000000001, groundHitTime}],
  Style["Initial X coordinate [m]", 10, Bold],
  styl                                pogrubiony
  {{xStart, 0}, -10, 10},
  Style["Initial Y coordinate [m]", 10, Bold],
  styl                                pogrubiony
  {{yStart, 20}, 1, 100},
  Style["Initial X velocity [m/s]", 10, Bold],
  styl                                pogrubiony
  {{vx, 10}, -30, 30},
  Style["Initial Y velocity [m/s]", 10, Bold],
  styl                                pogrubiony
  {{vy, 10}, -30, 30},
  Style["Object mass [kg]", 10, Bold],
  styl                        pogrubiony
  {{m, 1}, 1, 100},
  Style["Air resistance coefficient [kg/s]", 10, Bold],
  styl                                         pogrubiony
  {{k, 0.01}, 0, 0.2},    Style["Gravitational acceleration [m/s^2]", 10, Bold],
                          styl                                          pogrubiony
  {{g, 9.81}, 0.01, 100}], Style["Falling Body", 15, Bold], Top]
                           styl                     pogrub·· góra
```

Figure 8: Default data ranges

Enclosures:

❑ File with the program (Jędrzejczyk_Radosław_proj_2)