

Mathematical modelling and computer simulations in theory and practice

Documentation of final project

Title: SATELLITE AND GROUND STATIONS

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Field of studies: Informatics (sem.V)

Project Objective:

Project objective is to visualise satellite orbit around the earth, effective range of its detector and its' visibility from ground stations.

Description:

We want to describe a unique orbit of a body orbiting the Earth. We can neglect the mass of the orbiting object, because as long as we're not considering objects size of a moon any differences in trajectory aren't going to be noticeable.

In order to describe unique orbit we will use traditional orbital elements – Keplerian elements, those are:

1. Eccentricity (e) – shape of the ellipse.
2. Semi-major axis (a) – half the distance between the apoapsis and periapsis.
3. Inclination – tilt of the ellipse with respect to reference plane.
4. Longitude of the ascending node – orientation of the ascending node (point at which orbit is crossing reference plane, while body is moving up).
5. Argument of periapsis – angle from the ascending node to the periapsis.
6. True anomaly at given moment – angle from periapsis to the object.

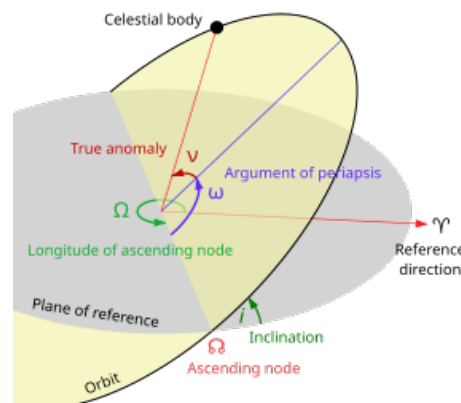


Figure 1: Keplerian orbit elements.

Knowing semi-major axis we can calculate semi-minor axis:

$$\sqrt{a^2(1-e^2)}$$

Knowing these values we can construct all possible trajectories around the body. We will start by describing the trajectory on the XY Plane (our reference plane):

$$f(t) = \begin{cases} x = a(\cos(t) - e) \\ y = b \sin(t) \\ z = 0 \end{cases}$$

In next step we're preparing rotation matrice using inclination, longitude of the ascending node and argument of periapsis, what will put orbit in the correct orientation. Knowing true anomaly at a given moment we can point current position of an object on the trajectory.

Additionally we will define values for hyphotetical detector on the sattelite:

- Range.
- View angle.
- Angle from nadir in the direction of movement.
- Angle from nadir perpendicular to the direction of movement.

Aforementioned inputs are given by sliders:

The image shows a software interface with two main sections: "Orbit elements" and "Satellite detector".

Orbit elements:

- Orbital eccentricity [-]**: A slider labeled "eccentricity" with a value of 0.
- Semi-major axis [km]**: A slider labeled "semiMajorAxis" with a value of 10000.
- Inclination [rad]**: A slider labeled "inclinationAngle" with a value of 0.785.
- Longitude of ascending node [rad]**: A slider labeled "longitudeAscendingNode" with a value of 0.
- Argument of periapsis [rad]**: A slider labeled "argumentPeriapsis" with a value of 0.
- True anomaly (current position) [rad]**: A slider labeled "trueAnomaly" with a value of 0.

Satellite detector:

- Angle of view [°]**: A slider labeled "viewAngleDeg" with a value of 30.
- Angle from nadir in the direction of movement [°]**: A slider labeled "nadirAngleDeg" with a value of 45.
- Angle from nadir perpendicular to the direction of movement [°]**: A slider labeled "perpAngleDeg" with a value of 45.
- Max detection range [km]**: A slider labeled "viewRange" with a value of 10000.

Figure 1: Inputs for satellite.

Additionally we can add ground stations and check altitude at which satellite is perceived for them. In order to define station we need to define:

1. Current date – position on Earth is defined by latitude and longitude but position of the satellite is based on celestial coordinates. We need to move latitude and longitude in correct postion relative to sky.
2. ID – in order to recognise them.
3. Latitude.
4. Longitude.

The image shows a software interface with two main sections: "Set correct date" and "Stations configuration".

Set correct date:

Day	Month	Year	Hour	Minute
20	1	2025	12	0

Stations configuration:

ID	Latitude [°]	Longitude [°]	Remove station by ID
"Gliwice"	50.2976	18.6766	"Gliwice"

Below the table are two buttons: "ADD" and "DELETE".

Figure 2: Inputs for ground stations.

Outputs:

As an output we get:

- Table presenting our stations positions, angle at which sattelite is visible relative to station, and visual information about it's visibility.
- Visualisation of sattelite position (red dot), its' orbit (black, dashed line), its detector field ov view (yellow cone), stations positions (green dots + IDs displayed).

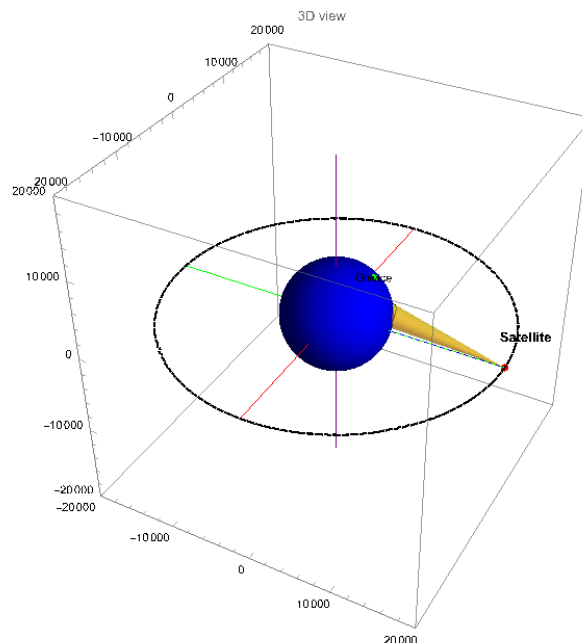


Figure 3: Example of visualisation.

We can define any orbit and detector field of view in this fashion.

List of stations				
ID	Latitude [°]	Longitude [°]	Sat altitude [°]	Visibility
Gliwice	50.2976	18.6766	17.969	■
another	-50.2976	18.6766	-73.6763	■

REFRESH TABLE

Figure 4: Exemplary table of stations.

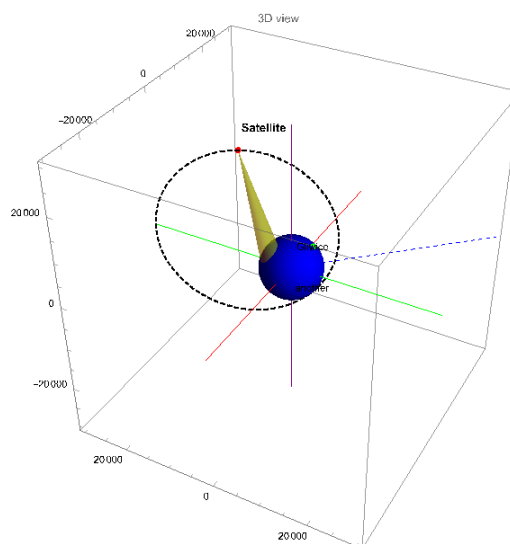


Figure 5: Sattelite in higly elliptical orbit.

Enclosures:

- ☐ File with the program (Jędrzejczyk_Radosław_project)