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Mathematical modelling and computer simulations in theory and practice

Documentation of laboratory task no 8

Title: DISTRIBUTION OF TEMPERATURE IN A BAR

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Field of studies: Informatics (sem.V)

Project Objective:
Project objective is to simulate a temperature gradient in a 1D bar at a given point in time.

Description:
Distribution of temperature in a 1D Bar can be described by difrential equation:

$$\frac{\partial u(x,t)}{\partial t} = k\,\frac{\partial^2 u(x,t)}{\partial x^2} \quad \text{for} \quad x \in [a,b],\ t \in [0,t^*],$$

Which can be solved for boundary conditions:

$$u(a,t) = \alpha(t),\ u(b,t) = \beta(t) \quad \text{for} \quad t \in [0,t^*],$$

And initial conditions:

$$u(x,0) = \phi(x) \quad \text{for} \quad x \in [a,b].$$

Solving diffrential equations can be computationaly heavy and it would be wise to use finite elements methods if we're going to make this problem more complicated, but in our case we will stay with Mathematica abilities to numericaly solve such problems.

We're going to use Dirichlet boundary conditions (constant temperature at the begining and end of a bar)

Inputs:
1. Function of x, describing initial temperature.
2. X coordinate of the bar begining 'a'.
3. X coordinate of the bar ending 'b'.
4. Temperature at the begining of the bar 'u[a,0]'.
5. Temperature at the end of the bar 'u[a,0]'.
6. Maximum time that can be checked on the simulation 'end time'
7. Additionaly we can check time in between using slider in Manipulate window.

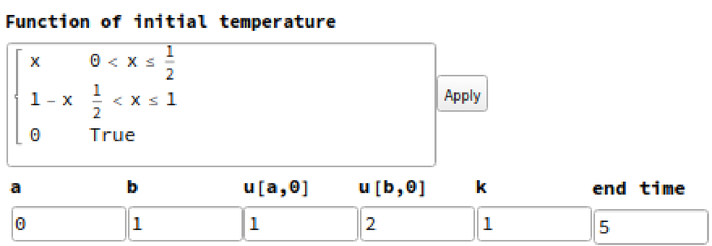

Figure 1: Input fields.

Outputs:
After pressing ‘Evaluate’ button our simulation will be created as solutions displayed as a plot and a gradient. Scales are adjusted automaticaly. Red color coresponds to the hottest place in simulation and blue to the coldest. Examples from instruction are solved below.

1. $a = 0$, $b = 1$, $t^* = 5$, $\alpha(t) = 0$, $\beta(t) = 0$, $\phi(x) = 2$ (uniform cooling of the bar from given initial temperature);

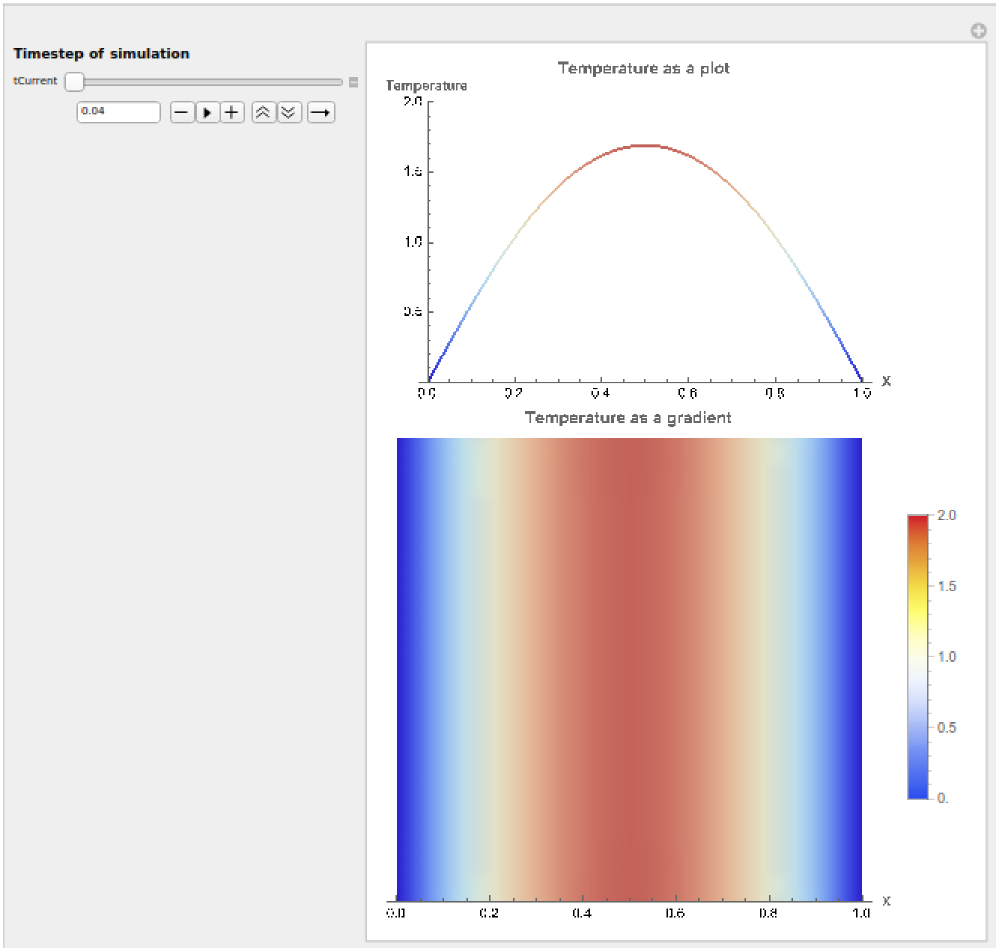

Figure 2: Part of the example 1 solution.

2. $a = 0$, $b = 1$, $t^* = 5$, $\alpha(t) = 2$, $\beta(t) = 2$, $\phi(x) = 0$ (uniform warming of the bar to given final temperature);

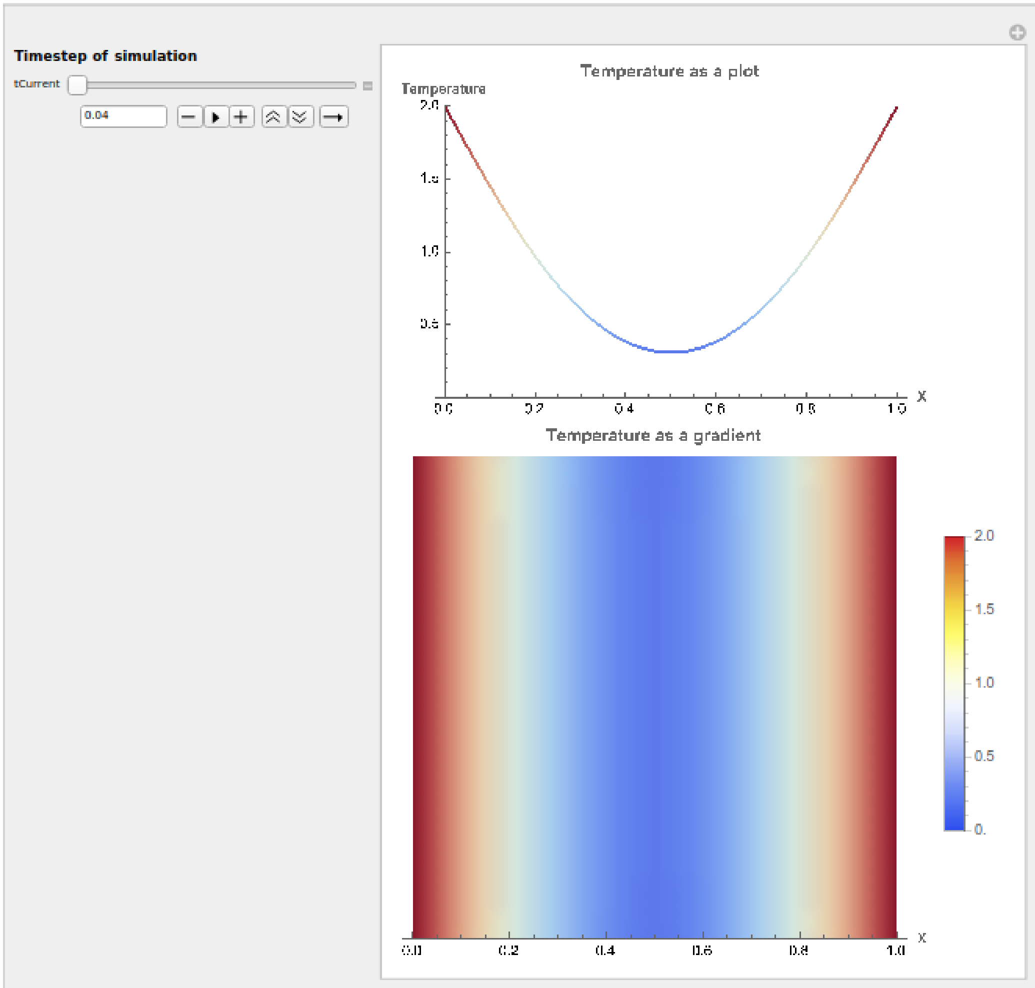

Figure 3: Part of the example 2 solution.

In both solutions above we can se that colors both on the gradient and plots corespond to each others. Those two examples are also basicaly the inverse of each other.

3. $a = 0$, $b = 1$, $t^* = 5$, $\alpha(t) = 0$, $\beta(t) = 2$, $\phi(x) = 0$ (warming the bar from the right hand side);

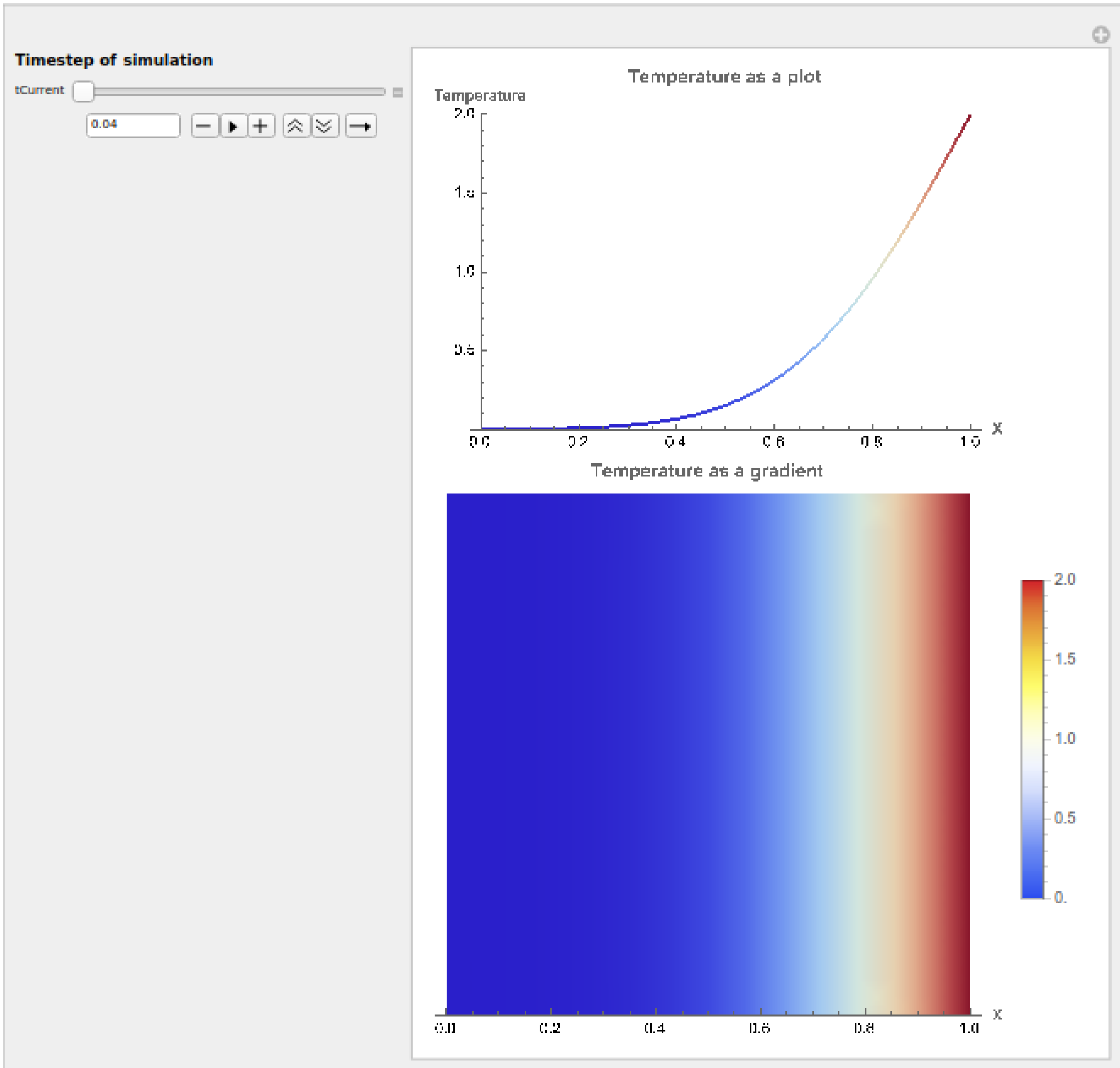

Figure 4: Part of the example 3 solution.
In the example above it's clearly visible that temperature rise faster when the diffrence in temperature is largest and this trend is much weaker where temperature difrences are very low.

We can also analyse more complex initial temperature functions.

4. $a = 0, b = 1, t^* = 5, \alpha(t) = 1, \beta(t) = 2, \phi(x) = \begin{cases} x & \text{for} \quad 0 < x \leq \frac{1}{2}, \\ 1 - x & \text{for} \quad \frac{1}{2} < x \leq 1, \end{cases}$
(nonuniform warming of the bar).

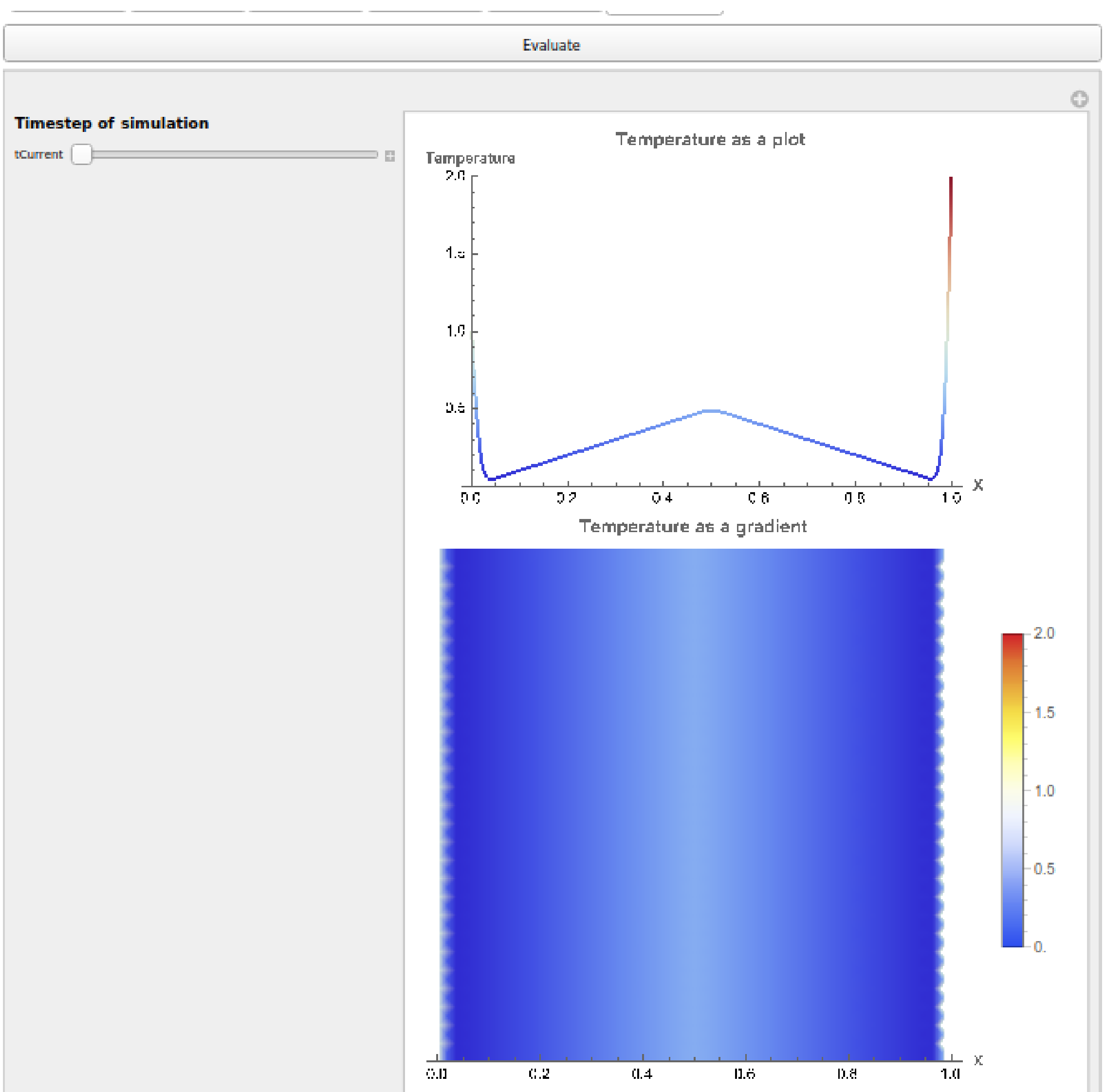

Figure 5: Example 4 initial conditions.

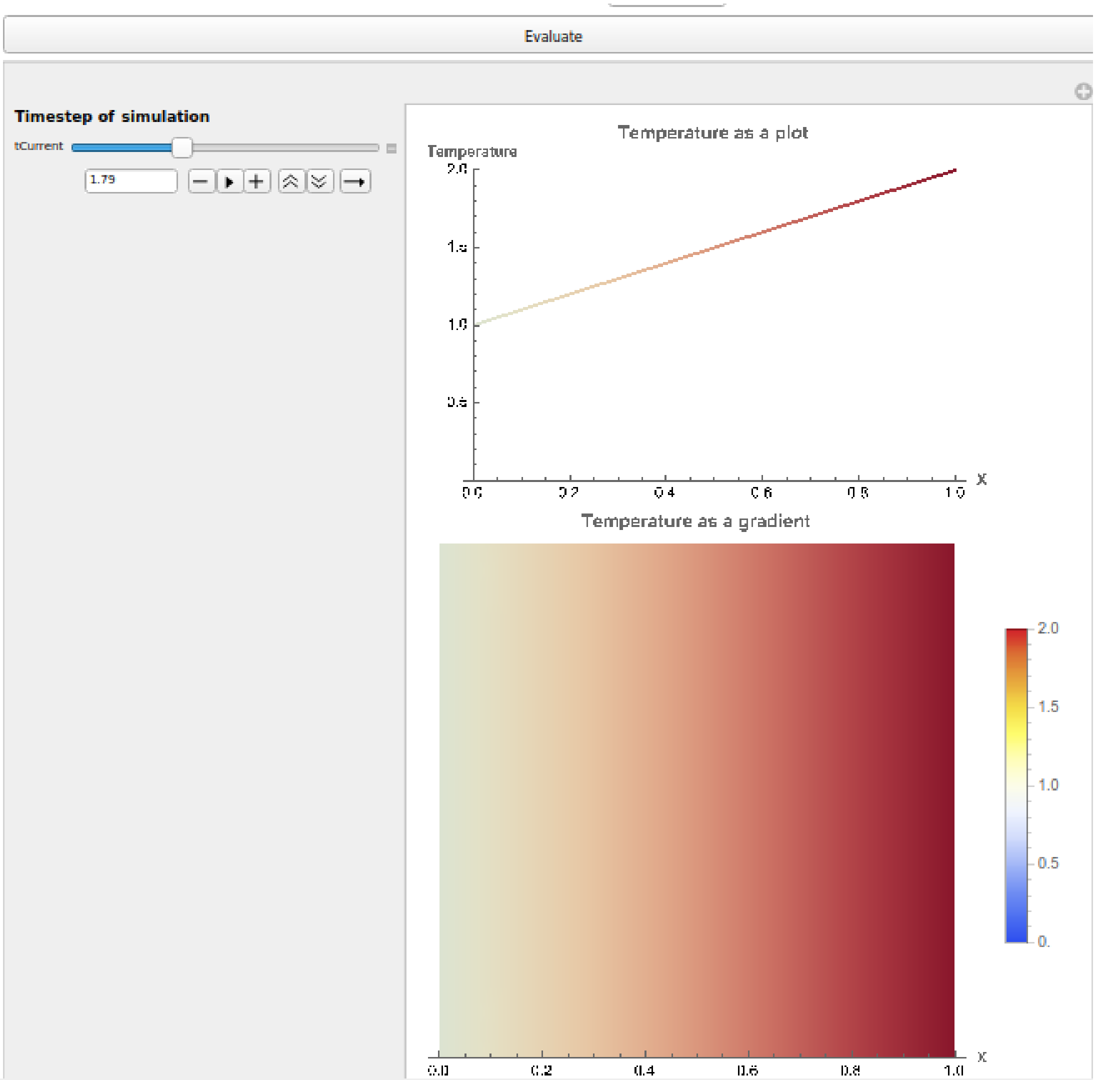

Figure 6: Part of the example 4 solution

In all of the examples steady conditions are finaly reached after some time. In those conditions heat gradient is linear through out the beam, what is expected from theory and therefore signalise that our solutions are probably correct.

Enclosures:

- File with the program (Jędrzejczyk_Radosław_proj_8)